

EXERCISE. 21 (A)**Question 1:**

$$\frac{\sec A - 1}{\sec A + 1} = \frac{1 - \cos A}{1 + \cos A}$$

Solution 1:

$$\begin{aligned}\text{LHS} &= \frac{\sec A - 1}{\sec A + 1} = \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1} \\ &= \frac{1 - \cos A}{1 + \cos A} = \text{RHS}\end{aligned}$$

Question 2:

$$\frac{1 + \sin A}{1 - \sin A} = \frac{\csc A + 1}{\csc A - 1}$$

Solution 2:

$$\begin{aligned}\text{LHS} &= \frac{1 + \sin A}{1 - \sin A} \\ \text{RHS} &= \frac{\csc A + 1}{\csc A - 1} = \frac{\frac{1}{\sin A} + 1}{\frac{1}{\sin A} - 1} \\ &= \frac{1 + \sin A}{1 - \sin A}\end{aligned}$$

Question 3:

$$\frac{1}{\tan A + \cot A} = \cos A \sin A$$

Solution 3:

$$\frac{1}{\tan A + \cot A} = \sin A \cos A$$

$$\begin{aligned}\text{LHS} &= \frac{1}{\tan A + \cot A} \\&= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} \\&= \frac{1}{1} \quad (\because \sin^2 A + \cos^2 A = 1) \\&= \sin A \cos A \\&= \text{RHS}\end{aligned}$$

Question 4:

$$\tan A - \cot A = \frac{1 - 2\cos^2 A}{\sin A \cos A}$$

Solution 4:

$$\begin{aligned}\tan A - \cot A &= \frac{\sin A}{\cos A} - \frac{\cos A}{\sin A} \\&= \frac{\sin^2 A - \cos^2 A}{\sin A \cos A} \\&= \frac{1 - \cos^2 A - \cos^2 A}{\sin A \cos A} \quad (\because \sin^2 A = 1 - \cos^2 A) \\&= \frac{1 - 2\cos^2 A}{\sin A \cos A}\end{aligned}$$

Question 5:

$$\sin^4 A - \cos^4 A = 2\sin^2 A - 1$$

Solution 5:

$$\begin{aligned}\sin^4 A - \cos^4 A &= (\sin^2 A)^2 - (\cos^2 A)^2 \\&= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A) \\&= \sin^2 A - \cos^2 A \\&= \sin^2 A - (1 - \sin^2 A) \\&= 2\sin^2 A - 1\end{aligned}$$

Question 6:

$$(1 - \tan A)^2 + (1 + \tan A)^2 = 2 \sec^2 A$$

Solution 6:

$$\begin{aligned} & (1 - \tan A)^2 + (1 + \tan A)^2 \\ &= (1 + \tan^2 A - 2 \tan A) + (1 + \tan^2 A + 2 \tan A) \\ &= 2(1 + \tan^2 A) \\ &= 2 \sec^2 A \end{aligned}$$

Question 7:

$$\operatorname{cosec}^4 A - \operatorname{cosec}^2 A = \cot^4 A + \cot^2 A$$

Solution 7:

$$\begin{aligned} \text{LHS} &= \operatorname{cosec}^4 A - \operatorname{cosec}^2 A \\ &= \operatorname{cosec}^2 A (\operatorname{cosec}^2 A - 1) \\ \text{RHS} &= \cot^4 A + \cot^2 A \\ &= \cot^2 A (\cot^2 A + 1) \\ &= (\operatorname{cosec}^2 A - 1) \operatorname{cosec}^2 A \\ \text{Thus, LHS} &= \text{RHS} \end{aligned}$$

Question 8:

$$\sec A (1 - \sin A) (\sec A + \tan A) = 1$$

Solution 8:

$$\begin{aligned} \text{LHS} &= \sec A (1 - \sin A) (\sec A + \tan A) \\ &= \frac{1}{\cos A} (1 - \sin A) \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) \\ &= \frac{(1 - \sin A)}{\cos A} \left(\frac{1 + \sin A}{\cos A} \right) = \left(\frac{1 - \sin^2 A}{\cos^2 A} \right) \\ &= \left(\frac{\cos^2 A}{\cos^2 A} \right) = 1 = \text{RHS} \end{aligned}$$

Question 9:

$$\cos \operatorname{ec} A (1 + \cos A) (\cos \operatorname{ec} A - \cot A) = 1$$

Solution 9:

$$\text{LHS} = \cos \operatorname{ec} A (1 + \cos A) (\cos \operatorname{ec} A - \cot A)$$

$$= \frac{1}{\sin A} (1 + \cos A) \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A} \right)$$

$$= \frac{(1 + \cos A)}{\sin A} \left(\frac{1 - \cos A}{\sin A} \right)$$

$$= \frac{1 - \cos^2 A}{\sin^2 A} \left(\frac{\sin^2 A}{\sin^2 A} \right) = 1 = \text{RHS}$$

Question 10:

$$\sec^2 A + \cos \operatorname{ec}^2 A = \sec^2 A \cos \operatorname{ec}^2 A$$

Solution 10:

$$\text{LHS} = \sec^2 A + \cos \operatorname{ec}^2 A$$

$$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\cos^2 A \cdot \sin^2 A}$$

$$= \frac{1}{\cos^2 A \cdot \sin^2 A} = \sec^2 A \cos \operatorname{ec}^2 A = \text{RHS}$$

Question 11:

$$\frac{(1 + \tan^2 A) \cot A}{\cos \operatorname{ec}^2 A} = \tan A$$

Solution 11:

$$\frac{(1 + \tan^2 A) \cot A}{\cos \operatorname{ec}^2 A}$$

$$= \frac{\sec^2 A \cot A}{\cos \operatorname{ec}^2 A} \left(\because \sec^2 A = 1 + \tan^2 A \right)$$

$$\begin{aligned} &= \frac{1}{\cos^2 A} \times \frac{\cos A}{\sin A} = \frac{1}{\cos A \sin A} \\ &= \frac{1}{\sin^2 A} = \frac{1}{\sin^2 A} \\ &= \frac{\sin A}{\cos A} = \tan A \end{aligned}$$

Question 12:

$$\tan^2 A - \sin^2 A = \tan^2 A \cdot \sin^2 A$$

Solution 12:

$$\begin{aligned} \text{LHS} &= \tan^2 A - \sin^2 A \\ &= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A} \\ &= \frac{\sin^2 A}{\cos^2 A} \cdot \sin^2 A = \tan^2 A \cdot \sin^2 A = \text{RHS} \end{aligned}$$

Question 13:

$$\cot^2 A - \cos^2 A = \cos^2 A \cdot \cot^2 A$$

Solution 13:

$$\begin{aligned} \text{LHS} &= \cot^2 A - \cos^2 A \\ &= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A = \frac{\cos^2 A(1 - \sin^2 A)}{\sin^2 A} \\ &= \cos^2 A \frac{\cos^2 A}{\sin^2 A} = \cos^2 A \cdot \cot^2 A = \text{RHS} \end{aligned}$$

Question 14:

$$(\operatorname{cosec} A + \sin A)(\operatorname{cosec} A - \sin A) = \cot^2 A + \cos^2 A$$

Solution 14:

$$\begin{aligned} &(\operatorname{cosec} A + \sin A)(\operatorname{cosec} A - \sin A) \\ &= \operatorname{cosec}^2 A - \sin^2 A \end{aligned}$$

$$= (1 + \cot^2 A) - (1 - \cos^2 A)$$

$$= \cot^2 A + \cos^2 A$$

Question 15:

$$(\sec A - \cos A)(\sec A + \cos A) = \sin^2 A + \tan^2 A$$

Solution 15:

$$(\sec A - \cos A)(\sec A + \cos A)$$

$$= \sec^2 A - \cos^2 A$$

$$= (1 + \tan^2 A) - (1 - \sin^2 A)$$

$$= \sin^2 A + \tan^2 A$$

Question 16:

$$(\cos A + \sin A)^2 + (\cos A - \sin A)^2 = 2$$

Solution 16:

$$\text{LHS} = (\cos A + \sin A)^2 + (\cos A - \sin A)^2$$

$$= \cos^2 A + \sin^2 A + 2 \cos A \cdot \sin A + \cos^2 A + \sin^2 A - 2 \cos A \cdot \sin A$$

$$= 2(\cos^2 A + \sin^2 A) = 2 = \text{RHS}$$

Question 17:

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$$

Solution 17:

$$\text{LHS} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$$

$$= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \left(\frac{1}{\tan A} + \tan A \right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)$$

$$= \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right) \left(\frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A} \right)$$

Question 18:

$$\frac{1}{\sec A + \tan A} = \sec A - \tan A$$

Solution 18:

$$\begin{aligned} & \frac{1}{\sec A + \tan A} \\ &= \frac{1}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A} \\ &= \frac{\sec A - \tan A}{\sec^2 A - \tan^2 A} \\ &= \sec A - \tan A \end{aligned}$$

Question 19:

$$\cos \text{ec} A + \cot A = \frac{1}{\cos \text{ec} A - \cot A}$$

Solution 19:

$$\begin{aligned} & \cos \text{ec} A + \cot A \\ &= \frac{\cos \text{ec} A + \cot A}{1} \times \frac{\cos \text{ec} A - \cot A}{\cos \text{ec} A - \cot A} \\ &= \frac{\cos \text{ec}^2 A - \cot^2 A}{\cos \text{ec} A - \cot A} = \frac{1 + \cot^2 A - \cot^2 A}{\cos \text{ec} A - \cot A} \\ &= \frac{1}{\cos \text{ec} A - \cot A} \end{aligned}$$

Question 20:

$$\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2\sec A \tan A + 2\tan^2 A$$

Solution 20:

$$\begin{aligned} & \frac{\sec A - \tan A}{\sec A + \tan A} \\ &= \frac{\sec A - \tan A}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A} \end{aligned}$$

$$\begin{aligned}
 &= \frac{(\sec A - \tan A)^2}{\sec^2 A - \tan^2 A} \\
 &= \frac{\sec^2 A + \tan^2 A - 2 \sec A \tan A}{1} \\
 &= 1 + \tan^2 A + \tan^2 A - 2 \sec A \tan A \\
 &= 1 - 2 \sec A \tan A + 2 \tan^2 A
 \end{aligned}$$

Question 21:

$$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

Solution 21:

$$\begin{aligned}
 &(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\
 &= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A \\
 &= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2 + 2 \\
 &= 1 + \operatorname{cosec}^2 A + \sec^2 A + 4 \\
 &= (1 + \cot^2 A) + (1 + \tan^2 A) + 2 \\
 &= 7 + \tan^2 A + \cot^2 A
 \end{aligned}$$

Question 22:

$$\sec^2 A \operatorname{cosec}^2 A = \tan^2 A + \cot^2 A + 2$$

Solution 22:

$$\begin{aligned}
 \text{LHS} &= \sec^2 A \operatorname{cosec}^2 A = \frac{1}{\cos^2 A \cdot \sin^2 A} \\
 \text{RHS} &= \tan^2 A + \cot^2 A + 2 = \tan^2 A + \cot^2 A + 2 \tan A \cdot \cot A \\
 &= (\tan A + \cot A)^2 = \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)^2 \\
 &= \left(\frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A} \right)^2 = \frac{1}{\cos^2 A \cdot \sin^2 A} \\
 \text{Hence, LHS} &= \text{RHS}
 \end{aligned}$$

Question 23:

$$\frac{1}{1 + \cos A} + \frac{1}{1 - \cos A} = 2 \operatorname{cosec}^2 A$$

Solution 23:

$$\begin{aligned} & \frac{1}{1 + \cos A} + \frac{1}{1 - \cos A} \\ &= \frac{1 - \cos A + 1 + \cos A}{(1 + \cos A)(1 - \cos A)} \\ &= \frac{2}{1 - \cos^2 A} \\ &= \frac{2}{\sin^2 A} \\ &= 2 \operatorname{cosec}^2 A \end{aligned}$$

Question 24:

$$\frac{1}{1 - \sin A} + \frac{1}{1 + \sin A} = 2 \sec^2 A$$

Solution 24:

$$\begin{aligned} & \frac{1}{1 - \sin A} + \frac{1}{1 + \sin A} \\ &= \frac{1 + \sin A + 1 - \sin A}{(1 - \sin A)(1 + \sin A)} \\ &= \frac{2}{1 - \sin^2 A} \\ &= \frac{2}{\cos^2 A} \\ &= 2 \sec^2 A \end{aligned}$$

Question 25:

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 \sec^2 A$$

Solution 25:

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$$

$$\begin{aligned}
 &= \frac{\cos \sec^2 A + \cos \sec A + \cos \sec^2 A - \cos \sec A}{\cos \sec^2 A - 1} \\
 &= \frac{2 \cos \sec^2 A}{\cot^2 A} \left(\because \cos \sec^2 A - 1 = \cot^2 A \right) \\
 &= \frac{2}{\frac{\sin^2 A}{\cos^2 A}} = \frac{2}{\cos^2 A} = 2 \sec^2 A
 \end{aligned}$$

Question 26:

$$\frac{\sec A}{\sec A + 1} + \frac{\sec A}{\sec A - 1} = 2 \cos \sec^2 A$$

Solution 26:

$$\begin{aligned}
 &\frac{\sec A}{\sec A + 1} + \frac{\sec A}{\sec A - 1} \\
 &= \frac{\sec^2 A - \sec A + \sec^2 A + \sec A}{\sec^2 A - 1} \\
 &= \frac{2 \sec^2 A}{\tan^2 A} \left(\because \sec^2 A - 1 = \tan^2 A \right) \\
 &= \frac{2}{\frac{\cos^2 A}{\sin^2 A}} = \frac{2}{\cos^2 A} = 2 \cos \sec^2 A
 \end{aligned}$$

Question 27:

$$\frac{1 + \cos A}{1 - \cos A} = \frac{\tan^2 A}{(\sec A - 1)^2}$$

Solution 27:

$$\begin{aligned}
 &\frac{1 + \cos A}{1 - \cos A} \\
 &= \frac{1 + \frac{1}{\sec A}}{1 - \frac{1}{\sec A}} = \frac{\sec A + 1}{\sec A - 1}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sec A + 1}{\sec A - 1} \times \frac{\sec A - 1}{\sec A - 1} \\
 &= \frac{\sec^2 A - 1}{(\sec A - 1)^2} = \frac{\tan^2 A}{(\sec A - 1)^2} \left(\because \sec^2 A - 1 = \tan^2 A \right)
 \end{aligned}$$

Question 28:

$$\frac{\cot^2 A}{(\operatorname{cosec} A + 1)^2} = \frac{1 - \sin A}{1 + \sin A}$$

Solution 28:

$$\begin{aligned}
 \text{R.H.S} &= \frac{1 - \sin A}{1 + \sin A} \\
 &= \frac{1 - \frac{1}{\operatorname{cosec} A}}{1 + \frac{1}{\operatorname{cosec} A}} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \\
 &= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \times \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A + 1} \\
 &= \frac{\operatorname{cosec}^2 A - 1}{(\operatorname{cosec} A + 1)^2} = \frac{\cot^2 A}{(\operatorname{cosec} A + 1)^2} \left(\because \operatorname{cosec}^2 A - 1 = \cot^2 A \right) \\
 &= \text{L.H.S}
 \end{aligned}$$

Question 29:

$$\frac{1 + \sin A}{\cos A} + \frac{\cos A}{1 + \sin A} = 2 \sec A$$

Solution 29:

$$\begin{aligned}
 &\frac{1 + \sin A}{\cos A} + \frac{\cos A}{1 + \sin A} \\
 &= \frac{(1 + \sin A)^2 + \cos^2 A}{\cos A (1 + \sin A)} \\
 &= \frac{1 + \sin^2 A + 2 \sin A + \cos^2 A}{\cos A (1 + \sin A)} \\
 &= \frac{1 + 2 \sin A + 1}{\cos A (1 + \sin A)}
 \end{aligned}$$

$$\begin{aligned} &= \frac{2(1 + \sin A)}{\cos A (1 + \sin A)} \\ &= 2 \sec A \end{aligned}$$

Question 30:

$$\frac{1 - \sin A}{1 + \sin A} = (\sec A - \tan A)^2$$

Solution 30:

$$\begin{aligned} &\frac{1 - \sin A}{1 + \sin A} \\ &= \frac{1 - \sin A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A} \\ &= \frac{(1 - \sin A)^2}{1 - \sin^2 A} \\ &= \frac{(1 - \sin A)^2}{\cos^2 A} \\ &= \left(\frac{1 - \sin A}{\cos A} \right)^2 \\ &= (\sec A - \tan A)^2 \end{aligned}$$

Question 31:

$$(\cot A - \operatorname{cosec} A)^2 = \frac{1 - \cos A}{1 + \cos A}$$

Solution 31:

$$\begin{aligned} \text{R.H.S} &= \frac{1 - \cos A}{1 + \cos A} \\ &= \frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A} \\ &= \frac{(1 - \cos A)^2}{1 - \cos^2 A} \\ &= \frac{(1 - \cos A)^2}{\sin^2 A} \end{aligned}$$

$$\begin{aligned}&= \left(\frac{1 - \cos A}{\sin A} \right)^2 \\&= (\operatorname{cosec} A - \cot A)^2 \\&= (\cot A - \operatorname{cosec} A)^2 \\&= \text{L.H.S}\end{aligned}$$

Question 32:

$$\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \left(\frac{\cos A}{1 + \sin A} \right)^2$$

Solution 32:

$$\begin{aligned}&\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \\&= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \times \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A + 1} \\&= \frac{\operatorname{cosec}^2 A - 1}{(\operatorname{cosec} A + 1)^2} \\&= \frac{\cot^2 A}{(\operatorname{cosec} A + 1)^2} \\&= \frac{\frac{\cos^2 A}{\sin^2 A}}{\left(\frac{1}{\sin A} + 1 \right)^2} \\&= \left(\frac{\cos A}{1 + \sin A} \right)^2\end{aligned}$$

Question 33:

$$\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$$

Solution 33:

$$\tan^2 A - \tan^2 B$$

$$\begin{aligned}
 &= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B} \\
 &= \frac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B} \\
 &= \frac{\sin A(1 - \sin^2 B) - \sin^2 B(1 - \sin^2 A)}{\cos^2 A \cos^2 B} \\
 &= \frac{\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B}{\cos^2 A \cos^2 B} \\
 &= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}
 \end{aligned}$$

Question 34:

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$$

Solution 34:

$$\begin{aligned}
 &= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} \\
 &= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)} \\
 &= \frac{\sin A(\sin^2 A + \cos^2 A - 2\sin^2 A)}{\cos A(2\cos^2 A - \sin^2 A - \cos^2 A)} \\
 &= \frac{\sin A(\cos^2 A - \sin^2 A)}{\cos A(\cos^2 A - \sin^2 A)} \\
 &= \frac{\sin A}{\cos A} \\
 &= \tan A
 \end{aligned}$$

Question 35:

$$\frac{\sin A}{1 + \cos A} = \operatorname{cosec} A - \cot A$$

Solution 35:

$$\frac{\sin A}{1 + \cos A}$$

$$\begin{aligned}
 &= \frac{\sin A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A} \\
 &= \frac{\sin A(1 - \cos A)}{1 - \cos^2 A} \\
 &= \frac{1 - \cos A}{\sin A} \\
 &= \frac{1}{\sin A} - \frac{\cos A}{\sin A} \\
 &= \operatorname{cosec} A - \cot A
 \end{aligned}$$

Question 36:

$$\frac{\cos A}{1 - \sin A} = \sec A + \tan A$$

Solution 36:

$$\text{L.H.S} = \frac{\cos A}{1 - \sin A}$$

$$\text{R.H.S} = \sec A + \tan A$$

$$\begin{aligned}
 &= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \frac{1 + \sin A}{\cos A} \\
 &= \frac{1 + \sin A}{\cos A} \left(\frac{1 - \sin A}{1 - \sin A} \right) = \left(\frac{1 - \sin^2 A}{\cos A(1 - \sin A)} \right) \\
 &= \frac{\cos^2 A}{\cos A(1 - \sin A)} = \frac{\cos A}{(1 - \sin A)} = \text{L.H.S}
 \end{aligned}$$

Question 37:

$$\frac{\sin A \tan A}{1 - \cos A} = 1 + \sec A$$

Solution 37:

$$\begin{aligned}
 &\frac{\sin A \tan A}{1 - \cos A} \\
 &= \frac{\sin A \tan A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A} \\
 &= \frac{\sin A \tan A(1 + \cos A)}{1 - \cos^2 A}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sin A \frac{\sin A}{\cos A} (1 + \cos A)}{\sin^2 A} \\
 &= \frac{1 + \cos A}{\cos A} \\
 &= \frac{1}{\cos A} + \frac{\cos A}{\cos A} \\
 &= \sec A + 1
 \end{aligned}$$

Question 38:

$$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$$

Solution 38:

$$\begin{aligned}
 &(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) \\
 &= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right) \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) \\
 &= \left(\frac{\sin A + \cos A - 1}{\sin A}\right) \left(\frac{\cos A + \sin A + 1}{\cos A}\right) \\
 &= \frac{(\sin A + \cos A - 1)(\sin A + \cos A + 1)}{\sin A \cos A} \\
 &= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cos A} \\
 &= \frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A - 1}{\sin A \cos A} \\
 &= \frac{1 + 2 \sin A \cos A - 1}{\sin A \cos A} \\
 &= \frac{2 \sin A \cos A}{\sin A \cos A} = 2
 \end{aligned}$$

Question 39:

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

Solution 39:

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

$$\begin{aligned} &= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A}} \\ &= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} = \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \\ &= \frac{1 + \sin A}{\cos A} \\ &= \sec A + \tan A \end{aligned}$$

Question 40:

$$\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$$

Solution 40:

$$\begin{aligned} &\sqrt{\frac{1 - \cos A}{1 + \cos A}} \\ &= \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A}} \\ &= \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} \\ &= \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} \\ &= \frac{1 - \cos A}{\sin A} \\ &= \operatorname{cosec} A - \cot A \end{aligned}$$

Question 41:

$$\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$$

Solution 41:

$$\sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\begin{aligned} &= \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 + \cos A}{1 + \cos A}} \\ &= \sqrt{\frac{1 - \cos^2 A}{(1 + \cos A)^2}} \\ &= \sqrt{\frac{\sin^2 A}{(1 + \cos A)^2}} \\ &= \frac{\sin A}{1 + \cos A} \end{aligned}$$

Question 42:

$$\sqrt{\frac{1 - \sin A}{1 + \sin A}} = \frac{\cos A}{1 + \sin A}$$

Solution 42:

$$\begin{aligned} &\sqrt{\frac{1 - \sin A}{1 + \sin A}} \\ &= \sqrt{\frac{1 - \sin A}{1 + \sin A} \times \frac{1 + \sin A}{1 + \sin A}} \\ &= \sqrt{\frac{1 - \sin^2 A}{(1 + \sin A)^2}} \\ &= \sqrt{\frac{\cos^2 A}{(1 + \sin A)^2}} \\ &= \frac{\cos A}{1 + \sin A} \end{aligned}$$

Question 43:

$$1 - \frac{\cos^2 A}{1 + \sin A} = \sin A$$

Solution 43:

$$1 - \frac{\cos^2 A}{1 + \sin A}$$

$$\begin{aligned}
 &= \frac{1 + \sin A - \cos^2 A}{1 + \sin A} \\
 &= \frac{\sin A + \sin^2 A}{1 + \sin A} \\
 &= \frac{\sin A (1 + \sin A)}{1 + \sin A} \\
 &= \sin A
 \end{aligned}$$

Question 44:

$$\frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A} = \frac{2 \sin A}{1 - 2 \cos^2 A}$$

Solution 44:

$$\begin{aligned}
 &\frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A} \\
 &= \frac{\sin A - \cos A + \sin A + \cos A}{\sin^2 A - \cos^2 A} \\
 &= \frac{2 \sin A}{1 - \cos^2 A - \cos^2 A} = \frac{2 \sin A}{1 - 2 \cos^2 A}
 \end{aligned}$$

Question 45:

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2 \sin^2 A - 1}$$

Solution 45:

$$\begin{aligned}
 &\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} \\
 &= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A + \cos A)(\sin A - \cos A)} \\
 &= \frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A}{\sin^2 A - \cos^2 A} \\
 &= \frac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A} \\
 &= \frac{2}{\sin^2 A - \cos^2 A} \quad [\sin^2 A + \cos^2 A = 1]
 \end{aligned}$$

$$= \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{\sin^2 A - (1 - \sin^2 A)}$$

$$\Rightarrow \frac{2}{2\sin^2 A - 1}$$

Question 46:

$$\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A}$$

Solution 46:

$$\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1} \quad [\operatorname{cosec}^2 A - \cot^2 A = 1]$$

$$= \frac{\cot A + \operatorname{cosec} A - [(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)]}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\cot A + \operatorname{cosec} A [1 - \operatorname{cosec} A + \cot A]}{\cot A - \operatorname{cosec} A + 1}$$

$$= \cot A + \operatorname{cosec} A$$

$$= \frac{\cos A}{\sin A} + \frac{1}{\sin A}$$

$$= \frac{1 + \cos A}{\sin A}$$

Question 47:

$$\frac{\sin \theta \tan \theta}{1 - \cos \theta} = 1 + \sec \theta$$

Solution 47:

$$\frac{\sin \theta \tan \theta}{1 - \cos \theta}$$

$$= \frac{\sin \theta \tan \theta}{1 - \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta}$$

$$= \frac{\sin \theta \tan \theta (1 + \cos \theta)}{1 - \cos^2 \theta}$$

$$\begin{aligned} &= \frac{\sin \theta \frac{\sin \theta}{\cos \theta} (1 + \cos \theta)}{\sin^2 \theta} \\ &= \frac{(1 + \cos \theta)}{\cos \theta} \\ &= \frac{1}{\cos \theta} + 1 \\ &= \sec \theta + 1 \end{aligned}$$

Question 48:

$$\frac{\cos \theta \cot \theta}{1 + \sin \theta} = \operatorname{cosec} \theta - 1$$

Solution 48:

$$\begin{aligned} &\frac{\cos \theta \cot \theta}{1 + \sin \theta} \\ &= \frac{\cos \theta \cot \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} \\ &= \frac{\cos \theta \cot \theta (1 - \sin \theta)}{1 - \sin^2 \theta} \\ &= \frac{\cos \theta \frac{\cos \theta}{\sin \theta} (1 - \sin \theta)}{\cos^2 \theta} \\ &= \frac{(1 - \sin \theta)}{\sin \theta} \\ &= \frac{1}{\sin \theta} - 1 \\ &= \operatorname{cosec} \theta - 1 \end{aligned}$$

EXERCISE. 21 (B)**Question 1:**

$$(i) \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

$$(ii) \frac{\cos^3 A + \sin^3 A}{\cos^3 A + \sin^3 A} + \frac{\cos^3 A - \sin^3 A}{\cos^3 A - \sin^3 A} = 2$$

$$(iii) \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = \sec A \operatorname{cosec} A + 1$$

$$(iv) \left(\tan A + \frac{1}{\cos A} \right)^2 + \left(\tan A - \frac{1}{\cos A} \right)^2 = 2 \left(\frac{1 + \sin^2 A}{1 - \sin^2 A} \right)$$

$$(v) 2 \sin^2 A + \cos^4 A = 1 + \sin^4 A$$

$$(vi) \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$$

$$(vii) (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$(viii) (1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$$

$$(ix) \frac{1}{\cos A + \sin A - 1} + \frac{1}{\cos A + \sin A + 1} = \operatorname{cosec} A + \sec A$$

Solution 1:

$$\begin{aligned} (i) \text{ LHS} &= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} \\ &= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}} = \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}} \\ &= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} = \frac{\cos^2 A - \sin^2 A}{(\cos A - \sin A)} \end{aligned}$$

$$= \sin A + \cos A = \text{RHS}$$

$$\begin{aligned} (ii) & \frac{\cos^3 A + \sin^3 A}{\cos^3 A + \sin^3 A} + \frac{\cos^3 A - \sin^3 A}{\cos^3 A - \sin^3 A} \\ &= \frac{(\cos^3 A + \sin^3 A)(\cos A - \sin A) + (\cos^3 A - \sin^3 A)(\cos A + \sin A)}{\cos^2 A - \sin^2 A} \\ &= \frac{\cos^4 A - \cos^3 A \sin A + \sin^3 A \cos A - \sin^4 A + \cos^4 A + \cos^3 A \sin A - \sin^3 A \cos A - \sin^4 A}{\cos^2 A - \sin^2 A} \end{aligned}$$

$$= \frac{2(\cos^4 A - \sin^4 A)}{\cos^2 A - \sin^2 A}$$

$$= \frac{2(\cos^2 A + \sin^2 A)2(\cos^2 A - \sin^2 A)}{(\cos^2 A - \sin^2 A)}$$

$$= 2(\cos^2 A + \sin^2 A)$$

$$= 2(\because \cos^2 A + \sin^2 A = 1)$$

$$(iii) \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

$$= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{1}{1 - \tan A}$$

$$= \frac{\tan^2 A}{\tan A - 1} + \frac{1}{\tan A(1 - \tan A)}$$

$$= \frac{\tan^3 A - 1}{\tan A(1 - \tan A)}$$

$$= \frac{(\tan A - 1)(\tan^2 A + 1 + \tan A)}{\tan A(\tan A - 1)}$$

$$= \frac{\sec^2 A + \tan A}{\tan A}$$

$$= \frac{1}{\frac{\cos^2 A}{\sin A}} + 1$$

$$= \frac{1}{\sin A \cos A} + 1$$

$$= \sec A \operatorname{cosec} A + 1$$

$$(iv) \left(\tan A + \frac{1}{\cos A} \right)^2 + \left(\tan A - \frac{1}{\cos A} \right)^2$$

$$= \left(\frac{\sin A + 1}{\cos A} \right)^2 + \left(\frac{\sin A - 1}{\cos A} \right)^2$$

$$= \frac{\sin^2 A + 1 + 2\sin A + \sin^2 A + 1 - 2\sin A}{\cos^2 A}$$

$$= \frac{2 + 2 \sin^2 A}{\cos^2 A}$$

$$= 2 \left(\frac{1 + \sin^2 A}{1 - \sin^2 A} \right)$$

$$(v) 2 \sin^2 A + \cos^4 A$$

$$= 2 \sin^2 A + (1 - \sin^2 A)^2$$

$$= 2 \sin^2 A + 1 + \sin^4 A - 2 \sin^2 A$$

$$= 1 + \sin^4 A$$

$$(vi) \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1 - 1}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= 0$$

$$(vii) \text{ LHS}$$

$$= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

$$= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right)$$

$$= \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right)$$

$$= \sin A \cos A$$

$$\text{RHS} = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} \cdot \frac{\sin A \cos A}{\sin A \cos A}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \sin A \cos A$$

LHS = RHS

$$\begin{aligned}
 & \text{(viii)} (1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 \\
 &= 1 + \tan^2 A \tan^2 B + 2 \tan A \tan B + \tan^2 A + \tan^2 B - 2 \tan A \tan B \\
 &= 1 + \tan^2 A + \tan^2 B + \tan^2 A \tan^2 B \\
 &= \sec^2 A + \tan^2 B (1 + \tan^2 A) \\
 &= \sec^2 A + \tan^2 B \sec^2 A \\
 &= \sec^2 A (1 + \tan^2 B) \\
 &= \sec^2 A \sec^2 B \\
 & \text{(ix)} \frac{1}{(\cos A + \sin A) - 1} + \frac{1}{(\cos A + \sin A) + 1} \\
 &= \frac{\cos A + \sin A + 1 + \cos A + \sin A - 1}{(\cos A + \sin A)^2 - 1} \\
 &= \frac{2(\cos A + \sin A)}{\cos^2 A + \sin^2 A + 2 \cos A \sin A - 1} \\
 &= \frac{2(\cos A + \sin A)}{1 + 2 \cos A \sin A - 1} = \frac{\cos A + \sin A}{\cos A \sin A} \\
 &= \frac{\cos A}{\cos A \sin A} + \frac{\sin A}{\cos A \sin A} \\
 &= \frac{1}{\sin A} + \frac{1}{\cos A} \\
 &= \operatorname{cosec} A + \sec A
 \end{aligned}$$

Question 2:

If $x \cos A + \sin A = m$ and

$x \sin A - y \cos A = n$, then prove that: $x^2 + y^2 = m^2 + n^2$

Solution 2:

$$\begin{aligned}
 & m^2 + n^2 \\
 &= (x \cos A + y \sin A)^2 + (x \sin A - y \cos A)^2 \\
 &= x^2 \cos^2 A + y^2 \sin^2 A + 2xy \sin A \cos A \\
 &\quad + x^2 \sin^2 A + y^2 \cos^2 A - 2xy \sin A \cos A \\
 &= x^2 (\cos^2 A + \sin^2 A) + y^2 (\cos^2 A + \sin^2 A) \\
 &= x^2 + y^2 \\
 & \text{Hence, } x^2 + y^2 = m^2 + n^2
 \end{aligned}$$

Question 3:

If $m = a \sec A + b \tan A$ and $n = a \tan A + b \sec A$, then prove that : $m^2 - n^2 = a^2 - b^2$

Solution 3:

Given,

$$m = a \sec A + b \tan A \text{ and } n = a \tan A + b \sec A$$

$$m^2 - n^2 = (a \sec A + b \tan A)^2 - (a \tan A + b \sec A)^2$$

$$= a^2 \sec^2 A + b^2 \tan^2 A + 2ab \sec A \tan A$$

$$- (a^2 \tan^2 A + b^2 \sec^2 A + 2ab \sec A \tan A)$$

$$= \sec^2 A (a^2 - b^2) + \tan^2 A (b^2 - a^2)$$

$$= (a^2 - b^2) [\sec^2 A - \tan^2 A]$$

$$= (a^2 - b^2) [\text{Since } \sec^2 A - \tan^2 A = 1]$$

$$\text{Hence, } m^2 - n^2 = a^2 - b^2$$

Question 4:

If $x = r \sin A \cos B$, $y = r \sin A \sin B$ and $z = r \cos A$, then prove that:

$$x^2 + y^2 + z^2 = r^2$$

Solution 4:

$$\text{LHS} = (r \sin A \cos B)^2 + (r \sin A \sin B)^2 + (r \cos A)^2$$

$$= r^2 \sin^2 A \cos^2 B + r^2 \sin^2 A \sin^2 B + r^2 \cos^2 A$$

$$= r^2 \sin^2 A (\cos^2 B + \sin^2 B) + r^2 \cos^2 A$$

$$= r^2 (\sin^2 A + \cos^2 A) = r^2 = \text{RHS}$$

Question 5:

If $\sin A + \cos A = m$ and $\sec A + \operatorname{cosec} A = n$, show that: $n(m^2 - 1) = 2m$

Solution 5:

Given:

$$\sin A + \cos A = m$$

and

$$\sec A + \operatorname{cosec} A = n$$

$$\begin{aligned}
 \text{Consider L.H.S} &= n(m^2 - 1) \\
 &= (\sec A + \operatorname{cosec} A) \left[(\sin A + \cos A)^2 - 1 \right] \\
 &= \left(\frac{1}{\cos A} + \frac{1}{\sin A} \right) [\sin^2 A + \cos^2 A + 2 \sin A \cos A - 1] \\
 &= \left(\frac{\cos A + \sin A}{\sin A \cos A} \right) (1 + 2 \sin A \cos A - 1) \\
 &= \frac{(\cos A + \sin A)}{\sin A \cos A} (2 \sin A \cos A) \\
 &= 2(\sin A + \cos A) \\
 &= 2m = \text{R.H.S}
 \end{aligned}$$

Question 6:

If $x = r \cos A \cos B$, $y = r \cos A \sin B$ and $z = r \sin A$, show that:

$$x^2 + y^2 + z^2 = r^2$$

Solution 6:

$$\begin{aligned}
 \text{LHS} &= (r \cos A \cos B)^2 + (r \cos A \sin B)^2 + (r \sin A)^2 \\
 &= r^2 \cos^2 A \cos^2 B + r^2 \cos^2 A \sin^2 B + r^2 \sin^2 A \\
 &= r^2 \cos^2 A (\cos^2 B + \sin^2 B) + r^2 \sin^2 A \\
 &= r^2 (\cos^2 A + \sin^2 A) = r^2 = \text{RHS}
 \end{aligned}$$

Question 7:

If $\frac{\cos A}{\cos B} = m$ and $\frac{\cos A}{\sin B} = n$ show that:

$$(m^2 + n^2) \cos^2 B = n^2.$$

Solution 7:

$$\begin{aligned}
 \text{LHS} &= (m^2 + n^2) \cos^2 B \\
 &= \left(\frac{\cos^2 A}{\cos^2 B} + \frac{\cos^2 A}{\sin^2 B} \right) \cos^2 B
 \end{aligned}$$

$$\begin{aligned}
 &= \left(\frac{\cos^2 A \sin^2 B + \cos^2 A \cos^2 B}{\cos^2 B \sin^2 B} \right) \cos^2 B \\
 &= \left(\frac{\cos^2 A \sin^2 B + \cos^2 A \cos^2 B}{\sin^2 B} \right) \\
 &= \frac{\cos^2 A (\sin^2 B + \cos^2 B)}{\sin^2 B} \\
 &= \frac{\cos^2 A}{\sin^2 B} \\
 &= n^2 \\
 \text{Hence, } (m^2 + n^2) \cos^2 B &= n^2.
 \end{aligned}$$

EXERCISE 21 (C)**Question 1:**

- (i) $\frac{\cos 22^\circ}{\sin 68^\circ}$ (ii) $\frac{\tan 47^\circ}{\cot 43^\circ}$
- (iii) $\frac{\sec 75^\circ}{\operatorname{cosec} 15^\circ}$ (iv) $\frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ}$
- (v) $\cos^2 40^\circ + \cos^2 50^\circ$
- (vi) $\sec^2 18^\circ - \cot^2 72^\circ$
- (vii) $\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ$
- (viii) $\sin 42^\circ \sin 48^\circ - \cos 42^\circ \cos 48^\circ$

Solution 1:

$$\begin{aligned}
 \text{(i)} \quad \frac{\cos 22^\circ}{\sin 68^\circ} &= \frac{\cos(90^\circ - 68^\circ)}{\sin 68^\circ} = \frac{\sin 68^\circ}{\sin 68^\circ} = 1 \\
 \text{(ii)} \quad \frac{\tan 47^\circ}{\cot 43^\circ} &= \frac{\tan(90^\circ - 43^\circ)}{\cot 43^\circ} = \frac{\cot 43^\circ}{\cot 43^\circ} = 1 \\
 \text{(iii)} \quad \frac{\sec 75^\circ}{\operatorname{cosec} 15^\circ} &= \frac{\sec(90^\circ - 15^\circ)}{\operatorname{cosec} 15^\circ} = \frac{\operatorname{cosec} 15^\circ}{\operatorname{cosec} 15^\circ} = 1 \\
 \text{(iv)} \quad \frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ} \\
 &= \frac{\cos(90^\circ - 35^\circ)}{\sin 35^\circ} + \frac{\cot(90^\circ - 55^\circ)}{\tan 55^\circ}
 \end{aligned}$$

$$= \frac{\sin 35^\circ}{\sin 35^\circ} + \frac{\tan 55^\circ}{\tan 55^\circ}$$

$$= 1 + 1 = 2$$

$$(v) \cos^2 40^\circ + \cos^2 50^\circ$$

$$= [\cos(90^\circ - 50^\circ)]^2 + \cos^2 50^\circ$$

$$= \sin^2 50^\circ + \cos^2 50^\circ$$

$$= 1$$

$$(vi) \sec^2 18^\circ - \cot^2 72^\circ$$

$$= [\sec(90^\circ - 72^\circ)]^2 - \cot^2 72^\circ$$

$$= \operatorname{cosec}^2 72^\circ - \cot^2 72^\circ$$

$$= 1$$

$$(vii) \sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ$$

$$= \sin(90^\circ - 75^\circ) \cos 75^\circ + \cos(90^\circ - 75^\circ) \sin 75^\circ$$

$$= \cos 75^\circ \cos 75^\circ + \sin 75^\circ \sin 75^\circ$$

$$= \cos^2 75^\circ + \sin^2 75^\circ = 1$$

$$(viii) \sin 42^\circ \sin 48^\circ - \cos 42^\circ \cos 48^\circ$$

$$= \sin(90^\circ - 48^\circ) \sin 48^\circ - \cos(90^\circ - 48^\circ) \cos 48^\circ$$

$$= \cos 48^\circ \sin 48^\circ - \sin 48^\circ \cos 48^\circ = 0$$

Question 2:

Evaluate

$$(i) \sin(90^\circ - A) \cos A + \cos(90^\circ - A) \sin A$$

$$(ii) \sin^2 35^\circ + \sin^2 55^\circ$$

$$(iii) \frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$$

$$(iv) \frac{2 \tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$$

$$(v) \cos^2 25^\circ + \cos^2 65^\circ - \tan^2 45^\circ$$

$$(vi) \frac{\cos^2 32^\circ + \cos^2 58^\circ}{\sin^2 59^\circ + \sin^2 31^\circ}$$

$$(vii) \left(\frac{\sin 77^\circ}{\cos 13^\circ} \right)^2 + \left(\frac{\cos 77^\circ}{\sin 13^\circ} \right)^2 - 2 \cos^2 45^\circ$$

$$(viii) \cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot 54^\circ}$$

Solution 2:

$$(i) \sin(90^\circ - A) \cos A + \cos(90^\circ - A) \sin A$$

$$= \cos A \cos A + \sin A \sin A$$

$$= \cos^2 A + \sin^2 A = 1$$

$$(ii) \sin^2 35^\circ + \sin^2 55^\circ$$

$$= [\sin(90^\circ - 55^\circ)]^2 + \sin^2 55^\circ$$

$$= \cos^2 55^\circ + \sin^2 55^\circ = 1$$

$$(iii) \frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$$

$$= \frac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} - 2$$

$$= \frac{\tan 36^\circ}{\tan 36^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2$$

$$= 1 + 1 - 2 = 0$$

$$(iv) \frac{2 \tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$$

$$= \frac{2 \tan(90^\circ - 37^\circ)}{\cot 37^\circ} - \frac{\cot(90^\circ - 10^\circ)}{\tan 10^\circ}$$

$$= \frac{2 \cot 37^\circ}{\cot 37^\circ} - \frac{\tan 10^\circ}{\tan 10^\circ}$$

$$= 2 - 1 = 1$$

$$(v) \cos^2 25^\circ + \cos^2 65^\circ - \tan^2 45^\circ$$

$$= \cos^2(90^\circ - 65^\circ) + \cos^2 65^\circ - \tan^2 45^\circ$$

$$= \sin^2 65^\circ + \cos^2 65^\circ - 1 = 1 - 1 = 0$$

$$(vi) \frac{\cos^2 32^\circ + \cos^2 58^\circ}{\sin^2 59^\circ + \sin^2 31^\circ}$$

$$= \frac{\cos^2(90^\circ - 58^\circ) + \cos^2 58^\circ}{\sin^2(90^\circ - 31^\circ) + \sin^2 31^\circ}$$

$$= \frac{\sin^2 58^\circ + \cos^2 58^\circ}{\cos^2 31^\circ + \sin^2 31^\circ}$$

$$= \frac{1}{1} = 1$$

$$(vii) \left(\frac{\sin 77^\circ}{\cos 13^\circ} \right)^2 + \left(\frac{\cos 77^\circ}{\sin 13^\circ} \right) - 2 \cos^2 45^\circ$$

$$= \left[\frac{\sin(90^\circ - 13^\circ)}{\cos 13^\circ} \right]^2 + \left[\frac{\cos(90^\circ - 13^\circ)}{\sin 13^\circ} \right] - 2 \cos^2 45^\circ$$

$$= \left[\frac{\cos 13^\circ}{\cos 13^\circ} \right] + \left[\frac{\sin 13^\circ}{\sin 13^\circ} \right] - 2 \left(\frac{1}{\sqrt{2}} \right)^2$$

$$= 1 + 1 - 1 = 1$$

$$(viii) \cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot 54^\circ}$$

$$\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot 54^\circ}$$

$$= \cos^2 26^\circ + \cos(90 - 26)^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot(90 - 36)^\circ}$$

$$= \cos^2 26^\circ + \sin 26^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\tan 36^\circ}$$

$$= \cos^2 26^\circ + \sin^2 26^\circ + 1$$

$$= 1 + 1$$

$$= 2$$

Question 3:

Show that:

$$(i) \tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ = 1$$

$$(ii) \sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ = 2$$

$$(iii) \frac{\sin 26^\circ}{\sec 64^\circ} + \frac{\cos 26^\circ}{\operatorname{cosec} 64^\circ} = 1$$

Solution 3:

$$(i) \tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ$$

$$= \tan(90^\circ - 80^\circ) \tan(90^\circ - 75^\circ) \tan 75^\circ \tan 80^\circ$$

$$= \cot 80^\circ \cot 75^\circ \tan 75^\circ \tan 80^\circ$$

$$= 1 [\text{As } \tan \theta, \cot \theta = 1]$$

$$(ii) \sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ = 2$$

$$\text{consider } \sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ$$

$$\Rightarrow \sin 42^\circ \sec(90^\circ - 42^\circ) + \cos 42^\circ \operatorname{cosec}(90^\circ - 42^\circ)$$

$$\Rightarrow \sin 42^\circ \cdot \operatorname{cosec} 42^\circ + \cos 42^\circ \sec 42^\circ$$

$$\Rightarrow \sin 42^\circ \cdot \frac{1}{\sin 42^\circ} + \cos 42^\circ \cdot \frac{1}{\cos 42^\circ}$$

$$\Rightarrow 1 + 1 = 2$$

$$(iii) \frac{\sin 26^\circ}{\sec 64^\circ} + \frac{\cos 26^\circ}{\operatorname{cosec} 64^\circ}$$

$$\begin{aligned}
 &= \frac{\sin 26^\circ}{\sec(90^\circ - 26^\circ)} + \frac{\cos 26^\circ}{\operatorname{cosec}(90^\circ - 26^\circ)} \\
 &= \frac{\sin 26^\circ}{\operatorname{cosec} 26^\circ} + \frac{\cos 26^\circ}{\sec 26^\circ} \\
 &= \sin^2 26^\circ + \cos^2 26^\circ \\
 &= 1
 \end{aligned}$$

Question 4:

Express each of the following in terms of angles between 0° and 45° :

(i) $\sin 59^\circ + \tan 63^\circ$

(ii) $\operatorname{cosec} 68^\circ + \cot 72^\circ$

(iii) $\cos 74^\circ + \sec 67^\circ$

Solution 4:

(i) $\sin 59^\circ + \tan 63^\circ$

$$= \sin(90^\circ - 31^\circ) + \tan(90^\circ - 27^\circ)$$

$$= \cos 31^\circ + \cot 27^\circ$$

(ii) $\operatorname{cosec} 68^\circ + \cot 72^\circ$

$$= \operatorname{cosec}(90^\circ - 22^\circ) + \cot(90^\circ - 18^\circ)$$

$$= \sec 22^\circ + \tan 18^\circ$$

(iii) $\cos 74^\circ + \sec 67^\circ$

$$= \cos(90^\circ - 16^\circ) + \sec(90^\circ - 23^\circ)$$

$$= \sin 16^\circ + \operatorname{cosec} 23^\circ$$

Question 5:

Show that:

(i) $\frac{\sin A}{\sin(90^\circ - A)} + \frac{\cos A}{\cos(90^\circ - A)} = \sec A \operatorname{cosec} A$

(ii) $\sin A \cos A - \frac{\sin A \cos(90^\circ - A) \cos A}{\sec(90^\circ - A)} - \frac{\cos A \sin(90^\circ - A) \sin A}{\operatorname{cosec}(90^\circ - A)} = 0$

Solution 5:

$$(i) \frac{\sin A}{\sin(90^\circ - A)} + \frac{\cos A}{\cos(90^\circ - A)}$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$$

$$= \frac{1}{\cos A \sin A}$$

$$= \sec A \operatorname{cosec} A$$

$$(ii) \sin A \cos A - \frac{\sin A \cos(90^\circ - A) \cos A}{\sec(90^\circ - A)} - \frac{\cos A \sin(90^\circ - A) \sin A}{\operatorname{cosec}(90^\circ - A)}$$

$$= \sin A \cos A - \frac{\sin A \sin A \cos A}{\operatorname{cosec} A} - \frac{\cos A \cos A \sin A}{\sec A}$$

$$= \sin A \cos A - \sin^3 A \cos A - \cos^3 A \sin A$$

$$= \sin A \cos A - \sin A \cos A (\sin^2 A + \cos^2 A)$$

$$= \sin A \cos A - \sin A \cos A (1)$$

$$= 0$$

Question 6:

For triangle ABC, show that:

$$(i) \sin \frac{A+B}{2} = \cos \frac{C}{2}$$

$$(ii) \tan \frac{B+C}{2} = \cot \frac{A}{2}$$

Solution 6:

(i) We know that for a triangle ΔABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\frac{\angle B + \angle A}{2} = 90^\circ - \frac{\angle C}{2}$$

$$\sin \left(\frac{A+B}{2} \right) = \sin \left(90^\circ - \frac{C}{2} \right)$$

$$= \cos \left(\frac{C}{2} \right)$$

(ii) We know that for a triangle ΔABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle C = 180^\circ - \angle A$$

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$$\begin{aligned}\tan\left(\frac{B+C}{2}\right) &= \tan\left(90^\circ - \frac{A}{2}\right) \\ &= \cos\left(\frac{A}{2}\right)\end{aligned}$$

Question 7:

Evaluate:

$$(i) \ 3 \frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\cos \operatorname{ec} 58^\circ}$$

$$(ii) \ 3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$$

$$(iii) \ \frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$$

$$(iv) \ \tan(55^\circ - A) - \cot(35^\circ + A)$$

$$(v) \ \operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A)$$

$$(vi) \ 2 \frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2} \cos 45^\circ$$

$$(vii) \ \frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2 \frac{\sin^2 75^\circ}{\cos^2 15^\circ}$$

$$(viii) \ \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ$$

$$(ix) \ 14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ$$

Solution 7:

$$\begin{aligned}(i) \ & 3 \frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\cos \operatorname{ec} 58^\circ} \\ &= 3 \frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\cos \operatorname{ec} 58^\circ} \\ &= 3 \frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\cos \operatorname{ec} 58^\circ} = 3 - 1 = 2\end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & 3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ \\
 &= 3\cos(90^\circ - 10^\circ) \operatorname{cosec} 10^\circ + 2\cos(90^\circ - 31^\circ) \operatorname{cosec} 31^\circ \\
 &= 3\sin 10^\circ \operatorname{cosec} 10^\circ + 2\sin 31^\circ \operatorname{cosec} 31^\circ \\
 &= 3 + 2 = 5
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ \\
 &= \frac{\sin(90^\circ - 10^\circ)}{\cos 10^\circ} + \sin(90^\circ - 31^\circ) \sec 31^\circ \\
 &= \frac{\cos 10^\circ}{\cos 10^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ} \\
 &= 1 + 1 = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & \tan(55^\circ - A) - \cot(35^\circ + A) \\
 &= \tan[90^\circ - (35^\circ + A)] - \cot(35^\circ + A) \\
 &= \cot(35^\circ + A) - \cot(35^\circ + A) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & \operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A) \\
 &= \operatorname{cosec}[90^\circ - (25^\circ - A)] - \sec(25^\circ - A) \\
 &= \sec(25^\circ - A) - \sec(25^\circ - A) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad & 2 \frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2} \cos 45^\circ \\
 &= 2 \frac{\tan(90^\circ - 33^\circ)}{\cot 33^\circ} - \frac{\cot(90^\circ - 20^\circ)}{\tan 20^\circ} - \sqrt{2} \left(\frac{1}{\sqrt{2}} \right) \\
 &= 2 \frac{\cot 33^\circ}{\cot 33^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 1 \\
 &= 2 - 1 - 1 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad & \frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2 \frac{\sin^2 75^\circ}{\cos^2 15^\circ} \\
 &= \frac{[\cot(90^\circ - 49^\circ)]^2}{\tan^2 49^\circ} - 2 \frac{[\sin(90^\circ - 15^\circ)]^2}{\cos^2 15^\circ}
 \end{aligned}$$

$$= \frac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2 \frac{\cos^2 15^\circ}{\cos^2 15^\circ}$$

$$= 1 - 2 = -1$$

$$\text{(viii)} \quad \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ$$

$$= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8 \left(\frac{1}{2}\right)^2$$

$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 2$$

$$= 1 + 1 - 2 = 0$$

$$\text{(ix)} \quad 14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ$$

$$= 14 \left(\frac{1}{2}\right) + 6 \left(\frac{1}{2}\right) - 5(1)$$

$$= 7 + 3 - 5 = 5$$

Question 8:

A triangle ABC is right angles at B; find the value of $\frac{\sec A \cdot \operatorname{cosec} A - \tan A \cdot \cot C}{\sin B}$

Solution 8:

Since, ABC is a right angled triangle, right angled at B.

So, $A + C = 90^\circ$

$$\frac{\sec A \cdot \operatorname{cosec} A - \tan A \cdot \cot C}{\sin B}$$

$$= \frac{\sec(90^\circ - C) \cdot \operatorname{cosec} C - \tan(90^\circ - C) \cdot \cot C}{\sin 90^\circ}$$

$$= \frac{\operatorname{cosec} C \cdot \operatorname{cosec} C - \cot C \cdot \cot C}{1}$$

$$= 1 \quad [\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1]$$

Question 9:

Find (in each case, given below) the value of x, if:

$$\text{(i)} \quad \sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$$

$$\text{(ii)} \quad \sin x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$(iii) \cos x = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$$

$$(iv) \tan x = \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$$

$$(v) \sin 2x = 2 \sin 45^\circ \cos 45^\circ$$

$$(vi) \sin 3x = 2 \sin 30^\circ \cos 30^\circ$$

$$(vii) \cos(2x - 6^\circ) = \cos^2 30^\circ - \cos^2 60^\circ$$

Solution 9:

$$(i) \sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$$

$$\sin x = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\sin x = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \sin 30^\circ$$

Hence, $x = 30^\circ$

$$(ii) \sin x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$\sin x = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\sin x = \frac{3}{4} + \frac{1}{4} = 1 = \sin 90^\circ$$

Hence, $x = 90^\circ$

$$(iii) \cos x = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$$

$$\cos x = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$\cos x = 0 = \cos 90^\circ$$

Hence, $x = 90^\circ$

$$(iv) \tan x = \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$$

$$\tan x = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}}$$

$$\tan x = \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Hence, $x = 30^\circ$

$$(v) \sin 2x = 2 \sin 45^\circ \cos 45^\circ$$

$$\sin 2x = 2 \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right)$$

$$\sin 2x = 1 = \sin 90^\circ$$

$$2x = 90^\circ$$

$$\text{Hence, } x = 45^\circ$$

$$(vi) 3x = 2 \sin 30^\circ \cos 30^\circ$$

$$\sin 3x = 2 \left(\frac{1}{2} \right) \left(\frac{\sqrt{3}}{2} \right)$$

$$\sin 3x = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$$3x = 60^\circ$$

$$\text{Hence, } x = 20^\circ$$

$$(vii) \cos(2x - 6^\circ) = \cos^2 30^\circ - \cos^2 60^\circ$$

$$\cos(2x - 6^\circ) = \cos^2(90^\circ - 60^\circ) - \cos^2 60^\circ$$

$$\cos(2x - 6^\circ) = \sin^2 60^\circ - \cos^2 60^\circ$$

$$\cos(2x - 6^\circ) = 1 - 2 \cos^2 60^\circ = 1 - 2 \left(\frac{1}{2} \right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\cos(2x - 6^\circ) = \frac{1}{2}$$

$$\cos(2x - 6^\circ) = \cos 60^\circ$$

$$(2x - 6^\circ) = 60^\circ$$

$$2x = 66^\circ$$

$$\text{Hence, } x = 33^\circ$$

Question 10:

In each case, given below, find the value of angle A, where $0^\circ \leq A \leq 90^\circ$.

$$(i) \sin(90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$$

$$(ii) \cos(90^\circ - A) \cdot \sec 77^\circ = 1$$

Solution 10:

$$(i) \sin(90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$$

$$\cos 3A \cdot \frac{1}{\sin 42^\circ} = 1$$

$$\cos 3A = \sin 42^\circ = \sin(90^\circ - 48^\circ) = \cos 48^\circ$$

$$3A = 48^\circ$$

$$A = 16^\circ$$

$$(ii) \cos(90^\circ - A) \cdot \sec 77^\circ = 1$$

$$\cos(90^\circ - A) \cdot \sec 77^\circ = 1$$

$$\sin A \cdot \frac{1}{\cos 77^\circ} = 1$$

$$\sin A = \cos 77^\circ = \cos(90^\circ - 13^\circ) = \sin 13^\circ$$

$$A = 13^\circ$$

Question 11:

Prove that:

$$(i) \frac{\cos(90^\circ - \theta) \cos \theta}{\cot \theta} = 1 - \cos^2 \theta$$

$$(ii) \frac{\sin \theta \sin(90^\circ - \theta)}{\cot(90^\circ - \theta)} = 1 - \sin^2 \theta$$

Solution 11:

$$(i) \text{ LHS} = \frac{\cos(90^\circ - \theta) \cos \theta}{\cot \theta} = \frac{\sin \theta \cos \theta}{\frac{\cos \theta}{\sin \theta}} = \sin^2 \theta = 1 - \cos^2 \theta$$

$$(ii) \text{ LHS} = \frac{\sin \theta \sin(90^\circ - \theta)}{\cot(90^\circ - \theta)} = \frac{\sin \theta \cos \theta}{\tan \theta} = \frac{\sin \theta \cos \theta}{\frac{\sin \theta}{\cos \theta}} = \cos^2 \theta = 1 - \sin^2 \theta$$

Question 12:

Evaluate:

$$\frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\cos^2 10^\circ - \tan^2 80^\circ}$$

Solution 12:

$$\begin{aligned}
 & \frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\sec^2 10^\circ - \tan^2 80^\circ} \\
 &= \frac{\sin 35^\circ \cdot \cos(90^\circ - 35^\circ) + \cos 35^\circ \cdot \sin(90^\circ - 35^\circ)}{\sec^2(90^\circ - 80^\circ) - \tan^2 80^\circ} \\
 &= \frac{\sin 35^\circ \cdot \sin 35^\circ + \cos 35^\circ \cdot \cos 35^\circ}{\sec^2 80^\circ - \tan^2 80^\circ} \\
 &= \frac{\sin^2 35^\circ + \cos^2 35^\circ}{\sec^2 80^\circ - \tan^2 80^\circ} = \frac{1}{1} = 1
 \end{aligned}$$

EXERCISE. 21 (D)**Question 1:**

Use tables to find sine of:

- (i) 21° (ii) $34^\circ 42'$
 (iii) $47^\circ 32'$ (iv) $62^\circ 57'$
 (v) $10^\circ 20' + 20^\circ 45'$

Solution 1:

- (i) $\sin 21^\circ = 0.3584$
 (ii) $\sin 34^\circ 42' = 0.5693$
 (iii) $\sin 47^\circ 32' = \sin(47^\circ 30' + 2') = 0.7373 + 0.0004 = 0.7377$
 (iv) $\sin 62^\circ 57' = \sin(62^\circ 54' + 3') = 0.8902 + 0.0004 = 0.8906$
 (v) $\sin(10^\circ 20' + 20^\circ 45') = \sin 30^\circ 65' = \sin 31^\circ 5' = 0.5150 + 0.0012 = 0.5162$

Question 2:

Use tables to find cosine of:

- (i) $2^\circ 4'$ (ii) $8^\circ 12'$
 (iii) $26^\circ 32'$ (iv) $65^\circ 41'$
 (v) $9^\circ 23' + 15^\circ 54'$

Solution 2:

- (i) $\cos 2^\circ 4' = 0.9994 - 0.0001 = 0.9993$
 (ii) $\cos 8^\circ 12' = 0.9898$
 (iii) $\cos 26^\circ 32' = \cos(26^\circ 30' + 2') = 0.8949 - 0.0003 = 0.8946$

$$(iv) \cos 65^\circ 41' = \cos (65^\circ 36' + 5') = 0.4131 - 0.0013 = 0.4118$$

$$(v) \cos (9^\circ 23' + 15^\circ 54') = \cos 24^\circ 77' = \cos 25^\circ 17' = \cos (25^\circ 12' + 5') = 0.9048 - 0.0006 = 0.9042$$

Question 3:

Use trigonometrical tables to find tangent of:

(i) 37° (ii) $42^\circ 18'$ (iii) $17^\circ 27'$

Solution 3:

(i) $\tan 37^\circ = 0.7536$

(ii) $\tan 42^\circ 18' = 0.9099$

(iii) $\tan 17^\circ 27' = \tan (17^\circ 24' + 3') = 0.3134 + 0.0010 = 0.3144$

Question 4:

Use tables to find the acute angle θ , if the value of $\sin \theta$ is:

(i) (i) 0.4848 (ii) 0.3827 (iii) 0.6525

Solution 4:

(i) From the tables, it is clear that $\sin 29^\circ = 0.4848$

Hence, $\theta = 29^\circ$

(ii) From the tables, it is clear that $\sin 22^\circ 30' = 0.3827$

Hence, $\theta = 22^\circ 30'$

(iii) From the tables, it is clear that $\sin 40^\circ 42' = 0.6521$

$\sin \theta - \sin 40^\circ 42' = 0.6525 - 0.6521 = 0.0004$

From the tables, diff of $2' = 0.0004$

Hence, $\theta = 40^\circ 42' + 2' = 40^\circ 44'$

Question 5:

Use tables to find the acute angle θ , if the value of $\cos \theta$ is:

(i) 0.9848 (ii) 0.9574 (iii) 0.6885

Solution 5:

(i) From the tables, it is clear that $\cos 10^\circ = 0.9848$

Hence, $\theta = 10^\circ$

(ii) From the tables, it is clear that $\cos 16^\circ 48' = 0.9573$

$\cos \theta - \cos 16^\circ 48' = 0.9574 - 0.9573 = 0.0001$

From the tables, diff of $1' = 0.0001$

Hence, $\theta = 16^\circ 48' - 1' = 16^\circ 47'$

(iii) From the tables, it is clear that $\cos 46^\circ 30' = 0.6884$

$\cos \theta - \cos 46^\circ 30' = 0.6885 - 0.6884 = 0.0001$

From the tables, diff of $1' = 0.0002$

Hence, $\theta = 46^\circ 30' - 1' = 46^\circ 29'$

Question 6:

Use tables to find the acute angle θ , if the value of $\tan \theta$ is:

(i) 0.2419 (ii) 0.4741 (iii) 0.7391

Solution 6:

(i) From the tables, it is clear that $\tan 13^\circ 36' = 0.2419$

Hence, $\theta = 13^\circ 36'$

(ii) From the tables, it is clear that $\tan 25^\circ 18' = 0.4727$

$\tan \theta - \tan 25^\circ 18' = 0.4741 - 0.4727 = 0.0014$

From the tables, diff of $4' = 0.0014$

Hence, $\theta = 25^\circ 18' + 4' = 25^\circ 22'$

(iii) From the tables, it is clear that $\tan 36^\circ 24' = 0.7373$

$\tan \theta - \tan 36^\circ 24' = 0.7391 - 0.7373 = 0.0018$

From the tables, diff of $4' = 0.0018$

Hence, $\theta = 36^\circ 24' + 4' = 36^\circ 28'$

EXERCISE. 21(E)

Question 1:

1. Prove the following identities:

$$(i) \frac{1}{\cos A + \sin A} + \frac{1}{\cos A - \sin A} = \frac{2 \cos A}{2 \cos^2 A - 1}$$

$$(ii) \operatorname{cosec} A - \cot A = \frac{\sin A}{1 + \cos A}$$

$$(iii) 1 - \frac{\sin^2 A}{1 + \cos A} = \cos A$$

$$(iv) \frac{1 - \cos A}{\sin A} + \frac{\sin A}{1 - \cos A} = 2 \operatorname{cosec} A$$

$$(v) \frac{\cot A}{1 - \tan A} + \frac{\tan A}{1 - \cot A} = 1 + \tan A + \cot A$$

$$(vi) \frac{\cos A}{1 - \tan A} + \tan A = \sec A$$

$$(vii) \frac{\sin A}{1 - \cos A} - \cot A = \operatorname{cosec} A$$

$$(viii) \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{\cos A}{1 - \sin A}$$

$$(ix) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{\cos A}{1 - \sin A}$$

$$(x) \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$$

$$(xi) \frac{1 + (\sec A - \tan A)^2}{\operatorname{cosec} A (\sec A - \tan A)} = 2 \tan A$$

$$(xii) \frac{(\operatorname{cosec} A - \cot A)^2 + 1}{\sec A (\operatorname{cosec} A - \cot A)} = 2 \cot A$$

$$(xiii) \cot^2 A \left(\frac{\sec A - 1}{1 + \sin A} \right) + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) = 0$$

$$(xiv) \frac{(1 - 2\sin^2 A)^2}{\cos^4 A - \sin^4 A} = 2\cos^2 A - 1$$

$$(xv) \sec^4 A (1 - \sin^4 A) - 2 \tan^2 A = 1$$

$$(xvi) \operatorname{cosec}^4 A (1 - \cos^4 A) - 2 \cot^2 A = 1$$

$$(xvii) (1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A) = 2$$

Solution 1:

$$\begin{aligned} (i) & \frac{1}{\cos A + \sin A} + \frac{1}{\cos A - \sin A} \\ &= \frac{\cos A + \sin A + \cos A - \sin A}{(\cos A + \sin A)(\cos A - \sin A)} \\ &= \frac{2 \cos A}{\cos^2 A - \sin^2 A} \\ &= \frac{2 \cos A}{\cos^2 A - (1 - \cos^2 A)} \\ &= \frac{2 \cos A}{2 \cos^2 A - 1} \end{aligned}$$

$$(ii) \operatorname{cosec} A - \cot A$$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= \frac{1 - \cos A}{\sin A}$$

$$= \frac{1 - \cos A}{\sin A} \times \frac{1 + \cos A}{1 + \cos A}$$

$$= \frac{1 - \cos^2 A}{\sin A (1 + \cos A)}$$

$$= \frac{\sin^2 A}{\sin A (1 + \cos A)}$$

$$= \frac{\sin A}{1 + \cos A}$$

$$(iii) 1 - \frac{\sin^2 A}{1 + \cos A}$$

$$= \frac{1 + \cos A - \sin^2 A}{1 + \cos A}$$

$$= \frac{\cos A + \cos^2 A}{1 + \cos A}$$

$$= \frac{\cos A (1 + \cos A)}{1 + \cos A}$$

$$= \cos A$$

$$(iv) \frac{1 - \cos A}{\sin A} + \frac{\sin A}{1 - \cos A}$$

$$= \frac{(1 - \cos A)^2 + \sin^2 A}{\sin A (1 - \cos A)}$$

$$= \frac{1 + \cos^2 A - 2 \cos A + \sin^2 A}{\sin A (1 - \cos A)}$$

$$= \frac{2 - 2 \cos A}{\sin A (1 - \cos A)}$$

$$= \frac{2(1 - \cos A)}{\sin A (1 - \cos A)}$$

$$= 2 \operatorname{cosec} A$$

$$(v) \frac{\cot A}{1 - \tan A} + \frac{\tan A}{1 - \cot A}$$

$$\begin{aligned} &= \frac{1}{\frac{\tan A}{1 - \tan A}} + \frac{\tan A}{1 - \frac{1}{\tan A}} \\ &= \frac{1}{\tan A(1 - \tan A)} + \frac{\tan^2 A}{\tan A - 1} \\ &= \frac{1 - \tan^3 A}{\tan A(1 - \tan A)} \\ &= \frac{(1 - \tan A)(1 + \tan A + \tan^2 A)}{\tan A(1 - \tan A)} \end{aligned}$$

$$= \frac{1 + \tan A + \tan^2 A}{\tan A}$$

$$= \cot A + 1 + \tan A$$

$$\text{(vi)} \quad \frac{\cos A}{1 + \sin A} + \tan A$$

$$= \frac{\cos A}{1 + \sin A} + \frac{\sin A}{\cos A}$$

$$= \frac{\cos^2 A + \sin A + \sin^2 A}{(1 + \sin A)\cos A}$$

$$= \frac{1 + \sin A}{(1 + \sin A)\cos A}$$

$$= \frac{1}{\cos A}$$

$$= \sec A$$

$$\text{(vii)} \quad \frac{\sin A}{1 - \cos A} - \cot A$$

$$= \frac{\sin A}{1 - \cos A} - \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A - \cos A + \cos^2 A}{(1 - \cos A)\sin A}$$

$$= \frac{1 - \cos A}{(1 - \cos A)\sin A}$$

$$= \frac{1}{\sin A}$$

$$= \operatorname{cosec} A$$

$$\begin{aligned}
 & \text{(viii)} \quad \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \\
 &= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{\sin A - (\cos A - 1)}{\sin A - (\cos A - 1)} \\
 &= \frac{(\sin A - \cos A + 1)^2}{\sin^2 A - (\cos A - 1)^2} \\
 &= \frac{\sin^2 A + \cos^2 A + 1 - 2\sin A \cos A - 2\cos A + 2\sin A}{\sin^2 A - \cos^2 A - 1 + 2\cos A} \\
 &= \frac{1 + 1 - 2\sin A \cos A - 2\cos A + 2\sin A}{-\cos^2 A - \cos^2 A + 2\cos A} \\
 &= \frac{2(1 - \cos A) + 2\sin(1 - \cos A)}{2\cos A(1 - \cos A)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1 + \sin A}{\cos A} \\
 &= \frac{1 + \sin A}{\cos A} \times \frac{1 - \sin A}{1 - \sin A} \\
 &= \frac{\cos^2 A}{\cos A(1 - \sin A)} \\
 &= \frac{\cos A}{1 - \sin A}
 \end{aligned}$$

$$\begin{aligned}
 & \text{(ix)} \quad \sqrt{\frac{1 + \sin A}{1 - \sin A}} \\
 &= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 - \sin A}{1 - \sin A}} \\
 &= \sqrt{\frac{1 - \sin^2 A}{(1 - \sin A)^2}} \\
 &= \sqrt{\frac{\cos^2 A}{(1 + \sin A)^2}} \\
 &= \frac{\cos A}{1 - \sin A}
 \end{aligned}$$

$$\begin{aligned}
 & \text{(x)} \quad \sqrt{\frac{1 - \cos A}{1 + \cos A}} \\
 &= \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 + \cos A}{1 + \cos A}}
 \end{aligned}$$

$$= \sqrt{\frac{1 - \cos^2 A}{(1 + \cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1 + \cos A)^2}}$$

$$= \frac{\sin A}{1 + \cos A}$$

$$(xi) \frac{1 + (\sec A - \tan A)^2}{\operatorname{cosec} A (\sec A - \tan A)}$$

$$= \frac{(\sec^2 A - \tan^2 A) + (\sec A - \tan A)^2}{\operatorname{cosec} A (\sec A - \tan A)}$$

$$= \frac{(\sec A - \tan A)(\sec A + \tan A) + (\sec A - \tan A)^2}{\operatorname{cosec} A (\sec A - \tan A)}$$

$$= \frac{(\sec A + \tan A) + (\sec A - \tan A)}{\operatorname{cosec} A}$$

$$= \frac{2 \sec A}{\operatorname{cosec} A}$$

$$= 2 \frac{\frac{1}{\sin A}}{\frac{1}{\sin A}}$$

$$= 2 \tan A$$

$$(xii) \frac{(\operatorname{cosec} A - \cot A)^2 + 1}{\sec A (\operatorname{cosec} A - \cot A)}$$

$$= \frac{(\operatorname{cosec} A - \cot A)^2 + (\operatorname{cosec}^2 A - \cot^2 A)}{\sec A (\operatorname{cosec} A - \cot A)}$$

$$= \frac{(\operatorname{cosec} A - \cot A)^2 + (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)}{\sec A (\operatorname{cosec} A - \cot A)}$$

$$= \frac{(\operatorname{cosec} A - \cot A) + (\operatorname{cosec} A + \cot A)}{\sec A}$$

$$= \frac{2 \operatorname{cosec} A}{\sec A}$$

$$= 2 \cot A$$

$$\begin{aligned}
 & \text{(xiii) } \cot^2 A \left(\frac{\sec A - 1}{1 + \sin A} \right) + \sec^2 A \left(\frac{\sec A - 1}{1 + \sec A} \right) \\
 &= \cot^2 A \left(\frac{\sec A - 1}{1 + \sin A} \times \frac{\sec A + 1}{\sec A + 1} \right) + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) \\
 &= \cot^2 A \left[\frac{\sec^2 A - 1}{(1 + \sin A)(\sec A + 1)} \right] + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) \\
 &= \cot^2 A \left[\frac{\tan^2 A}{(1 + \sin A)(\sec A + 1)} \right] + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) \\
 &= \frac{1}{(1 + \sin A)(\sec A + 1)} + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) \\
 &= \frac{1 + \sec^2 A (\sin A - 1)(1 + \sin A)}{(1 + \sin A)(\sec A + 1)} \\
 &= \frac{1 + \sec^2 A (\sin^2 A - 1)}{(1 + \sin A)(\sec A + 1)} \\
 &= \frac{1 + \sec^2 A (-\cos^2 A)}{(1 + \sin A)(\sec A + 1)} \\
 &= \frac{1 - 1}{(1 + \sin A)(\sec A + 1)} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 & \text{(xiv) } \frac{(1 - 2\sin^2 A)^2}{\cos^4 A - \sin^4 A} \\
 &= \frac{(1 - 2\sin^2 A)^2}{(\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)} \\
 &= \frac{(1 - 2\sin^2 A)^2}{1 - \sin^2 A - \sin^2 A} \\
 &= \frac{(1 - 2\sin^2 A)^2}{1 - 2\sin^2 A} \\
 &= 1 - 2\sin^2 A \\
 &= 1 - 2(1 - \cos^2 A) \\
 &= 2\cos^2 A - 1 \\
 & \text{(xv) } \sec^4 A (1 - \sin^4 A) - 2\tan^2 A
 \end{aligned}$$

$$= \sec^4 A (1 - \sin^2 A)(1 + \sin^2 A) - 2 \tan^2 A$$

$$= \sec^4 A (\cos^2 A)(1 + \sin^2 A) - 2 \tan^2 A$$

$$= \sec^2 A + \frac{\sin^2 A}{\cos^2 A} - 2 \tan^2 A$$

$$= \sec^2 A + \tan^2 A - 2 \tan^2 A$$

$$= \sec^2 A - \tan^2 A$$

$$= 1$$

$$(xvi) \operatorname{cosec}^4 A (1 - \cos^4 A) - 2 \cot^2 A$$

$$= \operatorname{cosec}^4 A (1 - \cos^2 A)(1 + \cos^2 A) - 2 \cot^2 A$$

$$= \operatorname{cosec}^4 A (\sin^2 A)(1 + \cos^2 A) - 2 \cot^2 A$$

$$= \operatorname{cosec}^2 A (1 + \cos^2 A) - 2 \cot^2 A$$

$$= \operatorname{cosec}^2 A + \frac{\cos^2 A}{\sin^2 A} - 2 \cot^2 A$$

$$= \operatorname{cosec}^2 A + \cot^2 A - 2 \cot^2 A$$

$$= \operatorname{cosec}^2 A - \cot^2 A$$

$$= 1$$

$$(xvii) (1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A)$$

$$= 1 + \cot A - \operatorname{cosec} A + \tan A + 1 - \sec A +$$

$$\sec A + \operatorname{cosec} A - \operatorname{cosec} A \sec A$$

$$= 2 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A} - \frac{1}{\sin A \cos A}$$

$$= 2 + \frac{\cos^2 A + \sin^2 A}{\sin A \cos A} - \frac{1}{\sin A \cos A}$$

$$= 2 + \frac{1}{\sin A \cos A} - \frac{1}{\sin A \cos A}$$

$$= 2$$

Question 2:

If $\sin A + \cos A = p$

and $\sec A + \operatorname{cosec} A = q$, then prove that: $q(p^2 - 1) = 2p$

Solution 2:

$$q(p^2 - 1) = (\sec A + \operatorname{cosec} A) [(\sin A + \cos A)^2 - 1]$$

$$\begin{aligned}
 &= (\sec A + \operatorname{cosec} A) \left[(\sin^2 A + \cos^2 A + 2 \sin A \cos A) - 1 \right] \\
 &= (\sec A + \operatorname{cosec} A) \left[(1 + 2 \sin A \cos A) - 1 \right] \\
 &= (\sec A + \operatorname{cosec} A) (2 \sin A \cos A) \\
 &= 2 \sin A + 2 \cos A \\
 &= 2P
 \end{aligned}$$

Question 3:

If $x = a \cos \theta$ and $y = b \cot \theta$, show that:

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

Solution 3:

$$\begin{aligned}
 &\frac{a^2}{x^2} - \frac{b^2}{y^2} \\
 &= \frac{a^2}{a^2 \cos^2 \theta} - \frac{b^2}{b^2 \cot^2 \theta} \\
 &= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \\
 &= \frac{1 - \sin^2 \theta}{\cos^2 \theta} \\
 &= \frac{\cos^2 \theta}{\cos^2 \theta} \\
 &= 1
 \end{aligned}$$

Question 4:

If $\sec A + \tan A = p$, show that:

$$\sin A = \frac{p^2 - 1}{p^2 + 1}$$

Solution 4:

$$\begin{aligned}
 &\frac{p^2 - 1}{p^2 + 1} \\
 &= \frac{(\sec A + \tan A)^2 - 1}{(\sec A + \tan A)^2 + 1}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{\sec^2 A + \tan^2 A + 2 \tan A \sec A - 1}{\sec^2 A + \tan^2 A + 2 \tan A \sec A - 1} \\
&= \frac{\tan^2 A + \tan^2 A + 2 \tan A \sec A}{\sec^2 A + \sec^2 A + 2 \tan A \sec A} \\
&= \frac{2 \tan^2 A + 2 \tan A \sec A}{2 \sec^2 A + 2 \tan A \sec A} \\
&= \frac{2 \tan A (\tan A + \sec A)}{2 \sec A (\tan A + \sec A)} \\
&= \sin A
\end{aligned}$$

Question 5:

If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that:

$$\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

Solution 5:

Given that, $\tan A = n \tan B$ and $\sin A = m \sin B$.

$$\Rightarrow n = \frac{\tan A}{\tan B} \text{ and } m = \frac{\sin A}{\sin B}$$

$$\begin{aligned}
&\therefore \frac{m^2 - 1}{n^2 - 1} \\
&= \frac{\left(\frac{\sin A}{\sin B}\right)^2 - 1}{\left(\frac{\tan A}{\tan B}\right)^2 - 1} \\
&= \frac{\tan^2 B (\sin^2 A - \sin^2 B)}{\sin^2 B (\tan^2 A - \tan^2 B)} \\
&= \frac{\sin^2 A - \sin^2 B}{\cos^2 B \left(\frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}\right)} \\
&= \frac{\cos^2 A (\sin^2 A - \sin^2 B)}{\sin^2 A \cos^2 B - (1 - \cos^2 B) \cos^2 A}
\end{aligned}$$

$$\begin{aligned}
 &= \frac{\cos^2 A (1 - \cos^2 A - 1 + \cos^2 B)}{\cos^2 B (\sin^2 A + \cos^2 A) - \cos^2 A} \\
 &= \frac{\cos^2 A (\cos^2 B - \cos^2 A)}{\cos^2 B - \cos^2 A} \\
 &= \cos^2 A
 \end{aligned}$$

Question 6:

(i) If $2 \sin A - 1 = 0$, show that:

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

(ii) If $4 \cos^2 A - 3 = 0$, Show that:

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

Solution 6:

(i) $2 \sin A - 1 = 0$

$$\Rightarrow \sin A = \frac{1}{2}$$

$$\text{We know } \sin 30^\circ = \frac{1}{2}$$

$$\text{So, } A = 30^\circ$$

$$\text{LHS} = \sin 3A = \sin 90^\circ = 1$$

$$\begin{aligned}
 \text{RHS} &= 3 \sin A - 4 \sin^3 A \\
 &= 3 \sin 30^\circ - 4 \sin^3 30^\circ
 \end{aligned}$$

$$= 3 \left(\frac{1}{2} \right) - 4 \left(\frac{1}{2} \right)^3$$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

$$\text{LHS} = \text{RHS}$$

(ii) $4 \cos^2 A - 3 = 0$

$$\Rightarrow 4 \cos^2 A = 3$$

$$\Rightarrow \cos^2 A = \frac{3}{4}$$

$$\Rightarrow \cos A = \frac{\sqrt{3}}{2}$$

$$\text{We know } \cos 30^\circ = \frac{\sqrt{3}}{2}$$

So, $A = 30^\circ$

$$\text{LHS} = \cos 3A = \cos 90^\circ = 0$$

$$\text{RHS} = 4\cos^3 A - 3\cos A$$

$$= 4\cos^3 30^\circ - 3\cos 30^\circ$$

$$= 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

LHS = RHS

Question 7:

Evaluate

$$(i) \ 2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right) + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$$

$$(ii) \ \sec 26^\circ \sin 64^\circ + \frac{\operatorname{cosec} 33^\circ}{\sec 57^\circ}$$

$$(iii) \ \frac{5\sin 66^\circ}{\cos 24^\circ} - \frac{2\cot 85^\circ}{\tan 5^\circ}$$

$$(iv) \ \cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ$$

$$(v) \ \sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$$

$$(vi) \ \frac{3\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$

$$(vii) \ 3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$$

$$(viii) \ \frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$$

Solution 7:

$$(i) \ 2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right) + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$$

$$= 2\left(\frac{\tan(90^\circ - 55^\circ)}{\cot 55^\circ}\right) + \left(\frac{\cot(90^\circ - 35^\circ)}{\tan 35^\circ}\right) - 3\left(\frac{\sec(90^\circ - 50^\circ)}{\operatorname{cosec} 50^\circ}\right)$$

$$= 2\left(\frac{\cot 55^\circ}{\cot 55^\circ}\right) + \left(\frac{\tan 35^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\operatorname{cosec} 50^\circ}{\operatorname{cosec} 50^\circ}\right)$$

$$= 2(1)^2 + 1^2 + -3$$

$$= 2 + 1 - 3$$

$$= 0$$

$$(ii) \sec 26^\circ \sin 64^\circ + \frac{\operatorname{cosec} 33^\circ}{\sec 57^\circ}$$

$$= \sec(90^\circ - 64^\circ) \sin 64^\circ + \frac{\operatorname{cosec}(90^\circ - 57^\circ)}{\sec 57^\circ}$$

$$= \operatorname{cosec} 64^\circ \sin 64^\circ + \frac{\sec 57^\circ}{\sec 57^\circ}$$

$$= 1 + 1 = 2$$

$$(iii) \frac{5 \sin 66^\circ}{\cos 24^\circ} - \frac{2 \cot 85^\circ}{\tan 5^\circ}$$

$$= \frac{5 \sin(90^\circ - 24^\circ)}{\cos 24^\circ} - \frac{2 \cot(90^\circ - 5^\circ)}{\tan 5^\circ}$$

$$= \frac{5 \cos 24^\circ}{\cos 24^\circ} - \frac{2 \tan 5^\circ}{\tan 5^\circ}$$

$$= 5 - 2 = 3$$

$$(iv) \cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ$$

$$= \cos(90^\circ - 50^\circ) \operatorname{cosec} 50^\circ + \sin(90^\circ - 40^\circ) \sec 40^\circ$$

$$= \sin 50^\circ \operatorname{cosec} 50^\circ + \cos 40^\circ \sec 40^\circ$$

$$= 1 + 1 = 2$$

$$(v) \sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$$

$$= \sin(90^\circ - 63^\circ) \sin 63^\circ - \cos 63^\circ \cos(90^\circ - 63^\circ)$$

$$= \cos 63^\circ \sin 63^\circ - \cos 63^\circ \sin 63^\circ$$

$$= 0$$

$$(vi) \frac{3 \sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$

$$= \frac{3 \sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ}$$

$$= \frac{3 \cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ}$$

$$= 3 - 1 = 2$$

$$(vii) 3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$$

$$= 3 \cos(90^\circ - 10^\circ) \operatorname{cosec} 10^\circ + 2 \cos(90^\circ - 31^\circ) \operatorname{cosec} 31^\circ$$

$$= 3\sin 10^\circ \operatorname{cosec} 10^\circ + 2\sin 31^\circ \operatorname{cosec} 31^\circ$$

$$= 3 + 2 = 5$$

$$\text{(viii)} \quad \frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$$

$$= \frac{\cos(90^\circ - 15^\circ)}{\sin 15^\circ} + \frac{\sin(90^\circ - 78^\circ)}{\cos 78^\circ} - \frac{\cos(90^\circ - 72^\circ)}{\sin 72^\circ}$$

$$= \frac{\sin 15^\circ}{\sin 15^\circ} + \frac{\cos 78^\circ}{\cos 78^\circ} - \frac{\sin 72^\circ}{\sin 72^\circ}$$

$$= 1 + 1 - 1 = 1$$

Question 8:

Prove that:

$$\text{(i)} \quad \tan(55^\circ + x) = \cot(35^\circ - x)$$

$$\text{(ii)} \quad \sec(70^\circ - \theta) = \operatorname{cosec}(20^\circ + \theta)$$

$$\text{(iii)} \quad \sin(28^\circ + A) = \cos(62^\circ - A)$$

$$\text{(iv)} \quad \frac{1}{1 + \cos(90^\circ - A)} + \frac{1}{1 - \cos(90^\circ - A)} = 2 \operatorname{cosec}^2(90^\circ - A)$$

$$\text{(v)} \quad \frac{1}{1 + \sin(90^\circ - A)} + \frac{1}{1 - \sin(90^\circ - A)} = 2 \sec^2(90^\circ - A)$$

Solution 8:

$$\text{(i)} \quad \tan(55^\circ + x) = \tan[90^\circ - (35^\circ - x)] = \cot(35^\circ - x)$$

$$\text{(ii)} \quad \sec(70^\circ - \theta) = \sec[90^\circ - (20^\circ + \theta)] = \operatorname{cosec}(20^\circ + \theta)$$

$$\text{(iii)} \quad \sin(28^\circ + A) = \sin[90^\circ - 62^\circ - A] = \cos(62^\circ - A)$$

$$\text{(iv)} \quad \frac{1}{1 + \cos(90^\circ - A)} + \frac{1}{1 - \cos(90^\circ - A)}$$

$$= \frac{1}{1 + \sin A} + \frac{1}{1 - \sin A}$$

$$= \frac{1 - \sin A + 1 + \sin A}{(1 + \sin A)(1 - \sin A)}$$

$$= \frac{2}{1 - \sin^2 A}$$

$$= \frac{2}{\cos^2 A}$$

$$= 2 \sec^2 A$$

$$= 2 \operatorname{cosec}^2 (90^\circ - A)$$

$$(v) \frac{1}{1 + \sin(90^\circ - A)} + \frac{1}{1 - \sin(90^\circ - A)}$$

$$= \frac{1}{1 + \cos A} + \frac{1}{1 - \cos A}$$

$$= \frac{1 - \cos A + 1 + \cos A}{(1 + \cos A)(1 - \cos A)}$$

$$= \frac{2}{1 - \cos^2 A}$$

$$= 2 \operatorname{cosec}^2 A$$

$$= 2 \sec^2 (90^\circ - A)$$

Question 9:

If A and B are complementary angles, prove that:

$$(i) \cot B + \cos B = \sec A \cos B (1 + \sin B)$$

$$(ii) \cot A \cot B - \sin A \cos B - \cos A \sin B = 0$$

$$(iii) \operatorname{cosec}^2 A + \operatorname{cosec}^2 B = \operatorname{cosec}^2 A \operatorname{cosec}^2 B$$

$$(iv) \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos B - \cos A}{\cos B + \cos A} = \frac{2}{2 \sin^2 A - 1}$$

Solution 9:

Since, A and B are complementary angles, $A + B = 90^\circ$

$$(i) \cot B + \cos B$$

$$= \cot(90^\circ - A) + \cos(90^\circ - A)$$

$$= \tan A + \sin A$$

$$= \frac{\sin A}{\cos A} + \sin A$$

$$= \frac{\sin A + \sin A \cos A}{\cos A}$$

$$= \frac{\sin A (1 + \cos A)}{\cos A}$$

$$= \sec A \sin A (1 + \cos A)$$

$$= \sec A \sin (90^\circ - B) [1 + \cos (90^\circ - B)]$$

$$= \sec A \cos B (1 + \sin B)$$

$$(ii) \cot A \cot B - \sin A \cos B - \cos A \sin B$$

$$= \cot A \cot (90 - A) - \sin A \cos (90 - A) - \cos A \sin (90 - A)$$

$$= \cot A \tan A - \sin A \sin A - \cos A \cos A$$

$$= 1 - (\sin^2 A + \cos^2 A)$$

$$= 1 - 1$$

$$= 0$$

$$(iii) \operatorname{cosec}^2 A + \operatorname{cosec}^2 B$$

$$= \operatorname{cosec}^2 A + [\operatorname{cosec} (90 - A)]^2$$

$$= \operatorname{cosec}^2 A + \sec^2 A$$

$$= \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$$

$$= \frac{\cos^2 A + \sin^2 A}{\sin^2 A \cos^2 A}$$

$$= \frac{1}{\sin^2 A \cos^2 A}$$

$$= \operatorname{cosec}^2 A [\sec (90^\circ - B)]^2$$

$$= \operatorname{cosec}^2 A \operatorname{cosec}^2 B$$

$$(iv) \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos B - \cos A}{\cos B + \cos A}$$

$$= \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos (90^\circ - A) - \cos (90^\circ - B)}{\cos (90^\circ - A) + \cos (90^\circ - B)}$$

$$= \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\sin A - \sin B}{\sin A + \sin B}$$

$$= \frac{(\sin A + \sin B)^2 + (\sin A - \sin B)^2}{(\sin A - \sin B)(\sin A + \sin B)}$$

$$= \frac{\sin^2 A + \sin^2 B + 2 \sin A \sin B + \sin^2 A + \sin^2 B - 2 \sin A \sin B}{\sin^2 A - \sin^2 B}$$

$$= 2 \frac{\sin^2 A + \sin^2 B}{\sin^2 A - \sin^2 B}$$

$$\begin{aligned}
 &= 2 \frac{\sin^2 A + \sin^2 (90^\circ - A)}{\sin^2 A - \sin^2 (90^\circ - A)} \\
 &= 2 \frac{\sin^2 A + \cos^2 B}{\sin^2 A - \cos^2 B} \\
 &= \frac{2}{\sin^2 A - (1 - \sin^2 A)} \\
 &= \frac{2}{2\sin^2 A - 1}
 \end{aligned}$$

Question 10:

Prove that

- (i) $\frac{1}{\sin A - \cos A} - \frac{1}{\sin A + \cos A} = \frac{2 \cos A}{2 \sin^2 A - 1}$
- (ii) $\frac{\cot^2 A}{\operatorname{cosec} A - 1} - 1 = \operatorname{cosec} A$
- (iii) $\frac{\cos A}{1 + \sin A} = \sec A - \tan A$
- (iv) $\cos A (1 + \cot A) + \sin A (1 + \tan A) = \sec A + \operatorname{cosec} A$
- (v) $(\sin A - \cos A)(1 + \tan A + \cot A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$
- (vi) $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$
- (vii) $(\sin A + \cos A)(\sec A + \operatorname{cosec} A) = 2 + \sec A \operatorname{cosec} A$
- (viii) $(\tan A + \cot A)(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = 1$
- (ix) $\cot^2 A - \cot^2 B = \frac{\cos^2 A - \cos^2 B}{\sin^2 A \sin^2 B} = \operatorname{cosec}^2 A - \operatorname{cosec}^2 B$

Solution 10:

$$\begin{aligned}
 \text{(i)} \quad & \frac{1}{\sin A - \cos A} - \frac{1}{\sin A + \cos A} \\
 &= \frac{\sin A + \cos A - \sin A + \cos A}{(\sin A - \cos A)(\sin A + \cos A)} \\
 &= \frac{2 \cos A}{\sin^2 A - \cos^2 A}
 \end{aligned}$$

$$= \frac{2 \cos A}{\sin^2 A - (1 - \sin^2 A)}$$

$$= \frac{2 \cos A}{2 \sin^2 A - 1}$$

$$(ii) \frac{\cot^2 A}{\operatorname{cosec} A - 1} - 1$$

$$= \frac{\cot^2 A - \operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}$$

$$= \frac{-\operatorname{cosec} A + \operatorname{cosec}^2 A}{\operatorname{cosec} A - 1}$$

$$= \frac{\operatorname{cosec} A (\operatorname{cosec} A - 1)}{\operatorname{cosec} A - 1}$$

$$= \operatorname{cosec} A$$

$$(iii) \frac{\cos A}{1 + \sin A}$$

$$= \frac{\cos A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A}$$

$$= \frac{\cos A (1 - \sin A)}{1 - \sin^2 A}$$

$$= \frac{\cos A (1 - \sin A)}{\cos^2 A}$$

$$= \frac{1 - \sin A}{\cos A}$$

$$= \sec A - \tan A$$

$$(iv) \cos A (1 + \cot A) + \sin A (1 + \tan A)$$

$$= \cos A + \frac{\cos^2 A}{\sin A} + \sin A + \frac{\sin^2 A}{\cos A}$$

$$= \sin A + \frac{\cos^2 A}{\sin A} + \cos A + \frac{\sin^2 A}{\cos A}$$

$$= \left(\frac{\cos^2 A + \sin^2 A}{\sin A} \right) + \left(\frac{\cos^2 A + \sin^2 A}{\cos A} \right)$$

$$= \frac{1}{\sin A} + \frac{1}{\cos A}$$

$$= \operatorname{cosec} A + \sec A$$

$$(v) (\sin A - \cos A)(1 + \tan A + \cot A)$$

$$= \sin A + \frac{\sin^2 A}{\cos A} + \cos A - \cos A - \sin A - \frac{\cos^2 A}{\sin A}$$

$$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

$$= \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$$

$$\text{(vi) LHS} = \sqrt{\sec^2 A + \operatorname{cosec}^2 A}$$

$$= \sqrt{\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}}$$

$$= \sqrt{\frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A}}$$

$$= \sqrt{\frac{1}{\sin^2 A \cos^2 A}}$$

$$= \sqrt{\frac{1}{\sin A \cos A}}$$

$$\text{RHS} = \tan A + \cot A$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{1}{\sin A \cos A}$$

$$\text{LHS} = \text{RHS}$$

$$\text{(vii) } (\sin A + \cos A)(\sec A + \operatorname{cosec} A)$$

$$= \frac{\sin A}{\cos A} + 1 + 1 + \frac{\cos A}{\sin A}$$

$$= 2 + \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$$

$$= 2 + \frac{1}{\sin A \cos A}$$

$$= 2 + \sec A \operatorname{cosec} A$$

$$\text{(viii) } (\tan A + \cot A)(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

$$= \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right) \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right)$$

$$\begin{aligned}
&= \left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \right) \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \\
&= \left(\frac{1}{\sin A \cos A} \right) \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right) \\
&= 1 \\
&\text{(ix) } \cot^2 A - \cot^2 B \\
&= \frac{\cos^2 A}{\sin^2 A} - \frac{\cos^2 B}{\sin^2 B} \\
&= \frac{\cos^2 A \sin^2 B - \cos^2 B \sin^2 A}{\sin^2 A \sin^2 B} \\
&= \frac{\cos^2 A (1 - \cos^2 B) - \cos^2 B (1 - \cos^2 A)}{\sin^2 A \sin^2 B} \\
&= \frac{\cos^2 A - \cos^2 A \cos^2 B - \cos^2 B + \cos^2 B \cos^2 A}{\sin^2 A \sin^2 B} \\
&= \frac{\cos^2 A - \cos^2 B}{\sin^2 A \sin^2 B} \\
&= \frac{1 - \sin^2 A - 1 + \sin^2 B}{\sin^2 A \sin^2 B} \\
&= \frac{-\sin^2 A + \sin^2 B}{\sin^2 A \sin^2 B} \\
&= \frac{\sin^2 B}{\sin^2 A \sin^2 B} - \frac{\sin^2 A}{\sin^2 A \sin^2 B} \\
&= \frac{1}{\sin^2 A} - \frac{1}{\sin^2 B} \\
&= \operatorname{cosec}^2 A - \operatorname{cosec}^2 B
\end{aligned}$$

Question 11:

If $4 \cos^2 A - 3 = 0$ and $0 \leq A \leq 90^\circ$, then prove that:

(i) $\sin 3A = 3 \sin A - 4 \sin^3 A$

(ii) $\cos 3A = 4 \cos^3 A - 3 \cos A$

Solution 11:

$$4 \cos^2 A - 3 = 0$$

$$\cos A = \frac{\sqrt{3}}{2}$$

We know $\cos 30^\circ = \frac{\sqrt{3}}{2}$

So, $A = 30^\circ$

$$(i) \text{ LHS} = \sin 3A = \sin 90^\circ = 1$$

$$\begin{aligned} \text{RHS} &= 3\sin A - 4\sin^3 A \\ &= 3\sin 30^\circ - 4\sin^3 30^\circ \\ &= 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3 \\ &= \frac{3}{2} - \frac{1}{2} \\ &= 1 \end{aligned}$$

LHS = RHS

$$(ii) \text{ LHS} = \cos 3A = \cos 90^\circ = 0$$

$$\begin{aligned} \text{RHS} &= 4\cos^3 A - 3\cos A \\ &= 4\cos^3 30^\circ - 3\cos 30^\circ \\ &= 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0 \end{aligned}$$

LHS = RHS

Question 12:

Find A, if $0^\circ \leq A \leq 90^\circ$ and:

$$(i) 2\cos^2 A - 1 = 0$$

$$(ii) \sin 3A - 1 = 0$$

$$(iii) 4\sin^2 A - 3 = 0$$

$$(iv) \cos^2 A - \cos A = 0$$

$$(v) 2\cos^2 A + \cos A - 1 = 0$$

Solution 12:

$$(i) 2\cos^2 A - 1 = 0$$

$$\Rightarrow \cos^2 A = \frac{1}{2}$$

$$\Rightarrow \cos A = \frac{1}{\sqrt{2}}$$

We know $\cos 45^\circ = \frac{1}{\sqrt{2}}$

Hence, $A = 45^\circ$

(ii) $\sin 3A - 1 = 0$

$$\Rightarrow \sin 3A = 1$$

We know $\sin 90^\circ = 1$

$$\therefore 3A = 90^\circ$$

Hence, $A = 30^\circ$

(iii) $4\sin^2 A - 3 = 0$

$$\Rightarrow \sin^2 A = \frac{3}{4}$$

$$\Rightarrow \sin A = \frac{\sqrt{3}}{2}$$

We know $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, $A = 60^\circ$

(iv) $\cos^2 A - \cos A = 0$

$$\Rightarrow \cos A (\cos A - 1) = 0$$

$$\Rightarrow \cos A = 0 \quad \text{Or} \quad \cos A = 1$$

We know $\cos 90^\circ = 0$ and $\cos 0^\circ = 1$

Hence, $A = 90^\circ$ or 0°

(v) $2\cos^2 A + \cos A - 1 = 0$

$$\Rightarrow 2\cos^2 A + 2\cos A - \cos A - 1 = 0$$

$$\Rightarrow 2\cos A (\cos A + 1) - 1(\cos A + 1) = 0$$

$$\Rightarrow (2\cos A - 1)(\cos A + 1) = 0$$

$$\Rightarrow \cos A = \frac{1}{2} \quad \text{or} \quad \cos A = -1$$

We know $\cos 60^\circ = \frac{1}{2}$

We also know that for no value of $A (0^\circ \leq A \leq 90^\circ)$, $\cos A = -1$.

Hence, $A = 60^\circ$

Question 13:

If $0^\circ < A < 90^\circ$; Find A, if:

$$(i) \frac{\cos A}{1 - \sin A} + \frac{\cos A}{1 + \sin A} = 4$$

$$(ii) \frac{\sin A}{\sec A - 1} + \frac{\sin A}{\sec A + 1} = 2$$

Solution 13:

$$(i) \frac{\cos A}{1 - \sin A} + \frac{\cos A}{1 + \sin A} = 4$$

$$\Rightarrow \frac{\cos A + \cos A \sin A + \cos A - \sin A \cos A}{(1 - \sin A)(1 + \sin A)} = 4$$

$$\Rightarrow \frac{2 \cos A}{1 - \sin^2 A} = 4$$

$$\Rightarrow \frac{2 \cos A}{\cos^2 A} = 4$$

$$\Rightarrow \frac{1}{\cos A} = 2$$

$$\Rightarrow \cos A = \frac{1}{2}$$

We know $\cos 60^\circ = \frac{1}{2}$

Hence, $A = 60^\circ$

$$(ii) \frac{\sin A}{\sec A - 1} + \frac{\sin A}{\sec A + 1} = 2$$

$$\Rightarrow \frac{\sin A \sec A + \sin A + \sec A \sin A - \sin A}{(\sec A - 1)(\sec A + 1)} = 2$$

$$\Rightarrow \frac{2 \sin A \sec A}{\sec^2 A - 1} = 2$$

$$\Rightarrow \frac{\sin A \sec A}{\tan^2 A} = 1$$

$$\Rightarrow \frac{\cos A}{\sin A} = 1$$

$$\Rightarrow \cot A = 1$$

We know $\cot 45^\circ = 1$

Hence, $A = 45^\circ$

Question 14:

Prove that:

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)\sec^2 A = \tan A$$

Solution 14:

L.H.S,

$$\begin{aligned} & (\operatorname{cosec} A - \sin A)(\sec A - \cos A)\sec^2 A \\ &= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \sec^2 A \\ &= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \sec^2 A \\ &= \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right) \sec^2 A \\ &= \frac{\sin A}{\cos A} = \tan A = \text{R.H.S} \end{aligned}$$