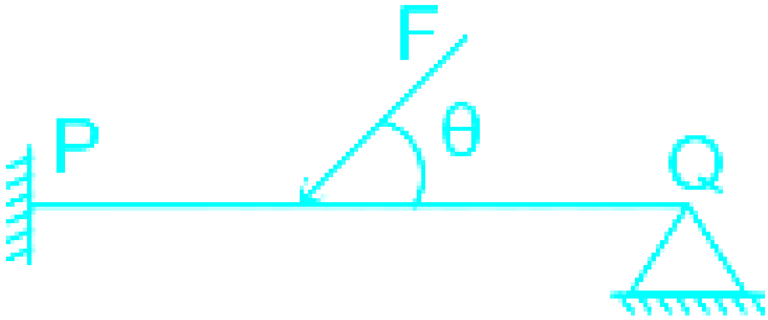
Structural Analysis Test - 2

1. Consider a beam PQ fixed at P, hinged at Q, and subjected to a load F as shown in figure (not drawn to scale). The static and kinematic degrees of indeterminacy, respectively, are



1. 2 and 1
2. 2 and 0
3. 1 and 2
4. 2 and 2

Ans. 1 : 2 and 1

<u>Explanation :-</u>

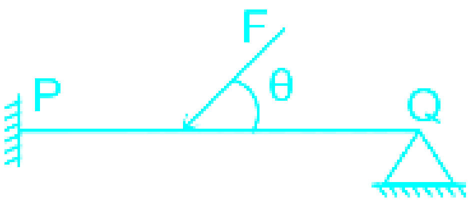
Static indeterminacy-

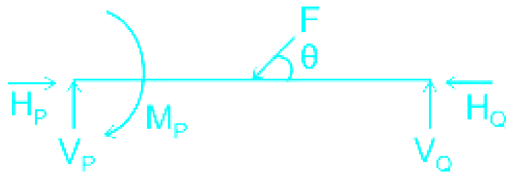
- It is defined as the difference between a total number of unknowns (Total member forces+ reactions) and the total number of available equations from the conditions of equilibrium.
- Degree of static indeterminacy = Total number of unknown forces - Number of equilibrium equations available
- If the degree of static indeterminacy = 0, it is known as a statically determinate structure.
- If the degree of static indeterminacy > 0, it is known as statically indeterminate structure.

Kinematic indeterminacy-

- The degree of kinematic indeterminacy is the minimum number of movements (degrees of freedom, DOF) with which the kinematic configuration of the overall structure can be defined, that is, the number of unknown independent movements of the structure.

Given data and Calculation-



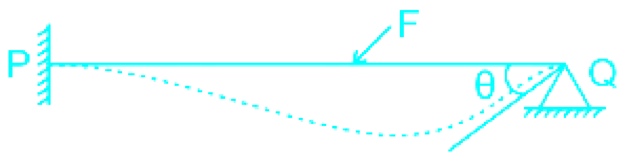


Total number of unknown forces, = (H_p, V_p, M_p , H_Q, V_Q) = 5

Number of equilibrium equations available = 3

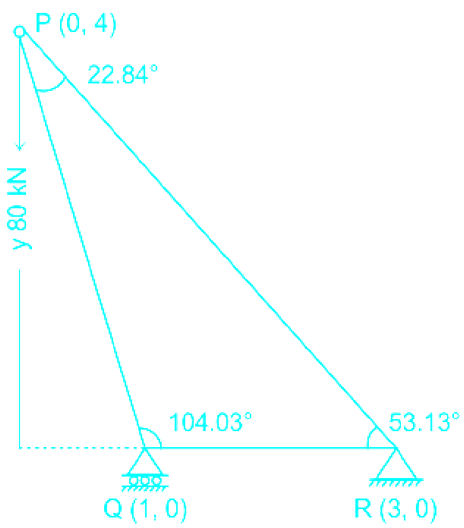
Degree of static indeterminacy = 5 - 3 = 2

As the joint P is fixed, no movement is allowed at that joint, joint Q is hinged so only rotation is allowed at that joint.



So the degree of kinematic indeterminacy = 1 (Rotation at joint Q)

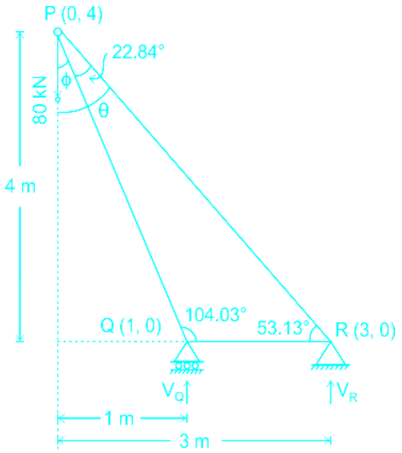
2. Mathematical idealization of a crane has three bars with their vertices arranged as shown in the figure with a load of 80 kN hanging vertically. The coordinates of the vertices are given in parentheses. The force in the member QR, FQR will be



1. 30 kN Compressive
2. 30 kN Tensile
3. 50 kN Compressive
4. 50 kN Tensile

Ans. 1 : 30 kN Compressive

Calculation:



$V_Q + V_R = 80$ ....(i)

$\Sigma M_R = 0$

$80 \times 3 = V_Q \times 2$

$\therefore V_Q = 120$ kN

From equation (i), $V_R = -40$ kN

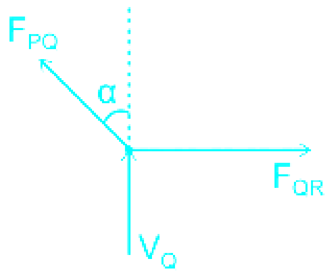
From fig, $\tan\phi = 1/4$

$$\Rightarrow \sin\phi = \frac{1}{\sqrt{17}}, \cos\phi = \frac{4}{\sqrt{17}}$$

$\therefore \tan\theta = 3/4$

$\Rightarrow \cos\theta = 4/5$

Considering joint Q,



$\alpha = 104.03° - 90° = 14.03°$

$\Sigma F_y = 0,$

$F_{PQ} \cos\alpha + V_Q = 0$

$F_{PQ} \cos 14.03 = -120$

$F_{PQ} = -123.6897$ kN

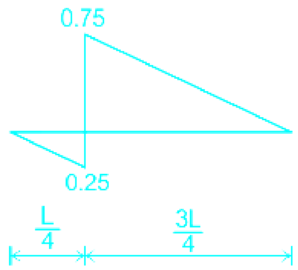
$\Sigma F_x = 0,$

$F_{PQ} \sin\alpha = FQR$

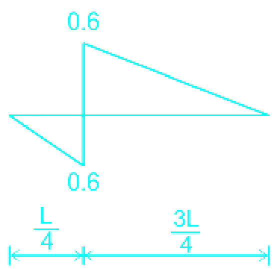
$F_{QR} = - 29.986$ kN

$F_{QR} \approx -30$ kN $= 30$ kN (Compressive)

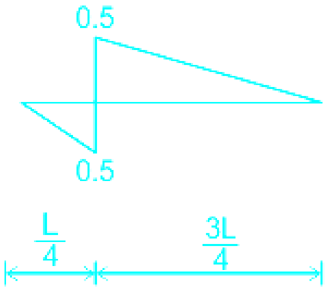
3. In a beam of length L, four possible influence line diagrams for shear force at a section located at a distance of l/4 from the left end support (marked as P, Q, R and S) are shown below. The correct influence line diagram is



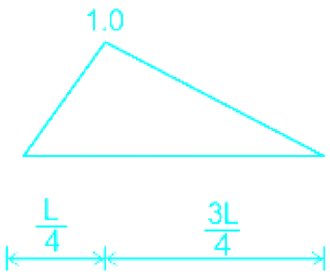
P



Q



R



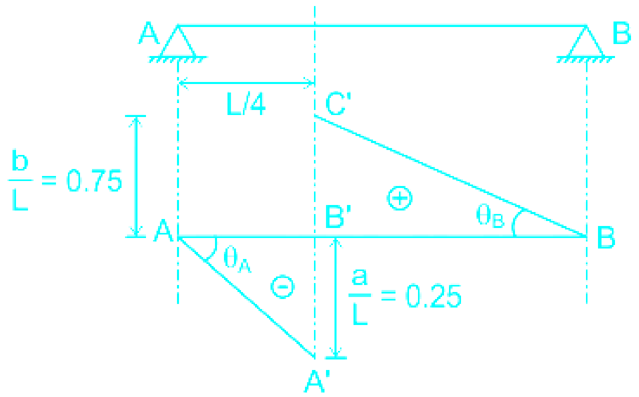
S

1. P
2. Q
3. R
4. S

Ans. 1 : P

Concept:

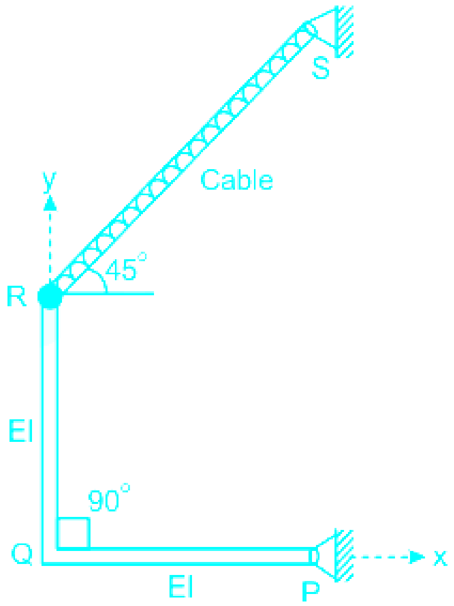
As per Muller Breslau Principle, to draw influence line diagram of shear force at any section, cut the beam at that section and lift in such a way that angle at joint A and B are equal, then corresponding deflected shape of the beam is itself an influence line diagram (do not remove support A and B). The ordinate of influence line diagram at that section will be (a + b)/L, where a and b are the length of the span on LHS and RHS of that section and L is the total length of beam.



$$\theta_A = \frac{A'B'}{AB'} = \frac{\frac{a}{L}}{\frac{L}{4}} = \frac{1}{L}$$

$$\theta_B = \frac{B'C'}{B'B} = \frac{\frac{b}{L}}{3\frac{L}{4}} = \frac{1}{L}$$

4. The degree of static indeterminacy of a rigid jointed frame PQR supported as shown in the figure is



1. Zero
2. One
3. Two
4. Unstable

Ans. 1 : Zero

Concept

Method 1: Using basic concept

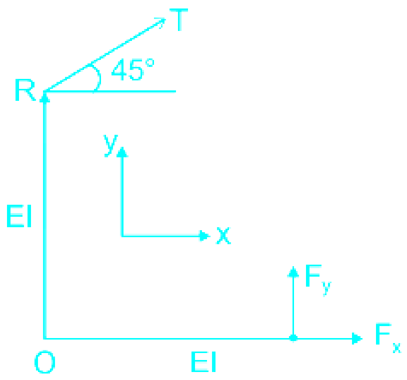
The degree of static indeterminacy means **no of unknowns beyond available equation of equilibrium equations.**

Segment RS is cable and cable can resist only tensile force, so there will be one unknown at joint R.

Further, Joint P is hinged and therefore, there will be only two reactions in x and y direction as shown in figure:

Total unknown = 1 (at Joint R) + 2 (at Joint P) = 3

For frame PQR, there are 3 Equations of Equilibrium available as shown in above figure, by using these equation these 3 unknown can be easily predicted.



Equilibrium equation:

$\sum F_x = 0, \sum F_y = 0, \sum M_p = 0$

$\therefore D_s = 3 - 3 = 0$

Method 2: Using direct formula:

Degree of Static Indeterminacy (D_s):

$D_s = D_{se} + D_{si}$

D_{se} = Degree of external static indeterminacy.

D_{si} = Degree of internal static indeterminacy.

D_{se} = Support Reactions – Number of Equilibrium equations

$D_{si} = 3\ \ C - R$

C = No of closed loops

R = No of releases = m - 1

Calculation:

No of reactions = 2 + 2 = 4 (2 reactions at joint S and P each)

No of equilibrium reactions = 3

$D_{se} = 4 - 3 = 1$

No of closed loops, C = 0

Releases

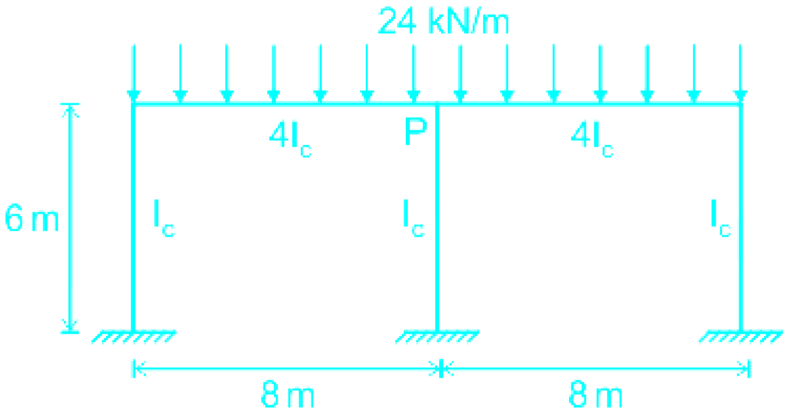
Moment releases at joint R = 2 - 1 = 1

$D_{si} = 3\ C - R = 3\ - 1 = -1$

$D_s = D_{se} + D_{si} = 1 - 1 = 0$

∴ Static Indeterminacy, $D_s = 0$

5. Considering the symmetry of a rigid frame as shown below, the magnitude of the bending moment (in kNm) at P (preferably using the moment distribution method) is

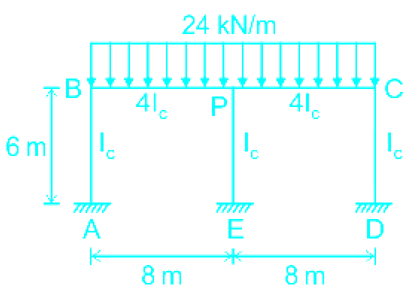


1. 170
2. 172
3. 176
4. 178

Ans. 3 : 176

Calculation:

Given:



Distribution Factor:

Joint	Member	Relative stiffness	Total relative stiffness	Distribution factor
B	BA	I/6	2I/3	1/4
	BP	4I/8		3/4
P	PB	4I/8	7I/6	3/7
	PE	I/6		1/7
	PC	4I/8		3/7
C	CP	4I/8	2I/3	3/4
	CD	I/6		1/4

Fixed End Moments:

$M_{AB} = M_{BA} = M_{PE} = M_{EP} = M_{CD} = M_{DC} = 0$

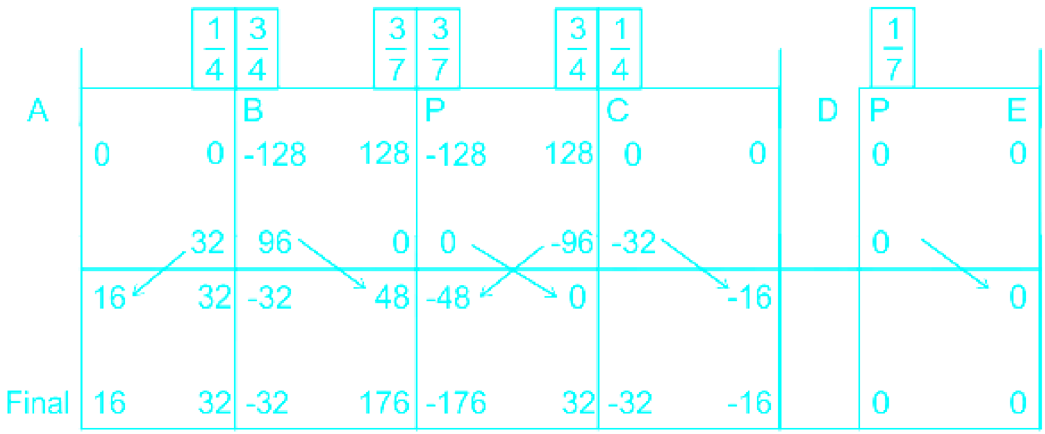
$$M_{BP} = -\frac{WL^2}{12} = -\frac{24\times 8^2}{12} = -128\ \text{kN.m}$$

$$M_{PB} = \frac{WL^2}{12} = \frac{24\times 8^2}{12} = 128\ \text{kN.m}$$

$$M_{PC} = -\frac{WL^2}{12} = -\frac{24\times 8^2}{12} = -128\ \text{kN.m}$$

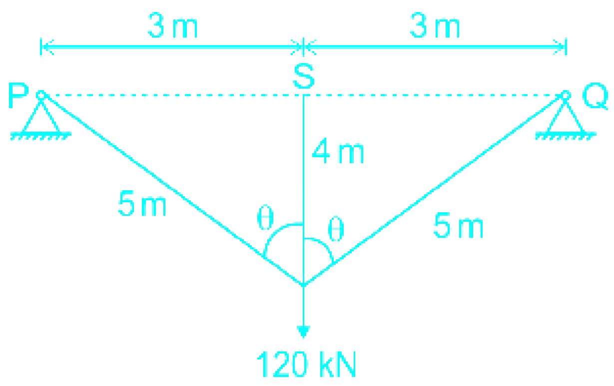
$$M_{CP} = \frac{WL^2}{12} = \frac{24\times 8^2}{12} = 128\ \text{kN.m}$$

Distribution Table:



Hence the magnitude of bending moment at P is 176 kNm

6. The tension (in kN) in a 10 m long cable, shown in the figure, neglecting its self-weight is

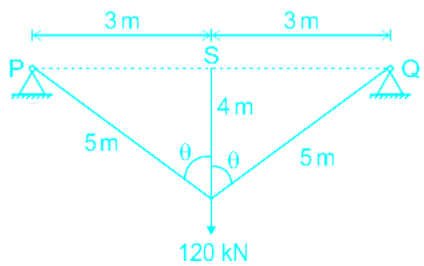


1. 120
2. 75
3. 60
4. 45

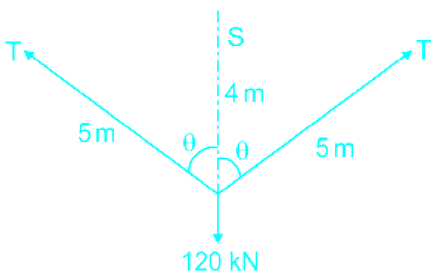
Ans. 2 : 75

Given,

Length of the cable = 10 m



After resolving the forces in the cable, we get the following figure



$\Sigma F_Y = 0$

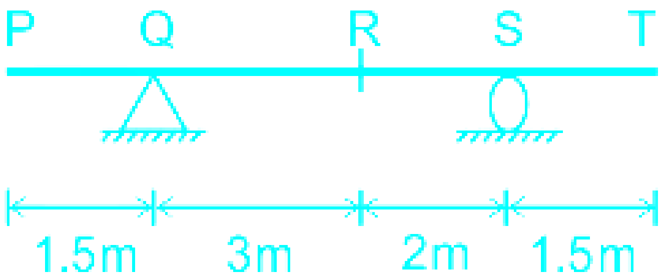
$2T \cos \theta = 120$

Where $\cos \theta = 4/5$

$2 \times T \times (4/5) = 120$

T = 75 kN

7. Distributed load(s) of 50 kN/m may occupy any position(s) (either continuously or in patches) on the girder PQRST as shown in the figure (not drawn to the scale)



The maximum negative (hogging) bending moment (in kN.m) that occurs at point R, is:

1. 22.50
2. 56.25
3. 93.75
4. 150.00

Ans. 2 : 56.25

<u>Concept:</u>

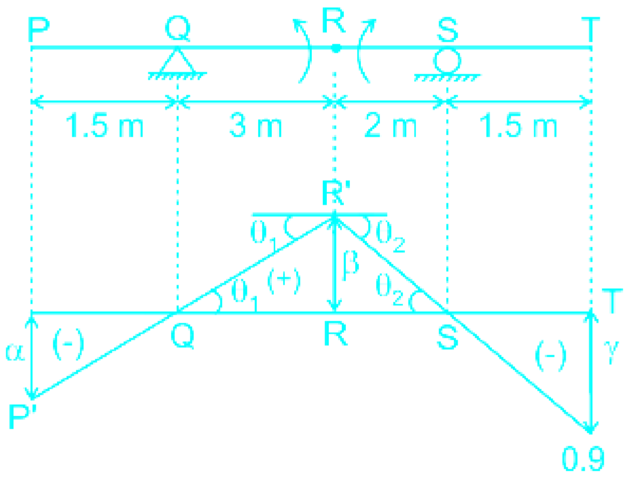
The value of any stress function can be found by drawing a qualitative ILD using the Muller-Breslau principle.

The **Muller Breslau Principle** for determinate structure states that the ordinate value of an influence line for any stress function on any structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint offered to correspond to the stress function from the structure and introducing a force that causes a unit displacement in the positive direction.

Given:

This is a determinate structure over which a UDL of 50 kN/m moves. The maximum negative moment is found by drawing ILD for the moment at R.

Using, the Muller-Breslau Principle, ILD is as shown:



$\theta_1 + \theta_2 = 1$ units

for small θ, tan θ ≈ sin θ ≈ θ

In ΔQRR' and SRR',

$\beta 3+\beta 2=1 \Rightarrow \beta=1.2\ units$

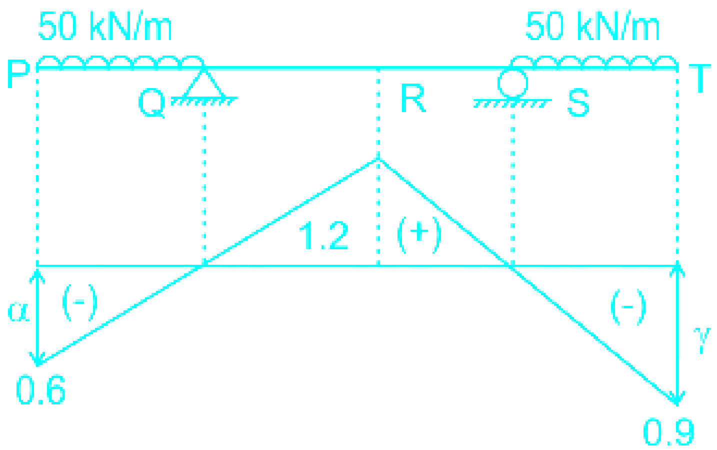
Again, ΔPQP' and ΔQRR' are similar

$\therefore \beta 3=\alpha 1.5 \Rightarrow \alpha=0.6\ units$

ΔSTT' and ΔSRR' are similar

$\therefore \gamma 1.5=\beta 2 \Rightarrow \gamma=0.9\ units$

For the maximum negative moment, the UDL should lie from P to Q and from S to T.



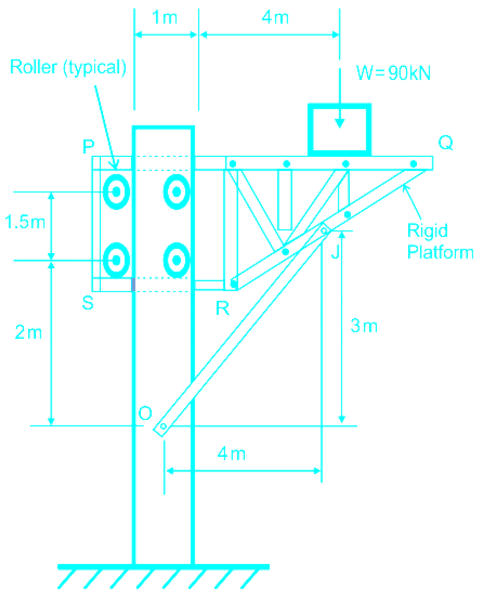
Maximum negative bending moment =

Important Points

To find out the value of stress function due to point loads we draw the ILD for that stress function and find out the ordinate of ILD at the location of the points load and the value of stress function is obtained by multiplying the ordinate of ILD with the value of point load and summed up.

The value of the stress function for UDL is obtained by multiplying the area under the ILD for stress function within the range of UDL with the load intensity (w).

8. A rigid weightless platform PQRS shown in the figure (not drawn to the scale) can slide freely in the vertical direction. The platform is held in position by the weightless member OJ and four weightless, frictionless rollers. Points O and J are pin connections. A block of 90 kN rests on the platform as shown in the figure.



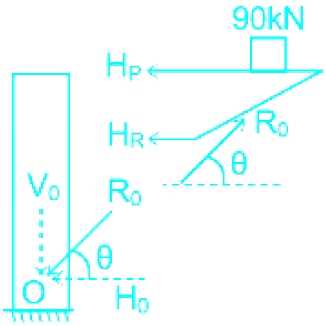
The magnitude of horizontal component of reaction (in kN) at pin O, is f

1. 90
2. 120
3. 150
4. 180

Ans. 2 : 120

Explanation:

Drawing FBD of the given frame –



$\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$

For vertical equilibrium of the frame,

$\sum F_y = 0 \Rightarrow R_0 \sin\theta - 90 = 0$

$\Rightarrow R_0 = \frac{90}{3} \times 5 = 150\ kN$

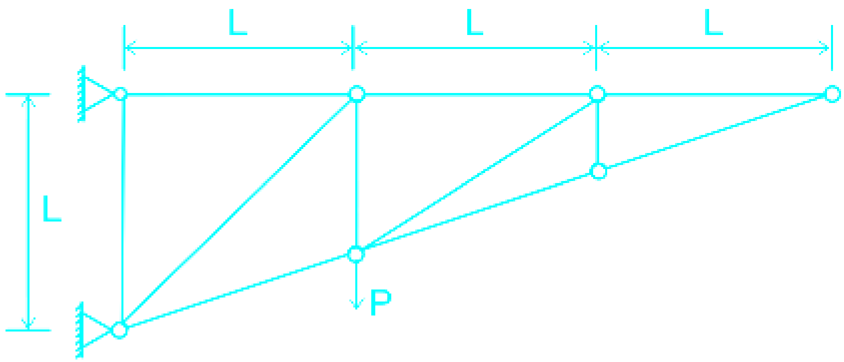
At point O, the horizontal component of reaction,

$H_0 = R_0 \cos\theta = 150 \times \frac{4}{5} = 120\ kN$

Important Points

- The **equilibrium** of the structure is decided based on **external force** only. The internal member forces do not impact the equilibrium of the structure.

9. Consider the planar truss shown in the figure (not drawn to the scale)



Neglecting self-weight of the members, the number of zero-force members in the truss under the action of the load P is

1. 6
2. 7
3. 8
4. 9

Ans. 3 : 8

<u>**Concept:**</u>

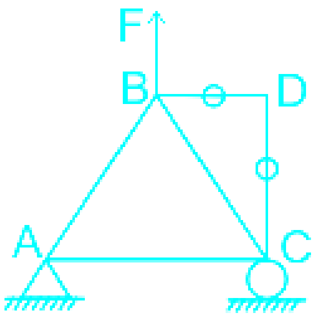
Planar Truss: When all the members and nodes of a truss lie within a 2-dimensional plane it is called a planar truss.

Members of a truss carry only axial force. If the member of a truss does not carry any force under some specific load conditions, those members are called **zero-force members**.

Identification of zero-force members:

1) If at any joint two members are non-collinear and no load is acting at the joint, then both the members will be zero-force members.

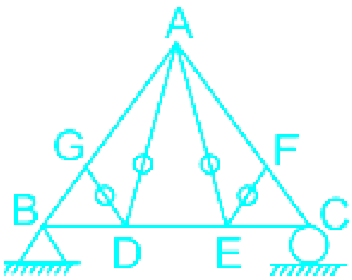
e.g.



In the truss shown above, there are two non-collinear members at joint D and there is no load acting at this joint, hence both the members BD and CD are zero force members.

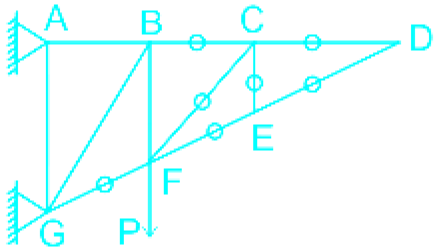
2) If at a truss joint, 3 members are meeting and two of them are collinear and no load is acting at the joint, then the non-collinear member will be a zero-force member.

e.g.



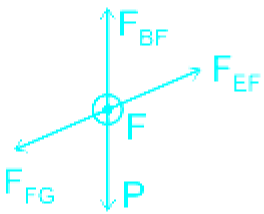
<u>**Calculation:**</u>

A 2D truss as shown below:



Using property 1 explained above, at joint D members CD and EC are zero force members. Similarly, EF and EC are zero force members, and at joint C, CF and BC are zero force members,

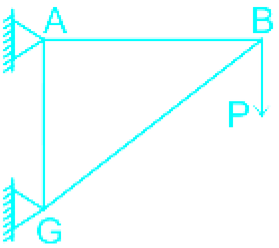
At joint F;



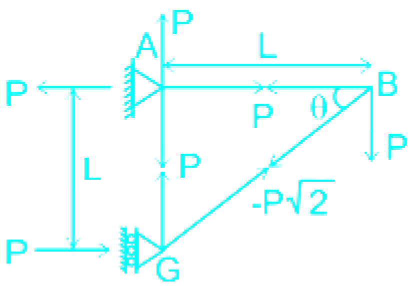
$\because \sum F_y = 0 \therefore F_{FG} = F_{EF}$

$\because \sum F_X = 0 \quad \therefore F_{BF} = P$

Thus the truss reduces to



This is an indeterminate truss (indeterminate to 1 degree) . Let the redundant be the vertical reaction at joint G.



$\theta = 45°$

At joint B ; $\sum F_y = 0 \Rightarrow F_{BG} \sin\theta + P = 0$

$\Rightarrow FBG = -P2$

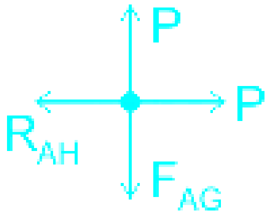
$\sum F_X = 0 \Rightarrow F_{BA} + F_{BG} \cos\theta = 0$

$\Rightarrow F_{BA} = P$

Negative sign in F_{BG} indicates compression.

At joint A, $\because \sum F_Y = 0 \Rightarrow F_{AG} = P$

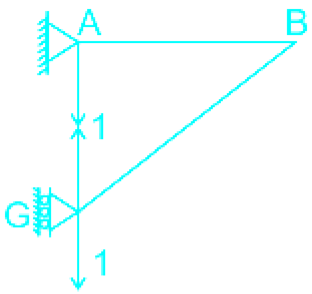
$\because \sum F_X = 0 \Rightarrow R_{AH} = 0$



For stability of truss, $\sum F_x = 0$

Thus horizontal reaction at G = P.

Apply a unit load at G and in the direction of redundant assumed.



By using property 1, explained above $F_{AB} = F_{BG} = 0$

By using the unit load method redundant R is given as

$$R = \frac{-\sum u_i\left(\frac{P_i L_i}{A_i E_i}\right)}{\sum\left(\frac{u_i^2 L_i}{A_i E_i}\right)}$$

Where,

u_i = Member force due to the unit load applied.

P_i = Member force due to external load.

$$\Rightarrow R = -\frac{(1 \times P \times \ell)}{1^2 \times \ell}$$

$\Rightarrow$ R = -P

$\therefore$ Force in GA = P + (-P) × ℓ = 0

Thus we have a total of 8 zero-force members.

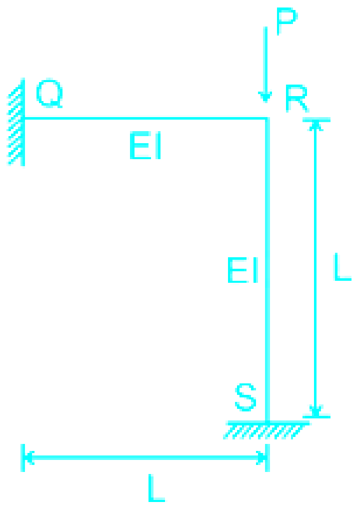
<u>**Key Points**</u>

Force in member GA can also be found using the fact that truss members carry the only axial force and if the axial deflection of any member is zero that member will be a zero force member.

In the given truss, due to hinge at A and G no axial deformation of the member AG is possible.

Hence GA is a zero force member.

10. The rigid-jointed plane frame QRS shown in the figure is subjected to a load P at the joint R. Let the axial deformations in the frame be neglected. If the support S undergoes a settlement of $\Delta = PL3\beta EI$, the vertical reaction at the support S will become zero when β is equal to



1. 0.1
2. 7.5
3. 3.0
4. 48.0

Ans. 2 : 7.5

<u>Explanation:</u>

Method 1: Moment Distribution method

Assume 'R' sinks by Δ

Fixed End Moments are given by

$$M_{F_{QR}} = \frac{-6EI\times\Delta}{L^2}$$

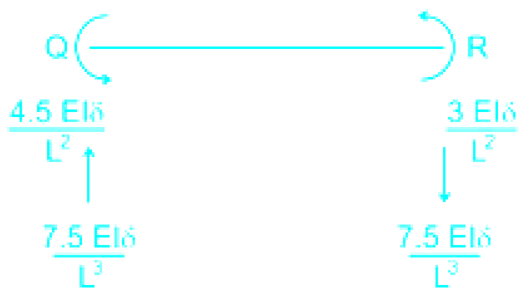
$$M_{F_{RQ}} = \frac{-6EI\times\Delta}{L^2}$$

$$M_{F_{RS}} = M_{F_{SR}} = 0$$

Distribution Factor at R is 0.5 & 0.5 for each end respectively due to symmetricity

	Q	R: 0.5	R: 0.5	S
FEM	$\frac{-6\,EI\delta}{L^2}$	$\frac{-6\,EI\delta}{L^2}$	0	0
Balance		$\frac{3\,EI\delta}{L^2}$	$\frac{3\,EI\delta}{L^2}$	
COM	$\frac{1.5\,EI\delta}{L^2}$			$\frac{1.5\,EI\delta}{L^2}$
Final end moments	$\frac{-4.5\,EI\delta}{L^2}$	$\frac{-3\,EI\delta}{L^2}$	$\frac{3\,EI\delta}{L^2}$	$\frac{1.5\,EI\delta}{L^2}$

By Analysis of beam QR,



$$\sum M_Q = 0$$

$$\frac{4.5 \times EI \times \Delta}{L^2} + \frac{3 \times EI \times \Delta}{L^2} = P \times L$$

$$\Delta = \frac{P \times L^3}{7.5\ \mathrm{EI}}$$

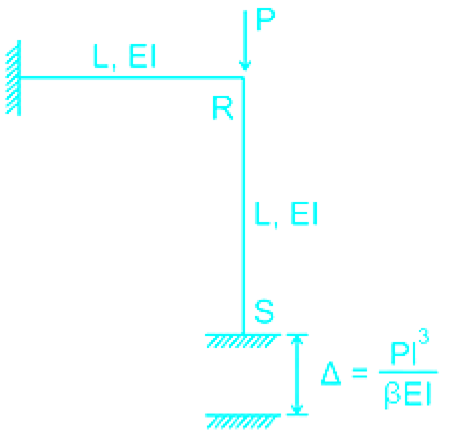
Method 2: Slope deflection method

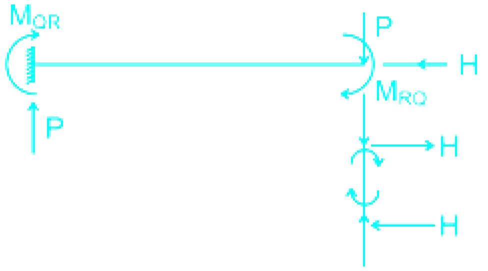
$$M_{QR} = \frac{2EI}{l}\left(\theta_R - \frac{3\Delta}{l}\right)$$

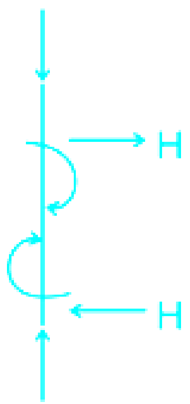
$$M_{RQ} = \frac{2EI}{l}\left(2\theta_R - \frac{3\Delta}{l}\right)$$

$$M_{RS} = \frac{2EI}{l}\left(2\theta_R\right)$$

If reaction at S is equal to zero







$M_{RQ} + M_{QR} + P \times l = 0$

$$\frac{6EI\theta_R}{l} - \frac{12EI\Delta}{l^2} + Pl = 0$$

$$\frac{6EI\theta_R}{l} - \frac{12EI}{l^2} \times \frac{Pl^3}{\beta EI} + Pl = 0$$

$$\frac{6EI\theta_R}{l} - \frac{12Pl}{\beta} + PL = 0 \text{ -----(i)}$$

From equilibrium of joint

$M_{RQ} + M_{RS} = 0$

$$\frac{8EIQ_R}{l} - \frac{6EI\Delta}{l^2} = 0$$

$$\frac{6EI\theta_R}{l} = \frac{6}{8}\left(\frac{6EI}{l^2} \times \frac{Pl^3}{\beta EI}\right)$$

$$\frac{6EI\theta_R}{l} = \frac{36}{8\beta}\frac{Pl}{} \text{ ----(ii)}$$

$\Rightarrow$ From (i) & (ii)

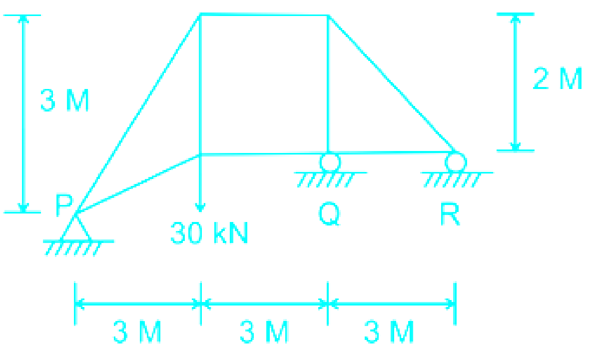
$$\frac{36Pl}{8\beta} - \frac{96Pl}{8\beta} Pl = 0$$

$$\Rightarrow \frac{60Pl}{8\beta} + Pl = 0$$

$$\Rightarrow 8\beta = 60$$

$$\beta = \frac{60}{8} = 7.5$$

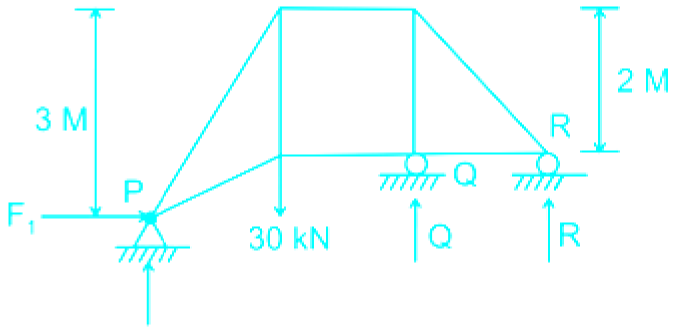
11. Consider the pin-jointed plane truss shown in the figure (not drawn to scale). Let RP, RQ, and RR denote the vertical reactions (upward positive) applied by the supports at P, Q, and R, respectively, on the truss. The correct combination of (R_P, R_Q, and R_R) is represented by



1. (30, - 30, 30) kN
2. (10, 30, - 10) kN
3. (20, 0, 10) kN
4. (0.60, -30) kN

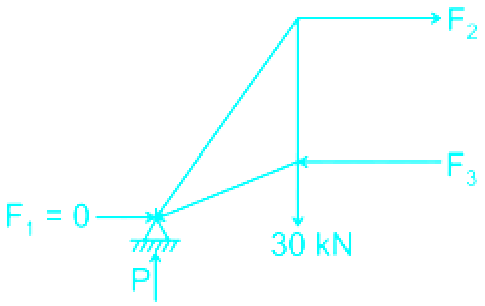
Ans. 1 : (30, - 30, 30) kN

Explanation:



$\sum FH=0$

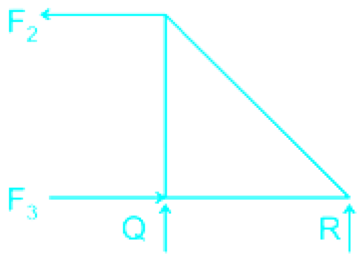
$F1=0.....(i)$



$\sum F_v = 0$

$P = 30 .....(ii)$

$\sum F_H = 0$



$F_2 = F_3$ ____(iii)

$\sum Mp = 0$

$F_2 \times 3 + 30 \times 3 - F_3 \times 1 = 0$

$F_2 \times 3 - F_3 \times 1 + 90 = 0$

$2F_2 = -90$

$\Rightarrow F_2 = -45$

$\sum MQ = 0$

$F_3 = 2 + R \times 3 = 0$

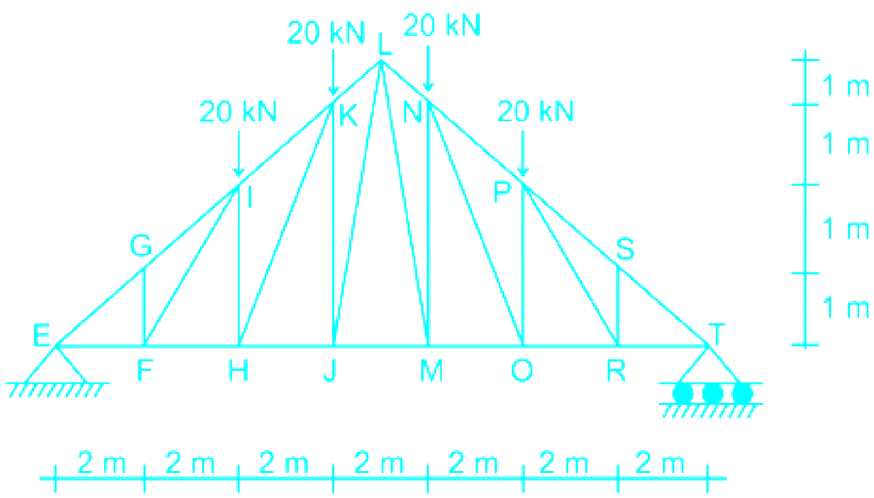
$-45 \times 2 + R \times 3 = 0$

$R = 30$

$Q + R = 0$

$\Rightarrow Q = -30$ kN

12. A plane truss is shown in the figure (not drawn to scale).



Which one of the options contains ONLY zero-force members in the truss?

1. FG, FI, HI, RS
2. FI, FG, RS, PR
3. FI, HI, PR, RS
4. FG, FH, HI, RS

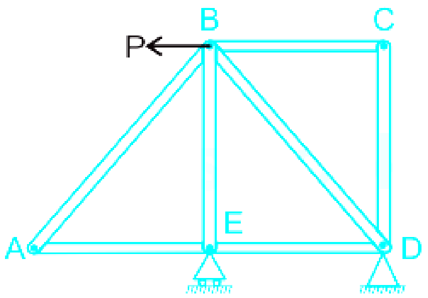
Ans. 2 : FI, FG, RS, PR

Concept:

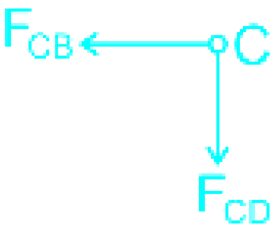
Case 1:

Consider the truss given the figure below.

The two members at joint C are connected together at a right angle and there is no external load on the joint.



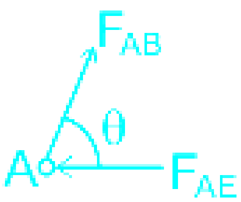
The free-body diagram of joint C, in the figure below, indicates that the force in each member must be zero in order to maintain equilibrium.



$\sum Fx = 0$, $FCB = 0$

$\sum Fy = 0$, $FCD = 0$

Furthermore, as in the case of joint A, in the figure below:



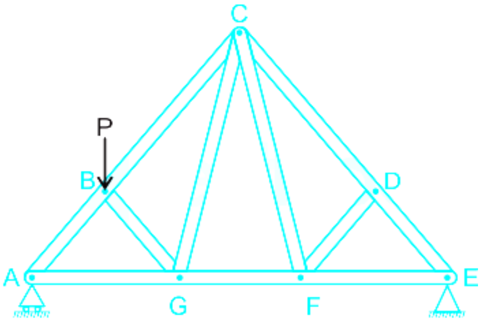
$\sum FY = 0$, $FAB \sin \theta = 0$, $F\ AB = 0$ kN

$\sum Fx = 0$, $- FAE + 0 = 0$, $FAE = 0$ kN

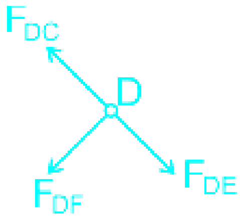
Case 2:

Consider the truss given the figure below.

Zero-force members also occur at joints having a geometry as joint D in the figure below.

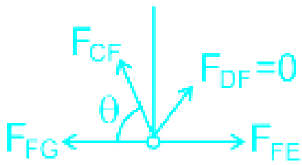


Here no external load acts on the joint, so that a force summation in the y-direction, shown in the figure below, which is perpendicular to the two collinear members, requires that FDF = 0.



$\sum Fy = 0$, $FDF = 0$

Using this result, FC is also a zero-force member, as indicated by the force analysis of joint F, in the figure below:

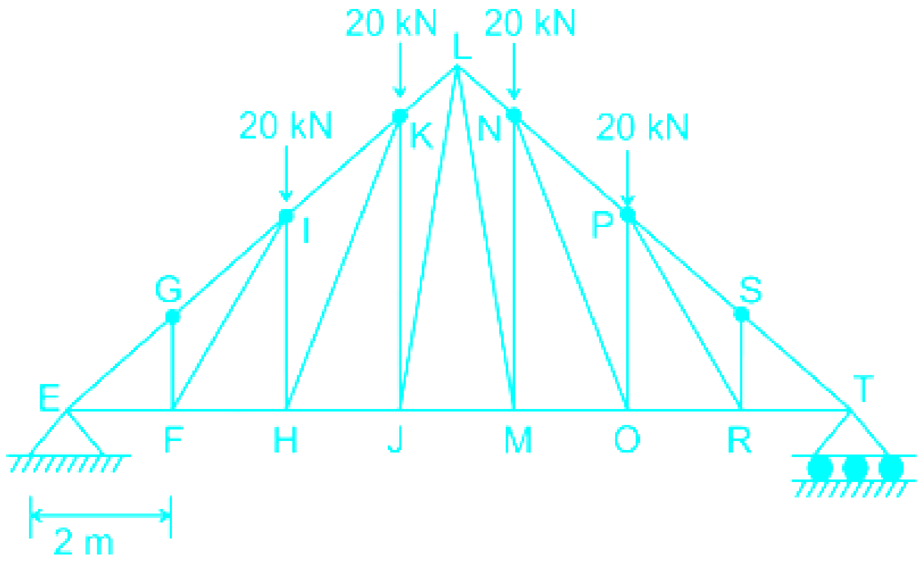


$\sum Fy = 0$, $FCF \sin \theta + 0 = 0$

$\therefore$ $FCF = 0$

Calculation:

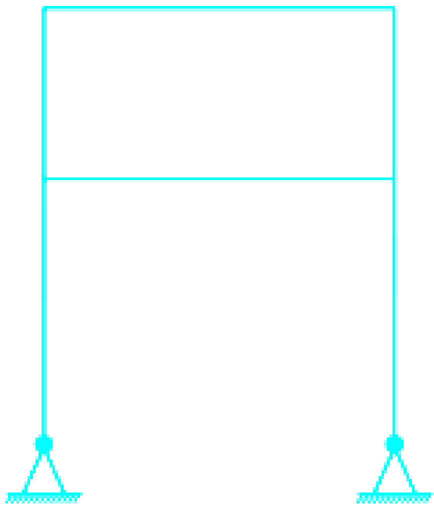
We know that, if three members are meeting at the joint, two of them are collinear and there is no point load acting on the joint, then the third member will carry zero force.



From the above statement,

We can say the GR, FI, RS, PR members will carry zero force

13. Consider the frame shown in figure.

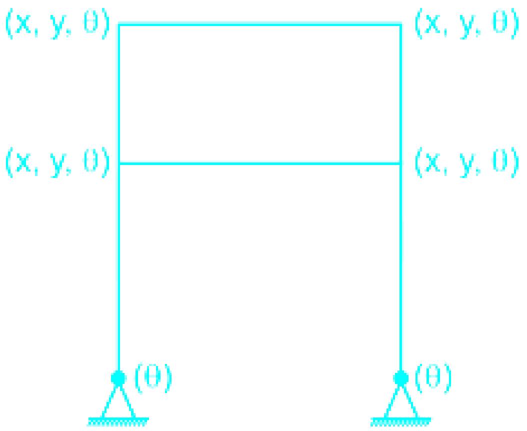


If the axial and shear deformations in different members of the frame are assumed to be negligible the reduction in the degree of kinematical indeterminacy would be equal to

1. 5
2. 6
3. 7
4. 8

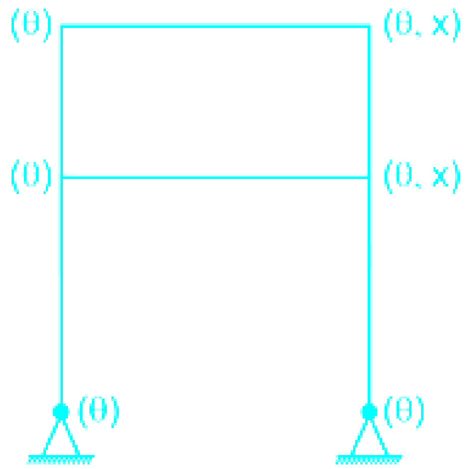
Ans. 2 : 6

Concept:



Total degree of freedom when axial and shear deformations in all members are allowed.

$D_k = 3 + 3 + 3 + 3 + 1 + 1 = 14$



When axial and shear deformations are negligible,

The degree of freedom equals to the number of members would be deducted i.e. 6

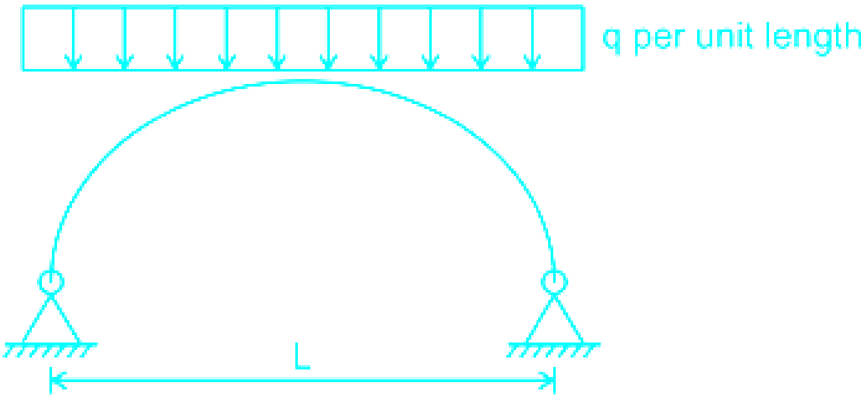
∴ Kinematic indeterminacy for the frame if axial and shear deformation is considered negligible, $(D_k) = 14 - 6 = 8$.

Hence, reduction in degree of kinematical indeterminacy = 14 – 8 = 6

Mistake Point:

Kindly do read the Question again as it is asked about the reduction in the degree of kinematical indeterminacy i.e 14 – 8 = 6

14. The figure shown a two-hinged parabolic arch of span L subjected to uniformly distributed load of intensity q per unit length



The maximum bending moment in the arch is equal to

1. $qL^2/8$
2. $qL^2/12$
3. Zero
4. $qL^2/10$

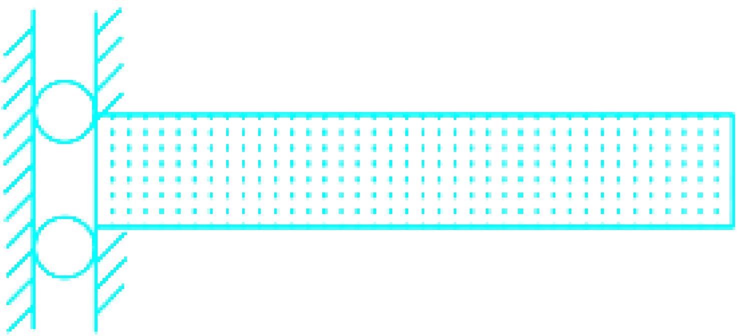
Ans. 3 : Zero

Concept:

If two-hinged parabolic is subjected to uniformly distributed load of intensity q per unit length.

The bending moment at anywhere in the arch is zero. So, the maximum bending moment in the arch is equal to zero.

15. A guided support as shown in the figure below is represented by three springs (horizontal, vertical and rotational) with stiffness k_x, k_y and k_θ respectively. The limiting values of kx, ky and k_θ are:



1. $\infty, 0, \infty$
2. ∞, ∞, ∞
3. $0, \infty, \infty$
4. $\infty, \infty, 0$

Ans. 1 : $\infty, 0, \infty$

Guided roller:

At guided roller support there are two reactions

1) **One horizontal reaction** (R_x)

2) **One moment reaction** (M_z)

The vertical guided roller has two reactions i.e it will have infinite stiffness in that direction

$\therefore K_x = \infty, K_\theta = \infty$

But the vertical guided roller support is free to move in vertical direction i.e it has zero stiffness in that direction

$\therefore K_y = 0$

Hence, $Kx = \infty$, $K\theta = \infty$ and $Ky = 0$