Chapter 5

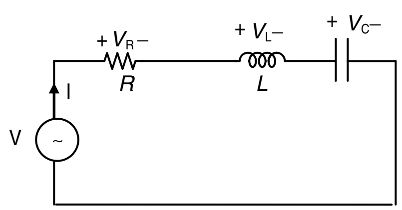
Resonance

CHAPTER HIGHLIGHTS

INTRODUCTION

Resonance in electrical circuits consisting of passive and active elements represents a particular state of the circuit when the current or voltage in the circuit is maximum or minimum with respect to the magnitude of excitation at a particular frequency the circuit impedance being either minimum or maximum at the power factor unity.

SERIES RESONANCE



The circuit is said to be in resonance if the current is in phase with the applied voltage.

The impedance $Z = R + j\omega L - \dfrac{1}{j\omega C}$

$$Z = R + j\left(\omega L - \frac{1}{\omega c}\right)$$

At resonance, the impedance is purely resistive

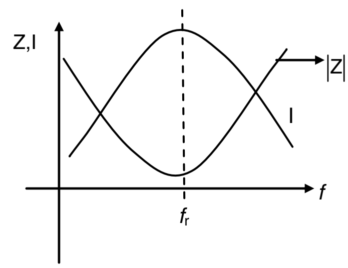
i.e., $$\omega L - \frac{1}{\omega C} = 0$$

$$\omega^2 = \frac{1}{LC}$$

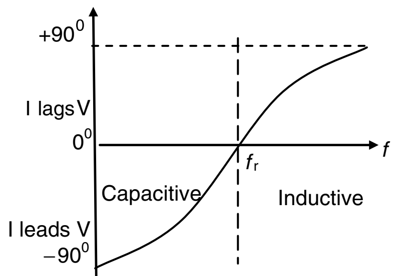
$$\omega = \frac{1}{\sqrt{LC}}$$

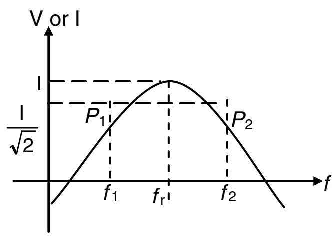
$f_r = \dfrac{1}{2\pi\sqrt{LC}}$ is the resonant frequency in Hz.

at resonance the current, $I = \dfrac{V}{R}$, that is, the current is maximum at resonance and the impedance is minimum.



- At zero frequency, X_C and Z are infinitely large, and X_L is zero because at zero frequency, the capacitor acts as an open circuit.
- As the frequency increases, X_C decreases and X_L increases. Since X_C is larger than X_L, at frequencies below the resonant frequency f_r, Z decreases along with X_C.
- At resonant frequency f_r, $X_C = X_L$ and $Z = R$.
  At frequencies above the resonant frequency f_r, X_L is larger than X_C, causing Z to increase.
- The phase angle as a function of frequency is shown in figure.





The bandwidth is the range of frequencies over which the current is equal to 70.7% of its value at the resonant frequency bandwidth, BW $= f_2 - f_1$.

where f_2 and f_1 are cut-off frequencies or half power frequencies.

At $f = f_1$ or f_2, the net reactance is R

i.e., $$\frac{1}{\omega_1 C} - \omega_1 L = R$$

and $$\frac{1}{\omega_2 C} - \omega_2 L = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = \frac{1}{\omega_2 C} - \omega_2 L$$

$$L(\omega_1 + \omega_2) = \frac{1}{C}\left[\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right]$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

$$\omega_r^2 = \omega_1 \omega_2$$

$$\omega_r = \sqrt{\omega_1 \omega_2}$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$f_2 - f_1 = \frac{R}{2\pi L}$$

Bandwidth, $$\text{BW} = \frac{R}{2\pi L}$$

$$f_1 = f_r - \frac{R}{4\pi L}$$

$$f_2 = f_r + \frac{R}{4\pi L}$$

The quality factor (Q factor) is the ratio of the reactance of the coil to its resistance at resonant frequency.

$$Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$$

$$\frac{f_2 - f_1}{f_r} = \frac{1}{Q}$$

$$Q = \frac{\omega L}{R} = \frac{1}{\omega CR}$$

$$\boxed{Q = \frac{f_r}{BW}}$$

Let V be the applied voltage across RLC circuit. At resonance, the current I is the voltage across L is $V_L = IX_L = \frac{V}{R}\omega_r L$.

$$V_L = QV$$

Similarly, $$V_C = VQ$$

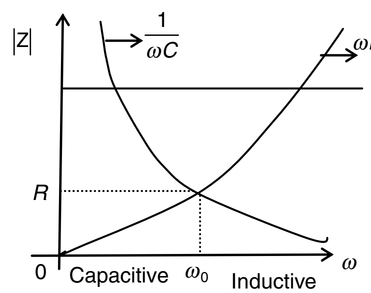
i.e., $$Q = \frac{V_L}{V} \text{ or } \frac{V_C}{V}$$

Quality Factor

A quality factor or figure of merit can be assigned to a component or to a complete circuit. It is defined as

$$Q = 2\pi\left(\frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}\right)$$

$$\boxed{Q = \frac{|X_L|}{R} = \frac{|X_C|}{R} = \frac{1}{R}\cdot\sqrt{\frac{L}{C}}}$$



- For $\omega < \omega_0 \Rightarrow$ series RLC circuit behaves like (RC circuit) capacitive circuit
  $\Rightarrow$ Leading power factor
  $\Rightarrow V_C = QV\angle -90°$
- For $\omega = \omega_0 \Rightarrow$ Resistive nature; pf = 1
- For $\omega > \omega_0 \Rightarrow$ inductive nature
  $\Rightarrow$ Lagging power factor
  $V_L = IZ_L$
  $V_L = QV\angle 90°$
  where $Q = \frac{\omega_0 L}{R}$
- **At Resonance:**
  $|V_L| = |V_C|$ and these are 180° out of phase.

Solved Examples

Example 1

In a series *RLC* circuit, the *Q* factor at resonance is 80. If all the component values are doubled, then the new *Q* factor is

(A) $Q^1 = 160$ (B) $Q^1 = 40$
(C) $Q^1 = 80^0$ (D) None of these

Solution

For a series *RLC* circuits,

$$Q = \frac{1}{R}\cdot\sqrt{\frac{L}{C}}$$

Given $R^1 = 2R$

$$L^1 = 2L$$

$$C^1 = 2C$$

$$Q^1 = \frac{Q}{2} = 40$$

Example 2

In a series *RLC* circuit, $R = 8\ \Omega$, $X_L = 16$, $X_C = 16$, and $V_{in} = 80$ V. The voltage across the capacitor is

(A) $V_C = 160\ \angle\ 90°$ (B) $V_C = 40\ \angle\ 90°$
(C) $V_C = 160\ \angle\ –90°$ (D) $V_C = 80\ \angle\ –90°$

Solution

$$V_C = QV\ \angle –90°$$

$$Q = \frac{|X_L|}{R} = \frac{16}{8} = 2$$

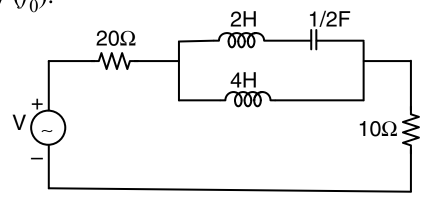
Given $V_{in} = V = 80$ V

$$V_C = 2 \times 80\ \angle –90°$$

$$= 160\ \angle –90°$$

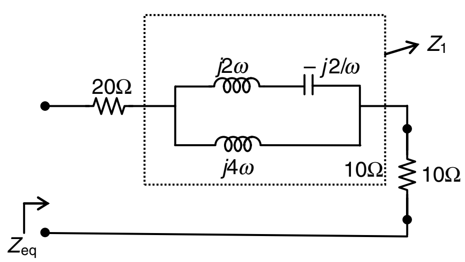
Example 3

For the circuit shown in figure, determine the resonant frequency (f_0).



(A) $f = 0$ Hz (B) $f = \frac{1}{2\pi}$ Hz
(C) $f = \frac{1}{\pi}$ Hz (D) None of these

Solution



$$Z_{eq} = 20 + Z_1 + 10$$

$$Z_1 = (j4\omega)\ ||[j(2\omega - 2/\omega)]$$

$$= \frac{j4\omega\ j(2\omega - 2/\omega)}{j4\omega + j2\omega - j2/\omega}$$

$$= \frac{j4\omega[2\omega - 2/\omega]}{6\omega - 2/\omega} \Rightarrow \frac{j4\omega[2\omega^2 - 2]}{6\omega^2 - 2}$$

$$= \frac{j4\omega[\omega^2 - 1]}{3\omega^2 - 1}$$

$$Z_{eq} = 20 + \frac{j\omega(\omega^2 - 1)\times 4}{3\omega^2 - 1} + 10$$

At resonance, imaginary part is equal to zero.

At $\omega = \omega_r$

$$\frac{\omega(\omega^2 - 1)\times 4}{3\omega^2 - 1} = 0$$

$$\Rightarrow \omega_r^2 - 1 = 0$$

$$\omega_r = 1$$

$$2\pi f_r = 1$$

$$f_r = \frac{1}{2\pi}\ \text{Hz}$$

Example 4

A series resonant circuit has $L = 10$ mH and $C = 100$ µF. The required *R* for the BW 16 Hz is

(A) $R = 16\ \Omega$ (B) $R = 0.16\ \Omega$
(C) $R = 160\ \Omega$ (D) $R = 1.6\ \Omega$

Solution

For a series *RLC* circuit, characteristic equation

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\text{BW} = \frac{R}{L}$$

$$\frac{R}{10\times 10^{-3}} = 16 \Rightarrow R = 0.16\ \Omega$$

Selectivity: The selectivity of a resonating circuit is defined by the ratio of f_0 (the resonance frequency) to the bandwidth of the circuit.

$$\text{Selectivity} = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}$$

Frequency at which V_C is Maximum

V_C and V_L are not maximum at resonant frequency in the case of series *RLC* resonance but at other frequency value. This can be obtained by

$$V_C = IX_C$$

$$= \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}}\ \frac{1}{\omega c}$$

where $I = \frac{V}{|Z|}$

The frequency at which V_C is maximum can be obtained by differentiating V_C w.r.t to ω

i.e., $$\frac{dV_c^2}{d\omega} = 0$$

$$\frac{dV_c^2}{d\omega} = 0 \Rightarrow \frac{-V\left[2\omega_0 (RC)^2 + 2(\omega_0^2 LC - 1).2\omega_0 LC\right]}{\left[(\omega_0 RC)^2 + (\omega_0^2 LC - 1)^2\right]^2}$$

However, $V \neq 0$,

$$2\omega_0^2 L^2 C + CR^2 - 2L = 0$$

By simplification, we get

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

where f_0 is at which voltage across capacitor is maximum.

The frequency at which V_L is maximum:

$$V_L = IX_L$$

$$= \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega c)^2}}\omega L$$

The frequency at which V_L is maximum is given by

$$\frac{dV_L^2}{d\omega} = 0$$

By the simplification, we get

$$2\omega^2 LC - \omega^2 (RC)^2 - 2 = 0$$

$$\omega^2 [2LC - (RC)^2] = 2$$

$$\omega^2 = \frac{2}{2LC - (RC)^2}$$

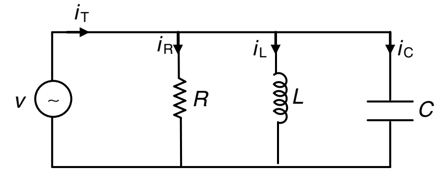
$$\boxed{f_{0L} = \frac{1}{2\pi} \times \frac{1}{\sqrt{LC - \frac{(RC)^2}{2}}}}$$

where f_{oL} is the frequency at which the voltage across the inductor is maximum.

Parallel Resonance

A circuit consisting of a parallel connection of R, L, and C is called a second-order parallel resonant circuit, and parallel resonance circuit is also called anti-resonance circuit. It acts as a band-reject filter.

Parallel Resonance



The admittance, $Y = G + \frac{1}{j\omega L} + j\omega C$

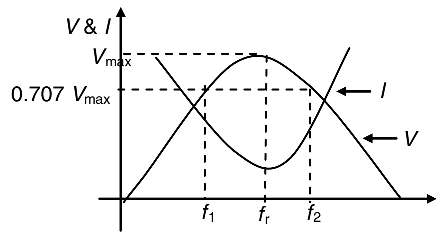
At resonance, 'Y' is purely real

i.e., $$j\left(\omega C - \frac{1}{\omega L}\right) = 0$$

$$\Rightarrow \omega C - \frac{1}{\omega L} = 0$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$



At resonance, the current is minimum and the impedance is maximum.

At lower half power frequency ω_1,

$$\omega_1 C - \frac{1}{\omega_1 L} = \frac{-1}{R}$$

$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

At upper half power frequency ω_2,

$$\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Bandwidth, $BW = \omega_2 - \omega_1 = \frac{1}{RC}$

The quality factor $Q = \frac{\omega_r}{BW}$

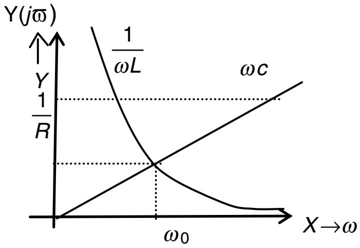
$$Q = \omega_r RC$$

$$Q = \frac{R}{\omega_r L}$$

At resonance, $I_L = I_r Q \ \angle -90^\circ$

$$I_C = I_r Q \ \angle 90^\circ$$

The resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ Hz

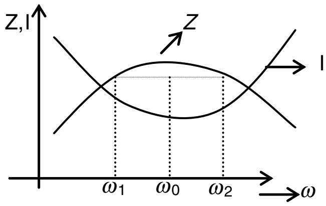


1. Below resonance, the circuit acts like an RL circuit $(\omega < \omega_o)$, that is, lagging power factor.
2. Above resonance, the circuit acts like an RC circuit $(\omega > \omega_o)$, that is, leading power factor.
3. At $\omega = \omega_0$, the circuit acts like an resistive nature.
4. BW $= \dfrac{1}{RC}$.
5. Quality factor $Q = R\sqrt{\dfrac{C}{L}}$
6. $\xi = \dfrac{1}{2Q}$

NOTE

Impedance of parallel resonant circuit is maximum at resonance. Current of parallel resonance circuit is minimum at resonance. $Z_{in} = R$ at $\omega = \omega_o$ is maximum.

$$I/\omega = \omega_o \ \frac{V_s}{R} \Rightarrow \text{minimum}$$



Example 5

A series resonant circuit has an inductive reactance of 1 kΩ, a capacitive reactance of 1 kΩ, and a resistance of 0.1 Ω. If the resonant frequency is 10 MHz, then the bandwidth of the circuit will be

(A) 1 kHz (B) 10 kHz
(C) 1 MHz (D) 0.1 kHz

Solution

$$\text{BW} = \frac{f_0}{Q} = \frac{f_0}{\left(\dfrac{\omega_0 L}{R}\right)}$$

$$\text{BW} = \frac{0.1 \times 10 \times 10^6}{1000} = 1 \text{ kHz}$$

Example 6

Assertion (A): A circuit containing reactance is said to be in resonance if the voltage across the circuit is in-phase with the current through it.

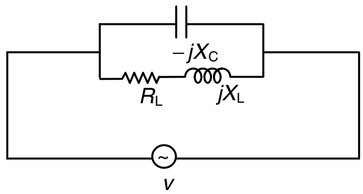
Reason (R): At resonance, the power factor of the circuit is zero.

(A) Both A and R are true and 'R' is the correct explanation of A.
(B) Both A and R are true but 'R' is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Solution

At resonance, the voltage across the circuit is in phase with the current through it and the power factor is '1'.
$\cos\phi = 1$ is the unity power factor.

Example 7



Find the resonant frequency for the tank circuit shown in figure.

Solution

The admittance is given by

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{-jX_C} = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{j}{X_C}$$

$$Y = \frac{R_L}{RL^2 + X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

At resonance, Y is purely real

$$\therefore \ \frac{1}{X_C} = \frac{X_L}{R_L^2 + X_L^2}$$

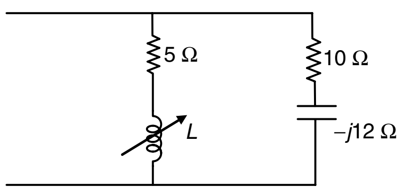
$$\omega C = \frac{\omega L}{R_L^2 + \omega^2 L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

Example 8

Find the value of L at which the circuit resonates at a frequency of 1,000 rad/s in the following circuit.



Solution

$$Y = \frac{1}{10 - j12} + \frac{1}{5 + jX_L}$$

$$Y = \frac{10 + j12}{10^2 + 12^2} + \frac{5 - jX_L}{25 + X_L^2}$$

$$Y = \frac{10}{10^2 + 12^2} + \frac{5}{25^2 + X_L^2} + j\left[\frac{12}{10^2 + 12^2} - \frac{X_L}{25 + X_L^2}\right]$$

At resonance, Y is purely real

i.e., $$\frac{X_L}{25 + X_L^2} = \frac{12}{10^2 + 12^2}$$

$$X_L^2 - 20.3X_L + 25 = 0$$

$$X_L = 18.98\ \Omega \text{ or } 1.32\ \Omega$$

$$X_L = \omega L = 18.98\ \Omega \text{ or } 1.32\ \Omega$$

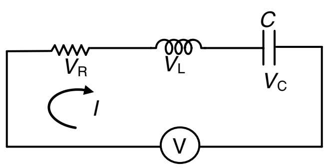
$$L = \frac{18.98}{1000} \text{ or } \frac{1.32}{1000}$$

$$L = 18.98 \text{ mH or } 1.32 \text{ mH}$$

Example 9

A series RLC circuit is supplied by 220 V and 50 Hz. At resonance, the voltage across the capacitor = 550 V and I = 1 A. Determine R, L, and C.

Solution



At resonance, $X_L = X_C$

$$I = \frac{V}{R} = \frac{220}{R}$$

$$\Rightarrow R = 220\ \Omega$$

$$V_C = I_0 X_C$$

$$550 = 1.\frac{1}{\omega_0 C}$$

$$C = \frac{1}{550 \times 2\pi \times 50} = 5.75\ \mu\text{F}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$L = \frac{1}{C}\left[\frac{1}{2\pi fr}\right]^2$$

$$L = 1.75 \text{ H}$$

Example 10

A series RLC circuit consists of a 50 Ω resistance, 0.2 H inductance, and 10 μF capacitor with an applied voltage of 20 V. Determine the resonant frequency. Find the Q factor of the circuit. Compute the lower and upper frequency limits and also find the bandwidth of the circuit.

Solution

Resonant frequency $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.2 \times 10 \times 10^{-6}}}$$

$$= 112.5 \text{ Hz}$$

Quality factor $$Q = \frac{\omega_r L}{R}$$

$$= \frac{2\pi \times 112.5 \times 0.2}{50}$$

$$= 2.83$$

Lower frequency limit $$f_1 = f_r - \frac{R}{4\pi L}$$

$$= 112.5 - \frac{50}{4 \times \pi \times 0.2}$$

$$= 92.6 \text{ Hz}$$

Upper frequency limit

$$f_2 = f_r + \frac{R}{4\pi L} = 112.5 + \frac{50}{4\pi \times 0.2}$$

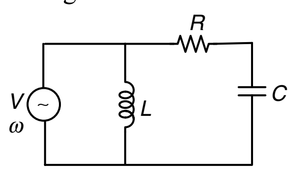
$$= 112.5 + 19.89$$

$$= 132.39 \text{ Hz}$$

Bandwidth, $\text{BW} = f_2 - f_1 = 39.79$ Hz

Example 11

Consider the following circuit



For what value of ω, the abovementioned circuit exhibits unity power factor?

(A) $\frac{1}{\sqrt{LC}}$ (B) $\frac{1}{\sqrt{LC + R^2C^2}}$

(C) $\frac{1}{\sqrt{LC - R^2C^2}}$ (D) $\frac{1}{RC}$

Solution

For unity power factor, imaginary part of impedance should be zero.

$$Z_{eq} = \frac{j\omega L\left(R - \frac{j}{\omega c}\right)}{R + j\omega L - j/\omega c}$$

$$Z_{eq} = \frac{j\omega l(R - j/\omega c)}{(R + j\omega L - j/\omega c)} \times \frac{(R - j\omega L + j/\omega c)}{(R - j\omega l + j/\omega c)}$$

At resonance, imaginary part equal to zero

$$R^2 - \frac{L}{C} + \frac{1}{\omega^2 C^2} = 0$$

By simplification, we get

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC - R^2C^2}}}$$

Filters

The general transfer functions for second-order filters:

	Filters		Transfer function
1	LPF	⇒	$\frac{P}{s^2 + as + b}$
2	HPF	⇒	$\frac{Ps^2}{s^2 + as + b}$
3	BPF	⇒	$\frac{Ps}{s^2 + as + b}$
4	BSF	⇒	$\frac{Ps^2 + q}{s^2 + as + b}$
5	APF	⇒	$\frac{s^2 + Ps + q}{s^2 + as + b}$

Coupled Circuits

Self-Inductance

When a current changes in a circuit, the magnetic flux linking the same circuit changes (and vice-versa) and an emf is induced in the circuit.

This induced emf is proportional to the rate of change of current:

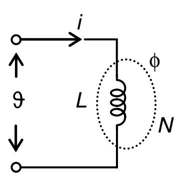
$$v = L.\frac{di}{dt} \quad (1)$$

where v is the induced voltage; $\frac{di}{dt} \rightarrow$ rate of change of current; and L is the self-inductance.

Inductance also expressed as

$$L = \frac{N\varphi}{I}$$

where N is the number of turns in the circuit and $\phi \Rightarrow L$ is the flux linkage.



$$v = L.\frac{d\left(\frac{N\varphi}{L}\right)}{dt}$$

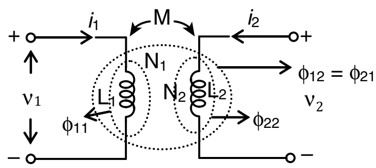
$$= N.\frac{d\varphi}{dt} \quad (2)$$

By equating Eqs (1) and (2)

$$L.\frac{di}{dt} = N.\frac{d\varphi}{di}$$

$$\boxed{L = N \cdot \frac{d\phi}{di}}$$

Mutual Inductance



The Induced voltage of coil-2 is given by

$$V_{L2} = N_2.\frac{d\varphi_{12}}{dt} \quad (1)$$

ϕ_{12} is related to the current of coil-1 and the induced voltage is proportional to the rate of change of i_1.

$$V_{L_2} = M.\frac{di_1}{dt} \quad (2)$$

By equating Eqs (1) with (2), we get

$$\boxed{M = N_2.\frac{\phi_{12}}{i_1} = N_1.\frac{\phi_{21}}{i_2}}$$

Coupling Coefficient

The amount of magnetic coupling is expressed by coefficient of coupling (k)

$$K = \frac{\text{useful flux}}{\text{total flux}} = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

$$\boxed{M = K\sqrt{L_1 L_2}}$$

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

For ideal circuits, $K = 1$ and practical circuits, range of K is $0 < K < 1$.

Leakage Factor

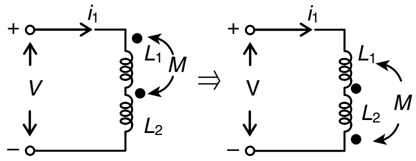
$$\text{Leakage factor} = \frac{\text{total flux}}{\text{useful flux}} = \frac{1}{K}$$

Dot conventions

1. When either or both the currents are entering at dotted terminal or leaving the dotted terminal, mutual inductance will be added.
2. If one current is entering and other is leaving the dotted terminal or vice-versa, then mutual inductance will be subtracted.

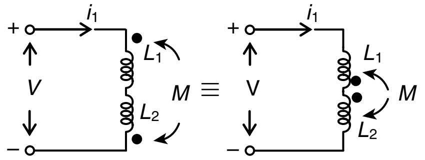
Series Connection of Coupled Coils

(i)



$$\boxed{L_{eq} = L = L_1 + L_2 + 2M \Rightarrow \text{aiding}}$$

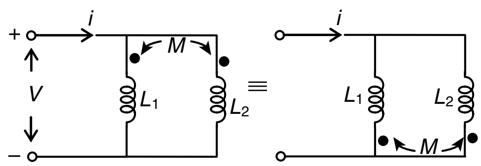
(ii)



$$\boxed{L_{eq} = L = L_1 + L_2 - 2M \Rightarrow \text{Opposition}}$$

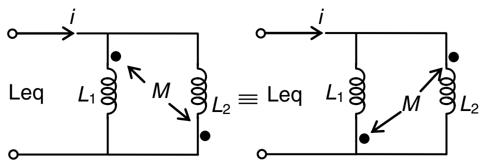
Magnetically Coupled Inductors in Parallel

(i) Magnetically aiding



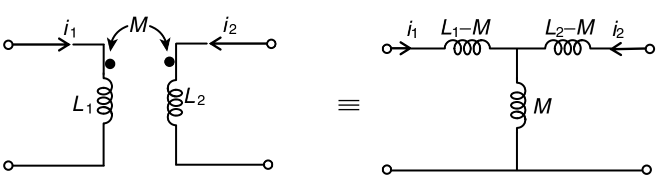
$$L_{eq} = \boxed{\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

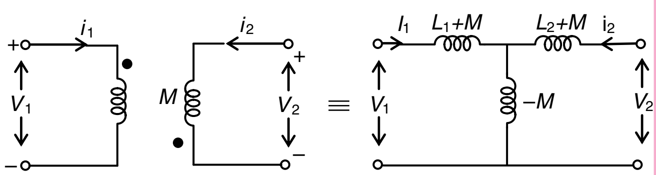
(ii) Magnetically opposition



$$L_{eq} = \boxed{\frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

Equivalent Circuit

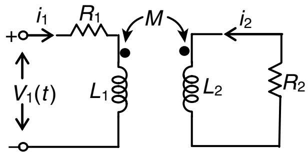




Transformer Coupling

Two inductors with self-inductances L_1 and L_2, mutual inductance M, and coefficient of coupling K are shown in figure, with dot convention

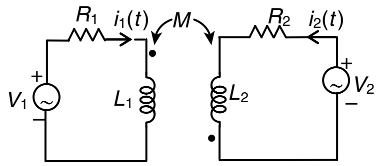
(i)



$$V_1(t) = R_1 i_1(t) + L_1 \frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$0 = R_2 i_2(t) + L_2 \frac{di_2}{dt} + M\frac{di_1(t)}{dt}$$

(ii)

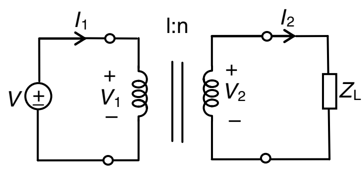


$$V_1 = R_1 i_1(t) + L_1 \frac{di_1(t)}{dt} - \mathrm{M}\frac{di_2(t)}{dt}$$

$$V_2 = R_2 i_2(t) + L_2 \frac{di_2(t)}{dt} - \mathrm{M}\frac{di_1(t)}{dt}$$

Ideal Transformer

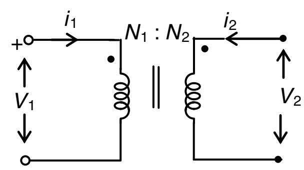
An ideal transformer is a unity coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.



$$V_1 = N_1 \frac{d\varphi}{dt} = v_2 = N_2 \frac{d\varphi}{dt}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{n}{1} = \frac{I_1}{I_2}$$

1. When $n = 1$, generally called the an isolation transformer.
2. $n > 1$, it is called step-up transformer ($V_2 > V_1$).
3. n < 1, it is called step-down transformer ($V_2 < V_1$).



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{-I_1}{I_2}$$

The input impedance as seen by the source is

$$Z_{in} = \frac{V_1}{I_1} = \frac{1}{n^2} \cdot \frac{V_2}{I_2}$$

$$\boxed{Z_{in} = \frac{Z_L}{n^2}}$$

$$\boxed{Z_{in} = Z_L \left(\frac{N_1}{N_2}\right)^2}$$

Example 12

Two couples coils have self-inductances L_1 = 15 mH and L_2 = 18 mH. The coefficient of coupling (k) being 0.75 in the air, find voltage in the second coil and flux of the first coil, provided the second coil has 500 turns and the circuit current is given by $i_1 = 5.\sin 314\,t$ A.

Solution

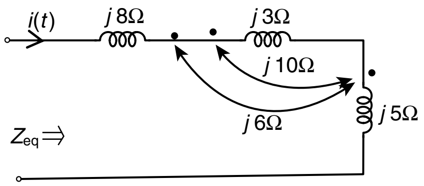
$$M = K\sqrt{L_1 K_2}$$
$$= 0.75\sqrt{15 \times 18} \times 10^{-3}$$
$$= 12.32 \times 10^{-3}\text{ H}$$

The voltage induced in the second coil is

$$V_{L_2} = M.\frac{di_1}{dt} = 12.32 \times 10^{-3}.\frac{d}{dt}(5\sin 314t)$$
$$= 12.32 \times 10^{-3} \times 5 \times 314 \times \cos 314t$$
$$V_{L2} = 19.34\cos 314t\text{ Volts}$$
$$M = \frac{N_2\varphi_{12}}{i_1}$$
$$M = \frac{500 \times k \times \varphi_1}{i_1}$$
$$\varphi = \frac{12.32 \times 10^{-3} \times 5\sin 314t \times 1}{0.75 \times 500}$$
$$\phi_1 = 16.42 \times 10^{-5}\sin 314\,t$$

Example 13

Impedance z, as shown in figure is



(A) $j29\ \Omega$ (B) $j9\ \Omega$
(C) $j19\ \Omega$ (D) $j24\ \Omega$

Solution

$$Z_{eq} = L_1 + L_2 + L_3 \pm 2M_{12} \pm 2M_{13} \pm 2M_{23}\ldots.$$
$$\therefore Z_{eq} = j8 + j3 + j5 + 2 \times j10 - 2 \times j6$$
$$= j16 + j8 = j24\ \Omega$$

Example 14

Two coils have self-inductances of 0.09 H and 0.01 H and a mutual inductance of 0.015 H. The coefficient of coupling between the coil is

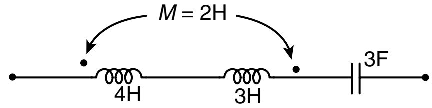
(A) 0.06 (B) 0.5 (C) 1.0 (D) 0.05

Solution

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.015}{\sqrt{0.09 \times 0.01}}$$
$$= \frac{0.015}{3 \times 10^{-2}}$$
$$= 0.5$$

Example 15

The resonant frequency of the given series circuit is



(A) $f = \frac{1}{4\pi}$ Hz (B) $f = \frac{1}{3\pi}$ Hz
(C) $f = \frac{1}{6\pi}$ Hz (D) $f = \frac{1}{2\sqrt{3}\pi}$ Hz

Solution

$$f = \frac{1}{2\pi\sqrt{L_{eq}c}}$$
$$L_{eq} = L_1 + L_2 \pm 2M$$
$$L_{eq} = 4 + 3 - 2 \times 2 = 3\text{ H}$$
$$f = \frac{1}{2\pi\sqrt{3 \times 3}}$$
$$f = \frac{1}{6\pi}\text{ Hz}$$

Network Functions and Laplace Transform

Laplace Transform

Let $f(t)$ be a time function, which is zero for $t \le 0$ and which is arbitrarily defined for $t > 0$.

Then, the Laplace transform of $f(t)$ is defined by

$$L\{f(t)\} = F(s) = \int_{0^+}^{\infty} f(t)e^{-st}.dt$$

In general,

$$\boxed{F(s) = \int_{-\infty}^{\infty} f(t).e^{-st}.dt}$$

The inverse Laplace transform can also be expressed as an integral

$$L^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{\sigma_o - j\infty}^{\sigma_o + j\infty} F(s).e^{st}ds.$$

Table 5.1 Laplace Transform pairs

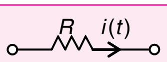
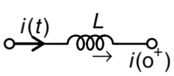
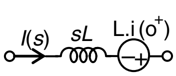
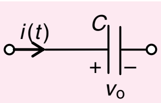
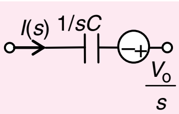
	$f(t)$	$F(s)$
1.	1	$1/s$
2.	$Au(t)$	A/s
3.	At	A/s^2
4.	e^{-at}	$\frac{1}{s+a}$
5.	$t.e^{-at}$	$\frac{1}{(s+a)^2}$
6.	$\sin at$	$\frac{a}{s^2+a^2}$
7.	$\cos at$	$\frac{s}{s^2+a^2}$
8.	$\frac{df(t)}{dt}$	$sF(s) - f(o+)$
9.	$\int_o^t f(t).dt$	$\frac{F(s)}{s}$
10.	$f(t-a)$	$e^{-aS}.F(s)$
11.	$\int f_1(\tau).f_2(t-\tau).d\tau$	$F_1(s).F_2(s)$

Initial-value and Final-value Theorems

i) $\underset{t\to o^+}{Lt}\ f(t) = f(o^+) = \underset{S\to\infty}{Lim}\{s.F(s)\}$

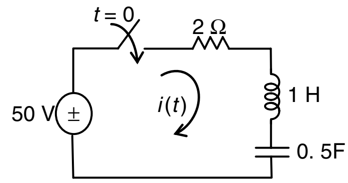
ii) $\underset{t\to\infty}{Lt}\ f(t) = f(\infty) = \underset{S\to o}{Lim}\{s.F(s)\}$

Table 5.2 Circuits in the S domain:

Time domain	S domain	S domain voltage
1. 		$R.I(s)$
2. 		$sLI(s) - L.i(o+)$
3. 		$\frac{I(s)}{sC} + \frac{V_o}{s}$

Example 16

In the following RLC circuit, there is no initial charge on the capacitor. If the switch is closed at $t = 0$, the resultant current is



(A) $i(t) = j25.e^{-t}.\cos t$ A (B) $i(t) = 50.e^{-t}\sin t$ A
(C) $i(t) = 25.e^{-t}\sin 2t$ A (D) None of these

Solution

Given $i(o^+) = 0$ and

$$V_c(o^+) = 0$$

For $t > 0$, apply KVL, the time domain equation of the given circuit is

$$R.i(t) + L.\frac{di(t)}{dt} + \frac{1}{c}\int_0^t i(t)\,dt = V. \quad (1)$$

Converting Eq. (1) into LPF

$$R.I(s) + sL.I(s) + \frac{1}{sC}.I(s) = V/s$$

$$\left(2 + S + \frac{2}{s}\right)\cdot I(s) = \frac{50}{s}$$

$$I(s) = \frac{50}{s^2 + 2s + 2}$$

By applying inverse Laplace transform to $I(s)$, we get

$$i(t) = j25\left\{e^{-1-jt} - e^{-1+jt}\right\}$$

$$= 50.e^{-t}.\sin t \text{ A}$$

NETWORK SYNTHESIS

Consider the following network, as shown in Figure 1.

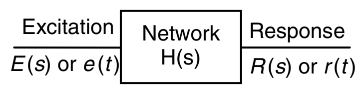


Figure 1 General Network.

The system function is defined as

$$\Rightarrow H(s) = \frac{R(s)}{E(s)}$$

Let us consider the following examples:

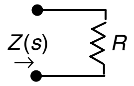
1. Driving point impedance $Z(s) = \frac{V(s)}{I(s)}$

2. Driving point admittance $V(S) = \frac{I(s)}{V(s)}$

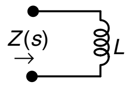
NOTE

Immittance = Impedance (or) admittance

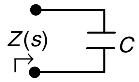
Driving Point Immittance of Simple Networks

1. 

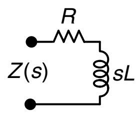
$\Rightarrow Z(s) = R;\ Y(s) = \frac{1}{R}$

2. 

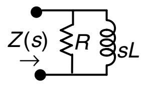
$\Rightarrow Z(s) = Ls;\ Y(s) = \frac{1}{sL}$

3. 

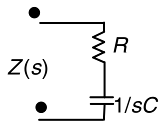
$\Rightarrow Z(s) = \frac{1}{sC};\ Y(s) = sC$

4. 

$\Rightarrow Z(s) = R + Ls;\ Y(s)\frac{1}{R+sL}$

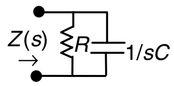
5. 

$\Rightarrow Z(S) = \frac{RLs}{R+Ls};\ y(s) = \frac{\frac{1}{L}+\frac{1}{R}s}{s}$

6. 

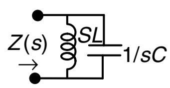
$\Rightarrow Z(s) = R + 1/sC$

$$Y(s) = \frac{sC}{1+RC\,s}$$

7. 

$\Rightarrow Z(S) = \frac{1}{Cs + 1/R}$

$$Y(s) = \frac{1}{R} + sC$$

8. 

$\Rightarrow Z(s) = \frac{sL}{1+s^2\,LC}$

$$Y(s) = \frac{1+s^2\,LC}{sL}$$

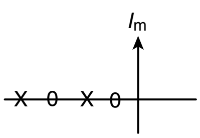
R-L Driving Point Impedance

The impedance function is of the form

$$Z(s) = K_o + K_v s + \sum_{i=1}^{n} \frac{K_i}{s+\sigma_i}$$

Properties

1. Poles and zeros of an *R–L* impedance lie on $-\sigma$ axis and alternate



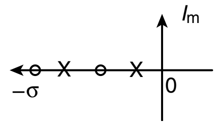
2. The residues are real and negative.
3. The singularity closest to the origin is a zero.

R–C Driving Point Impedance

The *R–C* impedance function has the general form $Z(s) = \frac{K_o}{s} + K_v + \sum_{i=1}^{n} \frac{Ki}{s+\sigma_i}$

Properties

1. The poles and zeroes lie on the negative real axis and alternate.



2. The residues of the poles, K_i, are real and positive.
3. *R–C* impedance ⇔ *R–L* admittance.
4. The singularity nearest the origin being a pole.

L–C Immittance Function

The general expansion is given by

$$Z(S) = \frac{K_o}{s} + \sum_{i=1}^{n} \frac{2K_i s}{s^2 + w_i^2} + ... + K_v s$$

Properties

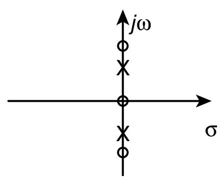
1. $Z(s)$ or $Y(s)$ is the ratio of odd to even or even to odd polynomials.
2. The poles and zeros are simple and lie on the $j\omega$ axis and alternate.
3. 

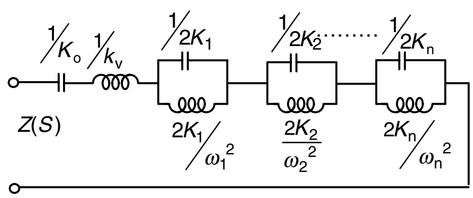
Figure 2 *L–C* Pole–Zero Realization.

4. The highest power of the numerator and denominator differ by unity.
5. The lowest power of the numerator and denominator differ by unity.
6. There is either a pole or a zero at the origin and at infinity.

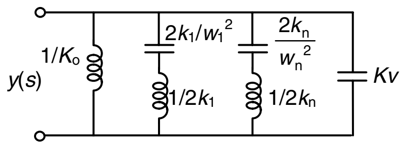
Network Function Have Four Canonical Forms

These are

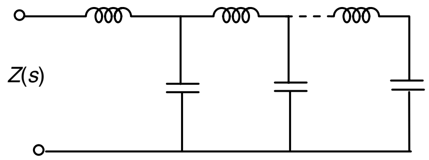
1. **Foster form-I**



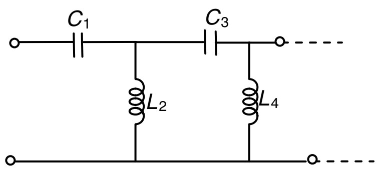
2. Foster form-II:



3. Cauer form-I:



4. Cauer form-II:



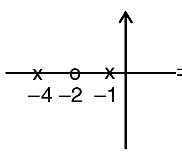
Example 17

The network function $F(s)=\dfrac{(s+2)}{(s+1)(s+4)}$ represents an

(A) *RL* impedance
(B) *RC* impedance
(C) *RC* impedance and an *RL* admittance
(D) *RC* admittance and *RL* impedance

Solution

Pole–zero realization:



$\Rightarrow$ pole nearby origin and so *RC* impedance

$\Rightarrow$ *RC* impedance $\Leftrightarrow$ *RL* admittance and vice-versa

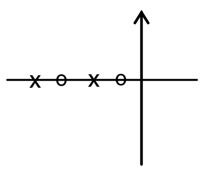
Example 18

The first critical frequency nearest the origin of the complex frequency plane for an *R*–*L* driving point impedance function will be

(A) a zero in the left-half plane
(B) a zero in the right-half plane
(C) a pole in the left-half plane
(D) either a pole or zero in the left-half plane

Solution

For *RL* impedance function



Therefore, critical frequency is a zero nearby origin in the left-half plane.

Example 19

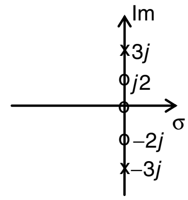
Driving point impedance $Z(S)=\dfrac{s(s^2+4)}{(s^2+9)}$ is not realizable because the

(A) number of zeros is more than the number of poles.
(B) poles and zeros lie on the imaginary axis.
(C) poles and zeros are not located on the real axis.
(D) poles and zeros do not alternated on imaginary axis.

Solution

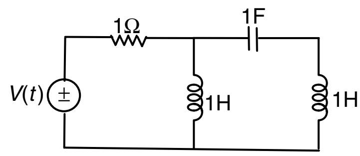
The pole–zero realization of $Z(S)$ is shown in the following figure, zeros $\Rightarrow Z=0$ and $Z=\pm j2=S$

Poles $S^2+9=0 \Rightarrow S=\pm j3$



Example 20

The driving point impedance of the following network is



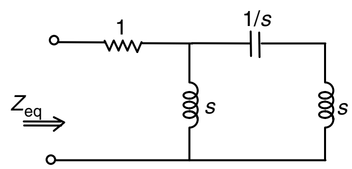
(A) $Z(S)=s+1$

(B) $Z(S)=\dfrac{2s^2+1}{s^3+2s^2+s+1}$

(C) $Z(S)=\dfrac{1}{s+1}$

(D) $Z(S)=\dfrac{s^3+2s^2+s+1}{2s^2+1}$

Solution



$$Z_{eq}=1+s\|(s+1/s)=1+\frac{s\left(s+\frac{1}{s}\right)}{s+s+\frac{1}{s}}$$

$$=\frac{s^3+2s^2+s+1}{2s^2+1}$$

Power Relationships In AC Circuit

In a passive AC circuit, let the instantaneous voltage be $V(t)=V_m\sin\omega t$. The current is given by $i(t)=I_m\sin(\omega t-\phi)$,

ϕ being the phase difference between voltage and current at any instant.

The instantaneous power p is given by

$$P = vi = V_m I_m . \sin\omega t . \sin(\omega t - \phi)$$

$$= \frac{1}{2} . V_m . I_m . \cos\phi . - \frac{1}{2} V_m . I_m . \cos(2\omega t - \phi) \quad (1)$$

Eq. (1) contains a double frequency term and the magnitude of the average value of this term is zero.

∴ The average power in the passive circuit is given by

$$P_{avg} = \frac{1}{2} . V_m . I_m . \cos\varphi$$

$$= \frac{V_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}} . \cos\varphi$$

$$P = V_{rms} . I_{rms.} \cos\phi = VI\cos\phi(W)$$

Loads	Power factor (cos ϕ)
R	Unity
L	zero lag
C	zero lead
$R - L$	lagging
$R - C$	leading

Apparent Power

S = voltage × current

$= V.I = V_{rms} I_{rms}$

Relative power $Q = V.I \sin\phi$ (VAR)

$S = \sqrt{P^2 + Q^2}$ VA

Average Value

$$X_{avg} = \frac{1}{T} \int_o^T x(t).dt$$

where T is the time period of periodic function $x(t)$.

rms or Effective Value

$$X_{rms} = \sqrt{\frac{1}{T} \int_o^T [x(t)]^2 .dt}$$

1. rms Value of $A \sin\omega t$ and $A \cos\omega t$ is $A/\sqrt{2}$
2. If $x(t) = a_o + (a_1 \cos\omega t + a_2 \cos 2\omega t + \ldots) + (b_1 \sin\omega t + b_2 \sin 2\omega t + \ldots\ldots)$

$$X_{rms} = \sqrt{a^2{}_o + \frac{1}{2}(a_1{}^2 + a^2{}_2 + \ldots) + \frac{1}{2}(b_1{}^2 + b_2{}^2 + \ldots.)}$$

Exercises

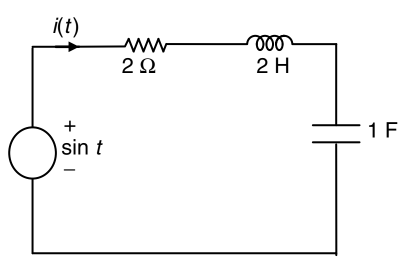
Practice Problems I

Direction for questions 1 to 30: Select correct alternative from the given choices.

1. A series RLC circuit has a resonance frequency of 1 kHz and a quality factor $Q = 100$. If each of R, L, and C is doubled from its original value, the new Q of the circuit is

(A) 25 (B) 50 (C) 100 (D) 200

2. The differential equation for the current i(t) in the following circuit is



(A) $2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i(t) = \sin t$

(B) $2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 2i(t) = \cos t$

(C) $2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i(t) = \cos t$

(D) $\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 2i(t) = \sin t$

3. In a series RLC circuit, R = 2 kΩ, L = 1 H, and $C = \frac{1}{400}$ μF. The resonant frequency is

(A) 2×10^4 Hz (B) $\frac{1}{\pi} \times 10^4$ Hz

(C) 10^4 Hz (D) $2\pi \times 10^4$ Hz

4. The current i(t) through a 10 Ω resistor in series with an inductance, is given by $i(t) = 3 + 4\sin(100t + 45°) + 4\sin(300t + 60°)$ A

The RMS value of the current and the power dissipated in the circuit are

(A) $\sqrt{41}$ A, 410 W, respectively

(B) $\sqrt{35}$ A, 350 W, respectively

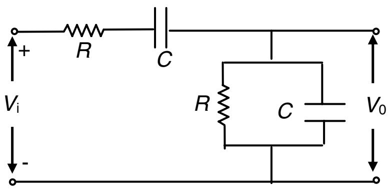
(C) 5 A, 250 W, respectively

(D) 11 A, 1,210 W, respectively

5. Two 2 H inductance coils are connected in series and are also magnetically coupled to each other, and the coefficient of coupling being 0.1. The total inductance of the combination can be

(A) 0.4 H (B) 3.2 H (C) 4.0 H (D) 3.6 H

6. The *RC* circuit shown in the figure is

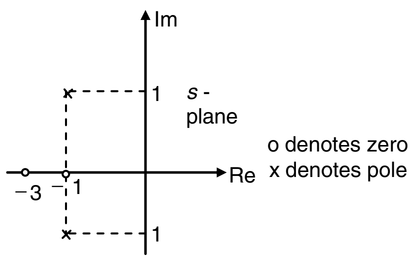


(A) a low-pass filter (B) a high-pass filter
(C) a band-pass filter (D) a band-reject filter

7. A certain series resonant circuit has a bandwidth of 1,000 Hz. If the existing coil is replaced by a coil with low Q, what happens to the bandwidth?

(A) It increases (B) It is zero
(C) It decreases (D) It remains the same

8. The driving point impedance $Z(s)$ of a network has the pole–zero locations, as shown in the figure. If $Z(0) = 3$, then $Z(s)$ is



(A) $\dfrac{3(s+3)}{s^2+2s+3}$ (B) $\dfrac{2(s+3)}{s^2+2s+2}$

(C) $\dfrac{3(s-3)}{s^2-2s-2}$ (D) $\dfrac{2(s-3)}{s^2-2s-3}$

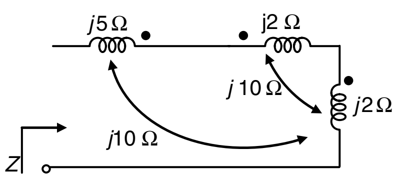
9. Consider the following statements S1 and S2

S1: At the resonant frequency the impedance of series *R–L–C* circuit is zero.

S2: In a parallel *G–L–C* circuit, increasing the conductance *G* results in increase in its *Q* factor. Which one of the following is correct?

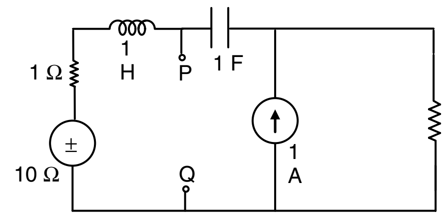
(A) S1 is false and S2 is true
(B) Both S1 and S2 are true
(C) S1 is true and S2 is false
(D) Both S1 and S2 are false

10. Impedance *Z*, as shown in the given figure, is



(A) $j29\ \Omega$ (B) $j9\ \Omega$ (C) $j19\ \Omega$ (D) $j39\ \Omega$

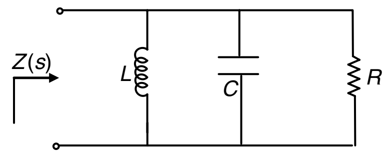
11. The Thevenin's equivalent impedance Z_{TH} between the nodes *P* and *Q* in the following circuit is



(A) 1 (B) $1+s+\dfrac{1}{s}$

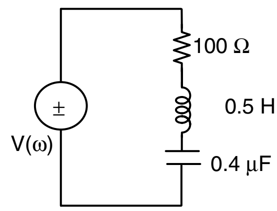
(C) $2+s+\dfrac{1}{s}$ (D) $\dfrac{s^2+s+1}{s^2+2s+1}$

12. The driving point impedance of the following network is given by $Z(s) = \dfrac{0.2s}{s^2+0.1s+2}$. The component values are



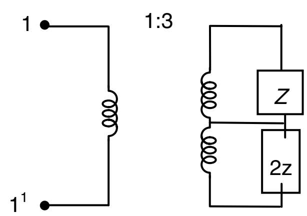
(A) $L = 5$ H, $R = 0.5\ \Omega$, $C = 0.1$ F
(B) $L = 0.1$ H, $R = 0.5\ \Omega$, $C = 5$ F
(C) $L = 5$ H, $R = 2\ \Omega$, $C = 0.1$ F
(D) $L = 0.1$ H, $R = 2\ \Omega$, $C = 5$ F

13. The resonant frequency for the series *RLC* circuit shown in the figure is



(A) 550 Hz (B) 670 Hz
(C) 1,100 Hz (D) 355 Hz

14. If an ideal centre tapped 1:3 transformer is loaded as shown in the figure, the impedance measured across the terminals 11' would be______.



(A) $\dfrac{2}{3}Z$ (B) Z (C) $\dfrac{Z}{9}$ (D) $\dfrac{Z}{3}$

15. What is the ratio of the heating effects of two current waves of equal peak value, one being sinusoidal and the other rectangular in waveform?

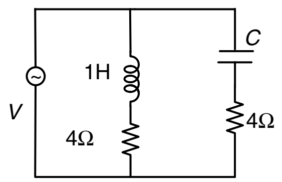
(A) 1:2 (B) $1:\sqrt{2}$ (C) $\sqrt{2}:1$ (D) 2:1

16. The form factor of a half wave and a full wave rectified sine wave are ______.

(A) 1.11, 1.57 (B) 1.57, 1.11

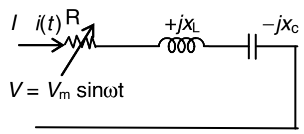
(C) 1.414, 1.11 (D) 1.11, 1.414

17. The value of capacitance C in the given AC circuit to make it a pure resistive circuit and for the supply current to be independent of its frequency is _____.



(A) $\frac{1}{12}$ F (B) $\frac{1}{8}$ F (C) $\frac{1}{4}$ F (D) $\frac{1}{16}$ F

18.

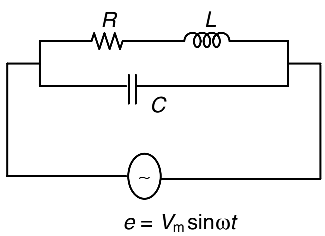


In the abovementioned circuit, if R is varied from 0 to ∞, the locus of the tip of the current phasor is

(A) circle (B) semi-circle

(C) exponential curve (D) sine curve

19.



The dynamic impedance of the abovementioned circuit at resonance is

(A) $\frac{R}{LC}$ (B) $\frac{1}{R}\sqrt{\frac{L}{C}}$ (C) $\frac{L}{RC}$ (D) $\frac{C}{RL}$

20. Match the following:

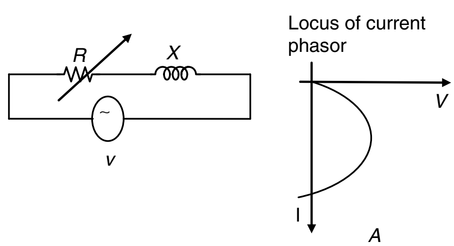


Figure 1

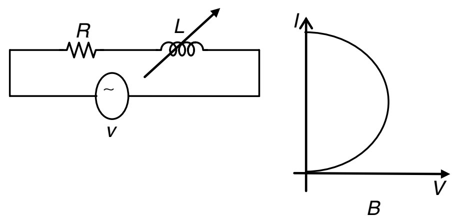


Figure 2

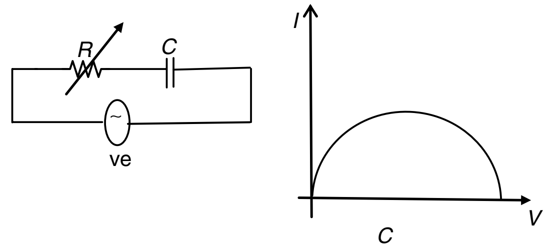


Figure 3

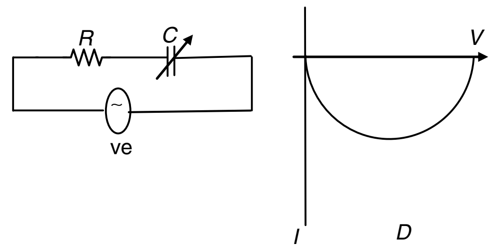


Figure 4

(A) 1 – D, 2 – A, 3 – B, 4 – C

(B) 1 – A, 2 – D, 3 – B. 4 – C

(C) 1 – B, 2 – C, 3 – A, 4 – D

(D) 1 – C, 2 – B, 3 – D, 4 – A

Direction for questions of 21 and 22:

A low-pass π-section filter consists of an inductance of 25 mH in the series arm and two capacitors 0.2 μF in the shunt arms.

21. The cut-off frequency is _____.

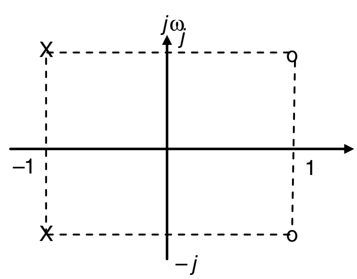
(A) $\frac{10^4}{\pi}$ (B) $\frac{1}{10^4\,\pi}$

(C) $\frac{5000}{\pi}$ (D) $\frac{1}{5000\,\pi}$

22. Design impedance is _____.

(A) 25 Ω (B) 250 Ω (C) 15 Ω (D) 2.5 Ω

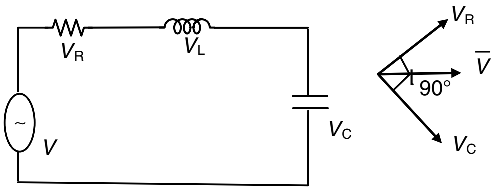
23. An impedance has the pole–zero pattern shown in figure. It must be composed of



(A) RLC elements (B) RL elements only

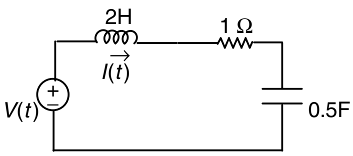
(C) RC elements only (D) LC elements only

24. For the series R–L–C circuit of Figure 1, phasor diagram (involving some phasors) is shown in Figure 2. The operating frequency of the circuit is



(A) equal to resonant frequency
(B) less than resonant frequency
(C) twice the resonant frequency
(D) greater than the resonant frequency

25. Which one of the following represents the state equation of the given R–L–C series circuit?



(A) $\begin{bmatrix} \varphi \\ q'c \end{bmatrix} = \begin{bmatrix} -2 & -0.5 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} \varphi \\ qc \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v(t)$

(B) $\begin{bmatrix} \varphi \\ q_c \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -05 & 0.5 \end{bmatrix} \begin{bmatrix} \varphi \\ qc \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t)$

(C) $\begin{bmatrix} \varphi \\ q_c \end{bmatrix} = \begin{bmatrix} -0.5 & -2 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} \varphi \\ qc \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v(t)$

(D) $\begin{bmatrix} \varphi \\ q_c \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.5 & 2 \end{bmatrix} \begin{bmatrix} \varphi \\ qc \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v(t)$

26. When two inductances are connected in series aiding the equivalent inductance of 14 H and in opposition is 6 H. Find out coefficient of coupling 'K'
(A) 0.408 (B) 0.534 (C) 0.628 (D) 0.707

27. If the secondary winding of the ideal transformer shown in the circuit of figure has 50 turns, the number of turns in the primary winding for maximum power transfer to the 3 Ω resister will be

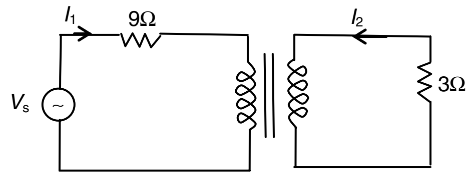
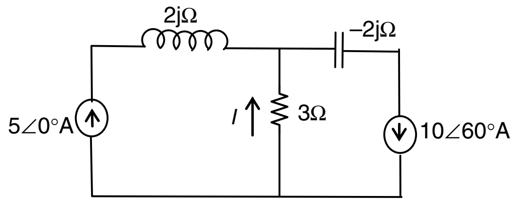


Figure Ideal Transformer 1:50

(A) 80 (B) 87 (C) 90 (D) 100

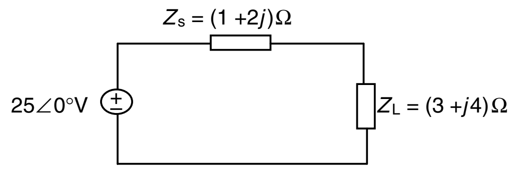
28. For the circuit in figure, the instantaneous current i(t) is



(A) $\frac{10\sqrt{3}}{2}\angle 90°$ A (B) $5\angle -60°$A

(C) $5\angle 60°$A (D) $\frac{10\sqrt{3}}{2}\angle -90°$A

29. An AC source of RMS voltage 25 V with internal impedance $Z_s = (1 + 2j)$ Ω feeds a load of impedance $Z_L = (3 + j4)$ Ω in the following figure. The reactive power consumed by the load is



(A) 60 VAR (B) 70 VAR
(C) 62.5 VAR (D) 78 VAR

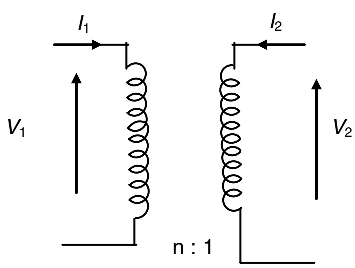
30. The network function $\frac{(s+1)(s+4)}{s(s+2)(s+6)}$ is a
(A) RL impedance function
(B) RC impedance function
(C) LC impedance function
(D) All the above

Practice Problems 2

Direction for questions 1 to 28: Select correct alternative from the given choices.

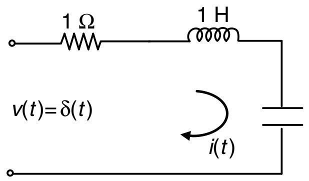
1. For parallel RLC circuit, which one of the following statements is not correct?
(A) The bandwidth of the circuit decreases if R is increased.
(B) The bandwidth of the circuit remains same if 'L' is increased.
(C) At resonance, input impedance is a real quantity.
(D) At resonance, the magnitude of the input impedance attains its minimum value.

2. The $ABCD$ parameters of an ideal n:1 transformer shown in the figure are $\begin{bmatrix} n & o \\ o & x \end{bmatrix}$. The value of x will be



(A) n (B) $\frac{1}{n}$ (C) n^2 (D) $\frac{1}{n^2}$

3. What is the total reactance of a series *RLC* circuit at resonance?
(A) equal to X_L (B) equal to X_C
(C) equal to R (D) zero

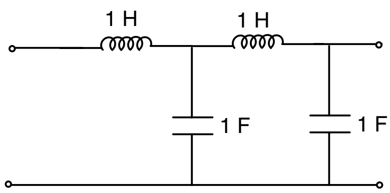
4.


The circuit shown in the figure is initially relaxed. The Laplace transform of the current $i(t)$ is

(A) $\frac{s+1}{s^2+2s+1}$ (B) $\frac{s+1}{s^2+s+1}$

(C) $\frac{s}{s^2+s+1}$ (D) $\frac{s}{s^2+2s+1}$

5. Driving point impedance of the network shown in the figure is



(A) $\frac{s^4+3s^2+1}{s^3+2s}$ (B) $\frac{s^2+1}{s(s^2+2)}$

(C) $\frac{s^4+3s^3+2s^2+1}{s^3+2s}$ (D) $\frac{s^2+1}{s+1}$

6. The two windings of a transformer have an inductance of 3 H each. If the mutual inductance between them is also 3 H, then
(A) transformation ratio is 3.
(B) it is an ideal transformer.
(C) it is a perfect transformer as the coefficient of coupling is 1.
(D) None of these

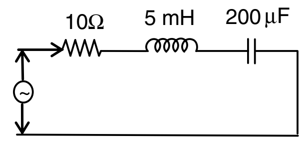
7. The complex power in a single-phase AC circuit is given by ______.
(A) VI (B) V^*I
(C) VI^* (D) V^*I^*

8. In a series resonance circuit, which of the following statements are true?
(i) PF is unity.
(ii) Voltage magnification takes place.
(iii) Current magnification takes place.
(iv) Resonant frequency depends on resistance.

(A) (i) and (ii) (B) (ii) and (iv)
(C) (i), (ii), and (iv) (D) (i) and (iii)

9. The condition for parallel resonance is
(A) net reactance is zero
(B) net susceptance is zero
(C) net reactive power is zero
(D) reactive component of net current is zero

10. In the circuit of the given figure, the magnitudes of V_L and V_c are equal, at what value of supply frequency ______.



(A) $\frac{1}{15\pi}$ (B) $\frac{2}{15}\pi$ (C) $\frac{10^3}{2\pi}$ (D) 2π

11. If the voltage and current in an AC circuit are given by $v = 200 \sin(\omega t + 30)$ and $i = 10 \sin(\omega t - 60)$, then the PF of circuit is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $\frac{1}{\sqrt{2}}$

12. If $v = 200 \sin(2\omega t + 30)$, $i = 10 \sin(\omega t + 30)$, then the phase difference between v and i is
(A) 0 (B) 30
(C) 60° (D) None of these

13. In a series *RLC* circuit for frequencies less than the resonant frequency, the circuit is ______.
(A) inductive (B) capacitive
(C) resistive (D) None of these

14. In a parallel resonance circuit for frequencies greater than the resonant frequency, the circuit is ______.
(A) inductive (B) capacitive
(C) resistance (D) None of these

15. The Q of a circuit can be increased by ______.
(A) increasing the BW (B) decreasing the BW
(C) increasing the R (D) None of these

16. In an *RLC* series circuit, the bandwidth is increased by ______.
(A) decreasing L (B) decreasing C
(C) increasing R (D) decreasing R

17. In an *RLC* series, resonant circuit at the half power points.
(A) the current is half of the current at resonance.
(B) the impedance is half of the impedance at resonance.
(C) the resistance is equal to the resultant reactance.
(D) None of these

18. Which of the following statements are true?
(i) The higher the value of Q, the more selective will be the circuit and lesser will be the bandwidth.
(ii) Impedance of parallel resonant circuit is maximum at resonance.

(A) i and ii (B) i only
(C) ii only (D) Both are false

19. Match the following:

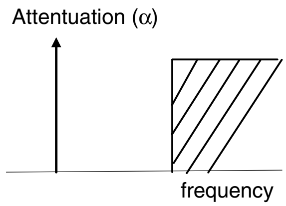


Figure 1

(A) Band pass (B) Low pass
(C) High pass (D) Band elimination

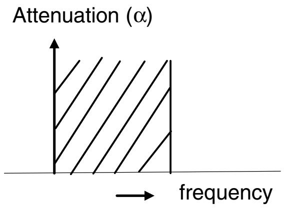


Figure 2

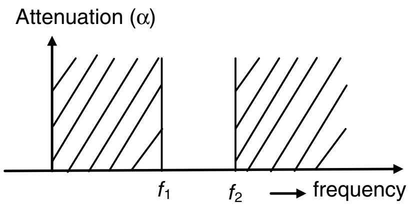


Figure 3

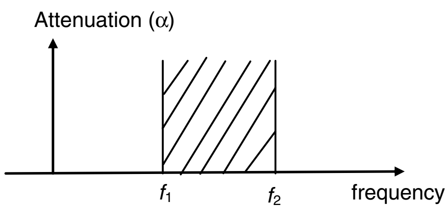


Figure 4

(A) 2– B, 1 – C, 3 – D, 4 – A
(B) 1 – B, 2 – C, 3 – A, 4 – D
(C) 1 – A, 2 – D, 3 – B, 4 – C
(D) 1 – A, 2 – D, 3 – C, 4 – B

20. Given that

$f_1 \rightarrow$ lower cut-off frequency
$f_2 \rightarrow$ higher cut-off frequency
Then, resonant frequency 'f_r', quality factor at resonance, and selectivity are given by

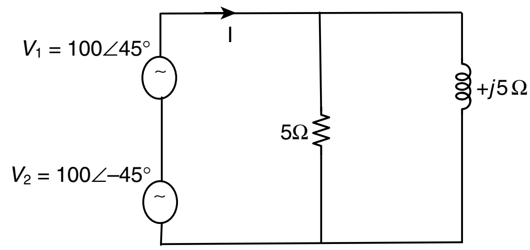
(A) $\sqrt{f_1 f_2}, \dfrac{f_2 - f_1}{f_r}, \dfrac{f_r}{f_2 - f_1}$

(B) $f_1 f_2, \dfrac{f_r}{f_2 - f_1}, \dfrac{f_2 - f_1}{f_r}$

(C) $\sqrt{f_1 f_2}, \dfrac{f_r}{f_2 - f_1}, \dfrac{f_2 - f_1}{f_r}$

(D) $\dfrac{1}{\sqrt{f_1 f_2}}, \dfrac{f_r}{f_2 - f_1}, \dfrac{f_2 + f_1}{f_2 - f_1}$

21. The phase angle of the current I with respect to the voltage V_2 in the circuit shown in the figure is



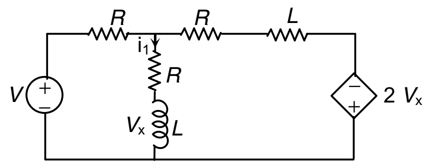
(A) 0° (B) –45° (C) +45° (D) +90°

22. The conditions for defining driving point functions are

(1) Response and excitation are applied to different terminals.
(2) The network should not contain independent sources.
(3) The network should be initially relaxed.

(A) (1), (2), and (3) (B) (1) and (3)
(C) (2) and (3) (D) (1) and (2)

23. Find the state equation for the following circuit.



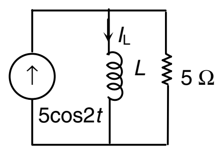
(A) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R + .5V$

(B) $L\dfrac{di_2}{dt} = -0.7V_x + 1.5i_2R - .5V$

(C) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R + .5V$

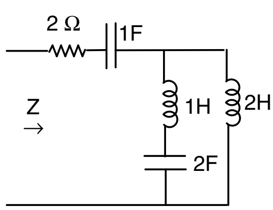
(D) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R - .5V$

24. What is the value of the inductance if current through it is given as $i(t) = 0.707 < -45°$. The input current is $5\cos 2t$.



(A) 2 H (B) 1 H (C) 5 H (D) 3 H

25. Find the driving point admittance of the following network.



(A) $\dfrac{3s^3+2s}{2s^4+6s^3+7s^2+4s+2}$

(B) $\dfrac{3s^2+2}{2s^4+5s^2+4s+2}$

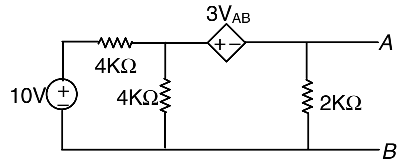
(C) $\dfrac{(3s^2+2)s}{2s^4+7s^3+6s^2+4s}$

(D) $\dfrac{5s^3+2s}{s^4+7s^3+6s^2+4s+2}$

26. An unknown impedance Z is connected across a voltage source $V=\sqrt{50}\cos(\omega t+50)$. The current flowing through the circuit is $\sqrt{18}\cos(10t+75)$. What is the value of Z?

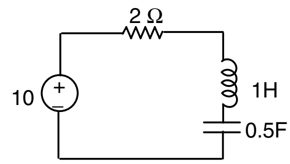
(A) 1.18 Ω resistor in series with 84.7 mH inductor.
(B) 1.18 Ω resistor in parallel with 84.7 mH inductor.
(C) 1.18 Ω resistor in series with 84.7 mF capacitor.
(D) 1.18 Ω resistor in parallel with 84.7 mF capacitor.

27. Find the Thevenin's resistance associated with the circuit.



(A) 1 kΩ (B) 0.45 kΩ
(C) 2 kΩ (D) 0.22 kΩ

28. Find the current i(t) through the following circuit.

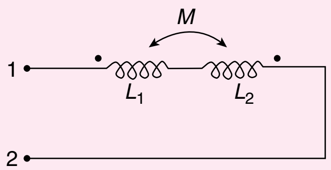


(A) $10e^{-t}\cos t$ (B) $\dfrac{10}{\sqrt{2}}e^{t}\cos\sqrt{2}t$

(C) $10e^{-t}\sin t$ (D) $\dfrac{10}{\sqrt{2}}e^{t}\sin\sqrt{2}\,t$

Previous Years' Questions

1. The equivalent inductance measured between terminal 1 and 2 for the circuit shown in figure is **[2004]**



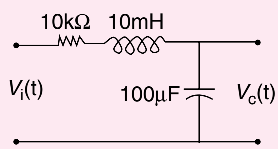
(A) L_1+L_2+M (B) L_1+L_2-M
(C) L_1+L_2+2M (D) L_1+L_2-2M

2. The transfer function $H(s)=\dfrac{V_0(s)}{V_i(S)}$ of an R–L–C circuit is given by $H(s)=\dfrac{10^6}{s^2+20s+10^6}$.

The quality factor (Q factor) of this circuit is **[2004]**

(A) 25 (B) 50
(C) 100 (D) 5,000

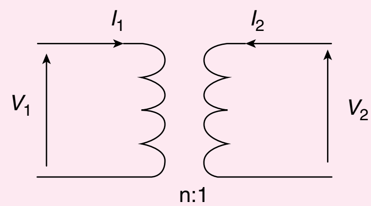
3. For the circuit shown in the figure of Q.35, the initial conditions are zero. Its transfer function $H(s)=\dfrac{V_c(s)}{V_i(S)}$ is **[2004]**



(A) $\dfrac{1}{s^2+10^6s+10^6}$ (B) $\dfrac{10^6}{s^2+10^6s+10^6}$

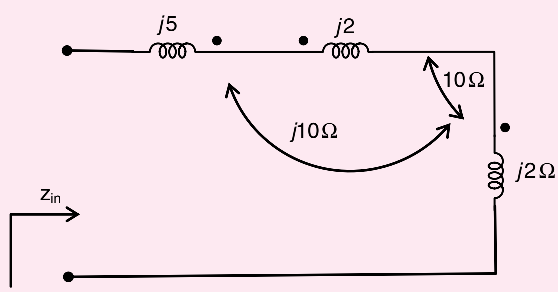
(C) $\dfrac{10^3}{s^2+10^3s+10^6}$ (D) $\dfrac{10^6}{s^2+10^6s+10^6}$

4. The $ABCD$ parameters of an ideal n:1 transformer shown in figure are $\begin{bmatrix} n & 0 \\ 0 & x \end{bmatrix}$. The value of x will be, **[2005]**



(A) n (B) $\dfrac{1}{n}$ (C) n^2 (D) $\dfrac{1}{n^2}$

5. Impedance Z as shown in figure is **[2005]**



(A) $j29\ \Omega$ (B) $j9\ \Omega$
(C) $j19\ \Omega$ (D) $j39\ \Omega$

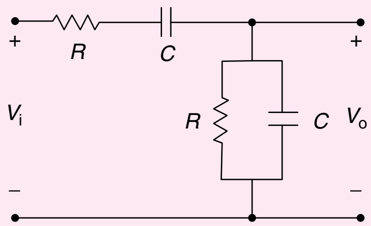
6. In a series RLC circuit $R = 2\ \text{k}\Omega$, $L = 1$ H, and $C = \frac{1}{400}\mu\text{F}$. The resonant frequency is **[2005]**

(A) 2×10^4 Hz (B) $\frac{1}{\pi} \times 10^4$ Hz
(C) 10^4 Hz (D) $2\pi \times 10^4$ Hz

7. The first and the last critical frequencies (singularities) of a driving point impedance function of a passive network having two kinds of elements are a pole and a zero, respectively. The abovementioned property will be satisfied by **[2006]**

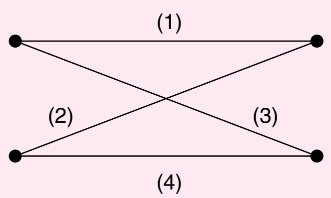
(A) RL network only
(B) RC network only
(C) LC network only
(D) RC as well as RL networks

8. The RC circuit shown in the figure is **[2007]**



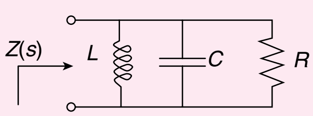
(A) a low-pass filter (B) a high-pass filter
(C) a band-pass filter (D) a band-reject filter

9. In the following graph, the number of trees (P) and the number of cut-sets (Q) are **[2008]**



(A) $P = 2$, $Q = 2$ (B) $P = 2$, $Q = 6$
(C) $P = 4$, $Q = 6$ (D) $P = 4$, $Q = 10$

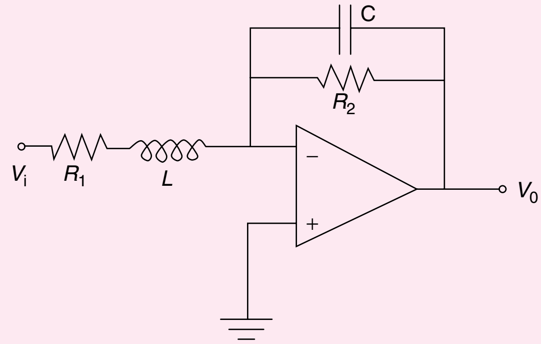
10. The driving point impedance of the following network **[2008]**



is given by $Z(s) = \frac{0.2s}{s^2 + 0.1s + 2}$. The component values are

(A) $L = 5$ H, $R = 0.5\ \Omega$, $C = 0.1$ F
(B) $L = 0.1$ H, $R = 0.5\ \Omega$, $C = 5$ F
(C) $L = 5$ H, $R = 2\ \Omega$, $C = 0.1$ F
(D) $L = 0.1$ H, $R = 2\ \Omega$, $C = 5$ F

11.



The OPAMP circuit shown in the abovementioned figure represents a **[2008]**

(A) high-pass filter (B) low-pass filter
(C) band-pass filter (D) band-reject filter

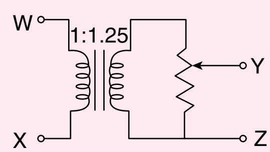
12. For parallel RLC circuit, which one of the following statements is not correct? **[2010]**

(A) The bandwidth of the circuit deceases if R is increased.
(B) The bandwidth of the circuit remains same if L is increased.
(C) At resonance, input impedance is a real quantity.
(D) At resonance, the magnitude of input impedance attains its minimum value.

13. The average power delivered to an impedance $(4 - j3)\ \Omega$ by a current $5\cos(100\pi + 100)$ A is **[2012]**

(A) 44.2 W (B) 50 W
(C) 62.5 W (D) 125 W

14. The following arrangement consists of an ideal transformer and an attenuator that attenuates by a factor of 0.8. An AC voltage $V_{wx1} = 100$ V is applied across WX to get an open circuit voltage V_{yz1} across YZ. Next, an AC voltage $V_{yz2} = 100$ V is applied across YZ to get an open circuit voltage V_{wx2} across WX. Then, V_{YZ1}/V_{wx1} and V_{wx2}/V_{yz2} are, respectively, **[2012]**

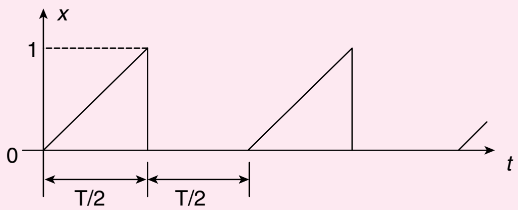


(A) 125/100 and 80/100
(B) 100/100 and 80/100
(C) 100/100 and 100/100
(D) 80/100 and 80/100

15. Two magnetically uncoupled inductive coils have Q factors q_1 and q_2 at the chosen operating frequency. Their respective resistances are R_1 and R_2. When connected in series, their effective Q factor at the same operating frequency is **[2013]**

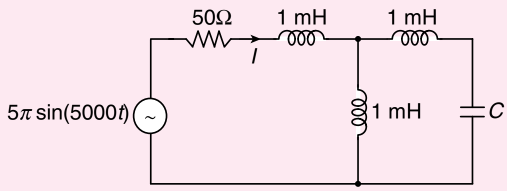
(A) $q_1 + q_2$
(B) $(1/q_1) + (1/q_2)$
(C) $(q_1R_1 + q_2R_2)/(R_1 + R_2)$
(D) $(q_1R_2 + q_2R_1)/(R_1 + R_2)$

16. A periodic variable x is shown in the figure as a function of time. The root-mean-square (rms) value of x is __________ **[2014]**

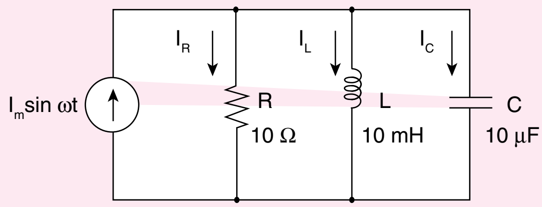


17. A series LCR circuit is operated at a frequency different from its resonant frequency. The operating frequency is such that the current leads the supply voltage. The magnitude of current is half the value at resonance. If the values of L, C, and R are 1 H, 1 F, and 1 Ω, respectively, the operating angular frequency (in rad/s) is_______ **[2014]**

18. In the circuit shown, the current I flowing through the 50 Ω resistor will be zero if the value of capacitor C (in μF) is ______. **[2015]**



19. The figure shows an RLC circuit with a sinusoidal current source



At resonance, the ratio $|I_L|/|I_R|$ i.e., the ratio of the magnitudes of the inductor current phasor and the resistor current phasor, is __________. **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** C	**3.** B	**4.** C	**5.** D	**6.** C	**7.** A	**8.** B	**9.** D	**10.** B
11. A	**12.** D	**13.** D	**14.** D	**15.** A	**16.** B	**17.** D	**18.** B	**19.** A	**20.** B
21. A	**22.** B	**23.** D	**24.** B	**25.** C	**26.** C	**27.** B	**28.** A	**29.** C	**30.** B

Practice Problems 2

1. D	**2.** B	**3.** D	**4.** C	**5.** A	**6.** C	**7.** C	**8.** A	**9.** B	**10.** C
11. C	**12.** D	**13.** B	**14.** B	**15.** B	**16.** A	**17.** C	**18.** A	**19.** B	**20.** C
21. A	**22.** C	**23.** A	**24.** C	**25.** A	**26.** C	**27.** D	**28.** C		

Previous Years' Questions

1. D	**2.** B	**3.** D	**4.** B	**5.** B	**6.** C	**7.** B	**8.** C	**9.** C	**10.** D
11. B	**12.** D	**13.** B	**14.** C	**15.** C	**16.** 0. 39 to 0. 42		**17.** 0. 45 to 0. 47		**18.** 20
19. 0.316									

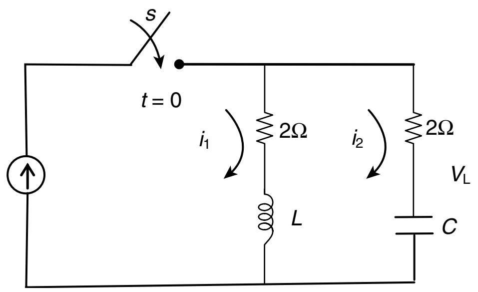
Test

Networks

Time: 60 Minutes

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. In the following circuit, if a constant current source of value I is switched on at $t = 0$, what are the values of currents i_2 and V_L at $t = 0$ with zero initial conditions?



(A) $0, I$ (B) $2I, I$ (C) $I, 0$ (D) $I, 2I$

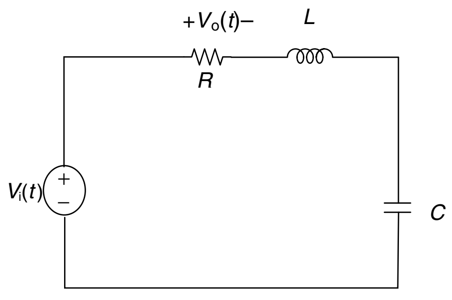
2. For a series *RLC* resonant circuit, what is the total reactance at the lower half power frequency?

(A) R (B) $\sqrt{2}R\angle 45°$
(C) $\frac{R}{\sqrt{2}}$ (D) $\sqrt{2}R\angle -45°$

3. A series *R–L–C* circuit is switched on to a step voltage V at $t = 0$. What are the initial and final values of the current in the circuit?

(A) $\frac{V}{R}, 0$ (B) $0, V/R$ (C) $0, \infty$ (D) $0, 0$

4. For the following network, function $G(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{5s}{s^2+5s+10}$; given R is 2 Ω. What are the values of L and C, respectively?
(A) $L = 0.4$ H, $C = 0.25$ F
(B) $L = 4$ H, $C = 2.5$ F
(C) $L = 0.25$ H, $C = 0.4$ F
(D) $L = 2.5$ H, $C = 4$ F

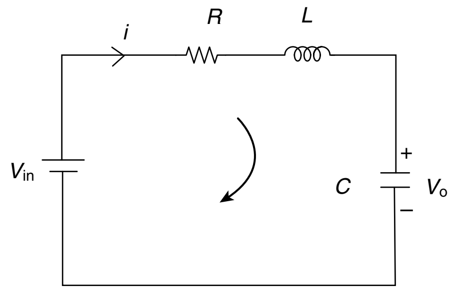


5. For a parallel *RLC* circuit, if $R = 25$ Ω, $L = 40$ H, and $C = 0.4$ F, what are the bandwidth and *Q*-factor, respectively?

(A) 0.1 rad/s, 2.5 (B) 0.1 rad/s, 4
(C) 0.625 rad/s, 4 (D) 10 rad/s, 2.5

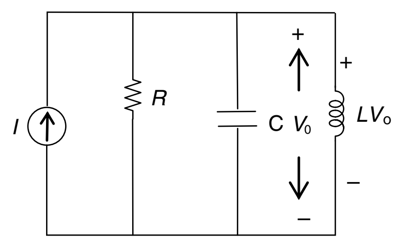
6. For an AC circuit if $v(t) = 50 \sin(\omega t + 30°)$ and $i(t) = 5 \sin(\omega t - 60°)$, the reactive power absorbed by the circuit is ______.
(A) 250 VAR (B) 100 VAR
(C) 125 VAR (D) 300 VAR

7.



The circuit represents a _____.
(A) LPF (B) HPF (C) BPF (D) BSF

8.



The circuits represents a _____.
(A) LPF (B) HPF (C) BPF (D) APF

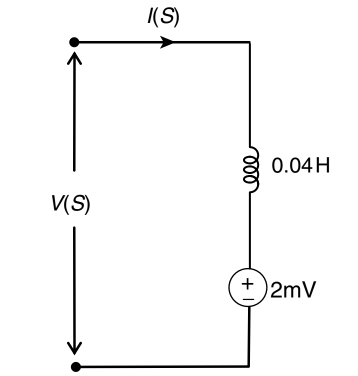
9. The resonant frequency of an *RLC* series circuit is 1 MHz, with the resonating capacitor of 100 pF. The bandwidth is 10 kHz. The effective value of the resistor R is ______.
(A) 8 Ω (B) 10 Ω (C) 14 Ω (D) 16 Ω

10. When a unit impulse voltage is applied to an inductor of 1 H, the energy supplied by the source is ______.

(A) $\frac{1}{4}$J (B) $\frac{1}{2}$J (C) 1J (D) 2J

11. The lowest and the highest critical frequencies of an *R–C* driving point impendence are, respectively, ______.
(A) a zero and a pole
(B) a pole and a pole
(C) a zero and a zero
(D) a pole and a zero

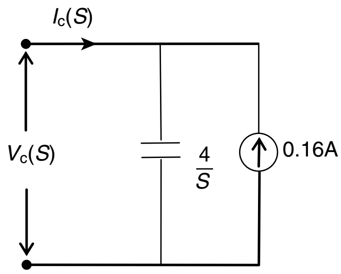
12.



In the abovementioned equivalent model of the inductor, the value of initial current is

(A) 20 mA (B) 80 mA (C) 50 mA (D) 0.5 A

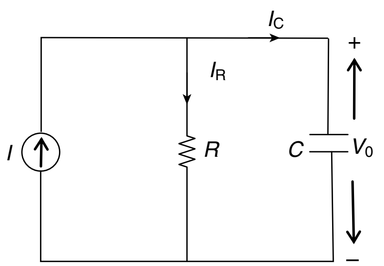
13.



The initial voltage across the capacitor is

(A) 0.4 V (B) 0.64 V (C) 1 V (D) 6.4 V

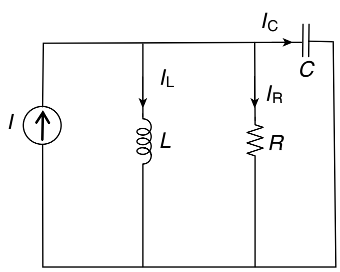
14.



If $I_R = 4$ A and $Ic = 3$ A, then the supply current I is

(A) 7 A (B) 1 A (C) 0.75 A (D) 5 A

Direction for questions 15 and 16:



15. If $I_L = 8$ A, $I_R = 3$ A, and $I_c = 4$ A, then the value of I is

(A) 5 A (B) 7 A
(C) 15 A (D) None of these

16. The power factor of the circuit in the abovementioned circuit is

(A) 0.6 lead (B) 0.6 lag
(C) 0.5 lag (D) 0.5 lead

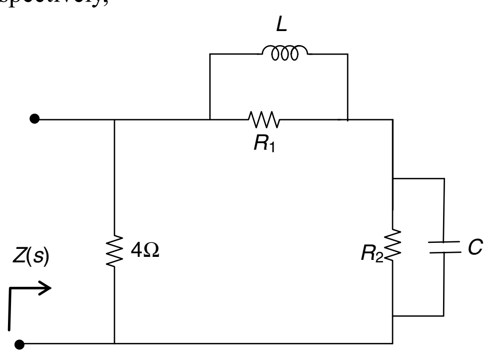
17. A unit step current of 1 A is applied to a network whose driving point impedance is $Z(s) = \dfrac{V(s)}{I(s)} = \left(\dfrac{s+2}{s+3}\right)^2$, then the steady state and initial voltage developed across the source are, respectively,

(A) $\frac{4}{9}$V, 1V (B) 1V, $\frac{4}{9}$V
(C) 0V, $\frac{4}{9}$V (D) $\frac{4}{9}$V, ∞V

18. The driving point impedance function $Z(s) = \dfrac{(s^2+3s+2)}{s(s+1.5)}$ can be realized as a

(A) *RC* admittance (B) *RC* impedance
(C) *LC* impedance (D) *RLC* impedance

19. Consider the following circuit $Z(s) = 3$ as $s \to \infty$ and $Z(s) = 2$ as $s \to 0$, then the values of R_1 and R_2, respectively,



(A) 12 Ω, 2 Ω (B) 0 Ω, 4 Ω
(C) 4 Ω, 12 Ω (D) 12 Ω, 4 Ω

20. The inductance matrix of a system of two mutually coupled inductors shown in Figure 1 is given by

$$L = \begin{bmatrix} 5 & -2 \\ -2 & 3 \end{bmatrix}$$

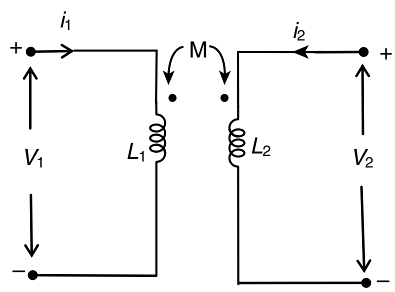


Figure 1

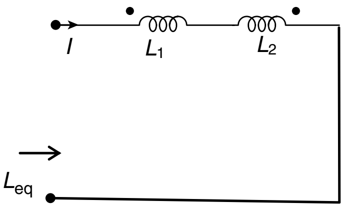
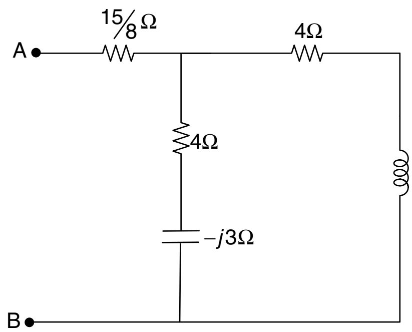


Figure 2

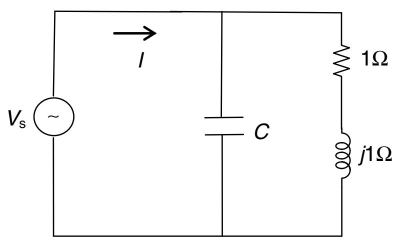
When the inductors are connected as shown in Figure 2, the equivalent inductance of the system is given by
(A) 8 H (B) 12 H (C) 4 H (D) 16 H

21. The total impedance of $Z(j\omega)$ of the following circuit is



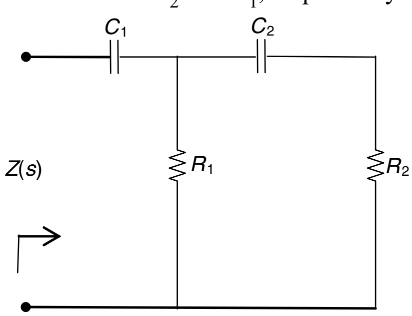
(A) $(0 + j5)\ \Omega$ (B) $(4 + j0)\ \Omega$
(C) $(5 + j0)\ \Omega$ (D) $(2 + j1.5)\ \Omega$

22. In the following circuit, for what value of 'C' will the current I be in phase with sinusoidal source voltage $V_s = 2 \sin 2t$?



(A) 4 F (B) $\frac{1}{4}$ F (C) $\frac{1}{2}$ F (D) 2 F

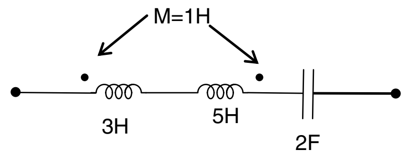
23. If $Z(s) = \dfrac{s^2 + 6s + 12}{s(s+3)}$ for the following circuit, then what is the value of R_2 and C_1, respectively?



(A) $\frac{2}{3}\Omega, \frac{1}{4}$F (B) $2\Omega, 2$F
(C) $2\Omega, \frac{1}{4}$F (D) $\frac{1}{2}\Omega, \frac{1}{4}$F

24. Two coils are coupled in such a way that the mutual inductance between them is 8 mH. If the inductances of the coils are 20 mH and 80 mH, respectively, the coefficient of coupling is
(A) 0.02 (B) 0.5 (C) 0.2 (D) 0.05

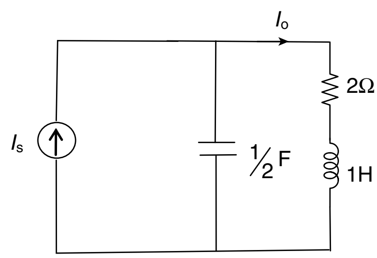
25. The resonant frequency of the given series circuit is



(A) $\dfrac{1}{2\pi\sqrt{3}}$ Hz (B) $\dfrac{1}{4\pi\sqrt{2}}$ Hz
(C) $\dfrac{1}{2\pi}$ Hz (D) $\dfrac{1}{4\pi\sqrt{3}}$ Hz

26. If the current flowing through a 10 Ω resistor is given as $i(t) = 3 + 4\cos 2\omega t - 5 \sin \omega t$ A, then the power consumed by the resistor is
(A) 295 W (B) 29.5 W (C) 250 W (D) 300 W

Direction for questions 27 and 28:
Consider the following circuit:

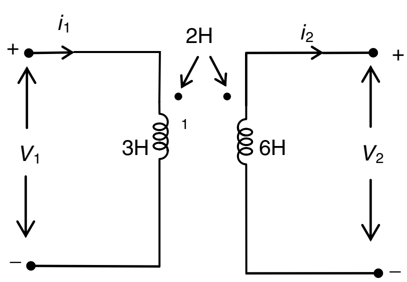


27. The transfer function $\dfrac{I_o(s)}{I_s(s)}$ is

(A) $\dfrac{s(s+4)}{(s+1)(s+3)}$ (B) $\dfrac{s+2}{s^2+2s+2}$
(C) $\dfrac{s(s+1)}{s(s+1)}$ (D) $\dfrac{2}{s^2+2s+2}$

28. The response of the system is
(A) undamped (B) under damped
(C) over damped (D) critically damped

Direction for questions 29 and 30:
Consider the following circuit:



29. If $i_1 = e^{-3t}$ A and $i_2 = 0$, then the voltage V_2 is

(A) $-4e^{-2t}$ V
(B) $\frac{2}{3}e^{-3t}$
(C) $6e^{-3t}$ V
(D) None of these

30. If $i_1 = 3$ A and $i_2 = 2\cos 3t$ A, then the voltage $V_1(t)$ is

(A) $6\cos 3t$ V
(B) $\frac{2}{3}\sin 3t$ V
(C) $12\sin 3t$ V
(D) $6\sin 3t$ V

Answer Keys

1. D	**2.** D	**3.** D	**4.** A	**5.** A	**6.** C	**7.** A	**8.** C	**9.** D	**10.** B
11. D	12. C	13. B	14. D	15. A	16. B	17. A	18. B	19. D	20. B
21. C	22. B	23. C	24. C	25. D	26. A	27. D	28. B	29. C	30. C