Chapter 3

Fluid Kinematics and Dynamics

CHAPTER HIGHLIGHTS

- *Fluid kinematics*
- *Lagrangian description of fluid flow*
- *Eulerian descriptionof fluid flow*
- *Scalar, vector and flow fields*
- *Fluid acceleration*
- *Types of fluid flow*
- *Description of flow pattern*
- *Basic principles of fluid flow*
- *Energy equations*
- *Force exerted by flowing fluid on a pipe bend*
- *Moment of momentum principle*
- *Flow through orifices*
- *Free liquid jet*
- *Vortex flow*

Fluid Kinematics

The description of the motion of fluids (or fluid flows) without necessarily considering the forces and moments that cause the motion is called *fluid kinematics*. The flow of fluid can be described by two ways:

1. Lagrangian description
2. Eulerian description.

Lagrangian Description of Fluid Flow

Here, individual fluid particles are identified (usually by specifying their initial spatial position of a given time) and the motion of each particle is observed as a function of time. Let the position of a fluid particle identified by $\vec{r_0}$. The position vector at any time 't' shall be $\vec{r} = r(\vec{r}_0, t)$,

Where $\vec{r}$ is the position vector of the fluid particle with respect to a fixed reference point at time t. Considering cartesian coordinates,

We have,

$$\vec{r}_0 = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$$

and $\vec{r} = x(r_0, t)\hat{i} + y(r_0, t)\hat{j} + z(r_0, t)\hat{k} = x\hat{i} + y\hat{j} + z\hat{k}$

Here, $\hat{i}, \hat{j}$ *and* $\hat{k}$ are unit vectors along the x, y, z directions respectively and r_0 denotes the point (x_0, y_0, z_0).

The velocity vector $\vec{v}$ having the scalar components u, v and w in the x, y and z directions respectively are given as follows:

$$\vec{v} = \left.\frac{\partial \vec{r}}{\partial t}\right|_{r_0}$$

$$= \hat{i}\left.\frac{\partial x}{\partial t}\right|_{r_0} + \hat{j}\left.\frac{\partial y}{\partial t}\right|_{r_0} + \hat{k}\left.\frac{\partial z}{\partial t}\right|_{r_0} = u\hat{i} + v\hat{j} + w\hat{k}$$

The acceleration vector $\vec{a}$ having the scalar components a_x, a_y and a_z in the x, y and z directions respectively are given as follows:

$$\vec{a} = \left.\frac{\partial^2 \vec{r}}{\partial t^2}\right|_{r_0}$$

$$= \hat{i}\left.\frac{\partial^2 x}{\partial t^2}\right|_{r_0} + \hat{j}\left.\frac{\partial^2 y}{\partial t^2}\right|_{r_0} + \hat{k}\left.\frac{\partial^2 z}{\partial t^2}\right|_{r_0}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Eulerian Description of Fluid Flow

In Eulerian method any point in the space, occupied by the fluid is selected and observations are made.

It is a average approach, as our concentration is on a particular space, point or section and all the particles passing from it are analysed as a bulk simultaneously.

The major advantage of the method is it consumes less time and computation required is also very less.

Scalar, Vector and Flow Fields

A *scalar field* is a region where at every point, a scalar function (scalar field variable) has a defined value.

Example: Pressure field of a fluid flow.

A *vector field* is a region where at every point, a vector function (vector field variable) has a defined value.

Example: Velocity field of a fluid in motion.

A *flow field* is a region in which the flow properties, i.e., velocity, pressure, etc., are defined at each and every point at any time instant. Two basic and important vector field variables of a flow are the velocity and acceleration fields.

Velocity Field

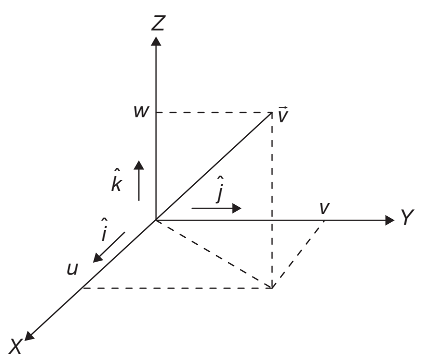
For a general three-dimensional fluid flow in Cartesian coordinates, the velocity vector is given by,

$$\vec{v} = \vec{v}_{(x,y,z,t)}$$

$$u(x, y, z, t)\,\vec{i} + v(x, y, z, t)\hat{j} + w(x, y, z, t)\hat{k}$$

The *speed* of the fluid,

$$v = |\vec{v}| = \sqrt{u^2 + v^2 + w^2}$$



A point in the fluid flow field where the velocity vector is zero is called a *stagnation point*.

Fluid Acceleration

Acceleration Field

For a general three-dimensional fluid flow in Cartesian coordinates, if $\vec{v}$ is the velocity field, then the *acceleration field* is given by:

$$\vec{a}(x, y, z, t) = \frac{\partial \vec{v}}{\partial t} + u\frac{\partial \vec{v}}{\partial x} + v\frac{\partial \vec{v}}{\partial y} + w\frac{\partial \vec{v}}{\partial z} \qquad (1)$$

The scalar components of the acceleration vector are:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Magnitude of the acceleration vector,

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Eq. (1) can be rewritten as

$$\vec{a}(x, y, z, t) = \frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = \frac{D\vec{v}}{Dt} \qquad (2)$$

The gradient (or del) operator, $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ and the operator $(\vec{v}\cdot\vec{\nabla}) = \frac{u\partial}{\partial x} + \frac{v\partial}{\partial y} + w\frac{\partial}{\partial z}$.

The components of the acceleration vector in cylindrical coordinates are:

$$a_r = \frac{\partial v_r}{\partial_t} + v_r\frac{\partial v_r}{\partial_r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial_z}$$

$$a_\theta = \frac{\partial v_\theta}{\partial_t} + v_r\frac{\partial v_\theta}{\partial_r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{dv_\theta}{\partial \theta}$$

$$a_z = \frac{\partial vz}{\partial t} + v_r\frac{\partial vz}{\partial r} + \frac{v_\theta}{r}\frac{\partial vz}{\partial \theta} + v_z\frac{\partial vz}{\partial z}$$

Local and convective derivative In Eq. (2), the operator $\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{v}\cdot\vec{\nabla})$ is called as the *total* (of material) or *substantial derivative.* The operator $\frac{\partial}{\partial t}$ is called the *local* or *temporal* or *unsteady derivative*, while the operator $(\vec{v}\cdot\vec{\nabla})$ is called the *convective derivative*. The local derivative represents the effect of unsteadiness while the convective derivative represents the variation due to the change in position of the fluid particle as it moves through a field with gradient (spatial change).

Local, Convective and Total Acceleration

In Eq. (2), the term $\frac{\partial \vec{v}}{\partial t}$ is called the *local* or *temporal* or *unsteady acceleration* whereas the term $(\vec{v}\cdot\vec{\nabla})\vec{v}$ is called the *convective* (*adjective*) *acceleration.* Eq. (2) elucidates that fluid particles experience acceleration due to:

1. Change in velocity with time (local acceleration)
2. Change in velocity with space (convective acceleration).

The acceleration vector $\vec{a}$ is called as the *total* or *material acceleration*.

Total acceleration = Local acceleration + Convective acceleration

SOLVED EXAMPLES

Example 1

The velocity field of a two-dimensional flow is given by $\vec{v} = 2xt\hat{i} + 2yt\hat{j}$, where t is in seconds. At t = 1 second, if the local and convective accelerations at any point (x, y) are denoted by $\vec{a}_l$ and $\vec{a}_c$ respectively, then

(A) $\vec{a}_l = 2\vec{a}_c$ (B) $\vec{a}_c = \vec{a}_l$
(C) $\vec{a}_c = \vec{a}_l = 0$ (D) $\vec{a}_c = 2\vec{a}_l$

Solution

From the velocity field description

$$u = 2xt$$
$$v = 2yt$$

x-component of the local acceleration, $a_{l,x} = \frac{\partial u}{\partial t} = 2x$

y-component of the local acceleration,

$$a_{l,y} = \frac{\partial v}{\partial t} = 2y$$

$$\vec{a}_l = a_{l,x}\hat{i} + a_{l,y}\hat{j}$$

$$= 2x\hat{i} + 2y\hat{j} \qquad (1)$$

x-component of the convective acceleration,

$$a_{c,x} = \frac{u\partial u}{\partial x} + \frac{v\partial u}{\partial y}$$
$$= 2xt \times 2t + 2yt \times 0 = 4xt^2$$

y-component of the convective acceleration,

$$a_{c,y} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$
$$= 2xt \times 0 + 2yt \times 2t$$
$$= 4yt^2$$

$$\vec{a}_c = a_c, x\hat{i} + a_c, y\hat{j}$$
$$= 4xt^2\,\hat{i} + 4yt^2\,\hat{j}$$

at t = 1 second,

$$\vec{a}_c = 4x\hat{i} + 4y\hat{j} \qquad (2)$$

From Eqs. (1) and (2), we have

$$\vec{a}_c = 2\vec{a}_l.$$

Hence, the correct answer is option (D).

Example 2

A two-dimensional velocity field is given by $\vec{v} = xy\hat{i} + 3xt\hat{j}$, where x and y are in metres, t is in seconds and $\vec{v}$ is in metres per second. The magnitude of the acceleration at x = 1 m, y = 0.5 m and t = 2 seconds is

(A) 6.25 m/s^2 (B) 8.663 m/s^2
(C) 12.25 m/s^2 (D) 6 m/s^2

Solution

From the velocity field description,

$$u = xy$$
$$v = 3xt$$

Now, $$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$
$$= 0 + xy \times y + 3xt + x$$
$$= xy^2 + 3x^2t$$

Now, $$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$
$$= 3x + xy \times 3t + 3xt \times 0$$
$$= 3x + 3xyt$$

at x = 1 m, y = 0.5 m and t = 2 seconds,

$$a_x = 1 \times (0.5)^2 + 3 \times 1 \times 2 = 6.25 \text{ m/s}^2$$
$$a_y = 3 \times 1 + 3 \times 1 \times 0.5 \times 2 = 6$$

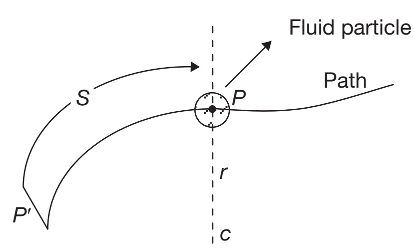
Magnitude of the acceleration,

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$
$$= \sqrt{(6.25)^2 + 6^2} = 8.663 \text{ m/s}^2.$$

Hence, the correct answer is option (B).

Tangential and Normal Acceleration

Consider a fluid particle moving along a path as shown in the following figure:



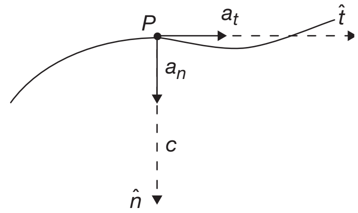
Let S denote the distance travelled by the particle along the path line relative to the reference point P', t denote time and V velocity.

$V = f(s, t)$ denote the speed of the particle. Let $\hat{t}$ be a unit vector tangential to the path at point P and let $\hat{n}$ be a unit vector normal to the path at point P and pointing inward towards the centre of curvature C. Let r denote the radius of curvature at point P.

The acceleration vector,

$$\boxed{\vec{a} = a_t\hat{t} + a_n\hat{n}}$$

$$\boxed{= \left(v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right)\hat{t} + \frac{v^2}{r}\hat{n}}$$



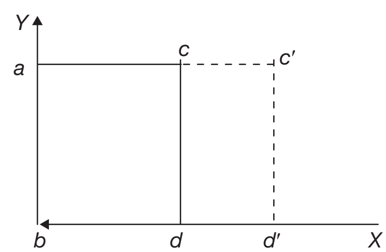
The *tangential component* of the acceleration vector, $a_t = \left(v\frac{\partial v}{\partial s}+\frac{\partial v}{\partial t}\right)$ and the *normal component*, $a_n = \frac{v^2}{r}$.

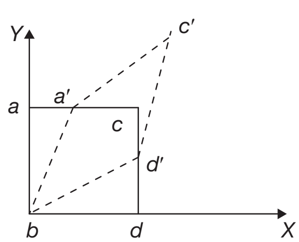
The component a_n is also called as the **centripetal acceleration**. The component a_n will be present anytime a fluid particle is moving on a curved path (velocity direction is changing) while the component a_t will be present whenever the fluid particle is changing speed (velocity magnitude is changing)

Fluid Flow Scenario (Only Steady Flows)	Tangential Acceleration or Deceleration	Normal Acceleration or Deceleration
Flow in a straight constant diameter pipe	Not present	Not present
Flow in a straight non-constant diameter pipe	Present	Not present
Flow in a curved constant diameter pipe	Not present	Present
Flow in a curved non-constant diameter pipe	Present	Present

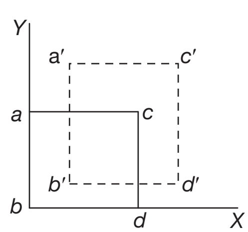
Translation, Deformation and Rotation of a Fluid Element

When a fluid element moves in space, several things may happen to it. Surely the moving fluid element undergoes translation, i.e., a linear displacement from one location to another. The fluid element in addition may undergo rotation, linear deformation or angular deformation.





Angular deformation



translation

In a two-dimensional flow field in cartesian coordinates, translation without deformation and rotation is possible if the velocity components u and v are neither a function of x nor of y. When a velocity component is a function of only one space coordinate along which that velocity component is defined, e.g., $u = u(x)$ and $v = v(y)$, then translation with linear deformation is possible. When $u = u(x, y)$ and $v = v(x, y)$, translation with angular and linear deformations is possible. It is also observed that when $u = u(x, y)$ and $v = v(x, y)$, rotation and angular deformation of a fluid element exists simultaneously. When $\frac{\partial v}{\partial x} = \frac{-\partial u}{\partial y}$, no angular deformation takes place and the situation is known as pure rotation. When $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$, the fluid element has angular deformation but no rotation about the Z-axis

Types of Fluid Flow

1. **Steady and unsteady flow:** In a *steady* fluid flow, fluid properties (such as density, pressure, etc.) and the flow characteristics (such as velocity, acceleration, etc.) at any point in the flow do not change with time. In a steady flow, the local derivative of the fluid property or fluid characteristic ϕ is zero, i.e., $\frac{\partial \phi}{\partial t} = 0$.

   Local acceleration is zero for steady flows

   Fluid flow through a pipe at a constant rate of discharge is a steady flow.

   In an *unsteady* fluid flow, some of the fluid properties or flow characteristics at any point in the flow changes with time. Fluid flow through a pipe at a varying rate of discharge is an unsteady flow.

2. **Uniform and non-uniform flows:** In a uniform fluid flow, the fluid properties or flow characteristics at any given time do not change with respect to space, i.e., from one point to another in the flow. Since for a uniform flow, there is no gradient (spatial change) the convective derivative of any fluid property of flow characteristic ϕ is zero, i.e., $(\vec{v}\cdot\vec{\nabla})\varphi = 0$.

   Convective acceleration is zero for uniform flows

   In uniform flows, the streamlines are straight and parallel.

   Fluid flow through a straight pipe of constant diameter is a uniform flow.

   In a non-uniform fluid flow, some of the fluid properties or flow characteristics at any given time changes with respect to space. Flow through a straight pipe of varying diameter is a non-uniform flow.

   Total acceleration is zero for steady uniform flows.

Flow Combinations

Type	Example
Steady uniform flow	Flow at a constant rate through a constant diameter pipe
Steady non-uniform flow	Flow at a constant rate through a non-constant diameter pipe
Unsteady uniform flow	Flow at a varying rate through a constant diameter pipe
Unsteady non-uniform flow	Flow at a varying rate through a pipe of varying cross-section

3. **One–, two–, and three–dimensional flows:** A flow is said to be *one*–, *two*–, or *three-dimensional* if one, two or three spatial dimensional are required to describe the velocity field.
4. **Inviscid and viscous flow:** A fluid flow in which the effects of viscosity (frictional effects) are absent is called as *inviscid* (*non-viscous*) fluid flow, whereas if the viscosity effects are present, then the fluid flow is called a *viscous* fluid flow. Flow of ideal fluids are inviscid flows while flow of real fluids are viscous flows.
5. **Rotational and irrotational flows:** A fluid flow is said to be *rotational* if the fluid particles while moving in the direction of flow rotate about their mass centres. If the fluid particle do not rotate, then the fluid flow is called as *irrotational* fluid flow. Fluid flow in a rotating tank is a rotational flow while fluid flow above a wash basin or drain hole of a stationary tank is an irrotational flow

   For an irrotational flow, the curl of the velocity vector is zero, i.e., $\vec{\nabla}\times\vec{v} = 0$ or curl $(\vec{v}) = 0$

6. **Compressible and incompressible flows:** If for a fluid flow, the density remains constant throughout the flow, i.e., $\frac{\partial\rho}{\partial t} = 0$, then the fluid flow is an *incompressible* fluid flow else it is a compressible fluid flow.
7. **Laminar and turbulent flow:** A flow is said to be laminar when the various fluid particles move in layers with one layer of fluid sliding smoothly over an adjacent layer, while in a turbulent flow fluid particles move in an entirely haphazard or disorderly manner, that results in a rapid and continuous mixing of the fluid leading to momentum transfer as flow occurs.

Example 3

The velocity field of a two-dimensional irrotational flow is represented by, $\vec{v} = \left(\frac{-x^2y^3}{3} + 2x - my\right)\hat{i} + \left(px - 2y - \frac{x^3y^2}{3}\right)\hat{j}$, where P and m are constants. If the value of P is equal to one, then the value of m for a streamline passing through the point (1, 2) is

(A) $\frac{-2}{3}$ (B) 0

(C) 3 (D) –1

Solution

From the velocity field relationship,

$$u = \frac{-x^2y^3}{3} + 2x - my$$

$$v = Px - 2y - \frac{x^3}{3}y^2$$

Since the flow is irrotational

$$\vec{\nabla}\times\vec{v} = 0$$

i.e.,

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

or $P - x^2y^2 = -x^2y^2 - m$

or $m = -P = -1$.

Hence, the correct answer is option (D).

Description of Flow Pattern

Streamline

A streamline is a curve that is everywhere tangent to the instantaneous local velocity vector. At a given instant of time, the tangent to a streamline at a particular point gives the direction of the velocity at that point. The fluid flow will always be along the streamlines and never cross it. At non-stagnation points, a streamline cannot interest itself nor can two streamlines cross each other. However, the two scenarios can be present at stagnation points.

The differential equation of a streamline in a three-dimensional flow $(\vec{v} = u\hat{i} + v\hat{j} + w\hat{k})$ is:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

For a two-dimensional flow $(\vec{v} = u\hat{i} + v\hat{j})$, the slope of the streamline is given as:

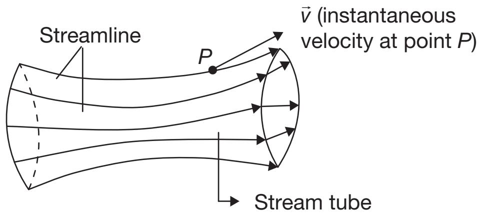
$$\frac{dy}{dx} = \frac{v}{u}$$

The pattern of streamline will be fixed in space for steady flows but need not be in the case of unsteady flows.

Stream Tube

An imaginary passage through which fluid flows and which is bounded by a bundle of streamline is called a *stream tube*. Fluid can enter or leave a streamline only through its ends but never across the stream tube's surface. At any instant in tube, the mass flow rate passing through any cross-sectional cut of a given stream tube will always be the same.

On steady flows, the shape and position of a stream tube does not change.



Streak Line

It is the locus of the fluid particles that have passed sequentially through a chosen point in the flow. It is also the curve generated by a tracer fluid, such as a dye, continuously injected in the flow field at the chosen point. An example of a streak line is the continuous smoke emitted by a chimney.

Path Line

It is the path followed by a fluid particle in motion. A path line can intersect with itself or two path lines can intersect with each other.

Streamline indicates the motion of bulk mass of fluid whereas the path line indicates the motion of a single fluid particle. A streak line indicates the motion of all the fluid particle along its length.

In a steady flow, the streamline, streak line and path line coincide if they pass through the same point.

Example 4

For a three-dimensional flow, if the velocity field is given by $\vec{v} = 4x\hat{i} + 6y\hat{j} - 10z\hat{k}$, then an equation for a streamline passing through the point (1, 4, 5) is

(A) $xyz = \frac{5}{4}$ (B) $xyz = \frac{1}{20}$

(C) $xyz = \frac{4}{5}$ (D) $xyz = 20$

Solution

From the velocity field representation, we have

$$u = 4x$$
$$v = 6y$$
$$w = -10z$$

For a streamline, $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

Taking $\frac{dx}{u} = \frac{dy}{v}$, we have $\frac{dx}{4x} = \frac{dy}{6y}$

Integrating, we get

$x^{6/4} = y \times C_1$, where C_1 is an integration constant.

Considering the point (1, 4, 5), we get

$$(1)^{6/4} = 4 \times C_1$$

i.e.,
$$C_1 = \frac{1}{4}$$

$$\therefore \quad x^{6/4} = \frac{y}{4} \qquad (1)$$

Taking $\frac{d_x}{u} = \frac{d_z}{w}$, we have $\frac{dx}{4x} = \frac{dz}{-10z}$,

Integrating, we get

$zx^{10/4} = C_2$, where C_2 is an integration constant.

Considering the point (1, 4, 5), we get $5 \times (1)^{10/4} = C_2$

That is, $C_2 = 5$

$$\therefore \quad zx^{10/4} = 5 \qquad (2)$$

Substituting Eq. (1) in Eq. (2), we get $zxy = 20$ as the equation of the streamline.

Hence, the correct answer is option (D).

Basic Principles of Fluid Flow

There are three basic principles used in the analysis of the problems of fluid in motion as noted below:

1. Principle of conservation of mass
2. Principle of conservation of energy
3. Principle of conservation of momentum

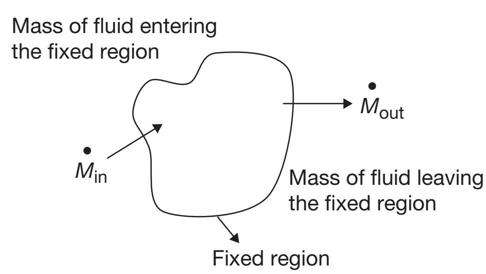
Principle of conservation of mass states that mass can neither be created nor destroyed.

Principle of conservation of energy states that energy can neither be created nor destroyed.

Principle of conservation of momentum or impulse momentum principle states that the impulse of the resultant force, or the product of the force and time increment during which it acts, is equal to the change in momentum of the body.

Continuity Equation

The continuity equation is actually mathematical statement of the principle of conservation of mass.



It may be stated that the rate of increase of the fluid mass contained within the region must be equal to the difference between the rate at which the fluid mass enters the region and the rate at which the fluid mass leaves the region. However if the flow is steady the rate of increase of fluid mass within the region equals zero.

Continuity Equation in Cartesian Coordinates

$$\frac{\delta\rho}{\delta t}+\frac{\delta(\rho u)}{\delta x}+\frac{\delta(\rho v)}{\delta y}+\frac{\delta(\rho w)}{\delta z}=0$$

Where u, v, w are the velocity components in x, y, and z directions ρ is the mass density of the fluid.

In vector notation the continuity equation can be expressed as

$$\frac{D\rho}{Dt}+\rho\nabla v=0$$

$$\frac{D\rho}{Dt}=\frac{\delta\rho}{\delta t}+u\frac{\delta\rho}{\delta x}+v\frac{\delta\rho}{\delta y}+w\frac{\delta\rho}{\delta z}$$

$$\Psi\cdot v=\text{div } v=\frac{\delta u}{\delta x}+\frac{\delta u}{\delta y}+\frac{\delta w}{\delta z}$$

Continuity Equation in Cylindrical Polar Coordinates

$$\frac{\delta\rho}{\rho t}+\frac{\delta(\rho v_r r)}{r\delta r}+\frac{\delta(\rho v_\theta)}{r\delta\theta}+\frac{\delta(\rho v_z)}{\delta t}=0$$

V_r, V_θ, V_z are the components of velocity V in the directions of r, θ, z at a point. ρ is the mass density of fluid.

Continuity Equation in Spherical Polar Coordinates

$$\frac{\delta\rho}{\delta t}+\frac{1}{r^2}\frac{\rho}{\rho_r}(\rho v_r r^2)+\frac{1}{r\sin\theta}\frac{\delta}{\delta\theta}(\rho v_\theta\sin\theta)$$
$$+\frac{1}{r\sin\theta}\frac{\delta}{\delta w}(\rho v_w)=0$$

Rotational Parameters

Angular Velocity

It is the rotational component about any axis. It may be defined as the average angular velocity of any two infinitesimal linear elements in the particle that are perpendicular to each other and to the axis of rotation.

For example, Z-axis:

$$\omega_z=\frac{1}{2}\left[\frac{\delta v}{\delta x}-\frac{\delta u}{\delta y}\right]$$

$$\vec{\omega}=\omega_x\hat{i}+\omega_y\hat{j}+\omega_z\hat{k}$$

$$|\bar{\omega}|=\left|\sqrt{\omega_x^2+\omega_y^2+\omega_z^2}\right|$$

$$\bar{\omega}=\frac{1}{2}|\bar{\nabla}\times\bar{v}|$$

Vorticity $(\bar{\Omega}(\text{or})\bar{\xi})$

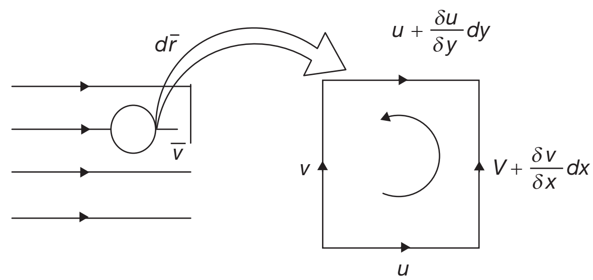
It is a vector quantity and gives us the actual measure of rotation of a fluid.

Vorticity vector is equal to twice the rate of rotation of angular velocity vector $\vec{\omega}$

$$\vec{\Omega}=2\vec{\omega}$$

Circulation (Γ)

It is defined as the counter clockwise line integral of velocity vector along a closed loop.



$$\Gamma=\oint\bar{v}.d\bar{r}\ (\text{or})\ \int dr\cdot v\cos\theta$$

In two-dimensional steady flow,

$$\bar{v}=u\hat{i}+v\hat{j}$$

$$dr=dx\hat{i}+dy\hat{j}$$

$$\Gamma=\int(u\hat{i}+v\hat{j}).(dx\hat{i}+dy\hat{j})$$

$$=\int udx+vdy$$

$$\Gamma=\int udx+\int\left(v+\frac{\delta v}{\delta x}\right)dx\,dy-\int\left(u+\frac{\delta u}{\delta y}dy\right)dx-\int vdy$$

$$\Gamma=\left(\frac{\delta v}{\delta x}-\frac{\delta u}{\delta y}\right)dxdy$$

$$\Gamma=\text{Vorticity}\times\text{Area}$$

$$\therefore\quad\frac{\delta v}{\delta x}-\frac{\delta u}{\delta y}=\Omega$$

$$dxdy=A$$

Velocity Potential Function

The velocity potential (ϕ) is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction.

$$\phi=f(x,y,z,t)$$

$$U=-\frac{\delta\phi}{\delta x};\ v=-\frac{\delta\phi}{\delta y};\ w=-\frac{\delta\phi}{\delta z}$$

For an incompressible fluid, if flow is steady then equation of continuity is given by,

$$\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}+\frac{\delta w}{\delta z}=0$$

Substituting values of u, v, w in terms of ϕ, we get

$$\frac{\delta^2\phi}{\delta x^2}+\frac{\delta^2\phi}{\delta y^2}+\frac{\delta^2\phi}{\delta z^2}=0$$

$\therefore\ \nabla^2\phi=0$, this equation is known as Laplace equation.

Physical Significance of Velocity Potential If $\nabla^2\phi = 0$, ϕ exists. If $\Delta^2\phi \neq 0$, ϕ does not exists but flow exists.

We know the rotational component along Z-axis is $\omega_z = \frac{1}{2}\left[\frac{\delta v}{\delta x} - \frac{\delta u}{\delta y}\right]$.

Substituting values for u, v in terms of ϕ, we get

$$\omega_z = \frac{1}{2}\left[\frac{\delta^2\phi}{\delta x \delta y} - \frac{\delta^2\phi}{\delta x \delta y}\right] = 0$$

$$\therefore \quad \omega_z = 0$$

If $\omega_z = 0$ and when ϕ exists, it represents irrotational fluid flow.

If $\omega_z \neq 0$ it represents rotational flow and also it denotes ϕ does not exists.

Equation of Equipotential Function Line

$$\frac{dy}{dx} = \frac{-u}{v}$$

Stream Function

The stream function Ψ (Greek 'Psi') is defined as a scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles (in the centre clockwise direction) to this direction.

Mathematically stream function may be defined as $\Psi = f(x, y, t)$ for unsteady flow.

$$\Psi = f(x, y) \text{ for steady flow.}$$

$$U = -\frac{\delta\psi}{\delta y}$$

$$V = \frac{\delta\psi}{\delta x}$$

In cylindrical polar coordinates

$$v_r = -\frac{\delta\psi}{r\,\delta\theta}$$

$$V_\theta = \frac{\delta\psi}{\delta r}$$

Cauchy–Rieman Equation

$$\frac{\delta\phi}{\delta x} = \frac{\delta\psi}{\delta y} - \frac{\delta\phi}{\delta y} = \frac{\delta\psi}{\delta x}$$

In polar coordinates,

$$\frac{\delta\phi}{\delta r} = \frac{\delta\psi}{r\,\delta\theta}$$

$$\frac{\delta\phi}{r\,\delta\theta} = -\frac{\delta\psi}{\delta r}$$

Streamline Equation and Flow Net

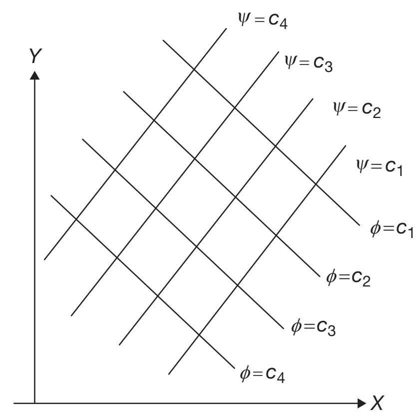
Streamline equation is given by $\frac{dy}{dx} = \frac{v}{u}$.

Discharge per unit width between two streamlines is given by

$$Q = \Psi_2 - \Psi_1$$

Streamlines and equipotential lines intersect each other orthogonally at all points of intersection.

A grid obtained by drawing a series of streamlines and equipotential lines is known as flow net. A flow net may be drawn for a two-dimensional irrotational flow and it provides a simple, yet valuable indication of flow pattern.



Elements of a flow net

[Potential function exists for irrotational flow only. The stream function applies to both rotational and irrotational flow]

Energy Equations

Forces Acting on Fluid in Motion

The various forces that may influence the motion of a fluid are due to gravity, pressure, viscosity, turbulence, surface tension and compressibility.

If a certain mass of fluid in the motion is influenced by all the above mentioned forces, then according to Newton's second law of motion the following equation of motion may be written as

$$M_a = F_g + F_p + F_v + F_t + F_s + F_e \quad (1)$$

In most of the problems of the fluids in motion the surface tension forces and compressibility forces are not significant. Hence these forces may be neglected.

So, Eq. (1) can be written as,

$$M_a = F_g + F_p + F_v + F_t \quad (2)$$

Eq. (2) is known as Reynolds's equations of motion which is useful in analysis of turbulent flows.

Further for laminar or viscous flows the turbulent forces also become less significant and hence may be neglected.

$$M_a = F_g + F_p + F_v \quad (3)$$

Eq. (3) is known as Navier–Stokes equation which is useful in analysis of viscous flow.

Further if the viscous forces are also of little significance, we may have

$$M_a = F_g + F_p \qquad (4)$$

Eq. (4) is known as Euler's equation of motion.

Euler's Equation of Motion

Consider a point $P(x, y, z)$ in a flowing mass of fluid at which let u, v and w be the velocity components in the directions x, y, and z respectively, ρ be the mass density of the fluid and p be the pressure intensity. Further, let x, y and z be the components of the body force per unit mass at same point, then:

Euler's equation of motion can be written as,

$$X - \frac{1}{\rho}\frac{\delta \rho}{\delta x} = \frac{\delta u}{\delta t} + u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} + w\frac{\delta u}{\delta z}$$

$$Y - \frac{1}{\rho}\frac{d\rho}{\delta y} = \frac{\delta v}{\delta t} + u\frac{\delta u}{\delta x} + v\frac{\delta v}{\delta y} + w\frac{\delta u}{\delta z}$$

$$Z - \frac{1}{\rho}\frac{\delta \rho}{\delta z} = \frac{\delta w}{\delta t} + u\frac{\delta w}{\delta x} + v\frac{\delta w}{\delta y} + w\frac{\delta w}{\delta z}$$

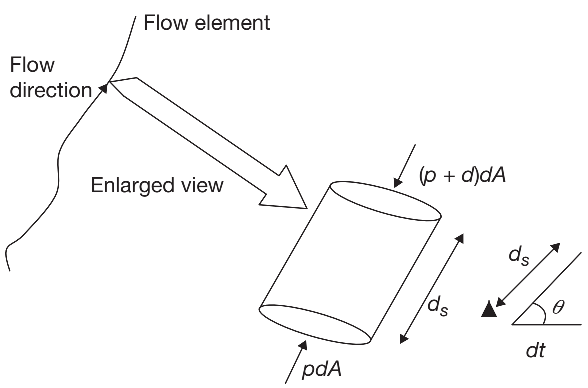
Integration of Euler's Equations

Euler's equations of motion can be integrated to yield energy equation.

The assumptions used are:

1. Flow is streamlined
2. Equation is applied where $\vec{\nabla} \times \vec{v} = 0$
3. Inviscid flow, $F_{\text{viscous}} = 0$

$$\frac{dp}{\rho} + ds\left[v\frac{\delta v}{\delta s} + \frac{\delta v}{\delta t}\right] + g\delta z = 0 \qquad (1)$$



Eq. (1) represents Euler's momentum equation for streamlined flow.

In case of steady flow, $\frac{\delta v}{\delta t} = 0$

We get, $$\iint \frac{dp}{\rho} + \int v + \int g\, dz = 0 \qquad (2)$$

Eq. (2) represents Euler's momentum equation for steady streamlined flow.

$$\int \frac{dp}{\rho} + \frac{v^2}{2} + gz = c \qquad (3)$$

Eq. (3) is known as Bernoulli's equation, which is applicable for steady irrotational flow of compressible fluids.

If the flowing fluid is incompressible, since the mass density is independent of pressure, then Eq. (3) becomes,

$$\frac{\rho}{\rho} + \frac{v^2}{2} + gz = c$$

Bernoulli's Theorem—Various Forms

First form:

$$p + \frac{1}{2}\rho v^2 + \rho gz = \text{Constant.}$$

This is energy per unit volume basis.

Second form:

$\frac{p}{\rho g} + \frac{v^2}{2g} + z =$ Constant. This is in the form of energy per unit weight basis.

$$\frac{p}{\rho g} = \text{Pressure head}$$

$$\frac{v^2}{2g} = \text{Velocity head}$$

$$Z = \text{Datum head}$$

It is representation of energy in terms of height of liquids column.

$\left(\frac{P}{\rho g} + z\right)$ is known as piezometric head.

Bernoulli's Equation

Bernoulli's equation is stated as follows:

$$\boxed{\frac{P}{\rho} + \frac{v^2}{2} + gz = C}$$

Where C is a constant. This equation is applicable only for a steady incompressible flow along a streamline and only in the inviscid regions (regions where viscous or frictional effects are negligibly small compared to inertial, gravitational and pressure effects) of flow. For point 1 and 2 along the same streamline, Bernoulli's equation can be written as:

$$\boxed{\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2}$$

Bernoulli's equation is not applicable in a flow section that involves a pump, turbine, from or any other machine or impeller since these devices destroy streamlines and transfer or extract energy to or from the fluid particles. This equation should also not be used for flow sections where significant temperature changes occur through heating or cooling sections.

NOTE

For a fluid flow, in general, the value of the constant C is different for different streamlines. However, if the flow is irrotational, constant C has the same value for all the streamlines in the flow. In other words, for irrotational flows, Bernoulli's equation becomes applicable across streamlines, i.e., between any two points in the flow region.

Bernoulli's equation and conservation of mechanical energy

The mechanical energy of a flowing fluid expressed on a unit-mass basis is,

$$\boxed{e_{\text{mech}} = \frac{P}{\rho} + \frac{v^2}{2} + gz}$$

Where, $\frac{P}{\rho}$ is the flow or pressure energy, $\frac{v^2}{2}$ is the kinetic energy and gz is the potential energy of the fluid, all per unit mass.

From Bernoulli's equation the following equation can be written

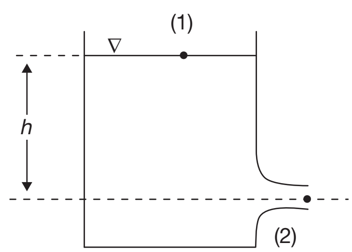
$$\boxed{E_{\text{mech}} = \text{Constant}}$$

Where, E_{mech} is the mechanical energy (sum of the kinetic, potential and flow energies) of a fluid particle is constant along a streamline in a steady, incompressible and inviscid flow. Hence, Bernoulli's equation can be taken as a 'conservation of mechanical energy principle'.

It is to be noted that the mechanical energy remains constant in an irrotational flow field.

Liquid discharge from a large tank.

A large tank open to the atmosphere is filled with a liquid to a height of h metres from the nozzle as shown in the following figure.



The flow is assumed to be incompressible and irrotational. The draining of the water is slow enough that the flow can be assumed to be steady (quasi-steady). Any losses in the nozzle are neglected. Point 1 is taken to be at the free surface of water and so $P_1 = P_{\text{atm}}$ and point 2 is taken to be at the centre of the outlet area of the nozzle and so $P_2 = P_{\text{atm}}$.

If A_1 and A_2 are the cross-sectional areas of the tank and nozzle respectively, then from the continuity equation, we have

$$A_1V_1 = A_2V_2 \qquad (1)$$

Since the tank is very large compared to the nozzle, we have $A_1 >>>> A_2$. Hence from Eq. (1), we have

$$V_1 \approx 0$$

From the Bernoulli's equation, we have

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Or $$V_2^2 = 2g(z_1 - z_2)$$

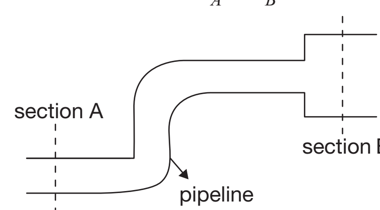
Or $$\boxed{V_2 = \sqrt{2gh}} \qquad (2)$$

Eq. (2) is called the *Torricelli equation.*

Example 5

Section A of the pipeline, shown in the figure below, has a diameter of 20 cm and a gauge pressure (P_A) of 40 kPa. The section is at an elevation of 120 m. The section B of the pipeline has a diameter of 40 cm and is at an elevation of 125 m. The volumetric flow rate of the liquid (density = 1100 kg/m^3) through the pipeline is 70 lit/s. If the frictional losses in the pipeline can be neglected and if P_B denotes the pressure of section B, then,

(A) flow is from B to A and $P_A - P_B = 51.395$ kPa
(B) flow is from A to B and $P_A - P_B = 51.395$ kPa
(C) flow is from A to B and $P_A - P_B = 28.605$ kPa
(D) flow is from B to A and $P_A - P_B = 28.605$ kPa



Solution

At section A, velocity of flow,

$$v_A = \frac{Q}{A_A}$$

$$= \frac{\frac{70}{1000}}{\frac{\pi}{4} \times \left(\frac{20}{100}\right)^2} = 2.228 \text{ m/s}$$

At section B, velocity of flow,

$$v_B = \frac{Q}{A_B} = \frac{\frac{70}{1000}}{\frac{\pi}{4} \times \left(\frac{40}{100}\right)^2} = 0.557 \text{ m/s}$$

Assuming the flow to be steady, Bernoulli's equation application between the two sections gives,

$$\frac{P_A}{\rho} + \frac{v_A^2}{2} + gz_A = \frac{P_B}{\rho} + \frac{v_B^2}{2} + gz_B \qquad (1)$$

Here $P_A = 40 \times 10^3$ Pa (gauge pressure)

$$z_A = 120 \text{ m}$$
$$z_B = 125 \text{ m}$$
$$\rho = 1100 \text{ kg/m}^3$$

Hence Eq. (1) gives,

$$\frac{40\times10^3}{1100}+\frac{(2.228)^2}{2}+9.81\times120$$
$$=\frac{P_B}{1100}+\frac{(0.557)^2}{2}+9.81\times125$$

Or $P_B = -11.395$ kPa (gauge pressure)

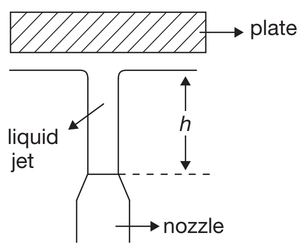
Since, $P_A > P_B$, flow is from A to B and

$$P_A - P_B = 40 - (-11.395)$$
$$= 51.395 \text{ kPa.}$$

Hence, the correct answer is option (B).

Example 6

A vertical jet of liquid (density = 850 kg/m^3) is issuing upward from nozzle of exit diameter 70 mm at a velocity of 15 m/s. A flat plate weighing 250 N is supported only by the jets impact. If all losses are neglected then the equilibrium height h of the plate above the nozzle exit is

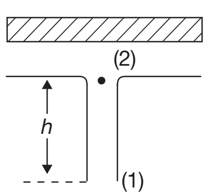


(A) 11.468 m (B) 6.434 m
(C) 9.682 m (D) 10.145 m

Solution

Mass flow rate, $\overset{\circ}{m} = \rho A v$

$$= 850 \times \frac{\pi}{4}\times\left(\frac{70}{1000}\right)^2\times15$$
$$= 49.068 \text{ kg/s}$$

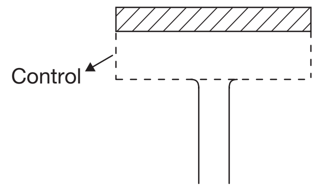


Applying Bernoulli's equation between points (1) and (2), we get

$$\frac{P_1}{\rho}+\frac{v_1^2}{2}+gz_1=\frac{P_2}{\rho}+\frac{v_2^2}{2}+gz_2$$

Here $P_1 = P_2 = P_{atm}$ and $z_2 - z_1 = h$

$$\therefore \quad v_2 = \sqrt{v_1^2 - 2gh}$$
$$= \sqrt{(15)^2 - 2\times9.81\times h}$$



Applying the linear momentum balance equation for the control volume shown above, we get

$$-250 = \overset{\circ}{m}(0 - v_2)$$

(momentum correction factor is assumed to be unity)

$$= -49.068\times\sqrt{(15)^2 - 2\times9.81\times h}$$
$$h = 10.145 \text{ m.}$$

Hence, the correct answer is option (D).

Types of Head of a Fluid in Motion

The Bernoulli's equation can be rewritten as:

$$\frac{p}{\rho g}+\frac{v^2}{2g}+z = \text{Constant}$$

Each term on the *LHS* of the above equation has the dimension of length and represents some kind of head of a flowing fluid.

1. **Pressure head:** It is the term $\frac{p}{\rho g}$ and it represents the height of a fluid column that is needed to produce the pressure p.
2. **Velocity head:** It is the term $\frac{v^2}{2g}$ and it represents the elevation needed for the fluid to reach the velocity v from rest during a frictionless free fall.
3. **Elevation head:** It is term z and it represents the potential energy of the fluid.

The sum of the pressure head and the elevation head, i.e., $\frac{p}{\rho g}+z$, is known as the piezometric head.

Static, Dynamic, Hydrostatic, Total and Stagnation Pressures

The Bernoulli's equation can be rewritten as:

$$p+\frac{\rho v^2}{2}+\rho gz = \text{Constant}$$

Each term on the LHS of the above equation has the units of pressure and represents some kind of pressure.

1. **Static pressure:** It is the term p and it represents the actual thermodynamic pressure of the fluid as it flows.
2. **Dynamic pressure:** It is the term $\frac{\rho v^2}{2}$ and it represents the pressure rise when the fluid is brought to a stop isentropically.

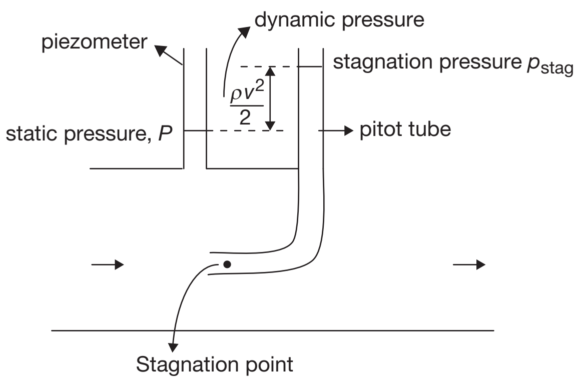
3. **Hydrostatic pressure:** It is the term ρgz. It is actually not a pressure although it does represent the pressure change possible due the potential energy variation of the fluid as a result of elevation changes.

Total pressure = Static + Dynamic + Hydrostatic pressures

Stagnation pressure = Static + Dynamic pressure

Stagnation pressure (p_{stag}) represents the pressure at a point where the fluid is brought to a complete stop isentropically.

$$\boxed{p_{stag} = p + \frac{\rho v^2}{2}}$$



Example 7

A two-dimensional irrotational flow has the velocity filed:

$$\vec{v} = ay\hat{i} + bx\hat{j}.$$

The angle made by the velocity vector at the point (1, 1) with the horizontal is

(A) 0° (B) 45°
(C) 30° (D) 60°

Solution

From the velocity field representation, we have $u = ay$, $v = bx$

Since the flow is irrotational, $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$

That is, $b = a$ (1)

Let the angle made by a velocity vector at point (x, y) is the flow field be θ.

$$\therefore \tan\theta = \frac{v}{u} \text{ (from slope of streamline)}$$

$$= \frac{bx}{ay} \quad (2)$$

Substituting Eq. (1) in Eq. (2), we get

$$\tan\theta = \frac{x}{y}$$

At point (1, 1), $\tan\theta = \frac{1}{1} = 1$

$$\therefore \theta = 45°.$$

Hence, the correct answer is option (B).

Force Exerted by Flowing Fluid on a Pipe Bend

As per **impulse-momentum theorem**, the impulse of a force on a body is equal to the change in linear momentum of the body in the duration of time for which the force acts.

That is, $\bar{F}dt = d\bar{p} = d(m\bar{v})$.

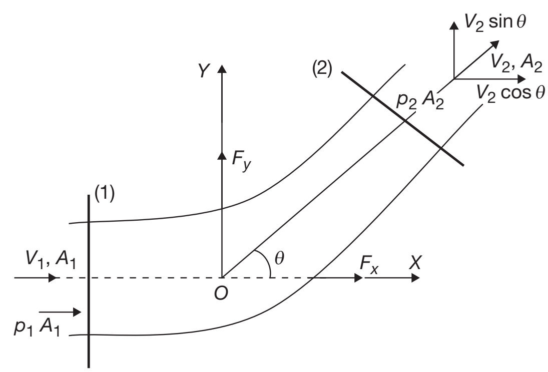
This can also be applied to forces acting on fluids.

Consequently, $\bar{F} = \frac{d\bar{p}}{dt} = \frac{d}{dt}(m\bar{v})$ = Rate of change of linear momentum.

For fluids, rate of change of linear momentum, $\frac{d\bar{p}}{dt} = \frac{d}{dt}(m\bar{v}) = \dot{m}(d\bar{v})$

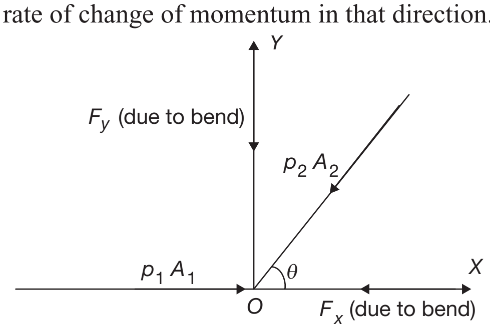
= Mass per second × (Change of velocity)
= (Density × Discharge) × Change of velocity
= $\rho Q(d\bar{v})$; $\boxed{F = \rho Q d\bar{v}}$

This equation can be used to determine the net force exerted by a flowing fluid on a pipe bend.



Consider a reducing elbow as shown in the figure. At the inlet section (1), pressure intensity = P_1, velocity of flow = V_1, along x-direction, area of cross-section = A_1, At the exit section (2), pressure intensity = Pp_2, velocity of flow = V_2 *at an angle* θ with X-axis and area of cross-section A_2. Let $\bar{F}$ be the force exerted by the flowing fluid on the bend, which can be resolved as $\bar{F}_x$ and $\bar{F}_y$ along the x and y directions respectively. As per Newton's third law of motion, the bend exerts an equal and opposite force $-\bar{F}$ on the fluid, which can be resolved as $-\bar{F}_x$ and $-\bar{F}_y$ in the x and y directions. The minus (–) sign shows that the direction of force exerted by the bend on fluid is opposite to corresponding force exerted by fluid on bend.

Along the x and y-directions, the forces on the fluid due to pressure of fluid and force exerted by bend, can be equated to the rate of change of momentum in that direction.



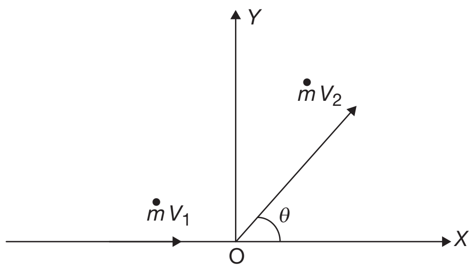
Forces on Fluid due to Pressures and Due to Bend

Net force on fluid is x direction is (let us call this P_x)

$$\boxed{P_x = p_1A_1 - p_2A_2 \cos\theta - F_x}$$

Net force on fluid in y-direction (let us call this P_y)

$$\boxed{P_y = -p_2A_2 \sin\theta - F_y}$$



Linear Momentum of Fluid at Inlet and Outlet

Time rate of change of linear momentum of fluid along X-axis,

$$= \dot{m}V_2 \cos\theta - \dot{m}V_1$$

$$= \dot{m}(V_2 \cos\theta - V_1)$$

$$= \rho Q\,(V_2 \cos\theta - V_1)\ [Q = \text{discharge in m}^3/\text{s}$$

$$\rho = \text{density in kg/m}^3]$$

Time rate of change of linear momentum of fluid along Y-axis,

$$= \dot{m}V_2 \sin\theta - 0$$

$$= \dot{m}V_2 \sin\theta$$

$$= \rho Q\, V_2 \sin\theta$$

Equate the net force on fluid in the x-direction to the time rate of change of linear momentum in the x-direction

$$\therefore\quad P_x = P_1A_1 - P_2A_2 \cos\theta - F_x = \rho Q\,(V_2 \cos\theta - V_1)$$

$$\Rightarrow\quad F_x = P_1A_1 - P_2A_2 \cos\theta - \rho Q\,(V_2 \cos\theta - V_1)$$

$$\Rightarrow\quad \boxed{F_x = P_1A_1 - P_2A_2\cos\theta - \rho Q\,(V_1 - V_2 \cos\theta)}$$

Therefore, $\boxed{F_x = P_1A_1 - P_2A_2\cos\theta - \rho Q\,(V_1 - V_2\cos\theta)}$ is the x-component of the force exerted by fluid on bend.

Similarly, equating the net force on fluid is the y-direction to the time rate of change of linear momentum is the y-direction,

$$P_y = -P_2A_2 \sin\theta - F_y = \rho Q\, V_2 \sin\theta$$

$$\therefore\quad F_y = -P_2A_2 \sin\theta - \rho Q\, V_2 \sin\theta$$

$$= -(P_2A_2 + \rho QV_2)\sin\theta$$

$\therefore$ $\boxed{F_y = -(P_2A_2 + \rho QV_2)\sin\theta}$ is the y-component of the force exerted by fluid on bend.

The net force (F) **exerted by fluid on bend** is given by,

$$\boxed{F = \sqrt{F_x^2 + F_y^2}}.$$

The angle (α) mode by the net force exerted by fluid on bend is given by,

$$\boxed{\tan\alpha = \frac{F_y}{F_x}}$$

Direction for solved examples 8 and 9:

The volumetric flow rate of a liquid of density 900 kg/m^3, flowing through a bent pipe, as shown in the figure, is 400 litres per second at the inlet of the pipe. The pipe which is bend by an angle θ has a constant diameter of 500 mm. The liquid is flowing in the pipe with a constant pressure of 500 kN/m^2. The horizontal component of the resultant force on the bend has a magnitude of 148325.358 N.

Example 8

The value of the angle θ is approximately

(A) 60° (B) 120°
(C) 30° (D) 45°

Solution

Let the subscripts 1 and 2 denote the inlet and outlet of the pipe respectively.

Diameter of the pipe, $d = 0.5$ m

Density of the fluid, $\rho = 900$ kg/m^3

Cross-sectional areas of the pipe,

$$A_1 = A_2 = \frac{\pi d^2}{4} = \frac{\pi \times (0.5)^2}{4} = 0.1963 \text{ m}^3$$

Given, pressures $p_1 = p_2$

$$= 500 \times 10^3 \text{ N/m}^2$$

Let $\vec{R}$ be the reaction force exerted by the bend on the control volume.

Now $\vec{R}$ would be equal and opposite in direction to the resultant force exerted in the bend. Let R_H and R_r be the magnitude of the respective horizontal and vertical components of $\vec{R}$.

Given, $R_H = 148325.358$ N

Now, mass flow rate,

$$\mathring{m} = \rho\, Q_1 = 900 \times 0.4 = 360 \text{ kg/s}$$

The flow is assumed to be steady flow. Also the weight of the pipe and the water in it is neglected. From the continuity equation, we can write

$$A_1v_1 = A_2v_2$$

Where v_1 and v_2 are velocities assuming uniform flow at inlet the (incompressible) liquid average and outlet. Given, volumetric flow rate

$$Q_1 = A_1V_1 = 0.4 \text{ m}^3/\text{s}$$

$$\therefore\quad V_1 = V_2 = \frac{0.4}{0.1963} = 2.0377 \text{ m/s}$$

The change in momentum in the direction of flow can be equated to

$$P_1A_1 + P_2A_2 \cos(180° - \theta) - R_H$$

∴ Therefore it becomes

$$P_1A_1 + P_2A_2 \cos(180° - \theta) - R_H$$

$$= (-v_2\cos(180° - \theta) - v_1)\ \overset{\circ}{m}$$

∴ cos (180°– θ) = (148325.358 – 360 × 2.0377 – 500 × 10^3 × 0.1763)

$$\therefore \cos(180° - \theta) = \frac{(148325.358) - 360 \times 2.0377 - 500 \times 10^3 \times 0.1963}{(500 \times 10^3 \times 0.1963 + 360 \times 2.0377)}$$

i.e., cos (180° – θ) = 0.5

or cos 180° – θ = 60°

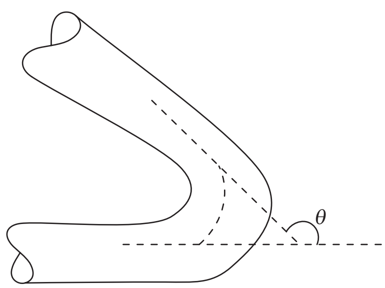
θ = 120°.

Hence, the correct answer is option (B).

Example 9

The magnitude of the resultant force on the bend is

(A) 148325.358 N (B) 85633. 17 N
(C) 0 N (D) 171270.11 N



Solution

Now cos(180° – θ) = 0.5

$$\sin(180° - \theta) = \sqrt{1 - \cos^2(180° - \theta)}$$
$$= 0.8660$$

The linear momentum equation in the y-direction

$$\sum F_y = \overset{\circ}{m}(V_{2,y} - V_{1,y}) \quad (2)$$

Here,
$$v_{1,y} = 0$$
$$V_{2,y} = V_2\sin(180° - \theta)$$
$$\sum F_y = -P_2A_2\sin(180° - \theta) + R_v$$

∴ Eq. (2) becomes

$$R_v - P_2A_2 \sin(180° - \theta)$$
$$= \overset{\circ}{m}\, v_2\sin(180° - \theta)$$

or,
$$R_v = 360 \times 2.0377 \times 0.8660 + 500 \times 10^3 \times 0.1963 \times 0.8660$$
$$= 85633.17 \text{ N}$$

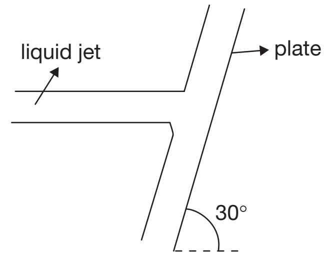
∴ Magnitude of the resultant force,

$$|\vec{R}| = \sqrt{R_H^2 + R_V^2}$$
$$= \sqrt{148325.358^2 + 85633.17^2}$$
$$= 171270.11 \text{ N.}$$

Hence, the correct answer is option (D).

Example 10

A 3.57 m diameter jet of liquid (density = 1100 kg/m^3/ from a nozzle steadily strikes a flat plate, inclined at an angle of 30° to the horizontal, as shown in the following figure.



If a horizontal force of 275.27 kN is applied on the plate to hold it stationary than the velocity of the liquid jet is

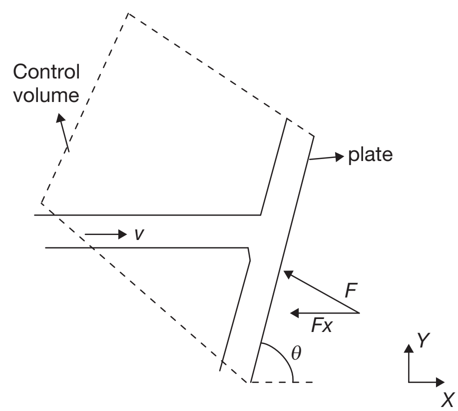
(A) 9.52 m/s (B) 3.37 m/s
(C) 90.63 m/s (D) 4.76 m/s

Solution

Let F be the force applied normally on the plate to hold it stationary. Let F_x be the horizontal component of the force F.

Given $F_x = 275.27 \times 10^3$ N

Linear momentum equation in the



direction normal to the plate yields:

$$-F = \overset{\circ}{m}\ (0 - v\cos(90 - \theta))$$

or
$$F = \overset{\circ}{m}\, v\sin\theta$$
$$= \rho A v^2 \sin\theta \quad (1)$$

Now here,

$$F_x = F\cos(90 - \theta) = F\sin\theta \quad (2)$$

Comparing Eqs. (1) and (2), we get $F_x = \rho A v^2 \sin^2\theta$

$$\text{So } 275.27 \times 10^3 = 1100 \times \frac{\pi}{4} \times (3.75)^2 \times V^2 \times (\sin 30°)^2$$

$\therefore \qquad V = 9.52$ m/s.

Hence, the correct answer is option (A).

Moment of Momentum Principle

The resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

Angular Momentum Equation

The general form of the angular momentum (or moment of momentum) equation that applies to a fluid, moving or deforming control volume is

$$\sum \vec{m} = \frac{\partial}{\partial t}\int_{cv}(\vec{r}\times\vec{v})\rho dv + \int_{cs}(\vec{r}\times\vec{v})\rho(\vec{v}_r\cdot\vec{n})dA \qquad (1)$$

Here, $\sum \bar{\vec{m}} = \sum(\vec{r}\times\vec{F})$ is the vector sum of the moment of all the forces acting on the control volume.

The term $\frac{\partial}{\partial_t}\int_{cv}(\vec{r}\times\vec{v})\rho dv$ represents the time rate of change of the angular momentum of the contents of the control volume and the term $\int_{cs}(\vec{r}\times\vec{v})\rho(\vec{v}\cdot\vec{n})dA$ represents the net flow rate of angular momentum out of the control surface by mass flow. For a fixed and non-deforming control volume, the angular momentum equation is,

$$\sum \vec{m} = \frac{\partial}{\partial t}\int_{cv}(\vec{r}\times\vec{v})\rho dv + \int_{cs}(\vec{r}\times\vec{v})\rho(\vec{v}_r\cdot\vec{n})dA$$

An approximate form the angular momentum equation written in terms of average properties becomes,

$$\sum \vec{m} = \frac{\partial}{\partial t}\int_{cv}(\vec{r}\times\vec{v})\rho dv + \sum_{out}\left(\vec{r}\times\mathring{m}\,\vec{v}_{avg}\right) - \sum_{in}\left(\vec{r}\times\mathring{m}\,\vec{v}_{avg}\right) \qquad (2)$$

For a steady flow, Eq. (2) reduces to

$$\sum \vec{m} = \sum_{out}(\vec{r}\times\dot{m}v_{avg}) - \sum_{in}\left(\vec{r}\times\mathring{m}\,\vec{v}_{avg}\right) \qquad (3)$$

Note that the term $\sum \vec{m}$ also represents the net torque acting on the control volume.

If the significant forces and momentum flows are in the same plane, then they would give rise to moments in the same plane. For such cases, Eq. (3) can be expressed in a scalar form as:

$$\sum \overline{m} = \sum_{out} r\,\mathring{m}v - \sum_{in} r\,\mathring{m}v$$

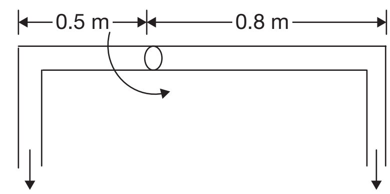
Where r represents the average normal distance between the point about which moments are taken and the line of action of the force or velocity provided that the same convention is followed for the moments. Moments in the counter clockwise position are positive and moments in the clockwise direction are negative.

Example 11

The sprinkler, shown in the following figure, has a frictionless shaft with equal flow in both the nozzles. If the water jets from the nozzles have a velocity of 10 m/s relative to the nozzles then the sprinkler rotates at an r p m of

(A) 32.19 (B) 318.31
(C) 139.48 (D) 73.46

Solution



Given, $r_A = 0.5$ m

$$r_B = 0.8 \text{ m}$$

Relative velocities, $v_{r,A} = 10$ m/s and

$$V_{r,B} = 10 \text{ m/s}$$

Let ω be the angular velocity of the sprinkler.

Absolute fluid velocity of A,

$$v_{a,A} = v_{r,A} + \omega r_A = 10 + 0.5\omega$$

Absolute fluid velocity of B,

$$v_{a,B} = v_{r,B} - \omega r_B = 10 - 0.8\omega.$$

NOTE

The jets of water coming out from the nozzle will exert a force in the opposite direction. So torque at B will be in the anticlockwise direction and torque at A will be in the clockwise direction. Since torque at B is greater than the torque at A, hence the sprinkler, if free, will rotate in the anticlockwise direction.

Since, there is no friction and no external torque is applied on the sprinkler, $\sum m = 0$.

Since, the moment of momentum of the water entering the sprinkler is zero,

$$\sum_{in} r\,\mathring{m}\,v = 0$$

$\therefore$ Similarly at exit it becomes,

$$\sum_{out} r\,\mathring{m}\,v = 0$$

or $\qquad -\mathring{m}_A r_A v_{a,A} + \mathring{m}_B r_B v_{a.B} = 0$

given, $\qquad \mathring{m}_A = \mathring{m}_B$

$\therefore$ –0.5 (10 +0.5 w) + 0.8 (10 –0.8ω) = 0

Or ω = 3.3708 rad/s.

If N is the speed of rotation of the sprinkler in rpm, then

$$\frac{2\pi N}{60} = 10 \text{ or } N = \frac{60 \times 3.3708}{2 \times \pi}$$

$$= 32.19 \text{ rpm.}$$

Hence, the correct answer is option (A).

Flow Through Orifices

A small opening of any cross-section, made on the bottom or sidewall of a tank through which a fluid can flow, is called an **orifice**.

Classification of Orifices

The various basis for classification of orifices are:

1. **Based on size of orifice:**
   - **(a) Small orifice:** If the head of liquid from the centre of orifice is more than five times the depth of orifice.
   - **(b) Large orifice:** If the head of liquid from the centre of orifice is less than five times the depth of orifice.
2. **Based on shape of cross-sectional area:**
   - (a) Circular orifice
   - (b) Triangle orifice
   - (c) Square orifice
   - (d) Rectangular orifice
3. **Based on shape of upstream edge of orifice:**
   - (a) Sharp edged orifice
   - (b) Bell–mouthed orifice
4. **Based on nature of discharge:**
   - (a) Free discharging orifices
   - (b) Drowned or submerged orifices, which are further classified as fully submerged orifices and partially submerged orifices.

When a jet of fluid flows out of a circular orifice, the area of cross-section of the jet keeps on decreasing and **becomes a minimum at the vena contracta and beyond that the jet diverges**. The location of minimum cross-sectional area (i.e., **vena contracta**) is approximately at a distance of half the diameter of the orifice from the tank. If the flow through the orifice is steady at a constant head H and the cross-sectional area of the tank is very large when compared to the cross-sectional area of the jet, it can be shown using Bernoulli's theorem that the theortical velocity of flow at the vena contracta $\boxed{V_T = \sqrt{2gH}}$, where g = acceleration due to gravity. The actual velocity of flow (V) at the vena contracta is less than this theoretical value, i.e., $V < V_T$.

The ratio $\frac{V}{V_T} = C_V$ = Coefficient of velocity.

Hence **coefficient of velocity (C_V)** is defined as the ratio of the actual velocity of flow at the vena contracta to the theoretical velocity of flow at the same location.

$$\therefore \quad \boxed{C_V = \frac{V}{V_T} = \frac{V}{\sqrt{2gH}}}$$

The value of C_V varies from 0.95 to 0.99 for various orifices and this value depends on:

1. Shape of orifice
2. Size of orifice and
3. On the head under which the flow takes place.

$$\therefore \quad C_V < 1$$

Coefficient of contraction (C_C) is defined as the ratio of area of cross-section of the jet at the vena contracta (a_c) to the cross-sectional area of orifice (a).

$$\therefore \quad \boxed{C_C = \frac{a_c}{a} < 1}$$

The value of C_C varies from 0.61 to 0.69 for various orifices and depends upon the same factors on which C_V depends.

Coefficient of discharges (C_d) is defined as the ratio of actual discharge from an orifice to the theoretically possible discharge through the orifice.

$$\therefore \quad C_d = \frac{Q_{\text{actual}}}{Q_{\text{theoretical}}}$$

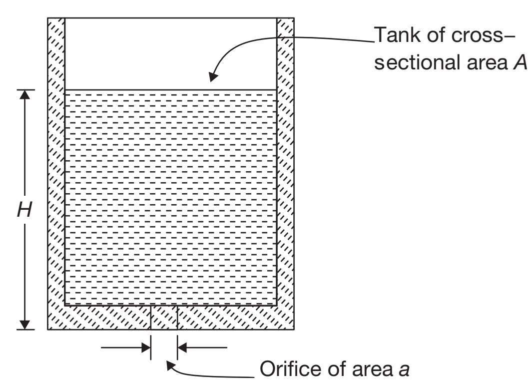
$$= \frac{\text{Actual cross-sectional area} \times \text{Actual velocity}}{\text{Theoretical cross-sectional area} \times \text{Theoretical velocity}}$$

$$= \frac{a_c \times V}{a \times V_T} = C_C \times C_V$$

$$\therefore \quad \boxed{C_d = C_C \times C_V}$$

The value of C_d varies from 0.61 to 0.65 for different orifices and depends on shape and size of orifice and the head under which the flow occurs.

Time for emptying a tank of uniform cross-sectional area through an orifice at its bottom:



At time $t = 0$, the height of liquid above orifice is H.

Using Bernoulli's equations, it can be shown that the theoretical time required for completely emptying the tank is,

$$T = \left(\frac{A}{a}\right)\sqrt{\frac{2H}{g}}.$$

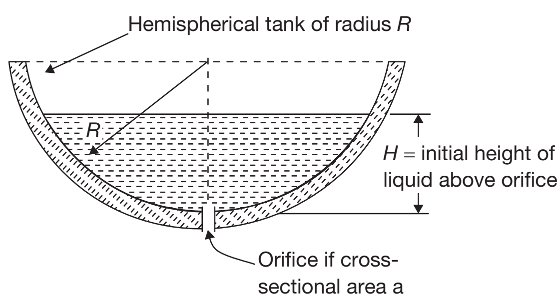
It may be noted that $\sqrt{\frac{2H}{g}}$ is the time needed for free fall from rest from a height of H.

If C_d is the coefficient of discharge through the nozzle, $T_{\text{ACTUAL}} = \left(\frac{A}{a}\right)\frac{1}{C_d}.\sqrt{\frac{2H}{g}}$ is the actual time taken for emptying the tank.

Also, the time needed for emptying the same tank from an initial height of liquid H_1 above orifice to a final height of liquid H_2 above orifice is given by,

$$T = \left(\frac{A}{a}\right)\frac{1}{C_d}.\sqrt{\frac{2}{g}}(\sqrt{H_1} - \sqrt{H_2})$$

Time for emptying a hemispherical tank through an orifice at the bottom:



If C_d is the coefficient of discharge through the orifice, it can be shown that the actual time needed for emptying the hemispherical tank is,

$$T_{\text{actual}} = \frac{\pi}{C_d a\sqrt{2g}}\left[\frac{4}{3}RH^{3/2} - \frac{2}{5}H^{5/2}\right]$$

Where

R = Radius of hemispherical tank

H = Initial height of liquid above orifice

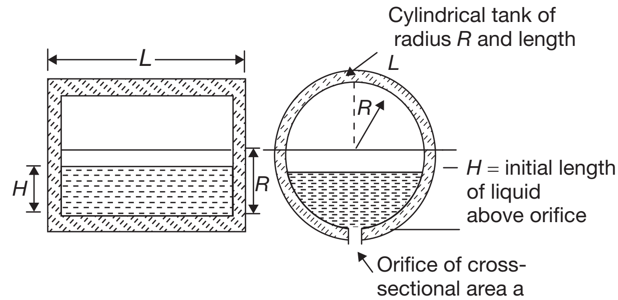
a = Cross-sectional area of orifice

g = Acceleration due to gravity

If initial height of liquid above orifice is H_1 and final height of liquid above orifice is H_2, then time needed for emptying the hemispherical tank is

$$T = \frac{2\pi}{C_d A\sqrt{2g}}\left[\frac{2}{3}R\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{1}{5}\left(H_1^{5/2} - H_2^{5/2}\right)\right]$$

Time for emptying a circular horizontal tank through an orifice at its bottom:



A horizontal cylindrical tank of radius R and length L is fitted with an orifice of cross-sectional area a at its bottom. The height of liquid above the nozzle is H. The coefficient of discharge through the nozzle is C_d.

Time for emptying the horizontal cylindrical tank is,

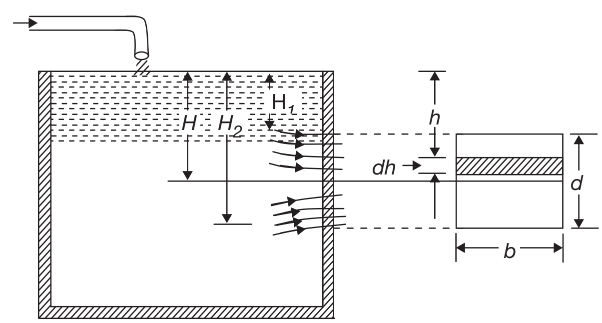
$$T = \frac{4L}{3C_d a\sqrt{2g}}\left[(2R)^{3/2} - (2R - H)^{3/2}\right]$$

If initial height of liquid above orifice is H_1 and final height of liquid above orifice is H_2, time required for decreasing the liquid level from H_1 to H_2 (i.e., emptying through orifice) is,

$$T = \frac{4L}{3C_d a\sqrt{2g}}\left[(2R - H_2)^{3/2} - (2R - H_1)^{3/2}\right]$$

Discharge through large rectangular orifice:

In a large rectangular orifice, there is a considerable variation of effective pressure head over the height of the orifice. Hence the velocity of liquid particles through the orifice is not constant.



Consider a large rectangular orifice of with b and height d, fitted to one vertical side of a large tank, discharging freely into atmosphere, under a constant H as shown in the figure.

Where

H_1 = Height of liquid above top edge of orifice

H_2 = Height of liquid above bottom edge of orifice

∴ Height of orifice, $d = H_2 - H_1$

b = Width of orifice

C_d = Coefficient of discharge of orifice

Area of a strip of orifice of height dh at a depth h below the free surface of liquid in the tank is $dA = bdh$

V = Theoretical velocity of flow through this strip $= \sqrt{2gh}$

$\therefore$ Discharge through the strip, dQ

$= C_d \times$ Area of strip $\times$ Velocity

$= C_d\,(bdh)\,\sqrt{2gh}$

$= C_d b\,\sqrt{2gh}\,dh$

$\therefore$ Total discharge through orifice,

$$Q = \int dQ = \int_{H_1}^{H_2} C_d b \sqrt{2gh}\;dh$$

$$\Rightarrow \boxed{Q = \frac{2}{3} C_d b \sqrt{2g}\left[H_2^{3/2} - H_1^{3/2}\right]}$$

is the actual discharging through the large orifice.

NOTE

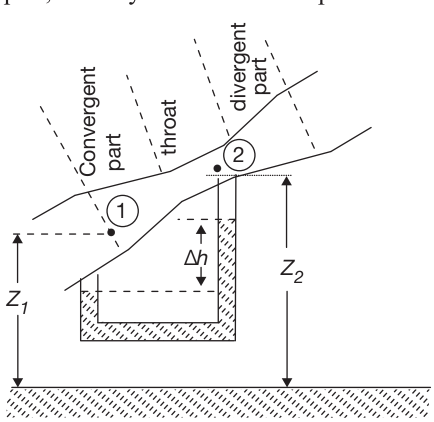
Velocity of approach is the velocity with which the liquid approaches the orifice. In the above expression for discharge Q over the rectangular orifice, velocity of approach V_a is taken as zero. If $V_a \neq 0$, then $H_{1\text{eff}} = \left(H_1 + \frac{V_a^2}{2g}\right)$ and $H_{2\text{eff}} = \left(H_2 + \frac{V_a^2}{2g}\right)$. In the expression for Q, H_1 and H_2 will get replaced to $H_{1\text{ eff}}$ and $H_{2\text{ eff}}$.

Practical Applications of Bernoulli's Equation

Venturimeter

It consists of two conical parts, the convergent part and the divergent part, with a small portion of uniform cross-section (with the minimum area), called the throat, in between the parts. The venturimeter is always used so that the upstream part of the flow takes place through the convergent part while the downstream part of the flow takes place through the divergent part.

In the convergent part, the velocity increases in the flow direction while the pressure decreases, with the velocity being maximum and pressure being is minimum at the throat. In the divergent part, velocity decreases while pressure increases.



From the Bernoulli's equation and the continuity equation, the velocity at the throat is obtained as follows:

$$V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g(h_1^* - h_2^*)}$$

Where, h_1^* and h_2^* are the piezometric heads at section 1 and 2 respectively and are given by:

$$h_1^* = \frac{p_1}{\rho g} + z_1 \qquad h_2^* = \frac{p_2}{\rho g} + z_2$$

The theoretical discharge or flow rate is given by,

$$Q = A_2 V_2 = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g(h_1^* - h_2^*)}$$

Here, $h_1^* - h_2^* = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$, where ρ_m is the density of the manometric fluid.

The actual discharge or flow rate is given by,

$$Q_{\text{actual}} = C_D \times Q$$

$$= C_D \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\Delta h\left(\frac{\rho_m}{\rho} - 1\right)}$$

Where, C_D is the coefficient of discharge or coefficient of venturimeter. C_D is always less than unity and lies between 0.95 and 0.98. The coefficient of discharge is introduced to account for the fact that the measured values of Δh for a real fluid will always be greater than that assumed for an ideal fluid due to frictional losses.

Example 12

A venturimeter with a throat diameter of 50 mm is used to measure the velocity of water in a horizontal pipe of 200 mm diameter. The pressure at the inlet of the venturimeter is 20 kPa and the vacuum pressure at the throat is 10 kPa. If frictional losses are neglected, then the flow velocity is

(A) 28 cm/s (B) 24.2 cm/s
(C) 14 cm/s (D) 48.5 cm/s

Solution

Given $p_1 = 20 \times 10^3\ P_a$

$$p_2 = -10 \times 10^3\ P_a$$

Since, the venturimeter would be horizontal, $z_1 = z_2$

$$\text{Now } h_1^* - h_2^* = \frac{p}{\rho g} + z_1 - \frac{p_2}{\rho g} - Z_2$$

$$= \frac{(20\times 10^3 + 10\times 10^3)}{1000\times g} = \frac{30}{g}$$

The flow velocity, $V_1 = \frac{A_2 V_2}{A_1}$

$$= \frac{A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\times(h_1^* - h_2^*)}$$

Here, $A_1 = \frac{\pi}{4}\left(\frac{200}{1000}\right)^2$

$$A_2 = \frac{\pi}{4}\left(\frac{50}{1000}\right)^2$$

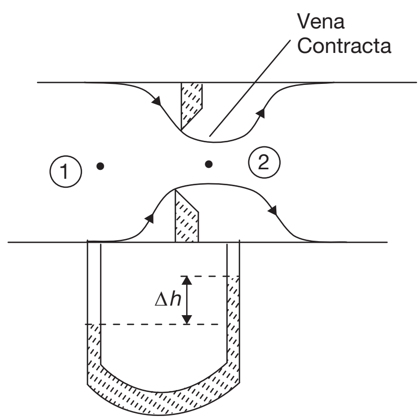
$$\therefore V_1 = \frac{50^2}{\sqrt{200^4 - 50^4}} \times \sqrt{60}$$

$$= 48.5 \text{ cm/s.}$$

Hence, the correct answer is option (D).

Orificemeter

An orificemeter is a thin circular plate with a sharp edged concentric circular hole in it.



The flow through the orificemeter from an upstream section contracts until a section downstream, where the vena contracta is formed, and then expands to fill the whole pipe. One of the pressure tappings is usually provided at the upstream of the orifice plate where the flow is uniform and the other is provided at the vena contracta. At the vena contracta, streamlines converge to a minimum cross-section.

The velocity of flow at the vena contracta,

$$V_2 = C_v \sqrt{\frac{2\rho g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h}{1 - \frac{A_2^2}{A_1^2}}}$$

Where, ρ_m is the density of the manometric liquid and C_V is the *coefficient of velocity.*

C_V is always less than unity. The coefficient of velocity is introduced to account for the fact that the pressure drop for a real fluid is always more due to friction that assumed for an inviscid flow.

The volumetric flow rate is given by $Q = A_2 V_2$

If the coefficient of contraction, C_c is defined as $C_c = \frac{A_2}{A_0}$, where A_0 is the area of the orifice, then

$$Q = C_d A_0 \sqrt{\frac{2g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h}{1 - C_c^2 \frac{A_0^2}{A_1^2}}}$$

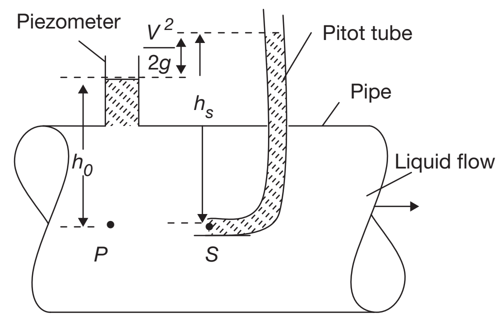
Where, the coefficient of discharge,

$$C_d = C_c$$

The coefficient of discharge of an orificemeter lies between 0.6 and 0.65.

Pitot Tube

It works on the principle that if the velocity of flow at a point becomes zero, the increase in the pressure at the point is due to conversion of kinetic energy into pressure energy. A pilot tube provides one of the most accurate methods for measuring the fluid velocity.



Point S is a stagnation point while point P is a point in the undisturbed flow both being at the same horizontal plane.

$$h_0 = \frac{p_0}{\rho g}\,;\; h_s = \frac{p_s}{\rho g}$$

Where, p_0 is the pressure at point P, i.e., static pressure and p_s is the stagnation pressure at point S.

$$\frac{p_0}{\rho g} + \frac{V^2}{2g} = \frac{p_s}{\rho g}$$

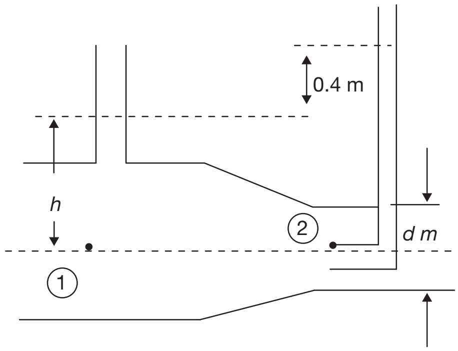
$$h_0 + \frac{V^2}{2g} = h_s$$

$$\boxed{V = \sqrt{2g(h_s - h_0)} = \sqrt{2g\Delta h}}$$

Where, Δh is the dynamic pressure head which is equal to the velocity head. It is to be noted that the pitot tube measures only the stagnation pressure and so the static pressure must be measured separately by using a piezometer. A pitot static tube however measures both static and stagnation pressures.

Example 13

Water is flowing through a pipe that contracts from a diameter of 0.15 m to d metres as shown in the following figure. The difference in manometer levels is 0.4 m. If the flow rate Q in the pipe is expressed in terms of the variable d as $Q = kd^n$, then



(A) $k = 0.0495$ and $n = 0$
(B) $k = 0.0495$ and $n = 2$
(C) $k = 7.848$ and $n = 0$
(D) $k = 6.164$ and $n = 2$

Solution

From Bernoulli's equation we have,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

Here $Z_1 = Z_2$

$V_2 = 0$ (stagnation point)

$$\therefore \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} \quad (1)$$

But $$\frac{p_1}{\rho g} = h$$

$$\frac{p_2}{\rho g} = h + 0.4$$

$$\therefore \frac{p_2}{\rho g} - \frac{p_1}{\rho g} = 0.4 \quad (2)$$

Substituting Eq. (2) in Eq (1), we have

$$\frac{V_1^2}{2g} = 0.4$$

$$V_1 = \sqrt{0.4 \times 2 \times 9.81} = 2.801 \text{ m/s}$$

$$Q = A_1 \times V_1$$

$$= \frac{\pi}{4} \times (0.15)^2 \times 2.801$$

$$= 0.0495 \text{ m}^3/\text{s}$$

$\therefore$ In the relationship,

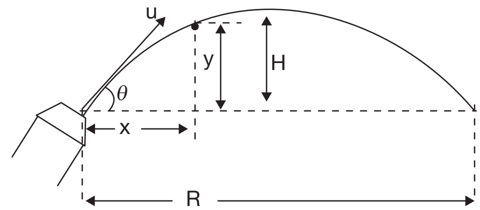
$$Q = kd^n$$

$$k = 0.0495 \text{ and } n = 0.$$

Hence, the correct answer is option (A).

Free Liquid Jet

A jet of liquid issuing from a nozzle in to the atmosphere is termed as a *free liquid jet*. The path traversed by a liquid jet under the action of gravity is called as its *trajectory* which would be a parabolic path.



Here, u is the velocity of the liquid jet and θ is the angle made by the jet with the horizontal. The equation of the jet is,

$$y = x\tan\theta - gx^2 \sec^2\theta / 2u^2.$$

1. Maximum height attained by the jet (H),

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

2. Time of flight (T),

$$T = \frac{2u \sin\theta}{g}$$

Time taken to reach the highest point is $= \dfrac{u \sin\theta}{g}$

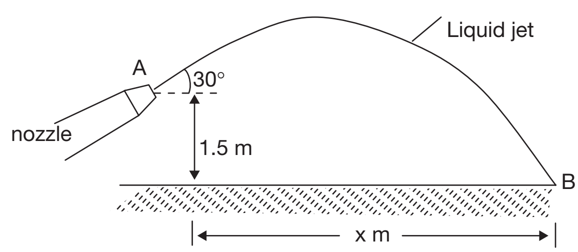
3. Horizontal range of the jet (R),

$$R = \frac{u^2 \sin 2\theta}{g}$$

Range is maximum when $\theta = 45°$ and its value is $\dfrac{u^2}{g}$.

Example 14

The flow rate of a liquid through a nozzle of diameter 50 mm is 18.62 lit/s. The nozzle is situated at a distance of 1.5 m from the ground and is inclined at an angle of 30° to the horizontal. The jet of liquid from the nozzle strikes the ground at a horizontal distance of



(A) 1.04 m
(B) 1.5 m
(C) 10 m
(D) 5 m

Solution

Area of the nozzle,

$$A = \frac{\pi}{4} \times \left(\frac{50}{1000}\right)^2 \text{ m}^2$$

Flow rate, $Q = 0.01862 \text{ m}^3/\text{s}$

$$\therefore u = \frac{Q}{A} = 9.483 \text{ m/s}.$$

Let the horizontal distance at which the jet strikes the ground be x.

If the coordinates of point A is set to (0, 0). Then the coordinates of point B will be $(x, -1.5)$.

The equation of the jet is,

$$y = x\tan\theta - \frac{gx^2 \sec^2\theta}{2u^2}$$

That is, $-1.5 = x \times \tan 30° - \dfrac{9.81 \times x^2 \times \sec^2 30°}{2 \times 9.483^2}$

$$0.07273\,x^2 - 0.5774x - 1.5 = 0$$

$\therefore$ $x = 10$ m.

Hence, the correct answer is option (C).

Vortex Flow

It is defined as the fluid flow along a curved path or the flow of a mass of fluid rotating about an axis.

Plane Circular Vortex Flows

These are flows with streamlines that are concentric circles. Considering a polar coordinate system, the velocity field of such a flow is defined as:

$$\boxed{V_\theta \neq 0 \text{ and } V_r = 0}$$

Where, V_θ and V_r are the tangential and radial components of the velocity respectively. For such flows V_θ is a function of r only and not θ.

Vortex flows can be mainly classified into two types:

1. Forced vortex flow
2. Free vortex flow

It is to be noted that a plane circular free vortex flow or a plane circular forced vortex flow will be simply referred to as respectively a free vortex flow or a forced vortex flow. Hence, all the characteristics of a plane circular vortex flow will be attributed sometimes to a free or forced vortex flow.

Forced Vortex Flow

It is defined as the vortex flow in which some external torque is employed to rotate the fluid mass. The tangential velocity of a fluid particle is given by, $V_\theta = r\,\omega$.

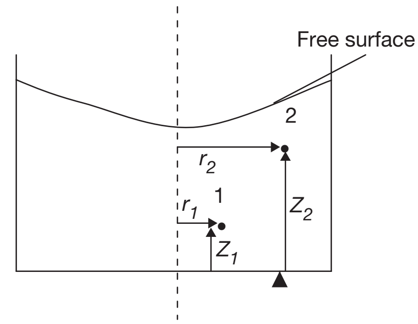
Where r is the distance of the fluid particle from the axis of rotation and ω is the angular velocity of the fluid particle. In a forced vortex flow all fluid particles rotate with the same angular velocity like a solid body and hence this flow is termed as a *solid body rotation*. A forced vortex is also called as a *flywheel vortex* or *rotational vortex*.

A forced vortex flow is a rotational flow (vorticity = 2ω). To maintain a forced vortex flow, mechanical energy has to be spent from outside and the total mechanical energy per unit mass is not constant. In such a flow, shear stress is zero at all points in the flow field since there is no relative motion. A forced vortex flow can be generated by rotating a vessel containing a fluid so that the angular velocity is the same at all points.

Examples:

1. Rotation of a liquid in a centrifugal pump.
2. Rotation of a gas in a centrifugal compressor
3. Rotation of water through the turbines runner

Consider two points 1 and 2 in a fluid having a forced vortex flow as shown in the following figure.



For the two points, the following equation is applicable.

$$\boxed{p_2 - p_1 = \frac{\rho}{2}\left(V_2^2 - V_1^2\right) - \rho g(Z_2 - Z_1)} \quad (1)$$

Where, $\boxed{V_1 = r_1\omega}$ and $\boxed{V_2 = r_2\omega}$

If the two points lie on the free surface of the liquid then $p_1 = p_2$ and Eq. (1) becomes,

$$\boxed{Z_2 - Z_1 = \frac{1}{2g}\left(V_2^2 - V_1^2\right)}$$

If additionally to the above case, point 1 lies on the axis of rotation, (i.e, $v_1 = r_1 \times w = 0 \times w = 0$), then

$$Z_2 - Z_1 = \frac{V_2^2}{2g} \quad \text{or} \quad \boxed{Z = \frac{\omega^2 r_2^2}{2g}} \quad (2)$$

Where, $Z = Z_2 - Z_1$

Since, Z varies with the square of r, Eq. (1) is an equation of a parabola consequently the free surface of the liquid is a paraboloid.

Cylindrical forced vortex: It can be generated by rotating a cylindrical vessel containing a fluid. At any horizontal plane, the tangential velocity, $V_\theta = r\,\omega$

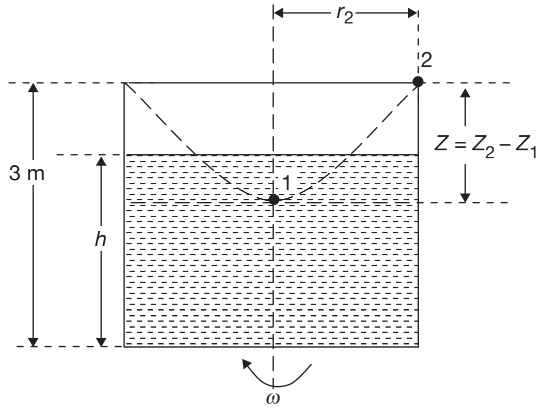
Spiral forced vortex: The superimposition of a purely radial flow with a plane circular forced vortex results in a spiral forced vertex flow.

Example 15

A cylindrical tank of diameter 1 m and height 3 m, which is open at the top, is filled with a liquid up to a certain depth. When the cylinder is rotated at 100 rpm. The liquid level is raised to be even with the brim. The depth of the liquid in the tank is

(A) 1.39 m (B) 2.3 m
(C) 3 m (D) 0.5 m

Solution



Let h be the depth of the liquid in the tank.

The points 1 and 2 are chosen as shown in the figure.

$$\text{Hence, } Z_2 - Z_1 = \frac{w^2 r_2^2}{2g}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60}$$

$$r_2 = 0.5 \text{ m}$$

$$\therefore Z = Z_2 - Z_1 = \frac{\left(\frac{2\pi \times 100}{60}\right)^2 \times (0.5)^2}{2 \times 9.81}$$

$$= 1.3973 \text{ m}$$

When the vessel is rotated, a paraboloid is formed.

Volume of air before rotation = Volume of air after rotation

$$\Rightarrow \quad \pi r_2^2 \times 3 - \pi r_2^2 \times h$$

$$= \frac{1}{2} \times \pi \times r_2^2 \times Z$$

or $$h = 3 - \frac{Z}{2} = 3 - \frac{1.3973}{2}$$

$$= 2.3 \text{ m.}$$

Hence, the correct answer is option (B).

Pressure forces on the top and bottom of a cylinder: Consider a cylinder of radius R and height H which is completely filled with a liquid. The cylinder is rotated about its vertical axis at a speed of ω rad/s.

Total pressure on the top of the cylinder,

$$F_T = \frac{\rho\omega^2}{4} \times \pi R^4$$

Total pressure force on the bottom of the cylinder (F_B) = Weight of the liquid in the cylinder + Total pressure force on the top of the cylinder (F_T)

That is, $$\boxed{F_B = \rho g \pi R^2 H + F_T}$$

Free Vortex Flow

A vortex flow in which no external torque is required to rotate the fluid mass is called a free vortex flow. The velocity field in a *free vortex flow* is described by,

$$\boxed{V_\theta = \frac{c}{r}}$$

Where c (called as the *strength of the vortex*) is a constant in the entire flow field. The above equation is derived from the fact that in a free vortex flow, as the external torque is zero, the time rate of change of angular momentum, i.e., the moment of momentum is zero.

A free vortex is also called as a *potential vortex* or *irrotational vortex*.

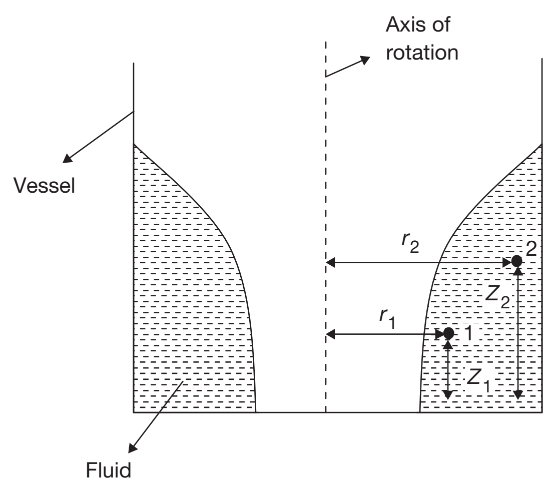
A free vortex flow is irrotational (zero vorticity). In this type of flow, the total mechanical energy per unit mass is constant in the entire flow field with no addition or destruction of mechanical energy in the flow field. In a free vortex flow, the fluid rotates due to either some previously imparted rotation or some internal action.

Examples:

1. Whirlpool in a river
2. Flow around a circular bend
3. Flow of liquid through an outlet provided at the bottom of a shallow vessel, e.g., wash tube, etc.

It is to be noted that Bernoulli's equation is applicable in the case of a free vortex flow.

Consider two points 1 and 2 in the fluid having radii r_1 and r_2 respectively from the axis of rotation and with heights Z_1 and Z_2 respectively from the bottom of the vessel as shown in the figure.



Since, Bernoulli's equation is applicable for free vortex flow, we can write,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

Example 16

In a free cylindrical vortex flow of air (Density = 1.2 kg/m^3), point A is located at a radius of 350 mm from the axis of rotation and at a height of 200 mm from the vessel bottom. Point B is however located at a radius of 500 mm and height 300 mm. If the velocity at point A is 20 m/s then the pressure difference between the points A and B is

(A) 121.22 P$_a$ (B) 10.29 P$_a$
(C) 12.35 P$_a$ (D) 25.62 P$_a$

Solution

Given,

$$r_A = 0.35 \text{ m}$$
$$Z_A = 0.2 \text{ m}$$
$$V_A = 20 \text{ m/s}$$
$$V_B = 0.5 \text{ m}$$
$$Z_B = 0.3 \text{ m}$$

For a free vortex flow,

$$V_r = \text{Constant}$$

$$\therefore \quad V_A r_A = V_B r_B.$$

or $$V_B = \frac{20 \times 0.35}{0.5} = 14 \text{ m/s}.$$

From Bernoulli's equation we have,

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$$

$$\frac{p_A}{\rho g} + \frac{20^2}{2 \times 9.61} + 0.2 = \frac{p_B}{\rho g} + \frac{14^2}{2 \times 9.81} + 0.3$$

$$\frac{p_B}{\rho g} - \frac{p_A}{\rho g} = 10.2975$$

or $$p_B - p_A = 10.2976 \times 9.81 \times 1.2 = 121.22 \text{ Pa}.$$

Hence, the correct answer is option (A).

Cylindrical free vortex: A cylindrical free vortex in a cylindrical coordinate system has the Z-axis directly vertically upwards where at each horizontal plane, there exists a planar free vortex motion with tangential velocity given by,

$$V_\theta = \frac{C}{r}.$$

Spiral free vortex: For a plane spiral free vortex two-dimensional flow, the tangential and radial velocity components at any point with respect to a polar coordinate system is inversely proportional to the radial coordinate at that point.

$\therefore$ In the flow field,

$$\boxed{V_\theta = \frac{C_1}{r} \qquad V_r = \frac{C_2}{r}}$$

Such a flow can be said to be the superimposition of a radial flow described by equation, $V_r = \frac{C_2}{r}$ with a free vortex flow.

If α is the angle between the velocity vector V and the tangential component of the velocity vector V_θ at any point then,

$$\boxed{\tan\alpha = \frac{V_r}{V_\theta} = \frac{C_2}{C_1}}$$

Now, $$\frac{V_r}{V\theta} = \frac{\frac{dr}{dt}}{rw} = \frac{\frac{dr}{dt}}{r\frac{d\theta}{dt}} = \frac{dr}{rd\theta}$$

$$\therefore \quad \boxed{\frac{dr}{rd\theta} = \tan\alpha}$$

This is the equation of the streamline in this flow. Integrating the above equation, it can be shown that

$$r = r_0 e^{\theta x \tan\alpha} = r_0 e^{\theta \frac{C_2}{C_1}}$$

Where, r_0 is the radius at $\theta = 0$. The above equation shows that the patterns of streamlines are logarithmic-spiral.

Example 17

An object, caught in a whirlpool, at a given instant is at a distance of 100 cm from the centre of the whirlpool. The two-dimensional velocity field of the whirlpool can be described by the tangential and radial components of the velocity such as V_θ and V_r respectively, where $V_\theta = -3V_r$. If after a certain period of time, the object is found to be at a distance of 4.32 m from the centre of the whirlpool, then the number of revolutions completed by the object from its original position is

(A) 3 (B) 1.5
(C) 4.5 (D) 1

Solution

The motion in a whirlpool can be simulated as a free vortex flow. Since $V_\theta \neq 0$ and $V_r \neq 0$ (for some finite radial location) the flow can be considered to a spiral free vortex flow.

Given, $r_0 = 100$ m

$$r = 4.32 \text{ m}$$

Now for a spiral free vortex flow,

$$r = r_0 e^{\theta c_2/c_1}$$
$$= r_0 e^{\theta Vr/V_\theta}$$

That is, $4.32 = 100 \times e^{\theta \times \left(\frac{1}{-3}\right)}$

or, $\theta = 9.425744$ radians

Now, 1 revolution = 2π radians

$\therefore$ Number of revolution completed by the object $= \frac{9.425744}{2\pi} = 1.5$ revolution.

Direction for solved examples 18 and 19:

The velocity profile for flow in a circular pipe is given as

$v = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$ where v is the velocity of any radius r, v_{max} is the velocity of the pipe axis and R is the radius of the pipe.

Example 18

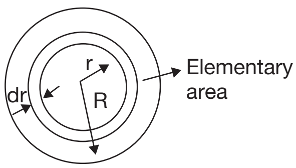
The average velocity of flow is given by

(A) v_{max} (B) $\frac{3}{4}v_{max}$

(C) $\frac{v_{max}}{4}$ (D) $\frac{v_{max}}{2}$

Solution

In a cross-section of the circular pipe, consider an elementary area dA in the form of a ring at a radius r and of thickness dr.



Then, $dA = 2\pi rdr$

Flow rate through the ring

$$= dQ = \text{Elemental area} \times \text{Local velocity}$$
$$= 2\pi rdr \times v$$

Total flow, $$Q = \int_0^R 2\pi rdr \cdot v$$

$$= \int_0^R 2\pi r v_{max}\left[1-\left(\frac{r}{R}\right)^2\right]dr$$

$$Q = \pi v_{max}\left(\frac{R^2}{2}\right) \quad (1)$$

Let, v_{avg} be the average velocity, then

$$Q = \pi R^2 \times v_{avg}$$

From Eq. (1) we have,

$$\pi\, v_{max}\left(\frac{R^2}{2}\right) = \pi R^2 v_{avg}$$

$$V_{avg} = \frac{v_{max}}{2}.$$

Hence, the correct answer is option (D).

Example 19

The value of the kinetic energy correction factor is

(A) 2 (B) 1.11

(C) 1.04 (D) 1

Solution

$$\alpha = \frac{1}{A}\int\left(\frac{v}{v_{avg}}\right)^3 dA$$

$$= \frac{1}{\pi R^2}\frac{8}{(V_{max})^3}\int_O^R V^3\, 2\pi rdr$$

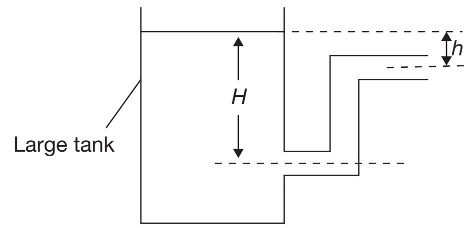
$$= \frac{16}{R^2}\int_O^R\left(1-\left(\frac{r}{R}\right)^2\right)^3 rdr$$

$$= \frac{16}{R^2}\times\left(\frac{R^2}{8}\right) = 2.$$

Hence, the correct answer is option (A).

Example 20

If the head losses in the pipe shown in the figure is h_2 metres, then the discharge velocity at the pipe exit is



(A) $\sqrt{2g(h-h_L)}$ (B) 0

(C) $\sqrt{2g(H-h_2)}$ (D) $\sqrt{2_g(H+h-h_L)}$

Solution

Let the height of the water surface from the bottom of the tank (chosen as the datum level) be L.

Consider point 1 to be the water surface of the tank and point 2 to be at the pipe exit.

Now, $P_1 = P_2 = P_{atm}$

The tank is considered to be very large such that $V_1 \approx 0$

Assuming the flow to be steady applying the energy equation between the two points we have

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+Z_1+h_P = \frac{P_2}{\rho g}+\frac{\alpha_2 V_2^2}{2g} + Z_2 + h_t + h_L \quad (1)$$

Since, no pump and turbine is involved, $h_p = h_t = 0$.

The kinetic correction factor are considered to be unity, i.e., $\alpha_1 = \alpha_2 = 1$.

Now the Eq. (1) can be written as:

$$L = \frac{V_2^2}{2g}+(L-h)+hL$$

$$V_2 = \sqrt{2g(h-h_L)}.$$

Hence, the correct answer is option (A).

Example 21

A hydraulic turbine is supplied with 5 M^3/s water at 420 kPa (gauge). A vacuum gauge fitted in the turbine discharge 4 on below the turbine inlet centre line shows a reading of 200 mm Hg. If the turbine shaft output power is 1200 kW and if the internal diameters of the supply and discharge pipe are identically 100 mm, then the power loss through the turbine is

(A) 2429.62 kW (B) 962.78 kW
(C) 1229.62 kW (D) 2162.78 kW

Solution

Let the subscripts S and D denote points in the suction and the discharge pipe respectively.

Given $P_s = 420$ kPa

$$Z_s = 4 \text{ m}$$

$Z_D = 0$ m (discharge pipe taken at the datum plane.)

$$W_{\text{turbine}} = 1200 \times 10^3 \text{ W}$$

The energy equation applied between the points S and D is as follows.

$$M\left(\frac{Ps}{\rho} + \alpha_s \frac{V_s^2}{2} + gZ_s\right) + W_{\text{pump}}$$

$$= M\left(\frac{P_D}{\rho} + \alpha_D \frac{V_D^2}{2} + gZ_D\right) + W_{\text{turbine}} + E_{\text{mech loss}} \quad (1)$$

Since, no pump is involved,

$$W_{\text{pump}} = 0.$$

The kinetic energy correction factors are assumed to be unity, i.e., $\alpha_S = \alpha_D = 1$.

Here, $Q = 5$ m$^{3/S}$

$$\therefore \overset{\circ}{m} = \rho Q = 1000 \times 5 = 5000 \text{ kg/s}$$

Now $P_D = -200$ mm kg

$$= \frac{-200}{1000} \times 13600 \times 9.81$$

$$= -26.6832 \text{ kPa}$$

Since, the supplies are discharge pipe have identical internal diameters, we have

$$V_S = V_0$$

$\therefore$ Eq. (1) becomes

$$5000 \times \left(\frac{420 \times 10^3}{1000} + 9.81 \times 4\right)$$

$$= 5000\left(\frac{-26.6832 \times 10^3}{1000}\right) + 1200 \times 10^3 + E_{\text{mech loss}}$$

$$E_{\text{mech loss}} = 1229.62 \text{ kW}.$$

Hence, the correct answer is option (C).

Stream Function

For an incompressible two-dimensional planar flow, the continuity equation reduces to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

A function $\psi(x, y)$, called the *stream function* can be defined such that whenever the velocity components are defined in terms of the stream function as shown below, the continuity Eq. (1) will always satisfied.

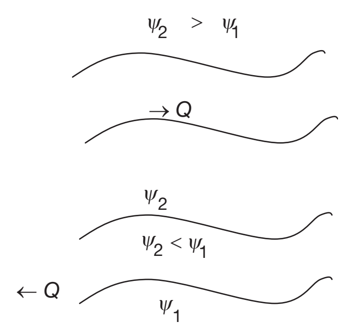
$$u = \frac{\partial \psi}{\partial y}, v = \frac{-\partial \psi}{\partial x} \quad (2)$$

Eq. (2) holds for rotational and irrotational regions of flow.

The volume rate of flow, Q, between two streamlines such as ψ_1 and ψ_2 is given by,

$$\boxed{Q = \psi_2 - \psi_1}$$

The relative value of ψ_2 with respect to ψ_1 will determine the flow direction as shown below:



Flow streamlines are curves of constant ψ.

Example 22

The velocity potential function of a two-dimensional incompressible and irrotational flow is $\phi = ax^3y - y^3x$. The value of a is

(A) 0 (B) 1
(C) 1/6 (D) 6

Solution

For an incompressible and irrotational flow, we have $\Delta^2 \phi = 0$

$$\text{or } \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\phi = ax^3y - y^3x \quad (1)$$

$$\frac{\partial \phi}{\partial x} = 3ax^2y - y^3$$

$$\frac{\partial^2 \phi}{\partial x^2} = 6axy \quad (2)$$

$$\frac{\partial \phi}{\partial y} = ax^3 - 3y^2x$$

$$\frac{\partial^2 \phi}{\partial y^2} = -6yx \quad (3)$$

Substituting Eqs. (2) and (3) in Eq. (1) we get

$$6axy - 6yx = 0$$

or $$a = 1.$$

Hence, the correct answer is option (B).

Example 23

A steady three-dimensional velocity field is given by:

$\vec{V} = axy^3\hat{i} + (10b - 3cy^4)\hat{j} + x^2y^2\hat{k}$. The condition under which the flow field will be incompressible is

(A) $a = 4c$ (B) $a = 0$
(C) $a = 12c$ (D) $b = c$

Solution

If the field is incompressible, then from the continuity equation we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

From the velocity field description,

$$u = axy^3$$
$$v = 10b - 3cy^4$$
$$w = x^2y^2$$

Substituting the above three equations in Eq. (1), we have

$$ay^3 - 12cy^3 + 0 = 0$$

or $$a = 12c.$$

Hence, the correct answer is option (C).

Example 24

An incompressible flow is represented by the velocity potential function $\phi = 4x^2 + 4y^2 + 17t$. For the flow, which one of the combinations of the following statement holds true?

I. Flow is physically possible.
II. Flow is physically not possible.
III. Flow satisfies the continuity equation.
IV. Flow does not satisfy the continuity equation.

(A) I and IV (B) I and III
(C) II and III (D) II and IV

Solution

$$\phi = 4x^2 + 4y^2 + 17t$$

$$u = \frac{\partial \phi}{\partial x} = 8x;\ V = \frac{\partial \phi}{\partial y} = 8y$$

The incompressible equation is

$$\frac{\partial u}{\partial n} + \frac{\partial v}{\partial y} = 0$$

Here, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 8 + 8 = 16 \neq 0$

Hence, the continuity equation is not satisfied and this implies that the flow is physically not possible.

Hence, the correct answer is option (D).

Example 25

Persons A, B and C claim that the functions $\phi = 5x^2 - 5y^2$, $\phi = 10\sin x$ and $\phi = 27xy$ respectively are valid potential functions. Which one of the following statements is ONLY correct regarding the claims?

(A) The claims of persons A and B are true.
(B) The claims of persons B and C are true
(C) The claims of persons A and C are true.
(D) The claims of person A is false.

Solution

For ϕ to be a valid potential function

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \text{ should be equal to zero.}$$

For $\phi = 5x^2 - 5y^2$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 10 - 10 = 0$$

Person A's claim is true.

For $\phi = 10\sin x$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -10\sin x + 0 = -10\sin x \neq 0$$

Person B's claim is not true.

For $\phi = 27xy$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 + 0 = 0$$

Person C's claim is true.

Hence, the correct answer is option (C).

Example 26

The stream function representing a two-dimensional flow is given by $\psi = \frac{ax^2y^2}{2} - 2xy - \frac{ax^4}{12} - \frac{y^4}{6}$.

If the flow is irrotational then the value of a is

(A) 0 (B) 2
(C) 0.5 (D) 12

Solution

If the flow is irrotational,

then $$\frac{\partial^2 \psi}{a x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \quad (1)$$

$$\frac{\partial \psi}{\partial x} = \frac{2axy^2}{2} - 2y - \frac{4x^3a}{12}$$

$$\frac{\partial^2 \psi}{\partial x^2} = ay^2 - x^2a \quad (2)$$

$$\frac{\partial \psi}{\partial y} = \frac{2ax^2y}{2} - 2x - \frac{4y^3}{6}$$

$$\frac{\partial^2 \psi}{\partial y^2} = ax^2 - 2y^2 \qquad (3)$$

Substituting Eqs. (2) and (3) in Eq. (1), we get

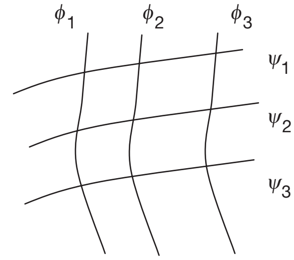
$$ay^2 - ax^2 + ax^2 - 2y^2 = 0$$

or $$a = 2.$$

Hence, the correct answer is option (B).

Flow Nets

A flow net is a grid obtained by drawing a set of streamlines and equipotential lines.



Flow nets are used to study two-dimensional irrotational flow especially in cases where the stream and velocity functions are unavailable or difficult to solve.

Exercises

1. Let the x and y components of velocity in steady, two-dimensional, incompressible flow be linear function of x and y such that $V = (ax + by)i + (cx + dy)j$ where a, b, c and d are constants. The condition for which the flow is irrotational is ________.

2. X-component of velocity in a two-dimensional incompressible flow is given by $u = y^2 + 4xy$. If Y-component of velocity 'v' equals zero at $y = 0$, the expression for 'v' is given by

(A) $4y$ (B) $2y^2$
(C) $-2y^2$ (D) $2xy$

3. Two flow patterns are represented by their stream functions ψ_1 and ψ_2 as given below:

$\psi_1 = x^2 + y^2$

$\psi_2 = 2xy$

If these two patterns are superimposed on one another, the resulting streamline pattern can be represented by one of the following:

(A) A family of parallel straight lines.
(B) A family of circles.
(C) A family of parabolas.
(D) A family of hyperbolas.

4. The relation that must hold for the flow to be irrotational is

(A) $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$ (B) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$

(C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ (D) $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

5. For a two-dimensional irrotational flow, the velocity potential is defined as $\phi = \log_e(x^2 + y^2)$. Which of the following is a possible stream function ψ, for this flow?

(A) $\frac{1}{2}\tan^{-1}(y/x)$ (B) $\tan^{-1}(y/x)$
(C) $2\tan^{-1}(y/x)$ (D) $2\tan^{-1}(x/y)$

6. A fluid flow is represented by the velocity field $\bar{V} = ax\,\bar{i} + ay\bar{j}$, where a is a constant. The equation of streamline passing through a point (1, 2) is

(A) $x - 2y = 0$ (B) $2x + y = 0$
(C) $2x - y = 0$ (D) $x + 2y = 0$

7. For a fluid flow through a divergent pipe of length L having inlet and outlet radii and R_1 and R_2 respectively and a constant flow rate of Q, assuming the velocity to be axial and uniform at any cross-section, the acceleration at the exit is

(A) $\frac{2Q(R_1 - R_2)}{\pi L R_2^{\,3}}$ (B) $\frac{2Q^2(R_1 - R_2)}{\pi L R_2^{\,3}}$

(C) $\frac{2Q^2(R_1 - R_2)}{\pi^2 L R_2^{\,5}}$ (D) $\frac{2Q^2(R_2 - R_1)}{\pi^2 L R_2^{\,5}}$

8. A closed cylinder having a radius R and height H is filled with oil of density ρ. If the cylinder is rotated about its axis at an angular velocity of ω, the thrust at the bottom of the cylinder is

(A) $\pi R^2 \rho g H$

(B) $\pi R^2 \frac{\rho \varpi^2 R^2}{4}$

(C) $\pi R^2(\rho\omega^2 R^2 + \rho g H)$

(D) $\pi R^2\left(\frac{\rho\omega^2 R^2}{4} + \rho g H\right)$

9. Of the possible irrotational flow functions given below, the incorrect relation is (where ψ = stream function and ϕ = velocity potential)

(A) $\psi = xy$
(B) $\psi = A(x^2 - y^2)$
(C) $\phi = ur\cos\theta + \frac{U}{r}\cos\theta$
(D) $\phi = \left(r - \frac{2}{r}\right)\sin\theta$

10. The curl of a given velocity field $(\nabla \times \vec{V})$ indicates the rate of

(A) increase or decrease of flow at a point.
(B) twisting of the lines of flow.
(C) deformation.
(D) translation.

11. The area of a 2 m long tapered duct decreases as $A = (0.5 - 0.2x)$ where 'x' is the distance in metres. At a given instant a discharge of 0.5 m^3/s is flowing in the duct and is found to increase at a rate of 0.2 m^3/s. The local acceleration (in m/s^2) at $x = 0$ will be
(A) 1.4 (B) 1.0
(C) 0.4 (D) 0.667

12. The velocity components in the x and y directions of a two-dimensional potential flow are u and v, respectively, then $\frac{\partial u}{\partial x}$ is equal to
(A) $\frac{\partial v}{\partial x}$ (B) $-\frac{\partial v}{\partial x}$
(C) $\frac{\partial v}{\partial y}$ (D) $-\frac{\partial v}{\partial y}$

13. A venturimeter of 20 mm throat diameter is used to measure the velocity of water in a horizontal pipe of 40 mm diameter. If the pressure difference between the pipe and throat sections is found to be 30 kPa then, neglecting frictional losses, the flow velocity is
(A) 0.2 m/s (B) 1.0 m/s
(C) 1.4 m/s (D) 2.0 m/s

14. A leaf is caught in a whirlpool. At a given instant, the leaf is at a distance of 120 m from the centre of the whirlpool. The whirlpool can be described by the following velocity distribution: $V_r = -\left(\frac{60\times10^3}{2\pi r}\right)$ m/s and $V_\theta = \frac{300\times10^3}{2\pi r}$ m/s, where r (in metres) is the distance from the centre of the whirlpool. What will be the distance of the leaf from the centre when it has moved through half a revolution?
(A) 48 m
(B) 64 m
(C) 120 m
(D) 142 m

15. Match List I (Example) with List II (Types of flow) and select the correct answer using the codes given

	List I		List II
a.	Flow in a straight long pipe with varying flow rate	1.	Uniform, steady
b.	Flow of gas through the nozzle of a jet engine	2.	Non-uniform, steady
c.	Flow of water through the hose of a fire fighting pump	3.	Uniform, unsteady
d.	Flow in a river during tidal bore	4.	Non-uniform, unsteady

Codes:

	a	b	c	d
(A)	1	4	3	2
(B)	3	2	1	4
(C)	1	2	3	4
(D)	3	4	1	2

16. Consider the following statements regarding a path line in fluid flow:
I. A path line is a line traced by a single particle over a time interval.
II. A path line shows the positions of the same particle at successive time instants.
III. A path line shows the instantaneous positions of a number of particles, passing through a common point, at some previous time instants.

Which of these statements are correct?
(A) I and III only (B) I and II only
(C) II and III only (D) I, II and III

17. In a two-dimensional velocity field with velocities u and v along the x and y-directions respectively, the convective a acceleration along the x-direction is given by
(A) $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$ (B) $u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}$
(C) $u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial y}$ (D) $v\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y}$

18. A two-dimensional flow field has velocities along the x and y-directions given by $u = x^2t$ and $v = -2xyt$ respectively, where t is time. The equation of streamlines is
(A) x^2y = Constant
(B) xy^2 = Constant
(C) xy = Constant
(D) Cannot be determined

19. A velocity field is given as $\overline{V} = 2y\hat{i} + 3x\hat{j}$ where x and y are in metres. The acceleration of a fluid particles at $(x, y) = (1, 1)$ in the x-direction is
(A) 0 (B) 5.00 m/s^2
(C) 6.00 m/s^2 (D) 8.48 m/s^2

20. The velocity in m/s at a point in a two-dimensional flow is given as $\overline{V} = 2\hat{i} + 3\hat{j}$. The equation of the streamline passing through the point is
(A) $3dx - 2dy = 0$ (B) $2x + 3y = 0$
(C) $3dx + 2dy = 0$ (D) $xy = 6$

21. An inert tracer is injected continuously from a point in an unsteady flow field. The locus of locations of all the tracer particles at an instance of time represents
(A) streamline (B) pathline
(C) streatube (D) streakline

22. A stream function is given by:
$\psi = 2x^2y + (x + 1)y^2$
The flow rate across a line joining points $A(3, 0)$ and $B(0, 2)$ is
(A) 0.4 units (B) 1.1 units
(C) 4 units (D) 5 units

23. Consider the following equations:

I. $A_1v_1 = A_2v_2$

II. $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$

III. $\int_s \rho v dA + \dfrac{\partial}{\partial t}\left(\int_v \rho dV\right) = 0$

IV. $\dfrac{1}{r}\dfrac{\partial}{\partial r}(rv_r) + \dfrac{\partial}{\partial z}(v_z) = 0$

Which of the above equations are forms of continuity equations? (Where u, v are velocities and V is volume)

(A) I only (B) I and II
(C) II and III (D) III and IV

24. Consider the following statements about velocity potential:

I. Velocity potential is a vector function similar to stream function.
II. It is a fully three-dimensional function and not limited to two coordinates.
III. Velocity potential does not exist at stagnation points.
IV. Velocity potential exists only if the flow is irrotational.

Which of these statements are correct?

(A) I, II and III (B) I, II and IV
(C) I, III and IV (D) II, III and IV

25. The predominant forces acting on an element of fluid in the boundary layer over a flat plate placed in a uniform stream include

(A) inertia and pressure forces.
(B) viscous and pressure forces.
(C) viscous and body forces.
(D) viscous and inertia forces.

26. The circulation 'Γ' around a circle of radius 2 units for the velocity field $u = 2x + 3y$ and $v = -2y$ is

(A) -6π units (B) -12π units
(C) -18π units (D) -24π units

27. In a nominal 90° triangular notch discharging under invariant head, the error in the estimated discharge due to 2% error in the vortex angle is _______.

28. A right angled triangular notch is used to measure the flow in a flume. If the head measured is 20 cm and C_d = 0.62, neglecting the velocity of approach, the discharge in lit/s is

29. The percentage error in the computed discharge over a triangular notch corresponding to an error of 1% in the measurement of the head over the notch would be

(A) 1.0 (B) 1.5
(C) 2.0 (D) 2.5

30. The equation $gz + \dfrac{v^2}{2} + \dfrac{P}{\rho}$ = constant along a streamline holds true for

(A) steady, frictionless, compressible fluid.
(B) steady, uniform, incompressible fluid.
(C) steady, frictionless, incompressible fluid.
(D) unsteady incompressible fluid.

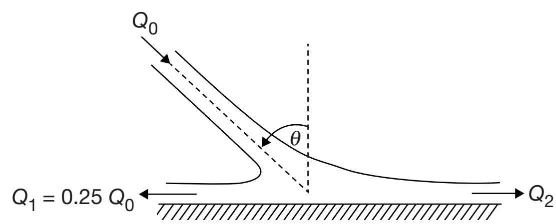
31. A nozzle discharging water under head H has n outlet area 'a' and discharge coefficient C_d = 1.0. A vertical plate is acted upon by the fluid force F_j when held across the free jet by the fluid force F_n when held against the nozzle to stop the flow. The ratio $\dfrac{F_j}{F_n}$ is

(A) $\dfrac{1}{2}$ (B) 1
(C) $\sqrt{2}$ (D) 2

32. A body moving through still water at 6 m/s produces a water velocity of 4 m/s at a point 1 m ahead. The difference in pressure between the nose and the point 1 m ahead would be

(A) 2000 N/m^2 (B) 1000 N/m^2
(C) 19620 N/m^2 (D) 98100 N/m^2

33. A horizontal jet strikes a frictionless vertical plate (the plan view is shown in the figure). It is then divided into two parts, as shown in the figure. If the impact loss is neglected, what is the value of θ?

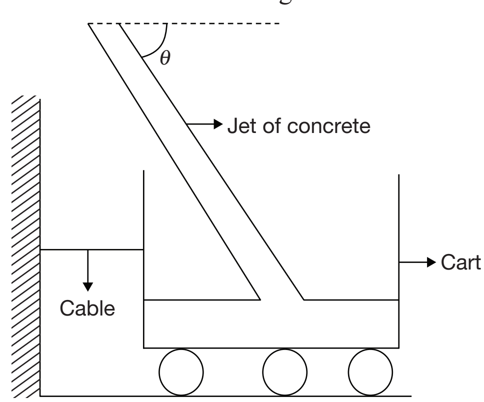


(A) 15° (B) 30°
(C) 45° (D) 60°

34. The x component of velocity in a two-dimensional compressible flow is given by $u = 1.5x$. At the point $(x, y) = (1, 0)$, the y component of velocity $v = 0$. The equation for the y component of velocity is

(A) $v = 0$ (B) $v = 1.5y$
(C) $v = -1.5x$ (D) $v = -1.5y$

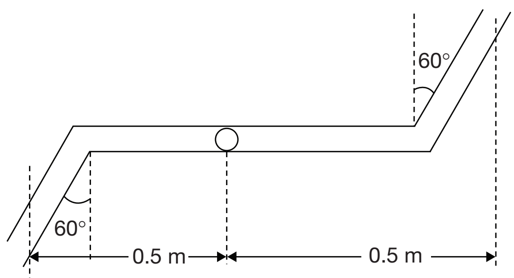
35. A 0.1 m diameter jet of concrete flows steadily at a velocity of 2 m/s into a cart which is attached to a wall by a cable as shown in the figure below.



The density of the concrete is 2200 kg/m^3. If at instant shown in the figure, the cart and the concrete in it together weighs 3560 N and the reaction force exerted by the ground on the cart is 3620 N, then the tension in the cable is

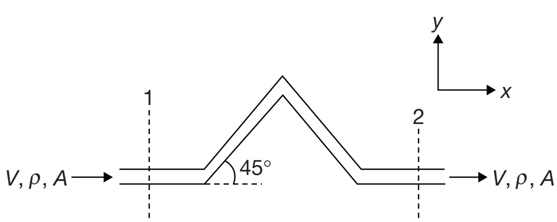
(A) 48.92 N (B) 34.31 N
(C) 11.65 N (D) 20.53 N

36. A sprinkler with equal arm lengths of 0.5 m, as shown in the following figure, discharges water at equal relative velocities through nozzles of equal diameters of 5 cm. The sprinkler freely rotates with no friction at a speed of 95.493 rpm. The torque (in Nm) required to hold the sprinkler stationary is



(A) 98.175 (B) 49.087
(C) 61.235 (D) 22.602

37. A frictionless fluid of density ρ flow through a bent pipe as shown below. If A is the cross-sectional area and V is the velocity of flow, the forces exerted on segment 1-2 of the pipe in the x and y directions are,



(A) ρAV^2; 0 (B) ρAV^2; $\sqrt{2}\rho\, AV^2$

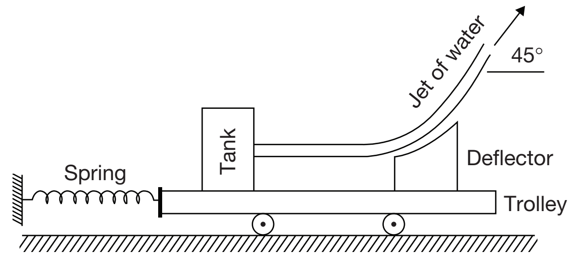
(C) 0; 0 (D) 0; $\frac{1}{\sqrt{2}}\rho AV^2$

38. The reading of differential manometer of a venturimeter, placed at 45° to the horizontal is 11 cm. If the venturimeter is turned to horizontal position, the manometer reading will be

(A) zero (B) $\frac{11}{\sqrt{2}}$ cm

(C) 11 cm (D) $11\sqrt{2}$ cm

39. A tank and a deflector are placed on a frictionless trolley. The tank issues water jet (mass density of water = 1000 kg/m^3), which strikes the deflector and turns by 45°. If the velocity of jet leaving the deflector is 4 m/s and discharge is 0.1 m^3/s, the force recorded by the spring will be



(A) 100 N (B) $100\sqrt{2}$ N
(C) 200 N (D) $200\sqrt{2}$ N

40. The velocity field for a flow is given by:

$\vec{V} = (5x + 6y + 7z)\hat{i} + (6x + 5y + 9z)\hat{j} + (3x + 2y + \lambda z)\hat{k}$ and the density varies as $\rho = \rho_0 \exp(-2t)$. In order that the mass is conserved, the value of λ should be

(A) −12 (B) −10
(C) −8 (D) 10

41. A cylindrical vessel is closed at the top and the bottom and has a diameter of 0.4 m and height 0.5 m. The vessel is completely filled with a liquid. When the vessel is rotated about its vertical axis with an angular speed of ω rad/s, the total pressure exerted by the liquid on the bottom is twice that exerted by the liquid on the top the vessel. The value of ω is

(A) 22.14 rad/s (B) 14 rad/s
(C) 44.29 rad/s (D) 28 rad/s

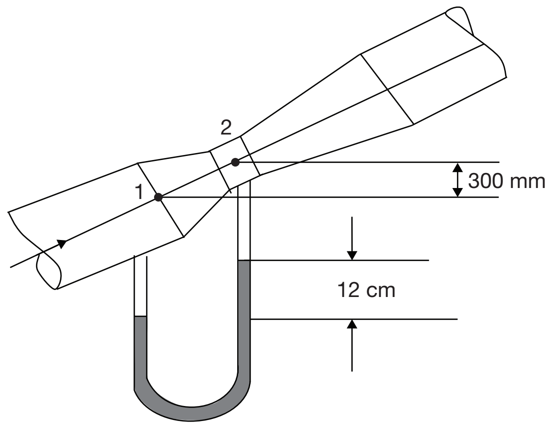
42. A glass tube with a 90° bend is open at both the ends. It is inserted into a flowing stream of oil, $S = 0.90$, so that one opening is directed upstream and the other is directed upward. Oil inside the tube is 50 mm higher than the surface of flowing oil. The velocity measured by the tube is, nearly,

(A) 0.89 m/s
(B) 0.99 m/s
(C) 1.40 m/s
(D) 1.90 m/s

43. If density of liquid $\rho = 1000$ kg/m^3 and area $A = 1$ m^2. Then flow rate Q at $t = 0$, $(x = 0, y = 0)$, is

(A) 100 (B) 1000
(C) 0 (D) Cannot be determined

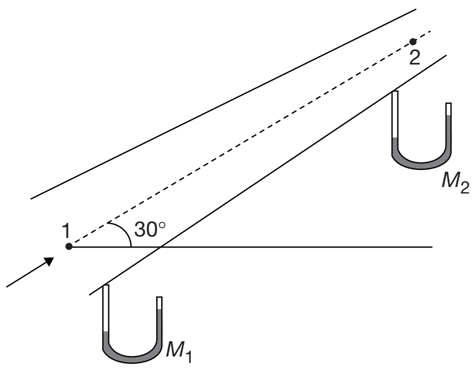
44. Water flows through an inclined venturimeter as shown in the figure. Inlet and throat diameters are 100 mm and 50 mm respectively. Inlet and throat sections have a level difference of 300 mm. The differential mercury manometer connected across inlet and throat indicates 12 cm of mercury level difference at a given flow rate. Coefficient of discharge is 0.99.



The rate of flow in lit/s is

(A) 14.76 (B) 12.85
(C) 10.91 (D) 8.86

45.



Water flows through a tapering pipe inclined at 30° to the horizontal. At points 1 and 2 manometers are connected. Point 1 is at an elevation of 1 m from ground level and 2 is 3 m from ground level. Diameter at section 1 and 2 are 15 cm and 10 cm respectively. Velocity at 1 is 6 m/s. If manometer M_2 reads 10 cm of mercury, the reading shown by manometer M_1 in cm of mercury is

(A) 79.5 (B) 65.6
(C) 58.3 (D) 49.4

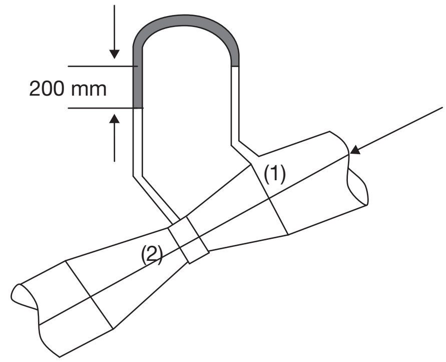
46. Mass flow rate of the oil in kg/s is

(A) 2.82 (B) 2.64
(C) 2.41 (D) 2.22

47. Power required to pump oil per 100 m length of pipe is

(A) 6.12 kW (B) 6.34 kW
(C) 6.63 kW (D) 6.82 kW

48. Water flows through an inclined pipe in which a venturimeter is installed for discharge measurement. The inlet and throat sections of the venturimeter have areas of cross sections 0.07 m^2 and 0.0177 m^2 respectively. An inverted *U*-tube manometer is used for measurement of differential pressure head. A liquid of specific gravity 0.7 is used in the manometer, which gives a reading of 250 mm. Inlet and throat sections have a level difference of 400 mm



Neglecting frictional losses the rate of flow through the pipe in m^3/s is

(A) 0.028 (B) 0.022
(C) 0.019 (D) 0.016

49. In a three-dimensional incompressible fluid flow, velocity components in *x* and *y* directions are:

$u = x^2 + y^2z^3$

$v = -(xy + yz + zx)$

Velocity component in the *z* direction is

(A) $-xz + \frac{z^2}{2} + f(x, y)$ (B) $-xz + \frac{z^2}{2}$

(C) $xz - \frac{z^2}{2} + C$ (D) $-x + z$

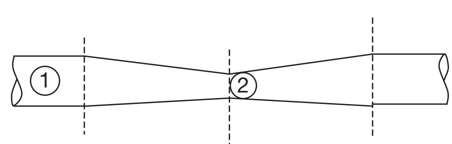
50. Match List I (Measuring devices) with List II (Measuring parameter) and select the correct answer using the codes:

	List I		List II
a.	Pitot tube	1.	Rate of flow measurement
b.	Micro-manometer	2.	Measurement of moderate pressure
c.	Venturimeter	3.	Velocity measurement
d.	Piezo-meter	4.	Easier measurement of large pressures

Codes:

	a	b	c	d
(A)	1	3	2	4
(B)	4	2	3	1
(C)	2	1	4	3
(D)	3	4	1	2

51. In a horizontal pipeline as shown in the figure, point 2 is a contraction with reduced area of cross-section. At point 1 the pressure head and velocity head are 60 cm and 4 cm respectively. If pressure head at point 2 is zero, the ratio of velocity at point 2 to that at point 1 is



(A) 2 (B) 3
(C) 4 (D) 6

52. For a flow, velocity components in the x and y directions are given by $u = y^2$, $v = -3x$.

Component of rotation about the Z-axis is

(A) $-(3 + 2y)$ (B) $(3 + 2y)$

(C) $\frac{-1}{2}(3+2y)$ (D) $\frac{1}{2}(3+2y)$

53. The velocity along the centre line of a nozzle of length 1.5 m is given by:

$v = 2t\left(1-\frac{x}{2l}\right)^2$ where l = length in m

v = Velocity in m/s,

t = Time in seconds

From the commencement of flow and x = distance from inlet. The value of local acceleration at x = 1 m when t = 5 seconds is

(A) 0.67 m/s^2 (B) 0.89 m/s^2

(C) 1.33 m/s^2 (D) 1.67 m/s^2

54. An orificemeter is calibrated with air in a geometrically similar model. Model to prototype scale ratio is $\frac{1}{4}$. The prototype has to carry water. Ratio of kinematic viscosity of air to water is 12.5. Dynamically similar flow will be obtained when the discharge ratio is

(A) 2.850 (B) 3.125

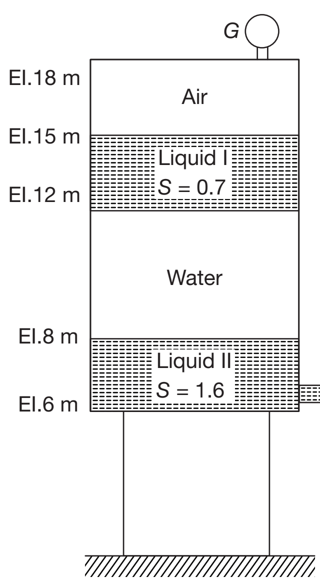
(C) 4.540 (D) 4.925

55. For a flow, the stream function is ψ. For the flow to be irrotational, the condition to be satisfied is

(A) $\frac{\partial\psi}{\partial x} - \frac{\partial\psi}{dy} = 0$ (B) $\frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial y} = 0$

(C) $\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2} = 0$ (D) $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$

56.



A tank installed at an elevation of 6 m contains a liquid of specific gravity 1.6, water and another liquid of specific gravity 0.7 space over the liquids contain air . The gauge G show a pressure of -17 kN/m^2. The elevation of liquid level in the piezometer A is

(A) 4.73 m (B) 6.73 m

(C) 8.73 m (D) 10.73 m

57. For the stream function $\psi = 3xy$, velocity at a point (1, 2) is

(A) $\sqrt{18}$ units (B) $\sqrt{35}$ units

(C) $\sqrt{45}$ units (D) $\sqrt{55}$ units

58. In a fluid, the velocity field is given by $V = (2x + 3y)i + (3z + 2x^2)j + (2t - 3z)k$. The speed at point (0, 1, 2) and at time t = 2 seconds, is

(A) 7.836 (B) 8.464

(C) 9.695 (D) 10.436

59. For a three-dimensional flow, the velocity components in m/s are given by:

$u = yz + t$

$v = xz - t$

$w = xy$

Total acceleration(in m/s^2) at a point (1, 1, 1) after 2 seconds is_______.

(A 4.96 (B) 4.28

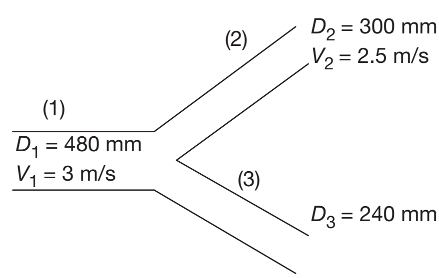
(C) 4.32 (D) 4.47

60. For the flow $v = 3xi - 3yj$, equation of streamline passing through (1, 2) is

(A) $xy = 2$ (B) $xy = 3$

(C) $\frac{x}{y} = 2$ (D) $\frac{x}{y} = 3$

61. A pipe of 480 mm diameter branches into two pipes of 300 mm and 240 mm diameter as shown in the figure. If average velocity of flows in 480 mm and 300 mm pipes are 3 m/s and 2.5 m/s respectively, average velocity of flow (in m/s) in 240 mm pipe is



(A) 8.8 (B) 8.2

(C) 8.0 (D) 8.9

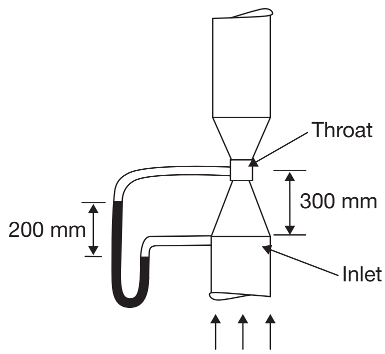
62. For a three-dimensional continuous flow, x and y components of velocity are, $u = 2xy$ and $v = 2yz$ respectively. Then z component of velocity is

(A) $-2yz - z^2 + f(x, y, t)$ (B) $2yz + z^2 + f(x, y, z)$

(C) $2yz + 2z^2 + f(x, y, t)$ (D) $2yz - z^2 + f(x, y, z)$

63. A two-dimensional flow field is given by $u = -3y$ and $v = -3x$. Discharge between the streamlines passing through points (2, 6) and (6, 6) is
(A) 16 units (B) 32 units
(C) 48 units (D) 64 units

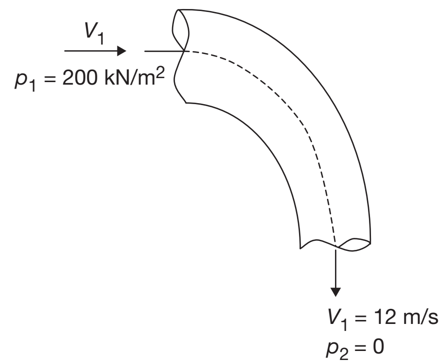
64. For a vertical venturimeter cross-sectional area at inlet is 0.07 m^2 and cross sectional area at throat is 0.0177 m^2. The venturimeter is used to measure discharge of oil of specific gravity 0.8. The height difference between inlet and throat is 300 mm. A U-tube manometer connected between throat and inlet shows a mercury level difference of 250 mm. Assuming a coefficient of discharge of 0.9, discharge through the venturimeter (in m^3/s) is______.



(A) 0.296 (B) 0.162
(C) 0.183 (D) 0.145

65. Water flows through a 100 mm diameter orifice used in a 200 mm diameter pipe at the rate of 0.016 m^3/s. If coefficient of contraction is 0.6 and coefficient of velocity is 1.0, the head difference between upstream section and vena contract section (in m of water) is______.
(A) 0.524 (B) 0.574
(C) 0.586 (D) 0.523

66. Water flows through a 90° reducer bend in a pipeline. The pressure at inlet is 200 kN/m^2 (gauge), when the cross-sectional area is 0.01 m^2. At the exit section when the cross sectional area is 0.0025 m^2, velocity is 12 m/s and pressure is atmospheric. Magnitude of force (in kN) acting on the bend is______ (Assume that the bend is in horizontal *XY*-plane).



(A) 2.23 (B) 2.12
(C) 2.42 (D) 2.82

Previous Years' Questions

1. In a steady flow through a nozzle, the flow velocity on the nozzle axis is given by $v = u_0(1 + 3x/L)i$, where x is the distance along the axis of the nozzle from its inlet plane and L is the length of the nozzle.

The time required for a fluid particle on the axis to travel from the inlet to the exit plane of the nozzle is **[GATE, 2007]**

(A) $\frac{L}{u_0}$ (B) $\frac{L}{3u_0}\text{In}4$

(C) $\frac{L}{4u_0}$ (D) $\frac{L}{2.5u_0}$

2. Which combination of the following statements about steady incompressible forced vortex flow is correct?
P. Shear stress is zero at all points in the flow.
Q. Vorticity is zero at all points in the flow.
R. Velocity is directly proportional to the radius from the centre of the vortex.
S. Total mechanical energy per unit mass is constant in the entire flow field.

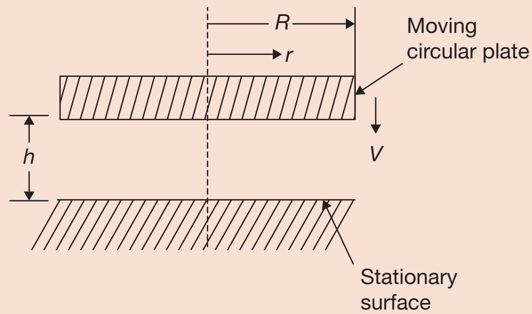
Select the correct answer using the given codes: **[GATE, 2007]**
(A) P and Q (B) R and S
(C) P and R (D) P and S

3. At two points 1 and 2 in a pipeline, the velocities are V and $2V$, respectively. Both the points are at the same elevation. The fluid density is ρ. The flow can be assume to be incompressible, inviscid, steady and irrotational. The difference in pressures P_1 and P_2 at points 1 and 2 is **[GATE, 2007]**
(A) $0.5\rho V^2$ (B) $1.5\rho V^2$
(C) $2\rho V^2$ (D) $3\rho V^2$

4. For the continuity equation given by $\vec{\nabla}\cdot\vec{V} = 0$ to be valid, where $\vec{V}$ is the velocity vector, which one of the following is a necessary condition? **[GATE, 2008]**
(A) Steady flow (B) Irrotational flow
(C) In viscid flow (D) Incompressible flow

Direction for questions 5 and 6:

The gap between a moving circular plate and a stationary surface is being continuously reduced, as the circular plate comes down at a uniform speed V towards the stationary bottom surface, as shown in the figure. In the process, the fluid contained between the two plates flows out radially. The fluid is assumed to be incompressible and inviscid. **[GATE, 2008]**



5. The radial velocity V_r at any radius r, when the gap width is h, is

(A) $v_r = \dfrac{Vr}{2h}$ (B) $v_r = \dfrac{Vr}{h}$

(C) $v_r = \dfrac{2Vh}{r}$ (D) $v_r = \dfrac{Vh}{r}$

6. The radial component of the fluid acceleration at $r = R$ is

(A) $\dfrac{3V^2R}{4h^2}$ (B) $\dfrac{V^2R}{4h^2}$

(C) $\dfrac{V^2R}{2h^2}$ (D) $\dfrac{V^2h}{2R^2}$

7. Consider steady, incompressible and irrotational flow through a reducer in a horizontal pipe where the diameter is reduced from 20 cm to 10 cm. The pressure in the 20 cm pipe just upstream of the reducer is 150 kPa. The fluid has a vapour pressure of 50 kPa and a specific weight of 5 kN/m^3. Neglecting frictional effects, the maximum discharge (in m^3/s) that can pass through the reducer without causing cavitation is **[GATE, 2009]**

(A) 0.05 (B) 0.16
(C) 0.27 (D) 0.38

8. You are asked to evaluate assorted fluid flows for their suitability in a given laboratory application. The following three flow choices, expressed in terms of the two-dimensional velocity fields in the XY-plane, are made available.

P. $u = 2y, v = -3x$
Q. $u = 3xy, v = 0$
R. $u = -2x, v = 2y$

Which flow(s) should be recommended when the application requires the flow to be incompressible and irrotational? **[GATE, 2009]**

(A) P and R (B) Q
(C) Q and R (D) R

9. Water ($\gamma_w = 9.879$ kN.m^3) flows with a flow rate of 0.3 m^3/s through a pipe AB of 10 m length and of uniform cross-section. The end B is above end A and the pipe makes an angle of 30º to the horizontal. For a pressure of 12 kN/m^2 at the end B, the corresponding pressure at the end A is **[GATE, 2009]**

(A) 12.0 kN/m^2
(B) 17.0 kN/m^2
(C) 56.4 kN/m^2
(D) 61.4 kN/m^2

10. Velocity vector of a flow field is given as $\vec{V} = 2xy\hat{i} - x^2z\hat{j}$. The vorticity vector at (1, 1, 1) is **[GATE, 2010]**

(A) $4\hat{i} - \hat{j}$ (B) $4\hat{i} - \hat{k}$

(C) $\hat{i} - 4\hat{j}$ (D) $\hat{i} - 4\hat{k}$

11. Match List I (Device) with List II (Uses) and select the answer using the codes given below the lists:

	List I		List II
a.	Pitot tube	1.	Measuring pressure in a pipe
b.	Manometer	2.	Measuring velocity of flow in a pipe
c.	Venturimeter	3.	Measuring air and gas velocity
d.	Anemometer	4.	Measuring discharge in a pipe

[GATE, 2010]

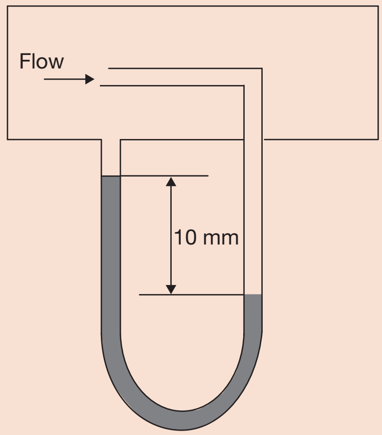
Codes:

	a	b	c	d
(A)	1	2	4	3
(B)	2	1	3	4
(C)	2	1	4	3
(D)	4	1	3	2

12. A streamline and an equi-potential line in a flow field **[GATE, 2011]**

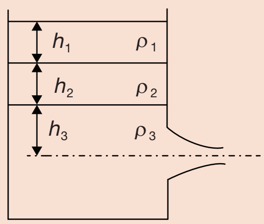
(A) are parallel to each other.
(B) are perpendicular to each other.
(C) intersect at an acute angle.
(D) are identical.

13. Figure shows the schematic for the measurement of velocity of air (density = 1.2 kg/m^3) through a constant area duct using a pitot tube and a water-tube manometer. The differential head of water (density = 1000 kg/m^3) in the two columns of the manometer is 10 mm. Take acceleration due to gravity as 9.8 m/s^2. The velocity of air in m/s is **[GATE, 2011]**



(A) 6.4 (B) 9.0
(C) 12.8 (D) 25.6

14. A large tank with a nozzle attached contains three immiscible, inviscid fluids as shown. Assuming that the changes in h_1, h_2 and h_3 are negligible, the instantaneous discharge velocity is **[GATE, 2012]**



(A) $\sqrt{2gh_3\left(1+\frac{\rho_1}{\rho_3}\frac{h_1}{h_3}+\frac{\rho_2}{\rho_3}\frac{h_2}{h_3}\right)}$

(B) $\sqrt{2g(h_1+h_2+h_3)}$

(C) $\sqrt{2g\left(\frac{\rho_1h_1+\rho_2h_2+\rho_3h_3}{\rho_1+\rho_2+\rho_3}\right)}$

(D) $\sqrt{2g\left(\frac{\rho_1h_2h_3+\rho_2h_3h_1+\rho_3h_1h_2}{\rho_1h_1+\rho_2h_2+\rho_3h_3}\right)}$

15. For a two-dimensional flow field, the stream function ψ is given as: $\Psi=\frac{3}{2}(y^2-x^2)$
The magnitude of discharge occurring between he streamline passing through points (0, 3) and (3, 4) is **[GATE, 2013]**

(A) 6 units (B) 3 units
(C) 1.5 units (D) 2 units

16. Water is coming out from a tap and falls vertically downwards. At the tap opening, the stream diameter is 20 mm with uniform velocity of 2 m/s. Acceleration due to gravity is 9.81 m/s^2. Assuming steady, inviscid flow, constant atmospheric pressure everywhere and neglecting curvature and surface tension effects, the diameter in mm of stream 0.5 m below the tap is approximately **[GATE, 2013]**

(A) 10 (B) 15
(C) 20 (D) 25

17. A plane flow has velocity components $u=\frac{x}{T_1}$, $v=-\frac{y}{T_2}$ and $w = 0$ along x, y and z directions respectively, where $T_1\,(\neq 0)$ and $T_2\,(\neq 0)$ are constants having the dimension of time. The given flow is incompressible if **[GATE, 2014]**

(A) $T_1=-T_2$ (B) $T_1=-\frac{T_2}{2}$
(C) $T_1=\frac{T_2}{2}$ (D) $T_1=T_2$

18. A particle moves along a curve whose parametric equations are: $x = t^3 + 2t$, $y = -3e^{-2t}$ and $z = 2\sin 5t$, where x, y and z show variations of the distance covered by the particle (in cm) with time t (in seconds). The magnitude of the acceleration of the particle (in cm/s^2) at $t = 0$ is ________. **[GATE, 2014]**

19. For an incompressible flow field $\vec{V}$, which one of the following conditions must be satisfied? **[GATE, 2014]**

(A) $\nabla\cdot\vec{V}=0$

(B) $\nabla\times\vec{V}=0$

(C) $(\vec{V}\cdot\nabla)\vec{V}=0$

(D) $\frac{\partial\vec{V}}{\partial t}+(\vec{V}\cdot\nabla)\vec{V}=0$

20. Consider the following statements regarding streamline(s):

I. It is a continuous line such that the tangent at any point on it shows the velocity vector at that point.
II. There is no flow across streamlines.
III. $\frac{dx}{u}=\frac{dy}{y}=\frac{dz}{w}$ is the differential equation of a streamline, where u, v and w are velocities in directions x, y and z respectively.
IV. In an unsteady flow, the path of a particle is a streamline.

Which one of the following combinations of the statements is true? **[GATE, 2014]**

(A) I, II, IV (B) II, III, IV
(C) I, III, IV (D) I, II, III

21. Consider a velocity field $\vec{V}=K(y\hat{i}+x\hat{k})$, where K is a constant. The vorticity Ω_z is **[GATE, 2014]**

(A) $-K$ (B) K
(C) $-K/2$ (D) $K/2$

22. List I lists a few devices while List II provides information about their uses. Match the devices with their corresponding use. **[GATE, 2014]**

	List I		List II
P.	Anemometer	**1.**	Capillary potential of soil water.
Q.	Hygrometer	**2.**	Fluid velocity at a specific point in the flow stream.
R.	Pitot Tube	**3.**	Water vapour content of air.
S.	Tensiometer	**4.**	Wind speed

Codes:

	P	Q	R	S
(A)	1	2	3	4
(B)	2	1	4	3
(C)	4	2	1	3
(D)	4	3	2	1

23. A venturimeter, having a diameter of 7.5 cm at the throat and 15 cm at the enlarged end, is installed in a horizontal pipeline of 15 cm diameter. The pipe carries an incompressible fluid at a steady rate of 30 litres per second. The difference of pressure head measured in terms of the moving fluid in between the enlarged and the throat of the venturimeter is observed to be 2.45 m. Taking the acceleration due to gravity as 981 m/s^2, the coefficient of discharge of the venturimeter (correct up to two places of decimal) is _______. **[GATE, 2014]**

24. A venturimeter having a throat diameter of 0.1 m is used to estimate the flow rate of a horizontal pipe having a diameter of 0.2 m. For an observed pressure difference of 2 m of water head and coefficient of discharge equal to unity, assuming that the energy losses are negligible, the flow rate (in m^3/s) through the pipe is approximately equal to **[GATE, 2014]**

(A) 0.500 (B) 0.150
(C) 0.050 (D) 0.015

25. In a two-dimensional steady flow field, in a certain region of the XY-plane, the velocity component in the x-direction is given by $v_x = x^2$ and the density varies as $\rho = \frac{1}{x}$. Which of the following is a valid expression for the velocity v_y component in the y-direction? **[GATE, 2015]**

(A) $V_y = -\frac{x}{y}$ (B) $V_y = \frac{x}{y}$
(C) $V_y = -xy$ (D) $V_y = xy$

26. The velocity components of a two-dimensional plane motion of a fluid are:

$$U = \frac{y^3}{3} + 2x - x^2 y \text{ and } v = xy^2 - 2y - \frac{x^3}{3}.$$

The correct statement is: **[GATE, 2015]**

(A) Fluid is incompressible and flow is irrotational.
(B) Fluid is incompressible and flow is rotational.
(C) Fluid is compressible and flow is irrotational.
(D) Fluid is compressible and flow is rotational.

27. Match the following pairs: **[GATE, 2015]**

	List I (Equation)		List II (Physical Interpretation)
P.	Ñ × $\vec{V} = 0$	**I.**	Incompressible continuity equation
Q.	Ñ. $\vec{V} = 0$	**II.**	Steady flow
R.	$\frac{D\vec{V}}{Dt} = 0$	**III.**	Irrotational flow
S.	$\frac{\delta \vec{V}}{\delta t} = 0$	**IV.**	Zero acceleration of fluid particle

(A) P–IV, Q–I, R–II, S–III
(B) P–IV, Q–III, R–I, S–II
(C) P–III, Q–I, R–IV, S–II
(D) P–III, Q–I, R–II, S–IV

28. The velocity field of an incompressible flow is given by $V = (a_1x + a_2y + a_3z)i + (b_1x + b_2y + b_3z)j + (c_1x + c_2y + c_3z)k$, where $a_1 = 2$ and $c_3 = -4$. The value of b_2 is ______. **[GATE, 2015]**

29. Water ($\rho = 1000$ kg/m^3) flows through a venturimeter with inlet diameter 80 mm and throat diameter 40 mm. The inlet and throat gauge pressures are measured to be 400 kPa and 130 kPa respectively. Assuming the venturimeter to be horizontal and neglecting friction, the inlet velocity (in m/s) is ______. **[GATE, 2015]**

30. If the fluid velocity for a potential flow is given by $V(x, y) = u(x, y)i + v(x, y)j$ with usual notations, then the slope of the potential line at (x, y) is **[GATE, 2015]**

(A) $\frac{v}{u}$ (B) $-\frac{u}{v}$
(C) $\frac{v^2}{u^2}$ (D) $\frac{u}{v}$

31. A Prandtl tube (Pitot-static tube with $C = 1$) is used to measure the velocity of water. The differential manometer reading is 10 mm of liquid column with a relative density of 10.

Assuming $g = 9.8$ m/s^2, the velocity of water (in m/s) is ______. **[GATE, 2015]**

32. A nozzle is so shaped that the average flow velocity changes linearly from 1.5 m/s at the beginning to 15 m/s at its end in a distance of 0.375 m. The magnitude of the convective acceleration (in m/s^2) at the end of the nozzle is _______. **[GATE, 2015]**

33. For steady incompressible flow through a closed-conduit of uniform cross-section, the direction of flow will always be **[GATE, 2015]**

(A) from higher to lower elevation.
(B) from higher to lower pressure.
(C) from higher to lower velocity.
(D) from higher to lower piezometric head.

Answer Keys

Exercises

1. $c = b$	**2.** C	**3.** A	**4.** A	**5.** C	**6.** C	**7.** C	**8.** D	**9.** C	**10.** C
11. C	**12.** D	**13.** D	**14.** B	**15.** B	**16.** B	**17.** A	**18.** D	**19.** C	**20.** A
21. D	**22.** C	**23.** B	**24.** D	**25.** D	**26.** B	**27.** 3.10% to 3.15%		**28.** 26.2	**29.** D
30. C	**31.** B	**32.** B	**33.** B	**34.** D	**35.** B	**36.** A	**37.** C	**38.** C	**39.** D
40. C	**41.** C	**42.** B	**43.** B	**44.** C	**45.** A	**46.** A	**47.** C	**48.** B	**49.** A
50. D	**51.** C	**52.** C	**53.** B	**54.** B	**55.** D	**56.** D	**57.** C	**58.** C	**59.** D
60. A	**61.** C	**62.** A	**63.** C	**64.** D	**65.** B	**66.** B			

Previous Years' Questions

1. B	**2.** B	**3.** B	**4.** D	**5.** A	**6.** B	**7.** B	**8.** D	**9.** D	**10.** D
11. C	**12.** B	**13.** C	**14.** A	**15.** B	**16.** B	**17.** D	**18.** 12	**19.** A	**20.** D
21. A	**22.** D	**23.** 0.95	**24.** C	**25.** C	**26.** A	**27.** C	**28.** 1.9 to 2.1		**29.** 6
30. B	**31.** 1.30 to 1.34		**32.** 540	**33.** D					