CHAPTER

7.2

RANDOM PROCESS

Statement for Question 1 - 4 :

A random process $X(t)$ has periodic sample functions as shown in figure where A, T and $4t_0 \leq T$ are constant but $\in$ is random variable uniformly distributed on the interval $(0, T)$.

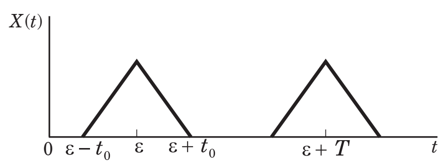

Fig. P7.2.1-4

1. The first order density function is

(A) $\begin{cases} \frac{T-2t_0}{T}\delta(x) + \frac{2t_0}{AT} & 0 \leq x < A \\ 0 & \text{else where} \end{cases}$

(B) $\frac{T-2t_0}{T} + \frac{2t_0}{AT}[u(x) - u(x-A)]$

(C) $\begin{cases} \frac{T+2t_0}{T}\delta(x) + \frac{2t_0}{AT} & 0 \leq x < A \\ 0 & \text{else where} \end{cases}$

(D) $\frac{T-2t_0}{T}\delta(x) + \frac{2T_0}{AT}$

2. The value of $E[X(t)]$ is

(A) $\frac{t_0 A}{2T}$

(B) $\frac{t_0 A}{T}$

(C) $\frac{t_0 A}{4T}$

(D) 0

3. The value of $E[X^2(t)]$ is

(A) $\frac{t_0 A^2}{T}$

(B) $\frac{t_0 A^2}{3T}$

(C) $\frac{2t_0 A^2}{3T}$

(D) 0

4. The value of σ_X^2 is

(A) $\frac{t_0 A}{T}\left[\frac{2}{3} - \frac{t_0}{T}\right]$

(B) $\frac{t_0 A^2}{T} - \frac{t_0}{T}$

(C) $\frac{t_0 A}{T}\left[\frac{2}{3} + \frac{t_0}{T}\right]$

(D) $\frac{t_0 A^2}{T}\left[\frac{2}{3} + \frac{t_0}{T}\right]$

5. An ergodic random power $x(t)$ has an auto-correlation function

$$R_{XX}(\tau) = 18 + \frac{2}{6+\tau^2} 1 + 4\cos(12\tau)$$

The $|\bar{X}|$ is

(A) $\pm\sqrt{18}$

(B) $\pm\sqrt{13}$

(C) $\pm\sqrt{17}$

(D) $\pm\sqrt{18} \pm \sqrt{17}$

6. For random process $\overline{X} = 6$ and

$$R_{XX}(t,\ t+\tau) = 36 + 25e^{-|\tau|}.$$

Consider following statements :

1. $X(t)$ is first order stationary.
2. $X(t)$ has total average power of 36 W.
3. $X(t)$ is a wide sense stationary.
4. $X(t)$ has a periodic component.

The true statement is/are

(A) 1, 2, and 4

(B) 2, 3, and 4

(C) 2 and 3

(D) only 3

7. A random process is defined by $X(t) + A$ where A is continuous random variable uniformly distributed on (0,1). The auto correlation function and mean of the process is

(A) 1/2 & 1/3 (B) 1/3 & 1/2

(C) 1 & 1/2 (D) 1/2 & 1

Statement for Question 8 - 9 :

A random process is defined by $Y(t) = X(t)\cos(\omega_0 t + \theta)$ where $X(t)$ is a wide sense stationary random process that amplitude modulates a carrier of constant angular frequency ω_0 with a random phase θ independent of $X(t)$ and uniformly distributed on $(-\pi / \pi)$.

8. The $E[Y(t)]$ is

(A) $E[X(t)]$ (B) $-E[X(t)]$

(C) 1 (D) 0

9. The autocorrelation function of $Y(t)$ is

(A) $R_{XX}(\tau)\cos(\omega_0\tau)$ (B) $\frac{1}{2}R_{XX}(\tau)\cos(\omega_0\tau)$

(C) $2R_{XX}(\tau)\cos(\omega_0\tau)$ (D) None of the above

Statement for Question 10 - 11 :

Consider a low-pass random process with a white-noise power spectral density $S_X(\omega) = \mathcal{N}/2$ as shown in fig.P7.2.10-11.

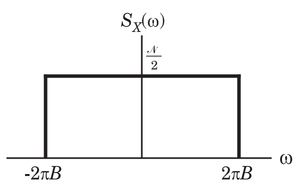

Fig.P7.2.10-11

10. The auto correlation function $R_X(\tau)$ is

(A) $2\mathcal{N}B$ sinc $(2\pi\beta\tau)$ (B) $\pi\mathcal{N}B$ sinc $(2\pi\beta\tau)$

(C) $\mathcal{N}B$ sinc $(2\pi\beta\tau)$ (D) None of the above

11. The power P_X is

(A) $2NB$ (B) πNB

(C) NB D) D$\frac{NB}{2\pi}$

12. If $X(t)$ is a stationary process having a mean value $E[X(t)] = 3$ and autocorrelation function $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$.

The variance of random variable $Y = \int_0^2 X(t)dt$ will be

(A) 1 (B) 2.31

(C) 4.54 (D) 0

13. A random process is defined by $X(t) = A\cos(\pi t)$ where A is a gaussian random variable with zero mean and variance σ_π^2. The density function of $X(0)$

(A) $\frac{1}{\sqrt{2\pi}\sigma_A} e^{-\frac{x^2}{2\sigma_A^2}}$ (B) $\sqrt{2\pi}\sigma_A e^{-\frac{x^2}{2\sigma_A^2}}$

(C) 0 (D) 1

Statement for Question 14-15 :

The two-level semi-random binary process is defined by

$X(t) = A$ or $-A$

where $(n-1)T < t < nt$ and the levels A and $-A$ occur with equal probability. T is a positive constant and $n = 0, \pm 1, \pm 2$

14. The mean value $E[X(t)]$ is

(A) 1/2 (B) 1/4

(C) 1 (D) 0

15. The auto correlation $R_{XX}(t_1 = 0.5T,\ t_2 = 0.7T)$ will be

(A) 1 (B) 0

(C) A^2 (D) $A^2/2$

16. A random process consists of three samples function $X(t, s_1) = 2$, $X(t, s_2) = 2\cos t_1$ and $X(t, s_3) = 3\sin t$- each occurring with equal probability. The process is

(A) First order stationary

(B) Second order stationary

(C) Wide-sense stationary

(D) Not stationary in any sense

Statement for Question 17 - 19 :

The auto correlation function of a stationary ergodic random process is shown in fig.P.7.2.17-19

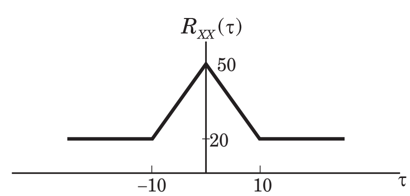

Fig. P7.2.17-19

17. The mean value $E[X(t)]$ is

(A) 50 (B) $\sqrt{50}$

(C) 20 (D) $\sqrt{20}$

18. The $E[X^2(t)]$ is

(A) 10 (B) $\sqrt{10}$

(C) 50 (D) $\sqrt{50}$

19. The variance σ_X^2 is

(A) 20 (B) 50

(C) 70 (D) 30

20. Two zero mean jointly wide sense stationary random process $X(t)$ and $Y(t)$ have no periodic components. It is know that $\sigma_X^2 = 5$ and $\sigma_Y^2 = 10$. The function, that can apply to the process is

(A) $R_{XX}(\tau) = 6u(\tau)e^{-3\tau}$ (B) $R_{YY}(\tau) = 5\left[\dfrac{\sin(3\tau)}{3\tau}\right]^2$

(C) $R_{XY}(\tau) = 9(1+2e^2)^{-1}$ (D) None of the above

21. A stationary zero mean random process $X(t)$ is ergodic has average power of 24 W and has no periodic component. The valid auto correlation function is

(A) $16 + 18\cos(3\tau)$ (B) $24\delta a^2(2\pi)$

(C) $\dfrac{e^{(-6\tau)}}{(1+3\tau^2)}$ (D) $24\delta(t-\tau)$

22. Air craft of Jet Airways at Ahmedabad airport arrive according to a poisson process at a rate of 12 per hour. All aircraft are handled by one air traffic controller. If the controller takes a 2 - minute coffee break, what is the probability that he will miss one or more arriving aircraft ?

(A) 0.33 (B) 0.44

(C) 0.55 (D) 0.66

23. Delhi airport has two check-out lanes that develop waiting lines if more than two passengers arrives in any one minute interval. Assume that a poission process describes the number of passengers that arrive for check-out. The probability of a waiting line if the average rate of passengers is 2 per minute, is

(A) 0.16 (B) 0.29

(C) 0.32 (D) 0.49

24. A complex random process $Z(t) = X(t) + jY(t)$ is defined by jointly stationary real process $X(t)$ and $Y(t)$. The $E[|Z(t)|^2]$ will be

(A) $2R_{XY}(0) + R_{XX}(0) + R_{YY}(0)$ (B) $R_{XX}(0) + R_{YY}(0)$

(C) $R_{XX}(0) - R_{YY}(0)$ (D) $R_{YY}(0) - R_{XX}(0)$

25. Consider random process $X(t) = A_0\cos(\omega_0 t + \theta)$ where A_0 and ω_0 are constant and θ is a random variable uniformly distributed on the interval $(0, \pi)$. The power in $X(t)$ is

(A) A^2 (B) $\frac{1}{2}A^2$

(C) $\frac{1}{4}A^2$ (D) 1

26. The non valid power spectral density function of a real random process is

(A) $\delta(\omega+\omega_0) - \delta(\omega-\omega_0)$ (B) $\dfrac{\omega^2}{\omega^2+25}$

(C) $\delta(\omega) + \dfrac{\omega^2}{\omega^2+16}$ (D) $\dfrac{\omega^2}{\omega^2+16}$

27. The valid power density spectrum is

(A) $\dfrac{\omega^2}{1+\omega^2+j\omega^2}$ (B) $\dfrac{\omega^2}{\omega^4+1} - \delta(\omega)$

(C) $e^{-(\omega-1)^2}$ (D) $\dfrac{\omega^2}{\omega^6+3\omega^2+3}$

28. A power spectrum is given as

$$\rho_{XX}(\omega) = \begin{cases} \dfrac{P}{1+(\omega/W)^2} & |\omega| < KW \\ 0 & |\omega| > KW \end{cases}$$

where $P, W,$ and K are real positive constants. The sums bandwidth of power spectrum is

(A) $W\sqrt{\dfrac{\tan + k}{k} - 1}$ (B) $W\sqrt{\dfrac{k}{\tan^{-1}k} - 1}$

(C) $W\sqrt{\dfrac{\tan^{-1}k}{k} + 1}$ (D) ∞

29. Consider the power spectrum given by

$$\rho_{XX}(\omega) = \begin{cases} P & |\omega| < W \\ 0 & |\omega| > W \end{cases}$$

where P and W are real positive constants. The rms bandwidth of the power spectrum is

(A) $\dfrac{W}{\sqrt{2}}$ (B) $\dfrac{W^2}{3}$

(C) $\dfrac{W}{\sqrt{3}}$ (D) $\dfrac{W}{2}$

30. For a random process $R_{XX}(\tau) = P\cos^4(\omega_0\tau)$ where P and ω_0 are constants. The power in process is

(A) P (B) $2P$

(C) $3P$ (D) $4P$

31. A random process has the power density spectrum $\rho_{XX}(\omega) = \frac{6\omega^2}{[1+\omega^2]^3}$. The average power in process is

(A) 1/4 (B) 3/8

(C) 5/8 (D) 1/2

32. A deterministic signal $A\cos(\omega_0 t)$, where A and ω_0 are real constants is added to a noise process $N(t)$ for which $\rho_{NN}(\omega) = \frac{W^2}{W^2+\omega^2}$ and $W > 0$ is a constant. The ratio of average signal power to average noise power is

(A) 1 (B) $\frac{A}{W}$

(C) $\frac{2A}{W}$ (D) $\frac{A^2}{W}$

33. The autocorrelation function of a random process $X(t)$ is

$$R_{XX}(t,\ t+\tau) = 12e^{Y\tau^2}\cos^2(24t)$$

The $R_{XX}(\tau)$ is

(A) $6e^{-4\tau^2}$ (B) $12e^{-4\tau^2}$

(C) $48e^{-4\tau^2}$ (D) None of the above

34. If $X(t)$ and $Y(t)$ are real random process, the valid power density spectrum $f_{XX}(\omega)$ is

(A) $\frac{6}{6+7\omega^3}$ (B) $\frac{4e^{-3|\tau|}}{1+\omega^2}$

(C) $3+j\omega^2$ (D) $18\delta(\omega)$

35. The cross correlation of jointly wide sense stationary process $X(t)$ and $Y(t)$ is $R_{XY}(\tau) = Au(\tau)e^{-W\tau}$ where $A > 0$ and $W > 0$ are constants. The $\rho_{XY}(\omega)$ is

(A) $\frac{A}{W^2-\omega^2}$ (B) $\frac{A}{W^2+\omega^2}$

(C) $\frac{A}{W+j\omega}$ (D) $\frac{A}{W-j\omega}$

36. A random process $X(t)$ is applied to a linear time invariant system. A response $Y(t) = X(t) - X(t-\tau)$ occurs when τ is a real constant. The system's transfer function is

(A) $1 - e^{j\omega\tau}$ (B) $2je^{-j\omega\tau/2}\sin\frac{\omega\tau}{2}$

(C) $2je^{-j\omega\tau/2}\cos\frac{\omega\tau}{2}$ (D) $1 + e^{-j\omega\tau}$

37. A random process $X(t)$ has an autocorrelation function $R_{XX}(\tau) = A^2 + Be^{-|\tau|}$ Where A and B are constants. A system have an input response

$$h(t) = \begin{cases} e^{-Wt} & 0 < t \\ 0 & t < 0 \end{cases}$$

where W is a real positive constant, which $X(t)$ is its input. The mean value of the response is

(A) $\frac{A}{W}$ (B) $\frac{A}{2W}$

(C) $\frac{2A}{W}$ (D) 0

38. In previous question if impulse response of system is

$$h(t) = \begin{cases} e^{-Wt}\sin(\omega_0 t) & 0 < t \\ 0 & t < 0 \end{cases}$$

where W and ω_0 are real positive constants, the mean value of response is

(A) $\frac{A\omega_0}{\omega_0^2+W^2}$ (B) $\frac{A}{2\omega_0}\left(\frac{1}{\omega_0^2+W^2}\right)$

(C) $\frac{2A}{\omega_0}\left(\frac{1}{\omega_0^2+W^2}\right)$ (D) $\frac{A}{2\omega_0}\left(\frac{1}{\omega_0^2+W^2}\right)$

39. A stationary random process $X(t)$ is applied to the input of a system for which $h(t) = 3u(t)t^2e^{-8t}$. If $E[X(t)] = 2$, the mean value of the system's response $Y(t)$ is

(A) $\frac{1}{128}$ (B) $\frac{1}{64}$

(C) $\frac{3}{128}$ (D) $\frac{1}{32}$

Statement for Question 40-41 :

A random process $X(t)$ is applied to a network with impulse response $h(t) = u(t)te^{-at}$ where $a > 0$ is a constant. The cross correlation of $X(t)$ with the output $Y(t)$ is known to have the same form $R_{XY}(\tau) = u(\tau)\tau e^{-a\tau}$

40. The auto correlation of $Y(t)$ is

(A) $\frac{4+a\tau}{4a^3}e^{-a|\tau|}$ (B) $\frac{1+a\tau}{3a^2}e^{-a|\tau|}$

(C) $\frac{4+a\tau}{8a^2}e^{-a|\tau|}$ (D) $\frac{1+a\tau}{4a^3}e^{-a|\tau|}$

41. The average power in $Y(t)$ is

(A) $\frac{1}{4a^3}$ (B) $\frac{1}{a^3}$

(C) $\frac{1}{3a^2}$ (D) None of the above

Statement for Question 42 - 43 :

A random noise $X(t)$ having a power spectrum $\rho_{XX}(\omega) = \frac{3}{49+\omega^2}$ is applied to a differentiator that has a transfer function $H(\omega) = j\omega$. The output is applied to a network for which $h(t) = u(t)t^2e^{-7t}$

42. The average power in $X(t)$ is

(A) 5/21 (B) 5/24

(C) 5/42 (D) 3/14

43. The power spectrum of $Y(t)$ is

(A) $\frac{4\omega^2}{(49+\omega^2)^3}$ (B) $\frac{12\omega^2}{(49+\omega^2)^4}$

(C) $\frac{42\omega^3}{(49+\omega^2)^2}$ (D) None of the above

44. White noise with power density $\mathcal{N}_0/2$ is applied to a lowpass network for which $|H(0)| = 2$. It has a noise bandwidth of 2 MHz. If the average output noise power is 0.1 W in a $1-\Omega$ resistor, the value of $\mathcal{N}_0$ is

(A) 12.5 nW/Hz (B) 12.5 μW/Hz

(C) 25 nW/Hz (D) 25 μW/Hz

45. An ideal filter with a mid-band power gain of 8 and bandwidth of 4 rad/s has noise $X(t)$ at its input with power spectrum $\rho_{XX}(\omega) = \frac{50}{\sqrt{8\pi}}e^{-\omega^2/8}$. The noise power at the network's output is $(F(2) = 0.9773)$

(A) 60.8 (B) 90.3

(C) 20.2 (D) 100.4

46. White noise with power density $\mathcal{N}_0/2 = 6$ μW/Hz is applied to an ideal filter of gain 1 and bandwidth W rad/s. If the output's average noise power is 15 watts, the bandwidth W is

(A) 2.5×10^{-6} (B) $2.5\pi\times10^{-6}$

(C) 5×10^{-6} (D) $\pi 5\times10^{-6}$

47. A system have the transfer function $|H(\omega)|^2 = \frac{1}{1+(\omega/W)^4}$ where W is a real positive constant. The noise bandwidth of the system is

(A) $\frac{1}{3}\pi W\sqrt{2}$ (B) $\frac{1}{4}\pi W\sqrt{2}$

(C) $\frac{1}{6}\pi W\sqrt{2}$ (D) None of the above

****

SOLUTION

1. (A) Let $\in$ have value e. Now $P\{X \le x|\in = e\} = F_X(X|\in = e)$ and for any $\in$ must be zero for $x<0$ because $x(t)$ is never negative. The event $\{X \le 0\}$ is satisfied whenever $x(t)$ is zero. This happens during the fraction of time $(T-2t_0)/T$. Hence $F_X(x|\in = e) = [(T-2T_0)/T\}u(x)$. For $0 \le x < A$ the additional time interval or fraction of time where $X \le x$ becomes 2 to $2t_0x/AT$.

Thus $F_X(x|\in = e) = \left(\frac{T-2t_0}{T}\right)u(x) + \frac{2t_0x}{AT},\ 0 \le x < A$

$= 1, A \le x$

$= 0, x < 0$

By differentiation

$$f_X(x|\in = e) = \left(\frac{T-2t_0}{T}\right)\delta(x) + \frac{2t_0}{AT},\ 0 \le x < A$$

$= 0$ else where

$$f_{X,e}(x,e) = f_X(x|\in = e)f_\in(e)$$

$$= \left(\frac{T-2t_0}{T^2}\right)\delta(x) + \frac{2T_0}{AT^2}, 0 \le x < A \text{ and } 0 < e < T$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,\in}(x,e)de$$

$$= \left(\frac{T-2t_0}{T}\right)\delta(x) + \frac{2t_0}{AT}. 0 \le x < A$$

$= 0$ elsewhere.

2. (B) $E[X(t)] = \int_{-\infty}^{\infty} xf_X(x)dx$

$$= \int_{-\infty}^{\infty} x\left(\frac{T-2t_0}{T}\right)\delta(x)dx + \int_0^A \frac{2t_0x}{AT}dx = \frac{t_0A}{T}$$

3. (C) $E[X^2(t)] = \int_{-\infty}^{\infty} x^2 f_X(x)dx = \int_0^A \frac{2t_0x^2}{AT} = \frac{2t_0A^2}{3T}$

4. (D) $\sigma_X^2 = E[X^2(t)] - \{E[X(t)]\}^2$

$$= \frac{2t_0A^2}{3T} - \frac{t_0^2A^2}{T^2} = \frac{t_0A^2}{T}\left[\frac{2}{3} - \frac{t_0}{T}\right]$$

5. (A) We know that (i) if $X(t)$ has a periodic component then $R_{XX}(\tau)$ will have a periodic component with the same period. (ii) if $E[X(t)] = \bar{X} \mp 0$ and $X(t)$ is ergodic with no periodic components then $\lim_{|\tau|\to\infty} R_{XX}(\tau) = \overline{X}^2$

Thus we get $|\overline{X}|^2 = 18$ or $\overline{X} = \pm\sqrt{18}$

6. (C) $\overline{X}$ = Constant and $R_{XX}(\tau)$ is not a function of t, so $X(t)$ is a wide sense stationary. So 1 is false & 3 is true.

$P_{XX} = R_{XX}(0) = 36 + 25 = 61.$

Thus 2 is false if $X(t)$ has a periodic component, then $R_{XX}(\tau)$ will have a periodic component with the same period. Thus 4 is false.

7. (B) $R_{XX}(t, t+\tau) = E[X(t)X(t+\tau)] = E[A^2] = \int_0^1 a^2 da = \frac{1}{3}$

$\overline{X} = E[X(t)] = E[A] = \int_0^1 a\, da = \frac{1}{2}$

8. (D) $E[Y(t)\} = E[X(t)\cos(\omega_0 t + \theta)]$

$= E_X[X(t)]\int_{-X}^{X} \cos(\omega_0 t + \theta)\frac{1}{2\pi}d\theta = 0$

where $E_X[\cdot]$ represent expectation with respect to X only

9. (B) $R_{YY}(t, t+\tau)$

$= E[X(t)\cos(\omega_0\tau + \theta)X(t+\tau)\cos(\omega_0 t + \theta + \omega_0\tau)]$

$= R_{XX}(\tau)\frac{1}{2}[\cos(\omega_0\tau) + \cos(2\omega_0 t + 2\theta + \omega_0\tau)]$

$= \frac{1}{2}R_{XX}(\tau)\cos(\omega_0\tau)$

10. (C) $S_X(\omega) = \frac{\mathcal{N}}{2}\text{rect}\left(\frac{\omega}{4\pi b}\right)$

We know that $R_X(\tau) \longleftrightarrow S_X(\omega)$

$\frac{W}{\pi}\sin(Wt) \longleftrightarrow \text{rect}\left(\frac{\omega}{2W}\right)$

Here $W = 2\pi B$

Hence $R_X(\tau) = \frac{2\pi B}{\pi}\frac{\mathcal{N}}{2}\sin(2\pi B\tau) = \mathcal{N}B\ \text{sinc}\ (2\pi B\tau)$

11. (C) $P_X = \overline{X^2} = R_X(0) = \text{N}B$ since $\text{sinc}\ (0) = 1$

12. (C) $E[Y] = E[\int_0^2 X(t)dt = \int_0^2 E[X(t)]dt = 3\int_0^2 dt = 6$

$E[Y^2] = E[\int_0^2 X(t)dt \int_0^2 X(u)du]] = \int_0^2\int_0^2 E[X(t)X(u)du\, dt]$

$= \int_0^2\int_0^2 R_{XX}(t-u)dt\, du = \int_0^2\int_0^2 [9 + 2e^{-|t-u|}]dt\, du$

$= 36 + 2\int_0^2\int_0^2 e^{-|t-u|}dt\, du = 4(10 + e^{-2})$

$\sigma_Y^2 = E[Y^2] - (E[Y])^2 = 4(1 + e^{-2}) = 4.541$

13. (A) For $t = 0, X(0) = A$, So $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_A}e^{-\frac{x^2}{2\sigma_A^2}}$

14. (D) $E[X(t)] = AP(A) + (-A)P(-A) = \frac{A}{2} - \frac{A}{2} = 0$

15. (C) Here $R_{XX}(t_1, t_2) = A^2$

If both t_1 and t_2 are in the same interval $(n-1)T < t, t_2 < nT, n = 0, \pm, \pm 2\ldots$

and $R_{XX}(t_1, t_2) = 0$ otherwise

Hence $R_{XX}(0.5T, 0.7T) = A^2$

16. (D) Let $x_1 = 2$, $x_2 = 2\cos t$ and $x_3 = 3\sin(t)$

Then $f_X(x) = \frac{1}{2}\delta(x - x_1) + \frac{1}{3}\delta(x - x_2) + \frac{1}{3}\delta(x - x_3)$

and $E[X(t)] = \int_{-\infty}^{\infty} x f_X(x)dx$

$= \int_{-\infty}^{\infty} x\left[\frac{1}{3}\delta(x - x_1) + \frac{1}{3}\delta(x - x_2) + \frac{1}{3}\delta(x - x_2)\right]$

$= \frac{1}{3}[2 + 2\cos t + 3\sin t]$

The mean value is time dependent so $X(t)$ is not stationary in any sense.

17. (D) We know that for ergodic with no periodic component

$\lim_{|\tau|\to\infty} R_{XX}(\tau) = \overline{X}^2$, Thus $\overline{X}^2 = 20$ or $\overline{X} = \sqrt{20}$

18. (C) $R_{XX}(0) = E[X^2(t)] = R_{XX}(0) = 50 = \overline{X^2}$

19. (D) $\sigma_X^2 = \overline{X^2} - \overline{X}^2 = 50 - 20 = 30$

20. Here $\overline{X} = 0$, $\overline{Y} = 0$, $R_{XX}(0) = 5, \sigma_Y^2 = R_{YY}(0) = 10$

For (A) : Function does not have even symmetry

For (B) : Function does not satisfy $R_{YY}(0) = 10$

For (C) : Function does not satisfy $|R_{XY}(\tau)| \le \sqrt{R_{XX}(0)R_{YY}(0)} = \sqrt{50}$

21. (D) For (A) : It has a periodic component.

For (B) ; It is not even in τ, total power is also incorrect.

For (C) It depends on t not even in τ and average power is ∞.

22. (A) P (miss/or more aircraft) $= 1 - P$(miss 0)

$= 1 - P\ (0\ \text{arrive}) = 1 - \frac{(\lambda t)^0 e^{-\lambda t}}{0!}$

$$E[Y(t)] = A\int_{-\infty}^{\infty} h(\xi)d\xi = A\int_{0}^{\infty} e^{-Wt}dt = \frac{A}{W}$$

38. (A) $\overline{X} = A$

$$E[Y(t)] = \overline{Y} = \overline{X}\int_{-\infty}^{\infty} h(t)dt = A\int_{0}^{\infty} e^{-Wt}\sin(\omega_0 t)dt = \frac{A\omega_0}{\omega_0^2 + W^2}$$

39. (C) $\overline{Y} = \overline{X}\int_{-\infty}^{\infty} h(t)dt = 2\int_{0}^{\infty} 3t^2 e^{-8t}dt = \frac{3}{128}$

40. (D) $R_{YY}(\tau) = \int_{-\infty}^{\infty} R_{XY}(\tau+\xi)h(\xi)d\xi$

$$= e^{-a\tau}\int_{-\infty}^{\infty} u(\xi)u(\xi+\tau)(\tau\xi+\xi^2)e^{-2a\xi}d\xi$$

There are two cases of interest $\tau \geq 0$ and $\tau < 0$ Since $R_{YY}(\tau)$ is an even function we solve only the ease $\tau \geq 0$

$$R_{YY}(\tau) = e^{-a\tau}\int_{0}^{\infty}(\tau\xi+\xi^2)e^{-2a\xi}d\xi = \frac{1+a\tau}{4a^3}e^{-a\xi}$$

41. (B) Power in $y(t) = R_{YY}(0) = \frac{1}{4a^3}$

42. (D) $P_{XX} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\rho_{XX}(\omega)d\omega = \frac{3}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{49+\omega^2} = \frac{3}{14}$

43. (B) $h_2 = 49t)t^2e^{-7t} \xleftrightarrow{F} \frac{2}{(7+j\omega)^3} = H_2(\omega)$

$$s_{YY}(\omega) = s_{XX}(\omega) = |H_1(\omega)H_2(\omega)|^2 = \frac{12\omega^2}{(49+\omega^2)^4}$$

44. (A) $P_{YY} = \frac{N_0|H(0)|^2 W_n}{2\pi} = 0.1$

So $N_0 = \frac{2\pi(0.1)}{|H(0)|^2 W_n} = \frac{2\pi(0.1)}{(2)^2 2\pi\times 2\times 10^6}$

$= 1.25\times 10^{-8}$ W/Hz $= 12.5$ nW/Hz

45. (A) $P_{YY} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\rho_{XX}(\omega)|H(\omega)|^2 d\omega$

$$= \frac{1}{2\pi}\int_{4}^{4}\frac{50}{\sqrt{8\pi}}e^{-\frac{\omega^2}{8}(8)d\omega} = \frac{200}{\pi}\int_{4}^{4}\frac{e^{-\frac{\omega^2}{2(u)}}}{\sqrt{2\pi(4)}}$$

$$= \frac{200}{\pi}[F(2)-F(-2)] = \frac{200}{\pi}[2F(2)-1)] = 60.8$$

46. (B) $P_{YY} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\rho_{XX}(\omega)|H(\omega)|^2 d\omega$

$$= \frac{1}{2\pi}\int_{-W}^{W} 6\times 10^{-6}d\omega = \frac{6\times 10^{-6}W}{\pi} = \frac{6\times 10^{-6}W}{\pi} = 15$$

So $W = 2.5\pi\times 10^6$

47. (B) Noise bandwidth $W_n = \frac{\int_{0}^{\infty}|H(\omega)|^2 d\omega}{|H(0)|^2}$

$$W_n = \int_{0}^{\infty}|H(\omega)|^2 d\omega \text{since } H(0) = 1 = \int_{0}^{\infty}\frac{d\omega}{1+(\omega/W)^4} = \frac{\pi W}{2\sqrt{2}}$$

****