PART SIX

ATOMIC AND NUCLEAR PHYSICS*

6.1. SCATTERING OF PARTICLES. RUTHERFORD-BOHR ATOM

• Angle θ at which a charged particle is deflected by the Coulomb field of a stationary atomic nucleus is defined by the formula:

$$\tan\frac{\theta}{2}=\frac{q_1q_2}{2bT}, \tag{6.1a}$$

where q_1 and q_2 are the charges of the particle and the nucleus, b is the aiming parameter, T is the kinetic energy of a striking particle.

• Rutherford formula. The relative number of particles scattered into an elementary solid angle $d\Omega$ at an angle θ to their initial propagation direction:

$$\frac{dN}{N}=n\left(\frac{q_1q_2}{4T}\right)^2\frac{d\Omega}{\sin^4(\theta/2)}, \tag{6.1b}$$

where n is the number of nuclei of the foil per unit area of its surface, $d\Omega=\sin\theta\, d\theta\, d\varphi$.

• Generalized Balmer formula (Fig. 6.1):

$$\omega=RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right),\quad R=\frac{me^4}{2\hbar^3}, \tag{6.1c}$$

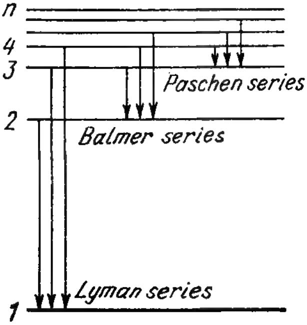

Fig. 6.1.

where ω is the transition frequency (in s^{-1}) between energy levels with quantum numbers n_1 and n_2, R is the Rydberg constant, Z is the serial number of a hydrogen-like ion.

6.1. Employing Thomson's model, calculate the radius of a hydrogen atom and the wavelength of emitted light if the ionization energy of the atom is known to be equal to $E = 13.6$ eV.

6.2. An alpha particle with kinetic energy 0.27 MeV is deflected through an angle of 60° by a golden foil. Find the corresponding value of the aiming parameter.

6.3. To what minimum distance will an alpha particle with kinetic energy $T = 0.40$ MeV approach in the case of a head-on collision to

(a) a stationary Pb nucleus;

(b) a stationary free Li^7 nucleus?

6.4. An alpha particle with kinetic energy $T = 0.50$ MeV is deflected through an angle of $\theta = 90°$ by the Coulomb field of a stationary Hg nucleus. Find:

* All the formulas in this Part are given in the Gaussian system of units.

(a) the least curvature radius of its trajectory;

(b) the minimum approach distance between the particle and the nucleus.

6.5. A proton with kinetic energy T and aiming parameter b was deflected by the Coulomb field of a stationary Au nucleus. Find the momentum imparted to the given nucleus as a result of scattering.

6.6. A proton with kinetic energy $T = 10$ MeV flies past a stationary free electron at a distance $b = 10$ pm. Find the energy acquired by the electron, assuming the proton's trajectory to be rectilinear and the electron to be practically motionless as the proton flies by.

6.7. A particle with kinetic energy T is deflected by a spherical potential well of radius R and depth U_0, i.e. by the field in which the potential energy of the particle takes the form

$$U = \begin{cases} 0 \text{ for } r > R, \\ -U_0 \text{ for } r < R, \end{cases}$$

where r is the distance from the centre of the well. Find the relationship between the aiming parameter b of the particle and the angle θ through which it deflects from the initial motion direction.

6.8. A stationary ball of radius R is irradiated by a parallel stream of particles whose radius is r. Assuming the collision of a particle and the ball to be elastic, find:

(a) the deflection angle θ of a particle as a function of its aiming parameter b;

(b) the fraction of particles which after a collision with the ball are scattered into the angular interval between θ and $\theta + d\theta$;

(c) the probability of a particle to be deflected, after a collision with the ball, into the front hemisphere $\left(\theta < \frac{\pi}{2}\right)$.

6.9. A narrow beam of alpha particles with kinetic energy 1.0 MeV falls normally on a platinum foil 1.0 μm thick. The scattered particles are observed at an angle of 60° to the incident beam direction by means of a counter with a circular inlet area 1.0 cm² located at the distance 10 cm from the scattering section of the foil. What fraction of scattered alpha particles reaches the counter inlet?

6.10. A narrow beam of alpha particles with kinetic energy $T = 0.50$ MeV and intensity $I = 5.0 \cdot 10^5$ particles per second falls normally on a golden foil. Find the thickness of the foil if at a distance $r = 15$ cm from a scattering section of that foil the flux density of scattered particles at the angle $\theta = 60°$ to the incident beam is equal to $J = 40$ particles/(cm²·s).

6.11. A narrow beam of alpha particles falls normally on a silver foil behind which a counter is set to register the scattered particles. On substitution of platinum foil of the same mass thickness for the silver foil, the number of alpha particles registered per unit time increased $\eta = 1.52$ times. Find the atomic number of platinum, assuming the atomic number of silver and the atomic masses of both platinum and silver to be known.

6.12. A narrow beam of alpha particles with kinetic energy $T = 0.50$ MeV falls normally on a golden foil whose mass thickness is $\rho d = 1.5$ mg/cm². The beam intensity is $I_0 = 5.0 \cdot 10^5$ particles per second. Find the number of alpha particles scattered by the foil during a time interval $\tau = 30$ min into the angular interval:

(a) 59-61°; (b) over $\theta_0 = 60°$.

6.13. A narrow beam of protons with velocity $v = 6 \cdot 10^6$ m/s falls normally on a silver foil of thickness $d = 1.0$ μm. Find the probability of the protons to be scattered into the rear hemisphere ($\theta > 90°$).

6.14. A narrow beam of alpha particles with kinetic energy $T = 600$ keV falls normally on a golden foil incorporating $n = 1.1 \cdot 10^{19}$ nuclei/cm². Find the fraction of alpha particles scattered through the angles $\theta < \theta_0 = 20°$.

6.15. A narrow beam of protons with kinetic energy $T = 1.4$ MeV falls normally on a brass foil whose mass thickness $\rho d = 1.5$ mg/cm². The weight ratio of copper and zinc in the foil is equal to 7 : 3 respectively. Find the fraction of the protons scattered through the angles exceeding $\theta_0 = 30°$.

6.16. Find the effective cross section of a uranium nucleus corresponding to the scattering of alpha particles with kinetic energy $T = 1.5$ MeV through the angles exceeding $\theta_0 = 60°$.

6.17. The effective cross section of a gold nucleus corresponding to the scattering of monoenergetic alpha particles within the angular interval from 90° to 180° is equal to $\Delta\sigma = 0.50$ kb. Find:

(a) the energy of alpha particles;

(b) the differential cross section of scattering $d\sigma/d\Omega$ (kb/sr) corresponding to the angle $\theta = 60°$.

6.18. In accordance with classical electrodynamics an electron moving with acceleration **w** loses its energy due to radiation as

$$\frac{dE}{dt} = -\frac{2e^2}{3c^3} \mathrm{w}^2,$$

where e is the electron charge, c is the velocity of light. Estimate the time during which the energy of an electron performing almost harmonic oscillations with frequency $\omega = 5 \cdot 10^{15}$ s^{-1} will decrease $\eta = 10$ times.

6.19. Making use of the formula of the foregoing problem, estimate the time during which an electron moving in a hydrogen atom along a circular orbit of radius $r = 50$ pm would have fallen onto the nucleus. For the sake of simplicity assume the vector **w** to be permanently directed toward the centre of the atom.

6.20. Demonstrate that the frequency ω of a photon emerging when an electron jumps between neighbouring circular orbits of a hydrogen-like ion satisfies the inequality $\omega_n > \omega > \omega_{n+1}$, where ω_n and ω_{n+1} are the frequencies of revolution of that electron around the nucleus along the circular orbits. Make sure that as $n \to \infty$ the frequency of the photon $\omega \to \omega_n$.

6.21. A particle of mass m moves along a circular orbit in a centrosymmetrical potential field $U(r) = kr^2/2$. Using the Bohr quantization condition, find the permissible orbital radii and energy levels of that particle.

6.22. Calculate for a hydrogen atom and a He^+ ion:

(a) the radius of the first Bohr orbit and the velocity of an electron moving along it;

(b) the kinetic energy and the binding energy of an electron in the ground state;

(c) the ionization potential, the first excitation potential and the wavelength of the resonance line ($n' = 2 \to n = 1$).

6.23. Calculate the angular frequency of an electron occupying the second Bohr orbit of He^+ ion.

6.24. For hydrogen-like systems find the magnetic moment μ_n corresponding to the motion of an electron along the n-th orbit and the ratio of the magnetic and mechanical moments μ_n/M_n. Calculate the magnetic moment of an electron occupying the first Bohr orbit.

6.25. Calculate the magnetic field induction at the centre of a hydrogen atom caused by an electron moving along the first Bohr orbit.

6.26. Calculate and draw on the wavelength scale the spectral intervals in which the Lyman, Balmer, and Paschen series for atomic hydrogen are confined. Show the visible portion of the spectrum.

6.27. To what series does the spectral line of atomic hydrogen belong if its wave number is equal to the difference between the wave numbers of the following two lines of the Balmer series: 486.1 and 410.2 nm? What is the wavelength of that line?

6.28. For the case of atomic hydrogen find:

(a) the wavelengths of the first three lines of the Balmer series;

(b) the minimum resolving power $\lambda/\delta\lambda$ of a spectral instrument capable of resolving the first 20 lines of the Balmer series.

6.29. Radiation of atomic hydrogen falls normally on a diffraction grating of width $l = 6.6$ mm. The 50th line of the Balmer series in the observed spectrum is close to resolution at a diffraction angle θ (in accordance with Rayleigh's criterion). Find that angle.

6.30. What element has a hydrogen-like spectrum whose lines have wavelengths four times shorter than those of atomic hydrogen?

6.31. How many spectral lines are emitted by atomic hydrogen excited to the n-th energy level?

6.32. What lines of atomic hydrogen absorption spectrum fall within the wavelength range from 94.5 to 130.0 nm?

6.33. Find the quantum number n corresponding to the excited state of He^+ ion if on transition to the ground state that ion emits two photons in succession with wavelengths 108.5 and 30.4 nm.

6.34. Calculate the Rydberg constant R if He^+ ions are known to have the wavelength difference between the first (of the longest wavelength) lines of the Balmer and Lyman series equal to $\Delta\lambda = 133.7$ nm.

6.35. What hydrogen-like ion has the wavelength difference between the first lines of the Balmer and Lyman series equal to 59.3 nm?

6.36. Find the wavelength of the first line of the He^+ ion spectral series whose interval between the extreme lines is $\Delta\omega = 5.18 \cdot 10^{15}$ s^{-1}.

6.37. Find the binding energy of an electron in the ground state of hydrogen-like ions in whose spectrum the third line of the Balmer series is equal to 108.5 nm.

6.38. The binding energy of an electron in the ground state of He atom is equal to $E_0 = 24.6$ eV. Find the energy required to remove both electrons from the atom.

6.39. Find the velocity of photoelectrons liberated by electromagnetic radiation of wavelength $\lambda = 18.0$ nm from stationary He^+ ions in the ground state.

6.40. At what minimum kinetic energy must a hydrogen atom move for its inelastic head-on collision with another, stationary, hydrogen atom to make one of them capable of emitting a photon? Both atoms are supposed to be in the ground state prior to the collision.

6.41. A stationary hydrogen atom emits a photon corresponding to the first line of the Lyman series. What velocity does the atom acquire?

6.42. From the conditions of the foregoing problem find how much (in per cent) the energy of the emitted photon differs from the energy of the corresponding transition in a hydrogen atom.

6.43. A stationary He^+ ion emitted a photon corresponding to the first line of the Lyman series. That photon liberated a photoelectron from a stationary hydrogen atom in the ground state. Find the velocity of the photoelectron.

6.44. Find the velocity of the excited hydrogen atoms if the first line of the Lyman series is displaced by $\Delta\lambda = 0.20$ nm when their radiation is observed at an angle $\theta = 45°$ to their motion direction.

6.45. According to the Bohr-Sommerfeld postulate the periodic motion of a particle in a potential field must satisfy the following quantization rule:

$$\oint p\, dq = 2\pi\hbar n,$$

where q and p are generalized coordinate and momentum of the particle, n are integers. Making use of this rule, find the permitted values of energy for a particle of mass m moving

(a) in a unidimensional rectangular potential well of width l with infinitely high walls;

(b) along a circle of radius r;

(c) in a unidimensional potential field $U = \alpha x^2/2$, where α is a positive constant;

(d) along a round orbit in a central field, where the potential energy of the particle is equal to $U = -\alpha/r$ (α is a positive constant).

6.46. Taking into account the motion of the nucleus of a hydrogen atom, find the expressions for the electron's binding energy in the ground state and for the Rydberg constant. How much (in per cent) do the binding energy and the Rydberg constant, obtained without taking into account the motion of the nucleus, differ from the more accurate corresponding values of these quantities?

6.47. For atoms of light and heavy hydrogen (H and D) find the difference

(a) between the binding energies of their electrons in the ground state;

(b) between the wavelengths of first lines of the Lyman series.

6.48. Calculate the separation between the particles of a system in the ground state, the corresponding binding energy, and the wavelength of the first line of the Lyman series, if such a system is

(a) a mesonic hydrogen atom whose nucleus is a proton (in a mesonic atom an electron is replaced by a meson whose charge is the same and mass is 207 that of an electron);

(b) a positronium consisting of an electron and a positron revolving around their common centre of masses.

6.2. WAVE PROPERTIES OF PARTICLES. SCHRÖDINGER EQUATION

- The de Broglie wavelength of a particle with momentum p:

$$\lambda = \frac{2\pi\hbar}{p}. \tag{6.2a}$$

- Uncertainty principle:

$$\Delta x \cdot \Delta p_x \geqslant \hbar. \tag{6.2b}$$

- Schrödinger time-dependent and time-independent equations:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + U\Psi,$$
$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E-U)\psi = 0, \tag{6.2c}$$

where Ψ is the total wave function, ψ is its coordinate part, ∇^2 is the Laplace operator, E and U are the total and potential energies of the particle. In spherical coordinates:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \varphi^2}. \tag{6.2d}$$

• Coefficient of transparency of a potential barrier $V(x)$:

$$D \approx \exp\left[-\frac{2}{\hbar}\int_{x_1}^{x_2} \sqrt{2m(V-E)}\,dx\right], \tag{6.2e}$$

where x_1 and x_2 are the coordinates of the points between which $V > E$.

6.49. Calculate the de Broglie wavelengths of an electron, proton, and uranium atom, all having the same kinetic energy 100 eV.

6.50. What amount of energy should be added to an electron to reduce its de Broglie wavelength from 100 to 50 pm?

6.51. A neutron with kinetic energy $T = 25$ eV strikes a stationary deuteron (heavy hydrogen nucleus). Find the de Broglie wavelengths of both particles in the frame of their centre of inertia.

6.52. Two identical non-relativistic particles move at right angles to each other, possessing de Broglie wavelengths λ_1 and λ_2. Find the de Broglie wavelength of each particle in the frame of their centre of inertia.

6.53. Find the de Broglie wavelength of hydrogen molecules, which corresponds to their most probable velocity at room temperature.

6.54. Calculate the most probable de Broglie wavelength of hydrogen molecules being in thermodynamic equilibrium at room temperature.

6.55. Derive the expression for a de Broglie wavelength λ of a relativistic particle moving with kinetic energy T. At what values of T does the error in determining λ using the non-relativistic formula not exceed 1% for an electron and a proton?

6.56. At what value of kinetic energy is the de Broglie wavelength of an electron equal to its Compton wavelength?

6.57. Find the de Broglie wavelength of relativistic electrons reaching the anticathode of an X-ray tube if the short wavelength limit of the continuous X-ray spectrum is equal to $\lambda_{sh} = 10.0$ pm?

6.58. A parallel stream of monoenergetic electrons falls normally on a diaphragm with narrow square slit of width $b = 1.0$ μm. Find the velocity of the electrons if the width of the central diffraction maximum formed on a screen located at a distance $l = 50$ cm from the slit is equal to $\Delta x = 0.36$ mm.

6.59. A parallel stream of electrons accelerated by a potential difference $V = 25$ V falls normally on a diaphragm with two narrow slits separated by a distance $d = 50$ μm. Calculate the distance between neighbouring maxima of the diffraction pattern on a screen located at a distance $l = 100$ cm from the slits.

6.60. A narrow stream of monoenergetic electrons falls at an angle of incidence $\theta = 30°$ on the natural facet of an aluminium single crystal. The distance between the neighbouring crystal planes parallel to that facet is equal to $d = 0.20$ nm. The maximum mirror reflection is observed at a certain accelerating voltage V_0. Find V_0

if the next maximum mirror reflection is known to be observed when the accelerating voltage is increased $\eta = 2.25$ times.

6.61. A narrow beam of monoenergetic electrons falls normally on the surface of a Ni single crystal. The reflection maximum of fourth order is observed in the direction forming an angle $\theta = 55°$ with the normal to the surface at the energy of the electrons equal to $T = 180$ eV. Calculate the corresponding value of the interplanar distance.

6.62. A narrow stream of electrons with kinetic energy $T = 10$ keV passes through a polycrystalline aluminium foil, forming a system of diffraction fringes on a screen. Calculate the interplanar distance corresponding to the reflection of third order from a certain system of crystal planes if it is responsible for a diffraction ring of diameter $D = 3.20$ cm. The distance between the foil and the screen is $l = 10.0$ cm.

6.63. A stream of electrons accelerated by a potential difference V falls on the surface of a metal whose inner potential is $V_i = 15$ V. Find:

(a) the refractive index of the metal for the electrons accelerated by a potential difference $V = 150$ V;

(b) the values of the ratio V/V_i at which the refractive index differs from unity by not more than $\eta = 1.0\%$.

6.64. A particle of mass m is located in a unidimensional square potential well with infinitely high walls. The width of the well is equal to l. Find the permitted values of energy of the particle taking into account that only those states of the particle's motion are realized for which the whole number of de Broglie half-waves are fitted within the given well.

6.65. Describe the Bohr quantum conditions in terms of the wave theory: demonstrate that an electron in a hydrogen atom can move only along those round orbits which accommodate a whole number of de Broglie waves.

6.66. Estimate the minimum errors in determining the velocity of an electron, a proton, and a ball of mass of 1 mg if the coordinates of the particles and of the centre of the ball are known with uncertainly 1 μm.

6.67. Employing the uncertainty principle, evaluate the indeterminancy of the velocity of an electron in a hydrogen atom if the size of the atom is assumed to be $l = 0.10$ nm. Compare the obtained magnitude with the velocity of an electron in the first Bohr orbit of the given atom.

6.68. Show that for the particle whose coordinate uncertainty is $\Delta x = \lambda/2\pi$, where λ is its de Broglie wavelength, the velocity uncertainty is of the same order of magnitude as the particle's velocity itself.

6.69. A free electron was initially confined within a region with linear dimensions $l = 0.10$ nm. Using the uncertainty principle, evaluate the time over which the width of the corresponding train of waves becomes $\eta = 10$ times as large.

6.70. Employing the uncertainty principle, estimate the minimum kinetic energy of an electron confined within a region whose size is $l = 0.20$ nm.

6.71. An electron with kinetic energy $T \approx 4$ eV is confined within a region whose linear dimension is $l = 1$ μm. Using the uncertainty principle, evaluate the relative uncertainty of its velocity.

6.72. An electron is located in a unidimensional square potential well with infinitely high walls. The width of the well is l. From the uncertainty principle estimate the force with which the electron possessing the minimum permitted energy acts on the walls of the well.

6.73. A particle of mass m moves in a unidimensional potential field $U = kx^2/2$ (harmonic oscillator). Using the uncertainty principle, evaluate the minimum permitted energy of the particle in that field.

6.74. Making use of the uncertainty principle, evaluate the minimum permitted energy of an electron in a hydrogen atom and its corresponding apparent distance from the nucleus.

6.75. A parallel stream of hydrogen atoms with velocity $v = 600$ m/s falls normally on a diaphragm with a narrow slit behind which a screen is placed at a distance $l = 1.0$ m. Using the uncertainty principle, evaluate the width of the slit δ at which the width of its image on the screen is minimum.

6.76. Find a particular solution of the time-dependent Schrödinger equation for a freely moving particle of mass m.

6.77. A particle in the ground state is located in a unidimensional square potential well of length l with absolutely impenetrable walls $(0 < x < l)$. Find the probability of the particle staying within a region $\frac{1}{3} l \leqslant x \leqslant \frac{2}{3} l$.

6.78. A particle is located in a unidimensional square potential well with infinitely high walls. The width of the well is l. Find the normalized wave functions of the stationary states of the particle, taking the midpoint of the well for the origin of the x coordinate.

6.79. Demonstrate that the wave functions of the stationary states of a particle confined in a unidimensional potential well with infinitely high walls are orthogonal, i.e. they satisfy the condition $\int\limits_0^l \psi_n \psi_{n'}\, dx = 0$ if $n' \neq n$. Here l is the width of the well, n are integers.

6.80. An electron is located in a unidimensional square potential well with infinitely high walls. The width of the well equal to l is such that the energy levels are very dense. Find the density of energy levels dN/dE, i.e. their number per unit energy interval, as a function of E. Calculate dN/dE for $E = 1.0$ eV if $l = 1.0$ cm.

6.81. A particle of mass m is located in a two-dimensional square potential well with absolutely impenetrable walls. Find:

(a) the particle's permitted energy values if the sides of the well are l_1 and l_2;

(b) the energy values of the particle at the first four levels if the well has the shape of a square with side l.

6.82. A particle is located in a two-dimensional square potential well with absolutely impenetrable walls ($0 < x < a$, $0 < y < b$). Find the probability of the particle with the lowest energy to be located within a region $0 < x < a/3$.

6.83. A particle of mass m is located in a three-dimensional cubic potential well with absolutely impenetrable walls. The side of the cube is equal to a. Find:

(a) the proper values of energy of the particle;

(b) the energy difference between the third and fourth levels;

(c) the energy of the sixth level and the number of states (the degree of degeneracy) corresponding to that level.

6.84. Using the Schrödinger equation, demonstrate that at the point where the potential energy $U(x)$ of a particle has a finite discontinuity, the wave function remains smooth, i.e. its first derivative with respect to the coordinate is continuous.

6.85. A particle of mass m is located in a unidimensional potential field $U(x)$ whose shape is shown in Fig. 6.2, where $U(0) = \infty$. Find:

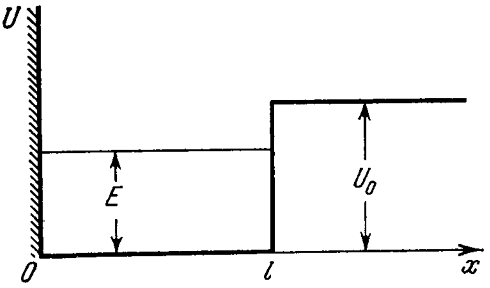

Fig. 6.2.

(a) the equation defining the possible values of energy of the particle in the region $E < U_0$; reduce that equation to the form

$$\sin kl = \pm kl \sqrt{\hbar^2/2ml^2U_0},$$

where $k = \sqrt{2mE}/\hbar$. Solving this equation by graphical means, demonstrate that the possible values of energy of the particle form a discontinuous spectrum;

(b) the minimum value of the quantity l^2U_0 at which the first energy level appears in the region $E < U_0$. At what minimum value of l^2U_0 does the nth level appear?

6.86. Making use of the solution of the foregoing problem, determine the probability of the particle with energy $E = U_0/2$ to be located in the region $x > l$, if $l^2U_0 = \left(\frac{3}{4}\pi\right)^2 \frac{\hbar^2}{m}$.

6.87. Find the possible values of energy of a particle of mass m located in a spherically symmetrical potential well $U(r) = 0$ for $r < r_0$ and $U(r) = \infty$ for $r = r_0$, in the case when the motion of the particle is described by a wave function $\psi(r)$ depending only on r.

Instruction. When solving the Schrödinger equation, make the substitution $\psi(r) = \chi(r)/r$.

6.88. From the conditions of the foregoing problem find:

(a) normalized eigenfunctions of the particle in the states for which $\psi(r)$ depends only on r;

(b) the most probable value r_{pr} for the ground state of the particle and the probability of the particle to be in the region $r < r_{pr}$.

6.89. A particle of mass m is located in a spherically symmetrical potential well $U(r) = 0$ for $r < r_0$ and $U(r) = U_0$ for $r > r_0$.

(a) By means of the substitution $\psi(r) = \chi(r)/r$ find the equation defining the proper values of energy E of the particle for $E < U_0$, when its motion is described by a wave function $\psi(r)$ depending only on r. Reduce that equation to the form

$$\sin kr_0 = \pm kr_0 \sqrt{\hbar^2/2mr_0^2U_0}, \text{ where } k = \sqrt{2mE}/\hbar.$$

(b) Calculate the value of the quantity $r_0^2U_0$ at which the first level appears.

6.90. The wavefunction of a particle of mass m in a unidimensional potential field $U(x) = kx^2/2$ has in the ground state the form $\psi(x) = Ae^{-\alpha x^2}$, where A is a normalization factor and α is a positive constant. Making use of the Schrödinger equation, find the constant α and the energy E of the particle in this state.

6.91. Find the energy of an electron of a hydrogen atom in a stationary state for which the wave function takes the form $\psi(r) = A(1 + ar)e^{-\alpha r}$, where A, a, and α are constants.

6.92. The wave function of an electron of a hydrogen atom in the ground state takes the form $\psi(r) = Ae^{-r/r_1}$, where A is a certain constant, r_1 is the first Bohr radius. Find:

(a) the most probable distance between the electron and the nucleus;

(b) the mean value of modulus of the Coulomb force acting on the electron;

(c) the mean value of the potential energy of the electron in the field of the nucleus.

6.93. Find the mean electrostatic potential produced by an electron in the centre of a hydrogen atom if the electron is in the ground state for which the wave function is $\psi(r) = Ae^{-r/r_1}$, where A is a certain constant, r_1 is the first Bohr radius.

6.94. Particles of mass m and energy E move from the left to the potential barrier shown in Fig. 6.3. Find:

(a) the reflection coefficient R of the barrier for $E > U_0$;

(b) the effective penetration depth of the particles into the region $x > 0$ for $E < U_0$, i.e. the distance from the barrier boundary to the point at which the probability of finding a particle decreases e-fold.

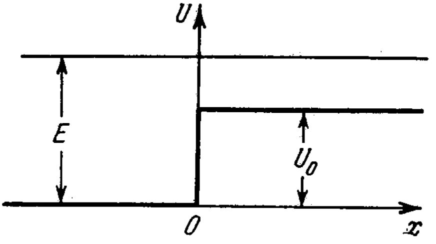

Fig. 6.3.

6.95. Employing Eq. (6.2e), find the probability D of an electron with energy E tunnelling through a potential barrier of width l and height U_0 provided the barrier is shaped as shown:

(a) in Fig. 6.4;

(b) in Fig. 6.5.

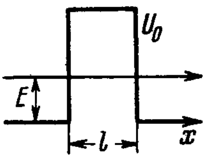

Fig. 6.4.

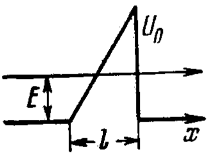

Fig. 6.5.

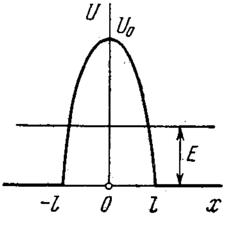

Fig. 6.6.

6.96. Using Eq. (6.2e), find the probability D of a particle of mass m and energy E tunnelling through the potential barrier shown in Fig. 6.6, where $U(x) = U_0(1 - x^2/l^2)$.

6.3. PROPERTIES OF ATOMS. SPECTRA

• Spectral labelling of terms: ${}^{\varkappa}(L)_J$, where $\varkappa = 2S + 1$ is the multiplicity, L, S, J are quantum numbers,

$$L = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \dots$$
$$(L):\ S,\ P,\ D,\ F,\ G,\ H,\ I,\ \dots$$

• Terms of alkali metal atoms:

$$T = \frac{R}{(n+\alpha)^2}, \tag{6.3a}$$

where R is the Rydberg constant, α is the Rydberg correction.

Fig. 6.7 illustrates the diagram of a lithium atom terms.

• Angular momenta of an atom:

$$M_L = \hbar\sqrt{L(L+1)}, \tag{6.3b}$$

with similar expressions for M_S and M_J.

• Hund rules:

(1) For a certain electronic configuration, the terms of the largest S value are the lowest in energy, and among the terms of S_{max} that of the largest L usually lies lowest;

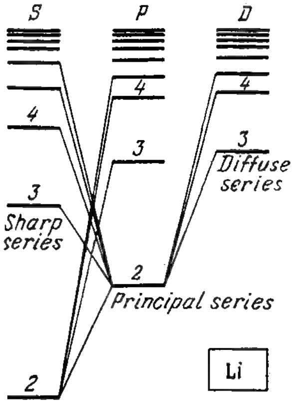

Fig. 6.7.

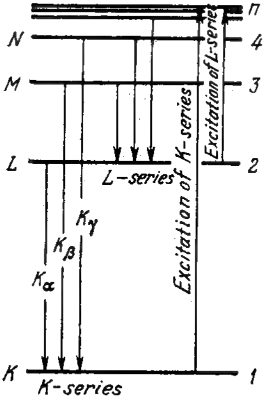

Fig. 6.8.

(2) for the basic (normal) term $J = |L - S|$ if the subshell is less than half-filled, and $J = L + S$ in the remaining cases.

• Boltzmann's formula:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT}, \tag{6.3c}$$

where g_1 and g_2 are the statistical weights (degeneracies) of the corresponding levels.

• Probabilities of atomic transitions per unit time between level *1* and a higher level *2* for the cases of spontaneous radiation, induced radiation, and absorption:

$$P_{21}^{sp} = A_{21}, \quad P_{21}^{ind} = B_{21}u_\omega, \quad P_{12}^{abs} = B_{12}u_\omega, \tag{6.3d}$$

where A_{21}, B_{21}, B_{12} are Einstein coefficients, u_ω is the spectral density of radiation corresponding to frequency ω of transition between the given levels.

• Relation between Einstein coefficients:

$$g_1B_{12} = g_2B_{21}, \quad B_{21} = \frac{\pi^2c^3}{\hbar\omega^3}A_{21}. \tag{6.3e}$$

• Diagram showing formation of X-ray spectra (Fig. 6.8).

• Moseley's law for K_α lines:

$$\omega_{K_\alpha} = \frac{3}{4}R(Z-\sigma)^2, \tag{6.3f}$$

where σ is the correction constant which is equal to unity for light elements.

- Magnetic moment of an atom and Landé g factor:

$$\mu = g\sqrt{J(J+1)}\,\mu_B, \qquad g = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}. \quad (6.3g)$$

- Zeeman splitting of spectral lines in a weak magnetic field:

$$\Delta\omega = (m_1 g_1 - m_2 g_2)\,\mu_B B/\hbar. \quad (6.3h)$$

- With radiation directed along the magnetic field, the Zeeman components caused by the transition $m_1 = m_2$ are absent.

6.97. The binding energy of a valence electron in a Li atom in the states $2S$ and $2P$ is equal to 5.39 and 3.54 eV respectively. Find the Rydberg corrections for S and P terms of the atom.

6.98. Find the Rydberg correction for the $3P$ term of a Na atom whose first excitation potential is 2.10 V and whose valence electron in the normal $3S$ state has the binding energy 5.14 eV.

6.99. Find the binding energy of a valence electron in the ground state of a Li atom if the wavelength of the first line of the sharp series is known to be equal to $\lambda_1 = 813$ nm and the short-wave cut-off wavelength of that series to $\lambda_2 = 350$ nm.

6.100. Determine the wavelengths of spectral lines appearing on transition of excited Li atoms from the state $3S$ down to the ground state $2S$. The Rydberg corrections for the S and P terms are -0.41 and -0.04.

6.101. The wavelengths of the yellow doublet components of the resonance Na line caused by the transition $3P \to 3S$ are equal to 589.00 and 589.56 nm. Find the splitting of the $3P$ term in eV units.

6.102. The first line of the sharp series of atomic cesium is a doublet with wavelengths 1358.8 and 1469.5 nm. Find the frequency intervals (in rad/s units) between the components of the sequent lines of that series.

6.103. Write the spectral designations of the terms of the hydrogen atom whose electron is in the state with principal quantum number $n = 3$.

6.104. How many and which values of the quantum number J can an atom possess in the state with quantum numbers S and L equal respectively to

(a) 2 and 3; (b) 3 and 3; (c) 5/2 and 2?

6.105. Find the possible values of total angular momenta of atoms in the states ^4P and ^5D.

6.106. Find the greatest possible total angular momentum and the corresponding spectral designation of the term

(a) of a Na atom whose valence electron possesses the principal quantum number $n = 4$;

(b) of an atom with electronic configuration $1s^2 2p3d$.

6.107. It is known that in F and D states the number of possible values of the quantum number J is the same and equal to five. Find the spin angular momentum in these states.

6.108. An atom is in the state whose multiplicity is three and the total angular momentum is $\hbar\sqrt{20}$. What can the corresponding quantum number L be equal to?

6.109. Find the possible multiplicities $\varkappa$ of the terms of the types

(a) ${}^{\varkappa}D_2$; (b) ${}^{\varkappa}P_{3/2}$; (c) ${}^{\varkappa}F_1$.

6.110. A certain atom has three electrons (s, p, and d), in addition to filled shells, and is in a state with the greatest possible total mechanical moment for a given configuration. In the corresponding vector model of the atom find the angle between the spin momentum and the total angular momentum of the given atom.

6.111. An atom possessing the total angular momentum $\hbar\sqrt{6}$ is in the state with spin quantum number $S = 1$. In the corresponding vector model the angle between the spin momentum and the total angular momentum is $\theta = 73.2°$. Write the spectral symbol for the term of that state.

6.112. Write the spectral symbols for the terms of a two-electron system consisting of one p electron and one d electron.

6.113. A system comprises an atom in ${}^2P_{3/2}$ state and a d electron. Find the possible spectral terms of that system.

6.114. Find out which of the following transitions are forbidden by the selection rules: ${}^2D_{3/2} \to {}^2P_{1/2}$, ${}^3P_1 \to {}^2S_{1/2}$, ${}^3F_3 \to {}^3P_2$, ${}^4F_{7/2} \to {}^4D_{5/2}$.

6.115. Determine the overall degeneracy of a $3D$ state of a Li atom. What is the physical meaning of that value?

6.116. Find the degeneracy of the states ${}^2P$, ${}^3D$, and ${}^4F$ possessing the greatest possible values of the total angular momentum.

6.117. Write the spectral designation of the term whose degeneracy is equal to seven and the quantum numbers L and S are interrelated as $L = 3S$.

6.118. What element has the atom whose K, L, and M shells and $4s$ subshell are filled completely and $4p$ subshell is half-filled?

6.119. Using the Hund rules, find the basic term of the atom whose partially filled subshell contains

(a) three p electrons; (b) four p electrons.

6.120. Using the Hund rules, find the total angular momentum of the atom in the ground state whose partially filled subshell contains

(a) three d electrons; (b) seven d electrons.

6.121. Making use of the Hund rules, find the number of electrons in the only partially filled subshell of the atom whose basic term is

(a) ${}^3F_2$; (b) ${}^2P_{3/2}$; (c) ${}^6S_{5/2}$.

6.122. Using the Hund rules, write the spectral symbol of the basic term of the atom whose only partially filled subshell

(a) is filled by 1/3, and $S = 1$;

(b) is filled by 70%, and $S = 3/2$.

6.123. The only partially filled subshell of a certain atom contains three electrons, the basic term of the atom having $L = 3$. Using the Hund rules, write the spectral symbol of the ground state of the given atom.

6.124. Using the Hund rules, find the magnetic moment of the ground state of the atom whose open subshell is half-filled with five electrons.

6.125. What fraction of hydrogen atoms is in the state with the principal quantum number $n = 2$ at a temperature $T = 3000$ K?

6.126. Find the ratio of the number of atoms of gaseous sodium in the state $3P$ to that in the ground state $3S$ at a temperature $T = 2400$ K. The spectral line corresponding to the transition $3P \to 3S$ is known to have the wavelength $\lambda = 589$ nm.

6.127. Calculate the mean lifetime of excited atoms if it is known that the intensity of the spectral line appearing due to transition to the ground state diminishes by a factor $\eta = 25$ over a distance $l = 2.5$ mm along the stream of atoms whose velocity is $v = 600$ m/s.

6.128. Rarefied Hg gas whose atoms are practically all in the ground state was lighted by a mercury lamp emitting a resonance line of wavelength $\lambda = 253.65$ nm. As a result, the radiation power of Hg gas at that wavelength turned out to be $P = 35$ mW. Find the number of atoms in the state of resonance excitation whose mean lifetime is $\tau = 0.15$ μs.

6.129. Atomic lithium of concentration $n = 3.6 \cdot 10^{16}$ cm^{-3} is at a temperature $T = 1500$ K. In this case the power emitted at the resonant line's wavelength $\lambda = 671$ nm ($2P \to 2S$) per unit volume of gas is equal to $P = 0.30$ W/cm^3. Find the mean lifetime of Li atoms in the resonance excitation state.

6.130. Atomic hydrogen is in thermodynamic equilibrium with its radiation. Find:

(a) the ratio of probabilities of induced and spontaneous radiations of the atoms from the level $2P$ at a temperature $T = 3000$ K;

(b) the temperature at which these probabilities become equal.

6.131. A beam of light of frequency ω, equal to the resonant frequency of transition of atoms of gas, passes through that gas heated to temperature T. In this case $\hbar\omega \gg kT$. Taking into account induced radiation, demonstrate that the absorption coefficient of the gas $\varkappa$ varies as $\varkappa = \varkappa_0 (1 - e^{-\hbar\omega/kT})$, where $\varkappa_0$ is the absorption coefficient for $T \to 0$.

6.132. The wavelength of a resonant mercury line is $\lambda = 253.65$ nm. The mean lifetime of mercury atoms in the state of resonance excitation is $\tau = 0.15$ μs. Evaluate the ratio of the Doppler line broadening to the natural linewidth at a gas temperature $T = 300$ K.

6.133. Find the wavelength of the K_α line in copper ($Z = 29$) if the wavelength of the K_α line in iron ($Z = 26$) is known to be equal to 193 pm.

6.134. Proceeding from Moseley's law find:

(a) the wavelength of the K_α line in aluminium and cobalt:

(b) the difference in binding energies of K and L electrons in vanadium.

6.135. How many elements are there in a row between those whose wavelengths of K_α lines are equal to 250 and 179 pm?

6.136. Find the voltage applied to an X-ray tube with nickel anticathode if the wavelength difference between the K_α line and the short-wave cut-off of the continuous X-ray spectrum is equal to 84 pm.

6.137. At a certain voltage applied to an X-ray tube with aluminium anticathode the short-wave cut-off wavelength of the continuous X-ray spectrum is equal to 0.50 nm. Will the K series of the characteristic spectrum whose excitation potential is equal to 1.56 kV be also observed in this case?

6.138. When the voltage applied to an X-ray tube increased from $V_1 = 10$ kV to $V_2 = 20$ kV, the wavelength interval between the K_α line and the short-wave cut-off of the continuous X-ray spectrum increases by a factor $n = 3.0$. Find the atomic number of the element of which the tube's anticathode is made.

6.139. What metal has in its absorption spectrum the difference between the frequencies of X-ray K and L absorption edges equal to $\Delta\omega = 6.85 \cdot 10^{18}$ s^{-1}?

6.140. Calculate the binding energy of a K electron in vanadium whose L absorption edge has the wavelength $\lambda_L = 2.4$ nm.

6.141. Find the binding energy of an L electron in titanium if the wavelength difference between the first line of the K series and its short-wave cut-off is $\Delta\lambda = 26$ pm.

6.142. Find the kinetic energy and the velocity of the photoelectrons liberated by K_α radiation of zinc from the K shell of iron whose K band absorption edge wavelength is $\lambda_K = 174$ pm.

6.143. Calculate the Landé g factor for atoms
(a) in S states; (b) in singlet states.

6.144. Calculate the Landé g factor for the following terms:
(a) $^6F_{1/2}$; (b) $^4D_{1/2}$; (c) $^5F_2$; (d) $^5P_1$; (e) $^3P_0$.

6.145. Calculate the magnetic moment of an atom (in Bohr magnetons)
(a) in ^1F state;
(b) in $^2D_{3/2}$ state;
(c) in the state in which $S = 1$, $L = 2$, and Landé factor $g = 4/3$.

6.146. Determine the spin angular momentum of an atom in the state D_2 if the maximum value of the magnetic moment projection in that state is equal to four Bohr magnetons.

6.147. An atom in the state with quantum numbers $L = 2$, $S = 1$ is located in a weak magnetic field. Find its magnetic moment if the least possible angle between the angular momentum and the field direction is known to be equal to 30°.

6.148. A valence electron in a sodium atom is in the state with principal quantum number $n = 3$, with the total angular momentum being the greatest possible. What is its magnetic moment in that state?

6.149. An excited atom has the electronic configuration $1s^2 2s^2 2p 3d$ being in the state with the greatest possible total angular momentum. Find the magnetic moment of the atom in that state.

6.150. Find the total angular momentum of an atom in the state with $S = 3/2$ and $L = 2$ if its magnetic moment is known to be equal to zero.

6.151. A certain atom is in the state in which $S = 2$, the total angular momentum $M = \sqrt{2}\hbar$, and the magnetic moment is equal to zero. Write the spectral symbol of the corresponding term.

6.152. An atom in the state $^2P_{3/2}$ is located in the external magnetic field of induction $B = 1.0$ kG. In terms of the vector model find the angular precession velocity of the total angular momentum of that atom.

6.153. An atom in the state $^2P_{1/2}$ is located on the axis of a loop of radius $r = 5$ cm carrying a current $I = 10$ A. The distance between the atom and the centre of the loop is equal to the radius of the latter. How great may be the maximum force that the magnetic field of that current exerts on the atom?

6.154. A hydrogen atom in the normal state is located at a distance $r = 2.5$ cm from a long straight conductor carrying a current $I = 10$ A. Find the force acting on the atom.

6.155. A narrow stream of vanadium atoms in the ground state $^4F_{3/2}$ is passed through a transverse strongly inhomogeneous magnetic field of length $l_1 = 5.0$ cm as in the Stern-Gerlach experiment. The beam splitting is observed on a screen located at a distance $l_2 = 15$ cm from the magnet. The kinetic energy of the atoms is $T = 22$ MeV. At what value of the gradient of the magnetic field induction B is the distance between the extreme components of the split beam on the screen equal to $\delta = 2.0$ mm?

6.156. Into what number of sublevels are the following terms split in a weak magnetic field:

(a) $^3P_0$; (b) $^2F_{5/2}$; (c) $^4D_{1/2}$?

6.157. An atom is located in a magnetic field of induction $B = 2.50$ kG. Find the value of the total splitting of the following terms (expressed in eV units):

(a) ^1D; (b) $^3F_4$.

6.158. What kind of Zeeman effect, normal or anomalous, is observed in a weak magnetic field in the case of spectral lines caused by the following transitions:

(a) $^1P \rightarrow {}^1S$; (b) $^2D_{5/2} \rightarrow {}^2P_{3/2}$; (c) $^3D_1 \rightarrow {}^3P_0$; (d) $^5I_5 \rightarrow {}^5H_4$?

6.159. Determine the spectral symbol of an atomic singlet term if the total splitting of that term in a weak magnetic field of induction $B = 3.0$ kG amounts to $\Delta E = 104$ μeV.

6.160. It is known that a spectral line $\lambda = 612$ nm of an atom is caused by a transition between singlet terms. Calculate the interval $\Delta\lambda$ between the extreme components of that line in the magnetic field with induction $B = 10.0$ kG.

6.161. Find the minimum magnitude of the magnetic field induction B at which a spectral instrument with resolving power $\lambda/\delta\lambda = 1.0 \cdot 10^5$ is capable of resolving the components of the spectral line $\lambda = 536$ nm caused by a transition between singlet terms. The observation line is at right angles to the magnetic field direction.

6.162. A spectral line caused by the transition $^3D_1 \rightarrow {}^3P_0$ experiences the Zeeman splitting in a weak magnetic field. When observed at right angles to the magnetic field direction, the interval between the neighbouring components of the split line is $\Delta\omega = 1.32 \cdot 10^{10}$ s^{-1}. Find the magnetic field induction B at the point where the source is located.

6.163. The wavelengths of the Na yellow doublet ($^2P \rightarrow {}^2S$) are equal to 589.59 and 589.00 nm. Find:

(a) the ratio of the intervals between neighbouring sublevels of the Zeeman splitting of the terms $^2P_{3/2}$ and $^2P_{1/2}$ in a weak magnetic field;

(b) the magnetic field induction B at which the interval between neighbouring sublevels of the Zeeman splitting of the term $^2P_{3/2}$ is $\eta = 50$ times smaller than the natural splitting of the term ^2P.

6.164. Draw a diagram of permitted transitions between the terms $^2P_{3/2}$ and $^2S_{1/2}$ in a weak magnetic field. Find the displacements (in rad/s units) of Zeeman components of that line in a magnetic field $B = 4.5$ kG.

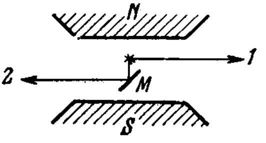

Fig. 6.9.

6.165. The same spectral line undergoing anomalous Zeeman splitting is observed in direction *1* and, after reflection from the mirror M (Fig. 6.9), in direction *2*. How many Zeeman components are observed in both directions if the spectral line is caused by the transition

(a) $^2P_{3/2} \rightarrow {}^2S_{1/2}$; (b) $^3P_2 \rightarrow {}^3S_1$?

6.166. Calculate the total splitting $\Delta\omega$ of the spectral line $^3D_3 \rightarrow {}^3P_2$ in a weak magnetic field with induction $B = 3.4$ kG.

6.4. MOLECULES AND CRYSTALS

• Rotational energy of a diatomic molecule:

$$E_J = \frac{\hbar^2}{2I} J(J+1), \tag{6.4a}$$

where I is the molecule's moment of inertia.

• Vibrational energy of a diatomic molecule:

$$E_v = \hbar\omega\left(v + \frac{1}{2}\right), \tag{6.4b}$$

where ω is the natural frequency of oscillations of the molecule.

• Mean energy of a quantum harmonic oscillator at a temperature T:

$$\langle E\rangle = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}. \tag{6.4c}$$

• Debye formula for molar vibrational energy of a crystal:

$$U = 9R\Theta\left[\frac{1}{8} + \left(\frac{T}{\Theta}\right)^4 \int_0^{\Theta/T} \frac{x^3 dx}{e^x - 1}\right], \tag{6.4d}$$

where Θ is the Debye temperature,

$$\Theta = \hbar\omega_{max}/k. \tag{6.4e}$$

• Molar vibrational heat capacity of a crystal for $T \ll \Theta$:

$$C = \frac{12}{5}\pi^4 R\left(\frac{T}{\Theta}\right)^3. \tag{6.4f}$$

• Distribution of free electrons in metal in the vicinity of the absolute zero:

$$dn = \frac{\sqrt{2}\, m^{3/2}}{\pi^2\hbar^3}\sqrt{E}\, dE, \tag{6.4g}$$

where dn is the concentration of electrons whose energy falls within the interval E, $E + dE$. The energy E is counted off the bottom of the conduction band.

• Fermi level at $T = 0$:

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}, \tag{6.4h}$$

where n is the concentration of free electrons in metal.

6.167. Determine the angular rotation velocity of an S_2 molecule promoted to the first excited rotational level if the distance between its nuclei is $d = 189$ pm.

6.168. For an HCl molecule find the rotational quantum numbers of two neighbouring levels whose energies differ by 7.86 meV. The nuclei of the molecule are separated by the distance of 127.5 pm.

6.169. Find the angular momentum of an oxygen molecule whose rotational energy is $E = 2.16$ meV and the distance between the nuclei is $d = 121$ pm.

6.170. Show that the frequency intervals between the neighbouring spectral lines of a true rotational spectrum of a diatomic molecule are equal. Find the moment of inertia and the distance between the nuclei of a CH molecule if the intervals between the neighbouring lines of the true rotational spectrum of these molecules are equal to $\Delta\omega = 5.47 \cdot 10^{12}$ s^{-1}.

6.171. For an HF molecule find the number of rotational levels located between the zeroth and first excited vibrational levels assuming rotational states to be independent of vibrational ones. The natural vibration frequency of this molecule is equal to $7.79 \cdot 10^{14}$ rad/s, and the distance between the nuclei is 91.7 pm.

6.172. Evaluate how many lines there are in a true rotational spectrum of CO molecules whose natural vibration frequency is $\omega = 4.09 \cdot 10^{14}$ s^{-1} and moment of inertia $I = 1.44 \cdot 10^{-39}$ g$\cdot$cm^2.

6.173. Find the number of rotational levels per unit energy interval, dN/dE, for a diatomic molecule as a function of rotational energy E. Calculate that magnitude for an iodine molecule in the state with rotational quantum number $J = 10$. The distance between the nuclei of that molecule is equal to 267 pm.

6.174. Find the ratio of energies required to excite a diatomic molecule to the first vibrational and to the first rotational level. Calculate that ratio for the following molecules:

Molecule	ω, 10^{14} s^{-1}	d, pm
(a) H_2	8.3	74
(b) HI	4.35	160
(c) I_2	0 40	267

Here ω is the natural vibration frequency of a molecule, d is the distance between nuclei.

6.175. The natural vibration frequency of a hydrogen molecule is equal to $8.25 \cdot 10^{14}$ s^{-1}, the distance between the nuclei is 74 pm. Find the ratio of the number of these molecules at the first excited vibrational level ($v = 1$) to the number of molecules at the first excited rotational level ($J = 1$) at a temperature $T = 875$ K. It should be remembered that the degeneracy of rotational levels is equal to $2J + 1$.

6.176. Derive Eq. (6.4c), making use of the Boltzmann distribution. From Eq. (6.4c) obtain the expression for molar vibration heat capacity $C_{V\ vib}$ of diatomic gas. Calculate $C_{V\ vib}$ for Cl_2 gas at the temperature 300 K. The natural vibration frequency of these molecules is equal to $1.064 \cdot 10^{14}$ s^{-1}.

6.177. In the middle of the rotation-vibration band of emission spectrum of HCl molecule, where the "zeroth" line is forbidden by the selection rules, the interval between neighbouring lines is $\Delta\omega = 0.79 \cdot 10^{13}$ s^{-1}. Calculate the distance between the nuclei of an HCl molecule.

6.178. Calculate the wavelengths of the red and violet satellites, closest to the fixed line, in the vibration spectrum of Raman scattering by F_2 molecules if the incident light wavelength is equal to $\lambda_0 = 404.7$ nm and the natural vibration frequency of the molecule is $\omega = 2.15 \cdot 10^{14}$ s^{-1}.

6.179. Find the natural vibration frequency and the quasielastic force coefficient of an S_2 molecule if the wavelengths of the red and violet satellites, closest to the fixed line, in the vibration spectrum of Raman scattering are equal to 346.6 and 330.0 nm.

6.180. Find the ratio of intensities of the violet and red satellites, closest to the fixed line, in the vibration spectrum of Raman scattering by Cl_2 molecules at a temperature $T = 300$ K if the natural vibration frequency of these molecules is $\omega = 1.06 \cdot 10^{14}$ s^{-1}. By what factor will this ratio change if the temperature is doubled?

6.181. Consider the possible vibration modes in the following linear molecules:

(a) CO_2 (O—C—O); (b) C_2H_2 (H—C—C—H).

6.182. Find the number of natural transverse vibrations of a string of length l in the frequency interval from ω to $\omega + d\omega$ if the propagation velocity of vibrations is equal to v. All vibrations are supposed to occur in one plane.

6.183. There is a square membrane of area S. Find the number of natural vibrations perpendicular to its plane in the frequency interval from ω to $\omega + d\omega$ if the propagation velocity of vibrations is equal to v.

6.184. Find the number of natural transverse vibrations of a right-angled parallelepiped of volume V in the frequency interval from ω to $\omega + d\omega$ if the propagation velocity of vibrations is equal to v.

6.185. Assuming the propagation velocities of longitudinal and transverse vibrations to be the same and equal to v, find the Debye temperature

(a) for a unidimensional crystal, i.e. a chain of identical atoms, incorporating n_0 atoms per unit length;

(b) for a two-dimensional crystal, i.e. a plane square grid consisting of identical atoms, containing n_0 atoms per unit area;

(c) for a simple cubic lattice consisting of identical atoms, containing n_0 atoms per unit volume.

6.186. Calculate the Debye temperature for iron in which the propagation velocities of longitudinal and transverse vibrations are equal to 5.85 and 3.23 km/s respectively.

6.187. Evaluate the propagation velocity of acoustic vibrations in aluminium whose Debye temperature is $\Theta = 396$ K.

6.188. Derive the formula expressing molar heat capacity of a unidimensional crystal, a chain of identical atoms, as a function of temperature T if the Debye temperature of the chain is equal to Θ. Simplify the obtained expression for the case $T \gg \Theta$.

6.189. In a chain of identical atoms the vibration frequency ω depends on wave number k as $\omega = \omega_{max} \sin (ka/2)$, where ω_{max} is the maximum vibration frequency, $k = 2\pi/\lambda$ is the wave number corresponding to frequency ω, a is the distance between neighbouring atoms. Making use of this dispersion relation, find the dependence of the number of longitudinal vibrations per unit frequency interval on ω, i.e. $dN/d\omega$, if the length of the chain is l. Having obtained $dN/d\omega$, find the total number N of possible longitudinal vibrations of the chain.

6.190. Calculate the zero-point energy per one gram of copper whose Debye temperature is $\Theta = 330$ K.

6.191. Fig. 6.10 shows heat capacity of a crystal vs temperature in terms of the Debye theory. Here C_{cl} is classical heat capacity, Θ is the Debye temperature. Using this plot, find:

(a) the Debye temperature for silver if at a temperature $T = 65$ K its molar heat capacity is equal to 15 J/(mol·K);

(b) the molar heat capacity of aluminium at $T = 80$ K if at $T = 250$ K it is equal to 22.4 J/(mol·K);

(c) the maximum vibration frequency for copper whose heat capacity at $T = 125$ K differs from the classical value by 25%.

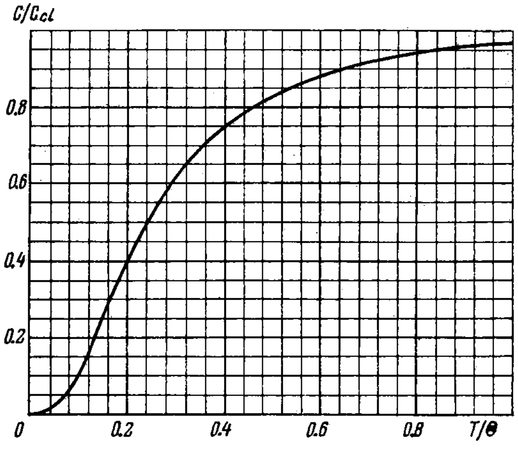

Fig. 6.10.

6.192. Demonstrate that molar heat capacity of a crystal at a temperature $T \ll \Theta$, where Θ is the Debye temperature, is defined by Eq. (6.4f).

6.193. Can one consider the temperatures 20 and 30 K as low for a crystal whose heat capacities at these temperatures are equal to 0.226 and 0.760 J/(mol·K)?

6.194. Calculate the mean zero-point energy per one oscillator of a crystal in terms of the Debye theory if the Debye temperature of the crystal is equal to Θ.

6.195. Draw the vibration energy of a crystal as a function of frequency (neglecting the zero-point vibrations). Consider two cases: $T = \Theta/2$ and $T = \Theta/4$, where Θ is the Debye temperature.

6.196. Evaluate the maximum values of energy and momentum of a phonon (acoustic quantum) in copper whose Debye temperature is equal to 330 K.

6.197. Employing Eq. (6.4g), find at $T = 0$:

(a) the maximum kinetic energy of free electrons in a metal if their concentration is equal to n;

(b) the mean kinetic energy of free electrons if their maximum kinetic energy T_{max} is known.

6.198. What fraction (in per cent) of free electrons in a metal at $T = 0$ has a kinetic energy exceeding half the maximum energy?

6.199. Find the number of free electrons per one sodium atom at $T = 0$ if the Fermi level is equal to $E_F = 3.07$ eV and the density of sodium is 0.97 g/cm³.

6.200. Up to what temperature has one to heat classical electronic gas to make the mean energy of its electrons equal to that of free electrons in copper at $T = 0$? Only one free electron is supposed to correspond to each copper atom.

6.201. Calculate the interval (in eV units) between neighbouring levels of free electrons in a metal at $T = 0$ near the Fermi level, if the concentration of free electrons is $n = 2.0 \cdot 10^{22}$ cm^{-3} and the volume of the metal is $V = 1.0$ cm^3.

6.202. Making use of Eq. (6.4g), find at $T = 0$:

(a) the velocity distribution of free electrons;

(b) the ratio of the mean velocity of free electrons to their maximum velocity.

6.203. On the basis of Eq. (6.4g) find the number of free electrons in a metal at $T = 0$ as a function of de Broglie wavelengths.

6.204. Calculate the electronic gas pressure in metallic sodium, at $T = 0$, in which the concentration of free electrons is $n = 2.5 \cdot 10^{22}$ cm^{-3}. Use the equation for the pressure of ideal gas.

6.205. The increase in temperature of a cathode in electronic tube by $\Delta T = 1.0$ K from the value $T = 2000$ K results in the increase of saturation current by $\eta = 1.4\%$. Find the work function of electron for the material of the cathode.

6.206. Find the refractive index of metallic sodium for electrons with kinetic energy $T = 135$ eV. Only one free electron is assumed to correspond to each sodium atom.

6.207. Find the minimum energy of electron-hole pair formation in an impurity-free semiconductor whose electric conductance increases $\eta = 5.0$ times when the temperature increases from $T_1 = 300$ K to $T_2 = 400$ K.

6.208. At very low temperatures the photoelectric threshold short wavelength in an impurity-free germanium is equal to $\lambda_{th} = 1.7$ μm. Find the temperature coefficient of resistance of this germanium sample at room temperature.

6.209. Fig. 6.11 illustrates logarithmic electric conductance as a function of reciprocal temperature (T in kK units) for some

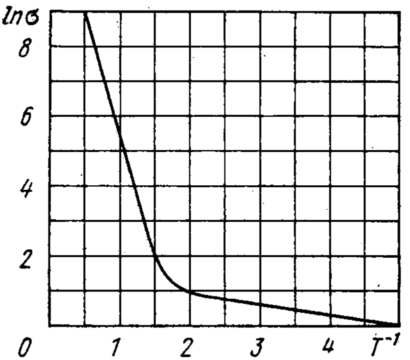

Fig. 6.11.

n-type semiconductor. Using this plot, find the width of the forbidden band of the semiconductor and the activation energy of donor levels.

6.210. The resistivity of an impurity-free semiconductor at room temperature is $\rho = 50\ \Omega\cdot\text{cm}$. It becomes equal to $\rho_1 = 40\ \Omega\cdot\text{cm}$ when the semiconductor is illuminated with light, and $t = 8$ ms after switching off the light source the resistivity becomes equal to $\rho_2 = 45\ \Omega\cdot\text{cm}$. Find the mean lifetime of conduction electrons and holes.

6.211. In Hall effect measurements a plate of width $h = 10$ mm and length $l = 50$ mm made of p-type semiconductor was placed in a magnetic field with induction $B = 5.0$ kG. A potential difference $V = 10$ V was applied across the edges of the plate. In this case the Hall field is $V_H = 50$ mV and resistivity $\rho = 2.5\ \Omega\cdot\text{cm}$. Find the concentration of holes and hole mobility.

6.212. In Hall effect measurements in a magnetic field with induction $B = 5.0$ kG the transverse electric field strength in an impurity-free germanium turned out to be $\eta = 10$ times less than the longitudinal electric field strength. Find the difference in the mobilities of conduction electrons and holes in the given semiconductor.

6.213. The Hall effect turned out to be not observable in a semiconductor whose conduction electron mobility was $\eta = 2.0$ times that of the hole mobility. Find the ratio of hole and conduction electron concentrations in that semiconductor.

6.5. RADIOACTIVITY

• Fundamental law of radioactive decay:

$$N = N_0 e^{-\lambda t}. \tag{6.5a}$$

• Relation between the decay constant λ, the mean lifetime τ, and the half-life T:

$$\lambda = \frac{1}{\tau} = \frac{\ln 2}{T}. \tag{6.5b}$$

• Specific activity is the activity of a unit mass of a radioisotope.

6.214. Knowing the decay constant λ of a nucleus, find:

(a) the probability of decay of the nucleus during the time from 0 to t;

(b) the mean lifetime τ of the nucleus.

6.215. What fraction of the radioactive cobalt nuclei whose half-life is 71.3 days decays during a month?

6.216. How many beta-particles are emitted during one hour by 1.0 μg of Na^{24} radionuclide whose half-life is 15 hours?

6.217. To investigate the beta-decay of Mg^{23} radionuclide, a counter was activated at the moment $t = 0$. It registered N_1 beta-particles by a moment $t_1 = 2.0$ s, and by a moment $t_2 = 3t_1$ the number of registered beta-particles was 2.66 times greater. Find the mean lifetime of the given nuclei.

6.218. The activity of a certain preparation decreases 2.5 times after 7.0 days. Find its half-life.

6.219. At the initial moment the activity of a certain radionuclide totalled 650 particles per minute. What will be the activity of the preparation after half its half-life period?

6.220. Find the decay constant and the mean lifetime of Co^{55} radionuclide if its activity is known to decrease 4.0% per hour. The decay product is nonradioactive.

6.221. A U^{238} preparation of mass 1.0 g emits $1.24 \cdot 10^4$ alpha-particles per second. Find the half-life of this nuclide and the activity of the preparation.

6.222. Determine the age of ancient wooden items if it is known that the specific activity of C^{14} nuclide in them amounts to 3/5 of that in lately felled trees. The half-life of C^{14} nuclei is 5570 years.

6.223. In a uranium ore the ratio of U^{238} nuclei to Pb^{206} nuclei is $\eta = 2.8$. Evaluate the age of the ore, assuming all the lead Pb^{206} to be a final decay product of the uranium series. The half-life of U^{238} nuclei is $4.5 \cdot 10^9$ years.

6.224. Calculate the specific activities of Na^{24} and U^{235} nuclides whose half-lifes are 15 hours and $7.1 \cdot 10^8$ years respectively.

6.225. A small amount of solution containing Na^{24} radionuclide with activity $A = 2.0 \cdot 10^3$ disintegrations per second was injected in the bloodstream of a man. The activity of 1 cm³ of blood sample taken $t = 5.0$ hours later turned out to be $A' = 16$ disintegrations per minute per cm³. The half-life of the radionuclide is $T = 15$ hours. Find the volume of the man's blood.

6.226. The specific activity of a preparation consisting of radioactive Co^{58} and nonradioactive Co^{59} is equal to $2.2 \cdot 10^{12}$ dis/(s·g). The half-life of Co^{58} is 71.3 days. Find the ratio of the mass of radioactive cobalt in that preparation to the total mass of the preparation (in per cent).

6.227. A certain preparation includes two beta-active components with different half-lifes. The measurements resulted in the following dependence of the natural logarithm of preparation activity on time t expressed in hours:

t	0	1	2	3	5	7	10	14	20
$\ln A$	4.10	3.60	3.10	2.60	2.06	1.82	1.60	1.32	0.90

Find the half-lifes of both components and the ratio of radioactive nuclei of these components at the moment $t = 0$.

6.228. A P^{32} radionuclide with half-life $T = 14.3$ days is produced in a reactor at a constant rate $q = 2.7 \cdot 10^9$ nuclei per second. How soon after the beginning of production of that radionuclide will its activity be equal to $A = 1.0 \cdot 10^9$ dis/s?

6.229. A radionuclide A_1 with decay constant λ_1 transforms into a radionuclide A_2 with decay constant λ_2. Assuming that at the initial moment the preparation contained only the radionuclide A_1, find:

(a) the equation describing accumulation of the radionuclide A_2 with time;

(b) the time interval after which the activity of radionuclide A_2 reaches the maximum value.

6.230. Solve the foregoing problem if $\lambda_1 = \lambda_2 = \lambda$.

6.231. A radionuclide A_1 goes through the transformation chain $A_1 \rightarrow A_2 \rightarrow A_3$ (stable) with respective decay constants λ_1 and λ_2. Assuming that at the initial moment the preparation contained only the radionuclide A_1 equal in quantity to N_{10} nuclei, find the equation describing accumulation of the stable isotope A_3.

6.232. A Bi^{210} radionuclide decays via the chain

$$Bi^{210} \xrightarrow[\lambda_1]{} Po^{210} \xrightarrow[\lambda_2]{} Pb^{206} \text{ (stable)},$$

where the decay constants are $\lambda_1 = 1.60 \cdot 10^{-6}$ s^{-1}, $\lambda_2 = 5.80 \cdot 10^{-8}$ s^{-1}. Calculate alpha- and beta-activities of the Bi^{210} preparation of mass 1.00 mg a month after its manufacture.

6.233. (a) What isotope is produced from the alpha-radioactive Ra^{226} as a result of five alpha-disintegrations and four β^--disintegrations?

(b) How many alpha- and β^--decays does U^{238} experience before turning finally into the stable Pb^{206} isotope?

6.234. A stationary Pb^{200} nucleus emits an alpha-particle with kinetic energy $T_\alpha = 5.77$ MeV. Find the recoil velocity of a daughter nucleus. What fraction of the total energy liberated in this decay is accounted for by the recoil energy of the daughter nucleus?

6.235. Find the amount of heat generated by 1.00 mg of a Po^{210} preparation during the mean lifetime period of these nuclei if the emitted alpha-particles are known to possess the kinetic energy 5.3 MeV and practically all daughter nuclei are formed directly in the ground state.

6.236. The alpha-decay of Po^{210} nuclei (in the ground state) is accompanied by emission of two groups of alpha-particles with kinetic energies 5.30 and 4.50 MeV. Following the emission of these particles the daughter nuclei are found in the ground and excited states. Find the energy of gamma-quanta emitted by the excited nuclei.

6.237. The mean path length of alpha-particles in air under standard conditions is defined by the formula $R = 0.98 \cdot 10^{-27} v_0^3$ cm, where v_0 (cm/s) is the initial velocity of an alpha-particle. Using this formula, find for an alpha-particle with initial kinetic energy 7.0 MeV:

(a) its mean path length;

(b) the average number of ion pairs formed by the given alpha-particle over the whole path R as well as over its first half, assuming the ion pair formation energy to be equal to 34 eV.

6.238. Find the energy Q liberated in β^-- and β^+-decays and in K-capture if the masses of the parent atom M_p, the daughter atom M_d and an electron m are known.

6.239. Taking the values of atomic masses from the tables, find the maximum kinetic energy of beta-particles emitted by Be^{10} nuclei and the corresponding kinetic energy of recoiling daughter nuclei formed directly in the ground state.

6.240. Evaluate the amount of heat produced during a day by a β^--active Na^{24} preparation of mass $m = 1.0$ mg. The beta-particles are assumed to possess an average kinetic energy equal to 1/3 of the highest possible energy of the given decay. The half-life of Na^{24} is $T = 15$ hours.

6.241. Taking the values of atomic masses from the tables, calculate the kinetic energies of a positron and a neutrino emitted by C^{11} nucleus for the case when the daughter nucleus does not recoil.

6.242. Find the kinetic energy of the recoil nucleus in the positronic decay of a N^{13} nucleus for the case when the energy of positrons is maximum.

6.243. From the tables of atomic masses determine the velocity of a nucleus appearing as a result of K-capture in a Be^7 atom provided the daughter nucleus turns out to be in the ground state.

6.244. Passing down to the ground state, excited Ag^{109} nuclei emit either gamma quanta with energy 87 keV or K conversion electrons whose binding energy is 26 keV. Find the velocity of these electrons.

6.245. A free stationary Ir^{191} nucleus with excitation energy $E = 129$ keV passes to the ground state, emitting a gamma quantum. Calculate the fractional change of gamma quanta energy due to recoil of the nucleus.

6.246. What must be the relative velocity of a source and an absorber consisting of free Ir^{191} nuclei to observe the maximum absorption of gamma quanta with energy $\varepsilon = 129$ keV?

6.247. A source of gamma quanta is placed at a height $h = 20$ m above an absorber. With what velocity should the source be displaced upward to counterbalance completely the gravitational variation of gamma quanta energy due to the Earth's gravity at the point where the absorber is located?

6.248. What is the minimum height to which a gamma quanta source containing excited Zn^{67} nuclei has to be raised for the gravitational displacement of the Mössbauer line to exceed the line width itself, when registered on the Earth's surface? The registered gamma quanta are known to have an energy $\varepsilon = 93$ keV and appear on transition of Zn^{67} nuclei to the ground state, and the mean lifetime of the excited state is $\tau = 14$ μs.

• Binding energy of a nucleus:

$$E_b = Zm_H + (A - Z)\, m_n - M, \tag{6.6a}$$

where Z is the charge of the nucleus (in units of e), A is the mass number, m_H, m_n, and M are the masses of a hydrogen atom, a neutron, and an *atom* corresponding to the given nucleus.

In calculations the following formula is more convenient to use:

$$E_b = Z\Delta_H + (A - Z)\Delta_n - \Delta, \tag{6.6b}$$

where Δ_H, Δ_n, and Δ are the mass surpluses of a hydrogen atom, a neutron, and an atom corresponding to the given nucleus.

• Energy diagram of a nuclear reaction

$$m + M \rightarrow M^* \rightarrow m' + M' + Q \tag{6.6c}$$

is illustrated in Fig. 6.12, where $m+M$ and $m'+M'$ are the sums of rest masses of particles before and after the reaction, $\tilde{T}$ and $\tilde{T}'$ are the total kinetic energies of particles before and after the reaction (in the frame of the centre of inertia), E^* is the excitation energy of the transitional nucleus, Q is the energy of the reaction, E and E' are the binding energies of the particles m and m' in the transitional nucleus, *1*, *2*, *3* are the energy levels of the transitional nucleus.

• Threshold (minimum) kinetic energy of an incoming particle at which an endoergic nuclear reaction

$$T_{th} = \frac{m+M}{M}\,|Q| \tag{6.6d}$$

becomes possible; here m and M are the masses of the incoming particle and the target nucleus.

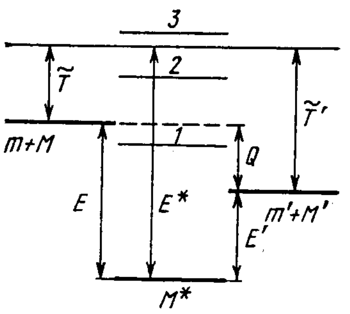

Fig. 6.12.

6.249. An alpha-particle with kinetic energy $T_\alpha = 7.0$ MeV is scattered elastically by an initially stationary Li^6 nucleus. Find the kinetic energy of the recoil nucleus if the angle of divergence of the two particles is $\Theta = 60°$.

6.250. A neutron collides elastically with an initially stationary deuteron. Find the fraction of the kinetic energy lost by the neutron

(a) in a head-on collision;

(b) in scattering at right angles.

6.251. Find the greatest possible angle through which a deuteron is scattered as a result of elastic collision with an initially stationary proton.

6.252. Assuming the radius of a nucleus to be equal to $R = 0.13\sqrt[3]{A}$ pm, where A is its mass number, evaluate the density of nuclei and the number of nucleons per unit volume of the nucleus.

6.253. Write missing symbols, denoted by x, in the following nuclear reactions:

(a) $B^{10}(x, \alpha)Be^8$;

(b) O^{17} (d, n) x;
(c) Na^{23} (p, x) Ne^{20};
(d) x (p, n) Ar^{37}.

6.254. Demonstrate that the binding energy of a nucleus with mass number A and charge Z can be found from Eq. (6.6b).

6.255. Find the binding energy of a nucleus consisting of equal numbers of protons and neutrons and having the radius one and a half times smaller than that of Al^{27} nucleus.

6.256. Making use of the tables of atomic masses, find:
(a) the mean binding energy per one nucleon in O^{16} nucleus;
(b) the binding energy of a neutron and an alpha-particle in a B^{11} nucleus;
(c) the energy required for separation of an O^{16} nucleus into four identical particles.

6.257. Find the difference in binding energies of a neutron and a proton in a B^{11} nucleus. Explain why there is the difference.

6.258. Find the energy required for separation of a Ne^{20} nucleus into two alpha-particles and a C^{12} nucleus if it is known that the binding energies per one nucleon in Ne^{20}, He^4, and C^{12} nuclei are equal to 8.03, 7.07, and 7.68 MeV respectively.

6.259. Calculate in atomic mass units the mass of
(a) a Li^8 atom whose nucleus has the binding energy 41.3 MeV;
(b) a C^{10} nucleus whose binding energy per nucleon is equal to 6.04 MeV.

6.260. The nuclei involved in the nuclear reaction $A_1 + A_2 \rightarrow$ $\rightarrow A_3 + A_4$ have the binding energies E_1, E_2, E_3, and E_4. Find the energy of this reaction.

6.261. Assuming that the splitting of a U^{235} nucleus liberates the energy of 200 MeV, find:
(a) the energy liberated in the fission of one kilogram of U^{235} isotope, and the mass of coal with calorific value of 30 kJ/g which is equivalent to that for one kg of U^{235};
(b) the mass of U^{235} isotope split during the explosion of the atomic bomb with 30 kt trotyl equivalent if the calorific value of trotyl is 4.1 kJ/g.

6.262. What amount of heat is liberated during the formation of one gram of He^4 from deuterium H^2? What mass of coal with calorific value of 30 kJ/g is thermally equivalent to the magnitude obtained?

6.263. Taking the values of atomic masses from the tables, calculate the energy per nucleon which is liberated in the nuclear reaction $Li^6 + H^2 \rightarrow 2He^4$. Compare the obtained magnitude with the energy per nucleon liberated in the fission of U^{235} nucleus.

6.264. Find the energy of the reaction $Li^7 + p \rightarrow 2He^4$ if the binding energies per nucleon in Li^7 and He^4 nuclei are known to be equal to 5.60 and 7.06 MeV respectively.

6.265. Find the energy of the reaction N^{14} (α, p) O^{17} if the kinetic energy of the incoming alpha-particle is $T_\alpha = 4.0$ MeV and the proton outgoing at an angle $\theta = 60°$ to the motion direction of the alpha-particle has a kinetic energy $T_p = 2.09$ MeV.

6.266. Making use of the tables of atomic masses, determine the energies of the following reactions:

(a) Li^7 (p, n) Be^7;
(b) Be^9 (n, γ) Be^{10};
(c) Li^7 (α, n) B^{10};
(d) O^{16} (d, α) N^{14}.

6.267. Making use of the tables of atomic masses, find the velocity with which the products of the reaction B^{10} (n, α) Li^7 come apart; the reaction proceeds via interaction of very slow neutrons with stationary boron nuclei.

6.268. Protons striking a stationary lithium target activate a reaction Li^7 (p, n) Be^7. At what value of the proton's kinetic energy can the resulting neutron be stationary?

6.269. An alpha particle with kinetic energy $T = 5.3$ MeV initiates a nuclear reaction Be^9 (α, n) C^{12} with energy yield $Q = +5.7$ MeV. Find the kinetic energy of the neutron outgoing at right angles to the motion direction of the alpha-particle.

6.270. Protons with kinetic energy $T = 1.0$ MeV striking a lithium target induce a nuclear reaction $p + Li^7 \rightarrow 2He^4$. Find the kinetic energy of each alpha-particle and the angle of their divergence provided their motion directions are symmetrical with respect to that of incoming protons.

6.271. A particle of mass m strikes a stationary nucleus of mass M and activates an endoergic reaction. Demonstrate that the threshold (minimal) kinetic energy required to initiate this reaction is defined by Eq. (6.6d).

6.272. What kinetic energy must a proton possess to split a deuteron H^2 whose binding energy is $E_b = 2.2$ MeV?

6.273. The irradiation of lithium and beryllium targets by a monoergic stream of protons reveals that the reaction $Li^7(p, n)Be^7 - 1.65$ MeV is initiated whereas the reaction $Be^9(p, n)B^9 - 1.85$ MeV does not take place. Find the possible values of kinetic energy of the protons.

6.274. To activate the reaction (n, α) with stationary B^{11} nuclei, neutrons must have the threshold kinetic energy $T_{th} = 4.0$ MeV. Find the energy of this reaction.

6.275. Calculate the threshold kinetic energies of protons required to activate the reactions (p, n) and (p, d) with Li^7 nuclei.

6.276. Using the tabular values of atomic masses, find the threshold kinetic energy of an alpha particle required to activate the nuclear reaction Li^7 (α, n) B^{10}. What is the velocity of the B^{10} nucleus in this case?

6.277. A neutron with kinetic energy $T = 10$ MeV activates a nuclear reaction C^{12} (n, α) Be^9 whose threshold is $T_{th} = 6.17$ MeV. Find the kinetic energy of the alpha-particles outgoing at right angles to the incoming neutrons' direction.

6.278. How much, in per cent, does the threshold energy of gamma quantum exceed the binding energy of a deuteron ($E_b = 2.2$ MeV) in the reaction $\gamma + H^2 \rightarrow n + p$?

6.279. A proton with kinetic energy $T = 1.5$ MeV is captured by a deuteron H^2. Find the excitation energy of the formed nucleus.

6.280. The yield of the nuclear reaction $C^{13}(d, n)N^{14}$ has maximum magnitudes at the following values of kinetic energy T_i of bombarding deuterons: 0.60, 0.90, 1.55, and 1.80 MeV. Making use of the table of atomic masses, find the corresponding energy levels of the transitional nucleus through which this reaction proceeds.

6.281. A narrow beam of thermal neutrons is attenuated $\eta = 360$ times after passing through a cadmium plate of thickness $d = 0.50$ mm. Determine the effective cross-section of interaction of these neutrons with cadmium nuclei.

6.282. Determine how many times the intensity of a narrow beam of thermal neutrons will decrease after passing through the heavy water layer of thickness $d = 5.0$ cm. The effective cross-sections of interaction of deuterium and oxygen nuclei with thermal neutrons are equal to $\sigma_1 = 7.0$ b and $\sigma_2 = 4.2$ b respectively.

6.283. A narrow beam of thermal neutrons passes through a plate of iron whose absorption and scattering effective cross-sections are equal to $\sigma_a = 2.5$ b and $\sigma_s = 11$ b respectively. Find the fraction of neutrons quitting the beam due to scattering if the thickness of the plate is $d = 0.50$ cm.

6.284. The yield of a nuclear reaction producing radionuclides may be described in two ways: either by the ratio w of the number of nuclear reactions to the number of bombarding particles, or by the quantity k, the ratio of the activity of the formed radionuclide to the number of bombarding particles. Find:

(a) the half-life of the formed radionuclide, assuming w and k to be known;

(b) the yield w of the reaction $Li^7(p, n)Be^7$ if after irradiation of a lithium target by a beam of protons (over $t = 2.0$ hours and with beam current $I = 10$ μA) the activity of Be^7 became equal to $A = 1.35 \cdot 10^8$ dis/s and its half-life to $T = 53$ days.

6.285. Thermal neutrons fall normally on the surface of a thin gold foil consisting of stable Au^{197} nuclide. The neutron flux density is $J = 1.0 \cdot 10^{10}$ part./(s·cm²). The mass of the foil is $m = 10$ mg. The neutron capture produces beta-active Au^{198} nuclei with half-life $T = 2.7$ days. The effective capture cross-section is $\sigma = 98$ b. Find:

(a) the irradiation time after which the number of Au^{197} nuclei decreases by $\eta = 1.0\%$;

(b) the maximum number of Au^{198} nuclei that can be formed during protracted irradiation.

6.286. A thin foil of certain stable isotope is irradiated by thermal neutrons falling normally on its surface. Due to the capture of neutrons a radionuclide with decay constant λ appears. Find the law describing accumulation of that radionuclide $N(t)$ per unit area of the foil's surface. The neutron flux density is J, the number of nuclei per unit area of the foil's surface is n, and the effective cross-section of formation of active nuclei is σ.

6.287. A gold foil of mass $m = 0.20$ g was irradiated during $t = 6.0$ hours by a thermal neutron flux falling normally on its surface. Following $\tau = 12$ hours after the completion of irradiation the activity of the foil became equal to $A = 1.9 \cdot 10^7$ dis/s. Find the neutron flux density if the effective cross-section of formation of a radioactive nucleus is $\sigma = 96$ b, and the half-life is equal to $T = 2.7$ days.

6.288. How many neutrons are there in the hundredth generation if the fission process starts with $N_0 = 1000$ neutrons and takes place in a medium with multiplication constant $k = 1.05$?

6.289. Find the number of neutrons generated per unit time in a uranium reactor whose thermal power is $P = 100$ MW if the average number of neutrons liberated in each nuclear splitting is $\nu = 2.5$. Each splitting is assumed to release an energy $E = 200$ MeV.

6.290. In a thermal reactor the mean lifetime of one generation of thermal neutrons is $\tau = 0.10$ s. Assuming the multiplication constant to be equal to $k = 1.010$, find:

(a) how many times the number of neutrons in the reactor, and consequently its power, will increase over $t = 1.0$ min;

(b) the period T of the reactor, i.e. the time period over which its power increases e-fold.

6.7. ELEMENTARY PARTICLES

- Total energy and momentum of a relativistic particle:

$$E = m_0c^2 + T, \quad pc = \sqrt{T(T + 2m_0c^2)}, \tag{6.7a}$$

where T is the kinetic energy of the particle.

- When examining collisions of particles it pays to use the invariant:

$$E^2 - p^2c^2 = m_0^2c^4, \tag{6.7b}$$

where E and p are the total energy and the total momentum of the system prior to collision, m_0 is the rest mass of the formed particle.

- Threshold (minimal) kinetic energy of a particle m striking a stationary particle M and activating the endoergic reaction $m + M \rightarrow m_1 + m_2 + \ldots$:

$$T_{th} = \frac{(m_1 + m_2 + \ldots)^2 - (m + M)^2}{2M} c^2, \tag{6.7c}$$

where $m, M, m_1, m_2, \ldots$ are the rest masses of the respective particles.

- Quantum numbers classifying elementary particles:

Q, electric charge,
L, lepton charge,
B, baryon charge,
T, isotopic spin, T_z, its projection,
S, strangeness, $S = 2\langle Q \rangle - B$,
Y, hypercharge, $Y = B + S$.

• Relation between quantum numbers of strongly interacting particles:

$$Q = T_z + \frac{Y}{2} = T_z + \frac{B+S}{2} \tag{6.7d}$$

• Interactions of particles obey the laws of conservation of the Q, L and B charges. In strong interactions the laws of conservation of S (or Y), T, and its projection T_z are also valid.

6.291. Calculate the kinetic energies of protons whose momenta are 0.10, 1.0, and 10 GeV/c, where c is the velocity of light.

6.292. Find the mean path travelled by pions whose kinetic energy exceeds their rest energy $\eta = 1.2$ times. The mean lifetime of very slow pions is $\tau_0 = 25.5$ ns.

6.293. Negative pions with kinetic energy $T = 100$ MeV travel an average distance $l = 11$ m from their origin to decay. Find the proper lifetime of these pions.

6.294. There is a narrow beam of negative pions with kinetic energy T equal to the rest energy of these particles. Find the ratio of fluxes at the sections of the beam separated by a distance $l = 20$ m. The proper mean lifetime of these pions is $\tau_0 = 25.5$ ns.

6.295. A stationary positive pion disintegrated into a muon and a neutrino. Find the kinetic energy of the muon and the energy of the neutrino.

6.296. Find the kinetic energy of a neutron emerging as a result of the decay of a stationary Σ^- hyperon ($\Sigma^- \rightarrow n + \pi^-$).

6.297. A stationary positive muon disintegrated into a positron and two neutrinos. Find the greatest possible kinetic energy of the positron.

6.298. A stationary neutral particle disintegrated into a proton with kinetic energy $T = 5.3$ MeV and a negative pion. Find the mass of that particle. What is its name?

6.299. A negative pion with kinetic energy $T = 50$ MeV disintegrated during its flight into a muon and a neutrino. Find the energy of the neutrino outgoing at right angles to the pion's motion direction.

6.300. A Σ^+ hyperon with kinetic energy $T_\Sigma = 320$ MeV disintegrated during its flight into a neutral particle and a positive pion outgoing with kinetic energy $T_\pi = 42$ MeV at right angles to the hyperon's motion direction. Find the rest mass of the neutral particle (in MeV units).

6.301. A neutral pion disintegrated during its flight into two gamma quanta with equal energies. The angle of divergence of gamma quanta is $\Theta = 60°$. Find the kinetic energy of the pion and of each gamma quantum.

6.302. A relativistic particle with rest mass m collides with a stationary particle of mass M and activates a reaction leading to formation of new particles: $m + M \rightarrow m_1 + m_2 + \ldots$, where the rest masses of newly formed particles are written on the right-hand side. Making use of the invariance of the quantity $E^2 - p^2c^2$, demonstrate that the threshold kinetic energy of the particle m required for this reaction is defined by Eq. (6.7c).

6.303. A positron with kinetic energy $T = 750$ keV strikes a stationary free electron. As a result of annihilation, two gamma quanta with equal energies appear. Find the angle of divergence between them.

6.304. Find the threshold energy of gamma quantum required to form

(a) an electron-positron pair in the field of a stationary electron;

(b) a pair of pions of opposite signs in the field of a stationary proton.

6.305. Protons with kinetic energy T strike a stationary hydrogen target. Find the threshold values of T for the following reactions:

(a) $p + p \to p + p + p + \tilde{p}$; (b) $p + p \to p + p + \pi^0$.

6.306. A hydrogen target is bombarded by pions. Calculate the threshold values of kinetic energies of these pions making possible the following reactions:

(a) $\pi^- + p \to K^+ + \Sigma^-$; (b) $\pi^0 + p \to K^+ + \Lambda^0$.

6.307. Find the strangeness S and the hypercharge Y of a neutral elementary particle whose isotopic spin projection is $T_z = +1/2$ and baryon charge $B = +1$. What particle is this?

6.308. Which of the following processes are forbidden by the law of conservation of lepton charge:

(1) $n \to p + e^- + \nu$; (4) $p + e^- \to n + \nu$;
(2) $\pi^+ \to \mu^+ + e^- + e^+$; (5) $\mu^+ \to e^+ + \nu + \tilde{\nu}$;
(3) $\pi^- \to \mu^- + \nu$; (6) $K^- \to \mu^- + \tilde{\nu}$?

6.309. Which of the following processes are forbidden by the law of conservation of strangeness:

(1) $\pi^- + p \to \Sigma^- + K^+$; (4) $n + p \to \Lambda^0 + \Sigma^+$;
(2) $\pi^- + p \to \Sigma^+ + K^-$; (5) $\pi^- + n \to \Xi^- + K^+ + K^-$;
(3) $\pi^- + p \to K^+ + K^- + n$; (6) $K^- + p \to \Omega^- + K^+ + K^0$?

6.310. Indicate the reasons why the following processes are forbidden:

(1) $\Sigma^- \to \Lambda^0 + \pi^-$; (4) $n + p \to \Sigma^+ + \Lambda^0$;
(2) $\pi^- + p \to K^+ + K^-$; (5) $\pi^- \to \mu^- + e^+ + \tilde{e}^-$;
(3) $K^- + n \to \Omega^- + K^+ + K^0$; (6) $\mu^- \to e^- + \nu_e + \tilde{\nu}_\mu$.