DAY THIRTY THREE

Nuclei

Learning & Revision for the Day

- Concept of Nucleus
- Radioactivity
- Mass Energy Relation
- Mass Defect and Binding Energy
- Nuclear Fission
- Nuclear Fusion

Concept of Nucleus

In every atom, the positive charge and mass is densely concentrated at the centre of the atom forming its **nucleus**. In nucleus, the number of protons is equal to the atomic number of that element and the remaining particles to fulfil the mass number are the neutrons.

Composition of Nucleus

Nucleus consists of protons and neutrons. Electrons cannot exist inside the nucleus. A proton is a positively charged particle having mass (m_p) of 1.007276 u and charge $(+e) = +1.602 \times 10^{-19}$ C.

For a neutral atom, **Number of proton (Z) = Number of electron**

This number is called the **atomic number**. A neutron is a neutral particle having mass $m_n = 1.008665$ u. The number of neutrons in the nucleus of an atom is called the **neutron number** N. The sum of the number of protons and neutrons is called the **mass number A.** Thus, $A = N + Z$.

Properties of Nucleus

Nuclear size

(a) Size of the nucleus is of the order of fermi (1 fermi $= 10^{-15}$ m).

(b) The radius of the nucleus is given by $R = R_0 \, A^{1/3}$,
where, $R_0 = 1.3$ fermi and A is the mass number.

Volume

The volume of nucleus is $V = \frac{4}{3} \pi \, (R_0 A^{1/3})^3$, where, $R_0 =$ radius of the nucleus.

Density

(a) $\text{Density} = \frac{\text{Mass of nucleus}}{\text{Volume of the nucleus}} = \frac{A m_p}{\frac{4}{3} \pi \, (R_0 A^{1/3})^3} = \frac{m_p}{\frac{4}{3} \pi R_0^3}$

where, $m_p = 1.6 \times 10^{-27}$ kg = mass of proton and $R_0 = 1.3$ fermi.

(b) Density of nuclear matter is of the order of 10^{17}kg/m^3.

(c) Density of nuclear matter is independent of the mass number.

Isotopes, Isobars and Isotones

Isotopes

Isotopes of an element are nuclides having same atomic number Z, but different mass number A (or different neutron number N) is called isotopes. ${}_1^1H$, ${}_1^2H$, ${}_1^3H$ and ${}_6^{11}C$, ${}_6^{12}C$, ${}_6^{14}C$, etc., are isotopes.

Isobars

Nuclides having same mass number A, but different atomic number Z are called isobars. In isobars number of protons Z as well as number of neutrons N differ but total nucleon (or mass) number $A = N + Z$ is the same. ${}_1^3H$, ${}_2^3He$ and ${}_6^{14}C$, ${}_7^{14}N$ are isobars.

Isotones

Nuclides with different atomic number Z and different mass number A, but same neutron number are called isotones. ${}_1^3H$, ${}_2^4He$ and ${}_{80}^{198}Hg$, ${}_{79}^{197}Au$ are examples of isotones.

Radioactivity

Radioactivity is the phenomenon of spontaneous emission of radiations by heavier nucleus. Some naturally occurring radioactive substances are uranium, thorium, polonium, radium, neptunium, etc. In fact, all elements having atomic number $Z > 82$ are radioactive in nature.

Radiations emitted by radioactive substances are of three types, namely (i) α-particles, (ii) β-particles and (iii) γ- rays.

- **α-particles** are positively charged particles with charge $q_\alpha = +2\,e$ and mass $m_\alpha = 4m_p$. Thus, α-particles may be considered as helium nuclei (or doubly charged helium ions). Ionising power of α-particles is maximum, but their penetrating power is minimum.
- **β-particles** are negatively charged particles with rest mass as well as charge same as that of electrons. But origin of β-particles is from the nucleus. Their ionising power is lesser than that of α-particles, but speed as well as penetrating power is much greater than that of α-particles. Generally, β-decay means β^-- decay.
- **γ-rays** are electromagnetic radiations of extremely short wavelengths. Thus, γ-rays travel with the speed of light. Their ionising power is least, but penetrating power is extremely high. These are not deflected either in an electric or a magnetic field.

Law of Radioactive Decay

According to Rutherford-Soddy's law for radioactive decay, 'The rate of decay of a radioactive material at any instant is proportional to the quantity of that material actually present at that time.'

Mathematically, $\frac{dN}{dt} \propto N$ or $\frac{dN}{dt} = -\lambda N$

Here, λ is a proportionality constant, known as the **decay constant** (or disintegration constant). Unit of λ is s^{-1} or day^{-1} or $year^{-1}$, etc.

It can be shown that number of nuclei present after time t is given by

$$N = N_0\, e^{-\lambda t}$$

where, N_0 = number of nuclei present at time $t = 0$.

Again, number of nuclei decayed in time t will be

$$N - N_0 = N_0\,[e^{-\lambda t} - 1]$$

= number of **daughter nuclei** produced at time t.

Half-Life Period ($T_{1/2}$)

It is the time in which, activity of the sample falls to one-half of its initial value.

Thus, for $t = \frac{T}{2}$, $N = \frac{N_0}{2}$ and $R = \frac{R_0}{2}$

- The half-life period is related to decay constant λ as
$$T_{1/2} = \frac{0.693}{\lambda}$$
- After n half-lives, the quantity of a radioactive substance left intact (undecayed) is given by
$$N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

Mean Life Period (τ)

- Mean life of a radioactive sample is the time, at which both N and R have been reduced to $\frac{1}{e}$ or e^{-1} or 36.8% of their initial values. It is found that $\tau = \frac{1}{\lambda}$.
- Half-life $T_{1/2}$ and mean life τ of a radioactive sample are correlated as, $T_{1/2} = 0.693\,\tau$ or $\tau = 1.44 T_{1/2}$.

Activity

The activity of a radioactive substance is defined as the rate of disintegration (or the count rate) of that substance.

Mathematically, activity is defined as

$$R = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = R_0\, e^{-\lambda t}$$

where, $R_0 = \lambda N_0$ = initial value of activity.

Units of activity are

- 1 becquerel = Bq = 1 disintegration per second (SI unit)
- 1 curie = 1 Ci = 3.7×10^{10} Bq
- 1 rutherford = 1 Rd = 10^6 Bq

Mass Energy Relation

In nuclear physics, mass is measured in **unified atomic mass units** (u), 1 u being one-twelfth of the mass of carbon-12 atom and equals 1.66×10^{-27} kg. It can readily be shown using $E = mc^2$ that, 1 u mass has energy 931.5 MeV

$$\text{Thus, } 1\text{ u} \equiv 931.5 \text{ MeV} \quad \text{or} \quad c^2 = 931.5 \text{ MeV/u}$$

A unit of energy may therefore be considered to be a unit of mass. For example, the electron has a rest mass of about 0.5 MeV.

If the principle of conservation of energy is to hold for nuclear reactions it is clear that mass and energy must be regarded as equivalent. The implication of $E = mc^2$ is that any reaction producing an appreciable mass decrease is a possible source of energy.

- At the rest, mass energy of each of electron and positron, is

  $$E_0 = m_0 c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \text{ J} = 0.51 \text{ MeV}$$

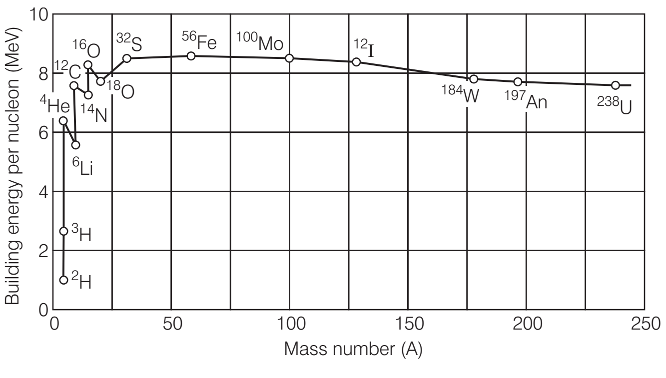
  Therefore, an energy of atleast 1.02 MeV is needed for pair production.

Mass Defect and Binding Energy

- The difference in mass of a nucleus and its constituent nucleons is called the mass defect of that nucleus. Thus, Mass defect, $\Delta M = Zm_p + (A - Z)m_n - M$
  where, M is the mass of a given nucleus.
- **Packing fraction** of an atom is the difference between mass of nucleus and its mass number per nucleon. Thus,
  Packing fraction $= \dfrac{M - A}{A}$.
- The energy equivalent of the mass defect of a nucleus is called its **binding energy.**
  Thus, binding energy, $\Delta E_b = \Delta M c^2$
  $= [Zm_p + (A - Z)m_n - M]\, c^2$

  If masses are expressed in atomic mass units, then
  $\Delta E_b = \Delta M \times 931.5$ MeV
  $= [Zm_p + (A - Z)m_n - M] \times 931.5$ MeV
- **Binding energy per nucleon** (ΔE_{bn}) is the average energy needed to separate a nucleus into its individual nucleons.

  Thus, $\quad \Delta E_{\text{bn}} = \dfrac{\Delta E_b}{A} = \dfrac{\Delta M \times 931}{A} \dfrac{\text{MeV}}{\text{Nucleon}}$



- The shown figure show binding energy per nucleon *versus* mass number. The nuclides showing binding energy per nucleon greater than 7.5 MeV/nucleon are stable.

NOTE
- Nucleons attract each other when they are separated by a distance of 10^{-14} m.
- The density of nucleus is of the order of 10^{17}.

Nuclear Fission

Nuclear fission is the process of splitting of a heavy nucleus ($^{235}_{92}$U or $^{239}_{94}$Pu) into two lighter nuclei of comparable masses along with the release of a large amount of energy ($\simeq 200$ MeV) after bombardment by slow neutrons.

A characteristic nuclear fission reaction equation for $^{235}_{92}$U is

$$^1_0n(\text{slow}) + {}^{235}_{92}\text{U} \longrightarrow {}^{236}_{92}\text{U} \longrightarrow {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3\,{}^1_0n + Q$$

In the fission of uranium, the percentage of mass converted into energy is about 0.1%.

Controlled Chain Reaction and Nuclear Reactor

- In the fission of one nucleus of $^{235}_{92}$U, on an average, $2\frac{1}{2}$ neutrons are released. These released neutrons may further, trigger more fissions causing more neutrons being formed, which in turn may cause more fission. Thus, a self sustained nuclear chain reaction is formed. To maintain the nuclear chain reaction at a steady (sustained) level, the extra neutrons produced, are absorbed by suitable neutron absorbents like cadmium or boron.
- Neutrons formed as a result of fission have an energy of about 2 MeV, whereas for causing further fission, we need slow thermal neutrons having an energy of about 0.3 eV. For this purpose, suitable material called a **moderator** is used, which slow down the neutrons. Water, heavy water and graphite are commonly used as moderators.
- A **nuclear reactor** is an arrangement in which nuclear fission can be carried out through a sustained and a controlled chain reaction and can be employed for producing electrical power, for producing different isotopes and for various other uses.
- Power of a reactor, $P = \dfrac{nE}{t}$, where n = number of atoms undergone fission in time t seconds and E = energy released in each fission.

Reproduction Factor

Reproduction factor (k) of a nuclear chain reaction is defined as

$$k = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss / Absorption of neutrons}}$$

- If $k = 1$, then the chain reaction will be steady and the reactor is said to be **critical**.
- If $k > 1$, then the chain reaction is accelerated and it may cause explosion in the reactor. Such a reactor is called **super-critical**.
- If $k < 1$, then chain reaction gradually slows down and comes to a halt. Such a reactor is called **sub-critical**.

The reactors giving fresh nuclear fuel which often exceeds the nuclear fuel used is known as **breeder reactor**.

Nuclear Fusion

Nuclear fusion is the process, in which two or more light nuclei combine to form a single large nucleus.

The mass of the single nucleus, so formed is less than the sum of the masses of parent nuclei and this difference in mass, results in the release of tremendously large amount of energy.

The fusion reaction going on in the central core of sun is a multistep process, but the net reaction is

$$4\,{}_1^1\text{H} + 2\,\varepsilon^- \longrightarrow {}_2^4\text{He} + 2\,\nu + 6\gamma + 26.7\text{ MeV}$$

When two positively charged particles (protons or deuterons) combine to form a larger nucleus, the process is hindered by the Coulombian repulsion between them.

To overcome the Coulombian repulsion, the charged particles are to be given an energy of atleast 400 keV.

For this, proton/deuterons must be heated to a temperature of about 3×10^9 K. Nuclear fusion reaction is therefore, known as **thermo nuclear fusion** reaction.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 Two nucleons are at a separation of 1 fm. The net force between them is F_1 if both are neutrons, F_2 if both are protons and F_3 if one is a proton and the other is a neutron.

(a) $F_1 > F_2 > F_3$ (b) $F_2 > F_1 > F_3$
(c) $F_1 = F_3 > F_2$ (d) $F_1 = F_2 > F_3$

2 A radioactive nucleus (initial mass number A and atomic number Z) emits 3α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be **→ AIEEE 2010**

(a) $\dfrac{A - Z - 8}{Z - 4}$ (b) $\dfrac{A - Z - 4}{Z - 8}$
(c) $\dfrac{A - Z - 12}{Z - 4}$ (d) $\dfrac{A - Z - 4}{Z - 2}$

3 The sequence of decay of a radioactive nucleus is $N_0 \xrightarrow{\alpha} N_1 \xrightarrow{\beta} N_2 \xrightarrow{\alpha} N_3 \xrightarrow{\alpha} N_4$. If nucleon number and atomic number of N_2 are 176 and 71 respectively, then what are their values for N_4 and N_0?

(a) 168, 67 and 180, 71 (b) 67, 168 and 180, 72
(c) 180, 67 and 72, 180 (d) None of these

4 A radioactive nucleus undergoes a series of decays according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72 respectively, these numbers of A_4 are

(a) 172, 69 (b) 177, 69 (c) 171, 69 (d) 172, 68

5 If $N_{t_1} = N_0\, e^{-\lambda t_1}$, then the number of atoms decayed during the time interval from t_1 and t_2 ($t_1 > t_2$), will be

(a) $N_{t_1} - N_{t_2} = N_0\,[e^{-\lambda t_1} - e^{-\lambda t_2}]$
(b) $N_{t_2} - N_{t_1} = N_0\,[e^{-\lambda t_2} - e^{-\lambda t_1}]$
(c) $N_{t_2} - N_{t_1} = N_0\,[e^{-\lambda t_2} - e^{\lambda t_1}]$
(d) None of the above

6 A radioactive sample decays by two different processes. Half-life for the first process is t_1 and for the second process is t_2. The effective half-life is

(a) $t_1 + t_2$ (b) $t_1 - t_2$ (c) $(t_1 + t_2)/2$ (d) $\dfrac{t_1 t_2}{t_1 + t_2}$

7 Half-life of a radioactive substance A is 4 days. The probability that a nucleus will decay in two half-lives is

(a) $\dfrac{1}{4}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{1}{2}$ (d) 1

8 The half-life of a radioactive substance is 20 min. The approximate time interval ($t_2 - t_1$) between the time t_2 when $\dfrac{2}{3}$ of it has decayed and time t_1 when $\dfrac{1}{3}$ of it had decayed is **→ AIEEE 2011**

(a) 14 min (b) 20 min (c) 28 min (d) 7 min

9 A sample of a radioactive element has a mass of 10 g at an instant $t = 0$. The approximate mass of this element in the sample after two mean lives is

(a) 1.35 g (b) 2.50 g (c) 3.70 g (d) 6.30 g

10 When uranium is bombarded with neutrons, it undergoes fission. The fission reaction can be written as

$${}_{92}\text{U}^{235} + {}_0n^1 \rightarrow {}_{56}\text{Ba}^{141} + {}_{36}\text{Kr}^{92} + 3X + Q\text{ (energy)}$$

where three particles names X are produced and energy Q is released. What is the name of the particle X? **→ JEE Main (Online) 2013**

(a) electron (b) α-particle (c) neutron (d) neutrino

11 On fission of one nucleus of U^{235}, the amount of energy obtained is 200 MeV. The power obtained in a reactor is 1000 kW. Number of nuclei fissioned per second in the reactor is

(a) 3.125×10^{16} (b) 6.25×10^{10}
(c) 3.125×10^{32} (d) 6.25×10^{20}

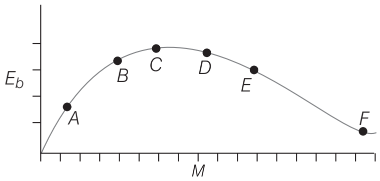
12 If M_o is the mass of an oxygen isotope $_8O^{17}$, M_p and M_n are the masses of a proton and a neutron, respectively the nuclear binding energy of the isotope is

(a) $(M_o - 8M_p)c^2$ (b) $(M_o - 8M_p - 9M_n)c^2$
(c) M_oc^2 (d) $(M_o - 17M_n)c^2$

13 The binding energies per nucleon of Li^7 and He^4 are 5.6 MeV and 7.06 MeV respectively, then the energy of the reaction $Li^7 + p = 2\,[_2He^4]$ will be

(a) 17.28 MeV (b) 39.2 MeV (c) 28.24 MeV (d) 1.46 MeV

14 The below is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei.



Consider four reactions **→[AIEEE 2010]**

(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
(iii) $D + E \rightarrow F + \varepsilon$ and (iv) $F \rightarrow D + E + \varepsilon$

where ε is the energy released. In which reactions is ε positive?

(a) (i) and (iv) (b) (i) and (iii)
(c) (ii) and (iv) (d) (ii) and (iii)

Direction (Q. Nos. 15-21) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (a), (b), (c) *and* (d) *given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

15 **Statement I** A certain radioactive substance has a half-life period of 30 days. Its disintegration constant is 0.0231 day^{-1}.

Statement II Decay constant varies inversely as half-life.

16 **Statement I** Half-life of a certain radioactive element is 100 days. After 200 days, fraction left undecayed will be 50%.

Statement II $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^n$, where symbols have usual meaning.

17 **Statement I** In a decay, daughter nucleus shifts two places to the left from the parent nucleus.

Statement II An alpha particle carries four units of mass.

18 **Statement I** Energy is released in nuclear fission.

Statement II Total binding energy of the fission fragments is larger than the total binding energy of the parent nucleus.

19 **Statement I** If half-life period and the mean-life of a radioactive element are denoted by T and T_m respectively, then $T < T_m$.

Statement II Mean-life $= \dfrac{1}{\text{decay constant}}$

20 **Statement I** Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

Statement II For heavy nuclei, binding energy for per nucleon increases with increasing Z while for light nuclei. It decreases with increasing Z.

21 **Statement I** A nucleus having energy E_1 decays by β^- emission to daughter nucleus having energy E_2, but β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.

Statement II To conserve energy and momentum in β-decay, atleast three particles must take part in the transformation. **→AIEEE 2011**

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 Consider $x \xrightarrow{-\alpha} y \xrightarrow{-\alpha} z$, where half-lives of x and y are z year and one month. The ratio of atoms of x and y when transient equilibrium $[T_{1/2}(x) > T_{1/2}(y)]$ has been established is

(a) 1 : 22 (b) 1 : 26 (c) 26 : 1 (d) 23 : 1

2 In a nuclear reactor, U^{235} undergoes fission liberating 200 MeV of energy per fission. The reactor has 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 yr, the total mass of uranium required is (Avogadro's number $= 6.02 \times 10^{26}$/K-mol, 1 eV $= 1.6 \times 10^{-19}$ J)

(a) 3.84×10^4 kg (b) 9.28×10^6 kg
(c) 3.84×10^8 kg (d) 9.28×10^4 kg

3 The half-life of a radioactive sample is 10 h. The total number of disintegration in 10th hour measured from a time when the activity was one Ci is

(a) 0.53×10^{-3} (b) 6.91×10^{13}
(c) 2.63×10^{-3} (d) 9.91×10^{13}

4 A piece of wood from the ruins of an ancient building was found to have a ^{14}C activity of 12 disintegrations per minute per gram of its carbon content. The ^{14}C activity of the living wood is 16 disintegrations per minute per gram. How long ago did the tree, from which the wooden sample came, die? Given, half-life of ^{14}C is 5760 yr?

(a) 2391 yr (b) 2300 yr (c) 2250 yr (d) 2261 yr

5 A radioactive sample S_1 having an activity of 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10μCi. The half-lives of S_1 and S_2 can be

(a) 20 yr and 5 yr, respectively
(b) 20 yr and 10 yr, respectively
(c) 10 yr each
(d) 5 yr each

6 The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially, they have the same number of atoms. Then,

(a) X will decay faster than Y
(b) Y will decay faster than X
(c) Y and X have same decay rate initially
(d) X and Y decay at same rate always

7 Deuteron is a bound state of a neutron and a proton with a binding energy $B = 2.2$ MeV. A γ-ray of energy E is aimed at a deuteron nucleus to try to break it into a (neutron + proton) such that the n and p move in the direction of the incident γ-ray. Where $E \neq B$. Then, calculate how much bigger that B must E be for such a process to happen?

(a) $\frac{B^2}{2mc^2}$ (b) $\frac{B}{2mc^2}$ (c) $\frac{B^2}{4mc^2}$ (d) $\frac{3B}{4mc^2}$

8 Assume that a neutron breaks into a proton and an electron. The energy released during this process is (mass of neutron = 1.6725×10^{-27} kg, mass of proton = 1.6725×10^{-27} kg, mass of electron = 9×10^{-31} kg)

→ **AIEEE 2012**

(a) 0.9 MeV (b) 7.10 MeV (c) 6.30 MeV (d) 5.4 MeV

9 A radioactive nucleus A with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. After sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by → **JEE Main 2017 (Offline)**

(a) $t = T\frac{\log 1.3}{\log_e 2}$ (b) $t = T \log 1.3$

(c) $t = \frac{T}{\log 1.3}$ (d) $t = \frac{T \log_e 2}{2\log 1.3}$

10 Half-lives of two radioactive elements A and B are 20 min and 40 min, respectively. Initially, the samples have equal number of nuclei. After 80 min, the ratio of decayed numbers of A and B nuclei will be

→ **JEE Main 2016 (Offline)**

(a) 1 : 16 (b) 4 : 1 (c) 1 : 4 (d) 5 : 4

ANSWERS

SESSION 1	**1** (c)	**2** (b)	**3** (a)	**4** (a)	**5** (a)	**6** (d)	**7** (b)	**8** (b)	**9** (a)	**10** (c)
	11 (a)	**12** (b)	**13** (a)	**14** (a)	**15** (a)	**16** (c)	**17** (b)	**18** (a)	**19** (b)	**20** (c)
	21 (a)									
SESSION 2	**1** (d)	**2** (a)	**3** (b)	**4** (a)	**5** (a)	**6** (b)	**7** (c)	**8** (a)	**9** (a)	**10** (d)

Hints and Explanations

SESSION 1

1 Nuclear force of attraction between any two nucleons (n-n, p-p, p-n) is same. The difference comes up only due to electrostatic force of repulsion between two protons.

$\therefore \quad F_1 = F_3 \neq F_2$

As, $\quad F_2 < F_3$ or F_1

$\therefore \quad F_1 = F_3 > F_2$

2 In positive β-decay a proton is transformed into a neutron and a positron is emitted.

$$p^+ \longrightarrow n^0 + e^+$$

Number of neutrons initially was $A - Z$

Number of neutrons after decay is $(A - Z) - 3 \times 2$ (due to α-particles) $- 2 \times 1$ (due to positive β-decay)

As [3×2 (due to α-particles) + 2 (due to positive β-decay)]

Hence, atomic number reduces by 8.

So, the ratio of number of neutrons to that of protons = $\frac{A - Z - 4}{Z - 8}$

3 As mass number of each α-particle is 4 units and its charge is 2 units, therefore for N_4

$A = 176 - 8 = 168$

and $\quad Z = 71 - 4 = 67$

Now, the charge of β is -1 and its mass number is zero.

So, $\quad A = 176 + 0 + 4 = 180$

and $\quad Z = 71 - 2 + 2 = 71$

4 As the mass number of each α-particle is 4 units and its charge is 2 unit. Therefore for A_4,
Mass number = 180 − 8 = 172
and $Z = 72 - 4 + 1$ (due to β^-) = 69

5 Since, $N_{t_1} = N_0 e^{-\lambda t_1}$ and $N_{t_2} = N_0 e^{-\lambda t_2}$

Then, the number of atoms decayed during the time interval t_1 to t_2 is

$$= N_{t_1} - N_{t_2} = N_0 [e^{-\lambda t_1} - e^{-\lambda t_2}]$$

6 As, $\lambda = \lambda_1 + \lambda_2$

$$\Rightarrow \quad \frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} = \frac{t_2 + t_1}{t_1 t_2}$$

or $$t = \frac{t_1 t_2}{t_1 + t_2}$$

7 After two half-lives 1/4th fraction of nuclei will remain undecayed. Or, 3/4th fraction will decay. Hence, the probability that a nucleus decays in two half-lives is 3/4.

8 $N_1 = N_0 - \frac{1}{3} N_0 = \frac{2}{3} N_0$,

$N_2 = N_0 - \frac{2}{3} N_0 = \frac{1}{3} N_0$

We have, $$\frac{N_2}{N_1} = \left(\frac{1}{2}\right)^n$$

Here, $n = 1$

$\therefore \; t_2 - t_1$ = one half-life = 20 min

9 The relation of mean-life and decay constant is,

$$t = 2\tau = \frac{2}{\lambda}, \text{ where } \tau = \frac{1}{\lambda}$$

Then we get from the equation,

$$m = m_0 e^{-\lambda t}$$

$$\Rightarrow \quad m = 10 \times e^{-\lambda \times 2/\lambda} = 10 \times e^{-2}$$

$$= 10 \times 0.135 = 1.35 \text{ g}$$

10 The fission of $_{92}U^{235}$ is represented by

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q$$

The name of the particle X is neutron $({}_0n^1)$.

11 Power received from the reactor is

$$P = 1000 \text{ kW}$$
$$= 1000 \times 1000 = 10^6 \text{ J s}^{-1}$$

Also, $1 \text{ MeV} = 1.6 \times 10^{-13}$ J

Number of nuclei fissioned per second

$$= \frac{10^6}{200 \times 1.6 \times 10^{-13}}$$
$$= 3.125 \times 10^{16} \text{ s}^{-1}$$

12 Binding energy

$$\text{BE} = (M_{\text{nucleus}} - M_{\text{nucleons}})c^2$$
$$= (M_o - 8M_p - 9M_n)c^2$$

13 The reaction is $_3Li^7 + {}_1p^1 \longrightarrow 2\,({}_2He^4)$

$$\therefore \quad E_p = 2E\,({}_2He^4) - E\,(\text{Li})$$
$$= 2\,(4 \times 7.06) - 7 \times 5.6$$
$$= 56.48 - 39.2 = 17.28 \text{ MeV}$$

14 Both fusion and fission reaction results into tremendous amount of energy release and nucleus/nuclei which has higher binding energy per nucleon than parent nuclei. So, option (a) is correct.

15 Half-life and decay constant for a nuclear reaction are related by a relation, which is

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$\Rightarrow \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{30} = 0.0231 \text{ day}^{-1}$$

16 Number of half-lives

$$n = \frac{t}{T} = \frac{200}{100} = 2$$

The fraction left undecayed is given by

$$\therefore \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2$$
$$= \frac{1}{4} = 25\%$$

17 On α-decay, charge number of parent nucleus decreases by 2 units. As classification or grouping of elements is based on charge number, hence daughter nucleus shifts two places to the left from the parent nucleus.

18 According to concept of binding energy, fission can occur because the total mass energy will decrease; that is ΔE_{bn} (binding energy) will increase. We see that for high mass nuclide $(A = 240)$, the binding energy per nucleon is about 7.6 MeV/nucleon. For the middle weight nuclides $(A = 120)$, it is about 8.5 MeV/nucleon. Thus, binding energy of fission fragments is larger than the total binding energy of the parent nucleus.

19 We know that half-life period T and decay constant λ are related by the equation.

$$T = \frac{0.6931}{\lambda} \quad \text{...(i)}$$

While mean-life T_m is related with λ by the equation

$$T_m = \frac{1}{\lambda} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$T = 0.6931\,T_m$$

or $$T < T_m$$

20 Here, Statement I is correct and Statement II is wrong, which can be directly concluded from binding energy nucleon curve.

21 In practicle situation, atleast three particles take place in transformation, so Energy of β-particle + Energy of third particle = $E_1 - E_2$
Hence, energy of β-particle $\le E_1 - E_2$

SESSION 2

1 $$N_2 = \frac{\lambda_1 N_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

When $(T_{1/2})_1 > (T_{1/2})_2$ at transient equilibrium, $\lambda_1 < \lambda_2$

$$e^{-\lambda_2 t} << e^{-\lambda_1 t}$$

$$\therefore \quad N_2 = \frac{\lambda_1 N_1 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1}$$
$$= \frac{\lambda_1 N_1}{\lambda_2 - \lambda_1}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{\lambda_2 - \lambda_1}{\lambda_1}$$

$$= \frac{\frac{0.693}{1} - \frac{0.693}{2 \times 12}}{\frac{0.693}{2 \times 12}} = \frac{23}{1}$$

2 Energy generated by the reactor $1000 \times 10^6 \text{ W} = 10^9 \text{ Js}^{-1}$

Total energy generated in 10 yr is

$$E = (10^9 \text{ Js}^{-1}) \times 10 \times 365 \times 24 \times 60 \times 60$$
$$= 1.97 \times 10^{30} \text{ MeV}$$

In the reactor 200 MeV energy is liberated in the fission of nucleus of U^{235} atom.

$\therefore$ Total number of U^{235} atoms required is

$$\frac{1.97 \times 10^{30}}{200} = 0.985 \times 10^{28}$$

1 kmol that is 235 kg of U^{235} has 6.02×10^{26} atoms Therefore, total mass of U^{235} having 0.985×10^{28} atoms is

$$\frac{235}{6.02 \times 10^{26}} \times (0.985 \times 10^{28})$$
$$= 3.84 \times 10^3 \text{ kg}$$

Since, efficiency of reactor is 10%, actual mass of U^{235} required is

$$(3.84 \times 10^3) \times \frac{100}{10} = 3.84 \times 10^4 \text{ kg}$$

3 As, $-\frac{dN}{dt} = \lambda N$,

$$N = \frac{3.7 \times 10^{10} \times 3.6 \times 10^4}{0.693}$$

$$\Delta N = -(N_0 e^{-\lambda \times 10 \times 3600} - N_0 e^{-\lambda \times 9 \times 3600})$$

$$(\because \Delta N = N_1 - N_2)$$

$$= \frac{3.7 \times 10^{14} \times 3.6}{0.693}[0.535 - 0.5]$$
$$= 6.91 \times 10^{13}$$

4 Given, $R = 12$ dis/min per g,

$$R_0 = 16 \text{ dis/min per g}$$

$$T_{1/2} = 5760 \text{ yr}$$

Let t be the time span of the tree.

According to radioactive decay law,

$$R = R_0 e^{-\lambda t} \quad \text{or} \quad e^{\lambda t} = \frac{R_0}{R}$$

Taking log on both the sides

$$\lambda t \log_e e = \log_e \frac{R_0}{R}$$

$$\lambda t = \left(\log_{10} \frac{16}{12}\right) \times 2.303$$

$$t = \frac{2.303 (\log 4 - \log 3)}{\lambda}$$

$$= 2391.20 \text{ yr} \approx 2391 \text{ yr}$$

5 We know that,

Activity $(A) = \lambda N_0$

For S_1, $A_{s_1} = 5\mu\text{Ci} = \lambda_1 2N_0$ ...(i)

For S_2, $A_{s_2} = 10\mu\text{Ci} = \lambda_2 N_0$ ...(ii)

As we know,

$$T_{s_1\,1/2} = \frac{0.693}{\lambda_1}$$

and $$T_{s_2\,1/2} = \frac{0.693}{\lambda_2}$$

Therefore, by dividing Eqs. (i) and (ii), we get

$$\frac{5}{10} = \frac{T_{s_2\,1/2}\, 2N_0}{T_{s_1\,1/2}\, N_0}$$

$$\Rightarrow \quad \frac{T_{s_2\,1/2}}{T_{s_1\,1/2}} = 4$$

So, only option (a) can be satisfied.

6 According to question,

$$T_{1/2}(X) = \tau(\lambda)$$

$$\Rightarrow \quad \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$$

or $$\lambda_Y = \frac{\lambda_X}{0.693}$$

$$\Rightarrow \quad \lambda_Y > \lambda_X$$

So, Y will decay faster than X.

7 Binding energy $B = 2.2$ MeV

From the energy conservation law,

$$E - B = K_n + K_p = \frac{p_n^2}{2m} + \frac{p_p^2}{2m} \quad \text{...(i)}$$

From conservation of momentum,

$$p_n + p_p = \frac{E}{c} \quad \text{...(ii)}$$

As $E = B$, Eq. (i), $\quad p_n^2 + p_p^2 = 0$

It only happen if $\quad p_n = p_p = 0$

So, the Eq. (ii) cannot satisfy and the process cannot take place.

Let $E = B + X$, where $X << B$ for the process to take place.

Put value of p_n from Eq. (ii) in Eq. (i), we get

$$X = \frac{\left(\frac{E}{c} - p_p\right)^2}{2m} + \frac{p_p^2}{2m}$$

or $$2p_p^2 - \frac{2Ep_p}{c} + \frac{E^2}{c^2} - 2mX = 0$$

Using the formula of quadratic equation, we get

$$p_p = \frac{\frac{2E}{c} + \sqrt{\frac{4E^2}{c^2} - 8\left(\frac{E^2}{c^2} - 2mX\right)}}{4}$$

For the real value $p_{p'}$ the discriminant is positive

$$\frac{4E^2}{c^2} = 8\left(\frac{E_2}{c_2} - 2mX\right)$$

$$16mX = \frac{4E^2}{c^2}$$

$$X = \frac{E^2}{4mc^2} \simeq \frac{B^2}{4mc^2}$$

8 According to given data, mass of neutron and proton are equal which do not permit the breaking up of neutron and proton. But if we take standard mass of neutron as 1.6750×10^{-27} kg, then

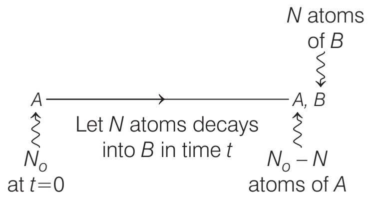
Energy released = mass defect $\times c^2$

$$= (m_n - m_p - m_e) \times c^2$$

$$= \frac{(1.6750 \times 10^{-27} - 1.6725 \times 10^{-27} - 9 \times 10^{-31})}{1.66 \times 10^{-27}} \times 931.5 \text{ MeV}$$

$$\approx 0.9 \text{ MeV}$$

9 Decay scheme is ,



Given, $\frac{N_B}{N_A} = 0.3 = \frac{3}{10}$

$$\Rightarrow \quad \frac{N_B}{N_A} = \frac{30}{100}$$

So, $\quad N_0 = 100 + 30 = 130$ atoms

By using $\quad N = N_0 e^{-\lambda t}$

We have, $\quad 100 = 130 e^{-\lambda t}$

$$\Rightarrow \quad \frac{1}{1.3} = e^{-\lambda t}$$

$$\Rightarrow \quad \log 1.3 = \lambda t$$

If T is half-life, then

$$\lambda = \frac{\log_e 2}{T}$$

$$\Rightarrow \quad \log 1.3 = \frac{\log_e 2}{T} \cdot t$$

$$\therefore \quad t = \frac{T \cdot \log (1.3)}{\log_e 2}$$

10 Given, 80 min = 4 half-lives of A = 2 half-lives of B.

Let the initial number of nuclei in each sample be N.

For radioactive element *A*,

N_A after 80 min $= \frac{N}{2^4}$

$\Rightarrow$ Number of A nuclides decayed

$$= N - \frac{N}{16} = \frac{15}{16} N$$

For radioactive element *B*,

N_B after 80 min $= \frac{N}{2^2}$

$\Rightarrow$ Number of B nuclides decayed

$$= N - \frac{N}{4} = \frac{3}{4} N$$

$\therefore$ Ratio of decayed numbers of A and B nuclei will be

$$\frac{(15/16)N}{(3/4)N} = \frac{5}{4}$$