23. Heat and Temperature

Short Answer

Answer.1

Yes, if two bodies are in thermal equilibrium in one frame, they will be in thermal equilibrium in all the frames. The two bodies in thermal equilibrium will be having same temperature and temperature is independent of frame of reference. Due to change in frame, if there is any change in the temperature of one body, the heat will flow from hotter body to colder body and as a result same change will be reflected in the temperature of other body. Therefore, the two bodies will be in thermal equilibrium even if the frame changes.

Answer.2

No, the temperature of a body doesn't dependent on the frame from which it is observed. Thermal motion can be considered as the random motion of the particles such as atoms, molecules etc. correlated with its thermal energy. The thermal energy of particles is in turn related to the temperature of that object. Thermal motion differs from ordinary mechanical motion because the particles within the object move in a random manner. The thermal motion doesn't dependent on the frame from which it is observed. As the driving mechanism behind temperature is the thermal motion, we can say that the temperature is also independent of the frame of reference.

Answer.3

Usually all substance expands uniformly with temperature except few fluids such as water. In general, when liquid is heated it expands with temperature as the kinetic energy of the micro particles increases. Due to the anomalous behavior of water, it's volume decreases with temperature from 0°C to 4°C and after 4°C like other fluids it expands with temperature. In a scenario where the temperature scale is yet to be defined, the uniform expansion behavior of mercury can be analyzed by performing a relative study of expansion of mercury with expansion of other fluids. Temperature scale just represents the relative level of magnitude of temperature. Hence, we cannot justify the statement that mercury expands uniformly before temperature scale was defined.

Answer.4

The ideal gas thermometer is based on the ideal gas law. So, it follows ideal gas equation PV=nRT, where R is a proportionality constant known as the ideal gas constant and P is pressure of the gas at constant volume V with n number of moles at temperature T. Therefore, we can say that, P = constant × T. According to this relation, if for a fixed volume, the pressure is directly proportional to the temperature of the gas. To verify this, there is no need to use kinetic theory of gases or any experimental results.

Answer.5

No, bulb of a thermometer can't be made of an adiabatic wall. In thermodynamics, a wall which does not permits thermal interaction is referred as adiabatic wall and a boundary wall that allows heat exchange between the system and surroundings is referred as diathermic wall. To measure the temperature of a body, the bulb of a thermometer is put in contact with it. Within sometime, the bulb reaches the body's temperature allowing us to determine the temperature of body. If the wall of a thermometer is adiabatic, then there will be no exchange of heat energy between two systems and temperature of the body cannot be measured. Therefore, the bulb should be made of diathermic wall not adiabatic wall.

Answer.6

The phenomena of anomalous expansion of water help marine animals to live inside a lake even when the surface of the lake freezes. Similar to all other liquids, volume of a given amount of water decreases with temperature until its temperature reaches 4 °C. Due to the anomalous behavior, the volume increases when the water is cooled below 4 °C and therefore the density decreases. When the temperature of surface water of lake reaches at 4 °C it sinks to the bottom as it becomes denser and water from bottom moves upward to the surface. When the temperature of bottom of lake reaches 4°C the circulation of water stops and at 0°C the surface water freezes. As the ice is less dense than water it floats on the surface of lake. This frozen water acts as an insulator of heat and check the further heat to escape. The water at the bottom of lake remains at 4 °C (constant temperature) as there is no circulation of water. Hence, marine animals can easily survive deep inside the lake even when the surface of the lake freezes.

Answer.7

Without considering all dependent parameters, we cannot reach a conclusion that brass shrinks on heating. Due to heat, during hot summer day, metals tend to expand. The coefficient of expansion differs from metal to metal. As the coefficient of linear expansion of aluminium is more than brass, it will expand comparatively more. As a result there will be an apparent decrease in length of the brass rod, as measured by the aluminium scale. It is not that brass shrinks on heating instead we can say on heating, aluminium expands more than brass.

Answer.8

Yes, we can make. Mercury and glass have equal coefficients of volume expansion which means they have equal volumetric expansion. The coefficient of volume expansion is defined as the fractional change of volume per Kelvin rise in temperature. The accepted value of coefficient of volumetric expansion of mercury is 181 x 10^{-6}/K. When there is a change in temperature, the increase in the volume of the glass tube would be zero. The rise in the glass tube's volume is equal to the difference of real increase in volume and increase in the volume of the container. Hence, mercury thermometer in a glass tube will give accurate reading at every temperature.

Answer.9

The density of water is related to temperature. At the temperature of 4°C the density of water is the highest. Atmospheric pressure is inversely proportional to altitude. So, when altitude increases, the atmospheric pressure decreases. According to the equation $P=h\rho g$, where ρ=density of fluid and P is the pressure of fluid, pressure depends on density. Therefore at 4°C, as the pressure is less, the water density at high altitude will be lower compared to sea level density.

Answer.10

A tightly closed metal lid of a glass bottle can be opened more easily if it is put in hot water for some time because coefficient of expansion of metal is greater than that of glass. So, when the metal lid comes in contact with hot water, it expands at a faster rate than the glass jar, which in turn forms a gap between the metal lid and the glass bottle. This gap increases as the metal lid expands, as a result glass bottle can be opened more easily.

Answer.11

In order to cool down the overheated engine, it is recommended to put water slowly and carefully. There are two reasons for the same. One is to avoid engine to crack down and second reason is safety. If overheated engine parts are cooled too rapidly, then there will be uneven thermal contraction in the parts. As a result, thermal stress is developed and potential cracking of the head and/or block may happen. The blow back pressure of putting cold water in an overheated engine can cause severe burns.

Answer.12

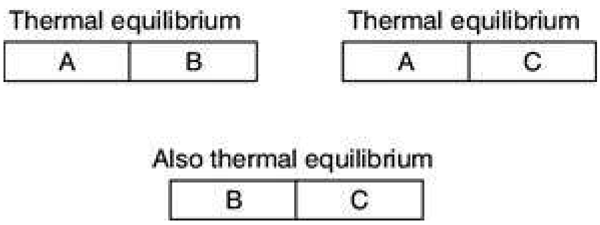
Yes, it possible for two bodies to be in thermal equilibrium if they are not in contact. By definition, even if two objects are not in direct contact, if they are in a closed system they will come into thermal equilibrium with each other if they are in thermal contact. Thermal equilibrium in a closed system is achieved by heat exchange. Suppose there are two bodies X and Y that are not in contact with each other. These two bodies are in contact with another body say, Z. Then according to the Zeroth law of thermodynamics the bodies X and Y will be in thermal equilibrium with each other as X is in thermal equilibrium with Z and Y is in thermal equilibrium with Z. Therefore, it is evident that two bodies can be in thermal equilibrium even if they are not in contact.

Answer.13

When heated, the volume of a spherical shell varies according to the equation $V_\theta = V_0 (1 + \gamma\theta)$. As the volume of spherical shell expands with rise of temperature, the volume referred here is volume of the material used to make up the shell. Here, the coefficient of expansion of volume is γ.

Objective I

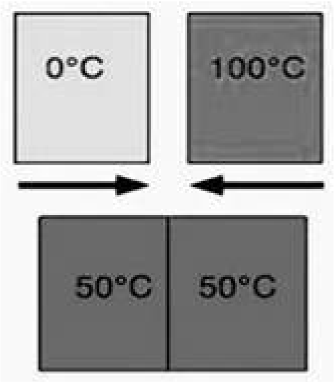
Answer.1



If thermal equilibrium exists between two systems means that those two systems are at the same temperature as that there is no heat exchange in between them.

Given that the system X is not in equilibrium with either Y or Z. If the relation between Y and Z given, we can state (b). But So we cannot say exact equilibrium condition between the systems.

Thus, due to insufficient information option (c) is correct.



Answer.2

conversion between Celsius and Fahrenheit

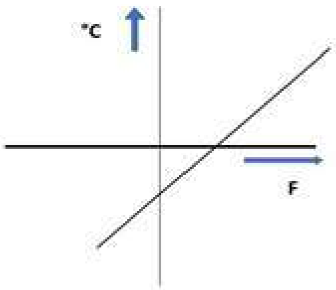
$$°C = (F - 32) * \frac{5}{9}$$

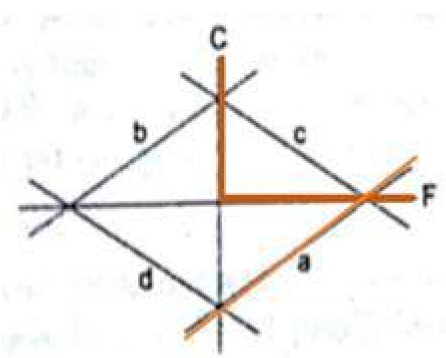
$$F = \left(°C * \frac{9}{5}\right) + 32$$

F = temperature in Fahrenheit

°C = temperature in Celsius)

If we draw the graph between Fahrenheit and Celsius, the curve will lies in the fourth quadrant and it look like the below fig.





(Note: change the scale of x and y-axis to get the exact graph which is shown in the question.)

Thus, a is the correct answer.

Answer.3

Fahrenheit to Kelvin conversion : $K = \frac{5}{9}(F - 32) + 273$

Celsius to Kelvin conversion : $°C = K - 273$

Where K is the temperature in Kelvin

F is the temperature in Fahrenheit

°C is the temperature in Celsius

Let T be the temperature

The relation between Fahrenheit, Kelvin is given by

$$\frac{T - 32}{9} = \frac{T - 273}{5}$$

$$T = \frac{(9 * 273) - (5 * 32)}{4}$$

$$T = 574.59$$

$$T = T - 273$$

• At 574.59 Fahrenheit the temperature equals to 574.59 Kelvin (574.59 °F= 574.59 K).

• In Celsius and Kelvin, we don't have any existing numerical value Which is having the same temperature value.

• Whereas in kelvin and platinum, because of made up with different types of materials as temperature measuring substances they do not agree with each other and not stand with the same temperature values.

Answer.4

For the proper function of the constant volume gas thermometer we have filled is with an ideal gas. The ideal condition for the gas is only available at high temperatures and low pressures. In ideal gas, particles move freely and do not interact with each other.

Answer.5

The dimension of the coefficient of

Linear expansion = $\frac{[L]}{[LT]} = \frac{1}{[T]} = \frac{1}{K} = K^{-1}$

Volume expansion=$\frac{[L^3]}{[L^3 T]} = \frac{1}{[T]} = \frac{1}{K} = K^{-1}$

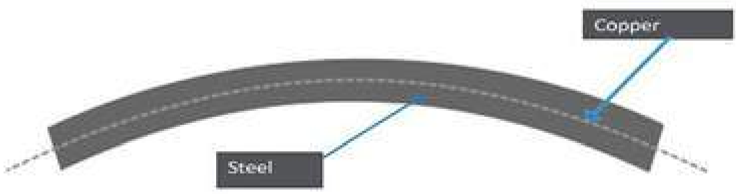
Thus, the linear and volume expansion of the gas have the same dimensions.

Hence, option A is the correct.

Answer.6

If the metal is heated it will expand, if the metal sheet is having a circular or any type of hole, it also a part of the metal. So, when it is getting heated, it will expand, and the whole size will be increased. Hence, the circular hole will become stronger.

Answer.7



Linear expansion coefficient of copper is greater than the steel ($\alpha_{copper} > \alpha_{steel}$). So, on heating copper gets more expansion than steel. So the bimetallic strip will bend with the copper on the convex side.

Linear expansion mathematically expressed as

$$\frac{\Delta L}{L} = \alpha \Delta T$$

Where,

ΔL = change in length

L= original length

α = Linear coefficient of thermal expansion

ΔT = change in temperature

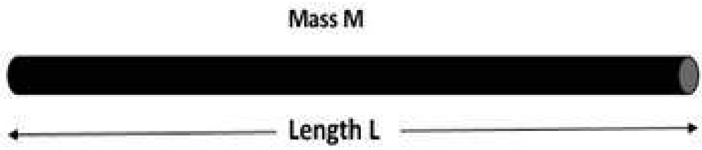
Linear expansion coefficient for steel and copper are mentioned in the below table.

Material	Fractional expansion per Fahrenheit × 10^{-6}	Fractional expansion per °C × 10^{-6}
Steel	7.2	13
Copper	9.4	17

Conclusion line missing??

Answer.8

Moment of inertia of a uniform rod is



$$I = \frac{1}{12} M L^2$$

when the temperature is increased by Δt then increased length is

$$L' = L(1 + a\Delta T)$$

the moment of inertia is

$$I' = \frac{M}{12}(L')^2$$

$$= \frac{M}{12} L^2 (1 + a\Delta T)^2$$

$$= \frac{ML^2}{12} (1 + a^2(\Delta T)^2 + 2a\Delta T)$$

there is only a slight change in length so a^2 $(\Delta T)^2$ value is very less, so we can ignore this term.

$$I' = I(1 + 2a\Delta T)$$

Hence, the increase in moment of inertia is

$$I' - I = 2a\Delta T$$

Hence, option C is the correct option.

Answer.9

The moment of inertia of uniform rod is $I = \frac{1}{12} ML^2$

Where,

I is the moment of inertia

M is the mass of the rod,

L is the length of the rod,

$$I' = \frac{M}{12} (L')^2$$

Where, $L' = L(1 + a\Delta T)$

L' is the new length of the rod after thermal expansion.

Putting it in the above relation, we get

$$= \frac{M}{12} L^2 (1 + a\Delta T)^2$$

$$= \frac{ML^2}{12}(1 + a^2(\Delta T)^2 + 2a\Delta T)$$

$$I' = I(1 + 2a\Delta T)$$

$$I' - I = 2a\Delta T$$

Hence, option C is the correct option

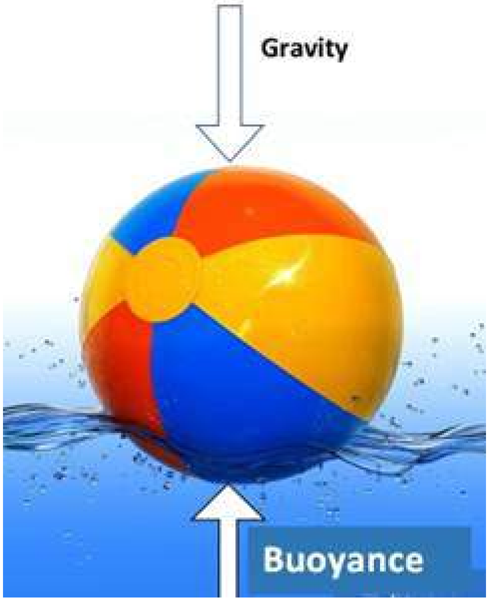
Answer.10

Temperature at the bottom of the lake is more than the temperature on the surface of the lake., Water gets its maximum density at 4°c. Above or below 4°c, the water is less dense. So the expected temperature at the bottom is 4°c.

Answer.11

Buoyancy is nothing but the force which is

responsible for the body to flow.



If the temperature is increased, then the density of the water will decrease, and the aluminium ball will start expanding leading to increase in its volume which leads to decrease in the volume displaced by the aluminium sphere. Therefore, the buoyancy will decrease.

Objective II

Answer.1

• when the body is in motion, it will exhibit kinetic energy. If the speed is decreased, then the kinetic energy will be decreased.

• Mechanical energy is the sum of both potential and kinetic energies. Due to a decrease in kinetic energy, mechanical energy also decreases.

• If the wheel gets slowdowns due to friction the mechanical energy is converted into internal energy leads to an increase in internal energy.

Answer.2

• Because of the friction, if the wheel B is brought into spinning, it will gain kinetic energy leads to an increase in mechanical energy.

• Due to friction heat will be generated which will lead to an increase in all energies.

• Internal energy is increasing due to a rise in temperature.

Answer.3

• Body A only exhibits potential energy

• Body B exhibits kinetic energy due to motion along with the train.

• Kinetic, mechanical (potential + kinetic) and total (all energies) of B are greater than A.

Answer.4

both Celsius and ideal gas scale measures temperature in Kelvin and ideal gas scale are called an absolute scale some of the time.

Answer.5

There is no heat loss in case of the adiabatic container. After sometime the heat will transfer to the solid object leads to an increase in the temperature of the object, and the temperature of the water will decrease.

Answer.6

Time period of the pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

T = time period of the pendulum

L = length of the pendulum

g = acceleration due to gravity

For Linear expansion:

$$\frac{\Delta L}{L} = \alpha \Delta T$$

Where ΔT is the change in temperature

α is the coefficient of linear expansion

If the temperature is increased due to the linear expansion, the length of the pendulum will increase which leads to increase in the time period.

Exercises

Answer.1

20° C

Explanation:

Given,

Steam point of the given scale, T_{steam}, is 80°,

Ice point of the given scale, T_{steam}, is 20°,

Temperature in centigrade scale corresponding to 32° , t=?

Formula used

The temperature of a mercury thermometer can be assumed to be in linear relation with its mercuric height or length. So the Temperature (T) can be represented as

$T = Al + B,$

where l represents the length; A, B are constants depending on the steam (80°) and ice point (20°) of this scale

The centigrade mercury scale temperature (t) can be represented as,

$t°C = al + b,$

Where l represents the length, a and b are constants depending on the steam (100°) and ice point (0°) of this scale.

We can conclude that, for fixed steam and ice points, the temperature is directly proportional to the mercuric height.

As both the scales use the mercuric height as the measuring parameter, these scales can be compared as;

$$\frac{(T - T_{ice})}{(T_{steam} - T_{ice})} = \frac{(t°C - t_{ice}°C)}{(t_{steam}°C - t_{ice}°C)}$$

Where T represents the temperatures in the given mercury scale and t represents that of the Centigrade scale.

By putting the given values in the above equations;

$$\frac{(32 - 20)}{(80 - 20)} = \frac{(t°C - 0°C)}{(100°C - 0°C)}$$

So,

$$t = 20°C$$

Hence, temperature t in centigrade scale corresponding to 32° of the given scale is 20°C (Ans.)

Answer.2

373.318K

Explanation:

Given,

Pressure at boiling point, P_{bp}, is 2.050 × 10^4 Pa

Pressure at triple point point, P_{tp}, is 1.500 × 10^4 Pa

Formula used

At constant volume(V), the relation between the temperature (T) and

pressure (P) of a volumetric thermometer can be represented as,

$$P \propto T$$

or

$$\frac{P}{T} = c$$

where c is a constant

The temperature at triple point pressure (P_{tp}) of water is standardized as, T_{tp} =273.16K. By comparing values at two states, the above equation can be written as,

$$\frac{P_{bp}}{T_{bp}} = \frac{P_{tp}}{T_{tp}}$$

Where,

Subscripts 'tp' and 'bp' represents triple point and boiling point respectively.

So, the temperature at the Boiling point,

$$T_{bp} = \frac{P_{bp}}{P_{tp}} \times T_{tp}$$

Substituting the values in the above formula, we get

$$T_{bp} = \frac{2.050 \times 10^4 \ Pa}{1.500 \times 10^4 \ Pa} \times 273.16K$$

Or,

$$T_{bp} = 373.18K$$

Hence, temperature at normal Boiling point, $T_{bp} = 373.18K$ (Ans.)

Answer.3

$600.95K$

<u>Explanation:</u>

Given

Pressure at Melting point= 2.20×Pressure at triple point of water

Formula used

The relation connecting the temperature (T) and pressure (P) of a volumetric thermometer can be represented as,

$$P = cT$$

To compare at two states, this equation can be written as,

$T = \frac{P}{P_{tp}} \times 273.16$K (eqn. 1)

Here Pressure, P at the melting point is given as 2.20 times the triple point of water P_{tp},

i.e, $P = 2.20 \times P_{tp}$

Putting this in eqn. 1, we get,

$$T = \frac{2.20 * P_{tp}}{P_{tp}} \times 273.16K$$

$= 600.95K$ (Ans.)

So, the melting point of lead is 600.95K

Answer.4

$54.64\ kPa$

Explanation:

Given

Pressure at triple point of water, P_{tp}, is 40kPa

Boiling point of water, T, is 100°C

Formula used

The relation connecting the temperature (T) and pressure ( ) of a volumetric thermometer can be represented as,

$T = cP$

Which leads us to the relation, P

$T = \frac{P}{P_{tp}} \times 273.16$K

where,

P_{tp} = Pressure at triple point of water= 40kPa

Here, we have to find the pressure at temperature 100°C (i.e, 373.15K), So substitute, T= 373.15K in the above equation

$$P = \frac{P_{tp} \times T}{273.16K}$$

$$= \frac{40\ kPa \times 373.15K}{273.16K}$$

i.e., $P = 54.64\ kPa.$ (Ans.)

Hence, the pressure measured at the boiling point of water is 54.64 kPa≈56 kPa

Answer.5

95.6 kPa.

Explanation:

Given,

The pressure at ice point, P_1, is 70kPa

Formula Used

The relation connecting the temperature (T) and pressure(P) of a volumetric thermometer can be represented as,

$T = \frac{P}{P_{tp}} \times 273.16$K (eqn. 1)

$T_1 = \frac{P_1}{P_{tp}} \times 273.16$K (eqn. 2)

$T_2 = \frac{P_2}{P_{tp}} \times 273.16$K (eqn. 3),

Where,

subscripts 1 and 2 in the above equations represents the ice point and steam point respectively. So,

$$T_1 = 273.15K, P_1 = 70kPa$$

$$T_2 = 373.15K,$$

$$P = \frac{T \times P_{tp}}{273.16K}$$

By dividing (eqn. 3) by (eqn. 2),

$$\frac{T_2}{T_1} = \frac{P_2 \times P_{tp} \times 273.16K}{P_1 \times P_{tp} \times 273.16K}$$

Or,

$$P_2 = \frac{T_2 \times P_1}{T_1}$$

$$= \frac{373.15K \times 70kPa}{273.15}$$

$$= 95.6\ kPa$$

P_2=95.6 kPa(Ans.)

Hence the pressure at steam point is 95.6 kPa ≈96 kPa

Answer.6

200°C

<u>Explanation:</u>

Given,

Pressure at ice point, P_{ice} , is 80 cm of Hg

Pressure at steam point, P_{steam}, is 90cm of Hg

Pressure of wax bath, P, is 100cm of Hg

Formula used

We have the relation between Pressure and temperature of a constant volume gas thermometer as

$$P \propto T$$

This can be written as

$$\frac{(P - P_{ice})}{(P_{steam} - P_{ice})} = \frac{(T - T_{ice})}{(T_{steam} - T_{ice})}$$

Where,

P_{ice} =Pressure at ice point=80cm of Hg

P_{steam}= Pressure at steam point = 90cm of Hg,

P=Pressure of wax bath=100cm of Hg,

T_{ice}=Ice point temperature= 0°C,

T_{steam}= Steam point temperature=100°C

T=?

Putting the given values in the above equation results,

$$T = \frac{100cm - 80cm}{90cm - 80cm} \times 100°C$$

$= 200°C$ (Ans.)

The temperature of the wax bath is 200° C

Answer.7

$307.125K$

Explanation:

Given:

Initial volume, V, is 1800CC

The poured out amount for compensation, V', is 200CC

Ice point temperature in Callender's compensated thermometer setup, $T_{0,}$ is 273K

Formula used:

From the ideal gas equation, for a constant pressure thermometer,

$V \propto T$

Where, V and T represents Volume and Temperature respectively.

Or, it can be written as,

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Here in the case of Callender's compensated air thermometer, the equation becomes,

$$\frac{V}{V-V'}=\frac{T}{T_0}$$

Where,

V= Initial volume= 1800CC

V'= The poured out amount for compensation=200CC

T_0=Ice point temperature= 273K

Hence, from the equation, the temperature of vessel,T

$$T=\frac{1800}{1800-200}\times 273=307.125K \text{ (Ans.)}$$

So, the temperature of the vessel =307.125K≈307K

Answer.8

60°C

Explanation:

Given

Resistance of thermometer at 0°C, R_0, is 80Ω

Resistance of thermometer at 100°C, R_{100}, is 90Ω

Formula used

Resistance varies linearly with temperature in a platinum resistance thermometer. The governing equation is,

$$T=\frac{R_T-R_0}{R_{100}-R_0}\times 100$$

Where,

R_T=resistance at which temperature is to be measured,

R_0=resistance at 0°C,

R_{100}=resistance at 100°C.

By substituting the values,

$$T = \frac{86-80}{90-80} \times 100 = 60°C \text{(Ans.)}$$

Hence the temperature at 86Ω is 60° C

Answer.9

$$\boldsymbol{R_0} = 20\,, \alpha = 3.806 \times 10^{-3}\,, \beta = -5.58 \times 10^{-7}$$

Explanation:

Given:

$R_{\theta 1}$= Resistance at 0°C=20.0Ω

$R_{\theta 2}$= Resistance at 100°C=27.5Ω

$R_{\theta 3}$= Resistance at 420°C=50.0Ω

R_0 = ?, α=?, β=?

Formula used

$$R_\theta = R_0\,(1 + \alpha\theta + \beta\theta^2)$$

The values for 3 resistance (R_θ) are given at 3 temperatures (θ),

So, at temperature 0° C,

$$R_{\theta 1} = R_0\,(1 + \alpha \times 0 + \beta \times 0^2), \text{(eqn. 1)}$$

At 100° C,

$$R_{\theta 2} = R_0\,(1 + \alpha \times 100 + \beta \times 100^2), \text{(eqn. 2)}$$

At 420° C,

$$R_{\theta 2} = R_0\,(1 + \alpha \times 420 + \beta \times 420^2), \text{(eqn. 1)}$$

Solving this three equation simultaneously for three unknowns,

From eqn. 1, we get,

$$R_{\theta 1} = R_0 = 20\Omega \text{(Ans.)}$$

Putting $R_0 = 20\Omega$ in eqn.2 and eqn.3, and substituting given values, we get,

Eqn.2 as $27.5\Omega = 20\Omega\,(1 + \alpha \times 100 + \beta \times 100^2)$, and

Eqn.3 as $50.0\Omega = 20\Omega(1 + \alpha \times 420 + \beta \times 420^2)$

Now, from eqn.2 after re arranging, we get,

$\alpha = \frac{0.375 - 100^2 \times \beta}{100}$, (eqn.4)

Putting this value in eqn. 3, we get

$$50.0\Omega = 20\Omega(1 + \left(\frac{0.375 - 100^2 \times \beta}{100}\right) \times 420 + \beta \times 420^2)$$

Or,

$$1.5 = 4.2 \times 0.375 - 100^2 \times \beta + 420^2 \times \beta$$

Or,

$\beta = -5.58 \times 10^{-7}$ (Ans.)

Putting the value of β in eqn.4 , we get

$$\alpha = \frac{0.375 - 100^2 \times -5.58 \times 10^{-7}}{100}$$

$\alpha = 3.806 \times 10^{-3}$ (Ans.)

Hence, the required values are

$\boldsymbol{R_0} = 20, \alpha = 3.806 \times 10^{-3}, \beta = -5.58 \times 10^{-7}$ (Ans.)

Answer.10

$10.0035m$

<u>Explanation:</u>

Given

The length of the lab at 0° C, L_0, is 10m

Change in temperature of slab from 0°C, θ is 35

Coefficient of linear expansion,

Formula used

The relation between the ength of the road at any other temperature can be written as,

$$L_t = L_0(1 + \alpha\theta)$$

Where

L_0= Length at a reference temperature= 10m

α= coefficient of linear expansion= $1.0\times10^{-5}°C^{-1}$

θ= Change in temperature= 35

$$L_t = 10m(1 + 1.0 \times 10^{-5}\ °C^{-1} \times 35)$$

Or

$L_t = 10.0035m$ (Ans.)

So, the length of the slab at 35°C is 10.0035m

Answer.11

0.99989cm

Explanation:

Given:

The temperature at which the steel meter is calibrated, t_1 is 20^oC

The temperature at which the scale is used, t_2 is 10^oC

The linear expansion coefficient of the scale is $1.1 \times 10^{-5}{}^oC^{-1}$.

Formula used:

The relation connecting the length at a specified temperature (L_t) and that of with a standardized temperature can be written as,

$$L_t = L_0(1 + \alpha\theta)$$

Where,

L_0= Length at a reference temperature,

α= coefficient of linear expansion,

θ= Change in temperature.

Here, 20°C can be taken as reference temperature and hence, L_0=length between adjacent centimeter markings=1cm= 0.01m

α=1.1× 10^{-5}°C^{-1},

θ= -10° C

Hence,

$$L_t = 0.01(1 + 1.1 \times 10^{-5} \times (-10))$$

=0.0099989m

=0.99989cm(Ans.)

The final length between 50cm and 51cm marks will be 0.99989cm

Answer.12

0.4cm

<u>Explanation:</u>

Given:

The initial length of the iron track, L_0, is 12 m

The initial temperature of the iron track is 18°C

The final temperature of the iron track is 48°C

The final length of the iron track be L_t

Formula used:

The relation connecting the length at a specified temperature (L_t) and that of with a standardized temperature can be written as,

$$L_t = L_0(1 + \alpha\theta)$$

L_0= Length at a reference temperature, α= coefficient of linear expansion, θ= Change in temperature.

L_0 =12m , α=11 ×10^{-6} °C^{-1}

Case 1: At temperature 18°C,

$$L_{18} = 12(1 + 11 \times 10^{-6} \times 18) = 12.00237m$$

Case 2: At temperature 48°C

$$L_{48} = 12(1 + 11 \times 10^{-6} \times 48) = 12.006336m$$

Now the difference between these two lengths should be the value of the gap.

So,

$$L_{48} - L_{18} = 12.006336 - 12.00237$$

$$= 3.966 \times 10^{-3} m$$

$= 0.4cm$(Ans.)

So then there must be a gap of 0.4cm between the rails.

Answer.13

2.0046cm

<u>Explanation:</u>

Given

Initial temperature of plate is 0°C

Final temperature of plate is 100°C

Diameter at 0°C, D_0 is 0.02m

Coefficient of linear expansion, α is 2.3×10^{-5} °C^{-1}

Temperature change from 0°C, θ is 100

Formula used

The relation between the diameter at a certain temperature with respect to that of a reference temperature can be written similar to the case of linear expansion.

$D_t = D_0(1 + \alpha\theta)$, where

D_t=Diameter at temperature t

D_0=Diameter at 0°C

α=Coefficient of linear expansion=2.3×10^{-5} °C^{-1}

θ=Temperature change from 0°C=100

Hence, for 100°C, the diameter

$D_{100} = 0.02(1 + 2.3 \times 10^{-5} \times 100) = 0.020046m$

$= 2.0046cm$ (Ans.)

The final diameter will be 2.0046 cm

Answer.14

a) 0.99977 b) 1.000249 c) 1.000959

Explanation:

Given,

Coefficient of linear expansion of Aluminium, α_{Al}, is 2.3×10^{-5}°C^{-1}

Coefficient of linear expansion of steel, α_S, is 1.1×10^{-5}°C^{-1}

The scale is calibrated at the reference temperature of 20°C

Formula used $L_t = L_0(1 + \alpha\theta)$

L_0= Length at a reference temperature,

α= coefficient of linear expansion,

θ= Change in temperature.

'Al' and 'S' will be used as subscripts to denote Aluminium and Steel respectively.

a) at 0°C

$$\frac{L_{Al}}{L_S} = \frac{L_0(1+\alpha_{Al}(-20))}{L_0(1+\alpha_S(-20))} = \frac{(1-2.3\ \times 10^{-5}C^{-1}\times 20)}{(1-1.1\ \times 10^{-5}C^{-1}\times 20)}$$

$$= 0.99977\ (\text{Ans.})$$

b) at 40°C

$$\frac{L_{Al}}{L_S} = \frac{L_0(1+\alpha_{Al}(20))}{L_0(1+\alpha_S(20))} = \frac{(1+2.3\ \times 10^{-5}C^{-1}\times 20)}{(1+1.1\ \times 10^{-5}C^{-1}\times 20)} = 1.000249\ (\text{Ans.})$$

c) at 100°C

$$\frac{L_{Al}}{L_S} = \frac{L_0(1+\alpha_{Al}(80))}{L_0(1+\alpha_S(80))} = \frac{(1+2.3\ \times 10^{-5}C^{-1}\times 80)}{(1+1.1\ \times 10^{-5}C^{-1}\times 80)} = 1.000959\ (\text{Ans.})$$

Answer.15

Let T_0 be the temperature at which the meter scale measures the correct length, T_0 = 16° = 289 K

Let the length of the meter scale be l

Given:

The coefficient of linear expansion of the steel meter scale (α), α = 11× 10^{-6} C^{-1}

a)

Given:

The temperature at which the scale measures during a summer day = T_S = 46° = 319 K

Therefore, Δ T = 319 – 289 = 30K

The change in length due to linear expansion as a consequence of rise in temperature is given as,

ΔL= lαΔT

So the above equation can be written as,

$$\frac{\Delta l}{l} = \alpha \Delta T$$

The percentage error is given as,

$$\% \text{ error} = \left(\frac{\Delta l}{l} \times 100\right)\%$$

$$\alpha \Delta \text{T} \times 100 = 11 \times 10^{-6} \times 30 \times 100 = 0.033\%$$

b)

Similarly,

The temperature at which the scale measures during a winter day = $T_W = 6° = 279$ K

Therefore, Δ T = 289 – 279 = 10K

The change in length due to linear expansion as consequence of fall in temperature is given as,

$\Delta L = l\alpha\Delta T$

So the above equation can also be written as,

$$\frac{\Delta l}{l} = \alpha \Delta T$$

The percentage error,

$$\% \text{ error} = \left(\frac{\Delta l}{l} \times 100\right)\%$$

$$\alpha \Delta \text{T} \times 100 = 11 \times 10^{-6} \times 10 \times 100 = 0.011\%$$

Answer.16

Given:

Temperature at which meter scale gives accurate reading: $T_1=20°$ C

Change in Length required: Δ L=0.055mm = 0.055×10^{-3} m.

Original Length: L= 1 m.

Coefficient of linear expansion of steel: $\alpha = 11 \times 10^{-6}\ °C^{-1}$.

Formula used:

Now, we have the direct formula for change in length due to thermal expansion:

$$\Delta L = \alpha L \Delta T$$

Where Δ L is the change in length.

Where Δ T is the change in temperature,

it can be $\mathbf{T_1+T_2}$ OR $\mathbf{T_1-T_2}$.

First let's take T_1-T_2.

$$0.055 \times 10^{-3} m = 11\, x\, 10^{-6}\,°C^{-1} \times 1\, m \times (20 - T_2)°C$$
$$\therefore \frac{0.055 \times 10^{-3} m}{11 \times 10^{-6}\,°C^{-1} \times 1\, m} = (20 - T_2)°C 5\,°C = 20\,°C - T_2$$
$$\therefore T_2 = (20 - 5)°C \therefore T_2 = 15°$$

Now, let's take T_1+T_2,

$$0.055 \times 10^{-3} m = 11 \times 10^{-6}\,°C^{-1} \times 1\, m \times (20 + T_2)°C$$
$$\therefore \frac{0.055 \times 10^{-3} m}{11 \times 10^{-6}\,°C^{-1} \times 1\, m} = (20 + T_2)°C \therefore 5\,°C = 20\,°C + T_2$$
$$\therefore T_2 = (20 + 5)°C \therefore T_2 = 25°$$

Hence, the range of the temperature within which this experiment can be performed with the given metre scale is 15° C to 25° C.

Answer.17

Given:

Density of water at 0 °C: ρ_0 = 0.998 g cm^{-3}

Density of water at 4 °C: ρ_4 = 1.000 g cm^{-3}

Temperature Range: θ = 4 °C

Formula used:

We know that the formula for volume expansion is:

$$V_\theta = V_0(1 + \gamma\theta)$$

Where V_θ is the volume at θ °C

V_0 is the volume at 0 °C

γ is the coefficient of volume expansion

Now, since $Density = \rho = \frac{Mass}{Volume} = \frac{M}{V}$

On substituting : $\therefore \frac{M}{\rho_\theta} = \frac{M}{\rho_0(1+\gamma\theta)} \therefore \rho_0 = \frac{\rho_\theta}{1+\gamma\theta} \therefore \rho_0 = \frac{\rho_4}{1+\gamma\times 4}$

$\therefore 0.998\ g\ cm^{-3} = 1.000\ g\ cm^{-3} \frac{1}{(1+\gamma\times 4)} \therefore 1+\ \gamma\times 4 = \frac{1}{0.998}$

$\therefore \gamma\times 4 = 1.002 - 1 \therefore \gamma = \frac{0.002}{4}\ °C^{-1} \therefore \boldsymbol{\gamma = 5\ x\ 10^{-4}\ °C^{-1}}$

As density decreases $\gamma = -5 \times 10^{-4}\ ^\circ C^{-1}$

Hence the average Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C is $\boldsymbol{\gamma = 5\ x\ 10^{-4}\ °C^{-1}}$

Answer.18

Given: Coefficient of Linear Expansion of iron rod : $\alpha_{Fe} = 12 \times 10^{-6}\ °C^{-1}$ Coefficient of Linear Expansion of Aluminium Rod : $\alpha_{Al} = 20 \times 10^{-6}\ °C^{-1}$ **Formula used:**

The formula for Linear Expansion is:

$$\boldsymbol{L' = L(1 + \alpha\Delta T)}$$

Let, L_{Fe} and L_{Al} be the original lengths of the iron and aluminium rods respectively.Let, L'_{Fe} and L'_{Al} be the changed lengths (ΔL) of iron and aluminium rods respectively when temperature is changed by ΔT.Now ,

$$L'_{Fe} = L_{Fe}(1 + \alpha_{Fe}\Delta T)\ and\ L'_{Al} = L_{Fe}(1 +\ \alpha_{Al}\Delta T)$$

Since , the difference in lengths is independent of temperature it shows that their difference is constant. That is, $L'_{Fe} - L'_{Al} = L_{Fe} - L_{Al}$

$\therefore L'_{Fe} - L'_{Al} = L_{Fe} + (L_{Fe} \times \alpha_{Fe}\Delta T) - L_{Al} - (L_{Al} \times \alpha_{Al}\Delta T)$
$\therefore\ L'_{Fe} - L'_{Al} = L_{Fe} - L_{Al} + (L_{Fe} \times \alpha_{Fe}\Delta T) - (L_{Al} \times \alpha_{Al}\Delta T)$
$\because Difference\ is\ independent\ of\ temperature\ \Delta T\ can\ be\ cut\ off.$ By

arranging the terms we get,$\therefore L_{Fe} \times \alpha_{Fe} = L_{Al} \times \alpha_{Al} \therefore \frac{L_{Fe}}{L_{Al}} = \frac{\alpha_{Al}}{\alpha_{Fe}}$

$$\therefore L_{Fe}:L_{Al} = \frac{23 \times 10^{-6}}{12 \times 10^{-6}}$$

$$\therefore \boldsymbol{L_{Fe}:L_{Al} = 23:12}$$

Hence, the ratio of the lengths of an iron rod and an aluminium rod for which the difference in the lengths is independent of temperature is 23:12.

Answer.19

Given: Temperature at which the pendulum shows correct time:T_1=20°CValue of gravitational acceleration at a place where T_1 is 20°C is: g_1= 9.8 m s^{-2} Value of g at different place is: g_2= 9.788 m s^{-2}Coefficient of linear expansion of steel: $\alpha = 12 \times 10^{-6}\ °C^{-1}$

Let, T_2 be the temperature at a place where value of g is 9.788 m s^{-2}.**Formula used:**Now the formula for time period is:$\boldsymbol{t = 2\pi\sqrt{\frac{l}{g}}}$

Where, l is the length of the rod and g is the acceleration due to gravity.For places where values of g is different ,say :

$$t_1 = 2\pi\sqrt{\frac{l_1}{g_1}}$$

And

$$t_2 = 2\pi\sqrt{\frac{l_2}{g_2}}$$

Where l_1 is the original length of the rod at T_1 and l_2 is the changed length at T_2.

Since, for obtaining correct time both the time periods should be same,That is: t_1=t_2

$\therefore 2\pi\sqrt{\frac{l_1}{g_1}} = 2\pi\sqrt{\frac{l_2}{g_2}} \therefore \sqrt{\frac{l_1}{g_1}} = \sqrt{\frac{l_2}{g_2}}$

Since length changes due to linear expansion, by formula of linear expansion : $\boldsymbol{L' = L(1 + \alpha\Delta T)}$

Where L' is the changed length. $\therefore l_2 = l_1(1 + \alpha\Delta T)$

Substituting this result in the equality above we get,$\therefore \sqrt{\frac{l_1}{g_1}} = \sqrt{\frac{l_1(1+\alpha\Delta T)}{g_2}}$

$\therefore \frac{l_1}{g_1} = \frac{l_1(1+\alpha\Delta T)}{g_2} \therefore \frac{g_2}{g_1} = 1 + 12 \times 10^{-6}\,°C^{-1} \times \Delta T$

$\therefore \frac{9.788}{9.800} - 1 = 12 \times 10^{-6}\,°C^{-1} \times \Delta T \therefore \frac{-1.2244 \times 10^{-3}}{12 \times 10^{-6}}\,°C = \Delta T$

$\therefore \Delta T = -102.04\,°C \because \boldsymbol{\Delta T = T2 - T1} \therefore T_2 - 20 = -102.04$

$\therefore T_2 = -102.04 + 20 \therefore T2 = -82.04\,°C$

Hence, at T_2 = -82.04 ° C the pendulum will give correct time when it is taken to a place where g is 9.788 m s^{-2}.

Answer.20

Given: Diameter of the hole in the aluminium plate : d_{Al}= 2.000 cmDiameter of the Steel sphere resting on the hole: d_{st}= 2.005 cmInitial temperature : T_1= 10 ° CCoefficient of linear expansion of aluminium : α_{Al} =23 × 10^{-6} $°C^{-1}$Coefficient of linear expansion of Steel : α_{st} = 11 × 10^{-6} $°C^{-1}$

Now, when the temperature of the entire system is increased thermal expansion will take place.**Formula used:**The formula for linear expansion is: $\boldsymbol{L' = L(1 + \alpha\Delta T)}$where L' is the changed length.Similarly, change in diameter of aluminium plate is:$d'_{Al} = d_{Al}(1 + \alpha_{Al}\Delta\mathrm{T})$

change in diameter of Steel sphere is:

$d'_{st} = d_{st}(1 + \alpha_{st}\Delta)$Where, d'_{Al} and d'_{st} is the changed diameter of aluminum plate and steel sphere respectively.*Now, for the steel ball to fall through the hole the changed diameter of the steel ball and the aluminum plate should be equal.*$\therefore d'_{Al} = d'_{st} \therefore d_{Al}(1 + \alpha_{Al}\Delta\mathrm{T}) = d_{st}(1 + \alpha_{st}\Delta)$

$\therefore 2\,cm + 2 \times 23 \times 10^{-6}\ cm\,°C^{-1} \times \Delta T$

$= 2.005\ cm + 2.005 \times 11 \times 10^{-6}\ cm\,°C^{-1} \times \Delta T$

$\therefore (2 - 2.005)cm = (22.055 \times 10^{-6} - 46 \times 10^{-6})\ cm\,°C^{-1} \times \Delta T$

$\therefore -5 \times 10^{-3} cm = -23.945 \times 10^{-6}\ cm\,°C^{-1} \times \Delta T$

$\therefore \Delta T = \frac{5 \times 10^{-3}}{23.945 \times 10^{-6}\,°C^{-1}} \therefore \Delta T = 208.811\ °C$*Now,* $\Delta T = T_1 - T_2$

$\therefore T_1 - T_2 = 208.811 \therefore T_2 = 208.811 + 10 \therefore T_2 = 218.811\ °C$

Hence the temperature at which the ball will fall down is T_2 ***= 218.811 ° C***

Answer.21

Given:Change in Temperature : ΔT= 40 ° C…(From- winter temperature is 0°C to working day temperature is 40°C)

Length of the Glass window : l_{Gl}= 20 cmBreadth of the Glass Window: b_{Gl} = 30 cm.Coefficient of linear expansion for glass :$\alpha_{gl} = 9.0 \times 10^{-6}\ °C^{-1}$

Coefficient of linear expansion for Aluminum: $\alpha_{Al} = 24 \times 10^{-6}\,°C^{-1}$.

Due to a drop in temperature, thermal contraction will take place(opposite of thermal expansion) and thus the dimensions of both aluminium frame and glass window will undergo contraction. In order to avoid crack in the glass window, the contracted size of both the frame and the window should be same.**Formula used:**We will see this numerically:

Formula for thermal contraction is:

$l' = l(1 - \alpha\Delta T)$Here, l' is the changed length and the value of α is negative (since its contraction).Now,Let the changed length of aluminium frame = Changed length of glass window.$\therefore l'_{Al} = l'_{gl}$

$\therefore l_{Al}(1 - \alpha_{Al}\Delta T) = l_{Gl}(1 - \alpha_{Gl}\Delta T)$

$\therefore l_{Al} = \frac{20\ cm \times (1 - 9.0 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40)} \therefore l_{Al} = \frac{20 \times 0.99964}{0.99904}\ cm$

$\therefore l_{Al} = 20.01\ cm$Now,Let the changed breadth of aluminium frame = Changed breadth of Glass window$\therefore b'_{Al} = b'_{Gl}$

$\therefore b_{Al}(1 - \alpha_{Al}\Delta T) = b_{Gl}(1 - \alpha_{Gl}\Delta T)$

$\therefore b_{Al} = \frac{30\ cm \times (1 - 9.0 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40)} \therefore l_{Al} = \frac{30 \times 0.99964}{0.99904}\ cm$

$\therefore b_{Al} = 30.018\ cm$Hence, the dimensions of the aluminium frame so that there is no stress on the glass in winter are **20.01 cm × 30.018 cm**

Answer.22

Given: Volume of Glass vessel at $T_1 = 20\ °C$ is : $V_g = 1000$ cc.

Coefficients of cubical expansion of mercury: $\gamma_{Hg} = 1.8 \times 10^{-4}\ °C^{-1}$.

Coefficients of cubical expansion of Glass : $\gamma_g = 9.0 \times 10^{-6}\ °C^{-1}$.

Now Let the Volume of mercury at $T_1 = 20°$ C be V_{Hg}. We need to find V_{Hg}.

Due to Change in temperature, Volume Expansion takes place.

When mercury is added in the glass, it consumes some amount of volume V_{Hg}, thus the remaining space is $\mathbf{V_g - V_{Hg}}$. **----(Initial).**

When volume expansion takes place the changed volume of glass and mercury inside it is V'_g and V'_{Hg} respectively. Thus the remaining space after the volume expansion is $\mathbf{V'_g - V'_{Hg}}$.**--(Final)**

The given condition says that the remaining space should not change with temperature which means: Initial = Final

$$V_g - V_{Hg} = V_g' - V_{Hg}'$$

Formula used : $\mathbf{V' = V(1 + \gamma\Delta T)}$ Where V' is the changed volume at T_2 and V is the initial volume at T_1.

For Glass: $V'_g = V_g(1 + \gamma_g\Delta T)$

For Mercury : $V'_{Hg} = V_{Hg}(1 + \gamma_{Hg}\Delta T)$

Substituting we get,

$$\therefore V_g - V_{Hg} = V_g(1 + \gamma_g\Delta T) - V_{Hg}(1 + \gamma_{Hg}\Delta T)$$
$$\therefore V_g - V_{Hg} = V_g + V_g\gamma_g\Delta T - V_{Hg} - V_{Hg}\gamma_{Hg}\Delta T$$
$$\therefore V_g - V_{Hg} = V_g - V_{Hg} + (V_g\gamma_g - V_{Hg}\gamma_{Hg})\Delta T \therefore (V_g\gamma_g - V_{Hg}\gamma_{Hg})\Delta T = 0$$
$$\therefore V_g\gamma_g = V_{Hg}\gamma_{Hg} \therefore V_{Hg} = \frac{V_g\gamma_g}{\gamma_{Hg}} \therefore V_{Hg} = \frac{1000\ cc \times 9.0 \times 10^{-6}\ °C^{-1}}{1.8 \times 10^{-4}\ °C^{-1}}$$
$$\therefore V_{Hg} = 50\ cc$$

Hence, the volume of mercury poured into the glass it at 20 ° C so that the volume of the remaining space does not change with temperature is **50 cc.**

Answer.23

Given:

Volume of water in Aluminium Can : V_w = 500 cm^3.

Area of the inner cross section of the can : A = 125 cm^2.

Initial Temperature while measuring: T_1 = 10 ° C.

Increased temperature : T_2 = 80 °C

Thus, Change in Temperature : Δ T = T_2-T_1 = 80-10= 70 ° C.

The coefficient of linear expansion of aluminium :α=23×10^{-6} °C^{-1}.

The Average coefficient of volume expansion of water:γ=3.2×10^{-4} °C^{-1}.

We need to find the increased height of the water level when temperature is increased to 80° C.

First Let's calculate the increased volume of water.**Formula Used:**

The formula for Volume Expansion of a body is:

$$\boldsymbol{V' = V(1 + \gamma \Delta T)}$$

Here V' is the changed volume of the body(in this case water) at T_2 due to expansion and V is the initial volume at T_1

Substituting the given values ,we get:$\therefore V_w' = V_w(1 + \gamma \Delta T)$

$\therefore V_w' = 500\ cm^3\ (\ 1 + 3.2\ \times\ 10^{-4}\ °C^{-1} \times 70\ °C)$

$\therefore V_w' = 500 \times 1.0224\ cm^3 \therefore V_w' = 511.2\ cm^3$

Thus the final volume of water after temperature increase is 511.2 cm^3.

Now, The aluminium can will be expanded too due to temperature increase.

But it's linear expansion is along it's length only. Which means that area expansion can be neglected.

Since we got the Final Volume and Initial volume of Water and neglecting area expansion we can find out the height of the increased water level.

$Rise\ in\ Volume\ of\ water = 511.2 - 500$

$\therefore\ Rise\ in\ Volume\ of\ water = 11.2\ cm^3$Now,

Rise in water level is given by : $\frac{\textbf{Rise in Volume}}{\textbf{Area of cross section}} = \frac{\mathbf{cm^3}}{\mathbf{cm^2}} = \mathbf{cm}$

$\therefore\ Rise\ in\ water\ level = \frac{11.2\ cm^3}{125\ cm^2}$

$\therefore\ Rise\ in\ Water\ level = 0.0896\ cm$

Hence the rise in the water level if the temperature increases to 80°C is 0.0896 cm.

Answer.24

Given: Volume of a glass vessel: V_g = 10cm x 10cm x 10cm = 1000 cm^3.Volume of mercury in the vessel : V_{Hg} = 1000 cm^3.(As it is filled completely in the glass vessel)Temperature at which glass is filled *completely* with mercury: T_1 = 0 °C.Increased temperature : T_2 = 10 ° C.Change in temperature : Δ T = T_2- T_1 = 10 ° C.*Volume of mercury overflowed:= 1.6 cm*^{3}Coefficient of linear expansion of glass : $\alpha_g = 6.5 \times 10^{-6}\ °C^{-1}$.**∴ Coefficient of Volume Expansion of Glass: γ_g: 3× α_g = 3× 6.5× $10^{-6}\ °C^{-1}$**We need to find Coefficient of volume expansion of mercury : γ_{Hg}**Formula used:**Formula for Volume expansion is :

$\boldsymbol{V' = V(1 + \gamma \Delta T)}$Where V' is the changed Volume of the body at T_2 and V is the volume of body at T_1.Now, the Volume of mercury overflown is equal **to the difference in changed volumes of mercury and glass respectively.**Numerically: $V'_{Hg} - V'_g = 1.6\ cm^3$ From the volume expansion formula,Changed Volume of Mercury: $V'_{Hg} = V_{Hg}(1+\gamma_{Hg}\Delta T)$Changed volume of Glass: $V'_g = V_g(1 + \gamma_g \Delta T)$Substituting the values , we get:

$\therefore V_{Hg}(1 + \gamma_{Hg}\Delta T) - V_g(1 + \gamma_g\ \Delta T) = 1.6\ cm^3$

$\therefore V_{Hg} + V_{Hg} \times \gamma_{Hg}\Delta T - V_g - V_g \times \gamma_g\ \Delta T = 1.6\ cm^3$

$\therefore 1000 + (1000\ \gamma_{Hg} \times 10) - 1000 - (1000 \times 3 \times 6.5 \times 10^{-6} \times 10) = 1.6$

$\therefore 10000 \times \gamma_{Hg} - 0.195 = 1.6 \therefore\ \gamma_{Hg} = \frac{1.6 + 0.195}{10000}$

$\therefore \gamma_{Hg} = 1.795 \times 10^{-4}\ °C^{-1}$

Calculate the coefficient of volume expansion of mercury is :

$\gamma_{Hg} = 1.795 \times 10^{-4}\ °C^{-1}$.

Note: The Steps marked in Bold are important for the numerical.

Answer.25

Given: At $_1$ = 0 °C,Density of Wood : ρ_w = 880 kg m.Density of Benzene: ρ_b = 900 kg m^{-3}.The coefficients of volume expansion of wood:γ_w =1.2×10^{-3}°C^{-1}The coefficients of volume expansion of benzene:γ_b=1.5×10^{-3} °C^{-1}.We need to find the temperature T_2 at which the piece of woo will just sink in benzene.Here we will be using Archimedes Law:**When the piece of wood just sinks in benzene it willdisplace some amount of benzene. In simple words the piece of wood starts to sink when it's density is equal to or greater than the density of benzene.Also Density of benzene decreases more rapidly with increase in temperature as compared to that of wood asγ_g > γ_w.**

Formula used:

Numerically:$\rho' = \frac{M}{V'}$

Where ρ' and V' are the changed (Due to volume expansion) Density and Volume respectively.

Formula for Volume expansion of a body is :

$$V' = V(1 + \gamma\, \Delta\, T)$$

Where V' is the changed volume due to change in temperature ΔT.

For Wood:

$$V_w' = V_w(1 + \gamma_w \Delta T)$$

For benzene:

$$V_b' = V_b(1 + \gamma_b \Delta T)$$

Now using Density-Volume Relation:

Changed Density of wood: $\rho_w' = \frac{M_w}{V_w'}$

Changed Density of benzene: $\rho_b' = \frac{M_b}{V_b'}$

We Equate the final densities of wood and benzene to obtain the required condition.

$$\therefore \ \rho_w' = \rho_b' \therefore \frac{M_w}{V_w'} = \frac{M_b}{V_b'} \therefore \frac{M_w}{V_w(1+\gamma_w \Delta T)} = \frac{M_b}{V_b(1+\gamma_b \Delta T)}$$

$$\therefore \frac{\rho_w}{(1+\gamma_w \Delta T)} = \frac{\rho_b}{(1+\gamma_b \Delta T)}$$

$$\therefore \frac{880}{1+1.2 \times 10^{-3} \times \Delta T} = \frac{900}{(1+1.5 \times 10^{-3} \times \Delta T)}$$

$$\therefore 880 \times (1+1.5 \times 10^{-3} \times \Delta T) = 900 \times (1+1.2 \times 10^{-3} \times \Delta T)$$

$$\therefore 880 + (1.32 \times \Delta T) = 900 + (1.08 \times \Delta T)$$

$$\therefore (1.32 - 1.08) \times \Delta T = 900 - 880 \therefore 0.24 \times \Delta T = 20$$

$\therefore \Delta T = T_2 - T_1 = \frac{20}{0.24} \therefore T_2 = 83.33 + 0°C \therefore T_2 = 83.33\ °C$At 83.33 ° C the piece of wood will just sink in benzene.

Answer.26

When the steel rod is heated up to 100 ° C , due to thermal expansion it's length will increase with temperature. Since it will be expanding freely , there won't be anything to oppose the expansion. Longitudinal Strain develops when there exists an opposing force to the expansion of the length.Hence, there will be zero longitudinal strain as it does not have opposing stress .If the rod was clamped at its ends or restricted by some object at it's ends then longitudinal strain would come in the picture

Answer.27

Given: Temperature at which rod is unstrained: T_1= 20 ° CIncreased Final Temperature : T_2 = 50 ° CChange in temperature : Δ T = $T_1 - T_2$ = 30 °CCoefficient of linear expansion of steel: α = 1.2 × $10^{-5}\ °C^{-1}$.**Formula used:**Now we know that formula for linear thermal expansion of a body is: $\boldsymbol{\Delta L = L\alpha\Delta T}$ Where ΔL is the Change in Length and L is the initial length at T_1.Substituting ,∴ $\frac{\Delta L}{L} = 1.2 \times 10^{-5} \times 30$ $\therefore \frac{\Delta L}{L} = 3.6 \times 10^{-4}$

But, Strain is Change in length divided by original length: $S = \frac{\Delta L}{L}$

$\therefore S = 3.6 \times 10^{-4}$

Hence, Longitudinal strain developed in the rod is 3.6 x 10

Answer.28

Given:

Cross-sectional area of the steel wire: A= 0.5 mm^2=0.5×10^{-6} m^2.Temperature at which wire is just taut: T_1= 20 ° C.Decrease in Temperature : T_2 = 0 °C.∴ Change in Temperature : Δ T = 20 ° C.Coefficient of linear expansion of steel: α_s = 1.2 × 10^{-5} $°C^{-1}$Young's modulus : Y= 2.0 × 10^{11} N m^{-2}.**Formula used:**

We know that, Young's Modulus: $Y = \frac{Stress}{Strain} \therefore Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$

$$\therefore \boldsymbol{Y} = \frac{\boldsymbol{F} \times \boldsymbol{L}}{\boldsymbol{A} \times \boldsymbol{\Delta L}}$$

Here, F is the force or tension between the wire and the fixed supports and A is the cross -section area.Also, Formula for Linear Expansion is (**in this case it's compression as temperature is reduced**):

$$\boldsymbol{\Delta L} = \boldsymbol{L\alpha\Delta T}$$

Where Δ L is the Change in Length due to decrease in temperature and L is the original length at T_1.Substituting the value of ΔL in the formula for Y, we get:

$$\therefore Y = \frac{F \times L}{A \times L\alpha\Delta T} \therefore Y = \frac{F}{A \times \alpha_s \times \Delta T} \therefore F = Y \times A \times \alpha_s \times \Delta T$$
$$\therefore F = 2.0 \times 10^{11} \text{N m}^{-2} \times 0.5 \times 10^{-6} \text{ m}^2 \times 1.2 \times 10^{-5} \text{ °C}^{-1} \times 20 \text{ ° C}$$
$\therefore \boldsymbol{F} = \boldsymbol{24\ N}$Hence , the tension in the wire when the temperature falls to 0°C is 24 N.

Answer.29

Given: Temperature at which rod is under zero tension : T_1 = 20 ° C.

Increased Temperature : T_2 = 100 ° C.Change in temperature : Δ T= $T_2 - T_1$ = 100-20 = 80 ° C.Area of cross section of the rod : A = 2.00 mm^2 = 2.00 × 10^{-6} m^2.

Coefficient of linear expansion of steel : α = 12.0 × 10^{-6} °C^{-1}

Young's modulus of steel: Y = 2.00 × 10^{11} N m^{-1}.**Formula used:**

We need to find the force on the clamps by the rod when the rod undergoes thermal expansion due to **increase** in temperature.

Formula for Linear Expansion:

$L' = L(1 + \alpha\Delta T)$Here, L' is the changed length at T_2 and L is the original length of the rod at T_1.

Thus, Change in length is $\Delta L = L' - L$

We get:

$\Delta L = L\ \alpha\Delta T$

Formula for Young's Modulus is:

$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$ ∴ $Y = \frac{F \times L}{A \times \Delta L}$Substituting Δ L:∴ $Y = \frac{F}{A \times \alpha \times \Delta T}$

$\therefore F = Y \times\ A \times \alpha \times \Delta T$

$\therefore F = \ 2.00\ \times\ 10^{11}\ \mathrm{N\,m^{-1}} \times 2.00 \times 10^{-6}\ \mathrm{m^2} \times 12.0\ \times\ 10^{-6}\ {}^{\circ}\mathrm{C}^{-1} \times 80\ {}^{\circ}\mathrm{C}$

$\therefore \mathbf{F = 384\ N}$

Hence when the temperature is increased to 100° C , the rod will exert a force of 384N on one of the champ.

Answer.30

Given: At T_1 = 0° CLength of two steel rods and one aluminium rod joined rigidly at T_1= lCoefficient of linear expansion of aluminium : α_aCoefficient of linear expansion of steel are : α_sYoung's modulus of aluminium : Y_αYoung's modulus of steel : Y_sIncreased Temperature : T_2 = θChange in temperature : Δ T= θ – 0 = θNow, since these rods are rigidly joined to each other when the temperature is increased these rods will undergo thermal expansion. The rate of thermal expansion for steel and aluminium will be different as they has different values of Coefficient of thermal expanion .While expanding the rods will exert force on each other.Hence ,due to thermal expansion the length of the rods will change , we will name the final length as l' at temperature θ .Thus Change in Length: Δ l= l'-l**Formulae used:**We know that:

$1. Young's\ Modulus: Y = \frac{Stress}{Strain}$ $2. Stress = \frac{F}{A}$ $3. Strain = \frac{\Delta l}{l}$

$4. \Delta l = l\alpha\Delta T \ldots\ldots(Linear\ Expansion\ Formula)$ $5. Y = \frac{F}{A\alpha\Delta T}$ Now,we know that : $Stress = Y \times \alpha\Delta T \ldots\ldots(1)$The Total Young's Modulus of whole system:$Total\ Y = \frac{Total\ Stress}{Total\ Strain}$Now,

$$\text{Total Stress} = \text{Stress due to 2 steel rods} + \text{Stress due to One aluminum rod}$$

$$\therefore Total\ Stress = (2 \times Y_s \times \alpha_s \times \theta) + (Y_a \times \alpha_a \times \theta) \ldots\ldots From\ (1)\ and\ Given$$

$$\therefore \boldsymbol{Total\ Stress = 2Y_s\alpha_s\theta + Y_a\alpha_a\theta \ldots\ldots\ldots(2)}$$ Now,

$$Total\ Young's\ Modulus = (2 \times Y_s) + Y_a$$

$$\therefore \boldsymbol{Total\ Y = 2Y_s + Y_a \ldots\ldots(3)}$$

Hence by 3rd Formula and equation (2),(3) $Total\ Strain = \frac{Total\ Stress}{Total\ Y}$

$$\therefore \frac{\Delta l}{l} = \frac{2Y_s\alpha_s\theta + Y_a\alpha_a\theta}{2Y_s + Y_a}$$

$$\therefore \Delta l = l \times \left(\frac{2Y_s\alpha_s\theta + Y_a\alpha_a\theta}{2Y_s + Y_a}\right)$$

$$\therefore l' - l = l \times \left(\frac{2Y_s\alpha_s\theta + Y_a\alpha_a\theta}{2Y_s + Y_a}\right)$$

$$\therefore l' = l\left(1 + \frac{2Y_s\alpha_s\theta + Y_a\alpha_a\theta}{2Y_s + Y_a}\right)$$

Hence, the length of the system when temperature is increase to θis

$\boldsymbol{l\left(1 + \frac{2Y_s\alpha_s\theta + Y_a\alpha_a\theta}{2Y_s + Y_a}\right)}$

Answer.31

Given: Initial pressure on the steel ball : P = 1.0 × 10^5 PaT_1= 20° CT_2 = 120° CChange in temperature : Δ T = 100° CVolume is Constant.Coefficient of linear expansion of steel : α = 12 × 10^{-6} °C^{-1}Bulk modulus of steel : B = 1.6 × 10^{11} N m^{-1}.**Formula used:** When a body is subject to pressure, its volume decreases and retains its volume when pressure is removed.Bulk modulus is the ratio of pressure and Strain.Formula for bulk modulus is:$\boldsymbol{B = P \times \left(\frac{V}{\Delta V}\right)}$ Here B is the Bulk Modulus, P is the Pressure inside the ball.We know that the formula for volume expansion of a body is$\boldsymbol{V' = V(1 + \gamma\Delta T)OR\Delta V = V\gamma\Delta T}$Where V' is the final or changed Volume when temperature is increased to T_2, V is the initial volume at T_1 and ΔV is the Change in Volume.ΔV = V' – V.Substituting value of ΔV in the equation of B, we get:$B = P \times \left(\frac{V}{V\gamma\Delta T}\right) \therefore B = \frac{P}{\gamma\Delta T}$

$\therefore P = B \times \gamma \times \Delta T$But γ = 3× α

$\therefore P = 1.6 \times 10^{11}$ N m^{-1} × 3 × 12×10^{-6} °C^{-1} × 100° C

$\therefore \mathbf{P = 5.76 \times 108\ Pa}$

The Pressure inside the ball is 5.76 × 10^8 Pa.

Answer.32

Given: Moment of inertia at 0° C = I_1Say temperature changes to θ .Change in temperature : Δ T = θ – 0 = θ**Formula Used:**

We know that,$I_1 = MR_0^2 \ldots .. (1)$ Where M is the mass of the body and R_0 is the radius of gyration at 0° C and I_1 is the initial moment of inertia at 0° .When temperature of the solid body increases, the radius of gyration also changes due to thermal expansion.Hence, formula for thermal expansion of radius of gyration is$R' = R_0(1 + \alpha\Delta T)$Here R' is the changed radius of gyration due to expansion.Let I be the changed moment of inertia when temperature is θ .$\therefore I = MR'^2 \therefore I = M\left(R_0(1 + \alpha\theta)\right)^2 \therefore I = MR_0^2(1 + \alpha\theta)^2$

$\therefore I = MR_0^2(1 + 2\alpha\theta + \alpha^2\theta^2)$Here, $\alpha^2\theta^2$is neglected, we get

$\therefore \boldsymbol{I = I_1(1 + 2\alpha\theta)} \ldots \ldots from\ (1)$

Hence proved that the moment of inertia of a solid body of any shape changes with temperature as I = I_1(1 + 2αθ).

Answer.33

Given: Coefficient of linear expansion: α = 2.4 × 10^{-5} °C^{-1}.Temperature change in winter: Δ T_1= 5° CTemperature change in summer : Δ T_2 = 45° C**Formula Used:**For torsional pendulum, the time period is given as:$\boldsymbol{T = 2\pi\sqrt{\frac{I}{K}}}$Here, I is the moment of inertia of the wire and k is the torque constant of the wire.Due to change in temperature the moment of inertia will also change.Hence,$I' = I_0(1 + 2\alpha\Delta T)$Here, I' is the changed moment of inertia and I_0 is the initial moment of inertia at 0 ° C**During winters:**

$I' = I_0(1 + 2\alpha\Delta T_1)$Substituting for time period in winter: t_1 $t_1 = 2\pi\sqrt{\frac{I'}{k}}$∴

$\therefore t_1 = 2\pi\sqrt{\frac{I_0(1 + 2 \times 2.4 \times 10^{-6} \times 5)}{k}}$ $\therefore t_1 = 2\pi\sqrt{\frac{I_0(1.000024)}{k}}$**During Summers** :$I' = I_0(1 + 2\alpha\Delta T_2)$Substituting for time period in summer: t_2

$t_2 = 2\pi\sqrt{\frac{I_0(1 + 2\alpha\Delta T_2)}{k}}$$\therefore t_2 = 2\pi\sqrt{\frac{I_0(1 + 2 \times 2.4 \times 10^{-5} \times 45)}{k}}$

$\therefore t_2 = 2\pi\sqrt{\frac{I_o(1.000216)}{k}}$Hence we know time period of torsional pendulum in winter and summer.Percentage change is given as:

$\% \, \boldsymbol{Change} = \frac{\boldsymbol{Change\ in\ quantity}}{\boldsymbol{Original\ Quantity}}$ $\therefore \% \, Change = \frac{t_2 - t_1}{t_1}$

Here, the numerator is the change in time period and the denominator is the original time period.$\therefore \% \, Change = \left(\frac{t_2}{t_1} - 1\right) \times 100$

$$\therefore \% \, Change = \left(\frac{2\pi\sqrt{\frac{I_o(1.00216)}{k}}}{2\pi\sqrt{\frac{I_0(1.000024)}{k}}} - 1\right) \times 100$$

$$\therefore \% \, Change = \left(\frac{\sqrt{1.00216}}{\sqrt{1.00024}} - 1\right) \times 100$$

$$\therefore \% \, Change = (1.00095931 - 1) \times 100$$

$\therefore \% \, Change = 9.59 \times 10^{-4} \times 100$$\therefore \% \, Change = 0.0959\%$Hence the percentage change in the time period between peak winter (5°C) and peak summer (45°C) is 0.0959%

Answer.34

Given:T_1 = 20 ° CT_2 = 50 ° CΔ T = 50-20= 30° CAngular Velocity: ω = constantCoefficient of linear expansion of iron : α = 1.2 × 10^{-5} °C^{-1}.**Formula used:**We know that:$\omega = \frac{v}{r}$Here, v is the velocity of the particle and r is the radius of the particle.When temperature is increased from 20° C to 50° C, the disc undergoes thermal expansion.Let **r** be the radius of particle at $\mathbf{T_1}$and **r'** be the **changed radius** of particle at $\mathbf{T_2}$.Let **v** be the velocity of particle at $\mathbf{T_1}$ and v' be the **velocity** of particle at $\mathbf{T_2}$.Hence,Angular Velocity at T_1: $\omega_1 = \frac{v}{r}$ Angular Velocity at T_2 : $\omega_2 = \frac{v'}{r'}$Now, we know that Thermal linear expansion of radius is:$r' = r(1 + \alpha\Delta T)$Angular velocity is constant even after heating the disc, $\omega = \frac{v}{r} = \frac{v'}{r'}$Substituting , we get$\therefore \frac{v}{r} = \frac{v'}{r(1 + \alpha\Delta T)}$ $\therefore v' = v \times (1 + 1.2 \times 10^{-5} \times 30)$$\therefore$ v' = v × 1.00036As we know that percentage change is : $\%\ \boldsymbol{Change} = \frac{\boldsymbol{Change\ in\ quantity}}{\boldsymbol{Original\ Quantity}}$

$\therefore \%\ Change = \left(\frac{v' - v}{v}\right) \times 100$$\therefore\ \%\ Change = \left(\frac{1.00036v - v}{v}\right) \times 100$

$\therefore \%\ Change = (1.00036 - 1) \times 100$$\therefore \%\ Change = 3.6 \times 10^{-4} \times 100$

$\%\ \boldsymbol{Change} = \mathbf{0.036}$

Hence the percentage change in the linear speed of a particle of the rim when the disc is slowly heated from 20°C to 50°C keeping the angular velocity constant is 0.036%.