Co-ordinate Geometry

Exercise – 6.1

Solution 1(i):

Let $A \equiv (x_1, y_1)$ and $B \equiv (x_2, y_2)$.

Here $x_1 = 3, y_1 = -4$ and $x_2 = -5, y_2 = 6$

$\therefore$ Using the distance formula,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore AB = \sqrt{(-5 - 3)^2 + (6 - (-4))^2}$$

$$\therefore AB = \sqrt{(-8)^2 + (10)^2}$$

$$\therefore AB = \sqrt{64 + 100}$$

$$\therefore AB = \sqrt{164}$$

$$\therefore AB = 2\sqrt{41}$$

$\therefore$ Distance between the two points A and B is $2\sqrt{41}$ units.

Solution 1(ii):

Let $G \equiv (x_1, y_1)$ and $H \equiv (x_2, y_2)$.

Here $x_1 = 10, y_1 = -8$ and $x_2 = -3, y_2 = -2$

$\therefore$ Using the distance formula,

$$GH = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore GH = \sqrt{(-3 - 10)^2 + (-2 - (-8))^2}$$

$$\therefore GH = \sqrt{(-13)^2 + (6)^2}$$

$$\therefore GH = \sqrt{169 + 36}$$

$$\therefore GH = \sqrt{205}$$

$\therefore$ Distance between the two points G and H is $\sqrt{205}$ units.

Solution 1(iii):

Let K ≡ (x_1, y_1) and L ≡ (x_2, y_2).

Here $x_1 = 0, y_1 = -5$ and $x_2 = -5, y_2 = 0$

∴ Using the distance formula,

$$KL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore KL = \sqrt{(-5 - 0)^2 + (0 - (-5))^2}$$

$$\therefore KL = \sqrt{(-5)^2 + (5)^2}$$

$$\therefore KL = \sqrt{25 + 25}$$

$$\therefore KL = \sqrt{50}$$

$$\therefore KL = 5\sqrt{2}$$

∴ Distance between the two points K and L is $5\sqrt{2}$ units.

Solution 1(iv):

Let I ≡ (x_1, y_1) and J ≡ (x_2, y_2).

Here $x_1 = 3.5, y_1 = 6.8$ and $x_2 = 1.5, y_2 = 2.8$

∴ Using the distance formula,

$$IJ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore IJ = \sqrt{(1.5 - 3.5)^2 + (2.8 - 6.8)^2}$$

$$\therefore IJ = \sqrt{(-2)^2 + (-4)^2}$$

$$\therefore IJ = \sqrt{4 + 16}$$

$$\therefore IJ = \sqrt{20}$$

$$\therefore IJ = 2\sqrt{5}$$

∴ Distance between the two points I and J is $2\sqrt{5}$ units.

Solution 1(v):

Let $M \equiv (x_1, y_1)$ and $N \equiv (x_2, y_2)$.

Here $x_1 = 0, y_1 = 0$ and $x_2 = -8, y_2 = -7$

$\therefore$ Using the distance formula,

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore MN = \sqrt{(-8 - 0)^2 + (-7 - 0)^2}$$

$$\therefore MN = \sqrt{(-8)^2 + (-7)^2}$$

$$\therefore MN = \sqrt{64 + 49}$$

$$\therefore MN = \sqrt{113}$$

$\therefore$ Distance between the two points M and N is $\sqrt{113}$ units.

Solution 2:

Let $A \equiv (5, 11)$, $B \equiv (-5, 13)$ and $C \equiv (3, 1)$

$\therefore$ Using the distance formula,

$$AB = \sqrt{(-5 - 5)^2 + (13 - 11)^2}$$

$$\therefore AB = \sqrt{(-10)^2 + (-2)^2}$$

$$\therefore AB = \sqrt{100 + 4}$$

$$\therefore AB = \sqrt{104} \qquad \ldots\ldots\ldots\ldots(i)$$

$$AC = \sqrt{(3 - 5)^2 + (1 - 11)^2}$$

$$\therefore AC = \sqrt{(-2)^2 + (-10)^2}$$

$$\therefore AC = \sqrt{4 + 100}$$

$$\therefore AC = \sqrt{104} \qquad \ldots\ldots\ldots\ldots(ii)$$

From (i) and (ii)
AB = AC

$\therefore$ The point (5, 11) is equidistant from the points
(-5, 13) and (3, 1).

Solution 3:

Let $L \equiv (4,-1)$, $M \equiv (1,-3)$ and $N \equiv (-2,-5)$

To prove points L, M and N are collinear it is enough to show that the sum of the lengths of the two line segments is equal to the length of the third line segment.

$\therefore$ Using the distance formula,

$$LM = \sqrt{(1-4)^2 + (-3-(-1))^2}$$

$$\therefore LM = \sqrt{(-3)^2 + (-2)^2}$$

$$\therefore LM = \sqrt{9+4}$$

$$\therefore LM = \sqrt{13} \qquad \ldots\ldots\ldots\ldots(i)$$

$$MN = \sqrt{(-2-1)^2 + (-5-(-3))^2}$$

$$\therefore MN = \sqrt{(-3)^2 + (-2)^2}$$

$$\therefore MN = \sqrt{9+4}$$

$$\therefore MN = \sqrt{13} \qquad \ldots\ldots\ldots\ldots(ii)$$

$$LN = \sqrt{(-2-4)^2 + (-5-(-1))^2}$$

$$\therefore LN = \sqrt{(-6)^2 + (-4)^2}$$

$$\therefore LN = \sqrt{36+16}$$

$$\therefore LN = \sqrt{52}$$

$$\therefore LN = 2\sqrt{13} \qquad \ldots\ldots\ldots\ldots(iii)$$

From (i), (ii) and (iii)

$LN = LM + MN$

$\therefore$ The points $L(4,-1)$, $M(1,-3)$ and $N(-2,-5)$ are collinear.

Solution 4:

Let $A \equiv (3,3)$, $B \equiv (-4,-1)$ and $C \equiv (3,-5)$

To prove that points A, B and C are the vertices of an isoceles triangle, it is enough to show that the sum of the lengths of the two line segments is equal.

$\therefore$ Using the distance formula,

$AB = \sqrt{(-4-3)^2 + (-1-3)^2}$

$\therefore AB = \sqrt{(-7)^2 + (-4)^2}$

$\therefore AB = \sqrt{49+16}$

$\therefore AB = \sqrt{65}$ ....(i)

$BC = \sqrt{(3-(-4))^2 + (-5-(-1))^2}$

$\therefore BC = \sqrt{(7)^2 + (-4)^2}$

$\therefore BC = \sqrt{49+16}$

$\therefore BC = \sqrt{65}$ ....(ii)

From (i) and (ii)

AB = BC

$\therefore$ The points $(3,3)$, $(-4,-1)$ and $(3,-5)$ are the vertices of an isosceles triangle.

Solution 5:

Let A(a, b) be the circumcentre of ΔPQR

$\therefore AP = AQ = AR \quad \ldots\ldots\ldots\ldots(i)$

$\therefore AP^2 = AQ^2$

$\therefore (a-2)^2 + (b-7)^2 = (a-(-5))^2 + (b-8)^2$

$\therefore a^2 - 4a + 4 + b^2 - 14b + 49 = a^2 + 10a + 25 + b^2 - 16b + 64$

$\therefore 14a - 2b + 36 = 0 \ldots\ldots\ldots\ldots(ii)$

$\therefore AP^2 = AR^2 \quad$ [from (i)]

$\therefore (a-2)^2 + (b-7)^2 = (a-(-6))^2 + (b-1)^2$

$\therefore a^2 - 4a + 4 + b^2 - 14b + 49 = a^2 + 12a + 36 + b^2 - 2b + 1$

$\therefore 16a + 12b - 16 = 0 \ldots\ldots\ldots\ldots(iii)$

Multiplying equation (ii) by 6 and adding it to equation (iii)

$$\begin{array}{r} 16a + 12b - 16 = 0 \\ 84a - 12b + 216 = 0 \\ \hline 100a \qquad + 200 = 0 \end{array}$$

$\therefore a = -2$

Substituting $a = -2$ in equation (ii) we get,

$14(-2) - 2b + 36 = 0$

$\therefore -28 - 2b + 36 = 0$

$\therefore 2b = 8$

$\therefore b = 4$

$\therefore$ The co-ordinates of the circumcentre are $(-2, 4)$.

Solution 6:

Let P(x, y) be equidistant from the points A(2, -4) and B(-2, 6)

Hence AP = BP

$\therefore AP^2 = BP^2$

$\therefore (x-2)^2 + (y+4)^2 = (x+2)^2 + (y-6)^2$

$\therefore x^2 - 4x + 4 + y^2 + 8y + 16 = x^2 + 4x + 4 + y^2 - 12y + 36$

$\therefore 8x - 20y + 20 = 0$

$\therefore 2x - 5y + 5 = 0$

$\therefore$ The relation between x and y is $2x - 5y + 5 = 0$.

Exercise – 6.2

Solution 1(i):

Let $Q \equiv (x_1, y_1)$ and $R \equiv (x_2, y_2)$.

Here $x_1 = -5, y_1 = 8,\ x_2 = 4, y_2 = -4,\ m = 2, n = 1$.

Let P ≡ (x, y)

By section formula for internal division,

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

$$\therefore x = \frac{2(4) + 1(-5)}{2+1}, \quad y = \frac{2(-4) + 1(8)}{2+1}$$

$$\therefore x = \frac{8-5}{3}, \quad y = \frac{-8+8}{3}$$

$$\therefore x = \frac{3}{3}, \quad y = \frac{0}{3}$$

$$\therefore x = 1, \quad y = 0$$

$\therefore$ Co-ordinates of point $P \equiv (1, 0)$.

Solution 1(ii):

Let Q ≡ (x_1, y_1) and R ≡ (x_2, y_2).

Here $x_1 = 1, y_1 = 7,\ x_2 = -3, y_2 = 1,\ m = 1, n = 2$

Let P ≡ (x, y)

By section formula for internal division,

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

$$\therefore x = \frac{1(-3) + 2(1)}{1+2}, \quad y = \frac{1(1) + 2(7)}{1+2}$$

$$\therefore x = \frac{-3+2}{3}, \quad y = \frac{1+14}{3}$$

$$\therefore x = \frac{-1}{3}, \quad y = \frac{15}{3}$$

$$\therefore x = \frac{-1}{3}, \quad y = 5$$

$\therefore$ Co-ordinates of point P ≡ $\left(-\frac{1}{3}, 5\right)$.

Solution 1(iii):

Let Q ≡ (x_1, y_1) and R ≡ (x_2, y_2).

Here $x_1 = 6, y_1 = -5,\ x_2 = -10, y_2 = 2,\ m = 3, n = 4$

Let P ≡ (x, y)

By section formula for internal division,

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

$$\therefore x = \frac{3(-10) + 4(6)}{3+4}, \quad y = \frac{3(2) + 4(-5)}{3+4}$$

$$\therefore x = \frac{-30+24}{7}, \quad y = \frac{6-20}{7}$$

$$\therefore x = \frac{-6}{7}, \quad y = \frac{-14}{7}$$

$$\therefore x = \frac{-6}{7}, \quad y = -2$$

$\therefore$ Co-ordinates of point P ≡ $\left(-\frac{6}{7}, -2\right)$.

Solution 2(i):

Let J ≡ (x_1, y_1) and L ≡ (x_2, y_2).

Here $x_1 = 4, y_1 = -5,\ x_2 = -6, y_2 = 7,\ m = 3, n = 5$

Let P ≡ (x, y)

By section formula for external division,

$$x = \frac{mx_2 - nx_1}{m-n}, \quad y = \frac{my_2 - ny_1}{m-n}$$

$$\therefore x = \frac{3(-6) - 5(4)}{3-5}, \quad y = \frac{3(7) - 5(-5)}{3-5}$$

$$\therefore x = \frac{-18 - 20}{-2}, \quad y = \frac{21 + 25}{-2}$$

$$\therefore x = \frac{-38}{-2}, \quad y = \frac{46}{-2}$$

$$\therefore x = 19, \quad y = -23$$

∴ Co-ordinates of point P ≡ (19, − 23).

Solution 2(ii):

Let J ≡ (x_1, y_1) and L ≡ (x_2, y_2).

Here $x_1 = -8, y_1 = -4,\ x_2 = 1, y_2 = 2,\ m = 1, n = 2$

Let P ≡ (x, y)

By section formula for external division,

$$x = \frac{mx_2 - nx_1}{m-n}, \quad y = \frac{my_2 - ny_1}{m-n}$$

$$\therefore x = \frac{1(1) - 2(-8)}{1-2}, \quad y = \frac{1(2) - 2(-4)}{1-2}$$

$$\therefore x = \frac{1 + 16}{-1}, \quad y = \frac{2 + 8}{-1}$$

$$\therefore x = -17, \quad y = -10$$

∴ Co-ordinates of point P ≡ (−17, − 10)

Solution 2(iii):

Let $J \equiv (x_1, y_1)$ and $L \equiv (x_2, y_2)$.

Here $x_1 = 5, y_1 = -3,\ x_2 = 0, y_2 = 9,\ m = 4, n = 3$

Let $P \equiv (x,\ y)$

By section formula for external division,

$$x = \frac{mx_2 - nx_1}{m-n}, \quad y = \frac{my_2 - ny_1}{m-n}$$

$$\therefore x = \frac{4(0) - 3(5)}{4-3}, \quad y = \frac{4(9) - 3(-3)}{4-3}$$

$$\therefore x = \frac{0 - 15}{1}, \quad y = \frac{36 + 9}{1}$$

$$\therefore x = -15, \quad y = 45$$

$\therefore$ Co-ordinates of point $P \equiv (-15,\ 45)$

Solution 3:

Let $J \equiv (x_1, y_1)$ and $L \equiv (x_2, y_2)$.

Here $x_1 = 3.5, y_1 = 9.5,\ x_2 = -1.5, y_2 = 0.5,$

Let $P \equiv (x,\ y)$

By midpoint formula,

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$

$$\therefore x = \frac{3.5 - 1.5}{2}, \quad y = \frac{9.5 + 0.5}{2}$$

$$\therefore x = \frac{2}{2}, \quad y = \frac{10}{2}$$

$$\therefore x = 1, \quad y = 5$$

$\therefore$ Co-ordinates of point $P \equiv (1, 5)$

Solution 4:

Let point (1, 3) divide the line joining the points (3, 6) and (-5, -6) internally in the ratio m:n.

Let (3, 6) $\equiv (x_1, y_1)$ and (-5, -6) $\equiv (x_2, y_2)$

Here $x_1 = 3, y_1 = 6, \ x_2 = -5, y_2 = -6$

Let (1, 3) $\equiv$ (x, y)

By section formula for internal division,

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$\therefore 1 = \frac{m(-5) + n(3)}{m+n}$$

$$\therefore 1 = \frac{-5m + 3n}{m+n}$$

$$\therefore m + n = -5m + 3n$$

$$\therefore 6m = 2n$$

$$\therefore \frac{m}{n} = \frac{1}{3}$$

$\therefore$ The point (1, 3) divides the line joining the points (3, 6) and (-5, -6) internally in the ratio 1:3.

Solution 5:

Let $P \equiv (-2,2)$ and $Q \equiv (6,-6)$.
Segment PQ is divided into four equal parts by the points A, B, and C.

Point A is the midpoint of segment PQ.
By the midpoint formula for point A

$$\left(\frac{-2+6}{2}, \frac{2-6}{2}\right) = \left(\frac{4}{2}, \frac{-4}{2}\right) = (2,-2)$$

$\therefore A \equiv (2,-2)$

Now point B is the midpoint of segment PA.
By midpoint formula for point B,

$$\left(\frac{-2+2}{2}, \frac{2-2}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0,0)$$

$\therefore B \equiv (0,0)$

Now point C is the midpoint of segment AQ
By midpoint formula for point C,

$$\left(\frac{6+2}{2}, \frac{-6-2}{2}\right) = \left(\frac{8}{2}, \frac{-8}{2}\right) = (4,-4)$$

$\therefore C \equiv (4,-4)$

The co-ordinates of the points which divide the line segment into four equal parts are (0, 0), (2, -2) and (4, -4) respectively.