Chapter 5 Quadratic Functions

Ex 5.3

Answer 1e.

Only terms having the same variable parts, called as like terms, can be combined. When like terms are combined, we either add or subtract their coefficients.

Therefore, when you add or subtract polynomials, you add or subtract the coefficients of like terms.

Answer 1gp.

Clear the parentheses in the given expression.

$$\left(t^2-6t+2\right)+\left(5t^2-t-8\right) \Rightarrow t^2-6t+2+5t^2-t-8$$

Rewrite the expression such that the like terms appear together.

$$t^2-6t+2+5t^2-t-8 \Rightarrow t^2+5t^2-6t-t+2-8$$

Add the like terms.

$$t^2+5t^2-6t-t+2-8 = 6t^2-7t-6$$

Therefore, the sum is $6t^2-7t-6$.

Answer 1q.

Consider the expression:

$$3^5 \cdot 3^{-1} \qquad \text{...... (1)}$$

We need to evaluate the expression (1).

From the expression (1), we have

$$\begin{aligned} 3^5 \cdot 3^{-1} &= 3^{5+(-1)} && \left[\text{Using product of powers:} a^m \cdot a^n = a^{m+n}\right] \\ &= 3^4 && \left[\text{Adding the powers}\right] \\ &= 81 \end{aligned}$$

Therefore the expression $3^5 \cdot 3^{-1} = \boxed{81}$.

Answer 2e.

Let us consider two polynomials

$$P(x) \text{ and } Q(x)$$

Difference of these polynomials is $P(x)-Q(x)$.

Then,

$$P(x)-Q(x)=P(x)+[-Q(x)] \qquad \text{Rewrite the expression}$$

Here $-Q(x)$ is also a polynomial.

Hence, polynomial subtraction is equivalent to polynomial addition.

Answer 2gp.

Consider the expression

$$(8d-3+9d^3)-(d^3-13d^2-4)$$

Now, find the difference.

Align like terms, and then add the opposite of the subtracted polynomial.

$$\begin{array}{r} (8d-3+9d^3) \\ -(d^3-13d^2-4) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} (8d-3+9d^3) \\ +\ \ -d^3+13d^2+4 \\ \hline \end{array}$$

Thus,

$$\begin{array}{l} \qquad (9d^3+8d-3) \\ +\ \ -d^3+13d^2+4 \\ \hline 9d^3-d^3+13d^2+8d-3+4 \qquad \text{Combine like terms} \\ =8d^3+13d^2+8d+1 \end{array}$$

Therefore,

$$\boxed{(8d-3+9d^3)-(d^3-13d^2-4)=8d^3+13d^2+8d+1}$$

Answer 2q.

Consider the expression:

$$(2^4)^2 \qquad \ldots\ldots (1)$$

We need to evaluate the expression (1).

From the expression (1), we have

$$\left(2^4\right)^2 = 2^{4 \cdot 2} \quad \left[\text{Using power of a power property:} \left(a^m\right)^n = a^{m \cdot n}\right]$$

$$= 2^8 \quad [\text{Multiplying the powers}]$$

$$= 256$$

Therefore the expression $\left(2^4\right)^2 = \boxed{256}$.

Answer 3e.

Clear the parentheses in the given expression.

$$\left(3x^2 - 5\right) + \left(7x^2 - 3\right) \Rightarrow 3x^2 - 5 + 7x^2 - 3$$

Rewrite the expression such that the like terms appear together.

$$3x^2 - 5 + 7x^2 - 3 \Rightarrow 3x^2 + 7x^2 - 5 - 3$$

Add the like terms.

$$3x^2 + 7x^2 - 5 - 3 = 10x^2 - 8$$

Therefore, the sum is $10x^2 - 8$.

Answer 3gp.

In order to multiply two polynomials, multiply the first polynomial by each term in the second polynomial.

$$(x + 2)(3x^2 - x - 5) = (x + 2)3x^2 + (x + 2)(-x) + (x + 2)(-5)$$

Use the distributive property to remove the parentheses.

$$(x + 2)3x^2 + (x + 2)(-x) + (x + 2)(-5) = 3x^3 + 6x^2 - x^2 - 2x - 5x - 10$$

Combine the like terms.

$$3x^3 + 6x^2 - x^2 - 2x - 5x - 10 = 3x^3 + 5x^2 - 7x - 10$$

Therefore, the product is $3x^3 + 5x^2 - 7x - 10$.

Answer 3q.

Consider the expression:

$$\left(\frac{2}{3^{-2}}\right)^2 \qquad \text{...... (1)}$$

We need to evaluate the expression (1).

From the expression (1), we have

$$\left(\frac{2}{3^{-2}}\right)^2 = \frac{2^2}{\left(3^{-2}\right)^2} \quad \left[\text{Using power of quotient property: } \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$$

$$= \frac{2^2}{3^{(-2)\cdot 2}} \quad \left[\text{Using power of a power property: } \left(a^m\right)^n = a^{m\cdot n}\right]$$

$$= \frac{2^2}{3^{-4}} \quad \left[\text{Multiplying the powers of the denominator}\right]$$

$$= 2^2 \times \frac{1}{3^4} \quad \left[\text{Using negative exponent property: } a^{-m} = \frac{1}{a^m}\right]$$

$$= \frac{2^2}{3^4} \quad \left[\text{Multiplying the two terms}\right]$$

$$= \frac{4}{81} \quad \left[\text{Dividing 4 by 81}\right]$$

$$= 0.049$$

Therefore the expression $\left(\frac{2}{3^{-2}}\right)^2 = \boxed{0.049}$.

Answer 4e.

Consider the expression

$$\left(x^2 - 3x + 5\right) - \left(-4x^2 + 8x + 9\right)$$

Now, find the difference of $\left(x^2 - 3x + 5\right) - \left(-4x^2 + 8x + 9\right)$

Thus,

$$\left(x^2 - 3x + 5\right) - \left(-4x^2 + 8x + 9\right) = x^2 - 3x + 5 + 4x^2 - 8x - 9$$

$$= \left(x^2 + 4x^2\right) + \left(-3x - 8x\right) + \left(5 - 9\right) \quad \text{Group like terms}$$

$$= 5x^2 - 11x - 4 \quad \text{Combine like terms}$$

Therefore,

$$\left(x^2 - 3x + 5\right) - \left(-4x^2 + 8x + 9\right) \boxed{= 5x^2 - 11x - 4}$$

Answer 4gp.

Consider the expression

$$(a-5)(a+2)(a+6)$$

Now, find the product of $(a-5)(a+2)(a+6)$.

Thus,

$$\begin{aligned}(a-5)(a+2)(a+6) &= (a-5)(a^2+6a+2a+12) && \text{Use distributive property}\\ &= (a-5)(a^2+8a+12) && \text{Group like terms}\\ &= a(a^2+8a+12)-5(a^2+8a+12) && \text{Use distributive property}\\ &= a^3+8a^2+12a-5a^2-40a-60 &&\\ &= a^3+3a^2-28a-60 && \text{Group like terms}\end{aligned}$$

Therefore, $\boxed{(a-5)(a+2)(a+6)=a^3+3a^2-28a-60}$

Answer 4q.

Consider the expression:

$$\left(\frac{2}{5}\right)^{-2} \quad \text{...... (1)}$$

We need to evaluate the expression (1).

From the expression (1), we have

$$\begin{aligned}\left(\frac{2}{5}\right)^{-2} &= \frac{2^{-2}}{5^{-2}} && \left[\text{Using power of quotient property:} \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0\right]\\ &= \frac{\frac{1}{2^2}}{\frac{1}{5^2}} && \left[\text{Using negative exponent property:} a^{-m} = \frac{1}{a^m}, a \neq 0\right]\\ &= \frac{1}{2^2}\cdot\frac{5^2}{1} && \left[\text{Using: } \frac{1}{a^m} \div \frac{1}{b^m} = \frac{1}{a^m}\cdot\frac{b^m}{1}\right]\\ &= \frac{25}{4} && [\text{Dividing 25 by 4}]\\ &= 6.25\end{aligned}$$

Therefore the expression: $\left(\frac{2}{5}\right)^{-2} = \boxed{6.25}$.

Answer 5e.

In subtracting two polynomials, we add the opposite of the polynomial that is being subtracted.

$$(4y^2 + 9y - 5) - (4y^2 - 5y + 3) \Rightarrow (4y^2 + 9y - 5) + (-4y^2 + 5y - 3)$$

Clear the parentheses in the expression

$$(4y^2 + 9y - 5) + (-4y^2 + 5y - 3) \Rightarrow 4y^2 + 9y - 5 - 4y^2 + 5y - 3$$

Rewrite the expression such that the like terms appear together.

$$4y^2 + 9y - 5 - 4y^2 + 5y - 3 \Rightarrow 4y^2 - 4y^2 + 9y + 5y - 5 - 3$$

Add the like terms.

$$4y^2 - 4y^2 + 9y + 5y - 5 - 3 = 14y - 8$$

Therefore, the difference is $14y - 8$.

Answer 5gp.

The expression $(xy - 4)^3$ is of the form $(a - b)^3$.

The cube of a binomial, $a - b$, can be expanded as $a^3 - 3a^2b + 3ab^2 + b^3$.

We have a as xy, and b as 4.

$$(xy - 4)^3 = (xy)^3 - 3(xy)^2(-4) + 3(xy)(-4)^2 + (-4)^3$$

Simplify.

$$(xy)^3 - 3(xy)^2(-4) + 3(xy)(-4)^2 + (-4)^3 = x^3y^3 + 12x^2y^2 - 48xy - 64$$

Therefore, the product is $x^3y^3 + 12x^2y^2 - 48xy - 64$.

Answer 5q.

Consider the expression:

$$(x^4y^{-2})(x^{-3}y^8) \qquad \text{......(1)}$$

We need to simplify the expression (1).

From the expression (1), we have

$$\left(x^4y^{-2}\right)\left(x^{-3}y^8\right)=\left(x^4\cdot\frac{1}{y^2}\right)\left(\frac{1}{x^3}\cdot y^8\right)$$ [Using negative exponent property: $a^{-m}=\frac{1}{a^m}, a\neq 0$]

$$=\left(\frac{x^4}{y^2}\right)\left(\frac{y^8}{x^3}\right)$$ [Multiplying: $\left(a^m\cdot\frac{1}{b^n}\right)\left(\frac{1}{a^p}\cdot b^q\cdot\right)=\left(\frac{a^m}{b^n}\right)\left(\frac{b^q}{a^p}\right)$]

$$=x^{4-3}\cdot y^{8-2}$$ [Using quotient of powers property: $\frac{a^m}{a^n}=a^{m-n}, a\neq 0$]

$$=xy^6$$ [Substracting the powers]

Therefore the simplest form of the expression: $\left(x^4y^{-2}\right)\left(x^{-3}y^8\right)=\boxed{xy^6}$.

Answer 6e.

Consider the expression

$$\left(z^2+5z-7\right)+\left(5z^2-11z-6\right)$$

Now, find the sum of $\left(z^2+5z-7\right)+\left(5z^2-11z-6\right)$.

Thus,

$$\left(z^2+5z-7\right)+\left(5z^2-11z-6\right)=z^2+5z-7+5z^2-11z-6$$

$$=\left(z^2+5z^2\right)+\left(5z-11z\right)+\left(-7-6\right)$$ Group like terms

$$=6z^2-6z-13$$ Combine like terms

Therefore, $\left(z^2+5z-7\right)+\left(5z^2-11z-6\right)=\boxed{6z^2-6z-13}$

Answer 6gp.

Consider the data

The model represents the average depth D (in feet) of new wells drilled and the average cost per foot C (in dollars) of drilling a new well.

$$D=109t+4010 \text{ and } C=0.542t^2-7.16t+79.4$$

Where in both models, t represents the number of years since 1980.

Now, write a model for the average total cost T of drilling a new well.

Clearly, $T=DC$

$$T=\left(109t+4010\right)\left(0.542t^2-7.16t+79.4\right)$$ Substitute C, D

Thus,

$$\begin{array}{ll} (0.542t^2 - 7.16t + 79.4) & \\ \times \qquad (109t + 4010) & \\ \hline 59.078t^3 - 780.44t^2 + 8654.6t & \text{Multiply } 0.542t^2 - 7.16t + 79.4 \text{ by } 109t \\ +2173.42t^2 - 28711.6t + 318394 & \text{Multiply } 0.542t^2 - 7.16t + 79.4 \text{ by } 4010 \\ \hline 59.078t^3 + 1392.98t^2 - 20057t + 318394 & \text{Combine like terms} \end{array}$$

Therefore, total cost is

$$\boxed{T = 59.078t^3 + 1392.98t^2 - 20057t + 318394}$$

Answer 6q.

Consider the expression:

$$\left(a^2b^{-5}\right)^{-3} \quad \ldots\ldots (1)$$

We need to simplify the expression (1).

From the expression (1), we have

$$\left(a^2b^{-5}\right)^{-3} = \left(a^2\right)^{-3}\left(b^{-5}\right)^{-3} \quad \left[\begin{array}{l}\text{Using power of a product property:} \\ (ab)^m = a^m b^m.\end{array}\right]$$

$$= a^{2\cdot(-3)}b^{(-5)\cdot(-3)} \quad \left[\begin{array}{l}\text{Using power of a power property:} \\ \left(a^m\right)^n = a^{m\cdot n}\end{array}\right]$$

$$= a^{-6}b^{15} \quad [\text{Multiplying the powers}]$$

Therefore the simplest form of the expression: $\left(a^2b^{-5}\right)^{-3} = \boxed{a^{-6}b^{15}}$.

Answer 7e.

Clear the parentheses in the given expression.

$$\left(3s^3 + s\right) + \left(4s^3 - 2s^2 + 7s + 10\right) \Rightarrow 3s^3 + s + 4s^3 - 2s^2 + 7s + 10$$

Rewrite the expression such that the like terms appear together.

$$3s^3 + s + 4s^3 - 2s^2 + 7s + 10 \Rightarrow 3s^3 + 4s^3 - 2s^2 + s + 7s + 10$$

Add the like terms.

$$3s^3 + 4s^3 - 2s^2 + s + 7s + 10 = 7s^3 - 2s^2 + 8s + 10$$

Therefore, the sum is $7s^3 - 2s^2 + 8s + 10$.

Answer 7q.

Consider the expression:

$$\frac{x^3y^7}{x^{-4}y^0} \qquad \ldots\ldots (1)$$

We need to simplify the expression (1).

From the expression (1), we have

$$\frac{x^3y^7}{x^{-4}y^0} = \frac{x^3y^7}{x^{-4}1} \qquad \left[\begin{array}{l}\text{Using zero exponent property:}\\ a^0 = 1, a \neq 0.\end{array}\right]$$

$$= \frac{x^3y^7}{\frac{1}{x^4}} \qquad \left[\begin{array}{l}\text{Using negative exponent property:}\\ a^{-m} = \frac{1}{a^{-m}}, a \neq 0.\end{array}\right]$$

$$= x^3y^7 \cdot x^4$$

$$= x^{4+3}y^7 \qquad \left[\begin{array}{l}\text{Using product of powers property:}\\ a^m \cdot a^n = a^{m+n}.\end{array}\right]$$

$$= x^7y^7 \qquad [\text{Adding the powers of } x]$$

$$= (xy)^7 \qquad \left[\begin{array}{l}\text{Using power of a product property:}\\ (ab)^m = a^m b^m.\end{array}\right]$$

Therefore the simplest form of the expression: $\frac{x^3y^7}{x^{-4}y^0} = \boxed{(xy)^7}$.

Answer 8e.

Consider the expression

$$(2a^2 - 8) - (a^3 + 4a^2 - 12a + 4)$$

Now, find the difference $(2a^2 - 8) - (a^3 + 4a^2 - 12a + 4)$

Thus,

$$\begin{aligned}(2a^2 - 8) - (a^3 + 4a^2 - 12a + 4) &= 2a^2 - 8 - a^3 - 4a^2 + 12a - 4 \\ &= (2a^2 - 4a^2) + (-8 - 4) - a^3 + 12a && \text{Group like terms} \\ &= -2a^2 - 12 - a^3 + 12a && \text{Combine like terms} \\ &= -a^3 - 2a^2 + 12a - 12 && \text{Rearrange the terms}\end{aligned}$$

Therefore,

$$(2a^2 - 8) - (a^3 + 4a^2 - 12a + 4) = \boxed{-a^3 - 2a^2 + 12a - 12}$$

Answer 8q.

Consider the expression:

$$\frac{c^3d^{-2}}{c^5d^{-1}} \quad \ldots\ldots (1)$$

We need to simplify the expression (1).

From the expression (1), we have

$$\frac{c^3d^{-2}}{c^5d^{-1}} = \frac{c^3\frac{1}{d^2}}{c^5\frac{1}{d^1}} \qquad \left[\text{Using negative exponent property: } a^{-m} = \frac{1}{a^{-m}}, a \neq 0.\right]$$

$$= \frac{\frac{c^3}{d^2}}{\frac{c^5}{d^1}} \qquad \left[\text{Multiplying: } a^m \frac{1}{b^n} = \frac{a^m}{b^n}.\right]$$

$$= \frac{c^3}{d^2}\cdot\frac{d^1}{c^5} \qquad \left[\text{Multiplying: } \left(\frac{a^m}{b^n}\right) \div \left(\frac{a^p}{b^q}\right) = \left(\frac{a^m}{b^n}\right)\cdot\left(\frac{b^q}{a^p}\right)\right]$$

$$= c^{3-5}d^{1-2} \qquad \left[\text{Using quotient of powers property: } \frac{a^m}{a^n} = a^{m-n}, a \neq 0\right]$$

$$= c^{-2}d^{-1} \qquad [\text{Substracting the powers}]$$

$$= \frac{1}{c^2}\frac{1}{d^1} \qquad \left[\text{Using negative exponent property: } a^{-m} = \frac{1}{a^{-m}}, a \neq 0.\right]$$

$$= \frac{1}{c^2d}$$

Therefore the simplest form of the expression: $\frac{c^3d^{-2}}{c^5d^{-1}} = \boxed{\frac{1}{c^2d}}$.

Answer 9e.

Clear the parentheses in the given expression.

$$\left(5c^2 + 7c + 1\right) + \left(2c^3 - 6c + 8\right) \Rightarrow 5c^2 + 7c + 1 + 2c^3 - 6c + 8$$

Rewrite the expression such that the like terms appear together.

$5c^2 + 7c + 1 + 2c^3 - 6c + 8 \Rightarrow 5c^2 + 2c^3 + 7c - 6c + 1 + 8$

Add the like terms.

$$5c^2 + 2c^3 + 7c - 6c + 1 + 8 = 5c^2 + 2c^3 + c + 9$$
$$\Rightarrow 2c^3 + 5c^2 + c + 9$$

Therefore, the sum is $2c^3 + 5c^2 + c + 9$.

Answer 9q.

Consider the polynomial function:

$$g(x) = 2x^3 - 3x + 1 \quad \text{...... (1)}$$

We need to graph the polynomial function (1).
The standard form of a cubic polynomial is:

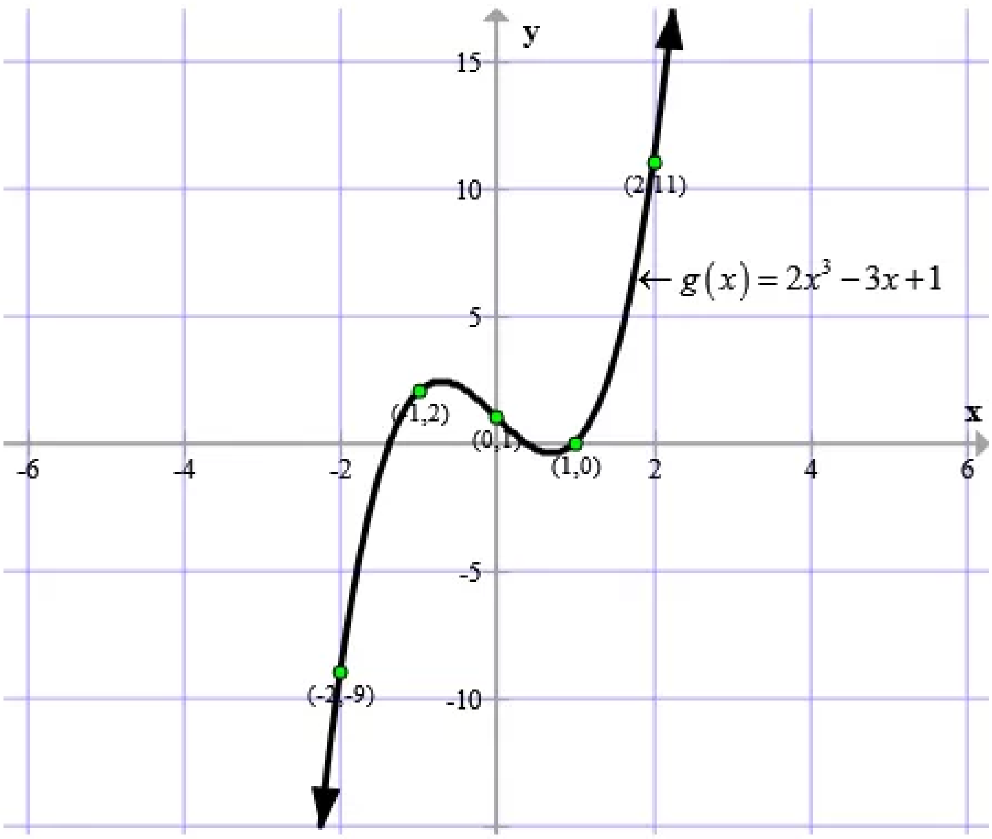
$$a_3x^3 + a_2x^2 + a_1x + a_0$$

where degree is 3, leading coefficient is a_3 and the constant term is a_0.

From (1), we have
The degree of the function is 3 and leading coefficient is 2.
To graph the function (1), we make a table of values and plot the corresponding points.
And then connect the points with a smooth curve.

x	-2	-1	0	1	2
y	-9	2	1	0	11

The graph of the polynomial function $g(x) = 2x^3 - 3x + 1$ is shown below:



Answer 10e.

Consider the expression

$$\left(4t^3-11t^2+4t\right)-\left(-7t^2-5t+8\right)$$

Now, find the difference of $\left(4t^3-11t^2+4t\right)-\left(-7t^2-5t+8\right)$

Thus,

$$\left(4t^3-11t^2+4t\right)-\left(-7t^2-5t+8\right)=4t^3-11t^2+4t+7t^2+5t-8$$

$$=4t^3+\left(-11t^2+7t^2\right)+\left(4t+5t\right)-8 \quad \text{Group like terms}$$

$$=4t^3-4t^2+9t-8 \quad \text{Combine like terms}$$

Therefore,

$$\left(4t^3-11t^2+4t\right)-\left(-7t^2-5t+8\right)=\boxed{4t^3-4t^2+9t-8}$$

Answer 10q.

Consider the polynomial function:

$$h(x)=x^4-4x+2 \qquad \ldots\ldots (1)$$

We need to graph the polynomial function (1).
The standard form of a quartic polynomial is:

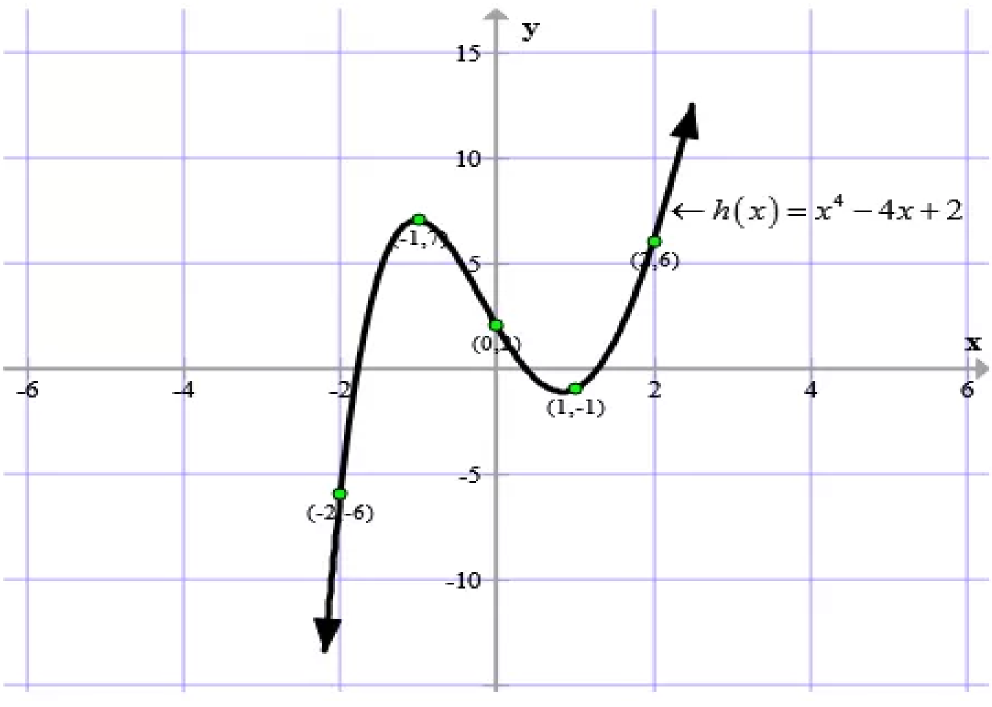
$$a_4x^4+a_3x^3+a_2x^2+a_1x+a_0$$

where degree is 4, leading coefficient is a_4 and the constant term is a_0.

From (1), we have
The degree of the function is 4 and leading coefficient is 1.
To graph the function (1), we make a table of values and plot the corresponding points.
And then connect the points with a smooth curve.

x	-2	-1	0	1	2
y	-6	7	2	-1	6

The graph of the polynomial function $h(x)=x^4-4x+2$ is shown below:



Answer 11e.

In subtracting two polynomials, we add the opposite of the polynomial that is being subtracted.

$$\left(5b-6b^3+2b^4\right)-\left(9b^3+4b^4-7\right) \Rightarrow \left(5b-6b^3+2b^4\right)+\left(-9b^3-4b^4+7\right)$$

Clear the parentheses in the expression

$$\left(5b-6b^3+2b^4\right)+\left(-9b^3-4b^4+7\right) \Rightarrow 5b-6b^3+2b^4-9b^3-4b^4+7$$

Rewrite the expression such that the like terms appear together.

$$5b-6b^3+2b^4-9b^3-4b^4+7 \Rightarrow 5b-6b^3-9b^3+2b^4-4b^4+7$$

Add the like terms.

$$5b-6b^3-9b^3+2b^4-4b^4+7 \Rightarrow 5b-15b^3-2b^4+7$$
$$\Rightarrow -2b^4-15b^3+5b+7$$

Therefore, the difference is $-2b^4-15b^3+5b+7$.

Answer 11q.

Consider the polynomial function:

$$f(x)=-2x^3+x^2-5 \qquad \text{...... (1)}$$

We need to graph the polynomial function (1).
The standard form of a cubic polynomial is:

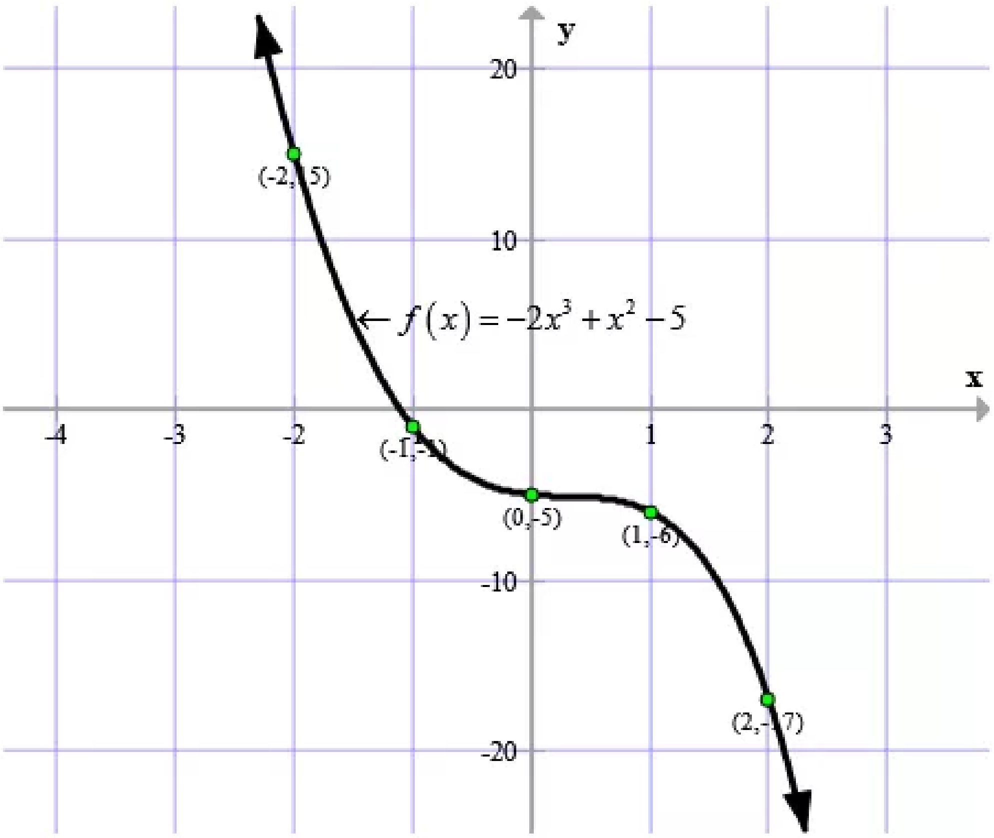
$$a_3x^3+a_2x^2+a_1x+a_0$$

where degree is 3, leading coefficient is a_3 and the constant term is a_0.

From (1), we have
The degree of the function is 3 and leading coefficient is -2.
To graph the function (1), we make a table of values and plot the corresponding points.
And then connect the points with a smooth curve.

x	-2	-1	0	1	2
y	15	-1	-5	-6	-17

The graph of the polynomial function $f(x)=-2x^3+x^2-5$ is shown below:



Answer 12e.

Consider the expression

$$(3y^2-6y^4+5-6y)+(5y^4-6y^3+4y)$$

Now, find the sum $(3y^2-6y^4+5-6y)+(5y^4-6y^3+4y)$.

Thus,

$$\begin{aligned}&(3y^2-6y^4+5-6y)+(5y^4-6y^3+4y)\\&=3y^2-6y^4+5-6y+5y^4-6y^3+4y\\&=3y^2+(-6y^4+5y^4)+5+(-6y+4y)-6y^3 &&\text{Group like terms}\\&=3y^2-y^4+5-2y-6y^3 &&\text{Combine like terms}\\&=-y^4-6y^3+3y^2-2y+5 &&\text{Rearrange the terms}\end{aligned}$$

Therefore,

$$(3y^2-6y^4+5-6y)+(5y^4-6y^3+4y)=\boxed{-y^4-6y^3+3y^2-2y+5}$$

Answer 12q.

Consider:

$$(x^3+x^2-6)-(2x^2+4x-8) \quad \text{..... (1)}$$

We need to perform the indicated operation.

First we write the opposite of the subtracted polynomial, and then adding the like terms.
Now from (1), we have

$$\begin{aligned}(x^3+x^2-6)-(2x^2+4x-8)&=x^3+x^2-6-2x^2-4x+8\\&=x^3-x^2-4x+2\end{aligned}$$

Therefore the solution of $(x^3+x^2-6)-(2x^2+4x-8)$ is: $\boxed{x^3-x^2-4x+2}$.

Answer 13e.

Clear the parentheses in the given expression.

$$\left(x^4 - x^3 + x^2 - x + 1\right) + \left(x + x^4 - 1 - x^2\right) \Rightarrow x^4 - x^3 + x^2 - x + 1 + x + x^4 - 1 - x^2$$

Rewrite the expression such that the like terms appear together.

$$x^4 - x^3 + x^2 - x + 1 + x + x^4 - 1 - x^2 \Rightarrow x^4 + x^4 - x^3 + x^2 - x^2 - x + x + 1 - 1$$

Add the like terms.

$$x^4 + x^4 - x^3 + x^2 - x^2 - x + x + 1 - 1 = 2x^4 - x^3$$

Therefore, the sum is $2x^4 - x^3$.

Answer 13q.

Consider:

$$\left(-3x^2 + 4x - 10\right) + \left(x^2 - 9x + 15\right) \qquad \ldots\ldots (1)$$

We need to perform the indicated operation.

From (1), we have

$$\begin{aligned}\left(-3x^2 + 4x - 10\right) + \left(x^2 - 9x + 15\right) &= -3x^2 + 4x - 10 + x^2 - 9x + 15 \\ &= -2x^2 - 5x + 5\end{aligned}$$

Therefore the solution of $\left(-3x^2 + 4x - 10\right) + \left(x^2 - 9x + 15\right)$ is: $\boxed{-2x^2 - 5x + 5}$.

Answer 14e.

Consider the expression

$$\left(8v^4 - 2v^2 + v - 4\right) - \left(3v^3 - 12v^2 + 8v\right)$$

Now, find the difference $\left(8v^4 - 2v^2 + v - 4\right) - \left(3v^3 - 12v^2 + 8v\right)$

Thus,

$$\begin{aligned}&\left(8v^4 - 2v^2 + v - 4\right) - \left(3v^3 - 12v^2 + 8v\right) \\ &= 8v^4 - 2v^2 + v - 4 - 3v^3 + 12v^2 - 8v \\ &= 8v^4 + \left(-2v^2 + 12v^2\right) + \left(v - 8v\right) - 4 - 3v^3 && \text{Group like terms} \\ &= 8v^4 + 10v^2 - 7v - 4 - 3v^3 && \text{Combine like terms} \\ &= 8v^4 - 3v^3 + 10v^2 - 7v - 4 && \text{Rearrange the terms}\end{aligned}$$

Therefore,

$$\left(8v^4 - 2v^2 + v - 4\right) - \left(3v^3 - 12v^2 + 8v\right) = \boxed{8v^4 - 3v^3 + 10v^2 - 7v - 4}$$

Answer 14q.

Consider:

$$(x-5)(x^2-5x+7) \qquad \text{..... (1)}$$

We need to perform the indicated operation.

From (1), we have

$$(x-5)(x^2-5x+7) = (x-5)x^2-(x-5)5x+(x-5)7$$
$$= x^3-5x^2-5x^2+25x+7x-35$$
$$= x^3-10x^2+32x-35$$

Therefore the solution of $(x-5)(x^2-5x+7)$ is: $\boxed{x^3-10x^2+32x-35}$.

Answer 15e.

We seek the difference $(8x^4-4x^3-x+2)-(2x^4-8x^2-x+10)$.

In subtracting two polynomials, we add the opposite of the polynomial that is being subtracted.

$$(8x^4-4x^3-x+2)-(2x^4-8x^2-x+10) \Rightarrow (8x^4-4x^3-x+2)+(-2x^4+8x^2+x-10)$$

Clear the parentheses in the expression.

$$(8x^4-4x^3-x+2)+(-2x^4+8x^2+x-10) \Rightarrow 8x^4-4x^3-x+2-2x^4+8x^2+x-10$$

Rewrite the expression such that the like terms appear together.

$$8x^4-4x^3-x+2-2x^4+8x^2+x-10 \Rightarrow 8x^4-2x^4-4x^3+8x^2+x-x-10+2$$

Add the like terms.

$$8x^4-2x^4-4x^3+8x^2+x-x-10+2 = 6x^4-4x^3+8x^2-8$$

This expression matches with the result given in choice **B**.

Answer 15q.

Consider:

$$(x+3)(x-6)(3x-1) \qquad \text{..... (1)}$$

We need to perform the indicated operation.

From (1), we have

$$(x+3)(x-6)(3x-1) = (x^2-6x+3x-18)(3x-1) \qquad \begin{bmatrix}\text{Multiplying 1st}\\ \text{and 2nd terms}\end{bmatrix}$$
$$= (x^2-3x-18)(3x-1)$$
$$= (x^2-3x-18)3x-(x^2-3x-18)1$$
$$= 3x^3-9x^2-54x-x^2+3x+18$$
$$= 3x^3-10x^2-51x+18$$

Therefore the solution of $(x+3)(x-6)(3x-1)$ is: $\boxed{3x^3-10x^2-51x+18}$.

Answer 16e.

Consider the expression,

$$x\left(2x^2-5x+7\right)$$

Find the product of the expression,

$$x\left(2x^2-5x+7\right)$$

Distributive Property:

$$a(b+c)=ab+ac$$

Now,

$$x\left(2x^2-5x+7\right)=x\cdot 2x^2+x(-5x)+x\cdot 7 \quad \text{Apply the distributive property}$$

$$=2x^3-5x^2+7x \quad \text{Simplify}$$

$$x\left(2x^2-5x+7\right)=2x^3-5x^2+7x$$

Therefore, $x\left(2x^2-5x+7\right)=\boxed{2x^3-5x^2+7x}$

Answer 16q.

Suppose on July $21,2004$ the national debt of the United State was about $\$7,282,000,000,000$. The population of the united states at that time was about $294,000,000$. Suppose the national debt was divided evenly among everyone in the United States. We need to find how much would each person owe.

Number of owe person $=$ national debt $\times$ population

$$=7,282,000,000,000\times 294,000,000 \quad \begin{bmatrix}\text{Substitute}\\ \text{values}\end{bmatrix}$$

$$=\left(728.2\times 10^{10}\right)\left(29.4\times 10^{7}\right) \quad \begin{bmatrix}\text{Write in}\\ \text{scientific}\\ \text{notation}\end{bmatrix}$$

$$=(728.2\times 29.4)\left(10^{10}\times 10^{7}\right) \quad \begin{bmatrix}\text{Use multiplication}\\ \text{property}\end{bmatrix}$$

$$=21414.96\times 10^{17} \quad \begin{bmatrix}\text{Product of powers}\\ \text{property}\end{bmatrix}$$

$$=2.14\times 10^{21} \quad \begin{bmatrix}\text{wrtten 21414.96 in}\\ \text{scientific notation}\end{bmatrix}$$

Therefore the number of each person owe is about: $\boxed{2.14\times 10^{21}}$ or $\boxed{21,400,000,000,000,000,000}$.

Answer 17e.

Use the distributive property to multiply $5x^2$ by $6x+2$.

$5x^2(6x+2) = 5x^2 \cdot 6x + 5x^2 \cdot 2$

Simplify.

$5x^2 \cdot 6x + 5x^2 \cdot 2 = 30x^3 + 10x^2$

Therefore, the product is $30x^3 + 10x^2$.

Answer 18e.

Consider the polynomial,

$$(y-7)(y+6)$$

Find the product of the polynomial.

$$(y-7)(y+6)$$

Distributive Property:

$$a(b+c) = ab + ac$$

Now,

$$\begin{aligned}(y-7)(y+6) &= (y \cdot y + y \cdot 6) - (7 \cdot y + 7 \cdot 6) && \text{Apply the Distributive property}\\ &= (y^2+6y) - (7y+42) && \text{Simplify}\\ &= y^2 + 6y - 7y - 42 \\ &= y^2 - y - 42 && \text{Combine like terms}\end{aligned}$$

Answer 19e.

In order to multiply two polynomials, multiply the first polynomial by each term in the second polynomial.

$(3z+1)(z-3) = (3z+1)z + (3z+1)(-3)$

Use the distributive property to remove the parentheses.

$(3z+1)z + (3z+1)(-3) = 3z^2 + z - 9z - 3$

Combine the like terms.

$3z^2 + z - 9z - 3 = 3z^2 - 8z - 3$

Therefore, the product is $3z^2 - 8z - 3$.

Answer 20e.

Consider the polynomial,

$$(w+4)(w^2+6w-11)$$

Find the product of the following polynomial,

$$(w+4)(w^2+6w-11)$$

Distributive Property:

$$a(b+c)=ab+ac$$

Now,

$$(w+4)(w^2+6w-11)=(w\cdot w^2+w\cdot 6w+w(-11))+4\cdot w^2+4\cdot 6w+4(-11)$$

Apply the distributive property

$$=w^3+6w^2-11w+4w^2+24w-44 \quad \text{Simplify}$$

$$=w^3+10w^2+13w-44 \quad \text{Combine like terms}$$

Answer 21e.

In order to multiply two polynomials, multiply the first polynomial by each term in the second polynomial.

$$(2a-3)(a^2-10a-2) = (2a-3)a^2+(2a-3)(-10a)+(2a-3)(-2)$$

Use the distributive property to remove the parentheses.

$$(2a-3)a^2+(2a-3)(-10a)+(2a-3)(-2) = 2a^3-3a^2-20a^2+30a-4a+6$$

Combine the like terms.

$$2a^3-3a^2-20a^2+30a-4a+6 = 2a^3-23a^2+26a+6$$

Therefore, the product is $2a^3-23a^2+26a+6$.

Answer 22e.

Consider the expression,

$$(5c^2-4)(2c^2+c-3)$$

Find the product of the following expression,

$$(5c^2-4)(2c^2+c-3)$$

Distributive Property:

$$a(b+c)=ab+ac$$

Now,

$$(5c^2-4)(2c^2+c-3)$$

$$=(5c^2\cdot 2c^2+5c^2\cdot c+5c^2(-3))-(4\cdot 2c^2+4\cdot c+4(-3))$$

Apply Distributive property

$$=10c^4+5c^3-15c^2-8c^2-4c+12 \quad \text{Simplify}$$

$$=10c^4+5c^3-23c^2-4c+12 \quad \text{Combine like terms}$$

Therefore, $\boxed{(5c^2-4)(2c^2+c-3)=10c^4+5c^3-23c^2-4c+12}$

Answer 23e.

In order to multiply two polynomials, multiply the first polynomial by each term in the second polynomial.

$(-x^2 + 4x + 1)x^2 + (-x^2 + 4x + 1)(-8x) + (-x^2 + 4x + 1)(3)$

Use the distributive property to remove the parentheses.

$-x^4 + 4x^3 + x^2 + 8x^3 - 32x^2 - 8x - 3x^2 + 12x + 3$

Combine the like terms.

$-x^4 + 12x^3 - 34x^2 + 4x + 3$

Therefore, the product is $-x^4 + 12x^3 - 34x^2 + 4x + 3$.

Answer 24e.

Consider the expression,

$$\left(-d^2+4d+3\right)\left(3d^2-7d+6\right)$$

Find the product of the following polynomial,

$$\left(-d^2+4d+3\right)\left(3d^2-7d+6\right)$$

Distributive Property:

$$a(b+c)=ab+ac$$

Now,

$$\left(-d^2+4d+3\right)\left(3d^2-7d+6\right)$$

$$=-\left(d^2\cdot 3d^2+d^2(-7d)+d^2\cdot 6\right)+\left(4d\cdot 3d^2+4d(-7d)+4d\cdot 6\right)+\left(3\cdot 3d^2+3(-7d)+3\cdot 6\right)$$

Apply the Distributive property

$$=-3d^4+7d^3-6d^2+12d^3-28d^2+24d+9d^2-21d+18 \quad \text{Simplify}$$

$$=-3d^4+19d^3-25d^2+3d+18 \quad \text{Combine like terms}$$

Answer 25e.

In order to multiply two polynomials, multiply the first polynomial by each term in the second polynomial.

$(3y^2 + 6y - 1)4y^2 + (3y^2 + 6y - 1)(-11y) + (3y^2 + 6y - 1)(-5)$

Use the distributive property to remove the parentheses.

$12y^4 + 24y^3 - 4y^2 - 33y^3 - 66y^2 + 11y - 15y^2 - 30y + 5$

Combine the like terms.

$12y^4 - 9y^3 - 854y^2 - 19y + 5$

Therefore, the product is $12y^4 - 9y^3 - 854y^2 - 19y + 5$.

Answer 26e.

Consider the expression,

$$(x^2-3x+4)-(x^3+7x-2)$$
$$=x^2-3x+4-x^3+7x-2$$

The error is, there is no change of sign for the term $7x-2$ which is a part of second polynomial.

Find the difference of the following expression,

$$(x^2-3x+4)-(x^3+7x-2)$$
$$(x^2-3x+4)-(x^3+7x-2)=x^2+(-3x-7x)+(4+2)-x^3$$

Group like terms

$$=x^2-10x+6-x^3$$ Combine like terms

$$=-x^3+x^2-10x+6$$ Rearrange the terms

Therefore, $\boxed{(x^2-3x+4)-(x^3+7x-2)=-x^3+x^2-10x+6}$

Answer 27e.

The expression $(2x-7)^3$ is of the form $(a-b)^3$. The cube of a binomial, $a-b$, can be expanded as $a^3-3a^2b+3ab^2+b^3$.

The error in the given simplification can be thus corrected by simplifying $(2x-7)^3$ using the expansion for $(a-b)^3$.

We have a as $2x$, and b as 7.

$$(2x-7)^3=(2x)^3-3(2x)^2(7)+3(2x)(7)^2-(7)^3$$
$$=8x^3-84x^2+294x-343$$

Thus, the simplified form of $(2x-7)^3$ is $8x^3-84x^2+294x-343$.

Answer 28e.

Consider the binomials,

$$(x+4)(x-6)(x-5)$$

Find the product of the following binomials $(x+4)(x-6)(x-5)$

$$(x+4)(x-6)(x-5)$$

Distributive Property:

$$a(b+c)=ab+ac$$

By the distributive property,

$$(x+4)(x-6)(x-5)=(x+4)((x\cdot x+x(-5))+(-6\cdot x-6(-5)))$$

Apply the distributive property

$$=(x+4)(x^2-6x-5x+30)$$ Simplify

$$=(x+4)(x^2-11x+30)$$ Combine like terms

$$=(x+4)(x\cdot x^2+x(-11x)+x\cdot 30)+(4\cdot x^2+4(-11x)+4\cdot 30)$$

Apply the distributive property

$$=x^3-11x^2+30x+4x^2-44x+120$$ Simplify

$$=x^3-7x^2-14x+120$$ Combine like terms

Answer 29e.

Follow the horizontal format of multiplication.

Start by multiplying $x+1$ and $x-7$. For this, multiply each term of $x+1$ by each term of $x-7$.

$$(x+1)(x-7)(x+3) = (x^2+x-7x-7)(x+3)$$
$$= (x^2-6x-7)(x+3)$$

Now, multiply x^2-6x-7 by each term in $x+3$.

$$(x^2-6x-7)(x+3) = (x^2-6x-7)x+(x^2-6x-7)3$$

Use the distributive property to remove the parentheses.

$$(x^2-6x-7)x+(x^2-6x-7)3 = x^3-6x^2-7x+3x^2-18x-21$$

Combine the like terms.

$$x^3-6x^2-7x+3x^2-18x-21 = x^3-3x^2-25x-21$$

Therefore, the product is $x^3-3x^2-25x-21$.

Answer 30e.

Consider the binomials,

$$(z-4)(-z+2)(z+8)$$

Find the product of the following binomials,

$$(z-4)(-z+2)(z+8)$$

Distributive Property:

$$a(b+c)=ab+ac$$

By the distributive property,

$$(z-4)(-z+2)(z+8)=(z-4)((-z\cdot z-z\cdot 8)+(2\cdot z+2\cdot 8))$$

Apply the Distributive property

$$=(z-4)(-z^2-8z+2z+16) \quad \text{Simplify}$$

$$=(z-4)(-z^2-6z+16) \quad \text{Combine like terms}$$

$$=(z\cdot(-z^2)+z(-6z)+z\cdot 16)+(-4\cdot(-z^2)-4(-6z)-4\cdot 16)$$

Apply the Distributive property

$$=-z^3-6z^2+16z+4z^2+24z-64 \quad \text{Simplify}$$

$$=-z^3-2z^2+40z-64 \quad \text{Combine like terms}$$

Answer 31e.

Follow the horizontal format of multiplication.
Start by multiplying $a-6$ and $2a+5$. For this, multiply each term of $a-6$ by each term of $2a+5$.

$$(a-6)(2a+5)(a+1) = (2a^2-12a+5a-30)(a+1)$$

$$= (2a^2-7a-30)(a+1)$$

Now, multiply $2a^2-7a-30$ by each term in $a+1$.

$$(2a^2-7a-30)(a+1) = (2a^2-7a-30)a+(2a^2-7a-30)1$$

Use the distributive property to remove the parentheses.

$$(2a^2-7a-30)a+(2a^2-7a-30)1 = 2a^3-7a^2-30a+2a^2-7a-30$$

Combine the like terms.

$$2a^3-7a^2-30a+2a^2-7a-30 = 2a^3-5a^2-37a-30$$

Therefore, the product is $2a^3-5a^2-37a-30$.

Answer 32e.

Consider the binomials,

$$(3p+1)(p+3)(p+1)$$

Find the product of the following binomials,

$$(3p+1)(p+3)(p+1)$$

Distributive Property:

$$a(b+c)=ab+ac$$

By the distributive property,

$$(3p+1)(p+3)(p+1)=(3p+1)((p\cdot p+p\cdot 1)+(3\cdot p+3\cdot 1))$$

Apply the Distributive property

$$=(3p+1)(p^2+p+3p+3) \quad \text{Simplify}$$

$$=(3p+1)(p^2+4p+3) \quad \text{Group like terms}$$

$$=((3p\cdot p^2+3p\cdot 4p+3p\cdot 3)+(1\cdot p^2+1\cdot 4p+1\cdot 3))$$

Apply the Distributive property

$$=3p^3+12p^2+9p+p^2+4p+3 \quad \text{Simplify}$$

$$=3p^3+13p^2+13p+3 \quad \text{Group like terms}$$

Answer 33e.

Follow the horizontal format of multiplication.
Start by multiplying $b - 2$ and $2b - 1$. For this, multiply each term of $b - 2$ by each term of $2b - 1$.

$$(b - 2)(2b - 1)(-b + 1) = \left(2b^2 - 4b - b + 2\right)(-b + 1)$$
$$= \left(2b^2 - 5b + 2\right)(-b + 1)$$

Now, multiply $2b^2 - 5b + 2$ by each term in $-b + 1$.
$(2b^2 - 5b + 2)(-b + 1) = (2b^2 - 5b + 2)(-b) + (2b^2 - 5b + 2)1$

Use the distributive property to remove the parentheses.
$(2b^2 - 5b + 2)(-b) + (2b^2 - 5b + 2)1 = -2b^3 + 5b^2 - 2b + 2b^2 - 5b + 2$

Combine the like terms.
$-2b^3 + 5b^2 - 2b + 2b^2 - 5b + 2 = -2b^3 + 7b^2 - 7b + 2$

Therefore, the product is $-2b^3 + 7b^2 - 7b + 2$.

Answer 34e.

Consider the binomials,

$$(2s+1)(3s-2)(4s-3)$$

Find the product of the following binomial,

$$(2s+1)(3s-2)(4s-3)$$

Distributive Property:

$$a(b+c) = ab + ac$$

By the distributive property,

$$(2s+1)(3s-2)(4s-3) = (2s+1)\left((3s\cdot 4s + 3s(-3)) + (-2\cdot 4s - 2(-3))\right)$$

Apply the distributive property

$$= (2s+1)\left(12s^2 - 9s - 8s + 6\right) \quad \text{Simplify}$$
$$= (2s+1)\left(12s^2 - 17s + 6\right) \quad \text{Group like terms}$$
$$= \left(2s\cdot 12s^2 + 2s(-17s) + 2s\cdot 6\right) + \left(1\cdot 12s^2 + 1(-17s) + 1\cdot 6\right)$$

Apply the distributive property

$$= 24s^3 - 34s^2 + 12s + 12s^2 - 17s + 6 \quad \text{Simplify}$$
$$= 24s^3 - 22s^2 - 55 + 6 \quad \text{Group like terms}$$

Answer 35e.

Follow the horizontal format of multiplication.

Start by multiplying $w - 6$ and $4w - 1$. For this, multiply each term of $w - 6$ by each term of $4w - 1$.

$$(w - 6)(4w - 1)(-3w + 5) = \left(4w^2 - 24w - w + 6\right)(-3w + 5)$$
$$= \left(4w^2 - 25w + 6\right)(-3w + 5)$$

Now, multiply $4w^2 - 25w + 6$ by each term in $-3w + 5$.
$(4w^2 - 25w + 6)(-3w + 5) = (4w^2 - 25w + 6)(-3w) + (4w^2 - 25w + 6)5$

Use the distributive property to remove the parentheses.
$(4w^2 - 25w + 6)(-3w) + (4w^2 - 25w + 6)5 = -12w^3 + 75w^2 - 18w + 20w^2 - 125w + 30$

Combine the like terms.
$-12w^3 + 75w^2 - 30w + 20w^2 - 125w + 30 = -12w^3 + 95w^2 - 143w + 30$

Therefore, the product is $-12w^3 + 95w^2 - 143w + 30$.

Answer 36e.

Consider the expression : $(4x-1)(-2x-7)(-5x-4)$.

We need to find the product of the binomial $(4x-1)(-2x-7)(-5x-4)$

$(4x-1)(-2x-7)(-5x-4)$

$= (4x-1)(10x^2+8x+35x+28)$ (by using distributive property)

$= (4x-1)(10x^2+43x+28)$ (by grouping like terms)

$= 40x^3+172x^2+112x-10x^2-43x-28$ (by distributive property)

$= 40x^3+162x^2+69x-28$ (by grouping like terms)

Hence the result is $\boxed{40x^3+162x^2+69x-28}$.

Answer 37e.

Follow the horizontal format of multiplication.

Start by multiplying $3q - 8$ and $-9q + 2$. For this, multiply each term of $3q - 8$by each term of $-9q + 2$.

$$(3q-8)(-9q+2)(q-2) = (-27q^2 + 72q + 6q - 16)(q-2)$$
$$= (-27q^2 + 78q - 16)(q-2)$$

Now, multiply $-27q^2 + 78q - 16$ by each term in $q - 2$.
$(-27q^2 + 78q - 16)(q - 2) = (-27q^2 + 78q - 16)q + (-27q^2 + 78q - 16)(-2)$

Use the distributive property to remove the parentheses.
$(-27q^2 + 78q - 16)q + (-27q^2 + 78q - 16)(-2) = -27q^3 + 78q^2 - 16q + 54q^2 - 156q + 32$

Combine the like terms.
$-27q^3 + 78q^2 - 16q + 54q^2 - 156q + 32 = -27q^3 + 132q^2 - 172q + 32$

Therefore, the product is $-27q^3 + 132q^2 - 172q + 32$.

Answer 38e.

Consider the expression : $(x+5)(x-5)$.

We need to find the product $(x+5)(x-5)$.

$$(x+5)(x-5)=x^2-5^2 \qquad \left(\text{Since } (a+b)(a-b)=a^2-b^2\right)$$
$$=x^2-25$$

Hence the result is $\boxed{x^2-25}$.

Answer 39e.

The expression $(w-9)^2$ is of the form $(a-b)^2$. The square of a binomial, $a-b$, can be expanded as $a^2-2ab+b^2$.

We have a as w, and b as 9.

$$(w-9)^2 = w^2-2\cdot w\cdot 9+9^2$$

Simplify.

$$w^2-2\cdot w\cdot 9+9^2 = w^2-18w+81$$

Therefore, the product is $w^2-18w+81$.

Answer 40e.

Consider the expression : $(y+4)^3$.

We need to find the product $(y+4)^3$.

$$(y+4)^3=y^3+3(y^2)(4)+3y(4^2)+4^3 \qquad \left(\text{Since } (a+b)^3=a^3+3a^2b+3ab^2+b^3\right)$$
$$=y^3+12y^2+48y+64$$

Hence the result is $\boxed{y^3+12y^2+48y+64}$.

Answer 41e.

The expression $(2c+5)^2$ is of the form $(a+b)^2$. The square of a binomial, $a-b$, can be expanded as $a^2-2ab+b^2$.

We have a as $2c$, and b as 5.

$$(2c+5)^2 = (2c)^2+2\cdot 2c\cdot 5+5^2$$

Simplify.

$$(2c)^2+2\cdot 2c\cdot 5+5^2 = 4c^2+20c+25$$

Therefore, the product is $4c^2+20c+25$.

Answer 42e.

Consider the expression : $(3t-4)^3$.

We need to find $(3t-4)^3$.

$$(3t-4)^3 = (3t)^3 - 3(3t)^2(4) + 3(3t)(4^2) - 4^3 \quad \left(\text{Since } (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3\right)$$
$$= 27t^3 - 108t^2 + 144t - 64$$

Hence the result is $\boxed{27t^3 - 108t^2 + 144t - 64}$.

Answer 43e.

The given expression is of the form $(a-b)(a+b)$. This product is equivalent to $a^2 - b^2$.

We have a as $5p-3$, and b as $5p+3$.
$(5p-3)(5p+3) = (5p)^2 - 3^2$

Simplify.
$(5p)^2 - 3^2 = 25p^2 - 9$

Therefore, the product is $25p^2 - 9$.

Answer 44e.

Consider the expression : $(7x-y)^3$.

We need to find the product $(7x-y)^3$.

$$(7x-y)^3 = (7x)^3 - 3(7x)^2 y + 3(7x)(y^2) - y^3 \quad \left(\text{Since } (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3\right)$$
$$= 343x^3 - 147x^2y + 21xy^2 - y^3$$

Hence the result is $\boxed{343x^3 - 147x^2y + 21xy^2 - y^3}$.

Answer 45e.

The given expression is of the form $(a+b)(a-b)$. This product is equivalent to $a^2 - b^2$.

We have a as $2a+9b$, and b as $2a-9b$.
$(2a+9b)(2a-9b) = (2a)^2 - (9b^2)$

Simplify.
$(2a)^2 - (9b^2) = 4a^2 - 81b^2$

Therefore, the product is $4a^2 - 81b^2$.

Answer 46e.

Consider the expression : $(3z+7y)^3$.

We need to find the product $(3z+7y)^3$.

$$(3z+7y)^3 = (3z)^3 + 3(3z)^2(7y) + 3(3z)(7y)^2 + (7y)^3 \quad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$= 27z^3 + 189z^2y + 441zy^2 + 343y^3$$

Hence the result is $\boxed{27z^3 + 189z^2y + 441zy^2 + 343y^3}$

Answer 47e.

The expression $(3x-2y)^2$ is of the form $(a-b)^2$. The square of a binomial, $a-b$, can be expanded as $a^2 - 2ab + b^2$.

We have a as $3x$, and b as $2y$.

$$(3x-2y)^2 = (3x)^2 - 2\cdot 3x\cdot 2y + (2y)^2$$

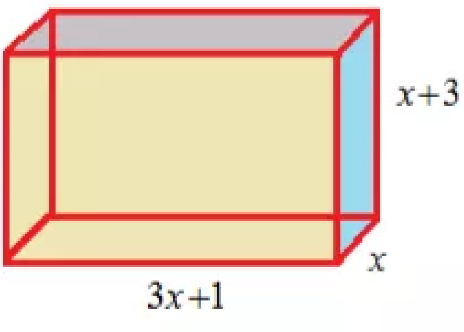
Simplify.

$$(3x)^2 - 2\cdot 3x\cdot 2y + (2y)^2 = 9x^2 - 12xy + 4y^2$$

This is equivalent to the expression given in choice **D**.

Answer 48e.

Consider the diagram :



Volume of the given cuboid is

$$V = (3x+1)x(x+3)$$
$$= (3x+1)(x^2+3x) \quad \text{(by distributive property)}$$
$$= 3x^3 + 9x^2 + x^2 + 3x \quad \text{(by distributive property)}$$
$$= 3x^3 + 10x^2 + 3x \quad \text{(by grouping like terms)}$$

Hence the result is $\boxed{3x^3 + 10x^2 + 3x}$.

Answer 49e.

Replace r with $x-4$, and h with $2x+3$ in the given equation.

$$V = \pi(x-4)^2(2x+3)$$

Rewrite $(x-4)^2$ using the expansion for $(a-b)^2$.

$$V = \pi\left(x^2-8x+16\right)(2x+3)$$

Multiply $x^2-8x+16$ by each term in $2x+3$.

$$\begin{aligned} V &= \pi\left[\left(x^2-8x+16\right)2x+\left(x^2-8x+16\right)3\right] \\ &= \pi\left[2x^3-16x^2+32x+3x^2-24x+48\right] \\ &= \pi\left[2x^3-13x^2+8x+48\right] \end{aligned}$$

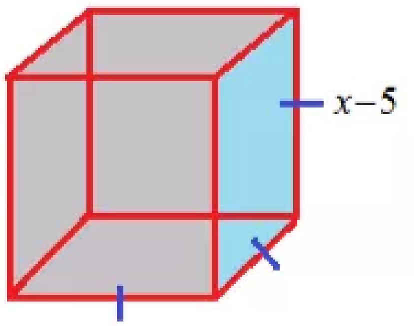
Clear the brackets using the distributive property.

$$V = 2\pi x^3-13\pi x^2+8\pi x+48\pi$$

Therefore, the volume of the given figure in standard form is $2\pi x^3-13\pi x^2+8\pi x+48\pi$.

Answer 50e.

Consider the diagram :



Volume of the given cube is

$$\begin{aligned} V &= (x-5)^3 \\ &= x^3-3x^2(5)+3x\left(5^2\right)-5^3 \quad \left(\text{Since } (a-b)^3 = a^3-3a^2b+3ab^2-b^3\right) \\ &= x^3-15x^2+75x-125 \end{aligned}$$

Hence the result is $\boxed{x^3-15x^2+75x-125}$.

Answer 51e.

We know that volume is the product of base area and height. For the given figure, the base area B is $(2x-3)^2$, and height h is $3x+4$.

Replace B with $(2x-3)^2$, and h with $3x+4$ in the given equation.

$$V = \frac{1}{3}(2x-3)^2(3x+4)$$

Rewrite $(2x-3)^2$ using the expansion for $(a-b)^2$.

$$V = \frac{1}{3}\left(4x^2-12x+9\right)(3x+4)$$

Multiply $4x^2-12x+9$ by each term in $3x+4$.

$$\begin{aligned} V &= \frac{1}{3}\left[\left(4x^2-12x+9\right)3x+\left(4x^2-12x+9\right)4\right] \\ &= \frac{1}{3}\left[12x^3-36x^2+27x+16x^2-48x+36\right] \\ &= \frac{1}{3}\left[12x^3-20x^2-21x+36\right] \end{aligned}$$

Clear the brackets using the distributive property.

$$V = 4x^3-\frac{20}{3}x^2-7x+12$$

Therefore, the volume of the given figure in standard form is $4x^3-\frac{20}{3}x^2-7x+12$.

Answer 52e.

Consider the expression : $(a+b)(a-b)=a^2-b^2$.

We need to verify $(a+b)(a-b)=a^2-b^2$.

$$\begin{aligned}(a+b)(a-b) &= a^2-ab+ba-b^2 \quad \text{(by distributive property)} \\ &= a^2-b^2\end{aligned}$$

Hence the result is $\boxed{a^2-b^2}$.

Answer 53e.

We can write $(a+b)^2$ as $(a+b)(a+b)$.

Multiply the first polynomial by each term in the second polynomial.
$(a+b)(a+b) = (a+b)a + (a+b)b$

Clear the parentheses using the distributive property.
$(a+b)a + (a+b)b = a^2 + ab + ab + b^2$

Combine the like terms.
$a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

The given special product pattern is thus verified.

Answer 54e.

Consider : $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

We need to verify $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

$$\begin{aligned}(a+b)^3 &= (a+b)(a+b)^2 \\ &= (a+b)(a^2+2ab+b^2) \\ &= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 && \text{(By distributive property)} \\ &= a^3 + 3a^2b + 3ab^2 + b^3 && \text{(by grouping like terms)}\end{aligned}$$

Hence the result is $\boxed{a^3 + 3a^2b + 3ab^2 + b^3}$.

Answer 55e.

We can write $(a-b)^3$ as $(a-b)(a-b)(a-b)$. First, do the multiplication $(a-b)(a-b)$.

$$\begin{aligned}(a-b)(a-b)(a-b) &= \left(a^2 - ab - ab + b^2\right)(a-b) \\ &= \left(a^2 - 2ab + b^2\right)(a-b)\end{aligned}$$

Multiply the first polynomial by each term in the second polynomial.
$(a^2 - 2ab + b^2)(a-b) = (a^2 - 2ab + b^2)a + (a^2 - 2ab + b^2)(-b)$

Clear the parentheses using the distributive property.
$(a^2-2ab+b^2)a+(a^2-2ab+b^2)(-b) = a^3-2a^2b+ab^2-a^2b+2ab^2-b^3$

Combine the like terms.
$a^3-2a^2b+ab^2-a^2b+2ab^2-b^3 = a^3-3a^2b+3ab^2-b^3$

The given special product pattern is thus verified.

Answer 56e.

Consider $p(x)=x^4-7x+14$ and $q(x)=x^2-5$.
Since the degree of the polynomial is the same as the degree of its terms with largest degree.

(a)

Need to find the degree of the polynomial $p(x)+q(x)$.

The sum of $p(x)$ and $q(x)$ is given by

$$\begin{aligned} p(x)+q(x) &= x^4-7x+14+x^2-5 \\ &= x^4+x^2-7x+9 \end{aligned}$$

Therefore $p(x)+q(x)$ is a polynomial with degree 4, because the term with largest degree (x^4) is $\boxed{4}$.

(b)

Need to find the degree of the polynomial $p(x)-q(x)$.

The difference of $p(x)$ and $q(x)$ is given by

$$\begin{aligned} p(x)-q(x) &= x^4-7x+14-(x^2-5) \\ &= x^4-7x+14-x^2+5 && \left[\text{Change the sign of each terms of } x^2-5 \text{ and drop the parentheses}\right] \\ &= x^4-x^2-7x+19 && [\text{Combine like terms}] \end{aligned}$$

Therefore $p(x)-q(x)$ is a polynomial with degree 4, because the term with largest degree (x^4) is $\boxed{4}$.

(c)

Need to find the degree of the polynomial $p(x)\cdot q(x)$

The product of $p(x)$ and $q(x)$ is given by

$$p(x)\cdot q(x) = (x^4 - 7x + 14)(x^2 - 5)$$

$$= (x^4 - 7x + 14)x^2 - (x^4 - 7x + 14)5 \quad \begin{bmatrix}\text{Distributive property,} \\ a(b+c) = a\cdot b + a\cdot c\end{bmatrix}$$

$$= x^6 - 7x^3 + 14x^2 - 5x^4 + 35x - 70 \quad [\text{Multiply the factors}]$$

$$= x^6 - 5x^4 - 7x^3 + 14x^2 + 35x - 70$$

Therefore $p(x)\cdot q(x)$ is a polynomial with degree 6, because the term with largest degree (x^6) is $\boxed{6}$.

(d)

Consider that the degree of $p(x)$ is m and the degree of $q(x)$ is n.

Also given that $m > n$

Therefore degree of $p(x)+q(x)$ is $\boxed{m}$, degree of $p(x)-q(x)$ is $\boxed{m}$ and the degree of $p(x)\cdot q(x)$ is $\boxed{m\cdot n}$.

Answer 57e.

a. If we observe the given polynomial factorizations, we can see that the first factor is $x - 1$ and the number of terms in the second factor is equivalent to the degree of the polynomial that is being factored. Also, the degree of the second factor is one less than the degree of the original polynomial.

Expand $x^5 - 1$ using this pattern.

$x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$

Verify the factorization by multiplying $x^4 + x^3 + x^2 + x + 1$ by each term in $x - 1$.

$$\left(x^4 + x^3 + x^2 + x + 1\right)(x - 1) = x^5 + x^4 + x^3 + x^2 + x - x^4 - x^3 - x^2 - x - 1$$

$$= x^5 - 1$$

Thus, $x^5 - 1$ can be factored as $(x - 1)(x^4 + x^3 + x^2 + x + 1)$.

Similarly, we can factor $x^6 - 1$ as $(x-1)(x^5 + x^4 + x^3 + x^2 + x + 1)$.

Verify by multiplying the factors.

$$\left(x^5 + x^4 + x^3 + x^2 + x + 1\right)(x-1) = x^6 + x^5 + x^4 + x^3 + x^2 - x - x^5 - x^4 - x^3 - x^2 - x - 1$$
$$= x^6 - 1$$

b. Any polynomial of the form $x^n - 1$ can be factored as $(x-1)(x^{n-1} + x^{n-2} + x^{n-3} + \cdots + 1)$.

Multiply the second polynomial by each term in the first polynomial.

$$\left(x^{n-1} + x^{n-2} + x^{n-3} + \cdots + 1\right)(x-1) = x^n + x^{n-1} + x^{n-2} + \cdots + x - x^{n-1} + x^{n-2} + x^{n-3} + \cdots + 1$$
$$= x^n - 1$$

Thus, the factorization for $x^n - 1$ works.

Answer 58e.

Consider $f(x) = (x+a)(x+b)(x+c)(x+d)$.

Need to show that the coefficient of x^3 is the sum of $a, b, c,$ and d and the constant term is the product of $a, b, c,$ and d.

Now

$$f(x) = (x+a)(x+b)(x+c)(x+d)$$
$$= \{(x+a)(x+b)\}\{(x+c)(x+d)\}$$
$$= \{x(x+b) + a(x+b)\}\{x(x+d) + c(x+d)\} \quad \left[\begin{array}{l}\text{Distributive property} \\ a(b+c) = a \cdot b + a \cdot c\end{array}\right]$$
$$= \left(x^2 + bx + ax + ab\right)\left(x^2 + dx + cx + cd\right)$$
$$= x^2\left(x^2 + dx + cx + cd\right) + bx\left(x^2 + dx + cx + cd\right) + ax\left(x^2 + dx + cx + cd\right) + ab\left(x^2 + dx + cx + cd\right)$$
$$= x^4 + dx^3 + cx^3 + cdx^2 + bx^3 + bdx^2 + bcx^2 + bcdx + ax^3 + adx^2 + acx^2 + acdx + abx^2 + abdx + abcx + abcd \quad [\text{Multiply the factors}]$$
$$= x^4 + (a+b+c+d)x^3 + (cd + bd + bc + ad + ac + ab)x^2 + (bcd + acd + abd + abc)x + abcd \quad [\text{Combine like terms}]$$

Therefore the coefficient of x^3 is given by $(a+b+c+d)$ and the constant term is $abcd$.

Hence coefficient of x^3 is the sum of a, b, c and d and the constant term is the product of a, b, c and d.

Answer 59e.

Let T represents the model for the total number of people attending institutes of higher education. The total number of people will be the sum of the number of males and the number of females.

Add the given two models.

$T = M + F$

$= 0.091t^3 - 4.8t^2 + 110t + 5000 + 0.19t^3 - 12t^2 + 350t + 3600$

Combine the like terms.

$M + F = 0.281t^3 - 16.8t^2 + 460t + 8600$

Therefore, the required model is $T = 0.281t^3 - 16.8t^2 + 460t + 8600$.

Answer 60e.

Consider the number of DVD players is $D = 4.11t + 4.44$

And average price per DVD player is $P = 6.82t^2 - 61.7t + 265$

Here t is the number of years since 1999.

Need to write a model for the revenue R from DVD sales.

And also find what the total revenue was in 2002.

To find the model for a total revenue R, multiply the given two models, obtain

$$\begin{array}{r} 6.82t^2 - \quad 61.7t + \quad 265 \\ \times \qquad\qquad 4.11t + \quad 4.44 \\ \hline 30.2808t^2 - 273.948t + 3676875 \\ 28.0302t^3 - 253.587t^2 + 1089.15t \qquad\qquad \\ \hline 28.0302t^3 - 223.3062t^2 + 815.202t + 3676875 \end{array}$$

Therefore the model for total revenue R can be expressed as

$\boxed{28.0302t^3 - 223.3062t^2 + 815.202t + 3676875}$, t is the number of years since 1999.

Now need to find the total revenue in 2002.

For this substitute $t = 3$ into the model, obtain

$28.0302t^3 - 223.3062t^2 + 815.202t + 3676875$

$= 28.0302(3)^3 - 223.3062(3)^2 + 815.202(3) + 3676875$ Substitute 3 for t

$= 28.0302(27) - 223.3062(9) + 815.202(3) + 3676875$

$= 756.8154 - 2009.7558 + 2445.606 + 3676875$

$= 3678067.6656$

Therefore the total revenue in 2002 is about $\boxed{3678067.6656}$ million dollars.

Answer 61e.

Replace F with $0.0116s^2 + 0.789$ in the given equation for P.
$P = 0.00267s(0.0116s^2 + 0.789)$

Clear the parentheses using the distributive property.
$P = 0.000030972s^3 + 0.00210663s$

Thus, the model for the power needed to keep the bicycle moving at speed s on level ground is $P = 0.000030972s^3 + 0.00210663s$.

Substitute 10 for s in $P = 0.000030972s^3 + 0.00210663s$.
$P = 0.000030972(10^3) + 0.00210663(10)$

Evaluate.
$P \approx 0.05$

Therefore, about 0.05 horse power is required to keep the bicycle moving at 10 miles per hour.

Answer 63e.

We know that average size of teams $= \dfrac{\text{total number of people in the teams}}{\text{number of teams}}$. It is given that L_m is the number teams for men and S_m is the average size of men's team. Then the total number of people in men's team will be $L_m \cdot S_m$. Similarly, the total number of people in women's team will be $L_w \cdot S_w$.

The total number of people N on the lacrosse team will be thus $L_m \cdot S_m + L_w \cdot S_w$.

Let us find $L_m \cdot S_m$ first.

$$L_m \cdot S_m = (5.57t + 182)\left(-0.127t^3 + 0.822t^2 - 1.02t + 31.5\right)$$

Multiply.

$$\begin{aligned} L_m \cdot S_m &= -0.70739t^4 + 4.57854t^3 - 5.6814t^2 + 175.455t - 23.114t^3 + 149.604t^2 - 185.64t + 5733 \\ &= -0.70739t^4 - 18.53546t^3 + 143.9226t^2 - 10.185 + 5733 \end{aligned}$$

Similarly, we get $L_w \cdot S_w$ as $-0.80764t^4 - 6.9156t^3 + 72t^2 + 137.935t + 4125.5$.

Now, find $N = L_m \cdot S_m + L_w \cdot S_w$. Substitute for $L_m \cdot S_m$ and $L_w \cdot S_w$.

$$N = -0.70739t^4 - 18.53546t^3 + 143.9226t^2 - 10.185 + 5733 + \left(-0.80764t^4 - 6.9156t^3 + 72t^2 + 137.935t + 4125.5\right)$$

On combining the like terms, we get
N as $-1.51503t^4 - 25.45106t^3 + 215.9226t^2 + 127.75t + 9858.5$.

Therefore, the model for the total number of people on lacrosse teams is
$N = -1.51503t^4 - 25.45106t^3 + 215.9226t^2 + 127.75t + 9858.5$.

Answer 64e.

Consider from 1970 to 2002, the circulation C (in millions) of Sunday news papers in the United States can be modeled by

$$C = -0.00105t^3 + 0.0281t^2 + 0.465t + 48.8 \qquad \ldots\ldots (1)$$

Here t is the number of years since 1970.

If number of years since 1970 is changed to 1975, the new value of t becomes $s = t + 5$.
Therefore

$$t = s - 5$$

Plugging $t = s - 5$ in the equation (1), obtain

$$C = -0.00105(s-5)^3 + 0.0281(s-5)^2 + 0.465(s-5) + 48.8$$

$$= -0.00105\left(s^3 - 15s^2 + 75s - 125\right) + 0.0281\left(s^2 - 10s + 25\right) + 0.465(s-5) + 48.8 \qquad \text{Expand binomials}$$

$$= -0.00105s^3 + 0.01575s^2 - 0.07875s + 0.13125 + 0.0281s^2 + 0.281s + 0.7025 + 0.465s - 2.325 + 48.8 \qquad \text{Multiply}$$

$$= -0.00105s^3 + 0.04385s^2 + 0.66725s + 47.30875 \qquad \text{Combine like terms}$$

Therefore

$$\boxed{C = -0.00105s^3 + 0.04385s^2 + 0.66725s + 47.30875}$$

Answer 65e.

Add 7 to both sides of the given equation.

$$2x - 7 + 7 = 11 + 7$$

$$2x = 18$$

Divide both the sides by 2.

$$\frac{2x}{2} = \frac{18}{2}$$

$$x = 9$$

The solution appears to be 9.

CHECK

Substitute 9 for x in the original equation.

$$2x - 7 = 11$$

$$2(9) - 7 \stackrel{?}{=} 11$$

$$18 - 7 \stackrel{?}{=} 11$$

$$11 = 11 \checkmark$$

The solution checks.

Therefore, the solution to the given equation is 9.

Answer 66e.

Consider the equation $10-3x=25$.
Need to solve the equation.

$$10-3x=25$$ Write the equation

$$10-10-3x=25-10$$ Subtract 10 from both sides of the equation

$$-3x=15$$

$$\frac{-3x}{3}=\frac{15}{3}$$ Divide both sides by 3

$$-x=5$$

$$x=-5$$ Multiply both sides by -1

Therefore the solution of the equation is $\boxed{x=-5}$.

Answer 67e.

Subtract $2t$ from each side of the given equation.

$$4t-7-2t=2t-2t$$

$$2t-7=0$$

Now, add 7 to each side.

$$2t-7+7=0+7$$

$$2t=7$$

Divide each side by 2.

$$\frac{2t}{2}=\frac{7}{2}$$

$$t=\frac{7}{2}$$

The solution appears to be $\frac{7}{2}$.

CHECK

Substitute $\frac{7}{2}$ for t in the original equation.

$$4t-7=2t$$

$$4\left(\frac{7}{2}\right)-7\stackrel{?}{=}2\left(\frac{7}{2}\right)$$

$$\frac{28}{2}-7\stackrel{?}{=}7$$

$$7=7\ \checkmark$$

The solution checks.

Therefore, the solution to the given equation is $\frac{7}{2}$.

Answer 68e.

Consider the equation $y^2 - 2y - 48 = 0$.
Need to solve the equation.

$y^2 - 2y - 48 = 0$	[Write the equation]
$y^2 - 8y + 6y - 48 = 0$	[In place of $-2y$, we write $-8y+6y$ so that product of coefficients of $-8y+6y$ is -48; $(-8)(6)=-48$]
$y(y-8)+6(y-8)=0$	[Take y common in y^2-8y; 6 common in $6y-48$]
$(y-6)(y-8)=0$	[Take $(y-8)$ common]
$y-6=0$ and $y-8=0$	Apply zero product property
$y=6$ and $y=8$	

Therefore the solutions of the equation are $\boxed{y=6,8}$.

Answer 69e.

The given equation is of the form $ax^2 + bx + c = 0$. Equations of this type can be solved using the zero product property only if the left side can be factored.

The trinomial on the left side of the given equation is of the form $x^2 + bx + c$, which when factored will be of the form $(x + m)(x + n)$, where the product of m and n gives c, and their sum gives b.

Compare $w^2 - 15w + 54$ with $x^2 + bx + c = 0$. The value of b is -15 and of c is 54. We need to find m and n such that their product is 54 and sum -15.

List the factors of 54 and find their sums.

Factors of 54: m, n	1, 54	2, 27	3, 18	6, 9	−1, −54	−2, −27	−3, −18	−6, −9
Sum of factors: $m + n$	55	29	21	15	−55	−29	−21	−15

From the table, it is clear that when m is -6 and n is -9, the product 54 and the sum is -15. The trinomial $w^2 - 15w + 54$ can be factored as $(x - 6)(x - 9)$.

The given equation thus becomes $(x - 6)(x - 9) = 0$.

Apply the zero product property. The property states that if the product of two expressions is zero, then one or both of the expressions equal zero.

$x - 6 = 0$ or $x - 9 = 0$

Solve the equations for x.

$x - 6 + 6 = 0 + 6$ or $x - 9 + 9 = 0 + 9$

$x = 6$ or $x = 9$

Therefore, the solutions to the given equation are 6 and 9.

Answer 70e.

Consider the equation $x^2+9x+14=0$.
Need to solve the equation.

$x^2+9x+14=0$	[Write the equation]
$x^2+7x+2x+14=0$	[In place of $9x$, we write $7x+2x$ so that product of coefficients of $7x+2x$ is 14; $(7)(2)=14$]
$x(x+7)+2(x+7)=0$	[Take x common in x^2+7x; 2 common in $2x+14$]
$(x+7)(x+2)=0$	[Take $(x+7)$ common]
$x+7=0$ and $x+2=0$	Apply zero product property
$x=-7$ and $x=-2$	

Therefore the solutions of the equation are $\boxed{x=-7,-2}$

Answer 71e.

The trinomial on the left side of the given equation is of the form az^2+bz+c, where a is 4, b is 21, and c is -18. The trinomial can be factored as $(kz+m)(lz+n)$, where k and l are factors of 4, and m and n are factors of c.

Since the leading coefficient 4 is positive, both k and l must be positive with $k \geq l$. Again, as c is negative, m and n must be of opposite signs.

Only the factors $4z-3$ and $z+6$ satisfy the conditions. The trinomial $4z^2+21z-18$ can thus be factored as $(4z-3)(z+6)$ and the given equation becomes $(4z-3)(z+6)=0$.

Apply the zero product property. The property states that if the product of two expressions is zero, then one or both of the expressions equal zero.

$4z-3=0$ or $z+6=0$

Solve the equations for z.

$4z-3+3=0+3$ or $z+6-6=0-6$

$z=\frac{3}{4}$ or $z=-6$

Therefore, the solutions to the given equation are -6 and $\frac{3}{4}$.

Answer 72e.

Consider the equation $9a^2 - 30a + 25 = 0$.
Need to solve the equation.

$$9a^2 - 30a + 25 = 0$$ [Write the equation]

$$(3a)^2 - 2\cdot 3a\cdot 5 + 5^2 = 0$$ [Converting the equation to the form $a^2 - 2ab + b^2$ which is equal to $(a-b)^2$]

$$(3a-5)^2 = 0$$

$$(3a-5)(3a-5) = 0$$

$$(3a-5) = 0 \text{ or } (3a-5) = 0$$ Apply zero product property

$$3a = 5 \text{ or } 3a = 5$$ Add 5 on both sides

$$\frac{3a}{3} = \frac{5}{3} \text{ or } \frac{3a}{3} = \frac{5}{3}$$ [Divide both sides by 3]

$$a = \frac{5}{3} \text{ or } a = \frac{5}{3}$$

Therefore the solutions of the equation are $\boxed{a = \frac{5}{3}, \frac{5}{3}}$

Answer 73e.

Number the equations in the given system.

$x + y - 2z = -4$ Equation 1

$3x - y + z = 22$ Equation 2

$-x + 2y + 3z = -9$ Equation 3

STEP 1 **Rewrite** the system as a linear system in two variables.

Let the two variables be y and z. To eliminate x from Equations 1 and 2, add -3 times Equation 1 to Equation 2.

$$\begin{array}{rcrcrcr} -3x & - & 3y & + & 6z & = & 12 \\ 3x & - & y & + & z & = & 22 \\ \hline & - & 4y & + & 7z & = & 34 \end{array}$$ new Equation 1

Add Equations 1 and 3.

$$\begin{array}{rcrcrcr} x & + & y & - & 2z & = & -4 \\ -x & + & 2y & + & 3z & = & -9 \\ \hline & & 3y & + & z & = & -13 \end{array}$$ new Equation 2

STEP 2 **Solve** the new linear system for both of its variables.

Add 4 times new Equation 2 to 3 times new Equation 1.

$$\begin{array}{rcrcr} -12y & + & 21z & = & 102 \\ 12y & + & 4z & = & -52 \\ \hline & & 25z & = & 50 \end{array}$$

Divide both the sides by 25 to isolate z.

$$\frac{25z}{25} = \frac{50}{25}$$
$$z = 2$$

Replace z with 2 in any of the new equations, say, new Equation 2 and solve for y.

$$3y + 2 = -13$$
$$3y = -15$$
$$y = -5$$

STEP 3 **Substitute** $y = -5$ and $z = 2$ into an original equation and solve for x.

Use Equation 1.

$x + (-5) - 2(2) = -4$

Solve for x.

$$x - 5 - 4 = -4$$
$$x - 9 = -4$$
$$x = 5$$

The solution to the given system of equations is $x = 5$, $y = -5$, and $z = 2$, or the ordered triple $(5, -5, 2)$.

Answer 74e.

Consider the system of equations

$x - 2y + z = -13$ ...... (1)

$-x + 4y + z = 35$ ...... (2)

$3x + 2y + 4z = 28$ ...... (3)

From equation (1)

$$x - 2y + z = -13$$

$$x = 2y - z - 13$$

Plugging $x = 2y - z - 13$ in the equation (2) and (3), obtain

$$-x + 4y + z = 35$$

$$-(2y - z - 13) + 4y + z = 35$$ Substitute $x = 2y - z - 13$

$$-2y + z + 26 + 4y + z = 35$$ Multiply by -1 and remove the parenthesis

$$2y + 2z = 35 - 26$$ Combine like terms

$$2y + 2z = 9$$ Simplify ...... (4)

Again

$$3x + 2y + 4z = 28$$

$$3(2y - z - 13) + 2y + 4z = 28$$ Substitute $x = 2y - z - 13$

$$6y - 3z - 39 + 2y + 4z = 28$$ Multiply by 3 and remove the parenthesis

$$8y + z = 28 + 39$$ Combine like terms

$$8y + z = 67$$ Simplify ...... (5)

From equation (4), we will get

$$2y + 2z = 9$$

$$2y = 9 - 2z$$ Subtract $2z$ from each side

$$y = \frac{9 - 2z}{2}$$ Divide by 2 on both sides

Putting $y = \frac{9 - 2z}{2}$ in the equation (5), we will get

$$8y + z = 67$$

$$\overset{2}{\cancel{8}}\left(\frac{9 - 2z}{\cancel{2}}\right) + z = 67$$ Substitute $y = \frac{9 - 2z}{2}$

$$4(9 - 2z) + z = 67$$ Simplify

$$36 - 8z + z = 67$$ Multiply by 4 and remove the parenthesis

$$-7z = 67 - 36$$ Combine like terms and subtract 36 from each side

$$-7z = 31$$

$$z = -\frac{31}{7}$$ Divide by -7 on both sides

Substitute $z=-\frac{31}{7}$ in $y=\frac{9-2z}{2}$, obtain

$$y=\frac{9-2z}{2}$$

$$y=\frac{9-2\left(-\frac{31}{7}\right)}{2} \qquad \text{Substitute } z=-\frac{31}{7}$$

$$=\frac{9+\frac{62}{7}}{2} \qquad \text{Multiply by -2 and remove the parenthesis}$$

$$=\frac{\frac{63+62}{7}}{2} \qquad \text{Simplify}$$

$$=\frac{125}{14}$$

Now putting $y=\frac{125}{14}$ and $z=-\frac{31}{7}$ in the equation (1), obtain

$$x-2y+z=-13$$

$$x-2\left(\frac{125}{14}\right)+\left(-\frac{31}{7}\right)=-13 \qquad \text{Substitute } y=\frac{125}{14} \text{ and } z=-\frac{31}{7}$$

$$x-\frac{125}{7}-\frac{31}{7}=-13$$

$$x=\frac{125}{7}+\frac{31}{7}-13 \quad \text{Add } \frac{125}{7} \text{ and } \frac{31}{7} \text{ on both sides}$$

$$=\frac{156}{7}-13 \qquad \text{Simplify}$$

$$=\frac{156-91}{7}$$

$$=\frac{65}{7}$$

Therefore the solutions of the equations are

$$\boxed{x=\frac{65}{7},\ y=\frac{125}{14} \text{ and } z=-\frac{31}{7}}$$

Answer 75e.

Number the equations in the given system.

$$\begin{aligned} 3x - y - 2z &= 20 && \text{Equation 1} \\ -x + 3y - z &= -16 && \text{Equation 2} \\ -2x - y + 3z &= -5 && \text{Equation 3} \end{aligned}$$

STEP 1 **Rewrite** the system as a linear system in *two* variables.

Let the two variables be y and z. To eliminate x from Equations 1 and 2, add 3 times Equation 2 to Equation 1.

$$\begin{array}{rcrcrcr} 3x & - & y & - & 2z & = & 20 \\ -3x & + & 9y & - & 3z & = & -48 \\ \hline & & 8y & - & 5z & = & -28 \end{array} \qquad \text{new Equation 1}$$

Add −2 times Equation 2 to Equation 3.

$$\begin{array}{rcrcrcr} 2x & - & 6y & + & 2z & = & 32 \\ -2x & - & y & + & 3z & = & -5 \\ \hline & - & 7y & + & 5z & = & 27 \end{array} \qquad \text{new Equation 2}$$

STEP 2 **Solve** the new linear system for both of its variables.

Add new Equation 1 and new Equation 2.

$$\begin{array}{rcrcr} 8y & - & 5z & = & -28 \\ -7y & + & 5z & = & 27 \\ \hline y & & & = & -1 \end{array}$$

Replace y with −1 in any of the new equations, say, new Equation 1 and solve for z.

$$\begin{aligned} 8(-1) - 5z &= -28 \\ -5z &= -20 \\ z &= 4 \end{aligned}$$

STEP 3 **Substitute** $y = -1$ and $z = 4$ into an original equation and solve for x.

Use Equation 2.

$-x + 3(-1) - 4 = -16$

Solve for x.

$$\begin{aligned} -x - 3 - 4 &= -16 \\ -x - 7 &= -16 \\ x &= 9 \end{aligned}$$

The solution to the given system of equations is $x = 9$, $y = -1$, and $z = 4$, or the ordered triple $(9, -1, 4)$.

Answer 76e.

Consider the matrix $\begin{bmatrix} 3 & -4 \\ 3 & 1 \end{bmatrix}$.

Need to evaluate the determinant of the matrix.

Since if $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 matrix, then the determinant of the matrix is given by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc .$$

Compare the matrix $\begin{bmatrix} 3 & -4 \\ 3 & 1 \end{bmatrix}$ with $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, obtain

$$a = 3, b = -4, c = 3 \text{ and } d = 1$$

Therefore the determinant of the given matrix is

$$\begin{aligned} \begin{vmatrix} 3 & -4 \\ 3 & 1 \end{vmatrix} &= 3\cdot1 - 3\cdot(-4) \\ &= 3 + 12 \\ &= \boxed{15} \end{aligned}$$

Answer 77e.

The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

From the given matrix, we get a as 5, b as 7, c as −4, and d as 9. Substitute for a, b, c, and d in the equation.

$$\begin{vmatrix} 5 & 7 \\ -4 & 9 \end{vmatrix} = 5(9) - 7(-4)$$

Evaluate.

$$\begin{aligned} 5(9) - 7(-4) &= 45 + 28 \\ &= 73 \end{aligned}$$

Therefore, the determinant of the given matrix evaluates to 73.

Answer 78e.

Consider the matrix $\begin{bmatrix} -1 & 8 & 0 \\ 3 & 4 & -3 \\ -5 & 2 & 1 \end{bmatrix}$.

We know that if $\begin{bmatrix} a & b & c \\ d & e & f \\ p & q & r \end{bmatrix}$ be a 3×3 matrix, then the determinant of the matrix is given by

$$\begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} = a(er-qf)-b(dr-pf)+c(dq-pe).$$

Therefore the determinant of the given matrix is

$$\begin{vmatrix} -1 & 8 & 0 \\ 3 & 4 & -3 \\ -5 & 2 & 1 \end{vmatrix} = (-1)\begin{vmatrix} 4 & -3 \\ 2 & 1 \end{vmatrix} - 8\begin{vmatrix} 3 & -3 \\ -5 & 1 \end{vmatrix} + 0\begin{vmatrix} 3 & 4 \\ -5 & 2 \end{vmatrix}$$

$$= (-1)[4\cdot1-2\cdot(-3)]-8[3\cdot1-(-5)\cdot(-3)]+0[3\cdot2-(-5)\cdot4]$$

$$= -(4+6)-8(3-15)+0 \quad [0 \text{ multiplied by any number is } 0]$$

$$= -10-8(-12)$$

$$= -10+96$$

$$= 86$$

Hence the result is $\boxed{86}$.

Answer 79e.

The determinant of a 3×3 matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is given by

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

From the given matrix, we get a as 2, b as 3, c as -4, d as -6, e as 1, f as 5, g as -3, h as -1, and i as -2. Substitute for the variables in the equation.

$$\begin{vmatrix} 2 & 3 & -4 \\ -6 & 1 & 5 \\ -3 & -1 & -2 \end{vmatrix} = (-4-45-24)-(12-10+36)$$

Evaluate.

$$(-4-45-24)-(12-10+36) = -73-38$$
$$= -111$$

Therefore, the determinant of the given matrix evaluates to -111.