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CHAPTER - 1

Complex Numbers

Rational Number + Irrational Number = Real Number

Real No. + Imaginary No. = Complex Number

Symbol i and Its Powers:-

i = iota

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = (i^2) \cdot i$

$= (-1) \cdot i = -i$

$i^4 = (i^2)^2$

$= (-1)^2 = 1$

$i^5 = (i^2)^2 \cdot i = (-1)^2 \cdot i$

$= i$

$i^6 = (i^2)^3$

$= (-1)^3 = -1$

$i^7 = (i^2)^3 \cdot i$

$= (-1)^3 i = -i$

$i^8 = (i^2)^4$

$= (-1)^4 = 1$

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$i^9 = (i^2)^4 \cdot i$
$= (-1)^4 \cdot i = i$

$i^{51} = (i^2)^{25} \cdot i$
$= (-1)^{25} \cdot i = -i$

$i^{520} = (i^2)^{260}$
$= (-1)^{260} = 1$

$i^{1001} = (i^2)^{500} \cdot i$
$= (-1)^{500} \cdot i = i$

$i^{201} = (i^2)^{100} \cdot i$
$= (-1)^{100} \cdot i = i$

$i^{204} = (i^2)^{102}$
$= (-1)^{102} = 1$

☆ **Imaginary Number :-**

Number i and its real multiples are called imaginary numbers. Eg. - $2i, 4i, 6i, -3i$ these are all Imaginary Number.

☆ **Complex Number :-**

Any number of the form $a+ib$, where a and b are real numbers and $i=\sqrt{-1}$ is called a complex number.

Eg. - $2+3i$, $3-4i$, $3+10i$, $40+70i$, $6-2i$ these are all Complex Number.

Purely Real = $z = 2+0i$
Purely Imaginary = $z = 0+3i$

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Note:-

Every Real is a complex but every Complex may not be Real.

★ Conjugate of Complex Number:-

Let the Complex Number be:-

$z = a+ib$ then $a-ib$ conjugate of Complex Number.

$z = a+ib$
$\bar{z} = a-ib$

E.g.-

(i) let- $z = 2+3i$
then- $\bar{z} = 2-3i$

(ii) $z = 2+0i$
$\bar{z} = 2-0i = 2$

(iii) $z = 2+3i$
$\bar{z} = 2-3i$
$(\bar{\bar{z}}) = 2+3i$

Note:-

(i) If z conjugate $= z$ then z is purely Real.

(ii) If z Conjugate equal to $-z$ then z is purely Imaginary.

(iii) $(\bar{\bar{z}})$ equal to z.

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Modulus of a Complex Number:-

Let $Z = a+ib$ be a Complex Number then the Modulus of a Complex No. Z.

$$|Z| = \sqrt{a^2+b^2}$$

E.g.-

(i) let $z = 2+3i$

$|Z| = \sqrt{(2)^2+(3)^2}$

$|Z| = \sqrt{4+9}$

$|Z| = \sqrt{13}$

(ii) $Z = 2-3i$

$|Z| = \sqrt{(2)^2+(-3)^2}$

$|Z| = \sqrt{4+9}$

$|Z| = \sqrt{13}$

E.g. $Z_1 = Z_2$ when,

$Z_1 = 2+3i$

$Z_2 = 2+3i$

$Z_1 = Z_2$, $x_1 = x_2$, $y_1 = y_2$

E.g.:

(i) $Z_1 = 2+3i$

$Z_2 = 2-3i$

$Z_1 \neq Z_2$

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Note :-

1. Different Complex No. May have Same Modulus.

2. Different Complex Can never have same Conjugate.

Let $z = a + ib$

$|z| = \sqrt{a^2 + b^2}$

$\bar{z} = a - ib$

$z \cdot \bar{z} = (a + ib)(a - ib)$

$z \cdot \bar{z} = a^2 - (ib)^2$

$z \cdot \bar{z} = a^2 - (-1b)^2 \quad \{\because i^2 = -1\}$

$= a^2 + b^2$

$z \cdot \bar{z} = (\sqrt{a^2 + b^2})^2$

$\boxed{z \cdot \bar{z} = |z|^2}$

Example 1.

Simplify $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

Sol. Consider $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

$= 2i^2 + 6(i)^2 \cdot i + 3(i^2)^8 - 6(i^2)^9 \cdot i + 4(i^2)^{12} \cdot i$

$= 2(-1) + 6(-1)^2 \cdot i + 3(-1)^8 - 6(-1)^9 \cdot i + 4(-1)^{12} \cdot i$

$= -2 + 6i + 3 - 6i + 4i$

$= 1+4i$

1) Real $\left(\frac{Z_1 Z_2}{\bar{Z}_1}\right)$ 2) Imaginary $\left(\frac{1}{Z_1 \bar{Z}_1}\right)$

Q. Let $z_1 = 2-i$, $z_2 = -2+i$ Find the Real Part

Sol. The given complex No.

$Z_1 = 2-i$ $Z_2 = -2+i$

$\bar{Z} = 2+i$

$$\frac{Z_1 Z_2}{\bar{Z}_1} = \frac{(2-i)(-2+i)}{2+i}$$

$$= \frac{-4+2i+2i-i^2}{2+i}$$

$$= \frac{-4+4i+1}{2+i} \quad \left\{\because i = -1\right\}$$

$$= \frac{-3+4i}{2+i}$$

$$= \frac{-3+4i}{2+i} \times \frac{2-i}{2-i}$$

$$= \frac{-6+3i+8i-4i^2}{2^2 - i^2}$$

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$$= \frac{-6 + 11i - 4(-1)^2}{4 - (-1)^2}$$

$$= \frac{-6 + 11i + 4}{4 + 1} = \frac{-2 + 11i}{5}$$

$$\frac{Z_1 Z_2}{\bar{Z}_1} = \left(\frac{-2}{5}\right) + \left(\frac{11}{5}\right) i$$

$$\text{Real}\left[\frac{Z_1 Z_2}{\bar{Z}_1}\right] = \frac{-2}{5}$$

Q. $Z_1 = 2 - i$, $Z_2 = -2 + i$ Find the Imaginary part.

Solu The given complex No.

$Z_1 = 2 - i$ $\qquad Z_2 = -2 + i$

$\bar{Z} = 2 + i$

$$\frac{1}{Z_1 \bar{Z}_1} = \frac{1}{(2-i)(2+i)}$$

$$= \frac{1}{(2)^2 - (i)^2} = \frac{1}{(2)^2 + 1}$$

$$= \frac{1}{4+1} = \frac{1}{5} + 0i$$

$$\text{Imaginary part}\left(\frac{1}{Z_1 \bar{Z}_1}\right) = 0$$

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Example :- 8

★ Find the multiplicative inverse of $2-3i$.

Sol. $z = 2-3i$

Multiplicative Inverse $z = \frac{1}{z}$

$$= \frac{1}{2-3i} \times \frac{2+3i}{2+3i}$$

$$= \frac{2+3i}{(2)^2-(3i)^2} = \frac{2+3i}{4-9i^2} \qquad \{i^2=-1\}$$

$$= \frac{2+3i}{4+9} = \frac{2+3i}{13}$$

$$= \left(\frac{2}{13}\right) + \left(\frac{3}{13}\right)i$$

★ Find the multiplicative inverse of

(1) $4-3i$

Sol. $z = 4-3i$

Multiplicative Inverse $z = \frac{1}{z}$

$$= \frac{1}{4-3i} \times \frac{4+3i}{4+3i}$$

$$= \frac{4+3i}{(4)^2-(3i)^2} = \frac{4+3i}{16-9i^2}$$

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$$= \frac{4+3i}{16-9(-1)} = \frac{4+3i}{16+9}$$

$$= \frac{4+3i}{25}$$

$$= \left(\frac{4}{25}\right)+\left(\frac{3}{25}\right)i$$

(2) $\sqrt{5}+3i$

Sol. $z = \sqrt{5}+3i$

Multiplicative Inverse $z = \frac{1}{z}$

$$= \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$$

$$= \frac{\sqrt{5}-3i}{(\sqrt{5})^2-(3i)^2} = \frac{\sqrt{5}-3i}{5-9(-1)}$$

$$= \frac{\sqrt{5}-3i}{5+9} = \frac{\sqrt{5}-3i}{14}$$

$$= \left(\frac{\sqrt{5}}{14}\right)+\left(\frac{-3}{14}\right)i$$

(3) $5-3i$

Sol. $z = 5-3i$

Multiplicative Inverse $z = \frac{1}{z}$

$$= \frac{1}{5-3i} \times \frac{5+3i}{5+3i}$$

$$= \frac{5+3i}{(5)^2 - 3(i)^2} = \frac{5+3i}{25+9}$$

$$= \frac{5+3i}{34}$$

$$= \left(\frac{5}{34}\right) + \left(\frac{3}{34}\right)i$$

(iv) $-i$

Sol. $z = -i$

Multiplicative Inverse $z = \frac{1}{z}$

$$= \frac{1}{-i} \times \frac{i}{i}$$

$$= \frac{i}{(-i)^2} = \frac{i}{1}$$

$$= 0 + i1$$

Q Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

Sol. $z = \frac{1+i}{1-i} - \frac{1-i}{1+i}$

$$= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)}$$

$$= \frac{((1)^2 + (i)^2 + 2i) - ((1)^2 + (i)^2 - 2i)}{(1)^2 - (i)^2}$$

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$$= \frac{(1-1+2i)-(1-1-2i)}{1+1}$$

$$= \frac{(0+2i)-(0-2i)}{2}$$

$$= \frac{2i+2i}{2} = \frac{4i}{2}$$

$Z = 2i$

$Z = 0+2i$

$|Z| = \sqrt{(0)^2+(2)^2}$

$|Z| = \sqrt{4}$

$|Z| = 2$

E.g. 16

Q. Find the smallest positive integer m for which $\left(\frac{1+i}{1-i}\right)^m = 1$

Sol. $\left(\frac{1+i}{1-i}\right)^m = 1$

$$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(1+i)^2}{1^2-i^2} = \frac{(1)^2+(i)^2+2i}{1+1}$$

$$= \frac{1-1+2i}{2} = \frac{0+2i}{2}$$

$$= \frac{2i}{2}$$

$Z = i$

$Z' = i' = i \neq 1$

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$Z^2 = i^2 = -1 \neq 1$
$Z^3 = i^3 = (i^2) \cdot i = -i \neq 1$
$Z^4 = i^4 = (i^2)^2 = 1$

Least value of $m = 4$

Q.19 If $(x+iy)^3 = u+iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 -$

Sol. $(x+iy)^3 = u+iv$

$= x^3 + 3(x)^2(iy) + 3x(iy)^2 + (iy)^3 = u+iv$

$= x^3 + 3x^2yi + 3x(-1)y^2 + (-i)y^3 = u+iv$

$= x^3 + 3x^2yi - 3xy^2 - y^3i = u+iv$

$= (x^3 - 3xy^2)\ (3x^2yi - y^3 i) = u+iv$

Equating Real and Imaginary part -

$x^3 - 3xy^2 = u$ — (1)
$3x^2yi - y^3i = iv$ — (2)

L.H.S.

$= \frac{u}{x} + \frac{v}{y}$

$= \frac{x^3 - 3xy^2}{x} + \frac{3x^2y\cancel{i} - y^3\cancel{i}}{y}$

$= \frac{\cancel{x}(x^2 - 3y^2)}{\cancel{x}} + \frac{\cancel{y}(3x^2\cancel{i} - y^2\cancel{i})}{\cancel{y}}$

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$= x^2 - 3y^2 + 3x^2 - y^2$

$= 4x^2 - 4y^2$

$= 4(x^2 - y^2)$

Q24 Find the square root of -

(i) $7+24i$

Sol. Let $\sqrt{7+24i} = x + iy$ — ①

Squaring both side.

$7+24i = (x+iy)^2$

$7+24i = x^2 + (iy)^2 + 2xyi$

$7+24i = (x^2-y^2) + 2xyi$

Equating Real and Imaginary part.

$x^2 - y^2 = 7$ — ②

$2xy = 24$ — ③

we know that.

$(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$

$(x^2+y^2)^2 = (7)^2 + (24)^2$

$(x^2+y^2)^2 = 49 + 576$

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$$(x^2+y^2)^2 = 625$$

$$x^2+y^2 = 25 \quad \text{——} \quad (4)$$

Adding (2) + (4)

$$2x^2 = 32$$
$$x^2 = 16$$
$$x = \pm 4$$

when $x = 4$

$$2(4)(y) = 24$$
$$8y = 24$$
$$y = 3$$

when $x = -4$

$$2(-4)(y) = 24$$
$$-8y = 24$$
$$y = -3$$

Square root $\sqrt{7+24i} = \pm(4+3i)$

(ii) $7-24i$

Sol. Let $\sqrt{7-24i} = x+iy$ —— (1)

Squaring both side -

$$7-24i = (x+iy)^2$$

$$7-24i = x^2+(iy)^2+2xyi$$

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$7 - 24i = (x^2 - y^2) + 2xyi$

Equaring Real and Imaginary part.

$x^2 - y^2 = 7$ — (2)
$2xy = -24$ — (3)

We know that

$(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$

$(x^2+y^2)^2 = (7)^2 + (-24)^2$

$(x^2+y^2)^2 = 49 + 576$

$(x^2+y^2)^2 = 625$

$x^2 + y^2 = 25$ — (4)

Adding (2) + (4),

$x^2 - y^2 = 7$
$x^2 + y^2 = 25$

$2x^2 = 32$
$x^2 = 16$
$x = \pm 4$

when $x = 4$
$2(4)(y) = -24$
$8y = -24$
$y = -3$

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when $x = -4$

$2(-4)(y) = -24$

$-8y = -24$

$y = 3$

Square root of $\sqrt{7-24i} = \pm(4-3i)$

(iii) $5+12i$

Sol. Let $\sqrt{5+12i} = x+iy$ — ①

Squaring both side

$5+12i = (x+iy)^2$

$5+12i = x^2+(iy)^2+2xyi$

$5+12i = (x^2-y^2)+2xyi$

Equaring Real and Imaginary part.

$x^2-y^2 = 5$ — (2)

$2xy = 12$ — (3)

we know that

$(x^2+y^2)^2 = (x^2-y^2)^2+(2xy)^2$

$(x^2+y^2)^2 = (5)^2+(12)^2$

$(x^2+y^2)^2 = 25+144$

$(x^2+y^2)^2 = 169$

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$x^2+y^2 = 13$ — (4)

Adding (2) + (4)

$$x^2-y^2 = 5$$
$$x^2+y^2 = 13$$
$$2x^2 = 18$$

$x^2 = 9$

$x = \pm 3$

when $x = 3$

$2(3)\, y = 12$

$6y = 12$

$y = 2$

when $x = -3$

$2(-3)\,y = 12$

$-6y = 12$

$y = -2$

Square root of $\sqrt{5+12i} = \pm(3+2i)$

iv) $5-12i$

sol. Let $\sqrt{5-12i} = x+iy$ — ①

squaring both side

$5-12i = (x+iy)^2$

$5-12i = x^2+(iy)^2+2xyi$

$5-12i = (x^2-y^2)+2xyi$

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Equating Real and Imaginary part

$(x^2-y^2) = 5$ —— (2)

$2xy = -12$ —— (3)

we know that

$(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$

$(x^2+y^2)^2 = (5)^2 + (-12)^2$

$(x^2+y^2)^2 = 25 + 144$

$(x^2+y^2)^2 = 169$

$x^2+y^2 = 13$ —— (4)

Adding (2) + (4)

$x^2-y^2 = 5$

$x^2+y^2 = 13$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$

when $x = 3$

$2(3)(y) = -12$

$6y = -12$

$y = -2$

when $x = -3$

$2(-3)(y) = -12$

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$-6y = -12$

$y = 2$

Square root of $\sqrt{5-12i} = \pm(3-2i)$

(v) $16+30i$

Sol. Let $\sqrt{16+30i} = x+iy$ — (1)

Squaring both side

$16+30i = (x+iy)^2$

$16+30i = x^2+(iy)^2+2xyi$

$16+30i = (x^2-y^2)+2xyi$

Equaring Real and Im. part.

$(x^2-y^2) = 16$ — (2)

$2xy = 30$ — (3)

We know that

$(x^2+y^2)^2 = (x^2-y^2)+(2xy)^2$

$(x^2+y^2)^2 = (16)^2+(30)^2$

$(x^2+y^2)^2 = 256+900$

$(x^2+y^2)^2 = 1156$

$x^2+y^2 = 34$ — (4)

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Add (2) + (4)

$x^2 - y^2 = 16$

$x^2 + y^2 = 34$

$2x^2 = 50$

$x^2 = 25$

$x = \pm 5$

when $x = 5$

$2(5)y = 30$

$10y = 30$

$y = 3$

when $x = -5$

$2(-5)y = 30$

$-10y = 30$

$y = -3$

Square root of $\sqrt{16+30i} = \pm(5+3i)$

Q. 10 If z_1 and z_2 are respectively $1-i$, $-2+4i$ show that $\operatorname{Im}\left[\frac{z_1 z_2}{\bar{z}_1}\right] = 2$

Sol. $z_1 - 1-i$

$z_2 = -2+4i$

$\bar{z}_1 = 1+i$

$$\left(\frac{z_1 z_2}{\bar{z}_1}\right) = \frac{(1-i)(-2+4i)}{1+i}$$

$$= \frac{-2+4i+2i-4i^2}{1+i}$$

$$= \frac{-2+6i-4(-1)}{1+i}$$

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$= \dfrac{-2+6i+4}{1+i}$

$= \dfrac{2+6i}{1+i} \times \dfrac{1-i}{1-i}$

$= \dfrac{2-2i+6i-6i^2}{1^2-i^2}$

$= \dfrac{2+4i+6}{1+1} = \dfrac{8+4i}{2}$

$= \dfrac{8}{2} + \dfrac{4i}{2}$

$= 4+2i$

$Im\left[\dfrac{Z_1 Z_2}{\bar{Z}_1}\right] = 2$

Evaluate $\left[i^{18} + \left(\dfrac{1}{i}\right)^{25}\right]^3$

$\left[i^{18} + \left(\dfrac{1}{i}\right)^{25}\right]^3$

$= \left[(i^2)^9 + \left(\dfrac{1 \times i}{i \times i}\right)^{25}\right]^3$

$= \left[(-1)^9 + \left(\dfrac{i}{i^2}\right)^{25}\right]^3$

$= \left[-1 + \left(\dfrac{i}{-1}\right)^{25}\right]^3$

$= \left(-1 - i^{25}\right)^3$

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$= \left[-1-(i^2)^{12} \cdot i\right]^3$

$= \left[-1-(-1)^{12}\, i\right]^3$

$= \left[-1-i\right]^3$

$= -\left[1+i\right]^3$

$= -\left[(1)^3 + 3(1)^2 i + 3(1)(i^2) + (i)^3\right]$

$= -\left[1 + 3i - 3 - i\right]$

$= -\left[-2+2i\right]$

$= (2-2i)$

(ii) $i^{49} + i^{19}$

Sol. $= i^9 + i^{19}$

$= (i^2)^4 \cdot i + (i^2)^9 \cdot i$

$= (-1)^4 \cdot i + (-1)^9 \cdot i$

$= i - i = 0$

(iii) $\left[-2-\frac{1}{3}i\right]^3$

Sol. $\left[-2-\frac{1}{3}i\right]^3$

$= -\left[2+\frac{1}{3}i\right]^3$

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$$= -\left[(2)^3 + 3(2)^2\frac{1}{3}i + 3(2)\left(\frac{1}{3}i\right)^2 + \left(\frac{1}{3}i\right)^3\right]$$

$$= -\left[8 + 3\times4\times\frac{1}{3}i + 6\times\left(\frac{1}{3}\right)^2 i^2 + \left(\frac{1}{3}\right)^3 i^3\right]$$

$$= -\left[8 + 4i + 6\times\frac{1}{9}(-1) + \frac{1}{27}\times(-i)\right]$$

$$= -\left[8 + 4i - \frac{6}{9} - \frac{1}{27}i\right]$$

$$= -\left[\frac{66}{9} - \frac{107}{27}i\right]$$

$$= \left(-\frac{22}{3} - \frac{107}{27}i\right)$$

Q Find the square root of :

(i) i

Sol. Let $\sqrt{i} = x + iy$ — (1)

Squaring both side

$$i = (x+iy)^2$$

$$i = x^2 + i^2y^2 + 2xyi$$

$$i = (x^2 - y^2) + 2xyi$$

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Equating Real and Imaginary part -

$(x^2 - y^2) = 0$ —— (2)

$2xy = 1$ —— (3)

We know that -

$(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$

$(x^2+y^2)^2 = (0)^2 + (1)^2$

$(x^2+y^2)^2 = 0 + 1$

$(x^2+y^2)^2 = 1$

$x^2+y^2 = \pm 1$ —— (4)

Adding (2) + (4)

$x^2 - y^2 = 0$
$x^2 + y^2 = 1$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{\sqrt{2}}$

when $x = \frac{1}{\sqrt{2}}$

$= 2\left(\frac{1}{\sqrt{2}}\right) y = 1$

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$= \sqrt{2} \times \sqrt{2}\left(\frac{1}{\sqrt{2}}\right) y = 1$

$\Rightarrow \sqrt{2} y = 1$

$y = \frac{1}{\sqrt{2}}$

when $x = -\frac{1}{\sqrt{2}}$

$2\left(\frac{-1}{\sqrt{2}}\right) y = 1$

$\sqrt{2} \times \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right) y = 1$

$-\sqrt{2} y = 1$

$y = \frac{-1}{\sqrt{2}}$

Square Root of $\sqrt{i} = \pm\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)$

(ii) $-i$

Sol. Let $\sqrt{-i} = x + iy$ — (1)

Squaring both side

$-i = (x+iy)^2$

$-i = x^2 + i^2y^2 + 2xyi$

$-i = (x^2 - y^2) + 2xyi$

Equating Real and Imaginary Part

$(x^2-y^2) = 0$ —— (2)

$2xy = -1$ —— (3)

we know that

$(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$

$(x^2+y^2)^2 = (0)^2 + (-1)^2$

$(x^2+y^2)^2 = 0+1$

$(x^2+y^2)^2 = 1$

$x^2+y^2 = 1$ —— (4)

Adding (2) + (4)

$2x^2 = +1$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$

when $x = \frac{1}{\sqrt{2}}$

$2\left(\frac{1}{\sqrt{2}}\right) y = -1$

$\sqrt{2}\times\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) y = -1$

$\sqrt{2}\, y = -1$

$y = \frac{-1}{\sqrt{2}}$

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when $x = \frac{-1}{\sqrt{2}}$

$2\left(\frac{-1}{\sqrt{2}}\right) y = -1$

$\sqrt{2} \times \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right) = -1$

$-\sqrt{2}y = -1$

$y = \frac{-1}{-\sqrt{2}} = \frac{1}{\sqrt{2}}$

Square Root of $\sqrt{-i} = \pm\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$

(iii) $1+i$

Sol. Let $\sqrt{1+i} = x+iy$ — (1)

Squaring both side

$1+i = (x+iy)^2$

$1+i = x^2 + i^2y^2 + 2xyi$

$1+i = (x^2-y^2) + 2xyi$

Equating Real and Imaginary Part

$(x^2-y^2) = 1$ —— (2)

$2xy = 1$ —— (3)

We know that

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$$(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$$

$$(x^2+y^2)^2 = (1)^2 + (1)^2$$

$$(x^2+y^2)^2 = 1+1$$

$$(x^2+y^2)^2 = 2$$

$$x^2+y^2 = \sqrt{2} \quad \text{---} \ (4)$$

Adding $(2) + (4)$

$$x^2 - y^2 = 1$$
$$x^2 + y^2 = \sqrt{2}$$

$$2x^2 = 1+\sqrt{2}$$

$$x^2 = \frac{1+\sqrt{2}}{2}$$

$$x = \pm\sqrt{\frac{1+\sqrt{2}}{2}}$$

when $x = \sqrt{\frac{1+\sqrt{2}}{2}}$,

$$2\left[\sqrt{\frac{1+\sqrt{2}}{2}}\right] y = 1$$

$$y = \frac{1}{2\left[\sqrt{\frac{1+\sqrt{2}}{2}}\right]}$$

$$y = \frac{\sqrt{2}}{2(\sqrt{1+\sqrt{2}})} \times \frac{\sqrt{1-\sqrt{2}}}{\sqrt{1-\sqrt{2}}}$$

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$$y = \frac{\sqrt{2} \times \sqrt{\sqrt{1}-\sqrt{1}}}{\sqrt{2} \times \sqrt{2} \times \sqrt{2-1}}$$

$$y = \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2} \times 1}$$

$$y = \sqrt{\frac{\sqrt{2}-1}{2}}$$

When $x = -\sqrt{\frac{1+\sqrt{2}}{2}}$

$$2\left(-\sqrt{\frac{1+\sqrt{2}}{2}}\right) y = 1$$

$$y = \frac{1}{2\left(-\sqrt{\frac{1+\sqrt{2}}{2}}\right)}$$

$$y = \frac{\sqrt{2}}{2\left(-\sqrt{\sqrt{2}+1}\right)}$$

$$y = \frac{\sqrt{2}}{-2\left(\sqrt{\sqrt{2}+1}\right)} \times \frac{\sqrt{\sqrt{2}-1}}{\sqrt{\sqrt{2}-1}}$$

$$y = \frac{1}{-2} \times \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2-1}}$$

$$y = \frac{\sqrt{\sqrt{2}-1}}{-\sqrt{2} \times 1} = -\sqrt{\frac{\sqrt{2}-1}{2}}$$

Square Root of $\sqrt{1+i} = \pm\left(\sqrt{\frac{1+\sqrt{2}}{2}} + \sqrt{\frac{\sqrt{2}-1}{2}}\, i\right)$

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(iv) $1-i$

Sol. Let $\sqrt{1-i} = x+iy$ — (1)

Squaring both side

$1-i = x^2+i^2y^2+2xyi$

$1-i = (x^2-y)^2+2xyi$

Equating Real and Imaginary part -

$x^2-y^2 = 1$ — (2)

$2xy = -1$ — (3)

we know that

$(x^2+y^2)^2 = (x^2-y^2)^2+(2xy)^2$

$(x^2+y^2)^2 = (1)^2+(-1)^2$

$(x^2+y^2)^2 = 1+1$

$(x^2+y^2)^2 = 2$

$x^2+y^2 = \sqrt{2}$ — (4)

Adding ~~Real~~ 2 and 4

$2x^2 = 1+\sqrt{2}$

$x^2 = \frac{1+\sqrt{2}}{2}$

$x = \pm\sqrt{\frac{1+\sqrt{2}}{2}}$

when $x = \sqrt{\frac{1+\sqrt{2}}{2}}$

$2\left(\sqrt{\frac{1+\sqrt{2}}{2}}\right) y = -1$

$y = \frac{-1}{2\left(\sqrt{\frac{\sqrt{2}+1}{2}}\right)}$

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$$y = \frac{-\sqrt{2}}{2(\sqrt{\sqrt{2}+1})} \times \frac{\sqrt{\sqrt{2}-1}}{\sqrt{\sqrt{2}-1}}$$

$$y = \frac{-\sqrt{2} \times \sqrt{\sqrt{2}-1}}{\sqrt{2} \times \sqrt{2} \times \sqrt{2-1}}$$

$$y = -\frac{\sqrt{\sqrt{2}-1}}{\sqrt{2} \times \sqrt{1}}$$

$$\cancel{y = -\sqrt{\frac{\sqrt{2}-1}{2}}}$$

when $x = -\sqrt{\frac{1+\sqrt{2}}{2}}$

$$2\left(-\sqrt{\frac{1+\sqrt{2}}{2}}\right) y = -1$$

$$\cancel{y = \frac{-1}{2\left(-\sqrt{\frac{1+\sqrt{2}}{2}}\right)}}$$

$$y = \frac{-\sqrt{2}}{-2\left(\sqrt{\frac{\sqrt{2}+1}{1}}\right)}$$

$$\cancel{y = \frac{-\sqrt{2}}{-2(\sqrt{\sqrt{2}+1})} \times \frac{\sqrt{\sqrt{2}-1}}{\sqrt{\sqrt{2}-1}}}$$

$$\cancel{y = \frac{+\sqrt{2}}{+\sqrt{2} \times \sqrt{2}(\sqrt{\sqrt{2}+1})} \times \frac{\sqrt{\sqrt{2}-1}}{\sqrt{\sqrt{2}-1}}}$$

$$\cancel{y = \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2} \times \sqrt{2-1}} = \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2} \times \sqrt{1}}}$$

$$y = \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2}} = \sqrt{\frac{\sqrt{2}-1}{2}}$$

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Square Root of $\sqrt{1-i} = \pm\left(\sqrt{\frac{1+\sqrt{2}}{2}} - \sqrt{\frac{\sqrt{2}-1}{2}}\, i\right)$

ucp $a+ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2 = \frac{[x^2+1]^2}{(2x^2+1)^2}$

Sol. We are given -

$$a+ib = \frac{(x+i)^2}{2x^2+1}$$

$$a+ib = \frac{x^2+i+2xi}{2x^2+1}$$

$$a+ib = \frac{(x^2-1)+2xi}{(2x^2+1)}$$

$$a+ib = \frac{(x^2-1)}{(2x^2+1)} + \frac{2x}{(2x^2+1)}i$$

Equating Real and Imaginary part -

$$a = \left(\frac{x^2-1}{2x^2+1}\right), \quad b = \frac{2x}{(2x^2+1)}$$

$$L.H.S. = a^2+b^2$$

$$= \frac{(x^2+(-1))^2}{(2x^2+(-1))^2} + \frac{(2x)^2}{(2x^2+1)^2}$$

$$= \frac{(x^2+(-1))^2}{(2x^2+1)^2} + \left(\frac{2x}{2x^2+1}\right)^2$$

$$= \frac{x^4+(+1)+(-2x^2)}{(2x^2+1)^2} + \frac{4x^2}{(2x^2+1)^2}$$

$$= \frac{x^4+1+(-2x^2)+4x^2}{(2x^2+1)^2}$$

$$= \frac{x^4+2x^2+1}{(2x^2+1)^2}$$

$$= \frac{(x^2+1)^2}{(2x^2+1)^2}$$

R.H.S.

Q Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

Sol. Let $Z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$

$$Z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$$Z = \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)}$$

$$Z = \frac{3+6i\sin\theta+2i\sin\theta+4i^2\sin^2\theta}{1-4i^2\sin\theta}$$

$$Z = \frac{3+8i\sin\theta+4i^2\sin^2\theta}{1-4i^2\sin\theta}$$

$$Z = \frac{3+8i\sin\theta-4\sin^2\theta}{1-4i^2\sin\theta}$$

$$Z = \frac{3-4\sin^2\theta+8i\sin\theta}{1+4\sin\theta}$$

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$$Z = \left(\frac{3-4\sin^2\theta}{1+4\sin^2\theta}\right) + \left(\frac{8\sin\theta}{1+4\sin^2\theta}\right)i$$

Z is purely Real when

$$\frac{8\sin\theta}{1+4\sin^2\theta} = 0 \Rightarrow \sin\theta = 0$$

$$\theta = n\pi, \ n \in Z$$

$$= 8\sin\theta$$

Q. If $x+iy = \frac{a+ib}{a-ib}$ prove that $x^2+y^2=1$

Sol. we are given

$$x+iy = \frac{a+ib}{a-ib}$$

$$|x+iy| = \left|\frac{a+ib}{a-ib}\right|$$

$$|x+iy| = \frac{|a+ib|}{|a-ib|}$$

$$\sqrt{x^2+y^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+(-b^2)}}$$

$$\sqrt{x^2+y^2} = 1$$

$$x^2+y^2 = 1$$

Q Find the value of $2x^4+5x^3+7x^2-x+41$ when $x = -2-\sqrt{3}i$.

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Sol We are given $x = -2-\sqrt{3}i$

$x+2 = -\sqrt{3}i$

we g Squaring both side

$(x+2)^2 = (-\sqrt{3}i)^2$

$x^2+(2)^2+2x(2) = 3i^2$

$x^2+4+4x = -3$

$x^2+4x+4+3 = 0$

$x^2+4x+7 = 0$ —— (1)

$$\begin{array}{r} 2x^2-3x+5 \\ x^2+4x+7 \overline{\big) 2x^4+5x^3+7x^2-x+41} \\ \underline{2x^4+8x^3+14x^2} \\ -\quad-\quad- \\ -3x^3-7x^2-x+41 \\ \underline{-3x^3-12x^2-21x} \\ +\quad+\quad+ \\ 5x^2+20x+41 \\ \underline{5x^2+20x+35} \\ -\quad-\quad- \\ 6 \end{array}$$

$2x^4+5x^3+7x^2-x+41 = (x^2+4x+7)(2x^2-3x+5)+6$

$= 0+6$

$= 6$

Q Find the value of $x^4-3x^3+3x^2+99x-95$ when x is $3-4i$.

Sol. We are given $x = 3-4i$

$x-3 = -4i$

Squaring both side

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$(x-3)^2 = (-4i)^2$

$x^2 + (3)^2 - 2x(3) = 16i^2$

$x^2 + 9 - 6x = -16$

$x^2 - 6x + 9 + 16 = 0$

$x^2 - 6x + 25 = 0$ — ①

$$
\begin{array}{r}
x^2 + 3x - 4 \\
x^2 - 6x + 25 \,\overline{\big)\, x^4 - 3x^3 + 3x^2 + 99x - 95} \\
x^4 - 6x^3 + 25x^2 \\
- \quad + \quad - \\
\hline
3x^3 - 22x^2 + 99x - 95 \\
3x^3 - 18x^2 + 75x \\
- \quad + \quad - \\
\hline
-4x^2 + 24x - 95 \\
-4x^2 + 24x - 100 \\
+ \quad - \quad + \\
\hline
5
\end{array}
$$

$(x^4 - 3x^3 + 3x^2 + 99x - 95) = (x^2 - 6x + 25)(x^2 + 3x - 4) + 5$

$= 0(x^2 + 3x - 4) + 5$

$= 0 + 5$

$= 5$

Q Find the value of $x^4 + 9x^3 + 35x^2 - x + 4$ wh $x = -5 + 2\sqrt{-4}$.

Sol. We are given -

$x = -5 + 2\sqrt{-4}$

$x = -5 + 2\sqrt{(4)(-1)}$

$x = -5 + 2\sqrt{4i^2}$

$x = -5 + 2 \times 2i$

$x = -5 + 4i$

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$x+5 = 4i$

Squaring both side.

$(x+5)^2 = (4i)^2$

$x^2+(5)^2+2x(5) = 16i^2$

$x^2+25+10x = 16i^2$

$x^2+10x+25 = -16$

$x^2+10x+25+16 = 0$

$x^2+10x+41 = 0 \quad —— (1)$

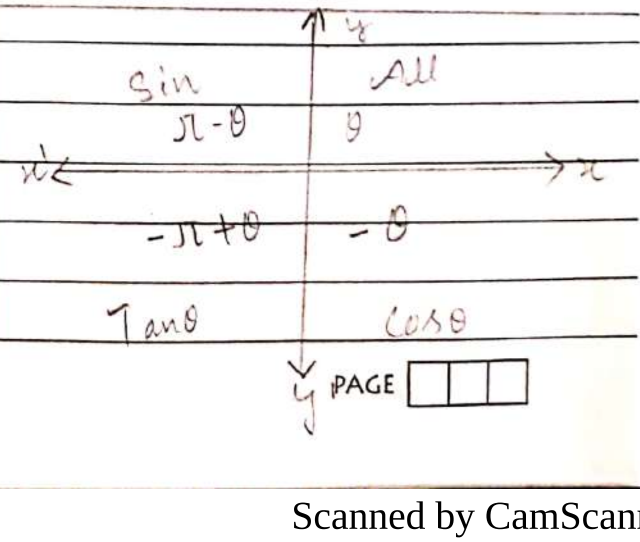
$$
\begin{array}{r}
x^2 - x + 4 \\
x^2+10x+41 \overline{\big) x^4+9x^3+35x^2-x+4} \\
x^4+10x^3+41x^2 \\
- \quad - \quad - \\
\hline
-x^3-6x^2-x+4 \\
-x^3-10x^2-41x+4 \\
+ \quad + \quad + \\
\hline
4x^2+40x+4 \\
4x^2+40x+164 \\
- \quad - \quad - \\
\hline
-160
\end{array}
$$

$$(x^4+9x^3+35x^2-x+4) = (x^2+10x+41)(x^2-x+4)+(-160)$$
$$= 0\,(x^2-x+4)+(-160)$$
$$= 0+(-160)$$
$$= -160$$

$0°$	$30°$	$45°$	$60°$	$90°$	$180°$
0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π

360

2π



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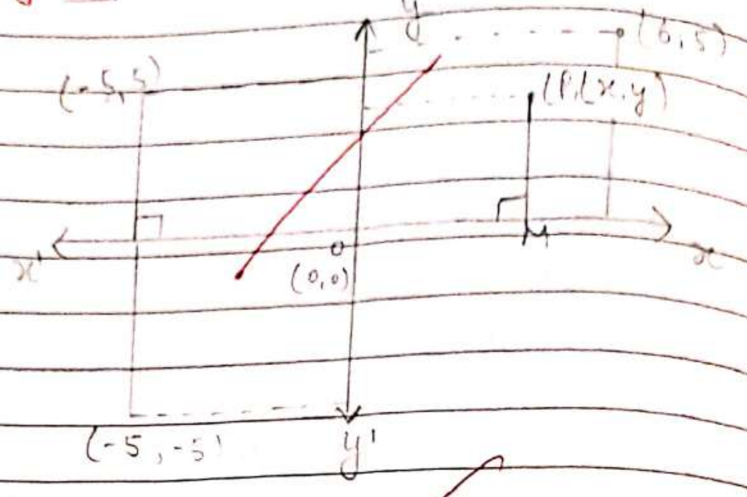
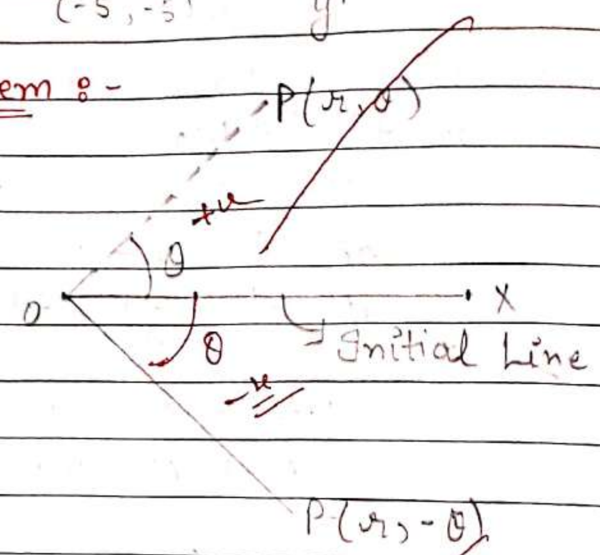
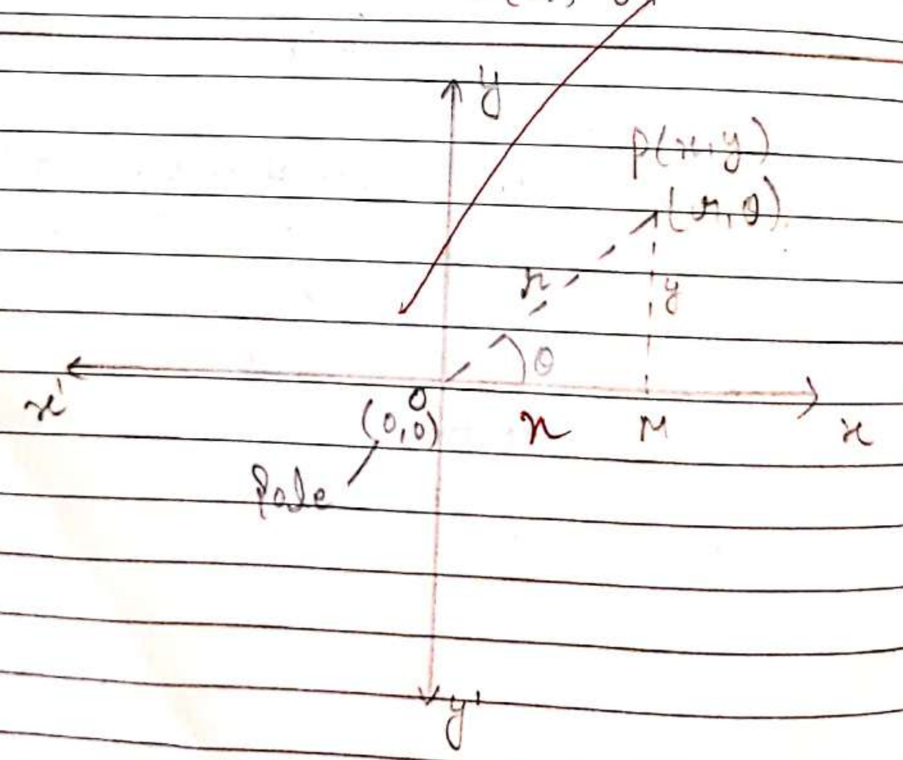
① Cartigen System :-



② Polar System :-





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$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$

$x = r\cos\theta$ ---- (1)

$y = r\sin\theta$ ---- (2)

Squaring and Adding -

$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$x^2+y^2 = r^2$

$r = \sqrt{x^2+y^2}$

$\Rightarrow \frac{(2)}{(1)}$ -

$\frac{y}{x} = \tan\theta$

$\theta = \tan\left(\frac{y}{x}\right)$

Q Represent the complex number $z = 1+i\sqrt{3}$ in the polar form.

Sol. We are given

$z = 1+i\sqrt{3}$

$1+i\sqrt{3} = r(\cos\theta + i\sin\theta)$

Equating R. and Im. part

$r\cos\theta = 1$ — (1)

$r\sin\theta = \sqrt{3}$ — (2)

Squaring and Adding -

$r^2\cos^2\theta + r^2\sin^2\theta = 1+3$

$r^2(\cos^2\theta + \sin^2\theta) = 4$

$r^2 = 4$

$r = \sqrt{4} = 2$

(1) $\Rightarrow$ $2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

(2) $\Rightarrow$ $2\sin\theta = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

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$$1+i\sqrt{3} = 2\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$$

Q. Represent the Complex No $-2+2i$ in the polar form.

Sol. We are given

$$z = -2+2i$$

$$-2+2i = r(\cos\theta + i\sin\theta)$$

Equating R and Im. part

$$r\cos\theta = -2 \quad \text{---} \quad (1)$$

$$r\sin\theta = 2 \quad \text{----} \quad (2)$$

Squaring and Adding:

$$r^2\sin^2\theta + r^2\cos^2\theta = (+2)^2 + (-2)^2$$

$$r^2(\sin^2\theta + \cos^2\theta) = 4 + 4$$

$$r^2 = 8$$

$$r = \sqrt{8}$$

$$r = 2\sqrt{2}$$

$$(1) \Rightarrow 2\sqrt{2}\cos\theta = -2$$

$$\cos\theta = \frac{-\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$(2) \Rightarrow 2\sqrt{2}\sin\theta = 2$$

$$\sin\theta = \frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\theta = \pi - \frac{\pi}{4}$$

$$\theta = \frac{3\pi}{4}$$

$$-2+2i = 2\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$$

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Q Find the principal value of argument of $-2-2i$.

Sol. we are given.

$z = -2-2i$

$-2-2i = r(\cos\theta + i\sin\theta)$

Equating R. and Im part

$r\cos\theta = -2$ ---- (1)

$r\sin\theta = -2$ ---- (2)

Squaring and Adding -

$r^2\cos^2\theta + r^2\sin^2\theta = (-2)^2 + (-2)^2$

$r^2(\cos^2\theta + \sin^2\theta) = 4+4$

$r^2 = 8$

$r = \sqrt{8}$

$r = 2\sqrt{2}$

V. good

(1) $\Rightarrow$ $2\sqrt{2}\cos\theta = -2$

$\cos\theta = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}}$

(2) $\Rightarrow$ $2\sqrt{2}\sin\theta = -2$

$\sin\theta = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}}$

8

$\theta = \frac{-\pi+\pi}{4}$

$= \frac{-3\pi}{4}$

$-2-2i = 2\sqrt{2}\left(\cos\left(\frac{-3\pi}{4}\right) + i\sin\theta\left(\frac{-3\pi}{4}\right)\right)$

Complex Numbers

Q Convert each of the complex numbers given below in the polar form.

(i) $-1+i$

Sol. Let $z = -1+i$

$-1+i = r(\cos\theta + i\sin\theta)$

Equating real and Imaginary part.

$r\cos\theta = -1 \quad \text{------} (1)$

$r\sin\theta = 1 \quad \text{-----} (2)$

Squaring and Adding -

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (1)^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$r^2 = 2$

$r = \sqrt{2}$

(1) $\Rightarrow \quad \cos\theta = \frac{-1}{r}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

(2) $\Rightarrow \quad \sin\theta = \frac{1}{r}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

$-1+i = \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

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(ii) $2-2i$

Sol. Let $z = 2-2i$

$2-2i = r(\cos\theta + i\sin\theta)$

Equating R and Im part.

$r\cos\theta = 2$ ---- (1)

$r\sin\theta = -2$ ---- (2)

Squaring and Adding.

$r^2\cos^2\theta + r^2\sin^2\theta = (2)^2 + (-2)^2$

$r^2(\cos^2\theta + \sin^2\theta) = 4+4$

$r^2 = 8$

$r = \sqrt{8} = 2\sqrt{2}$

(1) $\Rightarrow$ $\cos\theta = \frac{2}{r}$

$\cos\theta = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

(2) $\Rightarrow$ $\sin\theta = \frac{-2}{r}$

$\sin\theta = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$

$\theta = -\frac{\pi}{4}$

$2-2i = 2\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

(iii) $\frac{1+7i}{(2-i)^2}$

Sol. Let $z = \frac{1+7i}{(2-i)^2}$

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$$z = \frac{1+7i}{4+i^2-4i}$$

$$z = \frac{1+7i}{4-1-4i}$$

$$z = \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i}$$

$$z = \frac{(1+7i)(3+4i)}{3^2-(4i)^2}$$

$$z = \frac{3+4i+21i+28i^2}{9+16}$$

$$z = \frac{3+25i+28(-1)}{25}$$

$$z = \frac{3+25i-28}{25}$$

$$z = \frac{-25+25i}{25}$$

$$z = -1+i$$

$-1+i = r(\cos\theta + i\sin\theta)$

Equating R and Im. part

$r\cos\theta = -1$ ------ (1)

$r\sin\theta = 1$ --- (2)

Squaring R. and Im. and Adding -

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (1)^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$r^2 = 2$

$r = \sqrt{2}$

(1) $\Rightarrow$ $\cos\theta = \frac{-1}{r}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

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(2) $\Rightarrow \sin\theta = \frac{1}{r}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\frac{1+7i}{(2-i)^2} = \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

(iv) $\frac{1+3i}{1-2i}$

Sol. Let $z = \frac{1+3i}{1-2i}$

$z = \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$

$z = \frac{(1+3i)(1+2i)}{(1)^2 - (2i)^2}$

$z = \frac{1+2i+3i+6i^2}{1-4i^2}$

$z = \frac{1+5i-6}{1+4}$

$z = \frac{-5+5i}{5}$

$z = -1+i$

$-1+i = r(\cos\theta + i\sin\theta)$

Equating R. and Im. Part.

$r\cos\theta = -1$ ---- (1)

$r\sin\theta = 1$ ---- (2)

Squaring and Adding

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (1)^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$r^2 = 2$

$r = \sqrt{2}$

(1) $\Rightarrow$ $\cos\theta = \frac{-1}{r}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

(2) $\Rightarrow$ $\sin\theta = \frac{1}{r}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\left(\frac{1+3i}{1-2i}\right) = \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

(v) $1-i$

Sol. Let $z = 1-i$

$1-i = r(\cos\theta + i\sin\theta)$

Equating R. and Im. part

$r\cos\theta = 1$ ------ (1)

$r\sin\theta = -1$ ---- (2)

Squaring and Adding -

$r^2\cos^2\theta + r^2\sin^2\theta = (1)^2 + (-1)^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$r^2 = 2$

$r = \sqrt{2}$

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(1) $\Rightarrow$ $\cos\theta = \frac{1}{r}$

$\cos\theta = \frac{1}{\sqrt{2}}$

(2) $\Rightarrow$ $\sin\theta = \frac{-1}{r}$

$\sin\theta = \frac{-1}{\sqrt{2}}$

$\theta = -\frac{\pi}{4}$

$1-i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

(vi) $-1-i$

Sol. Let $z = -1-i$

$-1-i = r(\cos\theta + i\sin\theta)$

Equating R. and Im. part

$r\cos\theta = -1$ ..... (1)

$r\sin\theta = -1$ ..... (2)

Squaring and Adding.

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$r^2 = 2$

$r = \sqrt{2}$

(1) $\Rightarrow$ $\cos\theta = \frac{-1}{r}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

(2) $\Rightarrow$ $\sin\theta = \frac{-1}{r}$

$\sin\theta = \frac{-1}{\sqrt{2}}$

$\theta = -\pi + \frac{\pi}{4}$

$= -\frac{3\pi}{4}$

$-1-i = \sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$

(vii) $1-\sqrt{3}i$

Sol. Let $z = 1-\sqrt{3}i$

$1-\sqrt{3}i = r(\cos\theta + i\sin\theta)$

Equating R. and Im. part.

$r\cos\theta = 1$ ..... (1)

$r\sin\theta = -\sqrt{3}$ ---(2)

Squaring and Adding-

$r^2\cos^2\theta + r^2\sin^2\theta = (1)^2 + (-\sqrt{3})^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1+3$

$r^2 = 4$

$r = 2$

(1) $\Rightarrow$ $\cos\theta = \frac{1}{r}$

$\cos\theta = \frac{1}{2}$

(2) $\Rightarrow$ $\sin\theta = -\frac{\sqrt{3}}{r}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = -\frac{\pi}{3}$

$1-\sqrt{3}i = 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$

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(viii) $-1-\sqrt{3}i$

Sol. Let $z = -1-\sqrt{3}i$ $\Rightarrow$ $-1-\sqrt{3}i = r(\cos\theta + i\sin\theta)$

Equating R. and Im. part.

$r\cos\theta = -1$ ---- (1)

$r\sin\theta = -\sqrt{3}$ ---- (2)

Squaring and Adding.

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-\sqrt{3})^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1+3$

$r^2 = 4$

$r = 2$

(1) $\Rightarrow$ $\cos\theta = \frac{-1}{r}$

$\cos\theta = \frac{-1}{2}$

(2) $\Rightarrow$ $r\sin\theta = -\sqrt{3}$

$\sin\theta = \frac{-\sqrt{3}}{2}$

$\theta = -\pi + \frac{\pi}{3} = \frac{-2\pi}{3}$

$-1-\sqrt{3}i = 2\left(\cos\left(\frac{-2\pi}{3}\right) + i\left(\frac{-2\pi}{3}\right)\right)$

(ix) $-1+\sqrt{3}i$

Sol. Let $z = -1+\sqrt{3}i$

$-1+\sqrt{3}i = r(\cos\theta + i\sin\theta)$

Equating R. and Im. part -

$r\cos\theta = -1$ ----- (1)

$r\sin\theta = \sqrt{3}$ ---- (2)

Squaring and Adding -

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$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (\sqrt{3})^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = 1+3$$

$$r^2 = 4$$

$$r = 2$$

(1) $\Rightarrow$ $\cos\theta = \frac{-1}{r}$

$$\cos\theta = \frac{-1}{2}$$

(2) $\Rightarrow$ $\sin\theta = \frac{\sqrt{3}}{r}$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

$$-1+\sqrt{3}i = 2\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$$