H.C.F. and L.C.M. of Numbers

COMMON FACTOR

A *common factor* of two or more numbers is a number which divides each of them exactly.

For example, 4 is a common factor of 8 and 12.

HIGHEST COMMON FACTOR

Highest common factor (H.F.C.) of two or more numbers is the greatest number that divides each one of them exactly. For example, 6 is the highest common factor of 12, 18 and 24. Highest common factor is also called *Greatest Common Divisor* or *Greatest Common Measure*.

Symbolically, these can be written as H.C.F. or G.C.D. or G.C.M., respectively.

Methods of Finding H.C.F.

I. Method of Prime Factors

Step 1 Express each one of the given numbers as the product of prime factors.

[A number is said to be a *prime number* if it is exactly divisible by 1 and itself but not by any other number, e.g. 2, 3, 5, 7, etc. are prime numbers]

Step 2 Choose common factors.

Step 3 Find the product of these common factors. This is the required H.C.F. of given numbers.

Illustration 1 Find the H.C.F. of 70 and 90

Solution: $70 = 2 \times 5 \times 7$

$90 = 2 \times 5 \times 9$

Common factors are 2 and 5

$\therefore$ H.C.F. $= 2 \times 5 = 10$

Illustration 2 Find the H.C.F. of 3332, 3724 and 4508

Solution: $3332 = 2 \times 2 \times 7 \times 7 \times 17$

$3724 = 2 \times 2 \times 7 \times 7 \times 19$

$4508 = 2 \times 2 \times 7 \times 7 \times 23$

$\therefore$ H.C.F. $= 2 \times 2 \times 7 \times 7 = 196$

Illustration 3 Find the H.C.F. of 360 and 132

Solution: $360 = 2^3 \times 3^2 \times 5$

$132 = 2^2 \times 3^1 \times 11$

$\therefore$ H.C.F. $= 2^2 \times 3^1 = 12$.

Illustration 4 If $x = 2^3 \times 3^5 \times 5^9$ and $y = 2^5 \times 3^7 \times 5^{11}$, find H.C.F. of x and y

Solution: The factors common to both x and y are 2^3, 3^5 and 5^9

$\therefore$ H.C.F. $= 2^3 \times 3^5 \times 5^9$

II. Method of Division

***A. For two numbers*:**

Step 1 Greater number is divided by the smaller one.

Step 2 Divisor of (1) is divided by its remainder.

Step 3 Divisor of (2) is divided by its remainder.

This is continued until no remainder is left.

H.C.F. is the divisor of last step.

Illustration 5 Find the H.C.F. of 3556 and 3444

$$
\begin{array}{r}
3444\overline{)3556}(1 \\
3444 \\
\hline
112\overline{)3444}(30 \\
3360 \\
\hline
84\overline{)112}(1 \\
84 \\
\hline
28\overline{)84}(3 \\
84 \\
\hline
\times
\end{array}
$$

$\therefore$ H.C.F. $= 28$

B. For more than two numbers:

Step 1 Any two numbers are chosen and their H.C.F. is obtained.

Step 2 H.C.F. of H.C.F. (of (1)) and any other number is obtained.

Step 3 H.C.F. of H.C.F. (of (2)) and any other number (not chosen earlier) is obtained.

This process is continued until all numbers have been chosen. H.C.F. of last step is the required H.C.F.

Illustration 6 Find the H.C.F. of 13915, 9499 and 2553 by division method

Solution:

```
9499) 13915 (1
      9499
      ----
      4416 ) 9499 (2
             8832
             ----
              667 ) 4416 (6
                    4002
                    ----
                     414 ) 667 (1
                           414
                           ---
                           253 ) 414 (1
                                 253
                                 ---
                                 161 ) 253 (1
                                       161
                                       ---
                                        92 ) 161 (1
                                              92
                                             ---
                                              69 ) 92 (1
                                                   69
                                                   --
                                                   23 ) 69 (3
                                                        69
                                                        --
                                                        ×
```

Now, in the next step, we will find the H.C.F. of 23 and 2553

```
23) 2553 (111
    23
    --
     25
     23
     --
      23
      23
      --
      ×
```

Thus, H.C.F. of 13915, 9499 and 2553 = 23

Illustration 7 Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm.

Solution: Required length

= (H.C.F. of 700, 385, 1295) cm = 35 cm

COMMON MULTIPLE

A *common multiple* of two or more numbers is a number which is exactly divisible by each one of them.

For example, 32 is a common multiple of 8 and 16.

$$8 \times 4 = 32$$
$$16 \times 2 = 32.$$

LEAST COMMON MULTIPLE

The *least common multiple* of two or more given numbers is the least or lowest number which is exactly divisible by each of them.

For example, consider the two numbers 12 and 18.

Multiples of 12 are 12, 24, 36, 48, 72, ...

Multiples of 18 are 18, 36, 54, 72, ...

Common multiples are 36, 72, ...

∴ Least common multiple, i.e., L.C.M. of 12 and 18 is 36.

METHODS OF FINDING L.C.M.

A. Method of Prime Factors

Step 1 Resolve each given number into prime factors.

Step 2 Take out all factors with highest powers that occur in given numbers.

Step 3 Find the product of these factors. This product will be the L.C.M.

Illustration 8 Find the L.C.M. of 32, 48, 60 and 320

Solution:

$$32 = 2^5 \times 1$$
$$48 = 2^4 \times 3$$
$$60 = 2^2 \times 3 \times 5$$
$$320 = 2^6 \times 6$$
$$\therefore \text{ L.C.M.} = 2^6 \times 3 \times 5 = 960$$

B. Method of Division

Step 1 The given numbers are written in a line separated by common.

Step 2 Divide by any one of the prime numbers 2, 3, 5, 7, 11,…which will divide at least any two of the given numbers exactly. The quotients and the undivided numbers are written in a line below the first.

Step 3 Step 2 is repeated until a line of numbers (prime to each other) appears.

Step 4 Find the product of all divisors and numbers in the last line which is the required L.C.M.

Illustration 9 Find the L.C.M. of 12, 15, 20 and 54.

Solution:

2	12, 15, 20, 54
2	6, 15, 10, 27
3	3, 15, 5, 27
5	1, 5, 5, 9
	1, 1, 1, 9

L.C.M. $= 2 \times 2 \times 3 \times 5 \times 1 \times 1 \times 1 \times 9 = 540$

Note:

Before finding the L.C.M. or H.C.F., we must ensure that all quantities are expressed in the same unit.

Some Useful Short-cut Methods

1. H.C.F. and L.C.M. of Decimals

Step 1 Make the same number of decimal places in all the given numbers by suffixing zero(s) if necessary.

Step 2 Find the H.C.F./L.C.M. of these numbers without decimal.

Step 3 Put the decimal point (in the H.C.F./L.C.M. of step 2) leaving as many digits on its right as there are in each of the numbers.

Illustration 10 Find the L.C.M. of 1.2, 0.24 and 6

Solution: The given numbers can be written as

1.20, 0.24 and 6.00

Now, ignoring the decimal we find the L.C.M. of

120, 24 and 600

2	120, 24, 600
2	60, 12, 300
2	30, 6, 150
3	15, 3, 75
5	5, 1, 25
	1, 1, 5

$\therefore$ L.C.M. $= 2 \times 2 \times 2 \times 3 \times 5 \times 1 \times 5 = 600$

Thus, the required L.C.M. = 6.00, i.e., 6

Illustration 11 Find the H.C.F. of 6×16 and 13

Solution: The given numbers can be written as

6.16 and 13.00

Now, ignoring the decimals we find the H.C.F. of 616 and 1300

$$\begin{array}{r|l} 616 & 1300 \ (2 \\ & 1232 \\ \hline 68 & 616 \ (9 \\ & 612 \\ \hline 4 & 68 \ (17 \\ & 68 \\ \hline & \times \end{array}$$

$\therefore$ H.C.F. of 616 and 1300 is 4

Thus, the required H.C.F. = 0.04

2. L.C.M. and H.C.F. of Fractions

$$\text{L.C.M} = \frac{\text{L.C.M. of the numbers in numerators}}{\text{H.C.F. of the numbers in denominators}}$$

$$\text{H.C.F.} = \frac{\text{H.C.F. of the numbers in numerators}}{\text{L.C.M. of the numbers in denominators}}$$

Illustration 12 Find the L.C.M. of $\frac{2}{5}, \frac{3}{10}$ and $\frac{6}{25}$

Solution: L.C.M. of numerators 2, 3 and 6 is 6

H.C.F. of denominators 5, 10 and 25 is 5

$\therefore$ Required L.C.M. $= \dfrac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$

$= \dfrac{6}{5}$

Illustration 13 Find the H.C.F. of $\frac{4}{9}, \frac{10}{21}$ and $\frac{20}{63}$

Solution: H.C.F. of numerators 4, 10 and 20 is 2

L.C.M. of denominators 9, 21 and 63 is 63

$\therefore$ Required H.C.F. $= \dfrac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$

$= \dfrac{2}{63}$

Notes:

1. If the given set of numbers includes fractions as well as whole numbers, treat whole number too as fraction with 1 in its denominator.
2. The H.C.F. of a number of fractions is always a fraction, but the L.C.M. may be a fraction or an integer.

3. Product of two numbers

= L.C.M. of the numbers × H.C.F. of the numbers

Illustration 14 The H.C.F. and the L.C.M. of any two numbers are 63 and 1260, respectively. If one of the two numbers is 315, find the other number

Solution: The required number

$$= \frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{First number}} = \frac{1260 \times 63}{315} = 252$$

4. To find the greatest number that will exactly divide x, y and z.

Required number = H.C.F. of x, y and z.

Illustration 15 Find the greatest number that will exactly divide 200 and 320

Solution: The required greatest number

= H.C.F. of 200 and 320 = 40

5. To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively.

Required number = H.C.F. of $(x - a)$, $(y - b)$ and $(z - c)$.

Illustration 16 Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11, respectively

Solution: The required greatest number

= H.C.F. of (148 – 4), (246 – 6) and (623 – 11),

i.e., H.C.F. of 144, 240 and 612 = 12

6. To find the least number which is exactly divisible by x, y and z.

Required number = L.C.M. of x, y and z.

Illustration 17 What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Solution: The required smallest number

= L.C.M. of 36, 45, 63 and 80

= 5040

7. To find the least number which when divided by x, y and z leaves the remainders a, b and c, respectively. It is always observed that $(x - a) = (y - b) = (z - c) = k$ (say)

$\therefore$ Required number = (L.C.M. of x, y and z) – k.

Illustration 18 Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53, respectively

Solution: Since (36 – 25) = (48 – 37) = (64 – 53) = 11, therefore, the required smallest number

= (L.C.M. of 36, 48 and 64) – 11

= 576 – 11 = 565

8. To find the least number which when divided by x, y and z leaves the same remainder r in each case.

Required number = (L.C.M. of x, y and z) + r.

Illustration 19 Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5

Solution: The required smallest number

= (L.C.M. of 12, 16 and 18) + 5

= 144 + 5 = 149

9. To find the greatest number that will divide x, y and z leaving the same remainder in each case.

(A) When the value of remainder r is given:

Required number = H.C.F. of $(x - r)$, $(y - r)$ and $(z - r)$.

(B) When the value of remainder is not given:

Required number = H.C.F. of $|(x - y)|$, $|(y - z)|$ and $|(z - x)|$

Illustration 20 Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case

Solution: The required greatest number

= H.C.F. of (772 – 5) and (2778 – 5)

= H.C.F. of 767 and 2773

= 59

Illustration 21 Find the greatest number which on dividing 152, 277 and 427 leaves equal remainder.

Solution: The required greatest number

= H.C.F. of $|(x - y)|$, $|(y - z)|$ and $|(z - x)|$

= H.C.F. of $|(152 - 277)|$, $|(277 - 427)|$ and $|(427 - 152)|$

= H.C.F. of 125, 275 and 150

= 25

10. To find the n-digit greatest number which, when divided by x, y and z,

(A) leaves no remainder (i.e., exactly divisible)

Step 1 L.C.M. of x, y and $z = L$

Step 2 $L \overline{)\ n\text{-digit greatest number}\ (}$

Remainder = R

Step 3 Required number = n-digit greatest number $- R$

(B) leaves remainder K in each case.

Required number = (n-digit greatest number $- R$) + K.

Illustration 22 Find the greatest number of 4 digits which, when divided by 12, 18, 21 and 28, leaves 3 as a remainder in each case.

Solution: L.C.M. of 12, 18, 21 and 28 = 252

$$252 \overline{)\,9999\,(\,39}$$
$$\underline{9828}$$
$$171$$

$\therefore$ The required number = (9999 – 171) + 3 = 9931

Illustration 23 Find the greatest number of 4 digits which, when divided by 12, 15, 20 and 35 leaves no remainder

Solution: L.C.M. of 12, 15, 20 and 35 = 420

$$420 \overline{)\,9999\,(\,23}$$
$$\underline{9660}$$
$$339$$

$\therefore$ The required number = 9999 – 339 = 9663

11. To find the n-digit smallest number which when divided by x, y and z

(A) leaves no remainder (i.e., exactly divisible)

Step 1 L.C.M. of x, y and $z = L$

Step 2 $L \overline{)\,n\text{-digit smallest number}\,(}$

Remainder = R

Step 3 Required number = n-digit smallest number + $(L - R)$.

(B) leaves remainder K in each case.

Required number = n-digit smallest number + $(L - R) + k$.

Illustration 24 Find the least number of four digits which is divisible by 4, 6, 8 and 10.

Solution: L.C.M. of 4, 6, 8 and 10 = 120.

$$120 \overline{)\,1000\,(\,8}$$
$$\underline{960}$$
$$40$$

$\therefore$ The required number = 1000 + (120 – 40) = 1080.

Illustration 25 Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves remainder 3 in each case

Solution: L.C.M. of 12, 18, 21 and 28 = 252

$$252 \overline{)\,1000\,(\,3}$$
$$\underline{756}$$
$$244$$

$\therefore$ The required number

$$= 1000 + (252 - 244) + 3$$
$$= 1011.$$

Practice Exercises

Difficulty Level-1 (Based on Memory)

1. The L.C.M. of two numbers is 4800 and their HCF is 160. If one of the numbers is 480, then the second number is:

(a) 16 (b) 16000

(c) 160 (d) 1600

[Based on MAT, 2004]

2. What is the least number which when divided by 12, 18, 36, and 45 leaves remainders 8, 14, 32 and 41, respectively?

(a) 176 (b) 88

(c) 98 (d) 42

3. An electronic device makes a beep after every 60 s. Another device makes a beep after every 62 s. They beeped together at 10 a.m. The time when they will next make a beep together at the earliest is:

(a) 10.30 a.m. (b) 10.31 a.m.

(c) 10.59 a.m. (d) 11 a.m.

[Based on MAT, 2005]

4. Find the number of four-digit numbers that are divisible by 30 and 35 but not by 140.

(a) 21 (b) 22

(c) 43 (d) 44

5. Find the L.C.M. of $\frac{7}{14}$, $\frac{33}{21}$, $\frac{28}{3}$ and $\frac{15}{63}$.

(a) 220 (b) 4260

(c) 356 (d) None of these

6. Find the least common multiple of 6804 and 9828 given their highest common factor is 756.

(a) 84852 (b) 88452

(c) 85482 (d) 88542

7. H.C.F. of two numbers is 43 and their sum is 430. Total number of distinct pairs of two such numbers is:

(a) 5 (b) 2

(c) 6 (d) Data insufficient

8. Find the greatest possible length of the planks, if three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length?

(*a*) 8 m (*b*) 49 m

(*c*) 7 m (*d*) 63 m

9. A rectangular floor in my office has its area equal to 56 m^2. The minimum number of tiles required, if all the tiles are in square shape is:

(*a*) 15 (*b*) 9

(*c*) 14 (*d*) Cannot be determined

10. H.C.F. and L.C.M. of 24, 8^2, 16^2, 20^3 are:

(*a*) 2^3; 32000 (*b*) 2^4; 32000

(*c*) 2^4; 25600 (*d*) 2^2; 3200

11. L.C.M. of 35,85 and a number *k* is 7,735. The H.C.F. is 5. What is the least possible value of *k*?

(*a*) 65 (*b*) 25

(*c*) 325 (*d*) 13

12. HCF and LCM of two numbers are 21 and 4641, respectively. If one of the numbers lies between 200 and 300, then the two numbers are:

(*a*) 273, 357 (*b*) 273, 361

(*c*) 273, 359 (*d*) 273, 363

[Based on MAT (May), 2006]

13. LCM and HCF of two numbers are 84 and 21, respectively. If the ratio of the two numbers is 1:4, then the larger of the two numbers is:

(*a*) 12 (*b*) 48

(*c*) 84 (*d*) 108

[Based on MAT, 1997]

14. About the number of pairs which have 16 as their HCF and 136 as their LCM, we can definitely say that:

(*a*) only one such pair exists

(*b*) only two such pairs exist

(*c*) many such pairs exist

(*d*) no such pair exists

[Based on MAT, 2000]

15. L.C.M. of first 100 natural numbers is *N*. What is the L.C.M. of first 105 natural numbers?

(*a*) $5! \times N$ (*b*) $10403N$

(*c*) $105N/103$ (*d*) $4N$

16. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds, respectively. If they all change simultaneously at 8:20:00 hrs, then they will again change simultaneously at:

(*a*) 8:27:12 hrs (*b*) 8:27:24 hrs

(*c*) 8:27:36 hrs (*d*) 8:27:48 hrs

[Based on MAT, 2000]

17. The smallest perfect square number which is divisible by 8, 12 and 15 is:

(*a*) 2,500 (*b*) 3,600

(*c*) 3,721 (*d*) 6,400

[Based on MAT, 2000]

18. The greatest common divisor of 123456789 and 987654321 is:

(*a*) 1 (*b*) 3

(*c*) 9 (*d*) Greater than 9

[Based on JMET, 2011]

19. What is the least number of cut pieces of equal length that can be cut out of two lengths 10 m 857 mm and 15 m 87 mm?

(*a*) 174 (*b*) 172

(*c*) 164 (*d*) 184

20. L.C.M. of two numbers is 12 times their H.C.F. The sum of H.C.F. and L.C.M. is 403. If one number is 93, find the other.

(*a*) 134 (*b*) 124

(*c*) 128 (*d*) None of these

[Based on NMAT, 2006]

21. Let *x* denote the greatest 4-digit number which when divided by 6, 7, 8, 9 and 10 leaves a remainder of 4, 5, 6, 7 and 8, respectively. Then, the sum of the four-digits of *x* is:

(*a*) 25 (*b*) 18

(*c*) 20 (*d*) 22

[Based on JMET, 2009]

22. Find the greatest number of five digits which when divided by 8, 9 and 10 leaves 3 as remainder in each case.

(*a*) 99996 (*b*) 99723

(*c*) 99983 (*d*) None of these

23. A heap of stones can be made up into groups of 21. When made up into groups of 16, 20, 25 and 45, there are 3 stones left in each case. How many stones at least can there be in the heap?

(*a*) 7203 (*b*) 2403

(*c*) 3603 (*d*) 4803

24. Find the greatest number of four digits which must be added to 5231 so that the final number becomes exactly divisible by 12, 15, 27, 32 and 40.

(*a*) 7929 (*b*) 7829

(*c*) 9729 (*d*) 7729

25. Find the greatest number of four digits which is exactly divisible by 24, 28, 30 and 35.

(*a*) 9225 (*b*) 9240

(*c*) 9250 (*d*) 9260

26. Find the least number which when decreased by 11 is divisible by 14, 15, 21, 32 and 60.

(*a*) 4371 (*b*) 3271

(*c*) 3371 (*d*) 3360

27. In a morning walk, three person step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that they can cover the distance in complete steps?

(*a*) 122 m 40 cm (*b*) 123 m 45 cm

(*c*) 122 m 45 cm (*d*) 122 m 50 cm

[Based on MAT, 2011]

28. A, B and C are running on a circular track of 120 m at a speed of 5 m/s, 8 m/s, and 10 /s respectively. A, B and C all three are moving in the same direction. When will all the three meet again at the starting point?

(*a*) After 55 seconds (*b*) After 2 minutes

(*c*) After 5 minutes (*d*) After 40 seconds

[Based on MAT, 2013]

29. Let x denote the greatest 4-digit number which when divided by 6, 7, 8, 9 and 10 leaves a remainder of 4, 5, 6, 7 and 8 respectively. Then, the sum of the four-digits of x is:

(*a*) 25 (*b*) 18

(*c*) 20 (*d*) 22

[Based on SNAP, 2013]

Difficulty Level-2 (Based on Memory)

1. A simple mechanical device has four gears *A*, *B*, *C* and *D* such that they mesh in the order *A*—*B*—*C*—*D*. The number of teeth in these gears are 10, 21, 12, 17, respectively. If we start the mechanism from rest, how may revolutions will the largest gear have to turn before one full cycle is completed and the gears are in the position from where they started?

(*a*) 270 (*b*) 340

(*c*) 7140 (*d*) 285

2. A stamp collector likes to arrange and rearrange his collection in a symmetrical order. Sometimes he lines his stamps up in pairs, sometimes in threes, sometimes in fours, occasionally in fives, and once in a while in groups of six. After arranging them in any of these groups, he invariably has one stamp left over. When he arranges his stamps in groups of seven, he finds that there is not a single stamp to spare. How many stamps will be left over if he arranges them in groups of 8?

(*a*) 5 (*b*) 7

(*c*) 1 (*d*) 0

3. Three rings complete 60, 36 and 24 revolutions in a minute. They start from a certain point in their circumference downwards. By what time they come together again in the same position?

(*a*) 5 seconds (*b*) 6 seconds

(*c*) 8 seconds (*d*) 1 seconds

4. Six bells commence tolling together and toll at intervals of 2 seconds, 4 seconds, 6 seconds, 8 seconds, 10 seconds and 12 seconds, respectively. In 30 minutes, how many times do they toll together?

(*a*) 4 (*b*) 10

(*c*) 15 (*d*) 16

[Based on FMS (MS), 2006]

5. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. Minimum number of rows in which the trees may be planted is:

(*a*) 20 (*b*) 17

(*c*) 15 (*d*) 3

[Based on FMS, 2005]

6. Five bells begin to toll together and toll respectively at intervals of 6 seconds, 7 seconds, 8 seconds, 9 seconds and 12 seconds. How many times they will toll together in one hour, excluding the one at the start?

(*a*) 3 (*b*) 5

(*c*) 7 (*d*) 9

[Based on FMS, 2005]

7. A number which when divided by 10 leaves a remainder of 9, when divided by 9 leaves a remainder of 8, by 8 leaves a remainder of 7, etc., down to where, when divided by 2, it leaves a remainder of 1, is:

(*a*) 59 (*b*) 419

(*c*) 1259 (*d*) 2519

[Based on FMS, 2011]

8. The smallest positive number x, which leaves a remainder 1 when divided by 2, 3, 4 and 5, is:

(*a*) Greater than 75 (*b*) Divisible by 7

(*c*) A prime number (*d*) None of these

[Based on IIFT, 2005]

9. Five bells first begin to toll together and then at intervals of 3 seconds, 5 seconds, 7 seconds, 8 seconds and 10 seconds. Find after what interval they will again toll together. How many times do they toll together in one hour?

(*a*) 14 minutes, 3 times (*b*) 12 minutes, 4 times

(*c*) 14 minutes, 4 times (*d*) 12 minutes, 3 times

10. A chocolate dealer has to send chocolates of three brands to a shopkeeper. All the brands are packed in boxes of same size. The number of boxes to be sent is 96 of brand *A*, 240 of brand *B* and 336 of brand *C*. These boxes are to be packed in cartons of same size containing equal number of boxes. Each-carton should contain boxes of same brand of chocolates. What could be the minimum number of cartons that the dealer has to send?

(*a*) 20 (*b*) 48

(*c*) 42 (*d*) 38

[Based on XAT, 2010]

11. When asked for his taxi number, the driver replied, 'If you divided the number of my taxi by 2, 3, 4, 5 and 6 each time you will find a remainder of one. But, if you divide it by 11 the remainder is zero. You will also not find any other driver with a taxi having a lower number who can say the same'. What is the taxi number?

(*a*) 121 (*b*) 1001

(*c*) 1881 (*d*) 781

[Based on CAT, 2012]

12. A red light flashes 3 times per minute and a green light flashes 5 times in two minutes at regular intervals. If both lights start flashing at the same time, how many times do they flash together in each hour?

(*a*) 30 (*b*) 24

(*c*) 20 (*d*) 60

[Based on CAT, 2001]

13. Three pieces of cakes of weights $4\frac{1}{2}$ lbs, $6\frac{3}{4}$ lbs and $7\frac{1}{5}$ lbs, respectively, are to be divided into parts of equal weights. Further, each part must be as heavy as possible. If one part is served to each guest, then what is the maximum number of guests that could be entertained?

(*a*) 54 (*b*) 72

(*c*) 20 (*d*) None of these

[Based on CAT, 2001]

14. Let *S* be the set of integers *x* such that:

I. $100 \le x \le 200$

II. *x* is odd

III. *x* is divisible by 3 but not by 7

How many elements does *S* contain?

(*a*) 16 (*b*) 12

(*c*) 11 (*d*) 13

[Based on CAT, 2000]

15. Let $N = 1421 \times 1423 \times 1425$. What is the remainder when *N* is divided by 12?

(a) 0 (b) 9

(c) 3 (d) 6

[Based on CAT, 2000]

16. For two positive integers *a* and *b*, define the function *h*(*a*, *b*) as the greatest common factor (GCF) of *a*, *b*. Let *A* be a set of *n* positive integers. *G*(*A*) the GCF of the elements of set *A* is computed by repeatedly using the function *h*. The minimum number of times *h* is required to be used to compute *G* is:

(*a*) $\frac{1}{2}n$ (*b*) $(n-1)$

(*c*) n (*d*) None of these

[Based on CAT, 1999]

17. A is set of positive integers such that when divided by 2, 3, 4, 5, 6 leaves the remainder 1, 2, 3, 4, 5, respectively. How many integers between 0 and 100 belong to set A?

(a) 0 (b) 1

(c) 2 (d) None of these

[Based on CAT, 1998]

18. Three wheels can complete 60, 36, 24 revolutions per minute, respectively. There is a red spot on each wheel that touches the ground at time zero. After how much time, all these spots will simultaneously touch the ground again:

(*a*) $\frac{5}{3}$ seconds (*b*) $\frac{5}{3}$ seconds

(*c*) 6 seconds (*d*) 7.5 seconds

[Based on CAT, 1998]

19. Number of students who have opted for the subjects *A*, *B* and *C* are 60, 84 and 108, respectively. The examination is to be conducted for these students such that only the students of the same subject are allowed in one room. Also, the number of students in each room must be same. What is the minimum number of rooms that should be arranged to meet all these conditions?

(*a*) 28 (*b*) 60

(*c*) 12 (*d*) 21

[Based on CAT, 1998]

20. Three bells chime at intervals of 18 minutes, 24 minutes and 32 minutes, respectively. At a certain time, they begin to chime together. What length of time will elapse before they chime together again?

(*a*) 2 hrs and 24 minutes (*b*) 4 hrs and 48 minutes

(*c*) 1 hr and 36 minutes (*d*) 5 hrs

[Based on CAT, 1995]

21. Which is the least number that must be subtracted from 1856 so that the remainder when divided by 7, 12, and 16 is 4?

(*a*) 137 (*b*) 1361

(*c*) 140 (*d*) 172

[Based on CAT, 1994]

22. What is the smallest number which when increased by 5 is completely divisible by 8, 11 and 24?

(*a*) 264 (*b*) 259

(*c*) 269 (*d*) None of these

[Based on CAT, 1994]

23. The smallest number which, when divided by 4, 6 or 7 leaves a remainder of 2, is:

(a) 44 (b) 62

(c) 80 (d) 86

[Based on CAT, 1993]

24. The number of common terms in the sequences 17, 21, 25, …, 417 and 16, 21, 26, …, 466 is:

(a) 78 (b) 19

(c) 20 (d) 77

[Based on CAT, 2008]

Answer Keys

Difficulty Level-1

1. (*d*)	**2.** (*a*)	**3.** (*b*)	**4.** (*b*)	**5.** (*d*)	**6.** (*b*)	**7.** (*b*)	**8.** (*c*)	**9.** (*b*)	**10.** (*b*)	**11.** (*a*)	**12.** (*a*)	**13.** (*c*)
14. (*d*)	**15.** (*b*)	**16.** (*a*)	**17.** (*b*)	**18.** (*c*)	**19.** (*d*)	**20.** (*b*)	**21.** (*a*)	**22.** (*b*)	**23.** (*a*)	**24.** (*d*)	**25.** (*b*)	**26.** (*c*)
27. (*a*)	**28.** (*b*)	**29.** (*a*)										

Difficulty Level-2

1. (*b*)	**2.** (*a*)	**3.** (*a*)	**4.** (*c*)	**5.** (*b*)	**6.** (*c*)	**7.** (*d*)	**8.** (*c*)	**9.** (*c*)	**10.** (*b*)	**11.** (*a*)	**12.** (*a*)	**13.** (*d*)
14. (*d*)	**15.** (*c*)	**16.** (*b*)	**17.** (*b*)	**18.** (*c*)	**19.** (*d*)	**20.** (*b*)	**21.** (*d*)	**22.** (*b*)	**23.** (*d*)	**24.** (*c*)		

Explanatory Answers

Difficulty Level-1

1. (*d*) Second number $= \dfrac{\text{HCF} \times \text{LCM}}{\text{First number}}$

$= \dfrac{4800 \times 160}{480} = 1600.$

2. (*a*) L.C.M. of 12, 18, 36 and 45

$\Rightarrow$ L.C.M. $= 5 \times 3 \times 3 \times 2 \times 2 = 180$

$\Rightarrow$ Required answer $= 180 - 4 = 176$

Note: Here, $12 - 8 = 18 - 14$

$= 36 - 32 = 45 - 41 = 4.$

3. (*b*) L.C.M. of 60 seconds and 62 seconds is 1860 seconds = 31 minutes

$\therefore$ They will beep together at 10.31 a.m.

4. (*b*) If a number is divisible by 30 and 35, it is divisible by L.C.M. (30, 35) i.e., 210

If a number is divisible by 210 but not by 140, then it is not divisible by L.C.M. (210, 140) i.e., 420. So, the number is divisible by 210 but not by 420. The least and the greatest four digit multiples of 210 are 1050 and 9870, respectively

$\therefore$ The number of multiples of four-digit multiples of 210 are $+ \dfrac{9870 - 1050}{210} 1 = \dfrac{8820}{210} + 1 = 43$

Similarly, the number of four-digit multiples of 420 are $\dfrac{9660 + 1260}{420} + 1 = 21$

$\therefore$ The number of multiples of 210 which are not divisible by 420 are 43 – 21 i.e., 22.

5. (*d*) L.C.M. of $\left(\dfrac{7}{14}, \dfrac{23}{21}, \dfrac{28}{3}, \dfrac{15}{63}\right)$

$= \dfrac{\text{L.C.M. of } (7, 23, 28, 15)}{\text{H.C.F. of } (14, 21, 3, 63)}$

$$= \frac{15 \times 28 \times 11}{1} = 4620.$$

6. (b) We use the fact that, for any two numbers, a and b,

$$a \times b = \text{H.C.F.} \times \text{L.C.M}$$

$$\Rightarrow \quad \text{L.C.M.} = \frac{6804 \times 9828}{756}$$

$$= 9 \times 9828 = 88452.$$

7. (b) $43k + 43l = 430$

$\Rightarrow \quad 43(k + l) = 430$

$\Rightarrow \quad k + l = 10$

But k, l must be co-primes.

So $(k, l) = (1, 9)$ and $(3, 7)$.

8. (c) Required length = H.C.F. of $(42, 49, 63) = 7$ meters.

9. (b) $56 = 1 \times 56$

$\Rightarrow$ H.C.F. is 1, hence number of tiles = 56

$2 \times 28 \Rightarrow$ H.C.F. is 2, hence number of tiles

$= 1 + 14 = 15$

$4 \times 14 \Rightarrow$ H.C.F. is 2, therefore number of tiles

$= 2 + 7 = 9$

$7 \times 8 \Rightarrow$ H.C.F. is 1, therefore number of tiles

$= 7 + 8 = 15$

Hence, the minimum number of tiles can be 9 and the dimension of a tile is 2 meters each side. Thus, (b) is correct option.

10. (b) H.C.F. of $(24, 82, 162, 203) = 2^4$

L.C.M. of $(24, 82, 162, 203) = 28 \times 125 = 32000$.

11. (a) Prime factors of $35 = 5 \times 7$

$85 = 5 \times 17$

$\text{L.C.M.} = 7735 = 5 \times 7 \times 17 \times 13$

Since, H.C.F. = 5, Let k = 5a

$\therefore \quad 35 \times 85 \times 5a = 5 \times 7 \times 17 \times 13$

$\Rightarrow \quad a = 13$

$\therefore$ least possible value of $k = 5 \times 13 = 65$.

12. (a) Product of the numbers = H.C.F. $\times$ L.C.M. $= 21 \times 4641$

$= 21 \times 3 \times 7 \times 13 \times 17 = 3 \times 7 \times 3 \times 7 \times 13 \times 17$

$\therefore$ The numbers can be $3 \times 7 \times 13$ and $3 \times 7 \times 17$, i.e., 273 and 357.

13. (c) Let the numbers be x and y

$\therefore \quad xy = 84 \times 21$ and, $\frac{x}{y} = \frac{1}{4} \Rightarrow 4x = y$

Now, putting the value of y in the above equation, we have

$x \times 4x = 84 \times 21$

$\therefore \quad x = 21$ and, $y = 21 \times 4 = 84$

Hence, the larger number = 84.

14. (d) Using the relation,

First No. $\times$ Second No. = H.C.F. $\times$ L.C.M.

$= 16 \times 136 = 16 \times (8 \times 17)$

It is clear from here that no such pair is possible.

15. (b) If we look at the numbers $100 < N \leq 105$, we see only 101 and 103 do not have their factors in N (because these are primes). So, obviously the new L.C.M. will be $101 \times 103 \times N$.

16. (a) Let us first calculate L.C.M. of 48, 72, 108

2	48, 72, 108
2	24, 36, 54
2	12, 18, 27
3	6, 9, 27
3	2, 3, 9
	2, 1, 3

$\therefore$ L.C.M. of 48 seconds, 72 seconds, 108 seconds

$= 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 3 = 432$ seconds

Thus, the second time the three lights will change after 432 seconds = 7 minutes 12 seconds

Hence, next time the three lights will change simultaneously at

8:20:00 + 0:7:12 = 8:27:12 hrs.

17. (b) Let us first calculate L.C.M. of 8, 12, 15

$\therefore$ L.C.M. of $8, 12, 15 = 2 \times 2 \times 3 \times 2 \times 5 = 120$

Hence, the smallest perfect square number divisible by 8, 12, 15 i.e., by 120 is 3600.

18. (c) We notice that 9 divides both numbers. We can use Euler's algorithm to obtain the GCD

$$123456789 \overline{)\,987654321\,(\,8}$$

$$\underline{987456321}$$

$$9$$

If x divides A and x divides B, x divides the Rem $\left[\frac{A}{B}\right]$ where $(A > B)$

$\therefore$ The GCD is 9.

19. (d) H.C.F. of 10857 and 15087 is 141

The least number of cut pieces

$= (10857 + 15087) \div 141 = 184.$

20. (b) Let H.C.F. be x, then, L.C.M. $= 12x$

$\therefore \quad 12x + x = 403 \Rightarrow x = 31$

$\therefore \quad \text{L.C.M.} = 12 \times 31 = 372$

Other number $= 31 \times 272 \div 93 = 124.$

21. (a) The number will be a multiple of 6, 7, 8, 9, 10

L.C.M. of 6, 7, 8, 9, 10 = 2520

$\therefore$ Largest 4-digit number divided by this = 7560

$\therefore$ Required number = 7558

Sum of the digits of this number = 25.

22. (*b*) L.C.M. of 8, 9, 10 = 360

$$\begin{array}{r|l} 360 & 99999 \;(\; 277 \\ & 720 \\ \hline & 2799 \\ & 2520 \\ \hline & 2799 \\ & 2520 \\ \hline & 279 \end{array}$$

Greatest number of 5 digits which is divisible by 360 = 99999 – 279 = 99720

∴ Required number = 99729 + 3 = 99723.

23. (*a*) L.C.M. of 16, 20, 25, 45 = 3600

1st number = 3600 × 1 + 3 = 3603 which is not divisible by 21

2nd number = 3600 × 2 + 3 = 7203 which is divisible by 21.

24. (*d*) L.C.M. of 12, 15, 27, 32, 40 = 4320. Let us add 5231 to the greatest number of four digits and then divide by 4320 to find the remainder

$$\begin{array}{r|l} 4320 & 15230 \;(\; 3 \\ & 12960 \\ \hline & 2270 \end{array}$$

Required greatest number of four digits

= 9999 – 2270 = 7729.

25. (*b*) L.C.M. of 24, 28, 30 and 35

2	24,	28,	30,	35
2	12,	14,	15,	35
3	6,	7,	15,	35
5	2,	7,	5,	35
7	2,	7,	1,	7
	2,	1,	1,	1

= 2 × 2 × 2 × 3 × 5 × 7 = 840

Greatest number of four digits = 9999

Quotient when 9999 is divided by 840 is 11 and remainder is 759

∴ Greatest number of four digits in this case

= 9999 – 759 = 9240.

26. (*c*) Required number

= (L.C.M. of 14, 15, 21, 32, 60) + 11

= 3360 + 11 = 3371

27. (*a*) The minimum distance each person should walk = LCM of (80 cm, 85 cm and 90 cm)

= 12240 cm = 122 m 40 cm.

28. (*b*) Time taken by A to complete one round,

$$\frac{\text{Distance}}{\text{Speed}} = \frac{120}{5} = 24 \text{ seconds}$$

Similarly, time taken by B $= \frac{120}{8} = 15$ seconds

Time taken by C $= \frac{120}{10} = 12$ seconds

Time taken or the time after which all meet at a starting point.

= LCM of 24 seconds, 15 seconds and 12 seconds

2	24,	15,	12
2	12,	15,	6
2	6,	15,	3
3	3,	15,	3
	1,	5,	1

= 2 × 2 × 2 × 3 × 5 = 120 seconds = 2 minutes.

29. (*a*) The number will be a multiple of 6, 7, 8, 9, 10

LCM of 6, 7, 8, 9 and 10 = 2520

∴ Largest 4-digit number divisible by 2520 = 7560

∴ Required number = 7558

Sum of the digits of this number = 25.

DIFFICULTY LEVEL-2

1. (*b*) Let *n* be the minimum number of teeth advanced before all the wheels are in starting position again

Hence, *n* must be every divisible by 10, 21 12 and 17

Hence, n = L.C.M. = 7140

So, the required number of revolutions in largest gear = 7140/21 = 340.

2. (*a*) Number of tickets with the collector

= L.C.M. of (3, 4, 5, 6) + 1, i.e., 60 + 1 = 61

or one more than the multiple of 60 i.e., 61, 121, 181, 241, 301 etc

Since it must be exactly divisible by 7, so the collector has 301 stamps and when stamps are arranged in groups of 8, he will have 5 stamps left over.

3. (*a*) Time taken by each ring in one revolution are $\frac{60}{60}$ seconds, $\frac{60}{36}$ seconds and $\frac{60}{24}$ seconds respectively.

i.e., 1, $\frac{5}{3}$ and $\frac{5}{2}$ seconds

Requiring time = L.C.M. of 1, $\frac{5}{3}, \frac{5}{2}$ = 5 seconds.

4. (*c*) L.C.M. of 2, 4, 6, 8, 10, 12 = 120

Total no. of times = $\frac{30 \times 60}{120} = 15$

5. (*b*) H.C.F. of 21, 42, 56 = 7

$\therefore$ Number of rows = $\frac{119}{7} = 17$

6. (*c*) L.C.M. of 6, 7, 8, 9, 12 = 504 seconds

1 hr = 1 × 60 × 60 seconds = 3600

$\therefore$ Number of times = $\frac{3600}{504} \approx 7.$

7. (*d*) Every time the difference between divisor and remainder is 1. So, the number is of the form L.C.M. (10, 9, 8, 7, 6, 5, 4, 3, 2) $K - 1$

$= 2520\ K - 1$

when $K = 1$,

$2520 - 1 = 2519.$

8. (*c*) L.C.M. of 2, 3, 4, 5 = 60

$\therefore$ Smallest number will be 60 + 1 = 61

and 61 is a prime number.

9. (*c*) Required time interval

= L.C.M. of (3, 5, 7, 8 and 10)

= 840 seconds = 14 minutes

Number of times they will toll together in one hour

$= \frac{60}{14} = 4$ times (ignoring the fraction part).

10. (*b*) For minimum number of cartoons, there should be maximum number of chocolates in a cartoon that is H.C.F. of 96, 240 and 336, which is 48.

11. (*a*) The smallest number divisible by 2, 3, 4, 5 and 6 is their LCM, i.e., LCM (2, 3, 4, 5, 6) = 60

$\therefore$ The taxi number is of the form $(60n + 1)$. The options that satisfy the condition are (a) and (d). But the number has to be the smallest.

Therefore option (a).

12. (*a*) First light blinks, after 20 seconds, second light blinks after 24 seconds

Now, they blink together after LCM of 20 and 24 seconds = 120 seconds = 2 minutes

Hence, the number of times they blink together in an hour = 30.

13. (*d*) Total, the weight of three pieces

$= \left(\frac{9}{2} + \frac{27}{4} + \frac{36}{5}\right) = \frac{369}{20} = 18.45$ lbs

Required weight of a single piece is HCF of

$\left(\frac{9}{2}, \frac{27}{4}, \frac{36}{5}\right) = \frac{\text{HCF of}(9, 27, 36)}{\text{LCM of}(2, 4, 5)} = \frac{9}{20}$ lbs

$\therefore$ Number of guests $= \frac{18.45}{\frac{9}{20}} = \frac{18.45 \times 20}{9} = 41.$

14. (*d*) Numbers between 100 and 200, which are divisible by 3 are 102, 106, 109,..., 198 = 102 + $(n - 1) \times 3$

$\Rightarrow \quad n = 33$

Out of these 33 numbers, 17 are even and 16 are odd

Out of these 16 odd numbers, there are three numbers (= 105, 147, 189), which are divided by the LCM of (7, 3), i.e., 21

Hence, in all (16 − 3) = 13 numbers are contained in *S*.

15. (*c*) According to the remainder theorem, the remainders for the following expressions will be same

$\frac{1421 \times 1423 \times 1425}{12}$ or $\frac{5 \times 7 \times 9}{12}$ or $\frac{315}{12}$

$\Rightarrow$ Remainder = 3.

16. (*b*) It is clear that for n positive integers function $h(a, b)$ has to be used one time less than number of integers i.e., $(n-1)$ times.

17. (*b*) Required number of the set is calculated by the LCM of (2, 3, 4, 5, 6)−(common difference)

In this case, common difference

= (2 − 1) = (3 − 2) = (4 − 3) = (5 − 4) = (6 − 5) = 1.

$\therefore$ All integers of the set will be given by $(60n - 1)$

if $n = 1$, $(60n - 1) = 59$

if $n = 2$, $(60n - 1) = 119$

Since, range of the set A is between 0 and 100, hence there will exist only one number i.e., 59.

18. (*c*) Time taken by red spot on all the three wheels to touch the ground again simultaneously will be equal to the LCM of the time taken by the three wheels to complete one revolution

The first wheel completes 60 revolutions per minute. Therefore, to complete one revolution it takes

$\left(\frac{60}{60}\right) = 1$ seconds

Time taken by the second wheel to complete one revolution $= \frac{36}{60} = \frac{3}{5}$ seconds

And the time taken by the third wheel to complete one revolution $= \frac{24}{60} = \frac{2}{5}$ seconds

Hence, LCM of

$1, \frac{3}{5}, \frac{2}{5} = \frac{\text{LCM}(1, 3, 2)}{\text{HCF}(1, 5, 5)} = \frac{6}{1} = 6$ seconds

19. (*d*) Number of students that should be seated in each room is the HCF of 60, 84 and 108 that is 12.

$\therefore$ Number of rooms required for subjects A, B and $C = \frac{60}{12} = 5$ rooms, $\frac{84}{12} = 7$ rooms and $\frac{108}{12} = 9$ rooms, respectively

Hence, minimum number of rooms required to satisfy all the conditions

$= (5 + 7 + 9) = 21.$

20. (*b*) The bells will chime together again after a time that is equal to the LCM of 18, 24 and 32 = 288 minutes = 4 hrs and 48 minutes.

21. (*d*) LCM of (7, 12, 16) = 336

If we divide 1856 by 336, then remainder is 176. Since, it is given that remainder in this condition is 4. Hence, the least number to be subtracted $= (176 - 4) = 172.$

22. (*b*) Required number = LCM of (8, 11, 24) − 5

$= (264 - 5) = 259.$

23. (d) Required number = LCM of (4, 6, 7) + 2

$\Rightarrow \quad 84 + 2 = 86.$

24. (c) Both the sequences (17, 21, 25,…) and (16, 21, 26,…) are arithmetic progressions with common difference of 4 and 5, respectively

In both the sequences, first common term is 20. Hence, a new arithmetic sequence containing the common terms of both the series can be formed with a common difference of LCM of (4, 5) i.e., 20

$\therefore$ New sequence will be 21, 41, 61,…, 401

nth term $= a + (n - 1)\, d$

$401 = 21 + (n - 1)\, 20$

$\therefore \quad (n-1) = \frac{401 - 21}{20} = 19$

$\therefore \quad n = 20.$