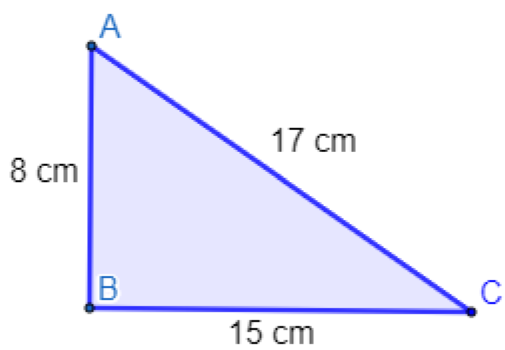
Trigonometry

Exercise 11.1

Q. 1. In right angle triangle ABC, 8 cm, 15 cm and 17 cm are the lengths of AB, BC and CA respectively. Then, find out sin A, cos A and tan A.

Answer : We have



Given: ∠ABC = 90°, AB = 8 cm, BC = 15 cm and CA = 17 cm

We know sin A is given by,

$$\sin A = \frac{perpendicular}{hypotenuse}$$

$\Rightarrow \sin A = \frac{BC}{AC}$

[∵, perpendicular is the side opposite to the angle A & hypotenuse is the side opposite to the right angle of that triangle]

$\Rightarrow \sin A = \frac{15}{17}$ …(i)

Also, cos A is given by

$$\cos A = \frac{base}{hypotenuse}$$

$\Rightarrow \cos A = \frac{AB}{AC}$

$\Rightarrow \cos A = \frac{8}{17}$ …(ii)

Now, tan A can be found out by two ways:

Method 1: tan A is given by,

$$\tan A = \frac{perpendicular}{base}$$

$\Rightarrow \tan A = \frac{BC}{AB}$

$\Rightarrow \tan A = \frac{15}{8}$

Method 2: tan A can also be written as,

$$\tan A = \frac{\sin A}{\cos A}$$

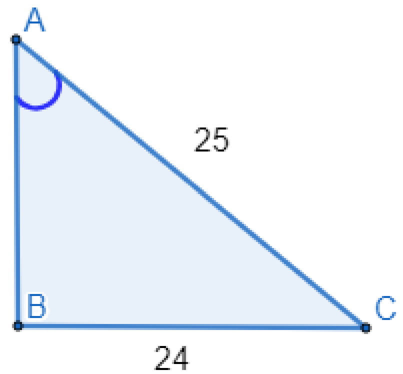
$\Rightarrow \tan A = \frac{15/17}{8/17}$ [from equations (i) & (ii)]

$\Rightarrow \tan A = \frac{15}{8}$

Thus, $\sin A = \frac{15}{17}$, $\cos A = \frac{8}{17}$ and $\tan A = \frac{15}{8}$.

Q. 3. In a right angle triangle ABC with right angle at B, in which a = 24 units, b = 25 units and ∠BAC = θ. Then, find cosθ and tanθ.

Answer : We have



In ∆ABC, ∠ABC = 90° and ∠BAC = θ.

Using this information, we can say

AC = hypotenuse of the triangle

BC = perpendicular (side opposite to the angle θ or ∠BAC)

Using Pythagoras theorem,

$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\Rightarrow (25)^2 = (24)^2 + (\text{base})^2$

$\Rightarrow (AB)^2 = 625 – 576$

$\Rightarrow (AB)^2 = 49$

$\Rightarrow AB = \sqrt{49} = 7$ units

So, we have AB = 7 units, BC = 24 units and AC = 25 units.

Thus, $\cos\theta = \frac{base}{hypotenuse} = \frac{AB}{AC}$

$\Rightarrow \cos\theta = \frac{7}{25}$

And $\tan\theta = \frac{perpendicular}{base} = \frac{BC}{AB}$

$\Rightarrow \tan\theta = \frac{24}{7}$

Thus, $\cos\theta = \frac{7}{25}$ and $\tan\theta = \frac{24}{7}$.

Q. 4

If $\cos A = \frac{12}{13}$, then find sin A and tan A

Answer : Given that,

$$\cos A = \frac{12}{13}$$

But ,

$$\cos A = \frac{base}{hypotenuse}$$

$$\Rightarrow \frac{base}{hypotenuse} = \frac{12}{13}$$

⇒ base = 12 and hypotenuse = 13

So, using Pythagoras theorem, we can say

$(hypotenuse)^2 = (perpendicular)^2 + (base)^2$

$\Rightarrow (perpendicular)^2 = (hypotenuse)^2 – (base)^2$

$\Rightarrow (perpendicular)^2 = (13)^2 – (12)^2$

$\Rightarrow (perpendicular)^2 = 169 – 144 = 25$

⇒ perpendicular = $\sqrt{25} = 5$

Using perpendicular = 5, base = 12 and hypotenuse = 13, we can find out sin A and tan A.

Sin A is given by

$$\sin A = \frac{perpendicular}{hypotenuse}$$

⇒ $\sin A = \frac{5}{13}$

And, tan A is given by

$$\tan A = \frac{perpendicular}{base}$$

⇒ $\tan A = \frac{5}{12}$

Thus, $\sin A = \frac{5}{13}$ and $\tan A = \frac{5}{12}$.

Q. 5. If 3 tanA = 4, then find sin A and cos A.

Answer : Given that, 3 tan A = 4

⇒ $\tan A = \frac{4}{3}$

But

$$\tan A = \frac{perpendicular}{base}$$

⇒ $\frac{perpendicular}{base} = \frac{4}{3}$

⇒ perpendicular = 4 and base = 3

So, using Pythagoras theorem, we can say

$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

⇒ $(\text{hypotenuse})^2 = (4)^2 + (3)^2$

⇒ $(\text{hypotenuse})^2 = 16 + 9 = 25$

⇒ hypotenuse = $\sqrt{25} = 5$

Using perpendicular = 4, base = 3 and hypotenuse = 5, we can find out sin A and cos A.

Sin A is given by

$$\sin A = \frac{perpendicular}{hypotenuse}$$

$\Rightarrow \sin A = \frac{4}{5}$

And, cos A is given by

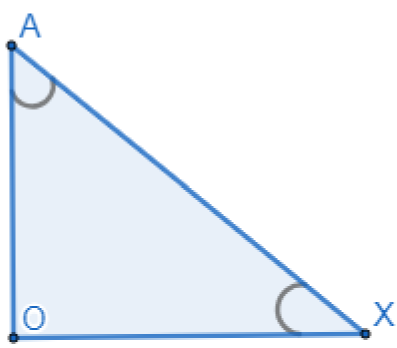
$$\cos A = \frac{base}{hypotenuse}$$

$\Rightarrow \cos A = \frac{3}{5}$

Thus, $\sin A = \frac{4}{5}$ and $\tan A = \frac{3}{5}$.

Q. 6. If ∠A and ∠X are acute angles such that cos A = cos X then show that ∠A =∠X.

Answer : We have



In this ΔAOX,

cos A = cos X

and cos A is given by,

$\cos A = \frac{AO}{AX}$ [$\because$, $\cos\theta = \frac{base}{hypotenuse}$]

Similarly, $\cos X = \frac{OX}{AX}$

$\Rightarrow \frac{AO}{AX} = \frac{OX}{AX}$

⇒ AO = OX [clearly, since denominator from either sides cancel each other]

Now, if sides AO and OX of ∆AOX are equal.

Then, ∠A = ∠X [∵, In a triangle, angles opposite to the equal sides are also equal]

Hence, ∠A = ∠X

Q. 7 A.

Given $\cot\theta = \frac{7}{8}$, **then evaluate**

A. $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

Answer : We have been given that,

$\cot\theta = \frac{7}{8}$,

And we have to solve for

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

We know the formula: $(x + y)(x - y) = x^2 - y^2$

Using this,

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-sin^2\theta}{1-cos^2\theta}$$

Also, we know the relationship between cos θ and sin θ which is given by

$\cos^2 \theta + \sin^2 \theta = 1$

$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$

So,

$\Rightarrow \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{cos^2\theta}{1-cos^2\theta}$

As we know,

$$\cot\ \theta = \frac{B}{P}$$

So, B = 7 and P = 8By Pythagoras theorem,$H^2 = P^2 + B^2$
$H^2 = 8^2 + 7^2$

= 64 + 49=113H = √113As we know,

$$\sin\ \theta = \frac{P}{H}\ and\ \cos\theta = \frac{B}{H}$$

So,

$$sin\ \theta = \frac{8}{\sqrt{113}}\ and\ cos\theta = \frac{7}{\sqrt{113}}$$

Now,

$$\frac{\cos^2\theta}{1 - cos^2\theta} = \frac{\left(\frac{7}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{\frac{49}{113}}{1 - \frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}}$$

$$= \frac{49}{64}$$

Q. 7 B.

Given $\cot\theta = \frac{7}{8}$, **then evaluate**

B. $\frac{(1+\sin\theta)}{\cos\theta}$

Answer :

Given that,

$\cos\theta = \frac{7}{8}$

To solve:

$\frac{(1+\sin\theta)}{\cos\theta}$

We need to find sin θ.

We know the relationship,

$\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta = \sqrt{(1 - \cos^2\theta)}$

$\Rightarrow \sin\theta = \sqrt{1 - \left(\frac{7}{8}\right)^2}$

$$\Rightarrow \sin\theta = \sqrt{1-\frac{49}{64}} = \sqrt{\frac{64-49}{64}} = \sqrt{\frac{15}{64}}$$

$$\Rightarrow \sin\theta = \frac{\sqrt{15}}{8}$$

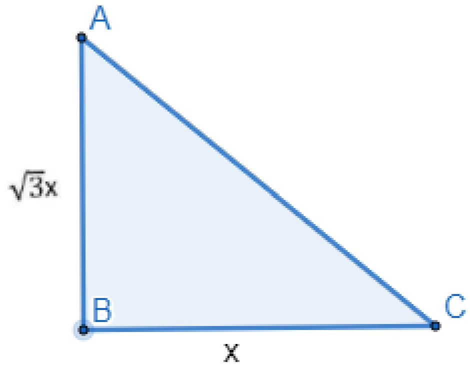
Now, $\frac{(1+\sin\theta)}{\cos\theta} = \frac{1+\frac{\sqrt{15}}{8}}{\frac{7}{8}}$

$$\Rightarrow \frac{(1+\sin\theta)}{\cos\theta} = \frac{8+\sqrt{15}}{7}$$

Q. 8 A. In a right angle triangle ABC, right angle is at B, if $\tan A = \sqrt{3}$ **then find the value of**

A. sin A cosC + cos A sin C

Answer : We have



Given that, tan A = √3/1

And tan A is given by,

$$\tan A = \frac{perpendicular}{base}$$

$$\Rightarrow \frac{perpendicular}{base} = \frac{\sqrt{3}}{1}$$

⇒ perpendicular = √3x and base = x

Then, we can use Pythagoras theorem in ΔABC,

$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

⇒ $(\text{hypotenuse})^2 = (\sqrt{3}x)^2 + (x)^2$

⇒ $(\text{hypotenuse})^2 = 3x^2 + x^2 = 4x^2$

⇒ hypotenuse = $\sqrt{(4x^2)} = 2x$

We have, AB = √3x, BC = x and AC = 2x.

Using these values,

$$\sin A = \frac{perpendicular}{hypotenuse}$$

⇒ $\sin A = \frac{BC}{AC}$

⇒ $\sin A = \frac{x}{2x}$

⇒ $\sin A = \frac{1}{2}$ …(i)

Similarly,

$$\sin C = \frac{AB}{AC}$$

⇒ $\sin C = \frac{\sqrt{3}x}{2x}$

⇒ $\sin C = \frac{\sqrt{3}}{2}$ …(ii)

Also,

$$\cos A = \frac{base}{hypotenuse}$$

⇒ $\cos A = \frac{AB}{AC}$

$\Rightarrow \cos A = \frac{\sqrt{3}x}{2x}$

$\Rightarrow \cos A = \frac{\sqrt{3}}{2}$ …(iii)

Similarly,

$\cos C = \frac{BC}{AC}$

$\Rightarrow \cos C = \frac{x}{2x}$

$\Rightarrow \cos C = \frac{1}{2}$ …(iv)

We have to solve: sin A cos C + cos A sin C.

Substituting equations (i), (ii), (iii) & (iv) in above,

$\sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

⇒ sin A cos C + cos A sin C = 1/4 + 3/4

⇒ sin A cos C + cos A sin C = 4/4 = 1

Thus, sin A cos C + cos A sin C = 1

Q. 8 B. In a right angle triangle ABC, right angle is at B, if $\tan A = \sqrt{3}$ **then find the value of**

B. cos A cos C – sin A sin C

Answer : To find: cos A cos C – sin A sin C.

From previous part of the question, we have

$\sin A = \frac{1}{2}$

$\sin C = \frac{\sqrt{3}}{2}$

$\cos A = \frac{\sqrt{3}}{2}$

$\cos C = \frac{1}{2}$

Using these values, we get

$\cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

⇒ cos A cos C – sin A sin C = √3/4 - √3/4 = 0

Thus, cos A cos C – sin A sin C = 0.

Exercise 11.2

Q. 1 A. Evaluate the following.

A. sin 45° + cos 45°

Answer : By trigonometric identities, we can say

sin 45° = 1/√2

and cos 45° = 1/√2

Adding them, we get

sin 45° + cos 45° = 1/√2 + 1/√2

⇒ sin 45° + cos 45° = 2/√2 = √2

Thus, sin 45° + cos 45° = √2.

Q. 1 B. Evaluate the following.

B. $\frac{\cos 45^{\circ}}{\sec 30^{\circ} + \operatorname{cosec} 60^{\circ}}$

Answer :

$$To\ Find: \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 60^\circ}$$

Trigonometric identities:

cos 45° = 1/√2

sec 30° = 1/cos 30° = 2/√3 [∵, cos 30° = √3/2]

cosec 60° = 1/sin 60° = 2/√3 [∵, sin 60° = √3/2]

Putting the values we get,

$$\frac{\cos 45^\circ}{\sec 30^\circ + cosec\ 60^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}}$$

$$\frac{\cos 45^\circ}{\sec 30^\circ + cosec\ 60^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{4}{\sqrt{3}}}$$

$$\frac{\cos 45^\circ}{\sec 30^\circ + cosec\ 60^\circ} = \frac{\sqrt{3}}{4\sqrt{2}}$$

$$\frac{\cos 45^\circ}{\sec 30^\circ + cosec\ 60^\circ} = \frac{\sqrt{3}}{4\sqrt{2}}.$$

Q. 1 C. Evaluate the following.

C. $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\cot 45^\circ + \cos 60^\circ - \sec 30^\circ}$

Answer :

By trigonometric identities,

cot 45° = 1/tan 45° = 1/1 = 1

sec 30° = 1/cos 30° = 2/√3 [∵, cos 30° = √3/2]

cosec 60° = 1/sin 60° = 2/√3 [∵, sin 60° = √3/2]

sin 30° = 1/2

tan 45° = 1

cos 60° = 1/2

Putting all these values in $\frac{\sin 30° + \tan 45° - cosec\, 60°}{\cot 45° + \cos 60° - \sec 30°}$, we get

$$\frac{\sin 30° \ + \ \tan 45° - cosec\ 60°}{\cot 45° \ + \ \cos 60° - \sec 30°} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{1 + \frac{1}{2} - \frac{2}{\sqrt{3}}}$$

Since, numerator is equal to denominator in the above calculation, we can say

$$\frac{\sin 30° \ + \ \tan 45° - cosec\ 60°}{\cot 45° \ + \ \cos 60° - \sec 30°} = 1$$

Q. 1 D. Evaluate the following.

D. 2tan^2 45° + cos^2 30° - sin^2 60°

Answer :. By trigonometric identities,

sin 60° = √3/2

tan 45° = 1

cos 30° = √3/2

Putting these values in 2 tan^2 45° + cos^2 30° - sin^2 60°, we get

2 tan^2 45° + cos^2 30° - sin^2 60° = 2(1)^2 + (√3/2)^2 – (√3/2)^2

⇒ 2 tan^2 45° + cos^2 30° - sin^2 60° = 2

Thus, 2 tan^2 45° + cos^2 30° - sin^2 60° = 2.

Q. 1 E. Evaluate the following.

E. $\frac{\sin^2 60° - \tan^2 60°}{\sin^2 30° + \cos^2 30°}$

Answer : We have to solve:

$$\frac{sec^2 60° - tan^2 60°}{sin^2 30° + cos^2 30°}$$

Recall the trigonometric identities,

$\sin^2 \theta + \cos^2 \theta = 1$

& $\sec^2 \alpha - \tan^2 \alpha = 1$

Put θ = 30° and α = 60°, we get

$\sin^2 30° + \cos^2 30° = 1$

& $\sec^2 60° - \tan^2 60° = 1$

So,

$$\frac{sec^2 60° - tan^2 60°}{sin^2 30° + cos^2 30°} = \frac{1}{1}$$

Thus,

$$\frac{sec^2 60° - tan^2 60°}{sin^2 30° + cos^2 30°} = 1$$

Q. 2 A. Choose the right option and justify your choice -

A) $\frac{2 \tan 30°}{1 + \tan^2 45°}$

A. sin 60°

B. cos 60°

C. tan 30°

D. sin 30°

Answer : we know,

tan 30° = 1/√3

tan 45° = 1

Then, putting these values in the question, we get

$$\frac{2\tan 30°}{1 + tan^2 45°} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + 1^2}$$

$$\Rightarrow \frac{2\tan 30°}{1 + tan^2 45°} = \frac{2 \times \frac{1}{\sqrt{3}}}{2} = \frac{1}{\sqrt{3}}$$

And we know, tan 30° = 1/√3

But, sin 60° = √3/2

cos 60° = 1/2

& sin 30° = 1/2

Thus, option (C) is correct.

Q. 2 B. Choose the right option and justify your choice –

B) $\frac{1 - \tan^2 45°}{1 + \tan^2 45°}$

A. tan 90°

B. 1

C. sin 45°

D. 0

Answer : We know that,

tan 45° = 1

So, using this value of tangent, we can write

$$\frac{1 - tan^2 45°}{1 + tan^2 45°} = \frac{1 - 1}{1 + 1} = 0$$

The answer has come out to be 0.

Thus, option (D) is correct.

Q. 2 C. Choose the right option and justify your choice -

C) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

A. cos 60°

B. sin 60°

C. tan 60°

D. sin 30°

Answer :

We know that,

tan 30° = 1/√3

So, using this value of tangent, we can write

$$\frac{2\tan 30^\circ}{1-tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{2\tan 30^\circ}{1-tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$$

$$\Rightarrow \frac{2\tan 30^\circ}{1-tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}}$$

$$\Rightarrow \frac{2\tan 30^\circ}{1-tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$\Rightarrow \frac{2\tan 30^\circ}{1-tan^2 30^\circ} = \frac{3}{\sqrt{3}}$$

$$\Rightarrow \frac{2\tan 30^\circ}{1-tan^2 30^\circ} = \sqrt{3}$$

And we know, tan 60° = √3

But, cos 60° = 1/2

sin 60° = √3/2

& sin 30° = 1/2

Thus, option (C) is correct.

Q. 3. Evaluate sin 60° cos 30° + sin 30°cos 60°. What is the value of sin(60° + 30°). What can you conclude?

Answer : Let us first solve sin 60° cos 30° + sin 30° cos 60°.

We know,

sin 60° = √3/2

cos 30° = √3/2

sin 30° = 1/2

& cos 60° = 1/2

$$\text{So, } \sin 60° \cos 30° + \sin 30° \cos 60° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$\Rightarrow \sin 60° \cos 30° + \sin 30° \cos 60° = \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4}$$

⇒ sin 60° cos 30° + sin 30° cos 60° = 1 …(i)

Now, for sin (60° + 30°):

sin (60° + 30°) = sin 90°

⇒ sin (60° + 30°) = 1 [∵, sin 90° = 1] …(ii)

By equations (i) & (ii), we can conclude that

sin (60° + 30°) = sin 60° cos 30° + sin 30° cos 60°

And infact in general, let 60° = x and 30° = y. Then, sin (x + y) = sin x cos y + sin y cos x

Q. 4. Is it right to say cos(60° + 30°) = cos 60° cos 30° - sin 60° sin 30°.

Answer : Let us solve Left-Hand-Side:

cos (60° + 30°) = cos 90°

⇒ cos (60° + 30°) = 0 [∵, cos 90° = 0]

Now, solve for Right-Hand-Side:

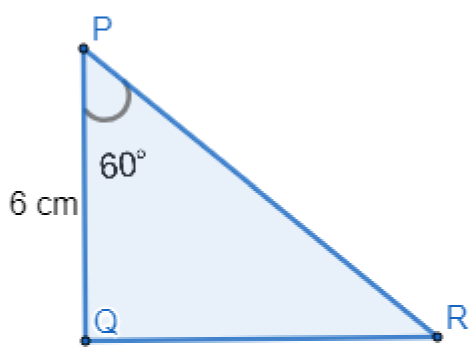
$$\cos 60° \cos 30° - \sin 60° \sin 30° = \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

⇒ $$\cos 60° \cos 30° - \sin 60° \sin 30° = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

⇒ cos 60° cos 30° - sin 60° sin 30° = 0

Q. 5. In right angle triangle ΔPQR, right angle is at Q and PQ = 6cms ∠RPQ = 60°. Determine the lengths of QR and PR.

Answer :



To find QR:

Since, tan θ = perpendicular/base

We know that,

$$\tan 60° = \frac{QR}{PQ}$$

$\Rightarrow \frac{QR}{6} = \sqrt{3}$ [∵, PQ = 6 cm & tan 60° = √3]

⇒ QR = 6√3

Now, PR can be found by two ways -

1st method: In ∆PQR, using Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$ [∵, $(hypotenuse)^2 = (perpendicular)^2 + (base)^2$]

$\Rightarrow PR^2 = 6^2 + (6\sqrt{3})^2$

$\Rightarrow PR^2 = 36 + 108 = 144$

⇒ PR = √144 = 12

2nd Method:

Since, cos θ = base/hypotenuse

We know that,

$$\cos 60° = \frac{PQ}{PR}$$

$$\Rightarrow \frac{PQ}{PR} = \frac{1}{2}$$

$$\Rightarrow \frac{PR}{PQ} = 2$$

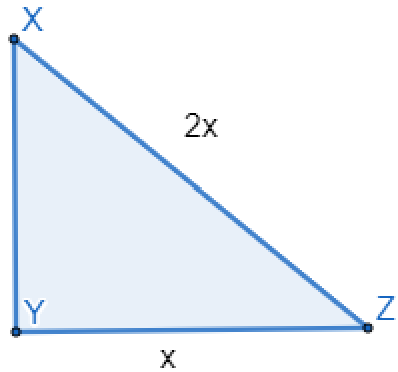
⇒ PR = 2 × PQ

⇒ PR = 2 × 6 = 12

Thus, QR = 6√3 cm and PR = 12 cm.

Q. 6. In right angle is at Y, YZ = x and XZ = 2x then determine ∠YXZ and ∠YZX.

Answer : We have



With given values, YZ = x and XZ = 2x, we can find out both angles.

For ∠YXZ:

Let ∠YXZ = θ, then

$$\sin\ \theta\ = \frac{YZ}{XZ}$$

$$\Rightarrow \sin\theta\ = \frac{x}{2x}$$

$$\Rightarrow \sin\theta\ = \frac{1}{2}$$

⇒ θ = $\sin^{-1}(1/2)$

⇒ θ = 30° [∵, sin 30° = 1/2]

For ∠YZX = α, then

$$\cos\alpha\ = \frac{YZ}{XZ}$$

$$\Rightarrow \cos\alpha\ = \frac{x}{2x}$$

$$\Rightarrow \cos\alpha\ = \frac{1}{2}$$

⇒ α = $\cos^{-1}(1/2)$

⇒ $\alpha = 60°$ [∵, cos 60° = 1/2]

Q. 7. Is it right to say that sin(A + B) = Sin A + Sin B? Justify your answer.

Answer : No, it is not correct to say that sin (A + B) = sin A + sin B.

Justification: Let's justify it by showing contradiction.

Let it be true that, sin (A + B) = sin A + sin B.

Now, let A = 30° and B = 60°

Then,

sin (30° + 60°) = sin 30° + sin 60°

⇒ sin 90° = sin 30° + sin 60°

⇒ $1 = 1/2 + \sqrt{3}/2$

⇒ $1 = (1 + \sqrt{3})/2$

But, it's not true.

$1 \neq (1 + \sqrt{3})/2$

Hence, we have a contradiction.

And therefore, it's not right to say that sin (A + B) = sin A + sin B.

Exercise 11.3

Q. 1 A. Evaluate

A) $\dfrac{\tan 36^\circ}{\cot 54^\circ}$

Answer : By trigometric identity, we have

tan (90° - θ) = cot θ

Replace θ = 54°

⇒ tan (90° - 54°) = cot 54°

⇒ tan 36° = cot 54° [∵, 90° - 54° = 36°]

Now, $\frac{\tan 36^\circ}{\cot 54^\circ} = \frac{\cot 54^\circ}{\cot 54^\circ}$

$\Rightarrow \frac{\tan 36^\circ}{\cot 54^\circ} = 1$

Q. 1 B. Evaluate

B) cos 12° - sin 78°

Answer : By trigonometric identity, we can say

cos (90° - θ) = sin θ

Now, just replace θ by 78°.

We get

cos (90° - 78°) = sin 78°

⇒ cos 12° = sin 78° [∵, 90° - 78° = 12°]

Now, cos 12° - sin 78° = sin 78° - sin 78°

⇒ cos 12° - sin 78° = 0

Q. 1. Evaluate

C) cosec 31° - sec 59°

Answer : By trigonometric identity, we can say

cosec (90° - θ) = sec θ

Replace θ by 59°.

We get

cosec (90° - 59°) = sec 59°

⇒ cosec 31° = sec 59°

Now, cosec 31° - sec 59° = sec 59° - sec 59°

⇒ cosec 31° - sec 59° = 0

Q. 1 D. Evaluate

D) sin 15° sec 75°

Answer : By trigonometric identities, we can say

sin (90° - θ) = cos θ

And sec θ = 1/cos θ

Replace θ by 75°.

We get

sin (90° - 75°) = cos 75°

⇒ sin 15° = cos 75° [∵, 90° - 75° = 15°]

And sec 75° = 1/cos 75°

Using these values, we can solve the given expression.

sin 15° sec 75° = sin 15°/cos 75°

⇒ sin 15° sec 75° = cos 75°/cos 75°

⇒ sin 15° sec 75° = 1

Q. 1 E. Evaluate

E. tan 26° tan 64°

Answer : By trigonometric identities, we can say

tan (90° - θ) = cot θ

And tan θ = 1/cot θ

Replace θ by 64°.

We get

tan (90° - 64°) = cot 64°

⇒ tan 26° = cot 64° [∵, 90° - 64° = 26°]

And tan 64° = 1/cot 64°

Using these values, we can solve for the given expression.

tan 26° tan 64° = cot 64° tan 64°

⇒ tan 26° tan 64° = cot 64°/cot 64°

⇒ tan 26° tan 64° = 1

Q. 2 A. Show that

A. tan 48° tan16° tan 42° tan 74° = 1

Answer : We have

LHS = tan 48° tan 16° tan 42° tan 74° = (tan 48° tan 42°)(tan 16° tan 74°)

We know the trigonometric identities, we can say

tan (90° - θ) = cot θ

And tan θ = 1/cot θ

First, replace θ by 42°.

tan (90° - 42°) = cot 42°

⇒ tan 48° = cot 42° …(1)

And tan 42° = 1/cot 42° …(2)

Now, replace θ by 74°.

tan (90° - 74°) = cot 74°

⇒ tan 16° = cot 74° …(3)

And tan 74° = 1/cot 74° …(4)

Using equations (1), (2), (3) & (4), we get

LHS = tan 48° tan 16° tan 42° tan 74° = (tan 48° tan 42°)(tan 16° tan 74°)

= (cot 42°/cot 42°)(cot 74°/cot 74°)

= 1 = RHS

Thus, tan 48° tan 16° tan 42° tan 74° = 1.

Q. 2 B. Show that

B. cos36°cos54° - sin36°sin54°

Answer : We have

LHS = cos 36° cos 54° - sin 36° sin 54°

We know the trigonometric identity,

cos (90° - θ) = sin θ

First, replace θ by 54°.

We get, cos (90° - 54°) = sin 54°

⇒ cos 36° = sin 54° …(1)

Now, replace θ by 36°.

We get, cos (90° - 36°) = sin 36°

⇒ cos 54° = sin 36° …(2)

Using equations (1) & (2), we get

LHS = cos 36° cos 54° - sin 36° sin 54° = sin 54° sin 36° - sin 54° sin 36°

= 0 = RHS

Thus, cos 36° cos 54° - sin 36° sin 54° = 0.

Q. 3. If tan2A = cot(A – 18°) where 2A is an acute angle. Find the value of A.

Answer : Given that, 2A is an acute angle.

⇒ 2A < 90°

So, using trigonometric identity, we can say that cot (90° - 2A) = tan 2A [∵, cot (90° - θ) = tan θ]

Now, replace tan 2A by cot (90° - 2A) in the given question.

tan 2A = cot (A – 18°)

⇒ cot (90° - 2A) = cot (A – 18°)

Now, we can compare the degrees from above, we get

90° - 2A = A – 18°

⇒ 2A + A = 90° + 18°

⇒ 3A = 108°

⇒ A = 108°/3

⇒ A = 36°

Thus, the value of A is 36°.

Q. 4. If tan A = cot B where A and B are acute angles, prove that A + B = 90°

Answer : Given that, A and B are acute angles.

⇒ A < 90° & B < 90°

So, using trigonometric identity, we can say

tan (90° - B) = cot B [∵, tan (90° - θ) = cot θ]

Replace cot B of RHS by tan (90° - B) in the given question.

tan A = cot B

⇒ tan A = tan (90° - B)

Now, comparing the degrees from the above, we get

A = 90° - B

⇒ A + B = 90°

Hence, proved that A + B = 90°.

Q. 5. If A, B and C are interior angles of a triangle ABC, then show that

$$\tan\left(\frac{A+B}{2}\right)=\cot\frac{C}{2}$$

Answer :

If A, B and C are interior angles of ∆ABC, then we can say that

A + B + C = 180° (by angle sum property of a triangle)

⇒ A + B = 180° - C

Take LHS:

$$tan\left(\frac{A\ +\ B}{2}\right)\ =\ \tan\left(\frac{180°-C}{2}\right)$$

$$\Rightarrow tan\left(\frac{A+B}{2}\right)\ =\ \tan\left(90°-\frac{C}{2}\right)$$

$$\Rightarrow tan\left(\frac{A+B}{2}\right)\ =\ \cot\frac{C}{2}$$

[∵, tan (90° - θ) = cot θ, where θ = C/2 here]

Hence, $tan\left(\frac{A+B}{2}\right)\ =\ \cot\frac{C}{2}$.

Q. 6. Express sin 75° + cos 65° in terms of trigonometric ratios of angles between 0º and 45º.

Answer : Given: sin 75° + cos 65°.

We can write,

75° = 90° - 15°

& 65° = 90° - 25°

Then, sin 75° + cos 65° = sin (90° - 15°) + cos (90° - 25°)

⇒ sin 75° + cos 65° = cos 15° + sin 25°

[∵, sin (90° - θ) = cos θ & cos (90° - θ) = sin θ]

In cos 15° + sin 25°, 15° & 25° both are angles between 0° and 45°.

Thus, answer is sin 75° + cos 65° = cos 15° + sin 25°.

Exercise 11.4

Q. 1 A. Evaluate the following :

A. (1 + tanθ + secθ)(1 + cotθ – cosecθ)

Answer : We have

$$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) = \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

⇒ $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) = \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$

⇒

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) = \frac{1}{\sin\theta\cos\theta}((\cos\theta + \sin\theta) + 1)((\sin\theta + \cos\theta) - 1)$

⇒

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) = \frac{1}{\sin\theta\cos\theta}((\cos\theta + \sin\theta)^2 - 1)$

[∵, (a + b)(a – b) = $a^2 - b^2$]

⇒

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) = \frac{1}{\sin\theta\cos\theta}(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1)$

[∵, $(a + b)^2 = a^2 + b^2 + 2ab$]

⇒

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) = \frac{1}{\sin\theta\cos\theta}(1 + 2\sin\theta\cos\theta - 1)$

[∵, $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow (1 + tan\,\theta + sec\,\theta)(1 + cot\,\theta - cosec\,\theta) = \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$

Thus, (1 + tan θ + sec θ)(1 + cot θ – cosec θ) = 2.

Q. 1 B. Evaluate the following :

B. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

Answer : We have

$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = ((\sin\theta + \cos\theta) + (\sin\theta - \cos\theta))^2 - 2(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$ [∵, $a^2 + b^2 = (a + b)^2 - 2ab$]

$\Rightarrow (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = (\sin\theta + \cos\theta + \sin\theta - \cos\theta)^2 - 2(\sin^2\theta - \cos^2\theta)$

[∵, $(a + b)(a - b) = a^2 - b^2$]

$\Rightarrow (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = (2\sin\theta)^2 - 2\sin^2\theta + 2\cos^2\theta$

$\Rightarrow (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 4\sin^2\theta - 2\sin^2\theta + 2\cos^2\theta$

$\Rightarrow (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2\sin^2\theta + 2\cos^2\theta$

$\Rightarrow (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2(\sin^2\theta + \cos^2\theta)$

$\Rightarrow (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$ [∵, $\sin^2\theta + \cos^2\theta = 1$]

Thus, $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$.

Q. 1 C. Evaluate the following :

C. $(\sec^2\theta - 1)(\mathrm{cosec}^2\theta - 1)$

Answer : We have

$(sec^2\,\theta - 1)(cosec^2\,\theta - 1) = (\frac{1}{cos^2\theta} - 1)(\frac{1}{sin^2\theta} - 1)$

$\Rightarrow (sec^2\,\theta - 1)(cosec^2\,\theta - 1) = (\frac{1-cos^2\theta}{cos^2\theta})(\frac{1-sin^2\theta}{sin^2\theta})$

$\Rightarrow (sec^2\,\theta - 1)(cosec^2\,\theta - 1) = (\frac{sin^2\theta}{cos^2\theta})(\frac{cos^2\theta}{sin^2\theta})$ [∵, $(1 - \cos^2\theta) = \sin^2\theta$ & $(1 - \sin^2\theta = \cos^2\theta$]

⇒ $(\sec^2 \theta - 1)(\text{cosec}^2 \theta - 1) = 1$

Thus, $(\sec^2 \theta - 1)(\text{cosec}^2 \theta - 1) = 1$.

Q. 2

Show that $(\text{cosec}\,\theta + \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$

Answer :

Using trigonometric identities,

cosec θ = 1/sin θ & cot θ = cos θ/sin θ

LHS = $(\text{cosec}\,\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1 - \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 - \cos\theta)^2}{sin^2\theta}$$

As we know, $\sin^2\theta = 1 - \cos^2\theta$

And $(1 - \cos^2 \theta) = (1 + \cos \theta)(1 - \cos \theta)$ [by $(a^2 - b^2) = (a + b)(a - b)$]

$\Rightarrow \sin^2 \theta = (1 + \cos \theta)(1 - \cos \theta)$ ...(i)

So, using equation (i),

Q. 2.

Show that $\left(\text{cosec}\,\theta + \cot\theta\right)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

Answer : Using trigonometric identities,

cosec θ = 1/sin θ & cot θ = cos θ/sin θ

LHS = $(\text{cosec}\,\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{sin^2\theta}$$

As we know, $\sin^2 \theta = 1 - \cos^2 \theta$

And $(1 - \cos^2 \theta) = (1 + \cos \theta)(1 - \cos \theta)$ [by $(a^2 - b^2) = (a + b)(a - b)$]

$\Rightarrow \sin^2 \theta = (1 + \cos \theta)(1 - \cos \theta)$ …(i)

So, using equation (i),

LHS = (cosec θ – cot θ)^2 = $\frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$ = RHS

Hence, we have got

(cosec θ – cot θ)^2 = $\frac{1-\cos\theta}{1+\cos\theta}$.

Q. 3.

Show that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Answer : Using trigonometric identity,

$\sin^2 A + \cos^2 A = 1$

$\Rightarrow \cos^2 A = 1 - \sin^2 A$

Take Left hand side:

LHS = $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$

$= \sqrt{\frac{(1 + \sin A)^2}{1 - sin^2 A}}$

$= \sqrt{\frac{(1 + \sin A)^2}{cos^2 A}}$

$= \frac{1 + \sin A}{\cos A}$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

= sec A + tan A = RHS

Thus,

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A.$$

Q. 4.

Show that $(\operatorname{cosec}\theta + \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Answer : Using trigonometric identity, we have

cot A = 1/tan A

Take left hand side,

$$\text{LHS} = \frac{1-tan^2 A}{cot^2 A-1}$$

$$= \frac{1-tan^2 A}{\frac{1}{tan^2 A}-1}$$

$$= \frac{1-tan^2 A}{\frac{1-tan^2 A}{tan^2 A}}$$

$$= tan^2 A \times \frac{1-tan^2 A}{1-tan^2 A}$$

= $\tan^2$ A = RHS

Thus, $\frac{1-tan^2 A}{cot^2 A-1} = \tan^2 A$.

Q. 5. Show that

$$\frac{1}{\cos\theta} - \cos\theta = \tan\theta.\sin\theta$$

Answer : Take left hand side of the given question:

LHS = 1/cos θ – cos θ

= (1 – $\cos^2$ θ)/cos θ

= $\sin^2$ θ/cos θ [∵, $\sin^2$ θ + $\cos^2$ θ = 1 ⇒ $\sin^2$ θ = 1 – $\cos^2$ θ]

= sin θ × sin θ/cos θ

= sin θ × tan θ [∵, tan θ = sin θ/cos θ]

= tan θ sin θ = RHS

Thus, $\frac{1}{\cos\theta} - \cos\theta = \tan\theta \sin\theta$.

Q. 6. Simplify secA(1 – sinA)(secA + tanA)

Answer : By trigonometric identities, sec A = 1/cos A & tan A = sin A/cos A

Using these identities, we have

$$\sec A\,(1 - \sin A)(\sec A + \tan A) = \frac{1}{\cos A}(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$

$$\Rightarrow \sec A\,(1 - \sin A)(\sec A + \tan A) = \frac{1}{\cos A}(1 - \sin A)\left(\frac{1 + \sin A}{\cos A}\right)$$

$$\Rightarrow \sec A\,(1 - \sin A)(\sec A + \tan A) = \frac{1}{\cos^2 A}(1 - \sin A)(1 + \sin A)$$

$$\Rightarrow \sec A\,(1 - \sin A)(\sec A + \tan A) = \frac{1}{\cos^2 A}(1 - \sin^2 A)$$

$\Rightarrow \sec A\,(1 - \sin A)(\sec A + \tan A) = \frac{\cos^2 A}{\cos^2 A}$ [∵, $\sin^2$ A + $\cos^2$ A = 1 ⇒ $\cos^2$ A = 1 – $\sin^2$ A]

⇒ sec A (1 – sin A)(sec A + tan A) = 1

Thus, sec A (1 – sin A)(sec A + tan A) = 1.

Q. 7. Prove that (sin A + cosec A)^2 + (cos A + sec A)2 = 7 + tan^2 A + cot^2 A

Answer : Take left hand side of the given equation:

LHS = (sin A + cosec A)^2 + (cos A + sec A)^2

Expanding the squares by formula: **(a + b)^2 = a^2 + b^2 + 2ab**

= sin^2 A + cosec^2 A + 2 sin A cosec A + cos^2 A + sec^2 A + 2 cos A sec A
Rearranging the terms, we get,

= (sin^2 A + cos^2 A) + 2 sin A cosec A + 2 cos A sec A + cosec^2 A + sec^2 A
we know that,

$$\text{cosec } A = \frac{1}{\sin A}, \sec A = \frac{1}{\cos A}$$

$$= 1 + 2\frac{\sin A}{\sin A} + 2\frac{\cos A}{\cos A} + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$$

$$= 1 + 2 + 2 + \frac{\cos^2 A + \sin^2 A}{\sin^2 A \cos^2 A}$$

= 5 + 1/(sin^2 A cos^2 A) …(i)

Now, take right hand side of the equation:

RHS = 7 + tan^2 A + cot^2 A

$$Using: \tan A = \frac{\sin A}{\cos A}, \cot A = \frac{\cos A}{\sin A}$$

$$= 7 + \frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A}$$

$$= 7 + \frac{1 - \cos^2 A}{\cos^2 A} + \frac{1 - \sin^2 A}{\sin^2 A}$$

$$= 7 + \frac{1}{\cos^2 A} - 1 + \frac{1}{\sin^2 A} - 1$$

$= 5 + \frac{sin^2 A + cos^2 A}{sin^2 A\ cos^2 A}$

= 5 + 1/($\sin^2$ A $\cos^2$ A) …(ii)

From equation (i) & (ii),

LHS = RHS

Hence, proved.

Q. 8. Simplify (1 – cosθ) (1 + cosθ) (1 + cot^{2}θ)

Answer : We have

(1 – cos θ)(1 + cos θ)(1 + $\cot^2$ θ) = [(1 – cos θ)(1 + cos θ)](1 + $\cot^2$ θ)

⇒ (1 – cos θ)(1 + cos θ)(1 + $\cot^2$ θ) = (1 – $\cos^2$ θ)(1 + $\cot^2$ θ) [∵, (a + b)(a – b) = $a^2 - b^2$]

⇒ (1 – cos θ)(1 + cos θ)(1 + $\cot^2$ θ) = $\sin^2$ θ × (1 + $\cos^2$ θ/$\sin^2$ θ) [∵, (1 – $\cos^2$ θ) = $\sin^2$ θ & $\cot^2$ θ = $\cos^2$ θ/$\sin^2$ θ]

⇒ (1 – cos θ)(1 + cos θ)(1 + $\cot^2$ θ) = $\sin^2$ θ × ($\sin^2$ θ + $\cos^2$ θ)/$\sin^2$ θ

⇒ (1 – cos θ)(1 + cos θ)(1 + $\cot^2$ θ) = $\sin^2$ θ + $\cos^2$ θ

⇒ (1 – cos θ)(1 + cos θ)(1 + $\cot^2$ θ) = 1 [∵, $\sin^2$ θ + $\cos^2$ θ = 1]

Thus, (1 – cos θ)(1 + cos θ)(1 + $\cot^2$ θ) = 1.

Q. 9. If secθ + tanθ = p, then what is the value of secθ – tanθ ?

Answer : Given that, sec θ + tan θ = p.

By trigonometric identity, we have

$\sec^2$ θ – $\tan^2$ θ = 1

So, $\sec^2$ θ – $\tan^2$ θ = 1

⇒ (sec θ – tan θ) (sec θ + tan θ) = 1

⇒ sec θ – tan θ = 1/(sec θ + tan θ)

⇒ sec θ – tan θ = 1/p [given]

Hence, sec θ – tan θ = 1/p.

Q. 10.

If cosesθ + cotθ = k then prove that $\cos\theta = \dfrac{k^2 - 1}{k^2 + 1}$

Answer : Given that, cosec θ + cot θ = k

$\Rightarrow \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = k$ [∵, cosec θ = 1/sin θ & cot θ = cos θ/sin θ]

⇒ (1 + cos θ)/sin θ = k

⇒ 1 + cos θ = k sin θ

Squaring both sides, we get

$(1 + \cos\theta)^2 = (k \sin\theta)^2$

$\Rightarrow (1 + \cos\theta)^2 = k^2 \sin^2\theta$

$\Rightarrow (1 + \cos\theta)^2 = k^2 (1 - \cos^2\theta)$ [∵, $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$]

$\Rightarrow (1 + \cos\theta)^2 = k^2 (1 - \cos\theta)(1 + \cos\theta)$ [∵, $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow 1 + \cos\theta = k^2 (1 - \cos\theta)$

$\Rightarrow 1 + \cos\theta = k^2 - k^2 \cos\theta$

$\Rightarrow k^2 \cos\theta + \cos\theta = k^2 - 1$

$\Rightarrow \cos\theta (k^2 + 1) = k^2 - 1$

$\Rightarrow \cos\theta = (k^2 - 1)/(k^2 + 1)$

Thus, $\cos\theta = (k^2 - 1)/(k^2 + 1)$.

Hence, proved.