CHAPTER

1

Number System

Number System

The system of numeration employing, then ten digits 0, 1, 2, 3, ..., 8, 9 is known as number system. A group of digits, denoting a number, is called a numeral.

A number may be a natural number, a whole number, an integer, a rational number, a real number etc. It is important to mention here that the above classification of numbers is not mutually exclusive, e.g. an integer can also be a whole number or a natural number.

Various Types of Numbers

1. **Natural Numbers** Counting numbers are called as natural numbers. The set of natural numbers is denoted by 'N'. Thus, $N = \{1, 2, 3, 4, 5, \ldots\}$ is the set of all natural numbers.
2. **Whole Numbers** If zero (0) is included in the set of natural numbers, then we get the set of whole numbers, denoted as W. Thus, $W = \{0, 1, 2, 3, 4, 5, \ldots\}$ is the set of whole numbers.
   - So, here, every natural number is a whole number.
   - But, zero (0) is the only whole number which is not a natural number.
3. **Even Numbers** The numbers which are divisible by 2 are called as even numbers. Such as 2, 4, 6, 8, 10, ... In general these are represented by $2m$, where $m \in N$.
4. **Odd Numbers** The number which are not divisible by 2 are called as odd numbers. Such as 1, 3, 5, 7, 9, ... In general, these are represented by $(2m \pm 1)$, where $m \in N$.
5. **Prime Numbers** A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and itself. For example, 2, 3, 5, 7, ... are some of the prime numbers.
   - If a number is not divisible by any of the prime number upto square root of that number, then it is a prime number.
   - 2 is the only even number which is prime.
   - The prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97 *i.e.*, there are 25 prime numbers upto 100.
6. **Co-prime Numbers** Two numbers x and y are said to be co-prime, if their HCF (highest common factor) is 1 *i.e.*, they do not have a common factor other than 1.
   For e.g., : (9, 2), (5, 6), (11, 15) are the pairs of co-prime numbers.
   - If x and y are any two co-primes, a number p is divisible by x as well as by y, then the number is also divisible by xy.
   - Co-prime are also called as relatively prime numbers.
   - **Twin Primes** Prime numbers which differ by 2 are called as twin primes. *e.g.*, (3, 5), (11, 13) are twin primes.
7. **Composite Numbers** All the natural numbers except 1 and the prime numbers constitute the set of composite numbers *e.g.*, 4, 6, 8, 9, ... all are composite numbers.
   - '1' is neither prime nor composites.
8. **Integers** All counting numbers and their negatives including zero form a new set of numbers which are called as integers. The set of integers is denoted by I.
   Thus, $I = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ is the set of integers.
   - So, here every natural number and whole number is an integer.
   - So, here $N \subset W \subset I$.

Some Subsets of Integers

1. **Positive Integers** The set of all positive integers is denoted by I^+. Thus, $I^+ = \{1, 2, 3, 4, 5, \ldots\}$ is a set of all positive integers. Here, we can see that N and I^+ are the same.
2. **Negative Integers** The set denoted by I^- is of all negative integers. Thus, $I^- = \{-1, -2, -3, -4, \ldots\}$ is a set of all negative integers.
   - Here, it is notable that 0 (zero) is neither positive nor negative.
3. **Non-negative Integers** The set $\{0, 1, 2, 3, 4, \ldots\}$ is called the set of non-negative integers.
4. **Non-positive Integers** The set $\{0, -1, -2, -3, -4, \ldots\}$ is the set of non-positive integer.
5. **Rational Numbers** The numbers which are expressed in the form of p/q; $p, q \in I$, $q \neq 0$ are called rational numbers, where p and q are co-primes. The set of rational numbers is represented by 'Q'. Thus, $Q = \{p/q : p$ and q are integers and co-primes, $q \neq 0\}$.
   e.g., $\frac{3}{5}, -\frac{3}{7}, 7, -6$ are rational numbers.

- Every natural number say x, can be written as $x/1$, so every natural number is a rational number.
- 0 (zero) can be written as 0/1.
- Every non-zero integer can be written as $K/1$, so every integer is a rational number.

Terminating and Repeating Decimals

Every rational number has a particular characteristic that when expressed in decimal form they are expressible either in terminating decimals or in repeating decimals.

e.g., $\frac{1}{5} = 0.5$ or $\frac{5}{4} = 1.25$ [terminating decimals]

$\frac{1}{3} = 0.333 = \bar{3}$ or $\frac{69}{550} = 0.125454 = 0.12\overline{54}$

[non-terminating but repeating decimals]

Irrational Numbers The number which cannot be expressed in the form p/q, where p and q both are integers and $q \neq 0$ are known as irrational numbers. The irrational numbers when expressed in decimal form are in non-terminating and non-repeating form. For example, $\sqrt{2}, \sqrt{5}, \sqrt{7}$ etc.

- Here, it is notable that exact value of π is not $\frac{22}{7}$, as $\frac{22}{7}$ is a rational number while π is irrational.
- $\frac{22}{7}$ and 3.14 are not exact value of π.
- Numbers like 0.101005001 ... is a non-terminating and non-repeating decimal. So, it is an irrational number.

Real Numbers The totality of all rational and all irrational numbers forms, the set of real numbers and is denoted by 'R'. Thus, all natural numbers, whole numbers, integers, rational and irrational numbers are real numbers.

Properties of Numbers and Operations on Them

Properties of Real Numbers

General Properties of R

1. If x and y are two real numbers, then either $x > y, y > x$ or $x = y$.
2. If x and y are two real numbers, then
   (i) if $x > y \Rightarrow \frac{1}{x} < \frac{1}{y}$
   (ii) $x > y \Rightarrow -x < -y$
   (iii) $x > y \Rightarrow x + a > y + a$
   (iv) $x > y \Rightarrow xa > ya$ when $a > 0$
3. If $xy = 0 \Rightarrow x = 0$ or $y = 0$

Properties of Addition on R

1. **Closure Property** The sum of two real number is always a real number i.e., $a \in R, b \in R \Rightarrow a + b \in R, \forall a, b \in R$
   Hence, R is closed for addition.
2. **Associative Law** If $a, b, c \in R$, then
   $$(a + b) + c = a + (b + c)$$
3. **Commutative Law** $a + b = b + a$ for all $a, b \in R$
4. **Additive Identity** Additive identity is the number which when added to any number of the set, then the number remains unaltered. Such as zero (0) is a real number such that $0 + a = a + 0 = a$ for all $a, 0 \in R$
   So, '0' is the additive identity in R
5. **Additive Inverse** It is a number in the set which when added to any number, then the result is additive identity. If $a \in R, -a \in R$ then $a + (-a) = 0$, then $-a$ is called as additive inverse of a

Properties of Multiplication on R

1. **Closure Property** R is closed for multiplication.
   i.e., $a, b \in R$ then ab or $ba \in R, \forall a, b \in R$
2. **Associative Law** If $a, b, c \in R$ then $(ab)c = a(bc)$.
3. **Commutative Law** If $a, b \in R$ then $ab = ba, \forall a, b \in R$
4. **Distributive Law** If $a, b, c \in R$ then $a(b + c) = ab + ac$
   So, multiplication is distributive over addition.
5. **Multiplicative Identity in R** It is a number which when multiplied by a real number, the number remains unaltered. i.e., if $a \in R$ then $1 \cdot a = a = a \cdot 1$ here 1 is the real number.
   So, '1' is the multiplicative identity in R
6. **Multiplicative Inverse** It is a number which when multiplied by the number the result is 1. If $a \in R$ then $a \cdot \frac{1}{a} = 1$, so $\frac{1}{a}$ is multiplicative inverse of a or *vice-versa*.
   - Zero has no reciprocal, thus has no multiplicative inverse.

Properties of Subtraction and Division on R

1. **Closure Property** R is closed for subtraction, if $a, b \in R$ then $a - b \in R$
2. Subtraction on R does not satisfies the commutative and associative laws.
3. Division of real number does not holds closure property since $3 \in R, 0 \in R$ but $\frac{3}{0} \neq R$

Properties of Integers (I)

Properties of Addition on I

1. **Closure Property** $\forall a, b \in I, a + b \in I$
2. **Commutative Property** $\forall a, b \in I, (a + b) = (b + a)$
3. **Associative Property** $\forall a, b, c \in I$
   $$(a + b) + c = a + (b + c)$$
4. **Additive Identity** $\forall a \in I\; a + 0 = 0 + a = a$
   So, $0 \in I$ and is called as additive identity of integers.
5. **Additive Inverse** $\forall a \in I$ there exists $-a \in I$ such that $a + (-a) = 0$, then, $(-a)$ is the additive inverse of a.

Properties of Subtraction on *I*

1. **Closure Property** *I* is closed for subtraction. Since, $a \in I, b \in I \Rightarrow a - b \in I$ for all $a, b \in I$.
2. In general the commutative property and associative properties are not followed by subtraction.
   Since, $3 - 2 \neq 2 - 3, \forall\, 2, 3 \in I$ and also
   $(a - b) - c \neq a - (b - c), \forall a, b, c \in I$

Properties of Multiplication on *I*

1. **Closure Property** *I* is closed for multiplication, as if $a, b \in I \Rightarrow a \times b = b \times a \in I$.
2. **Commutative Property** If $a, b \in I$, then $a \times b = b \times a$, so it holds commutative property.
3. **Associative Property** If $a, b, c \in I$, then $a(bc) = (ab)\, c \in I$.
4. **Multiplicative Identity** $\forall\, a \in I$.
   $a \times 1 = 1 \times a = a$ so 1 is the multiplicative identity of a.
5. **Distributive Property** $\forall\, a, b, c \in I$, then
   $a(b + c) = ab + ac$,
   so multiplication is distributive over addition.

Factors in Integers (*I*)

Let $a, b \in I$, then '*a*' is a factor of '*b*', if there exists an integer *q* such that $b = aq$. Also, here we can write a/b.

- If '*a*' is a factor of '*b*', then '*b*' is called a multiple of '*a*'.

Ordering in Integers (*I*)

Let '*x*' and '*y*' are any two integers, we say that $x > y$, if $x = y + z$ for some integer '*z*'.

- Also, if there are any two integers '*a*' and '*b*' and only one of the following holds
  (i) $a < b$ (ii) $a > b$ (iii) $a = b$

Properties of Rational Numbers

Properties of Addition on *Q*

1. **Closure Property** If a/b and c/d are two rational numbers and $a, b, c, d \in I$, then $\frac{a}{b} + \frac{c}{d}$ is also a rational number.
   i.e., if $\frac{a}{b}, \frac{c}{d} \in Q$, then $\left(\frac{a}{b} + \frac{c}{d}\right) \in Q$ Here, $b \neq 0, d \neq 0$.
2. **Commutativity** If $a/b, c/d, \in Q$, $a, b, c, d \in I$ and $b \neq 0, d \neq 0$, then $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$
3. **Associativity** If $\frac{a}{b}, \frac{c}{d}, \frac{e}{f} \in Q$ and $a, b, c, d, e, f \in I$ and $b \neq 0, d \neq 0, f \neq 0$, then $\left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right)$
4. **Additive Identity** For every $\frac{a}{b} \in Q$, there exists a unique value rational number $\frac{0}{c}, c \neq 0$
   i.e., 0 such that $\frac{a}{b} + \frac{0}{c} = \frac{0}{c} + \frac{a}{b} = \frac{a}{b} \in Q$
5. **Additive Inverse** $\forall\, \frac{a}{b} \in Q$, there exists a rational number $-\frac{a}{b}, b \neq 0$ such that $\frac{a}{b} + \left(-\frac{a}{b}\right) = 0$.

Properties of Multiplication on *Q*

1. **Closure Property** If $\frac{a}{b}, \frac{c}{d} \in Q$ and $a, b, c, d \in I$, then $\left(\frac{a}{b} \times \frac{c}{d}\right) \in Q$.
2. **Commutativity** If $\frac{a}{b}, \frac{c}{d} \in Q$ and $a, b, c, d \in I$, then $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$ and $b \neq 0, d \neq 0$.
3. **Associativity** If $\frac{a}{b}, \frac{c}{d}, \frac{e}{f} \in Q$ and $a, b, c, d \in I$, then
   $\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right), b \neq 0, d \neq 0, f \neq 0$.
4. **Multiplicative Identity** For every $\frac{a}{b} \in Q$ and $a, b \in I$, there exists a unique number $\frac{1}{1}$ such that $\frac{a}{b} \times \frac{1}{1} = \frac{1}{1} \times \frac{a}{b}$ $= \frac{a}{b} \in Q, b \neq 0$
5. **Multiplicative Inverse** For every $\frac{a}{b} \in Q$ and $a, b \in I, a \neq 0$ and $b \neq 0$, there exists a rational number $\frac{b}{a}$ such that $\frac{a}{b} \times \frac{b}{a} = 1$.
6. **Distributivity** If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then $\frac{a}{b}\left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \cdot \frac{c}{d} + \frac{a}{b} \cdot \frac{e}{f}$
   So, multiplication is distributive over addition.
   - Between any two rational numbers, there are an infinite number of rational numbers.
   - If '*a*' and '*b*' are any two rational number such that $a < b$, then $\frac{1}{2}(a + b)$ is a rational number between *a* and *b*.

Example 1. Write four rational number lying between $\frac{1}{4}$ and $\frac{1}{3}$.

(a) $\frac{13}{48}, \frac{9}{31}, \frac{7}{22}$ and $\frac{5}{16}$ (b) $\frac{13}{48}, \frac{9}{32}, \frac{7}{24}$ and $\frac{5}{16}$
(c) $\frac{13}{46}, \frac{9}{32}, \frac{7}{23}$ and $\frac{5}{17}$ (d) None of these

Sol. (*b*) Rational number between $\frac{1}{4}$ and $\frac{1}{3}$.

$$=\frac{1}{2}\left(\frac{1}{4}+\frac{1}{3}\right)=\frac{1}{2}\left(\frac{3+4}{12}\right)=\frac{1}{2}\times\frac{7}{12}=\frac{7}{24}$$

So, the number are $\frac{1}{4},\frac{7}{24},\frac{1}{3}$.

Now, we shall find a rational number between $\frac{1}{4}$ and $\frac{7}{24}$.

Rational number between $\frac{1}{4}$ and $\frac{7}{24}=\frac{1}{2}\left(\frac{1}{4}+\frac{7}{24}\right)$

$$=\frac{1}{2}\left(\frac{6+7}{24}\right)=\frac{13}{48}$$

So, the number are $\frac{1}{4},\frac{13}{48},\frac{7}{24},\frac{1}{3}$.

Again, we shall find a rational number between $\frac{13}{48}$ and $\frac{7}{24}$.

Rational number between $\frac{13}{48}$ and $\frac{7}{24}$ is

$$=\frac{1}{2}\left(\frac{13}{48}+\frac{7}{24}\right)=\frac{1}{2}\left(\frac{13+14}{48}\right)=\frac{27}{96}=\frac{9}{32}$$

Now, the number are $\frac{1}{4},\frac{13}{48},\frac{9}{32},\frac{7}{24},\frac{1}{3}$.

Again, we shall find one more rational number between $\frac{7}{24}$ and $\frac{1}{3}$.

$$=\frac{1}{2}\left(\frac{7}{24}+\frac{1}{3}\right)=\frac{1}{2}\left(\frac{7+8}{24}\right)=\frac{1}{2}\times\frac{15}{24}=\frac{15}{48}=\frac{5}{16}$$

So, the numbers are $\frac{1}{4},\frac{13}{48},\frac{9}{32},\frac{7}{24},\frac{5}{16},\frac{1}{3}$.

And four rational numbers between $\frac{1}{4}$ and $\frac{1}{3}$ are

$$\frac{13}{48},\frac{9}{32},\frac{7}{24}\text{ and }\frac{5}{16}$$

Important Points to be Remember

- A rational number is a terminating or non-terminating but recurring decimal.
- An irrational number is a non-terminating and non-repeating decimal.
- Every non-terminating and non-recurring decimal cannot be expressed as a rational number.
- Every terminating decimal or repeating decimal can be expressed as a rational number.
- If $a+\sqrt{b}=x+\sqrt{y}$, where a and x are rational and $\sqrt{b}$ and $\sqrt{y}$ are irrational, then $a=x$ and $b=y$.
- The set of irrational number is denoted by S e.g., $\pi,\sqrt{3},\sqrt{5},\sqrt{7}$ etc., are irrational number.

Absolute Value of a Real Number

The absolute value of a real number x is denoted by $|x|$. Thus,

$$|3|=3\text{ and }|-4|=4$$

If x is any real number, then $|x|=\begin{cases}x, \text{ when } & x>0\\ 0, \text{ when } & x=0\\ -x, \text{ when } & x<0.\end{cases}$

For example,

(a) if $x=5, |5|=5$
 if $x=0, |0|=0$
 Note if $x=-5, |-5|=-(-5)=5$

(b) $|x-2|=x-2$, if $x>2$
 $|x-2|=0$, if $x=2$
 $|x-2|=2-x$, if $x<2$

(c) If $n=x^2$; $\sqrt{n}=\sqrt{x^2}=|x|=x$, if $x>0$
 $=-x$, if $x<0$

Some Properties of the Absolute Values

1. $|x|\geq 0$ for all real x.
2. $|x|=a$ means $x=a$ or $x=-a$
3. $|x|>a$ means $x>a$ or $x<-a$
4. $\sqrt{x^2}=|x|=+x$, if $x>0-x$, if $x<0$
5. $|ab|=|a||b|$
6. $\left|\frac{a}{b}\right|=\frac{|a|}{|b|}$, if $b\neq 0$
7. $|a+b|\leq|a|+|b|$
8. $|a-b|\geq|a|-|b|$

Example 2. Find the value of x, when $|x-5|=5-x$.

(a) $x=5$ (b) $x<5$
(c) $x>5$ (d) $x=5$ or $x<5$

Sol. (*d*) As, $|x-5|=5-x$

or $|x-5|=-(x-5)$

Since, $|a|=-a$, if $a\leq 0$

so above is true, if $(x-5)\leq 0$ or $x\leq 5$ i.e., $x=5$ or $x<5$

Example 3. Evaluate $|3x-5|+|4-3x|$, when $x=1$

(a) 1 (b) 3 (c) −1 (d) −2

Sol. (b) $|3x-5|+|4-3x|=|3-5|+|4-3|$

$$=|-2|+|1|=2+1=3$$

So $|3x-5|+|4-3x|=3$ for $x=1$

Example 4. For what values of x, is the equality $x+|x|=2x$ is true?

(a) $x=0$ (b) $x\leq 0$ (c) $x>0$ (d) $x\geq 0$

Sol. (*d*) $x+|x|=2x \Rightarrow |x|=x$, which is true, if $x\geq 0$

So, $x+|x|=2x$, when $x\geq 0$

Example 5. Find the value of x which satisfy the inequalities $|x|\geq x$ and $2x-1>3$.

(a) all positive number
(b) all positive number greater than 2
(c) all negative number less than −2
(d) all negative number

Sol. (*b*) $|x|\geq x$ is true for all real values of x. Now, consider $2x-1>3$ or $2x>4$ or $x>2$.

So, the solution set is all positive number >2.

To Find the Unit's Place Digit of a given Exponential

1. In case of (0, 1, 5, 6) The unit's place digit is 0, 1, 5, 6, respectively.
2. In case of (4 and 9)
   (a) **If power is odd** The unit's place digit is 4 and 9 respectively.
   (b) **If power is even** The unit's place digit is 6 and 1 respectively.
3. In case of (2, 3, 7, 8) See the following example

To find the unit's place digit of $(134647)^{553}$

Step I 553 ÷ 4 gives 1 as remainder, this remainder is taken as new power.

Step II $(134647)^{553} \equiv (134657)^1 \equiv 7^1 = 7$

∴ The unit's place digit is 7.

or

Step I If on dividing the remainder obtained is zero, take 4 as new power instead of zero.

e.g. $(134647)^{552} \equiv (134647)^0 \equiv (7)^0 \equiv (7)^4 = 2401$

∴ The unit's place digit is 1.

Division on Numbers (Division Algorithm)

Let 'd' and 'b' be two integers such that $b \neq 0$ on dividing 'a' by 'b'. Let 'q' be the quotient and 'r' the remainder, then the relationship between a, b, q and r is $a = bq + r$.

or in general, we have

dividend = divisor × quotient + remainder.

Direct Divisibility Test

1. **Test for a number divisible by 2** If the number is an even number or has '0' in its unit place.
   e.g. The numbers like 17980, 314782, 2854, 316, 6148, as all are even numbers, so are divisible by 2.
2. **Test for a number divisible by 3** If the sum of the digits of the given number is divisible by 3.
   e.g. 23457 : Here, $2 + 3 + 4 + 5 + 7 = 21 \div 3 = 7$. Hence, 23457 is divisible by 3.
3. **Test for a number divisible by 4** If the number formed by the ten's place and unit place digits is divisible by 4 or last two digits are zero or divisible by 4.
   e.g. (a) 589372 : Here, $72 \div 4 = 18$. So, it is divisible by 4.
   (b) 378600 is divisible by 4.
4. **Test for a number divisible by 5** If the digit at unit place is 5 or 0, then the number is divisible by 5 e.g. 895, 700, etc.
5. **Test for a number divisible by 6** If a number is divisible by 2 and 3, then it is also divisible by 6.
   e.g. 759312. Here, as the last digit or unit digit is 2, so number is divisible by 2.
   Also, the sum of digit $7 + 5 + 9 + 3 + 1 + 2 = 27 \div 3 = 9$ is divisible by 3. Thus, the number is divisible by 6.
6. **Test for a number divisible by 7** If double of unit place digit of given number is subtracted from rest of digits and if the remainder is divisible by 7, then that number is divisible by 7.
   e.g. (a) $875 \rightarrow 87 - (2 \times 5) = 87 - 10 = 77 \div 7 = 11$
   Hence, 875 is divisible by 7.
   (b) $5103 \rightarrow 510 - 2 \times 3 = 504 \div 7 = 72$
   Hence, 5103 is divisible by 7.
   - Trick is applicable for number greater than 99.
7. **Test for a number divisible by 8** If the last three digits of a number are divisible by 8 or are 000, then the number is divisible by 8.
   e.g. $96432 \rightarrow 432 \div 8 = 54$, 16000 is divisible by 8.
8. **Test for a number divisible by 9** If the sum of all the digits of a number is completely divisible by 9, then the number is divisible by 9.
   e.g. $317349 \rightarrow 3 + 1 + 7 + 3 + 4 + 9 = 27 \div 9 = 3$
   So, 317349 is divisible by 9.
9. **Test for a number divisible by 10** If zero exists at the unit place, then the number is divisible by 10.
   e.g. 130, 15680
10. **Test for a number divisible by 11** If the difference between the sum of digits at even places and sum of digits at odd places is (0),then the number is divisible by 11.
    e.g. 1353 here
    Sum of the digits at odd places $= 1 + 5 = 6$
    Sum of the digits at even places $= 3 + 3 = 6$
    and difference of the sum $= 6 - 6 = 0$
    Hence, 1353 is divisible by 11.
    e.g. 123432166, so
    Sum of digits at odd places $= 1 + 3 + 3 + 1 + 6 = 14$
    Sum of digits at even places $= 2 + 4 + 2 + 6 = 14$
    Again, $14 - 14 = 0$, so number is divisible by 11.
    e.g. 10615, so
    Sum of digits at odd places $= 1 + 6 + 5 = 12$
    Sum of digits at even places $= 0 + 1 = 1$
    So, difference $= 12 - 1 = 11 \div 11 = 1$.
    So, it is divisible by11.

Theorem of Divisibility

1. If N is a composite number of the form $Na^p \cdot b^q \cdot c^r \ldots$, where a, b, c are primes, then the number of divisors of N, represented by m is given by $m = (p + 1)(q + 1)(r + 1) \ldots$
2. The sum of the divisors of N, represented by S is given by
   $S = (a^{p+1} - 1)(b^{q+1} - 1)(c^{r+1} - 1)/(a - 1)/(b - 1)(c - 1)$

Shortcut Methods for Multiplication

1. **Multiplication of a given number by 9, 99 etc.**

Trick Place as many zeros at the right of the multiplicand as is the number of nines and from the number, so formed, subtract it from the multiplicand and to get the result.

e.g., (i) Multiply 8886 by 9999

So, $8886 \times 9999 = 88860000 - 8886 = 88851114$

(ii) Multiply 56985 by 999

So, here $56985 \times 999 = 56985000 - 56985 = 56928015$.

2. **Multiplication of a given number by a power of 5**

Trick Put as many zeros to the right of the multiplicand as is the number of power of 5. Divide the number so formed by 2 to the same power as is the number of power of 5.

e.g., (i) Multiply 6798 by 125

Here, $6798 \times 125 = 6798 \times 5^3 = \frac{6798000}{2^3} = \frac{6798000}{8}$

$= 849750$

(ii) $87896 \times 625 = 87896 \times 5^4 = \frac{878960000}{2^4} = \frac{878960000}{16}$

$= 54935000$

3. **Square of numbers with unit digit 5**

Only for number like 5, 15, 25, 35, 45 etc. We will explain with process.

e.g., square of 45.

Then, find square of 5 *i.e.*, 25.

Multiply the other digit leaving 5.

With the number increased by one. *i.e.*, $4 \times 5 = 20$

and place it as 2025. *i.e.*, $(45)^2 = 2025$

e.g., square of 65

square of 5 *i.e.*, 25 and $6 \times 7 = 42$

So, square = 4225

e.g., square of 115

square of 5 = 25 and $11 \times 12 = 132$

So, square = 13225

e.g., square of 275

square of 5 = 25 and $27 \times 28 = 756$

So, square of 275 = 75625

Multiplication by Using Formulae

The formula given below are very useful for quick multiplication. Here, if *a*, *b* and *c* are real number, then

1. (i) $(a+b)^2 = (a+b)(a+b) = a^2 + b^2 + 2ab$
   (ii) $(a+b)^2 - 2ab = a^2 + b^2$
   (iii) $(a+b)^2 - (a^2+b^2) = 2ab$
2. (i) $(a-b)^2 = (a-b)(a-b) = a^2 + b^2 - 2ab$
   (ii) $(a-b)^2 + 2ab = a^2 + b^2$
   (iii) $(a-b)^2 - (a^2+b^2) = -2ab$
3. (i) $(a+b) = \sqrt{(a-b)^2 + 4ab}$
   (ii) $(a-b) = \sqrt{(a+b)^2 - 4ab}$
4. (i) $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$
   (ii) $(a+b)^2 - (a-b)^2 = 4ab$
5. $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$
6. (i) $a^2 - b^2 = (a-b)(a+b)$
   (ii) $a^2 + b^2 = (a+b)^2 - 2ab$
   (iii) $a^4 - b^4 = (a^2+b^2)(a^2-b^2)$
7. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
8. $(a-b)^2 + (b-c)^2 + (c-a)^2$
   $= 2(a^2 + b^2 + c^2 - ab - bc - ac)$
9. (i) $(a+b)^3 = a^3 + b^3 + 3ab(a+b) = a^3 + b^3 + 3a^2b + 3b^2a$
   (ii) $a^3 + b^3 = (a+b)^3 - 3ab(a+b) = (a+b)(a^2 + b^2 - ab)$
10. (i) $(a-b)^3 = a^3 - b^3 - 3ab(a-b) = a^3 - b^3 - 3a^2b + 3ab^2$
    (ii) $a^3 - b^3 = (a-b)^3 + 3ab(a-b) = (a-b)(a^2 + ab + b^2)$

Simplification

The word simplification refers to a procedure useful for converting long fractions or expressions in one fraction or numbers.

Trick For solving question or simplification the rule is 'VBODMAS'. The letter of word means
V—Under line pc. .on *i.e.*, bar; B—Brackets;
O—Of (Multiplication); D—Division;
M—Multiplication; A—Addition S—Subtraction

Example 6. Simplify $2 - [2 - \{2 - \overline{(2-2)}\}]$

(a) 0 (b) 2 (c) −2 (d) 4

Sol. (*b*) Here, $2 - [2 - \{2 - \overline{(2-2)}\}] = 2 - [2 - \{2 - 0\}]$

$= 2 - [2-2] = 2 - 0 = 2$

Example 7. If $x - \cfrac{5}{7 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{3}}}} = 3$, find x.

(a) $\frac{155}{224}$ (b) $2\left(\frac{155}{224}\right)$

(c) $3\left(\frac{155}{224}\right)$ (d) None of these

Sol. (*c*) Here, $x - \cfrac{5}{7 + \cfrac{1}{4 + \cfrac{1}{\frac{7}{3}}}} = x - \cfrac{5}{7 + \cfrac{1}{4 + \frac{3}{7}}}$

$= x - \cfrac{5}{7 + \cfrac{1}{\frac{31}{7}}} = x - \cfrac{5}{7 + \frac{7}{31}}$

$= x - \cfrac{5}{\frac{224}{31}} = x - \frac{31 \times 5}{224} = x - \frac{155}{224}$

So, $x - \frac{155}{224} = 3 \Rightarrow x = 3 + \frac{155}{224} = 3\left(\frac{155}{224}\right)$

Surds or Radicals

Let 'x' be a rational number and 'n' be a positive integer such that $\sqrt[n]{x}$ is irrational, then $\sqrt[n]{x}$ is called a radical of order n or surd and here 'x' is called the radicand.

A surd of order 2 is called a quadratic surd *i.e.*, $\sqrt{2}, \sqrt{3}$

A surd of order 3 is called a cubic surd. *i.e.*, $\sqrt[3]{2}, \sqrt[3]{3}$

A surd of order 4 is called a biquadratic surd *i.e.*, $\sqrt[4]{5}, \sqrt[4]{7}$

Points to be Remember

- A surd has no fraction under the radical sign.
- The radicand has no factor with exponent n.
- A surd is not equal to any surd of order lower than n.
- $\sqrt[n]{x}$ is a surd only if x is a rational and $\sqrt[n]{x}$ is irrational.
- When x is irrational or $\sqrt[n]{x}$ is rational, then $\sqrt[n]{x}$ is not a surd.

Laws of Radicals

The laws of indices which are applicable to the surds also are

(a) $(\sqrt[n]{x})^n = x$ (b) $\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$

(c) $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$ (d) $(\sqrt[n]{x})^m = (\sqrt[n]{x^m})$

(e) $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x} = \sqrt[n]{\sqrt[m]{x}}$

Types of Surds

Pure Surds A surd which only 1 as a rational factor, the other factor being irrational is called a pure surd.

e.g., $\sqrt{2}, \sqrt[3]{2}, \sqrt[4]{3}$ all are pure surds.

Mixed Surds A surd which is not a pure surd or has factor other than unity, the other factor being irrational is called a mixed surd. *e.g.*, $2\sqrt{3}, 5\sqrt[3]{12}, \frac{4}{3}\sqrt[3]{35}$ are mixed surds.

Comparison of Two Surds

1. If two surds are of the same power/order, then the one whose radicand is larger, is the larger of the two.

   e.g., $\sqrt{17} > \sqrt{13}, \sqrt[3]{21} > \sqrt[3]{16}$

2. If any two surds have different orders are to be compared then, we will first reduce them to the same but smallest order and then compare them.

- To change a surd of order 'n' into a surd of the order 'm'.
- Let $\sqrt[n]{x}$ be a surd of order n.
- Then, $\sqrt[n]{x} = \sqrt[m]{x^{m/n}}$ *i.e.*, $x^{1/n} = (x^{m/n})^{1/m}$

Example 8. Convert $\sqrt{2}$ into a surd of order 4.

(a) $\sqrt[4]{2}$ (b) $\sqrt[4]{8}$

(c) $\sqrt[4]{4}$ (d) None of these

Sol. (c) $\sqrt{2} = 2^{1/2} = (2^{4/2})^{1/4} = (2^2)^{1/4} = \sqrt[4]{4}$

Multiplication and Division of Surds

The like surds or unlike surds of same order can be multiplied or divided by other surd by using rules

(a) $\sqrt[n]{x} \times \sqrt[n]{y} = \sqrt[n]{xy}$ (b) $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$

If order are not same, then they are converted to the lowest common order by then multiplied and divided as required.

Example 9. Simplify $\sqrt[6]{12} \div \sqrt{3} \cdot \sqrt[3]{2}$

(a) $\sqrt[3]{3}$ (b) $\sqrt{3}$ (c) $\sqrt[3]{2}$ (d) $\sqrt[3]{\frac{1}{3}}$

Sol. (*d*) $\frac{\sqrt[6]{12}}{\sqrt{3} \cdot \sqrt[3]{2}}$; Now, here the oder of $\sqrt{3} \cdot \sqrt[3]{2}$ is 2 and 3 respectively.

So, $\text{LCM} = 6$

$$\sqrt{3} = 3^{1/2} = (3^{6/2})^{1/6} = (3^3)^{1/6} = (27)^{1/6} = \sqrt[6]{27}$$

$$\sqrt[3]{2} = (2)^{1/3} = (2^{6/3})^{1/6} = (2^2)^{1/6} = \sqrt[6]{4}$$

So, $$\frac{\sqrt[6]{12}}{\sqrt{3} \cdot \sqrt[3]{2}} = \frac{\sqrt[6]{12}}{\sqrt[6]{27} \cdot \sqrt[6]{4}} = \frac{\sqrt[6]{12}}{\sqrt[6]{27 \times 4}} = \sqrt[6]{\frac{12}{27 \times 4}} = \sqrt[6]{\frac{1}{9}}$$

$$\left[\left(\frac{1}{3}\right)^2\right]^{1/6} = \left[\frac{1}{3}\right]^{2/6} = \left[\frac{1}{3}\right]^{1/3} = \sqrt[3]{\frac{1}{3}}$$

Operations on Surds

Similar (like) **Surds** If two or more surds having the same irrational factor are known as similar/like surds.

e.g., $\sqrt{3}, 8\sqrt{3}, \frac{1}{7}\sqrt{3}$ are like surds.

Dissimilar (unlike) **Surds** If two or more surds have different irrational factor are known as dissimilar/unlike surds.

e.g., $\sqrt{3}, \sqrt{2}, \sqrt[3]{9}$ are unlike surds.

Addition of Like Surds

Like surds are added by using laws of numbers.

e.g., $2\sqrt{3} + 4\sqrt{3} + \frac{1}{2}\sqrt{3} = \left(2 + 4 + \frac{1}{2}\right)\sqrt{3} = \left(6 + \frac{1}{2}\right)\sqrt{3} = \frac{13}{2}\sqrt{3}$

Subtraction of Like Surds

Like surds can be subtracted by using laws of numbers.

e.g., $2\sqrt{3} + 4\sqrt{3} - \frac{1}{2}\sqrt{3}$

$$= \left(2 + 4 - \frac{1}{2}\right)\sqrt{3} = \left(6 - \frac{1}{2}\right)\sqrt{3}$$

$$= \left(\frac{12 - 1}{2}\right)\sqrt{3} = \frac{11}{2}\sqrt{3}$$

Rationalisation of Surds

Process of converting surd into rational number [illegible] as rationalisation of the surd.

When the product of two surds is a rational number, then each surd is called a rationalising factor of the other.

e.g., $\sqrt[3]{25} \times \sqrt[3]{5} = \sqrt[3]{25 \times 5} = 5$

So, $\sqrt[3]{25}$ and $\sqrt[3]{5}$ are rationalising factor of each other.

Monomial of Surds

A surd having only one term is known as a monomial surd, e.g., $\sqrt{2}, \sqrt{3}$ etc.

Binomial of Surds

A surd consisting of the sum of two monomial surds or the sum of a monomial surd and a rational number.

e.g., $(\sqrt{5}+\sqrt{2}), (\sqrt{5}-\sqrt{2}), (\sqrt[3]{3}-\sqrt{2})$ etc.

Trinomial Surd

A surd consisting of the sum of three monomial surds or two binomials. e.g., $(\sqrt{3}+\sqrt{3}+\sqrt{7})$, $(\sqrt{3}+2\sqrt{5}-\sqrt{7})$,

$$(\sqrt{5}+\sqrt{2})+(\sqrt{5}-\sqrt{3})=(2\sqrt{5}-\sqrt{3}+\sqrt{2})$$

Conjugate Surds

The binomial surds which differ only in the sign (+/−) between them are called as conjugate surds.

e.g., $(x+\sqrt{y})$ and $(x-\sqrt{y})$ are conjugate surds.

$(\sqrt{x}+\sqrt{y})$ and $(\sqrt{x}-\sqrt{y})$ are conjugate binomial surds.

$(\sqrt[3]{x}+\sqrt[3]{y})$ and $(\sqrt[3]{x}-\sqrt[3]{y})$ are conjugate binomial surds.

- Rationalising factor of $(x+\sqrt{y})$ is $(x-\sqrt{y})$.
- Rationalising factor of $(\sqrt{x}+\sqrt{y})$ is $(\sqrt{x}-\sqrt{y})$ etc.

Progressions

Sequence A sequence is a non-empty set X is defined to be a map.

$$f: N_n \to X \text{ or a map } f: N \to X$$

If $X=R$ the set of all real numbers, then f is called a real sequence. If $X=C$ the set of all complex numbers, then f is called a complex sequence.

If f is a sequence, then for any $K \in N_n$ or N according as f is a finite sequence or infinite sequence.

$$f(K)=a_k \in X \text{ where } X \text{ is either } R \text{ or } C$$

(i) $a_1, a_2, a_3, \ldots, a_n$, is a finite sequence and is denoted as $\{a_k\}_{k=1}^{n}$.

(ii) $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ is an infinite sequence and is denoted by $\{a_n\}_{n=1}^{\infty}$ or simply $\{a_n\}$.

Here, a_1 is called the first term and in case of finite sequence a_n is called the last term.

e.g., 2, 4, 6, 8, ..., 100 is a finite sequence.

1, 2, 3, 4, ... is an infinite sequence.

Progression Sequence following certain patterns are called progressions.

e.g., 2, 3, 4, 5, ... is a progression, here each term is increasing by 1.

Arithmetical Progression (AP)

In an AP the difference between any two consecutive terms is a constant. The constant 'd' is called the common difference. The first term of AP is represented by 'd' and the formula for the nth term is

$$a_n = a+(n-1)d$$

If an AP has first term $= a$ and common difference $= d$, then the general form of an AP is

$$a, a+d, a+2d, a+3d, \ldots, a+(n-1)d,$$

- Sum of the n terms of an AP is $S_n = \frac{n}{2}[2a+(n-1)d]$

Example 10. Find sum of the AP $2+4+6+8+10+12$

(a) 39 (b) 40 (c) 41 (d) 42

Sol. (d) Here, $a=2$, and $d=4-2=2$

Also $n=6$

So, $S_6=\frac{6}{2}[2(2)+(5)(2)]=3(4+10)=42$

Example 11. Find the sum of 20 terms of an AP whose first term is 4 and common difference is 3.

(a) 550 (b) 650
(c) 750 (d) None of these

Sol. (b) Here, $a=4, d=3, n=20$

So, $S_{20}=\frac{20}{2}[2(4)+19\times3]=10[8+57]=650$

- If first term $= a$ and last term $= a_n$ are given, then sum of n terms of an AP is $S_n = \frac{n(a_1+a_n)}{2}$

where, n = number of terms.

Example 12. In AP if the first term is 2, and last term is 29, then find the sum, if $n=10$.

(a) 155 (b) 156 (c) 157 (d) 158

Sol. (a) Here, $a_1=2, a_{10}=29, n=10$

So, $S_{10}=\frac{n}{2}(a_1+a_n)=\frac{10}{2}(a_1+a_{10})=\frac{10}{2}(2+29)=155$

Example 13. If the sum of n term of an AP is 525. If first term is 3 and last term is 39. Then, find n.

(a) 20 (b) 23 (c) 25 (d) 30

Sol. (c) Here, $S_n=525, a_1=3, a_n=39$

$$S_n=\frac{n}{2}(a_1+a_n) \Rightarrow 525=\frac{n}{2}(3+39)$$

$$525=\frac{n(42)}{2}=21n,\ n=\frac{525}{21} \Rightarrow n=25$$

- The nth term of a series is equal to the sum of n terms minus the sum of $(n-1)$ terms i.e., $a_n = S_n - S_{n-1}$

Example 14. If for an AP $S_{20}=210$ and $S_{19}=190$, then find the 20th term.

(a) 10 (b) 20 (c) 30 (d) 40

Sol. (b) Here, 20th term is $= S_{20}-S_{19}$

$$T_{20} = 20\text{th term} = 210-190=20$$

Arithmetic Mean

When three terms are in AP the middle term is called AM (Arithmetic mean).

So, if a, b, c are in AP. Then, $b = \frac{a+c}{2} \Rightarrow 2b = a + c$

'b' is AM between a and c.

Example 15. Find the AM between 3 and 9.

(a) 4 (b) 6
(c) 8 (d) None of these

Sol. (b) The arithmetic mean is $\frac{3+9}{2} = 6$

So, 3, 6, 9 are in AP

Exercise

1. The least prime number is
(a) 0 (b) 2 (c) 3 (d) 1

2. What is the total number of prime number less than 100?
(a) 25 (b) 30 (c) 24 (d) 26

3. The total prime numbers between 61 and 89 are
(a) 5 (b) 4 (c) 6 (d) 1

4. Which one of the following is a prime number? **(CDS 2011 II)**
(a) 161 (b) 171 (c) 173 (d) 221

5. Consider the following statements
I. A natural number is divisible by 2, if its last digit is divisible by 2.
II. A natural number is divisible by 2, if its last digit is either zero or 2.
III. A natural number is divisible by 2, if its last digit is even.
Of the above statement
(a) I and II are correct (b) I and III are correct
(c) II and III are correct (d) All of these

6. Which one of the following is rational?
(a) Area of a circle with radius $1/\pi$.
(b) Radius of circle with area $1/\pi$.
(c) Circumference of circle with radius $1/\pi$.
(d) Radius of circle with circumference $1/\pi$.

7. The product of a rational number and an irrational number is **(CDS 2010 II)**
(a) a natural number (b) an irrational number
(c) a composite number (d) a rational number

8. The value of $\log_2 \sqrt{2\sqrt{2\sqrt{2\ldots}}}$ upto infinity is
(a) 1/2 (b) 1 (c) 3/2 (d) 2

9. All prime factors of 182 are
(a) 2 and 13 (b) 2 and 7
(c) 2, 7 and 13 (d) None of these

10. Which one of the following numbers is not a square of any natural numbers? **(CDS 2010 I)**
(a) 5041 (b) 9852 (c) 1936 (d) 6241

11. Let 'a' and 'b' be natural number, not necessarily distinct. For all values of 'a' and 'b' the natural number would be
(a) $(a+b)$ (b) a/b
(c) $a-b$ (d) $\log(ab)$

12. Of the following
I. 1.5 II. $\sqrt{2}$ III. $2+3\sqrt{5}$ IV. $\sqrt{4}$
The number that are not rational are
(a) I and II (b) II and III (c) I only (d) II, III and IV

13. The next number in the sequence 7, 12, 19, 28, ... is
(a) 39 (b) 45 (c) 41 (d) 60

14. Let x be a real number such that $0 < x < 1$. Of the following the correct statement would be
(a) the positive square root of x is equal to x
(b) the positive square root of x is greater than x
(c) the positive square root of x is less than x
(d) the square of x is greater than x

15. x and y are two natural numbers such that x is less than y,q is the quotient and r is the remainder when y is divided by x. Therefore,
(a) $r = 0$ (b) $r < 0$
(c) $r > x$ (d) $0 \le r < x$

16. By adding x to 1254934, the resulting number becomes divisible by 11, while adding y to 1254934 makes the resulting number divisible by 3. Which one of the following is the set of values for x and y? **(CDS 2007 I)**
(a) $x = 1, y = 1$ (b) $x = 1, y = -1$
(c) $x = -1, y = 1$ (d) $x = -1, y = -1$

17. The real values of x for which $|x| < 3$ are
(a) $-3 \le x$ (b) $3 > x$
(c) $-3 < x < 3$ (d) None of these

18. If n is a natural number than $\sqrt{n}$ is
(a) always a whole number
(b) always a natural number
(c) sometimes a natural number and sometimes an irrational number
(d) always an irrational number

19. The numbers x, x + 2, x + 4 are all primes so x is
(a) 3 (b) 2 (c) 11 (d) 17

20. The minimum value of $(3y - 4x)$, if $-1 \le x \le 2$ and $1 \le y \le 3$ is
(a) 0 (b) +5 (c) −5 (d) −8

21. In a division operation, the divisor is 5 times the quotient and twice the remainder. If the remainder is 15, then what is the dividend? **(CDS 2007 II)**
(a) 175 (b) 185 (c) 195 (d) 250

22. The product of two numbers is y/x, if one of the number is $\frac{x}{y^2}$, then the other one is

(a) $\frac{x^2}{y}$ (b) $\frac{x}{y^2}$ (c) $\frac{y^2}{x^3}$ (d) $\frac{y^3}{x^2}$

23. Which one of the following is a correct statement?
(a) Decimal expansion of a rational number is terminating.
(b) Decimal expansion of a rational number is non-terminating.
(c) Decimal expansion of an irrational number is terminating.
(d) Decimal expansion of an irrational number is non-terminating and non-repeating.

24. Let p denote the product $2 \cdot 3 \cdot 5 \ldots 59 \cdot 61$ of all primes from 2 to 61. Consider the sequence $p + n\,(2 \le n \le 59)$. What is the number of primes in this sequence? (n is a natural number) **(CDS 2008 I)**
(a) 0 (b) 16 (c) 17 (d) 58

25. Zero is
(a) a natural number (b) a whole number
(c) a positive number (d) a negative integer

26. In between two rational numbers, there are
(a) a finite number of fractions
(b) precisely two fractions
(c) even number of rationals
(d) infinitely many numbers of fractions to previous column

27. If 'a' is an even positive integer and 'b' is an odd positive integer, then which of the following statements is true?
(a) $a(b-1)$ is even (b) $a(b-1)$ is odd
(c) $(a-1)(b-1)$ is even (d) $(a-1)b$ is even

28. Consider the following statements
I. Set of positive powers of 2 is closed under multiplication.
II. The set $\{1, 0, -1\}$ is closed under multiplication.
III. The number 35 has exactly four divisors.
IV. The set $\{1, 0, -1\}$ is closed under addition.
Of the above statement
(a) I, II, III are true (b) Only III is true
(c) All are false (d) All are true

29. A prime number greater than 11 will never end with
(a) 5 (b) 7 (c) 9 (d) 1

30. Consider the following statements
A number $a_1a_2a_3a_4a_5$ is divisible by 9, if
I. $a_1 + a_2 + a_3 + a_4 + a_5$ is divisible by 9.
II. $a_1 - a_2 + a_3 - a_4 + a_5$ is divisible by 9.
Which of the above statements is/are correct?
(a) Only I (b) Only II **(CDS 2009 I)**
(c) Both I and II (d) Neither I nor II

31. If 'P' is a prime number and P divides ab i.e., P/ab, where 'a' and 'b' are integers, then
(a) P/a or P/b (b) P/a and P/b
(c) $P/a-b$ (d) None of these

32. Which among the following is the largest four digit number that is divisible by 88? **(CDS 2010 II)**
(a) 9988 (b) 9966 (c) 9944 (d) 8888

33. The number 444 444 444 444 is divisible by
(a) 3, 11 (b) 3, 7
(c) 5, 11 (d) 9, 11

34. If 42 * 8 is a multiple of 9, then the digit represented by is
(a) 0 (b) 1 (c) 2 (d) 4

35. 3 is a factor of any number, if the sum of the digits is divisible by
(a) 0 (b) 5
(c) 2 (d) 3

36. The prime factors of 2310 are
(a) 2, 3, 5, 7, 11 (b) 2, 4, 6, 7, 11
(c) 2, 3, 4, 7 (d) None of these

37. Match list I with list II and select the correct answer using the codes given below the lists

Numbers	Product of prime factors
A. 1728	1. $7^2 \times 3^2$
B. 369	2. 7×3^4
C. 441	3. $2^6 \times 3^3$
D. 567	4. 7×3^3
	5. 41×3^2

Codes

	A	B	C	D
(a)	3	1	5	2
(b)	3	5	4	2
(c)	3	5	1	2
(d)	3	4	5	2

38. Consider the following statements
I. In a given whole number, if the sum of the odd numbered digit is equal to the sum of even numbered digits, then the number is divisible by 11.
II. In a given whole number, if the difference of sum of odd numbered digits and even numbered digits is divisible by 11, then the number is divisible by 11
Of these statements
(a) Only I is correct (b) Both I and II are wrong
(c) Only II is correct (d) Both I and II are correct

39. When n is divided by 4, the remainder is 3. What is the remainder when $2n$ is divided by 4?
(a) 0 (b) 2 (c) 6 (d) 3

40. Consider the following statements
If p is a prime such that $p+2$ is also a prime, then
I. $p(p+2)+1$ is a perfect square.
II. 12 is a divisor of $p+(p+2)$, if $p>3$.
Which of the above statements is/are correct?
(a) Only I (b) Only II **(CDS 2011 I)**
(c) Both I and II (d) Neither I nor II

41. Of the following statements for natural numbers a, b and c.
I. If 'a' is divisible by 'b' and 'b' is divisible by 'c', then a must be divisible by 'c'.
II. If 'a' is a factor of both 'b' and 'c', then 'a' must be a factor of '$b+c$'.
III. If 'a' is a factor of both 'b' and 'c', then 'a' must be a factor of '$b-c$'.

The true statements are

(a) I, II and III (b) I and II
(c) II and III (d) None of these

42. If $\dfrac{\frac{1}{4}-\frac{1}{6}-\frac{1}{48}}{\frac{1}{4}-\left(\frac{1}{6}-\frac{1}{48}\right)}+\dfrac{\frac{1}{4}\times\frac{1}{6}-\frac{1}{48}}{\frac{1}{4}\times\left(\frac{1}{6}-\frac{1}{48}\right)}=x$ the value of x is

(a) $\frac{20}{21}$ (b) $-\frac{21}{20}$ (c) $\frac{21}{20}$ (d) $-\frac{20}{21}$

43. If n is a positive integer, then what is the digit in the unit place of $3^{2n+1}+2^{2n+1}$? **(CDS 2010 II)**

(a) 0 (b) 3 (c) 5 (d) 7

44. What can be said about the expansion of $2^{12n}-6^{4n}$, where n is a positive integer? **(CDS 2010 I)**

(a) Last digit is 4 (b) Last digit is 8
(c) Last digit is 2 (d) Last two digits are zero

45. The value at * in $\frac{16}{7}\times\frac{16}{7}-\frac{*}{7}\times\frac{9}{7}+\frac{9}{7}\times\frac{9}{7}=1$ is

(a) 33 (b) −33 (c) −11 (d) 32

46. The value of $10\div\overline{4+6}\times 4$ is

(a) 4 (b) 1/4
(c) 5 (d) None of these

47. If $x\div 4+2\times 4=9$, then the value of x is

(a) 6 (b) 4 (c) 8 (d) 12

48. If $x+\dfrac{1}{2+\dfrac{4}{5\div 5+2}}=0$, then the value of x is

(a) 3/10 (b) −10/3 (c) −3/10 (d) 10/3

49. Consider the following statements

I. The product of any three consecutive integers is divisible by 6.

II. Any integer can be expressed in one of the three forms $3k$, $3k+1$, $3k+2$, where k is an integer.

Which of the above statements is/are correct? **(CDS 2011 II)**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

50. If the 14th term of an arithmetic series is 6 and 6th term is 14, then what is the 95th term?

(a) −75 (b) 75 (c) 80 (d) −80

51. The value of $\left(\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\dots\left(1-\frac{1}{n-1}\right)\left(1-\frac{1}{n}\right)$ is

(a) $\frac{1}{n}$ (b) $\frac{1}{2n}$
(c) $\frac{2}{n}$ (d) $\frac{3}{n}$

52. If $\frac{2^5\times 9^2}{8^2\times 3^5}=x$, the value of x is

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 6

53. How many $\frac{1}{8}$ s are there in $25\frac{1}{2}$?

(a) 204 (b) 404
(c) 104 (d) None of the above

54. If $\frac{a}{4}=\frac{b}{5}=\frac{c}{6}$, then the value of $\frac{a+b+c}{b}$ is

(a) 3 (b) 2 (c) 6 (d) 4

55. If $b=4, c=9$, then the value of $\sqrt{bc}+\sqrt{b}+5$ is

(a) 19 (b) 17 (c) 13 (d) 11

56. What least value must be given to ⊗, so that the number 84705 ⊗ 2 is divisible by 9? **(CDS 2009 II)**

(a) 0 (b) 1 (c) 2 (d) 3

57. If $a=3, b=9, c=10$, then the value of $\sqrt{13+a}+\sqrt{112+b}+\sqrt{c-1}$ is

(a) 15 (b) 18
(c) 16 (d) 10

58. The value of $[a-b+(b-a)]-[2a-2b+(b-2a)]$ is

(a) $b-2a$ (b) $a-2b$
(c) $b+2a$ (d) b

59. If $3.325\times 10^k=0.0003325$, the value of '$k$' is

(a) 4 (b) −4 (c) −3 (d) −2

60. What is the last digit in the expansion of $(2457)^{754}$? **(CDS 2009 I)**

(a) 3 (b) 7 (c) 8 (d) 9

61. The value of $(0.\overline{6}+0.\overline{8}+0.\overline{7})$ is

(a) $2\frac{1}{10}$ (b) $2\frac{1}{9}$
(c) $2\frac{1}{3}$ (d) None of these

62. If $1.6x=0.4y$, then the value of $\frac{x+y}{x-y}$ is

(a) 1.66 (b) −1.66 (c) 16.6 (d) −16.6

63. If $\sqrt{[0.04\times 0.4\times x]}=0.4\times 0.04\times\sqrt{y}$, then the value of x/y is

(a) 0.0016 (b) 0.16
(c) 0.016 (d) None of these

64. The unit digit in the product $(281\times 15\times 16\times 18)$ is

(a) 6 (b) 0 (c) 5 (d) 8

65. The unit digit in the product $(127)^{170}$ is

(a) 3 (b) 9 (c) 7 (d) 3

66. ABC is a triangle and AD is perpendicular to BC. It is given that the lengths of AB, BC, CA are all rational numbers. Which one of the following is correct? **(CDS 2011 I)**

(a) AD and BD must be rational.
(b) AD must be rational but BD need not be rational.
(c) BD must be rational but AD need not be rational.
(d) Neither AD nor BD need be rational.

67. The unit digit in the product $(7^{71}\times 6^{59}\times 3^{65})$ is

(a) 6 (b) 2 (c) 4 (d) 1

68. The unit digit in $(7^{27}-3^{14})$ is

(a) 0 (b) 7 (c) 4 (d) 6

69. If $-1\le x\le 3$ and $1\le y\le 3$, then the maximum value of $(3y-4x)$ is

(a) 18 (b) 13 (c) 5 (d) −6

70. If x is negative real number, then
(a) $|x| = x$
(b) $|x| = -x$
(c) $|x| = \frac{1}{x}$
(d) $|x| = -\frac{1}{x}$

71. If $\frac{x}{y} = \frac{3}{5}$, then the value of $\frac{x-y}{x+y}$ is
(a) $-\frac{1}{4}$
(b) $\frac{1}{4}$
(c) $\frac{1}{6}$
(d) -6

72. When $x = 2$, the value of $|2x-3| + |x-1|$ is
(a) 3
(b) -2
(c) 2
(d) -3

73. Consider the following assumption and two statements

Assumption A number '$ABCDE$' is divisible by 11.

Statement I $E - D + C - B + A$ is divisible by 11.

Statement II $E - D + C - B + A = 0$

Which one of the following is correct? **(CDS 2010 I)**
(a) Only statement I can be drawn from the assumption.
(b) Only statement II can be drawn from the assumption.
(c) Both the statements can be drawn from the assumption.
(d) Neither of the statements can be drawn from the assumption.

74. If x is real, then $\left|\frac{7-x}{3}\right| < 2$, if and only if
(a) $1 < x < 13$
(b) $-1 < x < 13$
(c) $x < 13$
(d) $x > 13$

75. A number is divisible by 25 only, if
(a) the last digits of the number is zero
(b) the last digit of the number is 5
(c) the last two digit of the number is divisible by 5
(d) the last two digit of the number is divisible by 25

76. When n is even, the product $n(n+1)(n+2)$ is divisible by
(a) 24
(b) 7
(c) 0
(d) None of these

77. A ten-digit number is divisible by 4 as well as by 5. What could be the possible digit at the ten's place in the given number? **(CDS 2007 I)**
(a) 0, 1, 2, 4, or 6
(b) 1, 2, 4, 6 or 8
(c) 2, 3, 4, 6 or 8
(d) 0, 2, 4, 6 or 8

78. If A is the set of squares of natural numbers and x and y are any two element of A, then the correct statement is
(a) $x + y$ belongs to A
(b) $x - y$ belongs to A
(c) $\frac{x}{y}$ belongs to A
(d) xy belongs to A

79. If $a^x = b, b^y = c$ and $c^z = a$, then xyz equals
(a) abc
(b) 1
(c) $\frac{1}{abc}$
(d) None of these

80. A three-digit number has digits h, t, u (from left to right) with $h > u$. If the digits are reversed and the number thus formed is subtracted from the original number, the unit's digit in the resulting number is 4. What are the other two digits of the resulting number from left to right? **(CDS 2008 II)**
(a) 5 and 9
(b) 9 and 5
(c) 5 and 4
(d) 4 and 5

81. Every composite number has
(a) no prime divisor
(b) atleast one prime divisor
(c) atleast two prime divisor
(d) one and only one prime divisor

82. If 'r' is a non-zero rational number and 'x' is an irrational number, then the product 'rx' is
(a) a rational number
(b) an integer
(c) an irrational number
(d) None of these

83. If 'a' is an odd integer, the number $a(a^2 - 1)$ is divisible by
(a) 8
(b) 32
(c) 24
(d) 16

84. Which of the following statement is true?
(a) Prime numbers are in GP.
(b) Prime numbers are in AP.
(c) Sum of any two prime numbers other than 2 is odd number.
(d) Sum of any two prime numbers other than 2 is an even integer.

85. If three sides of a right angled triangle are integers in their lowest form, then one of its sides is always divisible by **(CDS 2011 I)**
(a) 6
(b) 5
(c) 7
(d) None of these

86. Which of the following is necessarily true the square root?
(a) A positive number is real.
(b) A rational number is rational.
(c) An integer is an integer.
(d) A real number is positive.

87. If the sum of two numbers is 18 and sum of their square is 164, then the smallest number is
(a) 8
(b) 6
(c) 12
(d) 10

88. When a polynomial is divided by a linear polynomial then what is the remainder?
(a) Constant polynomial only.
(b) Zero polynomial only.
(c) Either constant or zero polynomial.
(d) Linear polynomial.

89. If 'p' is an integer greater than 3, then on dividing $p^{11} + 1$ by $p - 1$, we would get the remainder as
(a) 2
(b) 0
(c) -2
(d) -1

90. The value of $\frac{2.48 \times 2.48 - 1.52 \times 1.52}{0.96}$ is
(a) 4
(b) 0.96
(c) 16
(d) 15.04

91. If 'a' is proportional to b^2 and b is proportional to c^2, then a is proportional to
(a) c^6
(b) c^4
(c) $c^{2/3}$
(d) $c^{3/2}$

92. Which of the following is the correct statement?
(a) Sum of two rational numbers is always an integer.
(b) Sum of two irrational number is always an irrational number.
(c) Sum of a rational number and an irrational number is an irrational number.
(d) Square of an irrational number is always rational number.

93. Which one of the following is correct? **(CDS 2007 II)**
The number 222222 is
(a) divisible by 3 but not divisible by 7
(b) divisible by 3 and 7 but not divisible by 11
(c) divisible by 2 and 7 but not divisible by 11
(d) divisible by 3, 7 and 11

94. If a, b and c are real numbers such that $a < b$ and $c < 0$, then which of the statements is true?
(a) $(a/c) < (b/c)$ (b) $ac < bc$
(c) $(c/a) > (c/b)$ (d) $ac > bc$

95. If $\log x^2y^3 = a$ and $\log \frac{x}{y} = b$, then $\frac{\log x}{\log y}$ is equal to
(a) $\frac{a-2b}{a+3b}$ (b) $\frac{a+3b}{a-2b}$ (c) $\frac{a+2b}{a-3b}$ (d) $\frac{a-3b}{a+2b}$

96. 235. 235235235 ... is a/an
(a) integer (b) whole number
(c) rational number (d) irrational number

97. Find the value of 'a' and 'b'
$$3\frac{7}{a} \times b\frac{3}{15} = 8$$
(a) 2, 11 (b) 11, 2 (c) 1, 1 (d) 2, 1

98. The value of $\log_{10} 0.02$ lies between
(a) 0 and 1 (b) 0 and −1
(c) −2 and −3 (d) −2 and −1

99. The periodic decimal $0.272727... = 0.\overline{27}$ is the rational number
(a) 3/11 (b) 1/7 (c) 2/7 (d) 1/11

100. The angles of a triangle are in AP and the greatest angle is double the least. What is the ratio of angles in the radian measure? **(CDS 2009 I)**
(a) 2 : 3 : 4 (b) 1 : 2 : 3
(c) 3 : 3 : 6 (d) 4 : 5 : 7

101. Consider the following numbers.
I. 247 II. 203
Which of the above numbers is/are prime? **(CDS 2009 I)**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

102. If $x < 0 < y$, then which one of the following relations is correct?
(a) $\frac{1}{x^2} < \frac{1}{xy} < \frac{1}{y^2}$ (b) $\frac{1}{x^2} > \frac{1}{xy} < \frac{1}{y^2}$
(c) $\frac{1}{x} < \frac{1}{y}$ (d) $\frac{1}{x} > \frac{1}{y}$

103. Consider the following statements
I. If x and y are composite integers, so also is x + y.
II. If x and y are composite integers and x > y, then x − y is also a composite integer.
III. If x and y are composite integers, so also in xy.
Of the above, the correct statement are
(a) All the three (b) Only I and II
(c) Only III (d) None of these

104. If r and s are any real numbers such that $0 \le s \le 1$ and $r + s = 1$, then what is the maximum value of the product? **(CDS 2010 I)**
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

105. For a positive integer n, define $d(n)$ = The number of positive divisors of n. What is the value of $d(d(d(12)))$?
(a) 1 (b) 2 **(CDS 2011 I)**
(c) 4 (d) None of these

106. If x and y denote respectively, the area and the sum of the length of diagonals of a rectangle with length 1 unit and breadth $\frac{1}{2}$ unit, then which one of the following is correct? **(CDS 2007 II)**
(a) x and y are rational.
(b) x is rational and y is irrational.
(c) x is irrational and y is rational.
(d) x and y are both irrational.

Answers

1. (b)	2. (a)	3. (a)	4. (c)	5. (d)	6. (c)	7. (b)	8. (b)	9. (c)	10. (b)
11. (a)	12. (b)	13. (a)	14. (b)	15. (d)	16. (b)	17. (c)	18. (c)	19. (a)	20. (c)
21. (c)	22. (d)	23. (d)	24. (a)	25. (b)	26. (d)	27. (a)	28. (d)	29. (a)	30. (a)
31. (b)	32. (c)	33. (a)	34. (d)	35. (d)	36. (a)	37. (c)	38. (d)	39. (b)	40. (c)
41. (a)	42. (c)	43. (c)	44. (d)	45. (d)	46. (a)	47. (b)	48. (c)	49. (c)	50. (a)
51. (a)	52. (a)	53. (a)	54. (a)	55. (c)	56. (b)	57. (b)	58. (d)	59. (b)	60. (d)
61. (c)	62. (b)	63. (c)	64. (b)	65. (b)	66. (c)	67. (c)	68. (c)	69. (b)	70. (b)
71. (a)	72. (c)	73. (c)	74. (a)	75. (d)	76. (a)	77. (d)	78. (d)	79. (b)	80. (a)
81. (b)	82. (c)	83. (c)	84. (d)	85. (b)	86. (a)	87. (a)	88. (c)	89. (a)	90. (a)
91. (b)	92. (c)	93. (d)	94. (d)	95. (b)	96. (c)	97. (b)	98. (d)	99. (a)	100. (a)
101. (d)	102. (d)	103. (c)	104. (d)	105. (d)	106. (b)				

Hints and Solutions

1. Least prime number is 2, it is any even number which is prime.
2. Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
3. Prime numbers are 67, 71, 73, 79, 83.
4. (a) $161<(13)^2$

   If 161 is a prime number, then this is not divisible by any of the numbers 2, 3, 5, 7, 11. But, 161 is divisible by 7. Hence, 161 is not a prime number.

   (b) $171<(14)^2$

   For prime number 171, is not divisible by any of the numbers 2, 3, 5, 7, 11, 13. But, it is divisible by 3. Hence, 171 is not a prime number.

   (c) $173<(14)^2$

   For prime number 173 is not divisible by any of the numbers 2, 3, 5, 7, 11, 13. Hence, 173 is a prime number.

   (d) $221<(15)^2$

   For prime number 221, is not divisible by any of the numbers 2, 3, 5, 7, 11, 13. But it is divisible by 13. Hence, 221 is not a prime number.
5. All statements are true.
6. Circumference of circle $=2\pi R$ $\quad \left(\because R=\frac{1}{\pi}\right)$

   $$=2\pi\left(\frac{1}{\pi}\right)=2 \text{ and 2 is a rational number}$$
7. We know that the product of a rational number and an irrational number is an irrational number.
8. Let $x=\log_2\sqrt{2\sqrt{2\sqrt{2\ldots}}}$ upto infinity $\Rightarrow \sqrt{2\sqrt{2\sqrt{2\ldots}}}=2^x$

   $\Rightarrow \quad 2^x=\sqrt{2(2^x)}=\sqrt{2^{(x+1)}}=2^{\frac{(x+1)}{2}}$

   $\Rightarrow \quad x=\frac{(x+1)}{2} \Rightarrow 2x=x+1 \Rightarrow x=1$
9. As, $182=14\times 13=2\times 7\times 13$ all are prime
10. Any number is not a square, if the unit's place digit of number may be 2, 3, 7, 8.

    Hence, the number 9852 is not a square of any natural number.
11. $(a+b)$ always represent a natural number $\forall\, a,b\in N$.
12. $\sqrt{2}, 2+3\sqrt{5}$ both are irrational.
13. Each term is square of a number +3, *i.e.*, $7=2^2+3, 12=3^2+3$

    $19=4^2+3, 28=5^2+3, 6^2+3=36+3=39.$
14. If $x=0.1$, then $\sqrt{x}=\sqrt{0.1}=0316$. So, $\sqrt{x}>x$
15. As, $y=qx+r$, so $0\le r<x$.
16. Difference of sums of even and odd places digit of 1254934

    $=(1+5+9+4)-(2+4+3)=19-9=10$

    This number will be divisible by 11, after adding x, if $x=1$

    Also, the sum of digits of 1254934

    $=1+2+5+4+9+3+4=28$

    1254934 will be divisible by 3, after adding y, if $y=-1$
17. As, $|x|<3 \Rightarrow x<3$ or $-x<3 \Rightarrow x<3$ or $x>-3$; So, $-3<x<3$
18. Clearly, true *e.g.*, $\sqrt{4}=2$ (natural number)

    $\sqrt{2}=$irrational, where $2,4\in N$
19. When $x=3$, then the numbers are 3, 5, 7 and all are prime.
20. Given, $\quad -1\le x\le 2$

    $\Rightarrow \quad -4\le 4x\le 8$

    $\Rightarrow \quad 4\ge -4x\ge -8 \quad$ ...(i)

    and $\quad 1\le y\le 3$

    $\Rightarrow \quad 9\ge 3y\ge 3 \quad$ ...(ii)

    From Eqs. (i) and (ii),

    $13\ge 3y-4x\ge -5$

    $\therefore$ Minimum value of $(3y-4x)$ is -5.
21. Dividend $=D\times Q+R$

    Given, $\quad D=5Q$ and $D=2R$

    When $R=15, D=2\times 15=30 \therefore Q=\frac{D}{5}=\frac{30}{5}=6$

    $\therefore \quad$ Dividend $=30\times 6+15=195$
22. Other number $=\frac{\text{Product}}{\text{Given number}}=\frac{y/x}{x/y^2}=\frac{y}{x}\times\frac{y^2}{x}=\frac{y^3}{x^2}$
23. Decimal expansion of an irrational number is non-terminatin and non-repeating.
24. Given, $p=2\cdot 3\cdot 5\cdot\ldots 59\cdot 61=\ldots 0$

    Also, $\quad 2\le n\le 59$

    Now, we check the sequence $p+n$.

    Since, unit digit of p is zero. Therefore, for every even value o $n, (p+n)$ is always divisible.

    For odd value of $n=3,5,\ldots,59$

    Take $\quad n=3$

    $\therefore \quad p+n=p+3=(2\cdot 3\cdot 5\ldots 59\cdot 61+3)$

    $=3(2\cdot 5\ldots 59\cdot 61+1)$ which is divisible.

    Similarly, for even values of $n, p+n$ is divisible.

    Hence, it is clear that $p+n$ is always divisible by any numb So, there is no prime number exist in this sequence.
25. A whole number.
26. Infinitely many numbers of fractions.
27. As, 'a' is even and $b-1$ will be even, so $a(b-1)=$ even intege
28. All are true.
29. Then, it will be multiple of 5, hence not prime.
30. As, we know that a number $a_1a_2a_3a_4a_5$ is divisible by 9, if su of the digits, *i.e.*, $a_1+a_2+a_3+a_4+a_5$ is divisible by 9. Hen only statement I is true.
31. As p is prime, so P/a and P/b.
32. A number divisible by 88. It should be divisible by 8 and 11 In a given option number 9944 and 8888 is divisible by Hence, maximum number is 9944.

33. It satisfies divisibility rule of 3 and 11.

34. As, $4+2+\bullet+8=14+\bullet$; $42\bullet8$ is divisible by 9, if $14+\bullet$ divisible by 9. So, $14+\bullet=18$ nearest multiple of 9.

$\Rightarrow \quad \bullet=18-14=4$

35. Clearly true.

36. As, $\quad 2310=2\times3\times5\times7\times11$

37. As, we have $1728=2^6\times3^3$

$369=41\times3^2$; $441=7^2\times3^2$ and $567=7\times3^4$

38. Clearly, both statements satisfies divisibility rule of 11.

39. As, n is divided by 4 and say remainder is 3, if quotient is 'q' the

$n=4q+3 \Rightarrow 2n=8q+6$

if $\quad 2n=(8k+4)+2=4(2k+1)+2$

So, if $2n$ is divided by 4 the quotient is $2k+1$ and remainder is 2.

40. Taking $p=11$

$p+2=13$ (prime number)

I. $11\times13+1=144$ (a square number)

II. $11+13=24$ (12 is a divisor of 24)

Hence, both statements I and II are correct.

41. As, if a/b and b/c, then $b=ax$ and $c=by$ for $x,y\in N$

$\therefore \quad c=by=(ax)y=a(xy)$, so a/c

Also, a/b and a/c, then $b=ax, c=ay$

$\therefore \quad b+c=ax+ay=a(x+y)$, so $a/(b+c)$

as $\quad b-c=a(x-y) \Rightarrow a/(b-c)$

42. Here, $\dfrac{\frac{1}{4}-\frac{1}{6}-\frac{1}{48}}{\frac{1}{4}-\left(\frac{1}{6}-\frac{1}{48}\right)}+\dfrac{\frac{1}{4}\times\frac{1}{6}-\frac{1}{48}}{\frac{1}{4}\times\left(\frac{1}{6}-\frac{1}{48}\right)}=x$

$$\Rightarrow \quad x=\dfrac{\frac{12-8-1}{48}}{\frac{12-(8-1)}{48}}+\dfrac{\frac{1}{24}-\frac{1}{48}}{\frac{1}{4}\times\left(\frac{8-1}{48}\right)}$$

$$\Rightarrow \quad x=\dfrac{\frac{3}{48}}{\frac{5}{48}}+\dfrac{\frac{2-1}{48}}{\frac{7}{4\times48}}=\frac{3}{5}+\frac{4}{7}=\frac{21}{20}$$

43. $3^{2n+1}+2^{2n+1}=3^{2n}\cdot3+2^{2n}\cdot2$

If n is even, then unit's place digit in $3^{2n}\cdot3+2^{2n}\cdot2$ is 5

$\because$ Unit digit of $3^{2n}=3$ and unit digit of $2^{2n}=2$

Even, If n is odd, then unit's place digit in $3^{2n+1}+2^{2n+1}$ is 5.

$\therefore$ If n is a positive integer, then the unit's place digit in $3^{2n+1}+2^{2n+1}$ is 5.

44. $2^{12n}-6^{4n}=(2^{12})^n-(6^4)^n=(4096)^n-(1296)^n$

$=(4096-1296)[(4096)^{n-1}+(4096)^{n-2}(1296)+\ldots+(1296)^{n-1}]$

$=2800(k)$

Hence, last two digits are always be zero.

45. Here, $\frac{16}{7}\times\frac{16}{7}-\frac{\bullet}{7}\times\frac{9}{7}+\frac{9}{7}\times\frac{9}{7}=1$

Put $\quad \bullet=x$

$\Rightarrow \quad \frac{256}{49}-\frac{9x}{49}+\frac{81}{49}=1 \Rightarrow 256+81-9x=49$

$\Rightarrow \quad 9x=288 \Rightarrow x=32$

46. By BODMAS, $10\div\overline{4+6}\times4=10\div10\times4=1\times4=4$

47. $x\div4+2\times4=9 \Rightarrow \frac{x}{4}+8=9 \Rightarrow \frac{x}{4}=1$

$x=4$

48. $x+\dfrac{1}{2+\dfrac{4}{5\div5+2}}=0$

Then, $x+\dfrac{1}{2+\dfrac{4}{1+2}}=x+\dfrac{1}{2+\frac{4}{3}}=x+\frac{3}{10}=0 \Rightarrow x=-\frac{3}{10}$

49. I. The product of any three consecutive integers is divisible by 3! i.e., 6.

II. Here, $\quad 3k=\{\ldots-6,-3,0,3,6,\ldots\}$

$3k+1=\{\ldots-5,-2,1,4,7,\ldots\}$

and $\quad 3k+2=\{\ldots-4,-1,2,5,8,\ldots\}$

$\therefore \{3k,3k+1,3k+2\}=\{\ldots-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6\ldots\}$

Hence, it is true.

50. $\because T_{14}=6$

$\Rightarrow \quad a+13d=6 \quad$ ...(i)

and $\quad T_6=14$

$\Rightarrow \quad a+5d=14 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get $a=19, d=-1$

$\therefore \quad T_{95}=a+94d=19-94=-75$

51. $\frac{1}{2}\times\frac{2}{3}\times\ldots\times\frac{n-2}{n-1}\times\frac{n-1}{n}=\frac{1}{n}$

52. $\dfrac{2^5\times9^2}{8^2\times3^5}=\dfrac{2^5\times3^4}{2^6\times3^5}=\dfrac{1}{2\times3}=\dfrac{1}{6}$

53. Here, $25\frac{1}{2}=\frac{51}{2}$ or $25\frac{1}{2}=\frac{51\times8}{2\times8}=\frac{204}{8}$

So, answer is 204.

54. $\frac{a}{4}=\frac{b}{5}=\frac{c}{6}=k$ (say) $\Rightarrow a=4k, b=5k, c=6k$

So, $\quad \dfrac{a+b+c}{b}=\dfrac{4k+5k+6k}{5k}=\dfrac{15k}{5k}=3$

55. $\sqrt{bc}+\sqrt{b}+5=\sqrt{4\times9}+\sqrt{4}+5$

$=\sqrt{36}+\sqrt{4}+5=6+2+5=13$

56. The given number is divisible by 9, if sum of the digits is divisible by 3.

Here, sum of digits $=8+4+7+0+5+2+\otimes=26+\otimes$

If we $\otimes=1$, then $26+1=27$ is divisible by 9.

57. Here, $\sqrt{13+a}+\sqrt{112+b}+\sqrt{c-1}$

$=\sqrt{13+3}+\sqrt{112+9}+\sqrt{10-1}$

$=\sqrt{16}+\sqrt{121}+\sqrt{9}=4+11+3=18$

58. $[a-b+(b-a)]-[2a-2b+(b-2a)]$

$\Rightarrow \quad [a-b+b-a]-[2a-2b+b-2a]$

$\Rightarrow \quad 0-[2a-2a-b]=-0+b=b$

59. $10^k = \frac{0.0003325}{3.325} = \frac{3.325 \times 10^{-4}}{3.325} = 10^{-4}$ (given)

$\Rightarrow 10^k = 10^{-4}$ $\therefore$ $k = -4$

60. The last digit in the expansion of $(2457)^{754}$ is depend on $(7)^{754}$.

$(7)^{754} = (7^4)^{188} \times 7^2 = 1 \times 7^2 = (7)^2 = 49$

Hence, last digit is 9.

61. $0.\bar{6} + 0.\bar{8} + 0.\bar{7} = \frac{6}{9} + \frac{8}{9} + \frac{7}{9} = \frac{21}{9} = 2\frac{3}{9} = 2\frac{1}{3}$

62. $1.6x = 0.4y$ (given)

$\therefore$ $\frac{x}{y} = \frac{0.4}{1.6} = \frac{4}{16} = \frac{1}{4} = 0.25$

So, $\frac{x+y}{x-y} = \frac{x/y+1}{x/y-1} = \frac{0.25+1}{0.25-1} = -1.66$

63. $\sqrt{0.04 \times 0.4 \times x} = 0.4 \times 0.04 \times \sqrt{y}$ (given)

$\therefore$ $\frac{\sqrt{x}}{\sqrt{y}} = \frac{0.4 \times 0.04}{\sqrt{0.04 \times 0.4}}$ (squaring both sides)

$\frac{x}{y} = \frac{0.4 \times 0.4 \times 0.04 \times 0.04}{0.04 \times 0.4} = 0.016$

64. The required digit = unit digit in the product

$= 1 \times 5 \times 6 \times 8 = 0$.. Here, 0 is unit digit.

65. Since, unit digit in 7^4 is 1.

$\therefore 7^{168} = (7^4)^{42}$ give unit digit 1.

$\therefore 7^{170} = 7^{168} \times 7^2$ give unit digit in product $1 \times 7 \times 7 = 9$

66. Since, D is a point of BC. As, BC is rational, so BD must be rational but AD need not be rational.

67. Unit place in $7^4 = 1$, unit place in 7^{68} is 1..

$\therefore$ Unit place in $7^{68} \times 7^3 = 3$. Similarly, unit place in 6^{59} is 6 and unit place in 3^4 is 1 also in 3^{64} is 1.

$\therefore$ Unit place in $7^{71} \times 6^{59} \times 3^{65}$ is the unit place of $3 \times 6 \times 3 = 4$

68. Unit place in $7^4 = 1$, unit place in 7^{24} is 1.

$\therefore$ Unit place in $7^{24} \times 7^3 = 3$. Similarly, unit place in 3^4 is 1 also in 3^{12} is 1.

$\therefore$ Unit place in 3^{14} is 9. $\Rightarrow (7^{27} - 3^{14}) = 243 - 9 = 234$

$\therefore$ Unit place digit is 4.

69. $\because$ $1 \le y \le 3$

$\Rightarrow$ $3 \le 3y \le 9$ ...(i)

and $-1 \le x \le 3 \Rightarrow -4 \le 4x \le 12$

$\Rightarrow$ $-12 \le -4x \le 4$ ...(ii)

From Eqs. (i) and (ii), $-9 \le 3y - 4x \le 13$

Maximum value is 13.

70. Clearly, defined absolute value *i.e.*, $|x| = -x$.

71. $\frac{x-y}{x+y} = \frac{x/y-1}{x/y+1} = \frac{3/5-1}{3/5+1} = \frac{(3-5)/5}{(3+5)/5} = \frac{-2}{8} = -\frac{1}{4}$

72. $|2x-3| + |x-1| = |4-3| + |2-1| = 1+1 = 2$

73. We know that, if the difference of the sum of odd digits and sum of even digits is either 0 or multiple of 11, then the number is divisible by 11.

Given, number is $ABCDE$

Here, $A + C + E - (B + D) = 0$ or divisible by 11.

Hence, both statements are true.

74. As, $\left|\frac{7-x}{3}\right| < 2$ $(\because |x| < a \Rightarrow x < a \text{ or } -x < a)$

$\Rightarrow$ $\frac{7-x}{3} < 2$ or $-\left(\frac{7-x}{3}\right) < 2$

$\Rightarrow 7 - x < 6$ or $\frac{x-7}{3} < 2 \Rightarrow -x < -1$ or $x - 7 < 6$

$\Rightarrow$ $x > 1$ or $x < 13 \Rightarrow 1 < x < 13$

75. All other are incorrect, if last digit is zero, then 60 is not divisible by 25, if last digit is 5, then 35 is not divisible by 25, also third option is incorrect.

76. Let $n = 2$, even, so $n(n+1)(n+2) = 2 \times 3 \times 4 = 24$, which is divisible by 24.

77. Since, a ten-digit number is divisible by 4 as well as by 5, then this number must be divisible by 20.

We known that any number is divisible by 20, if last two digits is divisible by 20. If means unit place will be zero and ten's place may be 0, 2, 4, 6 or 8.

78. Let $x = a^2$ and $y = b^2$ for some $a, b \in N$, then $xy = a^2b^2 = (ab)^2$, where $ab \in N$.

So, $xy \in A$. But $\frac{x}{y} = \frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2 \notin A$ as $\frac{a}{b} \notin N$.

79. As, $a = c^z = (b^y)^z = [(a^x)^y]^z = a^{xyz}$ On comparing

$\Rightarrow$ $a = a^{xyz} \Rightarrow xyz = 1$

80. $\because$ Original number $= h \times 100 + t \times 10 + u$

Number obtained by reversing digits $= u \times 100 + t \times 10 + h$

$\therefore$ Required number $= (h \times 100 + t \times 10 + u)$

$- (u \times 100 + t \times 10 + h)$

$= 99(h - u)$

But the unit's place digit in above number is 4, therefore $(h-u)$ should be 6, then number is 594. Whose digits are 5, 9, 4 respectively.

81. Clearly true. (by definition)

82. An irrational number. (by property)

83. If $a = 3$, then $a^2 - 1 = 8$, so $a(a^2 - 1) = 24$, which is divisible by 24.

At $a = 5 \Rightarrow a(a^2 - 1) = 5 \cdot 24$,

At $a = 7 \Rightarrow a(a^2 - 1) = 7 \cdot 48$ ... etc.

Also, all other integer have a factor 24.

84. Clearly true. (by property)

85. Let the lowest sides of a right triangle be 3, 4, 5.

By Pythagoras theorem, $(3)^2 + (4)^2 = (5)^2$

Hence, one of its sides is always divisible by 5.

86. A positive number is real.

87. Let first number be x, second number $= 18 - x$, then

$x^2 + (18 - x)^2 = 164$ (by condition)

$\Rightarrow$ $x^2 + 324 + x^2 - 36x = 164$

$\Rightarrow 2x^2 - 36x + 160 = 0 \Rightarrow x^2 - 18x + 80 = 0 \Rightarrow (x-8)(x-10) = 0$

$\Rightarrow x = 8$ or $x = 10$, so smallest number is 8.

88. When a polynomial is divided by a linear polynomial, then the remainder is either constant or zero polynomial.

e.g.

$$(ax+b)\overline{)ax^2+bx+c(}\ x$$

$$\underline{_{-}ax^2\ {}_{+}bx}$$

$$c = \text{constant}$$

or

$$(ax+b)\overline{)ax^2+bx}\ x$$

$$\underline{_{-}ax^2\ {}_{\pm}bx}$$

$$0 = \text{zero}$$

89. $\because$ When $x^n + k$ is divided by $(x-1)$ the remainder is $1+k$, when $k<(x-1)$.

90. $\frac{(2.48)^2-(1.52)^2}{0.96} = \frac{(2.48-1.52)(2.48+1.52)}{0.96}$

$[\because a^2-b^2=(a-b)(a+b)] = \frac{0.96\times 4}{0.96} = 4$

91. $a \propto b^2, b \propto c^2, \Rightarrow a \propto (c^2)^2 = a \propto c^4$

92. As 2 is a rational number and $\sqrt{3}$ is irrational, then $2+\sqrt{3}$ is irrational.

93. Given, number is 222222.

Here, sum of digits $=2+2+2+2+2+2=12$, which is divisible by 3. So, given number is divisible by 3.

Now, sum of odd terms of digits $-$ Sum of even terms of digits $=6-6=0$, it is divisible by 11.

Since, in a number a digit repeated six times, then this number is divisible by 7, 11 and 13.

$\therefore$ The given number is divisible by 3, 7 and 11.

94. Since, $a<b \Rightarrow a-b<0$ Also, $c<0$

$\therefore \quad (a-b)c>0 \Rightarrow ac-bc>0 \Rightarrow ac>bc$

95. $\log x^2y^3 = a \Rightarrow \log x^2 + \log y^3 = a$

$\Rightarrow \quad 2\log x + 3\log y = a \quad$ ...(i)

as $\quad \log \frac{x}{y} = b$

$\log x - \log y = b \quad$ ...(ii)

Solving Eqs. (i) and (ii) as linear equation in $\log x$ and $\log y$, we get

$$\log x = \frac{a+3b}{5}, \log y = \frac{a-2b}{5}$$

$$\frac{\log x}{\log y} = \frac{\frac{a+3b}{5}}{\frac{a-2b}{5}} = \frac{a+3b}{a-2b}$$

96. As, number $235.235235235... = 235.\overline{235}$ being non-terminating but recurring is a rational number.

97. As, here when $a=11, b=2$, then

$$3\frac{7}{a}\times b\frac{3}{15} = 3\frac{7}{11}\times 2\frac{3}{15} = \frac{40}{11}\times\frac{33}{15} = 8$$

98. -2 and -1 $\quad (\because \log_{10} 0.02 = -1.69)$

99. Here, let $a = 0.\overline{27} = 0.272727...$ $\quad$ ...(i)

On multiply by 100

$100a = 27.272727 \quad$ ...(ii)

On subtracting Eqs. (i) and (ii), we get

$$99a = 27 \Rightarrow a = \frac{27}{99} = \frac{3}{11}$$

100. Let angles of a triangle in AP are

$a, a+d, a+2d$

Also, $\quad a+2d = 2a \quad$ (given condition)

$\Rightarrow \quad a = 2d \quad$ ...(i)

Also, $\quad a+a+d+a+2d = 180°$

$(\because$ sum of angles of triangle $= 180°)$

$\Rightarrow \quad 3a+3d = 180°$

$\Rightarrow \quad 3a + 3\left(\frac{a}{2}\right) = 180° \quad$ [from Eq. (i)]

$\Rightarrow \quad 9a = 360°$

$\Rightarrow \quad a = 40°, d = 20°$

$\therefore$ Ratio of angles $= 40° : 60° : 80° = 2:3:4$

101. Since, $247 < (16)^2$

If 247 is a prime number, then it is not divisible by any of the numbers 2, 3, 5, 7, 11, 13. But, 247 is divisible by 13. Hence, 247 is not a prime number.

If 203 is a prime number, then it is not divisible by any of the numbers 2, 3, 4, 5, 7, 11, 13. But 203 is divisible by 7. Hence, 203 is not a prime number.

102. As, $x<0<y \quad$ (given)

$\Rightarrow \quad x<0$ and $y>0$

$\therefore \quad \frac{1}{x}>0$ and $\frac{1}{y}<0$

$\therefore \quad \frac{1}{x}>\frac{1}{y}$

103. I. If $x=15$ and $y=14$, then $x+y=15+14=29$, which is a prime number. So, if x and y are composite, then $x+y$ is not always composite.

II. If $x=15$ and $y=14$, then $x-y=15-14=1$ which is neither prime nor composite, hence again $x-y$ is not always composite.

III. Third condition is satisfied for all measure.

Hence, it is correct.

104. Given, $r+s=1$

For maximum product, $r=s=\frac{1}{2} \therefore rs = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

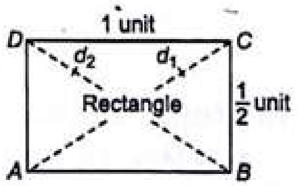
105. $d(d(d(12))) = d(d(6))$

[$\because$ positive integer divisor of $12 = 1, 2, 3, 4, 6, 12$]

$= d(4) \quad$ [$\because$ positive integer divisor of $6 = 1, 2, 3, 6$]

$= 3 \quad$ [$\because$ positive integer divisor of $4 = 1, 2, 4$]

106. Area $= x = 1\times\frac{1}{2} = \frac{1}{2} =$ Rational



$$d_1 = d_2 = \sqrt{1^2+(1/2)^2} = \frac{\sqrt{5}}{2}$$

$$\therefore y = d_1 + d_2 = \frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2}$$

$$= \frac{2\sqrt{5}}{2} = \sqrt{5} = \text{Irrational}$$