Chapter - 8
Exponents and Powers

Exercise

In questions 1 to 33, out of the four options, only one is correct. Write the correct answer.

1. In 2^n, n is known as

(a) Base (b) Constant (c) exponent (d) Variable

Solution:
(c) Exponent

As,
2 is the rational number which is the base here and n is the power of 2.
So, it is an exponent.

2. For a fixed base, if the exponent decreases by 1, the number becomes

(a) One-tenth of the previous number.
(b) Ten times of the previous number.
(c) Hundredth of the previous number.
(d) Hundred times of the previous number.

Solution:
(a) One-tenth of the previous number

Suppose for 10^6, when the exponent is decreased by 1, it becomes 10^5.
So,
$$\frac{10^5}{10^6} = \frac{1}{10}$$

3. 3^{-2} can be written as
(a) 3^2 (b) $\frac{1}{3^2}$ (c) $\frac{1}{3^{-2}}$ (d) $-\frac{2}{3}$

Solution:
(b) $\frac{1}{3^2}$

4. The value of $\frac{1}{4^{-2}}$ is

(a) 16 (b) 8 (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

Solution:

(a) 16

5. The value of $3^5 \div 3^{-6}$ is

(a) 3^5 (b) 3^{-6} (c) 3^{11} (d) 3^{-11}

Solution:

(c) 3^{11}

$$3^5 \div 3^{-6} = 3^{5+6}$$
$$= 3^{11}$$

6. The value of $\left(\frac{2}{5}\right)^{-2}$ is

(a) $\frac{4}{5}$ (b) $\frac{4}{25}$ (c) $\frac{25}{4}$ (d) $\frac{5}{2}$

Solution:

(c) $\frac{25}{4}$

$$\left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^{2}$$
$$= \frac{25}{4}$$

7. The reciprocal of $\left(\frac{2}{5}\right)^{-1}$is

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $-\frac{2}{5}$ (d) $-\frac{5}{2}$

Solution:

(b) $\frac{5}{2}$

8. The multiplicative inverse of 10^{-100} is

(a) 10 (b) 100 (c) 10^{100} (d) 10^{-100}

Solution:

(c) 10^{100}

9. The value of $(–2)^{2*3-1}$ is

(a) 32 (b) 64 (c) – 32 (d) – 64

Solution:

(c) – 32

$$(-2)^{2\times3+1} = (-2)^{2\times3-1}$$
$$= (-2)^5$$
$$= -32$$

10. The value of $(\frac{2}{3})^4$ is equal to

(a) $\frac{16}{81}$ (b) $\frac{81}{16}$ (c) $-\frac{16}{81}$ (d) $-\frac{81}{16}$

Solution:

(a) $\frac{16}{81}$

$$\left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

11. The multiplicative inverse of $(-\frac{5}{9})^{-99}$ is

(a) $(-\frac{5}{9})^{99}$ (b) $(\frac{5}{9})^{99}$ (c) $(-\frac{9}{5})^{99}$ (d) $(\frac{9}{5})^{99}$

Solution:

(a) $\left(\frac{-5}{9}\right)^{99}$

12. If x be any non-zero integer and m, n be negative integers, then $x^m \times x^n$ is equal to

(a) x^m (b) x^{m+n} (c) x^n (d) x^{m-n}

Solution:

(b) x^{m+n} (By the law of exponents)

13. If y be any non-zero integer, then y^0 is equal to

(a) 1 (b) 0 (c) – 1 (d) Not defined

Solution: Solution:

(a) 1 (By the law of exponent)

14. If x be any non-zero integer, then x^{-1} is equal to

(a) x (b) $\frac{1}{x}$ (c) – x (c) $-\frac{1}{x}$

Solution:

(b) $\frac{1}{x}$ (By the law of exponents)

15. If x be any integer different from zero and m be any positive integer, then x^{-m} is equal to

(a) x^m (b) $-x^m$ (c) $\frac{1}{x^m}$ (d) $-\frac{1}{x^m}$

Solution:

(c) $\frac{1}{x^m}$ (By the law of exponents)

16. If x be any integer different from zero and m, n be any integers, then $(x^m)^n$ is equal to

(a) x^{m+n} (b) x^{mn} (c) $x^{\frac{m}{n}}$ (d) x^{m-n}

Solution:

(b) x^{mn} (By the law of exponents)

17. Which of the following is equal to $\left(-\frac{3}{4}\right)^{-3}$?

(a) $\left(\frac{3}{4}\right)^3$ (b) $-\left(\frac{3}{4}\right)^{-3}$ (c) $\left(\frac{4}{3}\right)^{-3}$ (d) $\left(-\frac{4}{3}\right)^3$

Solution:

(d) $\left(\frac{-4}{3}\right)^3$

As,

$$\left(\frac{-3}{4}\right)^{-3} = \left(\frac{-4}{3}\right)^{3}$$

18. $\mathbf{\left(-\frac{5}{7}\right)^{-5}}$ **is equal to**

(a) $\mathbf{\left(\frac{5}{7}\right)^{-5}}$ **(b)** $\mathbf{\left(\frac{5}{7}\right)^{5}}$ **(c)** $\mathbf{\left(\frac{7}{5}\right)^{5}}$ **(d)** $\mathbf{\left(-\frac{7}{5}\right)^{5}}$

Solution:

(d) $\left(\frac{-7}{5}\right)^{5}$

As,

$$\left(\frac{-5}{7}\right)^{-5} = \left(\frac{-7}{5}\right)^{5}$$

19. $\mathbf{\left(-\frac{7}{5}\right)^{-1}}$ **is equal to**

(a) $\mathbf{\frac{5}{7}}$ **(b)** $\mathbf{-\frac{5}{7}}$ **(c)** $\mathbf{\frac{7}{5}}$ **(d)** $\mathbf{-\frac{7}{5}}$

Solution:

(b) $\frac{-5}{7}$

As,

$$\left(\frac{-7}{5}\right)^{-1} = \frac{-5}{7}$$

20. $\mathbf{(-9)^{3} \div (-9)^{8}}$ **is equal to**

(a) $\mathbf{(9)^{5}}$ **(b)** $\mathbf{(9)^{-5}}$ **(c)** $\mathbf{(-9)^{5}}$ **(d)** $\mathbf{(-9)^{-5}}$

Solution:

(d) $(-9)^{-5}$

As,

$$\left(-9\right)^{3} \div \left(-9\right)^{8} = \left(-9\right)^{3-8}$$
$$= \left(-9\right)^{-5}$$

21. For a non-zero integer x, $x^7 \div x^{12}$ is equal to

(a) x^5 (b) x^{19} (c) x^{-5} (d) x^{-19}

Solution:

(c) $\mathbf{x}^{-5}$

As,

$$x^{7} \div x^{12} = x^{7-12}$$
$$= x^{-5}$$

22. For a non-zero integer x, $(x^4)^{-3}$ is equal to

(a) x^{12} (b) x^{-12} (c) x^{64} (d) x^{-64}

Solution:

(b) x^{-12}

23. The value of $(7^{-1} – 8^{-1})^{-1} – (3^{-1} – 4^{-1})^{-1}$ is

(a) 44 (b) 56 (c) 68 (d) 12

Solution:

(a) 44

As,

$$\left(7^{-1} - 8^{-1}\right)^{-1} - \left(3^{-1} - 4^{-1}\right)^{-1} = \left(\frac{1}{7} - \frac{1}{8}\right)^{-1} - \left(\frac{1}{3} - \frac{1}{4}\right)^{-1}$$
$$= \left(\frac{1}{56}\right)^{-1} - \left(\frac{1}{12}\right)^{-1}$$
$$= 56 - 12$$
$$= 44$$

24. The standard form for 0.000064 is

(a) 64×10^4 (b) 64×10^{-4} (c) 6.4×10^5 (d) 6.4×10^{-5}

Solution:
(d) 6.4×10^{-5}

25. The standard form for 234000000 is

(a) 2.34×10^8 (b) 0.234×10^9 (c) 2.34×10^{-8} (d) 0.234×10^{-9}

Solution:
(a) 2.34×10^8

As,
$234000000 = 234 \times 10^6$
$= 2.34 \times 10^2 \times 10^6$
$= 2.34 \times 10^8$

26. The usual form for 2.03×10^{-5}

(a) 0.203 (b) 0.00203 (c) 203000 (d) 0.0000203

Solution:
(d) 0.0000203

27. $(\frac{1}{10})^0$ is equal to

(a) 0 (b) $\frac{1}{10}$ (c) 1 (d) 10

Solution:
(c) 1

As,
$a^0 = 1$ (by law of exponent)

28. $(\frac{3}{4})^5 \div \left(\frac{5}{3}\right)^5$ is equal to

(a) $(\frac{3}{4} \div \frac{5}{3})^5$ (b) $(\frac{3}{4} \div \frac{5}{3})^1$ (c) $(\frac{3}{4} \div \frac{5}{3})^0$ (d) $(\frac{3}{4} \div \frac{5}{3})^{10}$

Solution:

(a) $\left(\frac{3}{4} \div \frac{5}{3}\right)^5$

29. For any two non-zero rational numbers x and y, $x^4 \div y^4$ is equal to

(a) $(x \div y)^0$ (b) $(x \div y)^1$ (c) $(x \div y)^4$ (d) $(x \div y)^8$

Solution:
(c) $(x \div y)^4$

30. For a non-zero rational number p, $p^{13} \div p^8$ is equal to

(a) p^5 (b) p^{21} (c) p^{-5} (d) p^{-19}

Solution:
(a) p^5

(By law of exponent $(a)^m \div (a)^n = (a)^{m-n}$)

31. For a non-zero rational number z, $(z^{-2})^3$ is equal to

(a) z^6 (b) z^{-6} (c) z^1 (d) z^4

Solution:
(b) z^{-6}

(By the law of exponents: $(a^m)^n = a^{mn}$)

32. Cube of $-\frac{1}{2}$ is
(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $-\frac{1}{8}$ (d) $-\frac{1}{16}$

Solution:
(c) $\frac{-1}{8}$

33. Which of the following is not the reciprocal of $(\frac{2}{3})^4$?
(a) $(\frac{3}{2})^4$ (b) $(\frac{3}{2})^{-4}$ (c) $(\frac{2}{3})^{-4}$ (d) $(\frac{3^4}{2^4})$

Solution:

(b) $\left(\frac{3}{2}\right)^{-4}$

In questions 34 to 65, fill in the blanks to make the statements true.

34. The multiplicative inverse of 10^{10} is ___________.

Solution:

The multiplicative inverse of 10^{10} is $\underline{10^{-10}}$

35. $a^3 \times a^{-10}$ = ___________.

Solution:

$a^3 \times a^{-10} = a^{3+(-10)}$

$= a^{3-10}$

$= a^{-7}$

36. 5^0 = ___________.

Solution:

$5^0 = 1$

37. $5^5 \times 5^{-5}$ = ___________.

Solution:

$5^5 \times 5^{-5} = 5^{5+(-5)}$

$= 5^{5-5}$

$= 5^0$

$= 1$

38. The value of $\left(\frac{1}{2^3}\right)^2$ is equal to _________.

Solution:

The value of $\left(\frac{1}{2^3}\right)^2$ equal to $\left(\frac{1}{2^6}\right)$.

39. The expression for 8^{-2} as a power with the base 2 is _________.

Solution:

The expression for 8^{-2} as a power with the base 2 is $\underline{(2)^{-6}}$

40. Very small numbers can be expressed in standard form by using _________ exponents.

Solution:
Very small numbers can be expressed in standard form by using negative exponents.

41. Very large numbers can be expressed in standard form by using _________ exponents.

Solution:
Very large numbers can be expressed in standard form by using positive exponents.

42. By multiplying $(10)^5$ by $(10)^{-10}$ we get ________.

Solution:
By multiplying $(10)^5$ by $(10)^{-10}$ we get 10^{-5}

As,
$(10)^5 \times (10)^{-10} = 10^{5+(-10)}$
$= 10^{5-10}$
$= 10^{-5}$

43. $[(\frac{2}{13})^{-6} \div (\frac{2}{13})^3]^3 \times (\frac{2}{13})^{-9}$ = _______

Solution:

$$\left[\left(\frac{2}{13}\right)^{-6} \div \left(\frac{2}{13}\right)^{3}\right]^{3} \times \left(\frac{2}{13}\right)^{-9} = \left[\left(\frac{2}{13}\right)^{-6-3}\right]^{3} \times \left(\frac{2}{13}\right)^{-9}$$
$$= \left(\frac{2}{13}\right)^{-9\times 3} \times \left(\frac{2}{13}\right)^{-9}$$
$$= \left(\frac{2}{13}\right)^{-27} \times \left(\frac{2}{13}\right)^{-9}$$
$$= \left(\frac{2}{13}\right)^{-27-9}$$
$$= \left(\frac{2}{13}\right)^{-36}$$

44. Find the value $[4^{-1} + 3^{-1} + 6^{-2}]^{-1}$.

Solution:

$$\left[4^{-1}+3^{-1}+\ 6^{-2}\right]^{-1}=\left[\frac{1}{4}+\frac{1}{3}+\frac{1}{6^2}\right]^{-1}$$

$$=\left[\frac{9+12+1}{36}\right]^{-1}$$

$$=\left[\frac{22}{36}\right]^{-1}$$

$$=\frac{36}{22}$$

$$=\frac{18}{11}$$

45. $[2^{-1} + 3^{-1} + 4^{-1}]^0$ = _______

Solution:

$[2^{-1} + 3^{-1} + 4^{-1}]^0 = 1$ (Using law of exponent $a^0=1$)

46. The standard form of $\left[\frac{1}{100000000}\right]$ is _______.

Solution:

The standard form of $\frac{1}{100000000}$ is $\underline{1.0 \times 10^{-8}}$

47. The standard form of 12340000 is _______.

Solution:

The standard form of 12340000 is 1.234×10^7

48. The usual form of 3.41×10^6 is ________.

Solution:

The usual form of 3.41×10^6 is 3410000.

49. The usual form of 2.39461×10^6 is ________.

Solution:

The usual form of 2.39461 × 10^6 is 2394610.

50. If 36 = 6 × 6 = 6^2, then $\frac{1}{36}$ expressed as a power with the base 6 is ________.

Solution:

If 36 = 6 × 6 = 6^2, then $\frac{1}{36}$ expressed as a power with the base 6 is 6^{-2}.

51. By multiplying $(\frac{5}{3})^4$ by ________ we get 5^4.

Solution:

Let x be multiplied with $\left(\frac{5}{3}\right)^4$,

$$\left(\frac{5}{3}\right)^4 \times x = 5^4$$

$$\frac{5^4}{3^4} \times x = 5^4$$

$$x = 5^4 \times \frac{3^4}{5^4}$$

$$x = 3^4$$

$$x = 81$$

52. $3^5 \div 3^{-6}$ can be simplified as __________.

Solution:

$$3^5 \div 3^{-6} = 3^{5-(-6)}$$

$$= 3^{5+6}$$

$$= 3^{11}$$

53. The value of 3 × 10^{-7} is equal to ________.

Solution:

Given,

3 x 10^{-7} = 3.0 x 10^{-7}

Now, placing decimal seven place towards left of original position, we get 0.0000003.

So, the value of 3×10^{-7} is equal to 0.0000003.

54. To add the numbers given in standard form, we first convert them into numbers with __ exponents.

Solution:

To add the numbers given in standard form, we first convert them into numbers with equal exponents.

Ex:
$2.46 \times 10^6 + 24.6 \times 10^5 = 2.46 \times 10^6 + 2.46 \times 10^6$
$= 4.92 \times 10^6$

55. The standard form for 32,50,00,00,000 is __________.

Solution:
In standard form,
$32500000000 = 3250 \times 10^2 \times 10^2 \times 10^3$
$= 3250 \times 10^7$
$= 3.250 \times 10^{10}$ or 3.25×10^{10}

So, the standard form for 32500000000 is 3.25×10^{10}.

56. The standard form for 0.000000008 is __________.

Solution:
In standard form,
$0.000000008 = 0.8 \times 10^{-8}$
$= 8 \times 10^{-9}$
$= 8.0 \times 10^{-9}$

So, the standard form for 0.000000008 is 8.0×10^{-9}

57. The usual form for 2.3×10^{-10} is ____________.

Solution:
In usual form,
$2.3 \times 10^{-10} = 0.23 \times 10^{-11}$
$= 0.00000000023$

So, the usual form for 2.3×10^{-10} is 0.00000000023.

58. On dividing 85 by _________ we get 8.

Solution:

Let 8^5 be divided by x,

$$8^5 \div x = 8$$

$$8^5 \times \frac{1}{x} = 8$$

$$\frac{8^5}{8} = x$$

$$8^4 = x$$

59. On multiplying _________ by 2^{-5} we get 2^5.

Solution:

Let x be multiplied by 2^{-5},

$$x \times 2^{-5} = 2^5$$

$$x = \frac{2^5}{2^{-5}}$$

$$x = 2^{5+5}$$

$$x = 2^{10}$$

60. The value of $[3^{-1} \times 4^{-1}]^2$ is _________.

Solution:

$$\left[3^{-1} \times \ 4^{-1}\right]^2 = \left[\frac{1}{3} \times \frac{1}{4}\right]^2$$

$$= \left[\frac{1}{12}\right]^2$$

$$= \frac{1}{144}$$

61. The value of $[2^{-1} \times 3^{-1}]^{-1}$ is _________.

Solution:

$$\left[2^{-1} \times \ 3^{-1}\right]^{-1} = \left(\frac{1}{2} \times \frac{1}{3}\right)^{-1}$$

$$= \left(\frac{1}{6}\right)^{-1}$$

$$= 6$$

62. By solving $(6^0 – 7^0) \times (6^0 + 7^0)$ we get ________.

Solution:

$$\left(6^0 - 7^0\right)\times\left(6^0 + 7^0\right) = (1-1)\times(1+1)$$
$$= 0$$

63. The expression for 3^5 with a negative exponent is _________.

Solution:

The expression for 3^5 with a negative exponent is $\frac{1}{3^{-5}}$

64. The value for $(–7)^6 \div 7^6$ is _________.

Solution:

$$(-7)^6 \div 7^6 = 7^6 \div 7^6$$
$$= 1$$

65. The value of $[1^{-2} + 2^{-2} + 3^{-2}] \times 6^2$ is ________ .

Solution:

The value of $\left[1^{-2} + 2^{-2} + 3^{-2}\right] \times 6^2$ is 49.

$$\left[1^{-2} + 2^{-2} + 3^{-2}\right] \times 6^2 = \left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}\right]\times 6^2$$
$$= \left[1+\frac{1}{4}+\frac{1}{9}\right]\times 6^2$$
$$= \left(\frac{36+9+4}{36}\right)\times 36$$
$$= \frac{49}{36}\times 36$$
$$= 49$$

In questions 60 to 90, state whether the given statements are true (T) or false (F).

66. The multiplicative inverse of $(– 4)^{-2}$ is $(4)^{-2}$.

Solution:
False

As,
Multiplicative inverse of $(-4)^{-2} = (-4)^2$

67. The multiplicative inverse of ($\frac{3}{2}$)^2 is not equal to ($\frac{2}{3}$)$^{-2}$.

Solution:
True
As,
Multiplicative inverse of $\left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^{-2}$

68. $10^{-2} = \frac{1}{100}$

Solution:
True

69. 24.58 = 2 × 10 + 4 × 1 + 5 × 10 + 8 × 100

Solution:
False

R H S = 2 x 10 + 4 x 1 + 5 x 10 + 8 x 100
= 20+ 4 + 50+ 800
= 874

L H S ≠ R H S

70. 329.25 = 3 × 10^2 + 2 × 10^1 + 9 × 10^0 + 2 × 10^{-1} + 5 × 10^{-2}

Solution:
True

71. $(–5)^{-2} \times (–5)^{-3} = (–5)^{-6}$

Solution:
False

As,

$$(-5)^{-2} \times (-5)^{-3} = (-5)^{-2-3}$$
$$= (-5)^{-5}$$

72. $(-4)^{-4} \times (4)^{-1} = (4)^{5}$

Solution:
False

$$(-4)^{-4} \times (4)^{-1} = \left(\frac{1}{-4}\right)^{4} \times \frac{1}{4}$$
$$= \left(\frac{1}{4}\right)^{4} \times \frac{1}{4}$$
$$= \frac{1}{4^{5}}$$
$$= 4^{-5}$$

73. $\left(\frac{2}{3}\right)^{-2} \times \left(\frac{2}{3}\right)^{-5} = \left(\frac{2}{10}\right)^{10}$

Solution:
False

$$\left(\frac{2}{3}\right)^{-2} \times \left(\frac{2}{3}\right)^{-5} = \left(\frac{2}{3}\right)^{-2+(-5)}$$
$$= \left(\frac{2}{3}\right)^{-7}$$

74. $5^{0} = 5$

Solution:
False

We know that,
$a^0 = 1$
So,
$5^0 = 1$

75. $(-2)^{0} = 2$

Solution:
False

As, $a^0 = 1$

76. $(-\frac{8}{2})^0 = 0$

Solution:
False

$$\left(\frac{-8}{2}\right)^0 = 1$$

77. $(-6)^0 = -1$

Solution:
False

78. $(-7)^{-4} \times (-7)^2 = (-7)^{-2}$

Solution:
True

$$(-7)^{-4} \times (-7)^2 = (-7)^{-2}$$

$$\begin{aligned} LHS &= (-7)^{-4} \times (-7)^2 \\ &= (-7)^{-4+2} \\ &= (-7)^{-2} \end{aligned}$$

79. The value of $\frac{1}{4^{-2}}$ is equal to 16.

Solution:
True

$$\begin{aligned} \frac{1}{4^{-2}} &= 4^2 \\ &= 16 \end{aligned}$$

80. The expression for 4^{-3} as a power with the base 2 is 2^6.

Solution:
False

$$4^{-3} = (2)^{2\times -3}$$
$$= (2)^{-6}$$

81. $a^p \times b^q = (ab)^{pq}$

Solution:

False

$RHS = (ab)^{pq}$

Using law of exponents,

$(ab)^{pq} = (a)^{pq} \times (b)^{pq}$

$LHS \neq RHS$

82. $\frac{x^m}{y^m} = \left(\frac{y}{x}\right)^{-m}$

Solution:

True

$$\frac{x^m}{y^m} = \left(\frac{x}{y}\right)^m$$
$$= \left(\frac{y}{x}\right)^{-m}$$

83. $a^m = \frac{1}{a^{-m}}$

Solution:

True

84. The exponential form for $(-2)^4 \times \left(\frac{5}{2}\right)^4$ is 5^4.

Solution:

True

$$(-2)^4 \times \left(\frac{5}{2}\right)^4 = (-2)^4 \times \frac{5^4}{2^4}$$
$$= 2^{4-4} \times 5^4$$
$$= 2^0 \times 5^4$$
$$= 5^4$$

85. The standard form for 0.000037 is 3.7×10^{-5}.

Solution:
True
In standard form,
$0.000037 = 0.37 \text{ x } 10^{-4}$
$= 3.7 \text{ x } 10^{-5}$

86. The standard form for 203000 is 2.03×10^5

Solution:
True
In standard form,
$203000 = 203 \text{ x } 10 \text{ x } 10 \text{ x } 10$
$= 203 \text{ x } 10^3$
$= 2.03 \text{ x } 10^2 \text{ x } 10^3$
$= 2.03 \text{ x } 10^5$

87. The usual form for 2×10^{-2} is not equal to 0.02.

Solution:
False.
As,
$2 \times 10^{-2} = 0.02$

88. The value of 5^{-2} is equal to 25.

Solution:
False
$$5^{-2} = \frac{1}{5^2}$$
$$= \frac{1}{25}$$

89. Large numbers can be expressed in the standard form by using positive exponents.

Solution:
True
For example,

2360000 = 236 x 10 x 10 x 10 x 10
= 236 x 10^4
= 2.36 x 10^4 x 10^2
= 2.36 x 10^6

90. $a^m \times b^m = (ab)^m$

Solution:
True

91. Solve the following:
(i) 100^{-10} (ii) $2^{-2} \times 2^{-3}$ (iii) $\left(\frac{1}{2}\right)^{-2} \div \left(\frac{1}{2}\right)^{-3}$

Solution:

$$(i) 100^{-10}$$

$$100^{-10} = \frac{1}{100^{10}}$$

$$(ii)\ 2^{-2} \times 2^{-3}$$

$$2^{-2} \times 2^{-3} = 2^{-2-3}$$

$$= 2^{-5}$$

$$(iii) \left(\frac{1}{2}\right)^{-2} \div \left(\frac{1}{2}\right)^{-3}$$

$$\left(\frac{1}{2}\right)^{-2} \div \left(\frac{1}{2}\right)^{-3} = \left(\frac{1}{2}\right)^{-2-(-3)}$$

$$= \left(\frac{1}{2}\right)^{-2+3}$$

$$= \left(\frac{1}{2}\right)$$

92. Express $3^{-5} \times 3^{-4}$ as a power of 3 with positive exponent.

Solution:

Using identity,

$$3^{-5}\times 3^{-4} = 3^{-5-4}$$
$$= 3^{-9}$$
$$= \frac{1}{3^9}$$

93. Express 16^{-2} as a power with the base 2.

Solution:

We know that,

$$2\times2\times2\times2 = 16$$
$$16 = 2^4$$

Now,

$$16^{-2} = \left(2^4\right)^{-2}$$
$$= 2^{-8}$$

94. Express $\frac{27}{64}$ and $-\frac{27}{64}$ as powers of a rational number.

Solution:

We can write,

$$27 = 3^3$$
$$-27 = (-3)^3$$
$$64 = 4^3$$

Now,

$$\frac{27}{64} = \frac{3^3}{4^3}$$
$$= \left(\frac{3}{4}\right)^3$$

also,

$$\frac{-27}{64} = \frac{(-3)^3}{4^3}$$
$$= \left(\frac{-3}{4}\right)^3$$

95. Express $\frac{16}{81}$ and $-\frac{16}{81}$ as powers of a rational number.

Solution:

As,

16 = 4 x 4

And

81 = 9 x 9

Now,

$$\frac{16}{81} = \frac{4^2}{9^2}$$

$$= \left(\frac{4}{9}\right)^2$$

and,

$$\frac{-16}{81} = \frac{-(4)^2}{9^2}$$

$$= -\left(\frac{4}{9}\right)^2$$

96. Express as a power of a rational number with negative exponent.

(a) $[(\frac{-3}{2})^{-2}]^{-3}$

(b) $(2^5 \div 2^8) \times 2^{-7}$

Solution:

$$(a)\left[\left(\frac{-3}{2}\right)^{-2}\right]^{-3} = \left(\frac{-3}{2}\right)^{-2\times-3}$$

$$= \left(\frac{-3}{2}\right)^{6}$$

$$= \left(\frac{2}{-3}\right)^{-6}$$

$$(b)\left(2^5 \div 2^8\right) \times 2^{-7} = 2^{5-8} \times 2^{-7}$$

$$= 2^{-3-7}$$

$$= 2^{-10}$$

97. Find the product of the cube of (–2) and the square of (+4).

Solution:

As,

Cube of (-2)= $(-2)^3$

Square of (+4)= $(+4)^2$

Therefore,

Product = $(-2)^3$ x $(4)^2$

= (-8) x 16

= -128

98. Simplify:

(i) $\left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2}$

(ii) $\left[\left(\frac{-2}{3}\right)^{-2}\right]^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

(iii) $\frac{49 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}}$ $(z \neq 0)$

(iv) $(2^5 \div 2^8) \times 2^{-7}$

Solution:

$(i)\left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2}$

Using,

$a^{-m} = \frac{1}{a^m}$

So,

$4^2 + 2^2 + 3^2 = 16 + 4 + 9$

$= 29$

$$(ii)\left[\left(\frac{-2}{3}\right)^{-2}\right]^{-3}\times\left(\frac{1}{3}\right)^{-4}\times 3^{-1}\times\frac{1}{6}$$

$U \sin g\ identities,$

$$\left(\frac{-2}{3}\right)^{-2\times 3}\times 3^{4}\times\frac{1}{3}\times\frac{1}{6}$$

$$\left(\frac{-2}{3}\right)^{-6}\times 3^{4}\times\frac{1}{3}\times\frac{1}{6}$$

$$\left(\frac{3}{-2}\right)^{6}\times 3^{4}\times\frac{1}{3}\times\frac{1}{(2\times 3)}$$

$$\frac{(3)^{6}}{(-2)^{6}}\times 3^{4}\times\frac{1}{3^{2}}\times\frac{1}{2}$$

$$\frac{(3)^{10-2}}{2^{7}}$$

$$\frac{(3)^{8}}{2^{7}}$$

$$(iii)\frac{49\times z^{-3}}{7^{-3}\times 10\times z^{-5}}$$

$$\frac{49\times z^{-3}}{7^{-3}\times 10\times z^{-5}}=\frac{7^{2}\times z^{-3}}{7^{-3}\times 10\times z^{-5}}$$

$$=\frac{7^{2+3}\times z^{-3+5}}{10}$$

$$=\frac{7^{2+3}\times z^{-3+5}}{10}$$

$$=\frac{7^{5}\times z^{2}}{10}$$

$$=\frac{7^{5}}{10}z^{2}$$

$$(iv)\left(2^{5}\div 2^{8}\right)\times 2^{-7}$$

$$=\left(2^{5-8}\right)\times 2^{-7}$$

$$=\left(2^{-3}\right)\times 2^{-7}$$

$$=2^{-3-7}$$

$= 2^{-10}$

$= \frac{1}{2^{10}}$

$= \frac{1}{1024}$

99. Find the value of x so that

(i) $(\frac{5}{3})^{-2} \times (\frac{5}{3})^{-14} = (\frac{5}{3})^{-8x}$

(ii) $(-2)^3 \times (-2)^{-6} = (-2)^{2x-1}$

(iii) $(2^{-1} + 4^{-1} + 6^{-1} + 8^{-1})^x = 1$

Solution:

(i)

We have,

$$\left(\frac{5}{3}\right)^{-2} \times \left(\frac{5}{3}\right)^{-14} = \left(\frac{5}{3}\right)^{8x}$$

Now,

$a^m + a^n = a^{m+n}$

So,

$$\left(\frac{5}{3}\right)^{-2} \times \left(\frac{5}{3}\right)^{-14} = \left(\frac{5}{3}\right)^{8x}$$

$$\left(\frac{5}{3}\right)^{-2-14} = \left(\frac{5}{3}\right)^{8x}$$

Comparing both sides,

$-16 = 8x$

$-2 = x$

(*ii*) According to question,

$(-2)^3 \times (-2)^{-6} = (-2)^{2x-1}$

$(-2)^{3-6} = (-2)^{2x-1}$

$(-2)^{-3} = (-2)^{2x-1}$

Comparing both sides,

$-3 = 2x - 1$

$-2 = 2x$

$x = -1$

(*iii*) According to question,

$(2^{-1} + 4^{-1} + 6^{-1} + 8^{-1})^x = 1$

Using,

$a^{-m} = \frac{1}{a^m}$

So,

$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right)^x = 1$

$\left(\frac{25}{24}\right)^x = 1$

This can be possible only if $x = 0$ because $a^o = 1$.

100. Divide 293 by 10,00,000 and express the result in standard form.

Solution:

Using,

$a^{-m} = \frac{1}{a^m}$

Also,

$1000000 = 10^6$

So,

$\frac{293}{10^6} = 293 \times 10^{-6}$

$= 2.93 \times 10^{-4}$

101. Find the value of x^{-3} if $x = (100)^{1-4} \div (100)^0$.

Solution:

Consider:
$x = (100)^{1-4} \div (100)^0$

Now, simplify the above equation as follows:
$x = (100)^{-3} \div 1$
$x = (100)^{-3}$

Multiply both side by negative power 3, get:
$x^{-3} = [(100)^{-3}]^{-3}$
So, $x^{-3}=100^9$

102. By what number should we multiply $(–29)^0$ so that the product becomes $(+29)^0$.

Solution:
Let n be multiplied with $(–29)^0$ to get $(+29)^0$.
So, $n\times(–29)^0=(+29)^0$
$n\times1=1$
$n=1$
Hence, the number should be 1.

103. By what number should $(–15)^{-1}$ be divided so that quotient may be equal to $(–15)^{-1}$?

Solution:
Let $(–15)^{-1}$ be divided by n to get quotient $(–15)^{-1}$.

$$\frac{(-15)^{-1}}{n}=(-15)^{-1}$$

$$n=\frac{(-15)^{-1}}{(-15)^{-1}}$$

$$n=1$$

Hence, $(–15)^{-1}$ be divided by 1 to get quotient $(–15)^{-1}$.

104. Find the multiplicative inverse of $(–7)^{-2} \div (90)^{-1}$.

Solution:
As we know that a is called multiplicative inverse of b, if $a\times b=1$.
Given:
$(–7)^{-2} \div (90)^{-1}$
Simplify the above expression as follows:

$$(-7)^{-2} \div (90)^{-1} = \frac{1}{(7)^2} \div \frac{1}{(90)^1}$$

$$= \frac{1}{49} \div \frac{1}{90}$$

$$= \frac{1}{49} \times \frac{90}{1}$$

$$= \frac{90}{49}$$

Let $b = \frac{90}{49}$

So, $a \times \frac{90}{49} = 1$

$$a = \frac{49}{90}$$

Hence, the multiplicative inverse is $\frac{49}{90}$.

105. If $5^{3x-1} \div 25 = 125$, find the value of x.

Solution:

Consider the equation:

$5^{3x-1} \div 25 = 125$

Now, simplify the above equation as follows:

$5^{3x-1} \div 5^2 = 5^3$

$5^{3x-1-2} = 5^3$

Now, comparing the power of both side, get:

3x-1-2=3

3x=3+3

3x =6

x =2

Hence, the value of x is 2.

106. Write 39, 00, and 00,000 in the standard form.

Solution:

For standard form 390000000 = $39 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$$= 39 \times 10^7$$

$$= 3.9 \times 10^7 \times 10$$

$$= 3.9 \times 10^8$$

107. Write 0.000005678 in the standard form.

Solution:

For standard form, 0.000005678 = 0.5678 x 10^{-5} = 5.678 x 10^{-5} x 10^{-1} = 5.678 x 10^{-6} hence, 5.678 x 10^{-6} is the standard form of 0.000005678.

108. Express the product of 3.2 × 10^6 and 4.1 × 10^{-1} in the standard form.

Solution:

Product of 3.2 × 10^6 and 4.1 × 10^{-1}= $3.2\times10^6\times4.1\times10^{-1}$

$$=3.2\times4.1\times10^6\times10^{-1}$$

$$=13.12\times10^5$$

$$=1.312\times10^5\times10$$

$$=1.312\times10^6$$

109. Express $\frac{1.5\times10^6}{2.5\times10^{-4}}$ in the standard form.

Solution:

Consider the expression:

$$\frac{1.5\times10^6}{2.5\times10^{-4}}$$

Now, simplify the above expression as follows:

$$\frac{1.5\times10^6}{2.5\times10^{-4}}=\frac{15}{25}\times10^{6+4}$$

$$=\frac{3}{5}\times10^{10}$$

$$=0.6\times10^{10}$$

$$=6\times10^{10}\times10^{-1}$$

$$=6\times10^9$$

110. Some migratory birds travel as much as 15,000 km to escape the extreme climatic conditions at home. Write the distance in metres using scientific notation.

Solution:

Total distance travelled by migratory bird = 15000 km

$$=15000\times1000m$$

$$=15000000m$$

$$=15\times10^6 m$$

Hence, scientific notation of 15000 km is $15\times10^6 m$.

111. Pluto is 59,1,30,00, 000 m from the sun. Express this in the standard form.

Solution:

Distance between Pluto and Sun = 5913000000

Standard form of 5913000000 = 5913×10^6

$= 5.913 \times 10^6 \times 10^3$

$= 5.913 \times 10^9$

112. Special balances can weigh something as 0.00000001 gram. Express this number in the standard form.

Solution:

Weight = 0.00000001g

Standard form of 0.00000001g = $0.1 \times 10^{-7} g$

$= 1 \times 10^{-7} \times 10^{-1} g$

$= 1.0 \times 10^{-8} g$

113. A sugar factory has annual sales of 3 billion 720 million kilograms of sugar. Express this number in the standard form.

Solution:

Annual sales of a sugar factory = 3 billion 720 million kilograms

= 3720000kg

=$372 \times 10 \times 10 \times 10 \times 10$

$= 372 \times 10^4 kg$

$= 3.72 \times 10^4 \times 10^2 kg$

$= 3.72 \times 10^6 kg$

114. The number of red blood cells per cubic millimetre of blood is approximately 5.5 million. If the average body contains 5 litres of blood, what is the total number of red cells in the body? Write the standard form. (1 litre = 1,00,000 mm^3)

Solution:

The average body contain 5 L of blood.

Also, the number of red blood cells per cubic millimetre of blood is approximately 5.5 million.

Blood contained by body = 5 L = 5 x 100000 mm^3

Red blood cells = 5 x 100000 mm^3
Blood = 5.5 x 1000000 x 5 x 100000
= 55 x 5 x 10^{5+5}
= 275 x 10^{10}
= 2.75 x 10^{10} x 10^{2}
= 2.75 x 10^{12}

115. Express each of the following in standard form:

(a) The mass of a proton in gram is $\frac{1673}{1000000000000000000000000000}$

(b) A Helium atom has a diameter of 0.000000022 cm.

(c) Mass of a molecule of hydrogen gas is about 0.00000000000000000000334 tons.

(d) Human body has 1 trillon of cells which vary in shapes and sizes.

(e) Express 56 km in m.

(f) Express 5 tons in g.

(g) Express 2 years in seconds.

(h) Express 5 hectares in cm^2 (1 hectare = 10000 m^2)

Solution:

(a)
Given:

Mass of a proton in gram = $\frac{1673}{1000000000000000000000000000}$

Standard form= $\frac{1673}{10^{27}}$

$= 1673 \times 10^{-27} g$

$= 1.673 \times 10^{-27} \times 10^{3} g$

$= 1.673 \times 10^{-27+3} g$

$= 1.673 \times 10^{-24} g$

(b) A helium atom has a diameter of 0.000000022cm.
So, standard form of 0.000000022cm = $0.22 \times 10^{-7} cm$

$= 2.2 \times 10^{-7} \times 10^{-1} cm$

$= 2.2 \times 10^{-8} cm$

(c) Mass of a molecule of hydrogen gas is about 0.00000000000000000000334 tons
Standard form = 0.334×10^{-20}

$= 3.34 \times 10^{-20} \times 10^{-1}$

$= 3.34 \times 10^{-21}$

(d) Cells in human body = 1 trillon
1 trillion = 1000000000000
Now, standard form of 1000000000000 = 10^{12}

(e) Given:
56km = 56×1000m
=56000m

Now, standard form of 56000 m = 56×10^3
$= 5.6 \times 10^3 \times 10^1$
$= 5.6 \times 10^4$

(f) Given:
2yr = 2×365days
$= 2 \times 365 \times 24h$

$= 2 \times 365 \times 24 \times 60 \min$

$= 2 \times 365 \times 24 \times 60 \times 60s$

$= 63072000s$
Standard form of 63072000 = 63072×10^3
=$6.3072 \times 10^3 \times 10^4$
$= 6.3072 \times 10^7 s$

(h) Given:
5hec = $5 \times 10000m^2$
$= 5 \times 10000 \times 100 \times 100cm^2$
Now, standard form of $5 \times 10000 \times 100 \times 100cm^2$ = $5 \times 10^8 cm^2$

116. Find x so that $(\frac{2}{9})^3 \times (\frac{2}{9})^{-6} = (\frac{2}{9})^{2x-1}$

Solution:

Given:

$$\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2x-1}$$

Now, using law of exponents: $a^m \times a^n = a^{m+n}$

Then,

$$\left(\frac{2}{9}\right)^{3-6}=\left(\frac{2}{9}\right)^{2x-1}$$

$$\left(\frac{2}{9}\right)^{-3}=\left(\frac{2}{9}\right)^{2x-1}$$

On comparing, get:

-3=2x-1

-2=2x

x=-1

117. By what number should ($\frac{-3}{2}$)$^{-3}$ be divided so that the quotient may be ($\frac{4}{27}$)$^{-2}$?

Solution:

Let $\left(\frac{-3}{2}\right)^{-3}$ be divided by x to get $\left(\frac{4}{27}\right)^{-2}$ as quotient.

Then,

$$\left(\frac{-3}{2}\right)^{-3}\div x=\left(\frac{4}{27}\right)^{-2}$$

$$x=\left(\frac{-3}{2}\right)^{-3}\div\left(\frac{2^2}{3^3}\right)^{-2}$$

$$x=\left(\frac{-3}{2}\right)^{-3}\div\frac{(2)^{-4}}{(3)^{-6}}$$

$$x=\left(\frac{3}{2}\right)^{-3}\times\frac{(3)^{-6}}{(2)^{-4}}$$

$$x=\frac{(-3)^{-3}\times(3)^{-6}}{2^{-3}\times 2^{-4}}$$

$$x=\frac{3^{-9}}{2^{-7}}$$

$$x=\frac{2^7}{3^9}$$

In questions 118 and 119, find the value of n.

118. $\frac{6^n}{6^{-2}}$ = 6^3

Solution:

Given:

$$\frac{6^n}{6^{-2}} = 6^3$$

Now, using law of exponents:

$$\frac{6^n}{6^{-2}} = 6^3$$

$$6^{n+2} = 6^3$$

On comparing both side, get:
n+2=3
n=1

119. $\boldsymbol{\frac{2^n \times 2^6}{2^{-3}} = 2^{18}}$

Solution:

Given:

$$\frac{2^n \times 2^6}{2^{-3}} = 2^{16}$$

Now, using law of exponents: $a^{-m} = \frac{1}{a^m}$

$$2^n \times 2^6 \times 2^3 = 2^{18}$$

$$2^{n+9} = 2^{18}$$

On comparing both sides, get:
n+9=18
n=9

120. $\boldsymbol{\frac{125 \times x^{-3}}{5^{-3} \times 25 \times x^{-6}}}$

Solution:

Consider the expression:

$$\frac{125 \times x^{-3}}{5^{-3} \times 25 \times x^{-6}}$$

Using laws of exponents: $a^m + a^n = a^{n-m}$ and $a^{-m} = \frac{1}{a^m}$

$$\frac{125 \times x^{-3}}{5^{-3} \times 25 \times x^{-6}} = 5^3 \times 5^3 \times 5^{-2} \times x^{-3} \times x^6$$

$$= 5^4 \times x^3$$

$$= 625x^3$$

121. $\boldsymbol{\frac{16 \times 10^2 \times 64}{2^4 \times 4^2}}$

Solution:

Using laws of exponents: $a^m + a^n = a^{m-n}$ and $a^m \times a^n = a^{m+n}$

$$\frac{16\times10^2\times64}{2^4\times4^2} = 4^2\times10^2\times2^{-4}\times4^3\times4^{-2}$$

$$= 4^3\times10^2\times2^{-4}$$

$$= \left(2^2\right)^3\times10^2\times2^{-4}$$

$$= 2^6\times10^2\times2^{-4}$$

$$= 2^2\times10^2$$

$$= 4\times100$$

$$= 400$$

122. If $\frac{5^m\times 5^3\times5^{-2}}{5^{-5}} = 5^{12}$, find m.

Solution:

Given:

Using laws of exponents: $a^m + a^n = a^{m-n}$ and $a^{-m} = \frac{1}{a^n}$

Then,

$$5^m\times5^3\times5^{-2}\times5^5 = 5^{12}$$

$$5^m\times5^8\times5^{-2} = 5^{12}$$

$$5^m\times5^6 = 5^{12}$$

$$5^{m+6} = 5^{12}$$

On comparing both sides, get:

m+6=12

m=6

123. A new born bear weighs 4 kg. How many kilograms might a five year old bear weigh if its weight increases by the power of 2 in 5 years?

Solution:

Weight of new born bear = 4 kg

Weight increases by the power of 2 in 5 yr.

Weight of bear in 5 yr = $(4)^2$ = 16 kg

124. The cells of a bacteria double in every 30 minutes. A scientist begins with a single cell. How many cells will be there after

(a) 12 hours (b) 24 hours

Solution:

The cell of a bacteria in every 30 min =2

So, cell of a bacteria in 1 h = 2^2

(a) Cell of a bacteria in 12 h = $2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2$

$= 2^{24}$

(b) Similarly, bacteria in 24 h = $2^{24} \times 2^{24}$

$= 2^{24+24}$

$= 2^{48}$

125. Planet A is at a distance of 9.35 × 10^6 km from Earth and planet B is 6.27 × 10^7 km from Earth. Which planet is nearer to Earth?

Solution:

Distance between planet A and Earth = 9.35 x 10^6 km

Distance between planet B and Earth = 6.27 x 10^7 km

For finding difference between above two distances, we have to change both in same exponent of 10, that is 9.35 x.10^6 = 0.935 x 10^7, clearly 6.27 x 10^7 is greater.

Hence, planet A is nearer to Earth.

126. The cells of a bacteria double itself every hour. How many cells will there be after 8 hours, if initially we start with 1 cell. Express the answer in powers.

Solution:

The cell of a bacteria double itself every hour = 1+1=2= 2^1

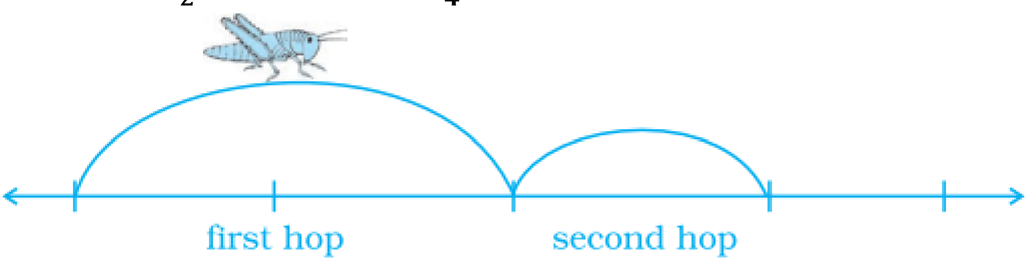
Since, the process started with 1 cell

So, the total number of cell in 8h = $2^1 \times 2^1 \times 2^1 \times 2^1 \times 2^1 \times 2^1 \times 2^1 \times 2^1$

$= 2^{1+1+1+1+1+1+1+1}$

$= 2^8$

127. An insect is on the 0 point of a number line, hopping towards 1. She covers half the distance from her current location to 1 with each hop. So, she will be at $\frac{1}{2}$ after one hop, $\frac{3}{4}$ after two hops, and so on.



(a) Make a table showing the insect's location for the first 10 hops.

(b) Where will the insect be after n hops?
(c) Will the insect ever get to 1? Explain.

Solution:

(a) On the basis of given information in the question, we can arrange the following table which shows the insect's location for the first 10 hops.

Number of hops	Distance Covered	Distance left	Distance covered
1.	$\frac{1}{2}$	$\frac{1}{2}$	$1-\frac{1}{2}$
2.	$\frac{1}{2}\left(\frac{1}{2}\right)+\frac{1}{2}$	$\frac{1}{4}$	$1-\frac{1}{4}$
3.	$\frac{1}{2}\left(\frac{1}{4}\right)+\frac{3}{4}$	$\frac{1}{8}$	$1-\frac{1}{8}$
4.	$\frac{1}{2}\left(\frac{1}{8}\right)+\frac{7}{8}$	$\frac{1}{16}$	$1-\frac{1}{16}$
5.	$\frac{1}{2}\left(\frac{1}{16}\right)+\frac{15}{16}$	$\frac{1}{32}$	$1-\frac{1}{32}$
6.	$\frac{1}{2}\left(\frac{1}{32}\right)+\frac{31}{32}$	$\frac{1}{64}$	$1-\frac{1}{64}$
7.	$\frac{1}{2}\left(\frac{1}{64}\right)+\frac{63}{64}$	$\frac{1}{128}$	$1-\frac{1}{128}$
8.	$\frac{1}{2}\left(\frac{1}{128}\right)+\frac{127}{128}$	$\frac{1}{256}$	$1-\frac{1}{256}$
9.	$\frac{1}{2}\left(\frac{1}{256}\right)+\frac{255}{256}$	$\frac{1}{512}$	$1-\frac{1}{512}$
10.	$\frac{1}{2}\left(\frac{1}{512}\right)+\frac{511}{512}$	$\frac{1}{1024}$	$1-\frac{1}{1024}$

(b) If we see the distance covered in each hopes

Distance covered in 1^{st} hop = $1-\frac{1}{2}$

Distance covered in 2nd hops = $1-\frac{1}{4}$

Distance covered in 3rd hops = $1-\frac{1}{8}$

….

Distance covered in n hops = $1-\left(\frac{1}{2}\right)^n$

(c) No, because for reaching 1, $\left(\frac{1}{2}\right)^n$ has to be zero for some finite n which is not possible.

128. Predicting the ones digit, copy and complete this table and answer the questions that follow.

Powers Table

x	**1^x**	**2^x**	**3^x**	**4^x**	**5^x**	**6^x**	**7^x**	**8^x**	**9^x**	**10^x**
1	**1**	**2**								
2	**1**	**4**								
3	**1**	**8**								
4	**1**	**16**								
5	**1**	**32**								
6	**1**	**64**								
7	**1**	**128**								
8	**1**	**256**								
Ones Digits of the Powers	**1**	**2,4,8,6**								

(a) Describe patterns you see in the ones digits of the powers.
(b) Predict the ones digit in the following:
1. 4^{12} 2. 9^{20} 3. 3^{17} 4. 5^{100} 5. 10^{500}

(c) Predict the ones digit in the following:
1. 31^{10} 2. 12^{10} 3. 17^{21} 4. 29^{10}

Solution:
(a) On the basis of given pattern in 1^x and 2^x , we can make more patterns for 3^x 4^x , 5^x ,6^x , 7^x , 8^x , 9^x , 10^x .
Thus, we have following table which shows all details about the patterns.

x	1^x	2^x	3^x	4^x	5^x	6^x	7^x	8^x	9^x	10^x
1	1	2	3	4	5	6	7	8	9	10
2	1	4	9	16	25	36	49	64	81	100
3	1	8	27	64	125	216	343	512	729	1000
4	1	16	81	256	625	1296	2401	4096	6561	10000
5	1	32	243	1024	3125	7776	16807	32768	59049	100000
6	1	64	729	4096	15625	46656	117649	262144	531441	1000000
7	1	128	2187	16384	78125	279936	823543	2097152	4782969	10000000
8	1	256	6561	65536	390625	1679616	5764801	16777216	43046721	100000000
Ones digits of the powers	1	2,4,8,6	3,9,7,1	4,6	5	6	7,9,3,1	8,4,2,6	9.1	0

(b) (i) ones digit in 4^{12} is 6.
(ii) one digit in 9^{20} is 1.
(iii) ones digit in 3^{17} is 3.
(iv) ones digit in 5^{100} is 5.
(v) ones digit in 10^{500} is 0.
(c) (i) Ones digit in 31^{10} is 1.
(ii) ones digit in 12^{10} is 4.
(iii) ones digit in 17^{21} is 7.
(iv) ones digit in 29^{10} is 1.

129. Astronomy The table shows the mass of the planets, the sun and the moon in our solar system.

Celestial Body	**Mass (kg)**	**Mass (kg) Standard Notation**
Sun	**1,990,000,000,000,000,000,000,000,000,000**	**1.99×10^{30}**
Mercury	**330,000,000,000,000,000,000,000**	
Venus	**4,870,000,000,000,000,000,000,000**	
Earth	**5,970,000,000,000,000,000,000,000**	
Mars	**642,000,000,000,000,000,000,000,000,000**	
Jupiter	**1,900,000,000,000,000,000,000,000,000**	
Saturn	**568,000,000,000,000,000,000,000,000**	
Uranus	**86,800,000,000,000,000,000,000,000**	
Neptune	**102,000,000,000,000,000,000,000,000**	
Pluto	**12,700,000,000,000,000,000,000**	
Moon	**73,500,000,000,000,000,000,000**	

(a) Write the mass of each planet and the Moon in scientific notation.

(b) Order the planets and the moon by mass, from least to greatest.
(c) Which planet has about the same mass as earth?

Solution:

Mass of each planet and moon in scientific notation given below:
Using law of exponents: $a^m \times a^n = a^{m+n}$

Sun = 199×10^{28}
$=1.99\times10^{28}\times10^{2}$
$=1.99\times10^{30}$

Mercury = 33×10^{22}
$=3.3\times10^{22}\times10$
$=3.3\times10^{23}$

Venus = 487×10^{22}
$=4.87\times10^{22}\times10^{2}$
$=4.87\times10^{24}$

Earth = 597×10^{22}
$=5.97\times10^{22}\times10^{2}$
$=5.97\times10^{24}$

Mars = 642×10^{27}
$=6.42\times10^{27}\times10^{2}$
$=6.42\times10^{29}$

Jupiter = 19×10^{26}
$=1.9\times10^{26}\times10$
$=1.9\times10^{27}$

Saturn = 568×10^{24}
$=5.68\times10^{24}\times10^{2}$
$=5.68\times10^{26}$

Uranus = 868×10^{23}
$=8.68\times10^{23}\times10^{2}$
$=8.68\times10^{25}$

Neptune = 102×10^{24}
$=1.02\times10^{24}\times10^{2}$
$=1.02\times10^{26}$

Pluto = 127×10^{20}
$=1.27\times10^{20}\times10^{2}$
$=1.27\times10^{22}$

Moon = 795×10^{20}

$= 7.95 \times 10^{20} \times 10^{2}$

$= 7.95 \times 10^{22}$

130. Investigating Solar System The table shows the average distance from each planet in our solar system to the sun.

Planet	**Distance from Sun (km)**	**Distance from Sun (km) Standard Notation**
Earth	**149,600,000**	**1.496×10^{8}**
Jupiter	**778,300,000**	
Mars	**227,900,000**	
Mercury	**57,900,000**	
Neptune	**4,497,000,000**	
Pluto	**5,900,000,000**	
Saturn	**1,427,000,000**	
Uranus	**2,870,000,000**	
Venus	**108,200,000**	

(a) Complete the table by expressing the distance from each planet to the Sun in scientific notation.
(b) Order the planets from closest to the sun to farthest from the sun.

Solution:

(a) Scientific notation of distance from Sun to:

Earth = 149600000 = $1496 \times 10^{5} = 1.496 \times 10^{8}$

Jupiter = 149600000 = $1496 \times 10^{5} = 1.496 \times 10^{8}$

Mars = 227900000 = $2279 \times 10^{5} = 2.279 \times 10^{8}$

Mercury = 59700000 $= 579 \times 10^{5} = 5.97 \times 10^{7}$

Neptune = 4497000000 = $4497 \times 10^{6} = 4.497 \times 10^{9}$

Pluto=5900000000 $= 59 \times 10^{8} = 5.9 \times 10^{9}$

Saturn = 1427000000 = $1427 \times 10^{6} = 1.427 \times 10^{9}$

Uranus = 2870000000 = $287 \times 10^{7} = 2.87 \times 10^{9}$

Venus = 108200000 = $1082 \times 10^{5} = 1.082 \times 10^{8}$

(b) Order of planet from closest to the Sun to farthest from the sun is given by:

Mercury < Venus < Earth < Mars < Jupiter < Saturn < Uranus < Neptune < Pluto

131. This table shows the mass of one atom for five chemical elements. Use it to answer the question given.

Element	Mass of atom (kg)
Titanium	7.95×10^{-26}
Lead	3.44×10^{-25}
Silver	1.79×10^{-25}
Lithium	1.15×10^{-26}
Hydrogen	1.674×10^{-27}

(a) Which is the heaviest element?
(b) Which element is lighter, Silver or Titanium?
(c) List all five elements in order from lightest to heaviest.

Solution:
Arrangement of masses of atoms in same power of 10 is given by:
Titanium $=7.95\times10^{-26}$
Lead $=34.4\times10^{-26}$
Silver $= 17.9\times10^{-26}$
Lithium $=1.15\times10^{-26}$
Hydrogen $=0.1674\times10^{-26}$
Therefore,
34.4 > 17.9 > 7.95>1.15>0.1674
(a) Lead is the heaviest element
(b) Silver $= 17.9\times10^{-26}$ and Titanium $= 7.95\times10^{-26}$, So, titanium is lighter.
(c) Arrangement of elements in order from lighest to haviest is given by:
Hydrogen < Lithium < Titanium < Silver < Lead

132. The planet Uranus is approximately 2,896,819,200,000 metres away from the Sun. What is this distance in standard form?

Solution:
Distance between the planet Uranus and the Sun is 2896819200000 m.
Standard form of 2896819200000 = 28968192 x 10 x 10 x 10 x 10 x 10
$= 28968192 \times 10^5$
$= 2.8968192 \times 10^{12}$ m

133. An inch is approximately equal to 0.02543 metres. Write this distance in standard form.

Solution:
Standard form of 0.02543 m $= 0.2543 \times 10^{-1}$ m $= 2.543 \times 10^{-2}$ m

Hence,' standard form of 0.025434s 2.543 x 10^{-2} m.

134. The volume of the Earth is approximately 7.67 × 10^{-7} times the volume of the Sun. Express this figure in usual form.

Solution:

Given:

Volume of the Earth is 7.67×10^{-7} times the volume of the Sun.

Usual form of 7.67×10^{-7} = 0.000000767

135. An electron's mass is approximately 9.1093826 × 10^{-31} kilograms. What is this mass in grams?

Solution:

Mass of electron = $9.1093826 \times 10^{-31} kg$

$= 9.1093826 \times 10^{-31} \times 1000 g$

$= 9.1093826 \times 10^{-31} \times 10^{3} g$

$= 9.1093826 \times 10^{-31+3} g$

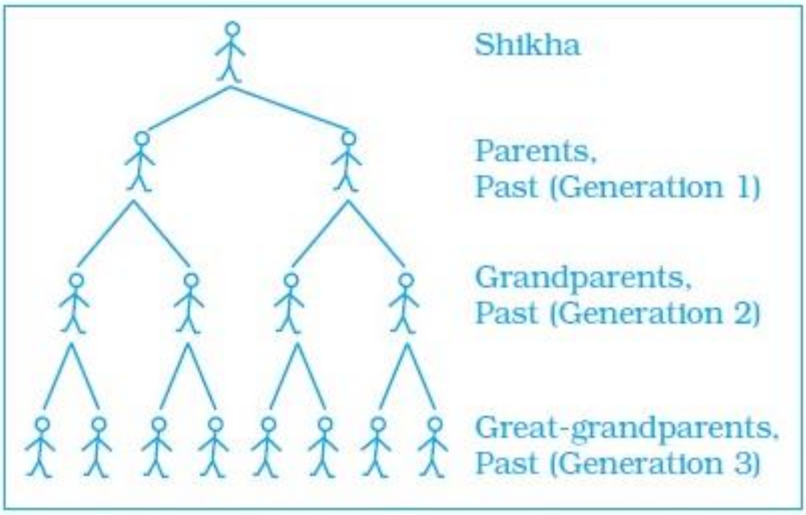
$= 9.1093826 \times 10^{-28} g$

136. At the end of the 20th century, the world population was approximately 6.1 × 10^{9} people. Express this population in usual form. How would you say this number in words?

Solution:

Given: At the end of the 20th century, the world population was 6,1 x 10^{9} (approx). People population in usual form = 6.1 x 10^{9} = 6100000000 Hence, population in usual form was six thousand one hundred million.

137. While studying her family's history. Shikha discovers records of ancestors 12 generations back. She wonders how many ancestors she has had in the past 12 generations. She starts to make a diagram to help her figure this out. The diagram soon becomes very complex.



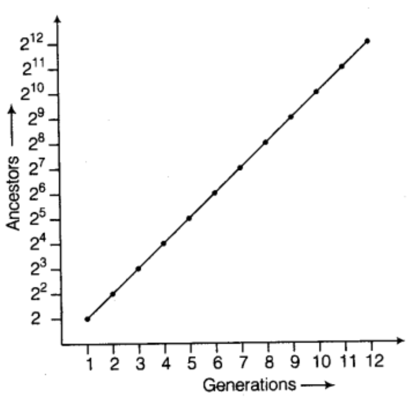
(a) Make a table and a graph showing the number of ancestors in each of the 12 generations.
(b) Write an equation for the number of ancestors in a given generation n.

Solution:

(a) On the basis of given diagram, we can make a table that shows the number of ancestors in each of the 12 generations.

Generations	Ancestors
1st	2
2nd	2^2
3rd	2^3
⋮	⋮
12th	2^{12}

Therefore, the graph that shows the relationship between generation and ancestor.



(b) On the basis of generation-ancestor graph, the number of ancestors in n generations will 2^n.

138. About 230 billion litres of water flows through a river each day. How many litres of water flows through that river in a week? How many litres of water flows through the river in an year? Write your answer in standard notation.

Solution:

Water flows through a river in each days = 230000000000 or 230 billion
Water flows through the river in a week = 7×230000000000
=1610000000000
=1610 billion
$=1.61\,10^{12}$ L

Water flows through the river in an year = 230000000000×365
$= 83950000000000$
$= 8.395 \times 10^{13} L$

139. A half-life is the amount of time that it takes for a radioactive substance to decay to one half of its original quantity.
Suppose radioactive decay causes 300 grams of a substance to decrease to 300×2^{-3} grams after 3 half-lives. Evaluate 300×2^{-3} to determine how many grams of the substance are left.
Explain why the expression 300×2^{-n} can be used to find the amount of the substance that remains after n half-lives.

Solution:

Since, 300 g of a substance is decrease to $300 \times 2^{-3} g$ after 3 half-lives.

So, to evaluate $300 \times 2^{-3} g = \frac{300}{8} g = 37.5g$

140. Consider a quantity of a radioactive substance. The fraction of this quantity that remains after t half-lives can be found by using the expression 3^{-t} .
(a) What fraction of substance remains after 7 half-lives?
(b) After how many half-lives will the fraction be $\frac{1}{243}$ of the original?

Solution:

(a) Since, 3^{-t} expression is used for finding the fraction of the quantity that remains after t half-lives.

(b) Given:
T half-lives = 3^{-t}

So, $\frac{1}{243} = 3^{-t}$

$\frac{1}{3^5} = \frac{1}{3^t}$

On comparing both sides, get:
t = 5 half-lives

141. One Fermi is equal to 10–15 metre. The radius of a proton is 1.3 Fermis . Write the radius of a proton in metres in standard form.

Solution:
The radius of a proton is 1.3 fermi.
One fermi is equal to 10^{-15} m.
So, the radius of the proton is 1.3 x 10^{-15} m.
Therefore, standard form of radius of the proton is 1.3 x 10^{-15} m.

142. The paper clip below has the indicated length. What is the length in standard form.

Solution:
Length of the paper clip = 0.05 m
In standard form, 0.05 m = 0.5 x 10^{-1} = 5.0 x 10^{-2} m
Hence, the length of the paper clip in standard form is 5.0 x 10^{-2} m

143. Use the properties of exponents to verify that each statement is true.
(a) $\frac{1}{4}(2^n) = 2^{n-2}$ (b) $4^{n-1} = \frac{1}{4}(4)^n$ (c) $25(5^{n-2}) = 5^n$

Solution:

(a) $\frac{1}{4}\left(2^n\right) = 2^{n-2}$

RHS = 2^{n-2}

$= 2^n \div 2^2$

$= \frac{2^n}{4}$

$= LHS$

(b) $4^{n-1} = \frac{1}{4}(4)^n$

LHS= 4^{n-1}

$= 4^n \div 4^1$

$= \frac{4^n}{4}$

$= RHS$

(c) $25\left(5^{n-2}\right) = 5^n$

LHS = $25\left(5^{n-2}\right)$

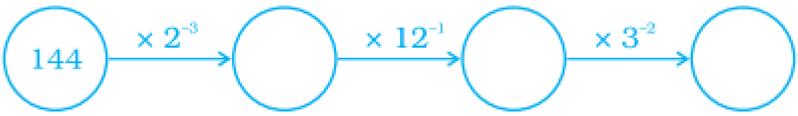
$= 5^2\left(5^n \div 5^2\right)$

$= 5^2 \times 5^n \times \frac{1}{5^2}$

$= 5^n$

$= RHS$

144. Fill in the blanks



Solution:

$144 \times 2^{-3} = 144 \times \frac{1}{8} = 18$

$18 \times 12^{-1} = 18 \times \frac{1}{12} = \frac{3}{2}$

$\frac{3}{2} \times 3^{-2} = \frac{3}{2} \times \frac{1}{3^2} = \frac{1}{2 \times 3} = \frac{1}{6}$

145. There are 864,00 seconds in a day. How many days long is a second? Express your answer in scientific notation.

Solution:

Total seconds in a day = 86400
So, a second is long as 1/86400 = 0.000011574
Scientific notation of 0.000011574= 1.1574 x 10^{-5}days

146. The given table shows the crop production of a State in the year 2008 and 2009. Observe the table given below and answer the given questions.

Crop	**2008 Harvest(Hectare)**	**Increase/Decrease (Hectare) in 2009**

Bajra	**1.4 × 103**	**– 100**
Jowar	**1.7 × 106**	**– 440,000**
Rice	**3.7 × 103**	**– 100**
Wheat	**5.1 × 105**	**+ 190,000 (**

a) For which crop(s) did the production decrease?
(b) Write the production of all the crops in 2009 in their standard form.
(c) Assuming the same decrease in rice production each year as in 2009, how many acres will be harvested in 2015? Write in standard form.

Solution:

(a) On the basis of given table, bajra, jowar and rice crops's production decreased.

(b) The production of all crop in 2009

Bajra=$1.4\times10^3 - 0.1\times10^3 = 1.3\times10^3$

Jowar = $1.7\times10^6 - 44\times10^4$ = $1.7\times10^6 - 0.44\times10^6 = 1.26\times10^6$

Rice = $3.7\times10^3 - 0.1\times10^3 = 3.6\times10^3$

Wheet = $5.1\times10^5 + 19\times10^4 = 5.1\times10^5 + 1.9\times10^5 = 7\times10^5$

(c) Incomplete information

147. Stretching Machine

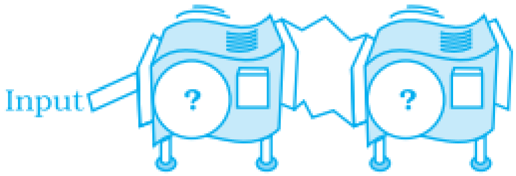
Suppose you have a stretching machine which could stretch almost anything. For example, if you put a 5 metre stick into a (× 4) stretching machine (as shown below), you get a 20 metre stick.
Now if you put 10 cm carrot into a (× 4) machine, how long will it be when it comes out? ×4 10 cm ?

Solution:

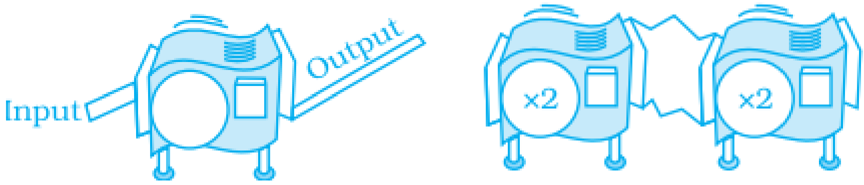
According to the question, if we put a 5 m stick into a (x 4) stretching machine, then machine produces 20 m stick.
Similarly, if we put 10 cm carrot into a (x 4) stretching machine, then machine produce 10 x 4= 40 cm stick.

148. Two machines can be hooked together. When something is sent through this hook up, the output from the first machine becomes the input for the second.

(a) Which two machines hooked together do the same work a ($\times 10^2$) machine does? Is there more than one arrangement of two machines that will work?



(b) Which stretching machine does the same work as two (× 2) machines hooked together?



Solution:

For getting the same work a $\left(\times 10^2\right)$ machine does, we have to $\left(\times 10^2\right)$ and $\left(\times 5^2\right)$ machines hooked together.

So, $\quad \times 10^2 = 100$

Similarly, $\times 2^2 \times 5^2 = \times 4 \times 25 = \times 100$

(b) If two machine $(\times 2)$ and $(\times 2)$ are hooked together to produce $\times 4$ then $(\times 4)$ single machine produce the same work.

149. Repeater Machine

Similarly, repeater machine is a hypothetical machine which automatically enlarges items several times.

For example, sending a piece of wire through a (× 2^4) machine is the same as putting it through a (× 2) machine four times. So, if you send a 3 cm piece of wire through a (× 2^4) machine, its length becomes 3 × 2 × 2 × 2 × 2 = 48 cm. It can also be written that a base (2) machine is being applied 4 times.

What will be the new length of a 4 cm strip inserted in the machine?

Solution:
According to the question, if we put a 3 cm piece of wire through a (x 2^4) machine, its length becomes 3 x 2 x 2 x 2 x 2 = 48 cm.
Similarly, 4 cm long strip becomes 4 x 2 x 2 x 2 x 2 = 64 cm.

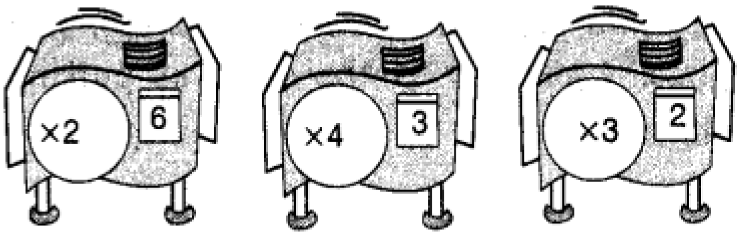
150. For the following repeater machines, how many times the base machine is applied and how much the total stretch is?

(a) (b) (c)

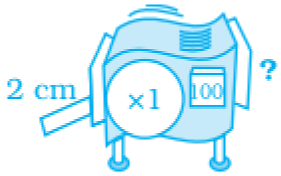
Solution:
In machine (a), (x 100^2) = 10000stretch. Since, it is two times the base machine.
In machine (b), (x 7^5) = 16807 stretch.
Since, it is fair times the base machine.
In machine (c), (x 5^7) = 78125stretch.
Hence, it is 7 times the base machine.

151. Find three repeater machines that will do the same work as a (×64) machine. Draw them, or describe them using exponents.

Solution:
As we know that, the possible factors of 64 are 2, 4, 8. :
If 2^6 =64, 4^3 =64 and 8^2 =64
Hence, three repeater machines that would work as a (x 64) will be (x 2^6), (x 4^3) and (x 8^2).
The diagram of (x 2^6), (x 4^3)and (x 8^2)is given below:



152. What will the following machine do to a 2 cm long piece of chalk?

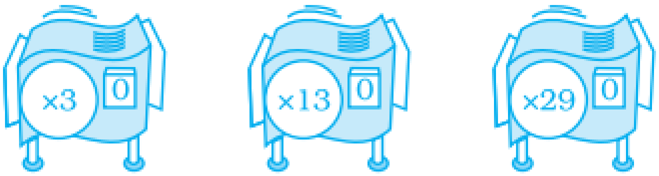


Solution:
The machine produce x 1^{100}=1
So, if we insert 2 cm long piece of chalk in that machine, the piece of chalk remains same.

153. In a repeater machine with 0 as an exponent, the base machine is applied 0 times.
(a) What do these machines do to a piece of chalk?

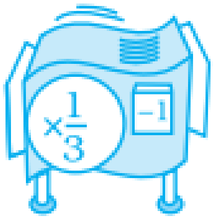
(b) What do you think the value of 6^0 is?

You have seen that a hookup of repeater machines with the same base can be replaced by a single repeater machine. Similarly, when you multiply exponential expressions with the same base, you can replace them with a single expression.



Solution:
(a) Since, $3^0=1, 13^0=1, 29^0=1$
Now, using law of exponents: $a^0=1$
So, machine $(\times 3^0), (\times 13^0)$ and $(\times 29^0)$ produce nothing on not change the pice 7 chalk.
(b) Using the law of exponents: $a^0=1$
Similarly, $(\times 6)$ machine does not change the piece.

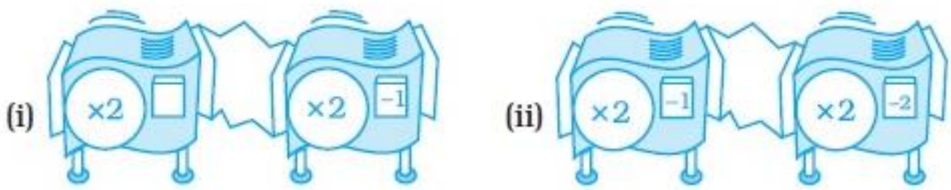
154. Shrinking Machine In a shrinking machine, a piece of stick is compressed to reduce its length. If 9 cm long sandwich is put into the shrinking machine below, how many cm long will it be when it emerges?



Solution:

According to the question, in a shrinking machine, a piece of stick is compressed to reduce its length. If 9 cm long sandwich is put into the shrinking machine, then the length of sandwich will be 9 x 1/ 3^{-1}= 9 x 3 = 27 cm.

155. What happens when 1 cm worms are sent through these hook-ups?



Solution:

(i) If 1cm worms are sent through $\left(\times 2^{1}\right)$ and $\left(\times 2^{-1}\right)$ machine, then the result comes with

$$1\times 2\times \frac{1}{2}=1cm$$

(ii) If 1 cm worms are sent through $\left(\times 2^{-1}\right)$ and $\left(\times 2^{-2}\right)$ hooked machine, the result comes with $1\times \frac{1}{2}\times \frac{1}{2\times 2}=\frac{1}{2\times 4}=\frac{1}{8}=0.125cm$

156. Sanchay put a 1cm stick of gum through a (1 × 3^{-2}) machine. How long was the stick when it came out?

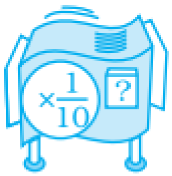
Solution:

If sanchay put a 1 cm stick of gum through a $\left(1\times 3^{-2}\right)$ machine.

Negation (-) sign in power shrews it is a shrinking maching. So, $1\times \frac{1}{3^{2}}=1\times \frac{1}{9}=\frac{1}{9}cm$

Hence, $\frac{1}{9}cm$ stick come out.

157. Ajay had a 1cm piece of gum. He put it through repeater machine given below and it came out $\frac{1}{100{,}000}$ cm long. What is the missing value?



Solution:

Ajay put a 1 cm piece of gum and came out $\frac{1}{100000}$.

So, it is a $\left(\times\frac{1^1}{10}\right)^5$ type shrinking machine.

Therefore missing value is 5.

158. Find a single machine that will do the same job as the given hook-up.

(a) a (× 2^3) machine followed by (× 2^{-2}) machine.

(b) a (× 2^4) machine followed by [×($\frac{1}{2}$)^2] machine.

(c) a (× 5^{99}) machine followed by a (5^{-100}) machine.

Solution:

(a) $\left(\times 2^3\right)$ machine followed by $\left(\times 2^{-2}\right)$ machine.

So, it produces $2\times2\times2\times\frac{1}{2}\times\frac{1}{2}=2^1$

Hence, $\left(\times 2^1\right)$ single machine can do the same job as the given hook-up.

(b) $\left(\times 2^4\right)$ machine followed by $\left(\times\left(\frac{1}{2}\right)^2\right)$ machine.

So, it produces $2\times2\times2\times2\times\frac{1}{2}\times\frac{1}{2}=4$

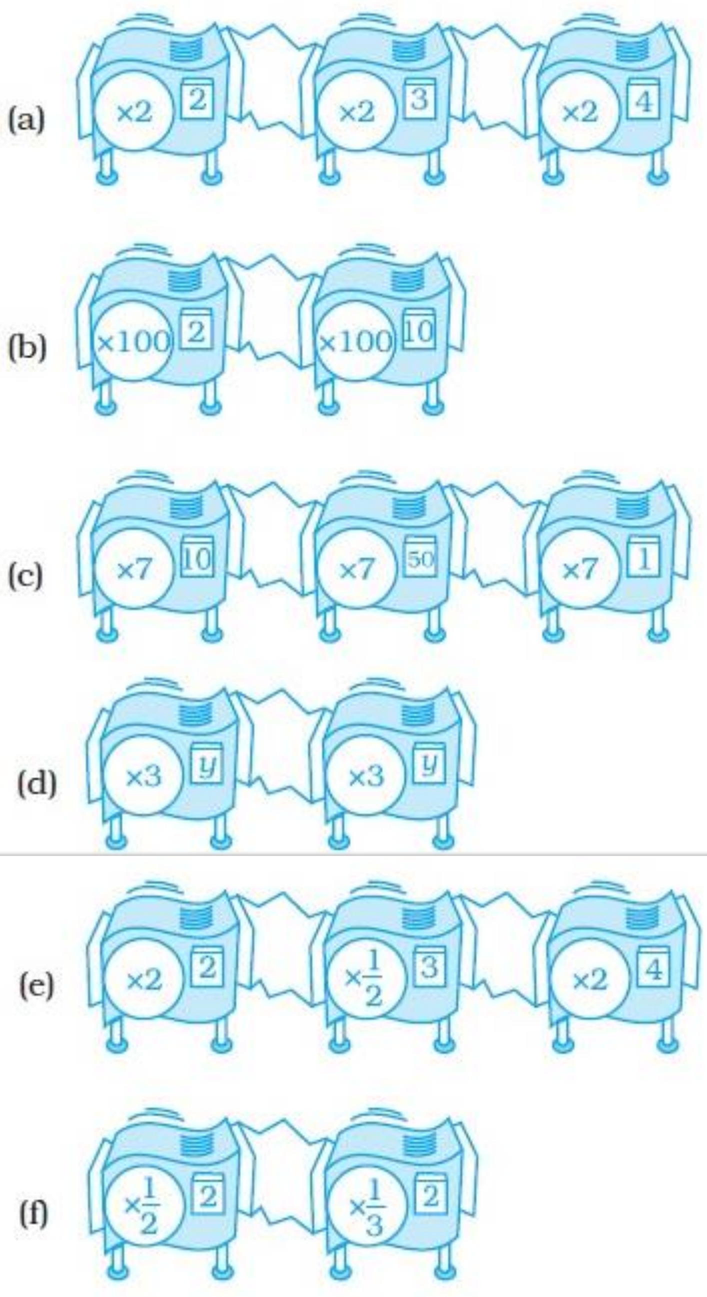
Hence, $\left(\times 2^2\right)$ single machine can do the same job as the given hook-up.

(c) $\left(\times 5^{99}\right)$ machine followed by $\left(\times(5)^{-100}\right)$ machine.

So, it produces $5^{99}\times\frac{1}{5^{100}}=\frac{1}{5}$

Hence, $\left(\times\frac{1}{5}\right)$ single machine can do the same job as the given hook-up.

159. Find a single repeater machine that will do the same work as each hook-up.



Solution:

Using law of exponents: $a^m \times a^n = a^{m+n}$

(a) Repeator machine can do the work is equal to $2^2 \times 2^3 \times 2^4 = 2^9$

So, $\left(\times 2^9\right)$ single machine can do the same work.

(b) Repeator machine can do the work is equal to $100^2 \times 100^{10}$

So, $\left(\times 100^{12}\right)$ single machine can do the same work.

(c) Repeator machine can do the work is equal to $7^{10} \times 7^{51} \times 7^{1} = 7^{61}$

So, $\left(\times 7^{61}\right)$ single machine can do the same work.

(d) Repeator machine can do the work is equal to $3^{y} \times 3^{y} = 3^{2y}$

So, $\left(\times 3^{2y}\right)$ single machine can do the same work.

(e) Repeator machine can do the work is equal to $2^{2} \times \left(\frac{1}{2}\right)^{3} \times 2^{4} = 2^{6} \times \frac{1}{2^{3}} = 2^{3}$

So, $\left(\times 2^{3}\right)$ single machine can do the same work.

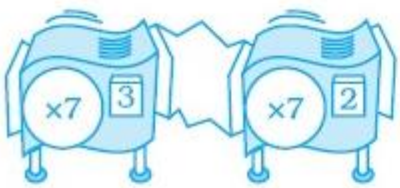
(f) Repeator machine can do the work is equal to

$$\left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{3}\right)^{2} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{2 \times 2 \times 3 \times 3} = \left(\frac{1}{6}\right)^{2}$$

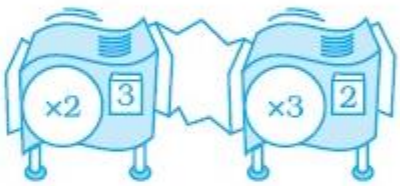
So, $\left(\times \left(\frac{1}{6}\right)^{2}\right)$ single machine can do the same work.

160. For each hook-up, determine whether there is a single repeater machine that will do the same work. If so, describe or draw it.

(a)

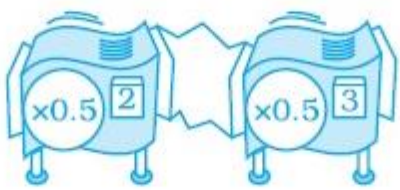


(b)



(c)

(d)



(e)

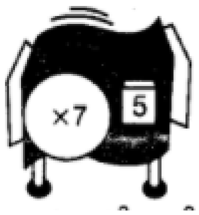
Solution:

Using law of exponents: $a^m \times a^n = a^{m+n}$

(a) Hook-up machine can do the work = $7^3 \times 7^2 = 7^5$

So, $\left(\times 7^5\right)$ Single machine can do the same work.

Diagram of single $\times 7^5$ machine.



(b) Hook-up machine can do the work = $2^3 \times 3^2 = 8 \times 9 = 72$

So, it is not possible for a single machine to do the same work.

Sice, $\left(\times 8^2\right) = 64$ and $\left(\times 9^2\right) = 81$

(c) Hook-up machine can do the work = $2^2 \times \left(\frac{1}{3}\right)^3 \times 5^4 = 4 \times \frac{1}{27} \times 625 = \frac{2500}{27} = 92.59$

So, $(\times 0.5)^5$ machine can for the same work.

(d) Hook-up machine's work = $(0.5)^2 \times (0.5)^3 = (0.5)^5$

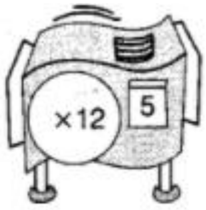
So, $\left(\times (0.5)^5\right)$ machine can for the same work,

Diagram of single $\times (0.5)^5$ machine.

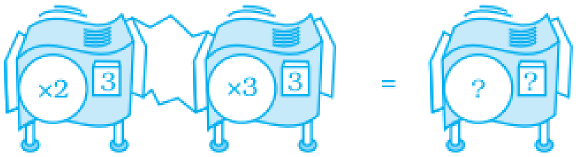
(e) Hook-up machine can do the work = $12^2 \times 12^3 = 12^5$

So, $\times (12)^5$ machine can do the same work.

Diagram of $\times 12^5$ single machine.



161. Shikha has an order from a golf course designer to put palm trees through a (× 2^3) machine and then through a (× 3^3) machine. She thinks she can do the job with a single repeater machine. What single repeater machine should she use?



Solution:

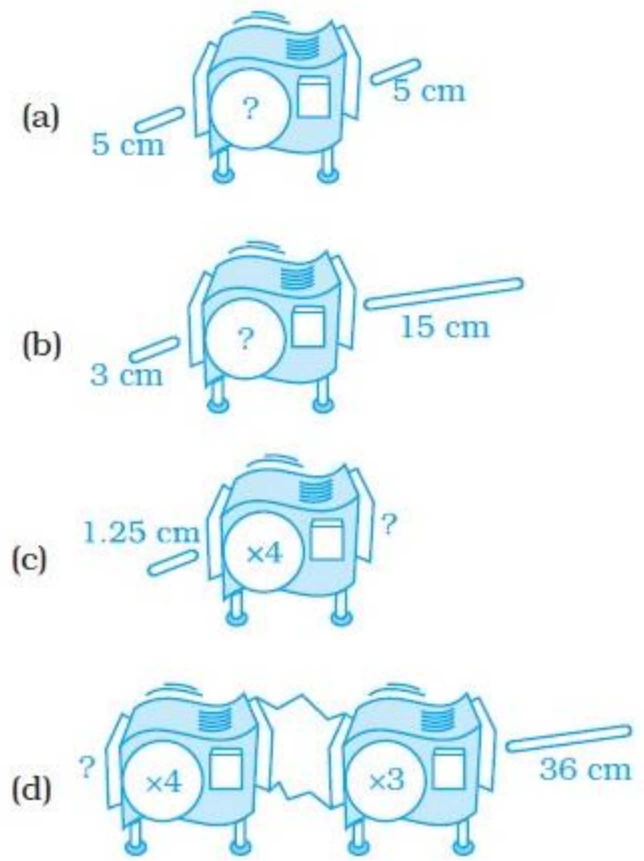
The work done by hook-up machine is equal to 2 x 2 x 2 x 3 x 3 x 3 = 216 = 6^3 So, she should use (x 6^3) single machine for the purpose.

162. Neha needs to stretch some sticks to 25^2 times their original lengths, but her (×25) machine is broken. Find a hook-up of two repeater machines that will do the same work as a (×25^2) machine. To get started, think about the hookup you could use to replace the (×25) machine.

Solution:

Work done by single machine (x 25^2) = 25 x 25 = 625 or 5 x 5 x 5 x 5 or 52 x 52
Hence, (x 5^2) and (x 5^2) hook-up machine can replace the (x 25) machine.

163. Supply the missing information for each diagram.



Solution:

(a) If 5 cm long piece is inserted in single machine, then it produce same 5 cm long piece.

So, it is $(\times 1)$ repeated machine.

Thus, ? = 1

(b) If 5 cm long piece is inserted in single machine, then it produce same 15 cm long piece.

So, it is $(\times 5)$ repeated machine.

Thus, ? = 5

(c) If 1.25 cm long piece is inserted in $(\times 4)$ repeated machine, then it produce same 1.25×4 =10 cm long piece.

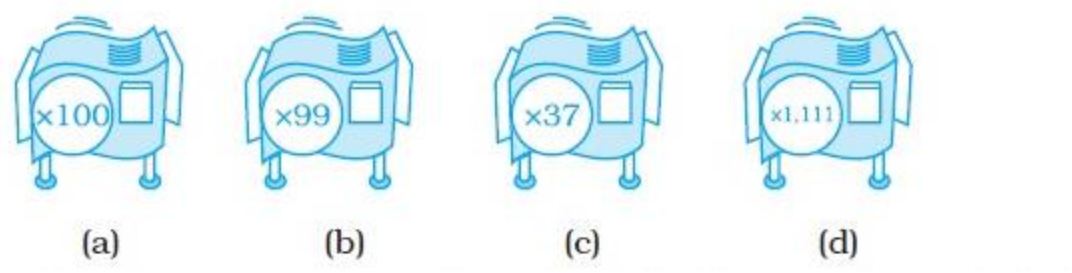
Thus, ? = 10cm

(d) If x cm long piece is inserted in $(\times 4)$ and $(\times 3)$ hooked machine, then it produce same 36 cm long piece.

So, $x \times 4 \times 3 = 36$

x=3cm

164. If possible, find a hook-up of prime base number machine that will do the same work as the given stretching machine. Do not use (× 1) machines.



Solution:

(a) Single machine work = 100

Hook-up machine of prime base number that do the same work down by x 100

$= 2^2 \text{ x } 5^2$

$=4\times25$

$= 100$

(b) x 99 = 3^2 x 111 hook-up machine.

(c) x 37 machine cannot do the same work.

(d) x 1111 = 101 x 11 hook-up machine.

165. Find two repeater machines that will do the same work as a (× 81) machine.

Solution:

Two repeater machines that do the same work as (x 81) are (x 3^4) and (x 9^2).

Since, factor of 81 are.3 and 9.

166, Find a repeater machine that will do the same work as a ($\times \frac{1}{8}$) machine.

Solution:

Machine, $\frac{1}{8}$ are $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$.

So, $\frac{1}{8} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3$

Hence, $\left(\times \frac{1}{2^3}\right)$ repeater machine can do the same work as a $\left(\times \frac{1}{8}\right)$ machine.

167. Find three machines that can be replaced with hook-ups of (× 5) machines.

Solution:

Since, $5^2 = 25$, $5^3 = 125$, $5^4 = 625$

Hence, (x 5^2), (x 5^3)and (x 5^4) machine can replace (x 5) hook-up machine.

168. The left column of the chart lists the lengths of input pieces of ribbon. Stretching machines are listed across the top. The other entries are the outputs for sending the input ribbon from that row through the machine from that column. Copy and complete the chart.

Input Length	**Machine**			
	× 2			
	1	**5**		
3				**15**
	14		**7**	

Solution:

In the given table, the left column of chart list is the length of input piece of ribbon. Thus, the outputs for sending the input ribbon are given in the following table.

Input length	**Machine**		
	×2	×10	×5
5	1	5	**2.5**
3	**6**	**30**	15
7	14	**70**	**35**

169. The left column of the chart lists the lengths of input chains of gold. Repeater machines are listed across the top. The other entries are the outputs you get when you send the input chain from that row through the repeater machine from that column. Copy and complete the chart.

Input Length	**Repeater Machine**		
	$\times 2^3$		
	40		**125**
2			
		162	

Solution:

In the given table, the left column of the chart lists is the length of input chains of gold. Thus, the output we get when we send the input chain from the row through the repeater machine are detailed in the following table.

Input length	×3	×12	×9
13.3	40	**160**	125
2	**6**	**24**	**18**
13.5	**141**	162	**121**

170. Long back in ancient times, a farmer saved the life of a king's daughter. The king decided to reward the farmer with whatever he wished. The farmer, who was a chess champion, made an unusal request:
"I would like you to place 1 rupee on the first square of my chessboard, 2 rupees on the second square, 4 on the third square, 8 on the fourth square, and so on, until you have covered all 64 squares. Each square should have twice as many rupees as the previous square." The king thought this to be too less and asked the farmer to think of some better reward, but the farmer didn't agree.
How much money has the farmer earned?
[Hint: The following table may help you. What is the first square on which the king will place at least Rs 10 lakh?]

Position of Square on chess board	Amount(in Rs)
1st square	1
2nd square	2
3rd square	4

Solution:
Given:
A 8×8 grid.
Now, find the sum of each row.
1^{st} row= $2^0+2^1+2^2+...+2^7=255$
2^{nd} row $=2^8+2^9+2^{10}+...+2^{15}$
$=2^8\left(2^0+2^1+2^2+...+2^7\right)$
$=2^8\times 255$
$=255\times 255$
$=65280$
3^{rd} row= $2^{16}\times 255=16711680$

171. The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Compare their diameters by division.

Solution:

Diameter of the sun = $1.4\times10^9 m$

Diameter of the Earth = 1.275610^7 m

For comparison, to change diameter in same power of 10 that is $1.2756\times10^7 = 0.012756\times10^9$

Therefore, if we divide of sun by diameter of earth, get:

$$\frac{1.4\times10^9}{0.012756\times10^9} = 110$$

Hence, diameter of sun is 110 times the diameter of Earth.

172. Mass of Mars is 6.42×10^{28} kg and mass of the Sun is 1.99×10^{30} kg. What is the total mass?

Solution:

Mass of Mars = 6.42×10^{29} kg

Mass of the Sun = 1.99×10^{30} kg

Total mass of Mars and Sun together = $6.42 \times 10^{29} + 1.99 \times 10^{30}$

$= 6.42 \times 10^{29} + 19.9 \times 10^{29} = 26.32 \times 10^{29}$ kg

173. The distance between the Sun and the Earth is 1.496×10^8 km and distance between the Earth and the Moon is 3.84×10^8 m. During solar eclipse the Moon comes in between the Earth and the Sun. What is distance between the Moon and the Sun at that particular time?

Solution:

The distance between the Sun and the Earth is 1.496 x 10s km

$= 1.496 \times 10^8 \times 10^3$ m $= 1496 \times 10^8$ m

The distance between the Earth and the Moon is 3.84×10^8 m.

The distance between the Moon and the Sun at particular time (solar eclipse) = $(1496 \times 10^8 - 3.84 \times 10^8)$m = 1492. 16 $\times 10^8$ m

174. A particular star is at a distance of about 8.1×10^{13} km from the Earth. Assuring that light travels at 3×10^8 m per second, find how long does light takes from that star to reach the Earth.

Solution:

The distance between star and Earth = $8.1\times10^{13} km = 8.1\times10^{13}\times10^3 m$

Since, light travel at $3\times10^8 m$ per second.

So, time taken by light to reach the Earth = $\frac{8.1\times10^{13}\times10^{3}}{3\times10^{8}}$

$=\frac{8.1\times10^{16}}{3\times10^{8}}$

$=\frac{8.1}{3}\times10^{8}$

$=2.7\times10^{8}s$

175. By what number should (–15)$^{-1}$ be divided so that the quotient may be equal to (–5)$^{-1}$?

Solution:

Let x be the number divide $(-15)^{-1}$ to get $(-5)^{-1}$ as a quotient.

So,

$(-15)^{-1}\div x=(-5)^{-1}$

$\frac{1}{-15}\times\frac{1}{x}=\frac{1}{-5}$

$\frac{1}{x}=\frac{1}{-5}\div\frac{1}{-15}$

$\frac{1}{x}=\frac{1}{-5}\times\frac{-15}{1}$

$\frac{1}{x}=3$

$x=\frac{1}{3}$

176. By what number should (–8)$^{-3}$ be multiplied so that that the product may be equal to (–6)$^{-3}$?

Solution:

Let x be the number multiplied with $(-8)^{-3}$ to get the product equal to $(-6)^{-3}$.

$$x \times (-8)^{-3} = (-6)^{-3}$$

$$x = \frac{(-6)^{-3}}{(-8)^{-3}}$$

$$x = \frac{(-8)^{3}}{(-6)^{3}}$$

$$x = \frac{512}{216}$$

$$x = \frac{64}{27}$$

177. Find x.

(1) $\left(\frac{-1}{7}\right)^{-5} \div \left(\frac{-1}{7}\right)^{-7} = (-7)^{x}$

(2) $\left(\frac{2}{5}\right)^{2x+6} \times \left(\frac{2}{5}\right)^{3} = \left(\frac{2}{5}\right)^{x+2}$

(3) $2^{x} + 2^{x} + 2^{x} = 192$

(4) $\left(\frac{-6}{7}\right)^{x-7} = 1$

(5) $2^{3x} = 8^{2x+1}$

(6) $5^{x} + 5^{x-1} = 750$

Solution:

(i) Consider the equation:

$$\left(-\frac{1}{7}\right)^{5} + \left(-\frac{1}{7}\right)^{-7} = (-7)^{x}$$

Using law of exponents: $a^{m} + a^{n} = (a)^{m-n}$

Then,

$$\left(-\frac{1}{7}\right)^{-5+7} = (-7)^{x}$$

$$\left(-\frac{1}{7}\right)^{2} = (-7)^{x}$$

$$(-7)^{-2} = (-7)^{x}$$

On comparing power of (-7), get:

x= -2

(ii) Consider the equation:

$$\left(\frac{2}{5}\right)^{2x+6} \times \left(\frac{2}{5}\right)^{3} = \left(\frac{2}{5}\right)^{x+2}$$

Using law of exponents: $a^{m} \times a^{n} = a^{m+n}$

Then,

$$\left(\frac{2}{5}\right)^{2x+6+3} = \left(\frac{2}{5}\right)^{x+2}$$

On comparing power of $\left(\frac{2}{5}\right)$, get:

2x+6+3=x+2
2x+9=x+2
x = -7

(iii) Consider the equation:

$$2^x + 2^x + 2^x = 192$$

Now, simplify it as follows:

$$2^x (1+1+1) = 192$$

$$2^x = \frac{192}{3}$$

$$2^x = 64$$

$$2^x = 2^6$$

On comparing power of (2), get:
x = 6

(iv) Consider the equation:

$$\left(-\frac{6}{7}\right)^{x-7} = 1$$

Using law of exponents, $x^0 = 1$
Then,

$$\left(-\frac{6}{7}\right)^{x-7} = 1$$

It is possible only if x= 7

So, $\left(-\frac{6}{7}\right)^{7-7} = 1$

$$\left(-\frac{6}{7}\right)^{0} = 1$$

$$1 = 1$$

Hence, x = 7

(v) Consider the equation:

$$2^{3x} = 8^{2x+1}$$

Simplify it as follows:

$$2^{3x} = 2^{3(2x+1)}$$

On comparing the power of 2, get:
3x = 3(2x+1)
x = 2x +1
x = -1

(vi) Consider the equation:

$5^x + 5^{x-1} = 750$

Now, simplify it as follows:

$$5^x + \frac{5^x}{5} = 750$$

$$5^x\left(1+\frac{1}{5}\right) = 750$$

$$5^x = 750\times\frac{5}{6}$$

$$5^x = 125\times 5$$

$$5^x = 625$$

$$5^x = 5^4$$

On comparing the power of 5, get:

x = 4

178. If a = – 1, b = 2, then find the value of the following:

(1) $a^b + b^a$ (2) $a^b – b^a$ (3) $a^b \times b^2$ (4) $a^b \div b^a$

Solution:

(i) Given:

$a^b + b^a$

If a = -1 and b = 2,

Then:

$$(-1)^2 + (2)^{-1} = 1+\frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}$$

(ii) Given:

$a^b – b^a$

If a = -1 and b = 2,

Then:

$$(-1)^2 - 2^{-1} = 1-\frac{1}{2^1} = \frac{2-1}{2} = \frac{1}{2}$$

(iii) Given:

$a^b \times b^2$

If a = -1 and b = 2,

Then:

$$(-1)^2 \times (2)^{-1} = 1\times\frac{1}{2} = \frac{1}{2}$$

(iv) Given:

$a^b \div b^a$

If a = -1 and b = 2,

Then:

$$(-1)^2 \div (2)^{-1} = 1 \div \frac{1}{2^1} = 1 \times 2 = 2$$

179. Express each of the following in exponential form:

(1) $-\frac{1296}{14641}$ (2) $\frac{-125}{343}$ (3) $\frac{400}{3969}$ (4) $\frac{-625}{10000}$

Solution:

(i) Given:

$$\frac{-1296}{14641}$$

Since, $6 \times 6 \times 6 \times 6 = 1296 = 6^4$ and $11 \times 11 \times 11 \times 11 = 14641 = 11^4$

Exponential form of $\frac{-1296}{14641} = -\frac{6^4}{11^3} = -\left(\frac{6}{11}\right)^4$

(ii) Given:

$$\frac{-125}{343}$$

Since, $-5 \times -5 \times -5 = -125 = (-5)^3$ and $7 \times 7 \times 7 = 343 = (-7)^3$

Exponential form of $\frac{-125}{343} = \frac{-5^3}{(7)^3} = \left(\frac{-5}{7}\right)^3$

(iii) Given:

$$\frac{400}{3969}$$

Since, $20 \times 20 = 400 = 20^2$ and $63 \times 63 = 3969 = 63^2$

Exponential form of $\frac{400}{3969} = \frac{20^2}{63^2} = \left(\frac{20}{63}\right)^2$

(iv) Given:

$$\frac{-625}{10000}$$

Since, $5 \times 5 \times 5 \times 5 = 625 = 5^4$ and $10 \times 10 \times 10 \times 10 = 1000 = 10^4$

Exponential form of $\frac{-625}{10000} = \frac{-5^4}{10^4} = -\left(\frac{1}{2}\right)^4$

180. Simplify:

(1) $\left(\frac{1}{2}\right)^2 - \left[\left(\frac{1}{4}\right)^3\right]^{-1} = \times\ 2^{-3}$

(2) $\left(\frac{4}{3}\right)^{-2} - \left[\left(\frac{3}{4}\right)^2\right]^{(-2)}$

(3) $\left(\frac{4}{13}\right)^4 \times \left(\frac{13}{7}\right)^2 \times \left(\frac{7}{4}\right)^3$

(4) $\left(\frac{1}{5}\right)^{45} \times \left(\frac{1}{5}\right)^{-60} - \left(\frac{1}{5}\right)^{28} \times \left(\frac{1}{5}\right)^{-48}$

(5) $\frac{9^3 \times 27 \times t^4}{3^{-2} \times 3^4 \times t^2}$

(6) $\frac{3^{-2^2} \times 5^{2^{-3}} \times t^{-3^2}}{3^{-2^5} \times 5^{3^{-2}} \times t^{-4^3}}$

Solution:

(i) Given:

$$\left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^3\right]^{-1} \times 2^{-3} = \left(\frac{1}{4} - \frac{1}{64}\right)^{-1} \times 2^{-3}$$

$$= \left(\frac{16-1}{64}\right)^{-1} \times 2^{-3}$$

$$= \left(\frac{15}{64}\right)^{-1} \times 2^{-3}$$

$$= \frac{64}{15} \times \frac{1}{8}$$

$$= \frac{8}{15}$$

(ii) $\left[\left(\frac{4}{3}\right)^{-2} - \left(\frac{3}{4}\right)^{-2}\right] = \left[\left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]^{-2} = 0^{-2} = 0$

(iii)

$$\left(\frac{4}{13}\right)^4 \times \left(\frac{13}{7}\right)^2 \times \left(\frac{7}{4}\right)^3 = \frac{4^4}{13^4} \times \frac{13^2}{7^2} \times \frac{7^3}{4^3}$$

$$= 4^4 \times 4^{-3} \times 13^2 \times 13^{-4} \times 7^3 \times 7^{-2}$$

$$= 4^{4-3} \times 13^{2-4} \times 7^{3-2}$$

$$= 4^1 \times 13^{-2} \times 7^1$$

$$= \frac{28}{169}$$

(iv) $\left(\frac{1}{5}\right)^{45} \times \left(\frac{1}{5}\right)^{-60} - \left(\frac{1}{5}\right)^{28} \times \left(\frac{1}{5}\right)^{-43} = \frac{1}{5^{45}} \times \frac{1}{5^{-60}} - \frac{1}{5^{28}} \times \frac{1}{5^{-43}}$

$$= \frac{1}{5^{45-60}} - \frac{1}{5^{28-43}}$$

$$= \frac{1}{5^{-15}} - \frac{1}{5^{-15}}$$

$$= 5^{15} - 5^{15}$$

$$= 0$$

(v)

$$\frac{9^3 \times 27 \times t^4}{3^{-2} \times 3^4 \times t^2} = \frac{\left(3^2\right)^3 \times 3^3 \times t^4}{3^{-2} \times 3^4 \times t^2}$$

$$= 3^6 \times 3^3 \times 3^2 \times 3^{-4} \times t^4 \times t^{-2}$$

$$= 3^{11-4} \times t^{4-2}$$

$$= 3^7 \times t^2$$

(vi)

$$\frac{\left(3^{-2}\right)^2 \times \left(5^2\right)^{-3} \times \left(t^{-3}\right)^2}{\left(3^{-2}\right)^5 \times \left(5^3\right)^{-2} \times \left(t^{-4}\right)^3} = \frac{3^{-4} \times 5^{-6} \times t^{-6}}{3^{-10} \times 5^{-6} \times t^{-12}}$$

$$= 3^{-4} \times 3^{10} \times 5^{-6} \times 5^6 \times t^{-6} \times t^{12}$$

$$= 3^{-4+10} \times 5^{6+6} \times t^{-6+12}$$

$$= 3^6 \times 5^0 \times t^6$$

$$= \left(3t\right)^6$$