46. The Nucleus

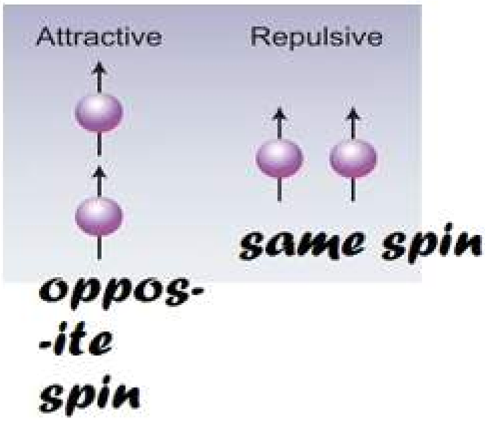
Short Answer

Answer.1

Neutrons exert attractive forces, but are neutral in nature. So there will be no charge inside the nucleus and hence electrons will not face any attraction. Thus the atom in overall will become unstable.

Answer.2

The answer can be yes as well as no. The fact is that if the neutrons have the same spin, then the force will be of same magnitude (repulsive) and if they have opposite spin, then force will be of different magnitude(attractive). Such forces are often referred to as tensor forces.

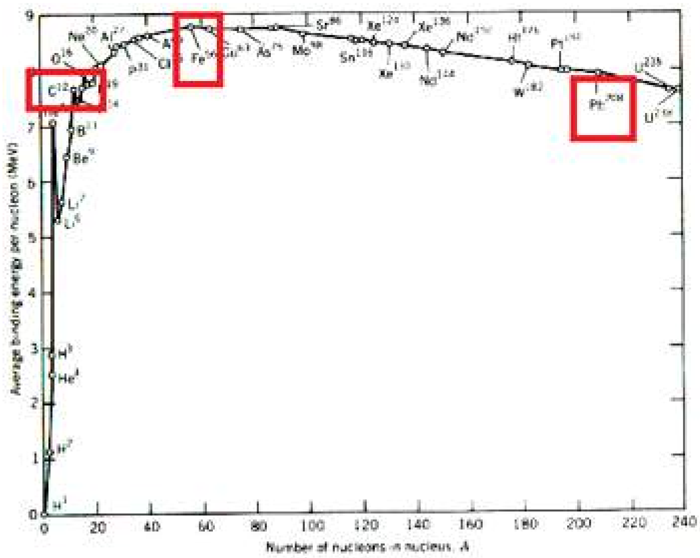


Answer.3

Generally, when we talk about nuclear forces, we restrict our measurement to femtometer(fm)which is a small unit of measurement. But when we discuss of the forces between molecules, we make our measurement in Å which is quite larger than fm. Thus we neglect the nuclear forces between the protons in a H-molecule.

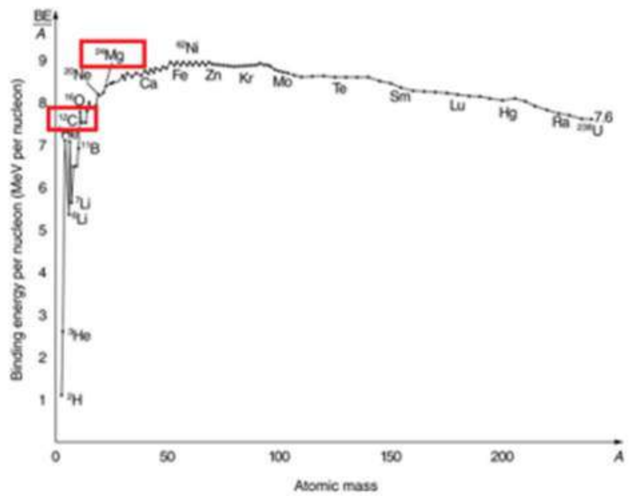
Answer.4

It will be easier to take out nucleon from carbon and lead as iron is most stable among them because it has the highest binding energy per nucleon and thus it is difficult to remove nucleon from the nucleus of iron.



Answer.5

More energy will be liberated in case of ^{24}Mg because it has more binding energy per nucleon as shown in the graph as compared to ^{12}C nuclei.



Answer.6

Cathode Rays	Beta Rays
1. They contain electron	1. They also consist of electron
2. They get created when high voltage is applied across the electrodes	2. They get created with nuclear fission of radioactive element
3. The electrons in cathode ray get generated through metals(as they conduct electricity)	3. The electrons in beta ray gets generated when neutron breaks to give electron and proton (during nuclear fission)

We will not be able to distinguish between them as they are not visible to human eye because both of the rays consists of electrons.

Answer.7

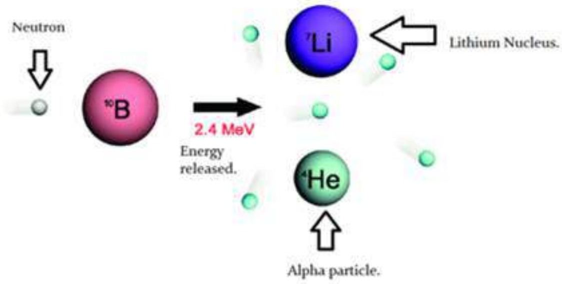
This appears to be because of mass defect which is commonly known as Einstein's mass defect theory whose proof is yet to be done.

Answer.8

The answer is yes. This is because in case of beta decay an electron comes out from the atom and thus the atom gets oppositely charged as it becomes an ion.

Answer.9

- When boron having atomic no.5 and atomic mass 10 is bombarded with neutron, the atomic no. decreases by 2 i.e.5-2=3 and the atomic mass decreases by 2 times of atomic no. i.e.4. Thus Lithium with atomic no.3 and mass number 6.



Answer.10

The answer is no. This is because in a gamma decay, neither the proton number nor the neutron number changes. Only the quantum numbers of nucleons changes.

Answer.11

-The lighter part will have more kinetic energy.

There can be 2 cases, if the nucleus is at rest, then the two parts will have same linear momentum. If the nucleus is not at rest, then the heavier one will have more linear momentum (as kinetic energy is inversely proportional to momentum).

Answer.12

-Since the initial separation between helium nuclei are large (in terms of Å), thus nuclear reaction cannot start on their own.

Objective I

Answer.1

This is because the unit which is widely used in describing mass in nuclear as well as atomic physics is unified atomic mass unit(amu).

This amu is equal to $1/12^{th}$ of the mass of a carbon-12 isotope. Thus for a carbon atom in the ground state, the mass will be equal to 12amu.

Option(b) is not appropriate as because the mass will not be less than 12u (as it is exactly equal to 12 u).

Option(c) is also not appropriate as the mass cannot be more than 12u (as it is exactly equal to 12 u).

Option(d) is not appropriate as the definition describes that the mass is equal to $1/12^{th}$ of carbon-12 isotope. So, it will not depend upon graphite or charcoal.

Answer.2

This is because in a nucleus, the mass number is the sum of the total numbers of protons and neutrons together.

Thus (c) will be the correct answer as number of nucleons refers to the total number of protons and neutrons.

Answer.3

^{12}C and ^{14}C are the isotopes of carbon. An isotope of an element has the same atomic number but the mass number is different.

The number 12 and 14 in the carbon atom refers to the mass number i.e. the total number of neutrons and protons.

Thus the mass number will increase when the number of neutrons increases. So, the answer is (c).

Option (a) is incorrect as increase in the number of electrons does not change the mass number.

Option (b) is also incorrect. If the number of protons increases but not the number of electrons, then the atomic number of the element will change.

Option (d) is not correct as increasing the number of electrons does not change the mass number.

Answer.4

The fact is that the atomic number is the number of protons and electrons in an atom and mass number is the number of neutrons and electrons, so it can be sometimes more or sometimes less than the atomic number. For example - Hydrogen atom has 1 proton and 1 electron and thus has atomic and mass number as equal.

Option(a) is not correct as it can't be less than the atomic number because atomic number is the number of protons and mass number is the sum of protons and neutrons.

Option(b) is not appropriate as it can't be always greater than its atomic number as because sometimes the number of the number of protons is less as compared to the electrons that is in case of anion.

Option (c) is not appropriate as mass number is not always equal to its atomic number.

Answer.5

As we know that the radius of a nucleus is

$$R = R_0 A^{1/3}$$

Applying log on both the sides, we have,

$$Ln(R) = ln(R_0) \times 1/3\ ln(A)$$

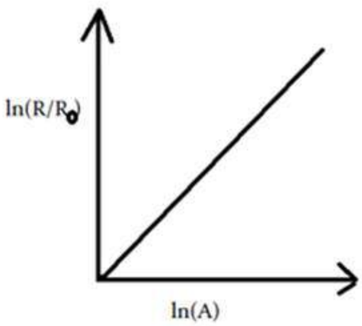
Or $\ln\left(\frac{R}{R_0}\right) = \frac{1}{3}\ln(A)$

Thus the above equation gives us an equation of a straight line

$$y = mx + c$$

Here, $y=\ln(R/R_0)$ and $x=\ln(A)$.

Since in the above equation, c=0.Therefore, the graph obtained will be a straight line passing through the origin.



Thus (a) is the correct answer.

(b) is not the correct answer as the equation of a parabola is

$y = 4ax^2$

(c) is not correct as the equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Answer.6

As the distance of separation is equal, so the forces that will act between the particles will be same as because

Mass of neutron=Mass of proton=$1.67 \times 10^{-27} kg$

Thus, the forces can be represented as,

Fpp=Fnn=Fpn= $\frac{kQq}{r}$

$$or \ \frac{k \times 1.67 \times 10^{-19} \times 1.67 \times 10^{-19}}{1fm}$$

Therefore, as the distance of separation is equal, thus they will have same magnitude of forces.

So, (b) is the correct answer.

Answer.7

Nuclear forces are much stronger as compared to gravitational and electromagnetic forces.

As neutrons does not have any charge, thus the force between 2 neutron and between a proton and a neutron will be zero as

$$Fpn = Fnn = \frac{(kqQ)}{r} = 0\ (as\ charge\ of\ neutron\ is\ zero)$$

Also, the force between 2 protons is columbic in nature. This coulomb force is weaker as compared to the nuclear forces between proton-proton and neutron-proton. Therefore, option (d) is the correct option.

Option (a) is not correct as the force between proton-proton can't be greater than those of neutron-neutron and proton-neutron.

Option (b) is not correct as the forces can't be same because coulomb force and nuclear force are not equal in magnitude.

Option (c) also can't be correct as because force between proton-neutron and neutron-neutron are all nuclear force.

Answer.8

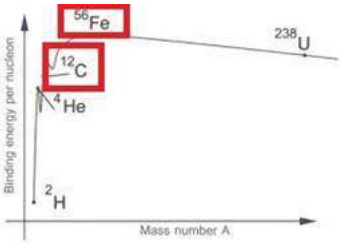
The correct answer will be (b).This is because electromagnetic force is $10^{36} stronger$.Whereas nuclear force is $10^{25} stronger$.

Option (a) is not correct as nuclear forces and electromagnetic forces can't be equal because electromagnetic force is much stronger than nuclear force.

Option (c) is not correct as nuclear force is not stronger than electromagnetic force.

Option (d) is also not correct as they differ by a large margin.

Answer.9



Clearly from graph iron(Fe)has more binding energy per nucleon than carbon

Thus the value of binding energy will increase as the mass number increases.

Answer.10

The correct answer is (d). All the other options are correct about binding energy per nucleon.

This is because inside the nucleus, the nucleons are in rest. So, there is no question of kinetic energy for those particles at rest.

D. all the nuclei decay.

Answer.11

The correct answer is (c). This is because, the formula for average life calculation is,

$$t_{av} = \frac{t\left(\frac{1}{2}\right)}{0.693}$$

So the decay will be more than half as dividing the half-life with 0.693 will give the result to be more than half of the active nuclei will decay.

Answer.12

Atomic number is the number of protons while atomic mass number is the sum of protons and neutrons. The particles emitted will not be proton, electron or neutron since there is no change in the atomic number and mass number. Hence, the emitted particle must be photon.

Answer.13

Negative beta decay is a process in which an unstable nucleus, formed with more number of neutrons than needed for stability, tries to go towards stability by converting a neutron to a proton. In this process, the unstable nucleus emits an electron and an antineutrino i.e. $n \rightarrow p + e + antineutrino$. Thus, we can say that a neutron in the nucleus decays, emitting an electron.

Answer.14

The intensity of radiation emitted by radioactive source also decreases with time. Half-life of the given material is 2 hours. Let the radiation starts at t=0. Initial intensity of radiation is 64 times the safe intensity level.

At t=2 hour, the intensity of radiation will be half (due to half-life) which is now 32 times the safe intensity level.

At t=4 hour, the intensity of radiation will be 16 times the safe level.

Doing this, t=6 h $\Longrightarrow$ 8 times; t=8 h $\Longrightarrow$ 4 times; t=10 h $\Longrightarrow$ 2 times and t=12 h $\Longrightarrow$ 1 times the safe intensity level. In other words, $2^6 = 64$ and thus the time will be 6*2 hour= 12 hour. Therefore, the minimum time is 12 hours after which the intensity of radiation is equal to the permissible safe level and we can work safely with the material.

Answer.15

The half-life of a radioactive material is given by

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

The average life of a radioactive material is given by

$$t_{av} = \frac{1}{\lambda}$$

Answer.16

An α-particle is ${}^4_2He$. ${}^4_2He$ is bombarded on ${}^{14}_7N$ and thus ${}^{17}_8O$ is formed with an emission of a particle given by following equation

$${}^{14}_7N + {}^4_2He \rightarrow {}^{17}_8O + {}^1_1x$$

The particle emitted is ${}^1_1x$ which is equal to a proton since $Z = 1$.

Answer.17

A neutron is converted to a proton with creation of an electron and an antineutrino during beta-decay. The rest mass of antineutrino is approximately zero (though unknown in physics till now) while the rest mass of an electron is $9.11 * 10^{-31}$kg. Thus the mass of a beta particle is much less when compared to the mass of Co atom. Hence even after 540 days in a container, the weight of the material will be approximately equal to 10g.

Answer.18

The separation between the alpha particle and the recoiling nucleus is measured as x in time t when uranium atom is kept in a stationary train. If decay takes place in a moving train, the distance between the alpha particle and the recoiling nucleus is now measured, will remain x in time t. This is because the decay process and the passenger are in same time frame.

OBJECTIVE.

Answer.1

The reaction in which the nuclear energy is obtained by breaking a heavy nucleus into two or more light nuclei is called as nuclear fission reaction. Highly fissionable material like ${}^{236}_{92}U$ are not found in nature. However, the natural Uranium contains 0.7% of ${}^{235}_{92}U$ which has high probability of absorbing a slow neutron and thus forming ${}^{236}_{92}U$. Thus, a heavy nucleus ${}^{235}_{92}U$ is bombarded by thermal neutrons which results in formation of ${}^{236}_{92}U$. This highly fissionable material now breaks up and nuclear fission reaction occurs.

Objective II

Answer.1

The average radius R of a nucleus is given by

$$R = R_0 A^{1/3} \cdots (1)$$

Where A is the mass number and $R_0 = 1.1 * 10^{-15}$m.

The Volume of the nucleus V is given by (Putting equation (1))

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi {R_0}^3 A \cdots (2)$$

Since, the number of protons and neutrons $(Z + N = A)$ are nearly same, say m, the Mass M is approximately equal to

$$M = mA \cdots (3)$$

Density is given by

$$\rho = \frac{M}{V} = \frac{3m}{4\pi {R_0}^3} \cdots (from\ equation\ (2) and\ (3))$$

The above equation of density says that it is independent of A. So as mass number A increases, density do not change.

Answer.2

Heavier nuclei having large mass number A has large nuclei radius$\left(R \propto A^{1/3}\right)$. Attractive nuclear force, which has small range, is not effective in heavier nuclei due to the large nuclei radius. Repulsive Coulomb forces between protons will be now effective since these forces have longer range. Repulsive force causes instability of nuclei.

Stability is achieved by having more neutrons than protons in the nuclei. More neutrons (not exerting electric repulsion) in the nuclei or larger N/Z ratio do make the range between the nucleon-pairs comparable to the nuclear forces. Hence, attractive nuclear forces dominate over repulsive coulomb forces and thus stability is achieved. Therefore, Option C and D both are correct.

A. neutron is a composite particle made of a proton and an electron whereas proton is a fundamental particle

B. neutron is an uncharged particle whereas proton is a charged particle

C. neutron has larger rest mass than the proton

D. weak forces can operate in a neutron but not in a proton.

Answer.3

When a free neutron decays to a proton, an electron and an antineutrino are created i.e. $n \rightarrow p + e + antineutrino$. The rest mass of neutron is larger than that of proton and thus the Q-value is positive. If we consider a free proton decays to a neutron, a positron and a neutrino are created i.e. $p \rightarrow n + e^{+} + neutrino$. The Q-value is thus negative which is impossible. Also, a lower mass particle cannot be decayed into a large mass particle. Hence, a free proton does not decay to a neutron.

Answer.4

Since an electron and an antineutrino are emitted from a pure beta-active material, these particles do not have same energy due to their different masses. The beta particles are ejected when a neutron is converted into a proton and thus we can't say that beta particles were already present in the nucleus. The mass of antineutrino is unknown. Hence, Option A, B and C are not correct. Nuclei with same mass number but different atomic number are called *isobars.* The beta decay process is ${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + e + antineurtino$. It is prevalent that, $A = Z + N$ (Before beta decay) while $A = (Z + 1) + (N - 1) = Z + N$ (after beta decay). Thus we say that the active nucleus changes to one of its isobars after the beta decay.

Answer.5

α-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He$ $\Rightarrow$ Element changes.

β^{+}-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-1}^{\ \ A}Y + e^{+} + neutrino$ $\Rightarrow$ Element changes.

β^{-}-decay: ${}_{Z}^{A}X \rightarrow {}_{Z+1}^{\ \ A}Y + e^{-} + antineutrino$ $\Rightarrow$ Element changes.

γ-decay: This decay is a radioactive process in which the excited nucleus comes down to its ground energy level by emitting photons. The element does not change in this process.

Answer.6

α-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He$ $\Rightarrow$ Atomic number decreases by 2.

β^{+}-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-1}^{\ \ A}Y + e^{+} + neutrino$ $\Rightarrow$ Atomic number decreases by 1.

β^{-}-decay: ${}_{Z}^{A}X \rightarrow {}_{Z+1}^{\ \ A}Y + e^{-} + antineutrino$ $\Rightarrow$ Atomic number increases by 1.

γ-decay: This decay is a radioactive process in which the excited nucleus comes down to its ground energy level by emitting photons. Atomic number does not change.

Answer.7

Magnetic field gets deflected when there is charge/current within its surroundings.

α-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He \Rightarrow {}_{2}^{4}He$ causes deflection in magnetic field.

β^{+}-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + e^{+} + neutrino \Rightarrow e^{+}$ causes deflection in magnetic field.

β^{-}-decay: ${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + e^{-} + antineutrino \Rightarrow e^{-}$ causes deflection in magnetic field.

γ-decay: This decay is a radioactive process in which the excited nucleus comes down to its ground energy level by emitting photons. Photons are not deflected by magnetic field.

Answer.8

Photons constitute electromagnetic waves. Since gamma rays are photons emitted during nuclear transitions, they are electromagnetic waves.

Answer.9

In a lithium vapor at room temperature, the distance between two lithium nuclei is larger when compared to the short-range attractive nuclear forces. Thus, repulsive coulomb forces will be effective and this does not allow the two nuclei to come very close to form a carbon nucleus. If we want to combine two lithium nuclei to form a carbon nucleus, we need a temperature of the order of 10^9 K.

For nuclei with A > 100,

A. the binding energy of the nucleus decreases on an average as A increases

B. the binding energy per nucleon decreases on an average as A increases

C. if the nucleus breaks into two roughly equal parts, energy is released

D. if two nuclei fuse to form a bigger nucleus, energy is released.

Answer.10

For A=50 to A=80, the binding energy per nucleon increases on an average. For A>80, the binding energy per nucleon decreases on an average. Therefore, Option B is correct. For heavy nuclei with A>100, the unstable nucleus can break into two roughly equal parts with release of energy to attain stability. Therefore, option C is correct. For heavy nuclei with A>100, it is impossible to combine two nuclei to form a bigger nucleus. Lighter nuclei with A<100 can be combined to form a bigger nucleus with release of energy. Therefore, option D is not correct for A>100.

Exercises

Answer.1

Given mass of nucleus(M) = Am_p

We know that,

$mp = 1.007276\, u$ (mass of proton)

where μ is the atomic mass unit ,$u = 1.6605402 \times 10^{-27}\, kg$

Radius of nucleus(R) $= R_0\, A^{\frac{1}{3}}$

$= 1.1 \times 10^{-15}\, A^{\frac{1}{3}}\, m$

Volume(V) $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(R_0 A^{\frac{1}{3}}\right)^3$

$= \frac{4}{3}\pi R_0^3 A$

Density(D) $= \frac{Mass(M)}{Volume(V)}$

$$= \frac{A \times 1.007276 \times 1.6605402 \times 10^{-27}}{\frac{4}{3} \times \pi \times R_0^3 \times A}$$

= 3.0000688×10^{17} kg/m^3

Specific gravity $= \frac{Density(in\,SI)}{1000}$

= 3.0000688×10^{14}.

Hence, the density of matter in kg m^{-3} inside a nucleus is 3.0000688×10^{17} kg/m^3 and its specific gravity is 3.0000688×10^{14}.

Answer.2

Given mass of star(M) = 4×10^{30} kg

We know,

Density of nuclear matter(D) = 2.3×10^{17} kg/m^3

$$Density \ = \frac{Mass}{Volume}$$

$$Volume \ = \frac{Mass}{Density}$$

$$= \frac{4 \times 10^{30}}{2.3 \times 10^{17}} m^3$$

$$= \frac{4}{2.3} \times 10^{13} m^3$$

Again, Volume $= \frac{4}{3}\pi R^3$ [As the star is assumed to be a sphere]

[R = Radius of the neutron star]

So equating the expressions of volume we can say,

$$\frac{4}{3}\pi R^3 \ = \frac{4}{2.3} \times 10^{13}$$

$$\Rightarrow R^3 \ = \frac{3 \times 10^{13}}{2.3 \times \pi}$$

$\Rightarrow R = \left(\frac{3\times10^{13}}{2.3\times\pi}\right)^{\frac{1}{3}}$

$\Rightarrow R = 16071.2964\, m$

$\Rightarrow R = 16.07\, km$

Hence radius of the star is 16.07 km

Answer.3

We know that an alpha particle consists of two protons and two neutrons.

Mass of proton = 1.007276 u

Mass of neutron = 1.008665 u

Where u = 1.6605402 × 10^{-27} kg (Atomic mass unit)

Let the mass of the alpha particle be M

∴ Mass defect is ΔM,

$$\Delta M = (no.of\ proton \times Mass\ of\ proton + no.of\ nuetron \times mass\ of\ neutron) - M$$

$$\Delta M = (2 \times 1.007276\text{ u} + 2 \times 1.008665\text{ u}) - M$$

Given, Binding energy = 28.2 MeV

Binding energy is also equal to ΔMc^2

So,

$$\Delta Mc^2 = 28.2$$

$\Delta M = \frac{28.2}{c^2}$ [Note $c^2 = 931.5\, MeV/u$]

$$\Delta M = 0.030273u$$

$$(2 \times 1.007276u + 2 \times 1.008665u) - M = 0.030273u$$

$$M = [(2 \times 1.007276 + 2 \times 1.008665) - 0.030273\,]u$$

$$M = 4.0016\, u$$

Hence, the mass of an alpha particle is 4.0016u

Answer.4

Given,

Atomic mass of ^{7}Li = 7.0160 u

Mass of proton = 1.007276 u

Atomic mass of ^{4}He = Mass of α particle = 4.0026 u

$\therefore Mass\ defect(\Delta M) = (mass\ of\ reactants) - (mass\ of\ products)$

$Mass\ defect(\Delta M) = [(7.0160 + 1.007276)u - 2 \times (4.0026\ u)]$

So, ΔM = 0.018076u

Binding Energy $= \Delta Mc^2$ [Where $c^2 = 931.5\ MeV/u$]

$= 0.018076 \times 931.5\ MeV$

$= 16.83\ MeV$

Hence, the binding energy for the reaction is 16.83 MeV

Answer.5

^A_ZX , here X = elementA = Mass Number[No. of protons + neutrons]

Z = Atomic Number [No. of protons]

Therefore, Number of neutrons(N)= A – Z

Now coming to the problem,

$^{197}_{79}Au$ means that A = 197 and Z = 79

N = A – Z = 197 – 79 = 118

Binding energy $= [(Z_{mp} + N_{mn}) - M]c^2$, m_p = mass of proton

m_n= mass of neutron

M = Atomic Mass

c^2 = (Speed of light)^2

= 931.5 MeV/u

Binding energy

$$= [(79 \times 1.007276\ +\ 118 \times 1.008665) - 196.96]\ \times\ 931.5\ MeV$$

$= \ 1525.12\ MeV$

Number of nucleons = No. of protons + neutrons = A = 197

Binding energy per nucleon $= \frac{total\ binding\ energy}{no\ of\ nucleons} = \frac{1525.12}{197} =\ 7.741$

Hence, the binding energy per nucleon = 7.741

Answer.6

a) $U^{238} \rightarrow Th^{234} + He_2^4$ [Reaction for emitting α-particle]

$_2He^4$ = alpha particle

$Energy\ released\ \ = [mass\ of\ reactant - mass\ of\ products] \times c^2$

$= [M_u - (M_\alpha\ +\ M_{Th})] \times c^2$,

M_u = mass of Uranium

M_α = mass of α-particle

M_{th} = mass of Thorium

c^2= (Speed of light)^2 = 931.5 MeV/u

$$= [238.0508 - (234.04363 + 4.00260)]u \times 931.5\frac{MeV}{u}$$

$E\ = 4.2569\ MeV$

The energy released is 4.2569 MeV

b) $U^{238} \rightarrow Th^{234} + 2p^1 + 2n^1$

$Energy\ released\ \ = [mass\ of\ reactant - mass\ of\ products] \times c^2$

$= [M_u - \{2 \times (M_p + M_n) + M_{Th}\}] \times c^2]$

M_p = mass of proton=$1.007276u$

M_n = mass of neutron=$1.008665u$

$$= [238.0508 - 2 \times (1.007276 + 1.008665 + 234.04363)]u \times 931.5 \frac{MeV}{u}$$

$$= 23.019\ MeV$$

The energy released is 23.019 MeV

Answer.7

$Ra^{223} \rightarrow Pb^{209} + C^{14}$

$Energy\ released\ = [mass\ of\ reactant - mass\ of\ products] \times c^2$

$Energy\ released\ = [M_{Ra} - (M_{Pb} + M_C)\ \}] \times c^2]$

M_{Pb} = mass of Pb^{209}=208.981 u

M_C = mass of C^{14} =14.003 u

M_{Ra} = mass of Ra^{223}=223.018 u

$$= [223.018 - (208.981 + 14.003)] \times 931.5\ MeV$$

$$= 31.671\ MeV$$

The energy released is 31.671 MeV

8. Question

Show that the minimum energy needed to separate a proton from a nucleus with Z protons and N neutrons is

$\Delta E = (M_{Z-1,\ N} + M_H - M_{Z,\ N})\ c^2$

Where $M_{Z, N}$ = mass of an atom with Z protons and N neutrons in the nucleus and M_H = mass of a hydrogen atom. This energy is known as proton-separation energy.

Answer.9

$X_{Z,N} \rightarrow X_{Z-1,N} + H_1$, X = element under consideration

[Note: Hydrogen has no neutrons in the nucleus]

Now from the above equation we can easily write the energy equation:

$$\Delta E = (mass\ of\ products - mass\ of\ reactans)c^2$$

Hence

$$\Delta E = (M_{Z-1,N} + M_H - M_{Z,N})c^2$$

Answer.10

$X_{Z,N} \rightarrow X_{Z,N-1} + n_0^1$

Energy released = Mass difference × c^2

c^2 = (speed of light)^2 = 931.5 MeV/u

$$\Delta E = (mass\ of\ products - mass\ of\ reactans)c^2$$

$$= (M_{Z,N-1} + M_N - M_{Z,N}) \times c^2$$

$$\therefore \Delta E = (M_{Z,N-1} + M_N - M_{Z,N}) \times c^2$$

Answer.11

$P^{32} \rightarrow S^{32} + \bar{v}_0^0 + \beta_1^0$

Sum of energy of antineutrino and β-particle is must be equal to the energy difference between the P^{32} and S^{32} nuclei

$P^{32} - S^{32} \rightarrow + \bar{v}_0^0 + \beta_1^0$

Energy $=$ $(mass\ of\ P^{32} - mass\ of\ S^{32}) \times c^2$

where c^2= (Speed of light)^2 = 931.5 MeV/u

$$= (31.974 \text{ u} - 31.972 \text{ u}) \times 931.5 \frac{\text{MeV}}{\text{u}}$$

$$= 0.002 \times 931.5\, MeV$$

$$= 1.863\, MeV$$

Hence, the of sum of energy of antineutrino and β-particle is 1.863 MeV

Answer.12

a) Given, half-life = 14 minutes = 840 seconds

We know, half life $= \frac{\ln 2}{\lambda}$, where λ = decay constant

$\lambda = \frac{\ln 2}{840}$ = 8.25×10^{-4} s^{-1}

b) $n_0^1 \rightarrow p_1^1 + \beta_{-1}^0$

[equation of neutron undergoing β decay]

$$Energy\ released = [mass\ of\ reactant - mass\ of\ products] \times c^2$$

Energy $= \left[M_n - \left(M_p + M_\beta\right)\right] \times c^2$

M_n = mass of Neutron

M_p = mass of proton

M_β = mass of β-particle

c^2= (Speed of light)^2 = 931.5 MeV/u

$$= [1.008665 - (1.007276 + 0.0005486)] \times 931.5\, MeV$$

$$= 0.78283\, MeV$$

= 782.83 KeV

Answer.13

a) As one alpha particle is produced so the mass number will decrease by 4 and the atomic number by 2,

Resultant reaction:

$$Ra^{226} \rightarrow \alpha_2^4 + Rn^{222}$$

b) As fluorine is produced so the atomic number increases by 1, which suggests β-decay and usually β decay occurs with a loss of an antineutrino

$$O_8^{19} \rightarrow F_9^{19} + \beta_{-1}^{0} + \overline{v}_0^0$$

c) A similar reaction as the previous one, atomic number decreases by one so β^+ emission occurs

$$Al_{13}^{25} \rightarrow Mg_{12}^{25} + \beta_1^0 + \overline{v}_0^0$$

Answer.14

$$Cu^{64} \rightarrow Ni^{61} + e^+ + v$$

a) Maximum energy of positron refers to zero K.E of the neutrino

So when the energy of positron is 0.150 MeV as energy is conserved,

Energy of neutrino $= Max\ K.E\ of\ positron - Present\ energy\ of\ positron$

$$= 0.650 - 0.150\ MeV$$

$$= 0.500\ MeV$$

b) For a photon,

$momentum = \frac{Energy}{c}$, where c = speed of light

Energy of neutrino in SI = 0.500 × 10^6 × 1.6 × 10^{-19} J

Momentum = $\frac{0.500 \times 10^6 \times 1.6 \times 10^{-19}}{3\times10^8}$ $kgms^{-1}$

= 2.67 × 10^{-22} $kgms^{-1}$

Answer.15

a) Equations:

1) $K_{19}^{40} \to Ca_{20}^{40} + e_{-1}^{0} + energy$ [β^- emission]

2) $K_{19}^{40} \to Ar_{18}^{40} + e_{+1}^{0} + energy$ [β^+ emission]

3) $K_{19}^{40} + e_{-1}^{0} \to Ar_{18}^{40} + energy$ [electron capture]

b)

$Q\ value\ = [Mass\ of\ reactants - Mass\ of\ products]c^2$,

where c^2 = 931.5 MeV/u

case 1:

$Q\ = [39.9640 - 39.9626] \times\ 931.5\ MeV$

$=\ 1.3041\ MeV$

case 2:

$Q\ = [39.9640 - (39.9624 + 2 \times 0.005486)] \times 931.5\ MeV$

$=\ 0.4683\ MeV$

Case 3:

$Q\ = [39.9640 - 39.9624] \times 931.5\ MeV$

$=\ 1.490\ MeV$

Note: For energy calculations we use the masses of the nuclei of the reactions but as we are given atomic masses we add and subtract appropriate number of electrons to get the above expressions. For more details, one should look through the derivation of Q value for the three cases.

Answer.16

Considering the data given in the question only 2 isotopes are stable

$Li_3^6 + n \rightarrow Li_3^7$ $[Stable]$

$Li_3^7 + n \rightarrow Li_3^8$ $[Unstable]$

So Li_3^8 ejects and electron to convert to more stable Be and which in turn ejects two α particles

$Li_3^8 \rightarrow Be_4^8 + e^- + energy$

$Be_4^8 \rightarrow 2He_2^4$

Answer.17

$C_6^{11} \rightarrow B_5^{11} + \beta_1^0$

The maximum energy for the positron will be equal to the energy due to the mass defect (ΔM)

ΔM = [11.0114-11.0093] ×931 MeV

= 1.955 MeV

Answer.17

Energy equation for the reaction $^{224}Ra^* \rightarrow {}^{224}Ra + \gamma$ will be

$(M_{Ra^*} - M_{Ra}) \times c^2 = 0.217$ Mev

$(M_{Ra^*} - 224.020196\ \mathrm{u}) \times 931 \frac{\mathrm{MeV}}{\mathrm{u}} = 0.217\ MeV$

$M_{Ra^*} - 224.020196\ \mathrm{u} = 0.00023333u$

$M_{Ra^*} = 224.02042u$

Now for the first equation:

$^{228}Th \rightarrow {}^{224}Ra^{*} + \alpha$

K.E of α particle $= (Mass\ of\ reactants - mass\ of\ products) \times\ c^2$

$= (M_{Th} -\ M_{Ra^*}\ - M_\alpha) \times c^2$

$=\ [228.028726 - 224.02042 - 4.00260] \times 931\ MeV$

$=\ 5.3038\ MeV$

Answer.18

Given: $^{12}N \rightarrow {}^{12}C^{*} + e^{+} + v$

$^{12}C^{*} \rightarrow {}^{12}C + \gamma$

Adding the two reactions:

$N^{12} \rightarrow C^{12} + e^{+} + v + \gamma(4.43MeV)$

Max K.E of β-particle =$\{[\text{mass of } {}^{12}N - \text{mass of } {}^{12}C] \times c^2\} - 4.43\text{Mev}$

$=\ [12.018613 - 12] \times 931 - 4.43$

$=\ 12.89\ MeV$

Note: for max K.E β-particle, K.E of ν is considered zero

Answer.19

a) Half-life $= \frac{\ln 2}{decay\ constant}$

$= \frac{ln\,2}{1.8 \times 10^{-4}}\ s$

$=\ 3850.81\ s$

$\approx\ 64\ min.$

b) Average-life $= \frac{1}{decay\ constant}$

$= \frac{1}{1.8 \times 10^{-4}}\ s$

$=\ 5555.56\ s$

$\approx 92\ min.$

c) We know,

$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$

where A= Activity of the substance

A_0 = Initial activity

N = number of half lives

According to the problem,

Present activity = (1-0.25) A_0

= $0.75A_0$

So,

$$\frac{0.75A_0}{A_0} = \left(\frac{1}{2}\right)^n$$

$$\ln 0.75 = n \times ln\left(\frac{1}{2}\right)$$

$$N = 0.415 \times 3850.81s$$

$$= 1598.23\ s$$

Answer.20

a) 198 g of Au contains $6.023{\times}10^{23}$ atoms (Avogadro's Number)

So 1µg of Au contains $\frac{6.023\times10^{23}}{198} \times 10^{-6}$ atoms

= 3.041×10^{15} atoms

Activity = λN , λ = decay constant = $\frac{\ln 2}{half\ life}$

$$= \frac{\ln 2 \times 3.041 \times 10^{15}}{2.7 \times 24 \times 3600}\ \frac{distintegrations}{second}$$

$$= \frac{\ln 2 \times 3.041 \times 10^{15}}{2.7 \times 24 \times 3600 \times 3.7 \times 10^{10}} Ci$$

$$= 0.244\ Ci$$

b) We know,

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$$

where A= Activity of the substance

A_0 = Initial activity

N = number of half lives

Here $n = \frac{7}{2.7}$

$= 2.592$

$A = 0.5^{2.592} \times Ao$

$= 0.5^{2.592} \times 0.244\ Ci$

$= 0.0404\ Ci$

Answer.21

Given,

Half-life = 8 days, A_0 = 20μCi

a) We know that,

$A = A_0 e^{-\lambda t}$

$$\lambda = \frac{\ln 2}{half\ life}$$

Where,

A= Activity of the substance

A_0 = Initial activity

t = time

λ = decay constant

So,

$$\lambda = \frac{ln\ 2}{8}$$

$\lambda = 0.086\ day^{-1}$

$A = A_0 e^{-\lambda t}$

$A = 20 \times 10^{-6} \times e^{-0.086 \times 4}$

$= 14.14\ \mu Ci$

b) Decay constant in per second $= \frac{0.086}{24 \times 3600}$ s^{-1}

$= 1.4 \times 10^{-6} s^{-1}$

Answer.22

Given,

The decay constant of ^{238}U is 4.9×10^{-18} s^{-1}

a) Average life $= \frac{1}{decay\ constant}$

$= 2.04 \times 10^{17}\ s$

$= 6.49 \times 10^{9}\ years$

b) Half life $= \frac{\ln 2}{decay\ constant}$

$= 1.414 \times 10^{17}\ s$

$= 4.48 \times 10^{9}\ years$

c) 9×10^{9} years = 2 half-lives(approximately)

We know

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$$

where A= Activity of the substance

A_0 = Initial activity

N = number of half lives

N = 2, in this case.

So it is evident that the sample activity will decrease by a factor of 4

Answer.23

Initial Activity = 500

Final activity = 200

a) We know that,

$A = A_0 e^{-\lambda t}$

$$\lambda = \frac{\ln 2}{half\ life}$$

Where,

A= Activity of the substance

A_0 = Initial activity

t = time=50 mins

λ = decay constant

so,

$200 = 500 \times e^{-50\times 60\times \lambda}$

$$e^{-50\times 60\times \lambda} = \frac{2}{5}$$

$$-3000\lambda = \ln\left(\frac{2}{5}\right)$$

$-3000\lambda = -0.9161$

$\lambda = 3.05 \times 10^{-4}\ s^{-1}$

b) Half life $= \frac{\ln 2}{3.05\times 10^{-4}}$

$= 2272.61\ s$

$= 38\ min$

Answer.24

We know that,

$$A = A_0 e^{-\lambda t}$$

$$\lambda = \frac{\ln 2}{half\ life}$$

Where,

A= Activity of the substance=$1 \times 10^6\ s^{-1}$

A_0 = Initial activity=$4 \times 10^6\ s^{-1}$

t = time=20h

λ = decay constant

So according to the problem,

$$1 \times 10^6 = 4 \times 10^6 \times e^{-\lambda \times 20h}$$

$$e^{-20\lambda} = 0.25$$

Now we have to evaluate activity after 100 hours

$$A = 4 \times 10^6 \times\ e^{-100\lambda}$$

$$A = 4 \times 10^6 \times \left(e^{-20\lambda}\right)^5$$

$$A = 4 \times 10^6 \times (0.25)^5$$

A = 3.9×10^3 disintegrations per second

Hence, the activity after 100 hours is 3.9×10^3 disintegrations per second.

Answer.25

Molecular weight of $RaCl_2 = 1(\ 226) + 2(35.5)\ = 297$

So, 297 g of $RaCl_2$ contains 6.023×10^{23} atoms

0.1 g contains $= \frac{6.023\times10^{23}\times0.1}{297}$

$= 2.027 \times 1020\ atoms(A)$

Now for decay constant(λ) $= \frac{\ln 2}{half\ life} = \frac{ln2}{1602\ years} = \frac{ln2}{5.052\times10^{10}}$

$= 1.37 \times 10^{-11}\ s^{-1}$

Activity = λA

where

A= no. of atoms

λ = decay constant

$activity = 1.37 \times 10^{-11} \times 2.027 \times 10^{20}$

$= 2.8 \times 10^{9} \frac{disintegrations}{s}$

Answer.26

We know that,

$A = A_0 e^{-\lambda t}$

$\lambda = \frac{\ln 2}{half\ life}$

Where,

A= Activity of the substance

A_0 = Initial activity

t = time

λ = decay constant

Given,

Half-life = 10h, A_0=1 Ci

Activity after 9 hours $= e^{\left(-\frac{ln2}{10}\right)\times 9}$

= 0.536 Ci

Number of atoms left after 9 hours $= \frac{Activity\ in\ 9\ hrs}{decay\ constant}$

$$= \frac{0.536 \times 3.7 \times 10^{10} \times 3600}{\frac{\ln 2}{10}}$$

$$= 1.03 \times 10^{15}\ atoms$$

Activity after 10 hours = 0.5 Ci [As 10h is half-life]

Number of atoms left after 10 hours $= \frac{Activity\ in\ 10\ hrs}{decay\ constant}$

$$= \frac{0.5 \times 10 \times 3.7 \times 10^{10} \times 3600}{\ln 2}$$

$$= 9.60 \times 10^{14}\ atoms$$

Hence, net disintegrations at the 10^{th} hour = $(1.03\times10^{15}-9.60\times10^{14})$

= 6.92×10^{13} atoms

Answer.27

Given, $t_{1/2}$ = 14.3 days

⇒The disintegration rate, $\lambda = \frac{ln2}{14.3}s^{-1}$

t = 1 month = 30 days

$A_{\circ}$ = 800 disintegrations/sec

According to Law of Radioactivity, rate of disintegration of a radioactive isotope at time t will decay exponentially with rate of disintegration initially.

So, $A = A_{\circ}e^{-\lambda t}$

$$= 800e^{-\frac{0.693\times30}{14.3}}$$

$= 187$disintegrations/sec

Hence, selling rate will be 187 rupees.

Answer.28

Given, ^{57}Co decays by β^+ and γ- emission. Rate of emission of gamma rays is 5.0×10^9.

As the emission rate reduce to half of the given value. So the amount of emission should reduce to half of the original amount. -

∵ Time elapsed for drop of emission rate to half is half life of the β -emission.

Answer.29

Given, ^6C decays to ^5B.

(a) $^{11}_6C \rightarrow {}^{11}_5B + \beta^+$

So, it is β^+ decay.

(b) $t_{1/2}$ = 20.3 min.

$\lambda = \frac{ln2}{20.3}$ min^{-1}

= $0.9C_\circ$, $C_f = 0.1C_\circ$

$$C_f = C_i\, e^{-\frac{0.693 \times t}{20.3}}$$

So, $0.1C_\circ = 0.9C_\circ e^{-\frac{0.693 \times t}{20.3}}$

C_i

$$e^{-\frac{0.693 \times t}{20.3}} = \frac{0.1}{0.9}$$

Taking $\log_e$ both sides, we get

⇒

$\Rightarrow -\frac{0.693 \times t}{20.3} = \ln\left(\frac{0.1}{0.9}\right)$

$\therefore -\frac{0.693 \times t}{20.3} = \ln\left(\frac{1}{9}\right)$

So, $t = \frac{20.3 \times 2.197}{0.693}$

$t = 64.36\, min.$

Answer.30

Give $\quad_{1/2} = 12.3$ y

n, t

(a) Activity $= \frac{dN}{dt} = \lambda N = \frac{ln2}{12.3} \times 4 \times 1023\, \frac{dis}{y}$

$= 2.25 \times 1022\, dis/y$

(b) $\frac{dN}{dt} = 2.25 \times 10^{22} \frac{dis}{y}$

$= 2.57 \times 10\text{^}18\, \frac{dis}{h}$

$\therefore N = 2.57 \times 10^{18} \times 10$

= 2.57×10^{19} atoms will decay in next 10 hours.

(c) No. of atoms initially, $N_{\circ} = 4 \times 10^{23}$

$N = N_{\circ} e^{\frac{-0.693 \times 6.15}{12.3}}$

$= 2.82 \times 1023$ atoms remained

So, No. of atoms decayed $= (4 - 2.82) \times 10^{23}$

$= 1.18 \times 10^{23}$

Answer.31

Given, Count received at 1m from source = 50000 counts/cm^{2}sec

As it is a point source, total nuclei radiated from the source

= Counts received per unit area × total area

$= 4\pi r^2 \times 50000 \dfrac{counts}{sec}$

So, $\frac{dN}{dt} = 4 \times 3.14 \times 1^2 \times 10^4 \times 50000$

$= 6.28 \times 10^9 \; counts/sec$

No. of active nuclei $= 6 \times 10^{16} \; counts$

Now, $\frac{dN}{dt} = N\lambda$

$6.28 \times 10^9 = 6 \times 1016 \times \lambda$

$\therefore \lambda = 1.0467 \times 10^{-7} s^{-1}$

Answer.32

Half-life of any decay means the time taken to reduce the no. of atoms to half of the initial value. So, even for chain of the processes, half-life will be a unique value for a particular decay.

No. of atoms of ^{238}U$= \frac{6\times10^{23}\times2\times10^{-3}}{238} = 0.0504 \times 10^{20}$

No. of atoms of ^{206}Pb $= \frac{6\times10^{23}\times0.6\times10^{-3}}{206} = 0.017 \times 10^{20}$

Total no. of ^{238}U atoms initially $= (0.0504 + 0.017) \times 10^{20}$

$= 0.67 \times 10^{20}$

$$\lambda = \frac{ln2}{4.47 \times 10^9} = 0.1555 \times 10^{-9}$$

Now, $N = N_\circ e^{-\lambda t}$

$\Rightarrow 0.05 \times 10^{20} = 0.067 \times 10^{20} e^{-0.155 \times 10^{-9} t}$

Taking $\log_e$ on both sides, we get

$$ln\left(\frac{0.05}{0.067}\right) = -0.1555 \times 10^{-9} t$$

$\therefore t = 1.92 \times 10^9 y$

Answer.33

Given, activity $A = 12.3 \frac{dis}{sec}$

Initial activity $A_\circ = 15.3 \frac{dis}{sec}$

Half-life of ^{14}C $t_{\frac{1}{2}} = 5730\ y$

Disintegration rate $\lambda = \frac{ln2}{5730}\ y^{-1}$

According to Law of Radioactivity,

No. of radioactive sample at time t,$N = N_\circ e^{-\lambda t}$

$\therefore 12.3 = 15.3 e^{-\frac{ln2}{5730} t}$

So, $\frac{12.3}{15.3} = e^{\frac{-ln2\, t}{5730}}$

Taking ln on both sides, we get

$\Rightarrow \ln\left(\frac{12.3}{15.3}\right) = \frac{-ln2\, t}{5730}$

So, $t = \frac{5730 \times 0.218}{ln2}$

$\Rightarrow t = 1804.2\ y$

Answer.34

Let initial activity of the bottle = A_o

Activity of the bottle found on the mountain, $A = A_o e^{-\lambda t}$

Also, $A = A_o e^{-\frac{ln2}{12.5} \times 8} \times 0.015$

Comparing both we get,

$$A_o e^{-\frac{ln2}{12.5} \times 8} \times 0.015 = A_o e^{-\lambda t}$$

$$-\frac{ln2 \times 8}{12.5} + ln(0.015) = -\frac{ln2}{12.5}t$$

$$(t-8)\frac{ln2}{12.5} + ln(0.015) = 0$$

$\Rightarrow t = 83.74\ y$

Answer.35

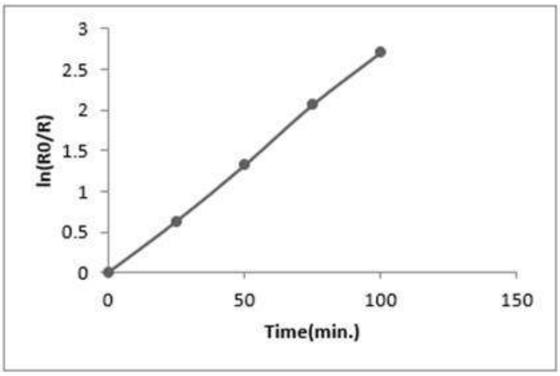
We take R_o at time $t=t_o$ i.e. $R_o = 30\times 10$

(a) $ln(\frac{R_0}{R_1}) = ln(\frac{30\times10^9}{30\times10^9}) = 0$

$$ln(\frac{R_0}{R_2}) = ln(\frac{30 \times 10^9}{16 \times 10^9}) = 0.63$$

$$ln(\frac{R_0}{R_3}) = ln(\frac{30 \times 10^9}{8 \times 10^9}) = 1.32$$

$$ln(\frac{R_0}{R_4}) = ln(\frac{30 \times 10^9}{3.8 \times 10^9}) = 2.067$$



$$ln(\frac{R_0}{R_5}) = ln(\frac{30 \times 10^9}{2 \times 10^9}) = 2.71$$

(b) As $\ln(R_0/R) = \lambda t$

So, slope of this curve will give the value of λ

$\therefore \lambda = 0.027 \text{ min}^{-1}$

(c) $t_{\frac{1}{2}} = \frac{ln2}{\lambda} = 25.67 min.$

Answer.36

Given, $t_{1/2} = 1.3\times 10^9$ y

Activity, A= 160 counts s^{-1}

$$= 160 \times 3600 \times 24 \times 365 \frac{counts}{y}$$

As $A = \lambda N$

$$N = \frac{160 \times 3600 \times 24 \times 365 \times 1.3 \times 10^9}{ln2} counts = 9.46 \times 10^{18}$$

As 6.023×10^{23} atoms are present in 40g

$$\Rightarrow 1atom = \frac{40}{6.023 \times 10^{23}}$$

$$9.46 \times 10^{18} \, atoms = \frac{40 \times 9.46 \times 10^{18}}{6.023 \times 10^{23}} = 0.00063$$

∴ The relative abundance of ^{40}K in natural potassium= (2×0.00063×100) % = 0.12%

Answer.37

$^{197}{}_{80}Hg \rightarrow {}^{197}{}_{79}Au$

(a) A proton is converted to a neutron; a neutrino is emitted.

(b) As given: By Mosley's law, $\sqrt{v} = \alpha(Z - b)$

$$\sqrt{\frac{c}{\lambda}} = 4.95 \times 10^7 \times (79 - 1) = 4.95 \times 10^7 \times 78$$

$$\frac{3 \times 10^8}{\lambda} = (4.95 \times 10^7 \times 78)2$$

$$\therefore \lambda = 20pm$$

Answer.38

Given, rate of radioactive isotope production= R

Rate of decay= λN (∵ According to law of Radioacivity)

As activity decreases exponentially and after a time t >> $t_{1/2}$, the number of active nuclei will become constant and rate of decay will also become constant.

So, $(dN/dt)_{produce}=(dN/dt)_{decay}=R$

R= λN

$\Rightarrow N=\frac{R}{\lambda}=\frac{Rt1/2}{ln2}$

Answer.39

Let $N_\circ$ is the radioactive isotope present at time t=0

N be the radioactive isotope present at time t

And λ be the disintegration constant.

By Law of Radioactivity,

$N = N_\circ e^{-\lambda t}$

No. of particles decay = $N_\circ$-N= $N_\circ$ (1- $e^{-\lambda t}$)

As the production starts at t=0.

So, $\left(\frac{dN}{dt}\right)_{produce} = \left(\frac{dN}{dt}\right)_{decay} = R$

Activity $A_\circ = \lambda N_\circ = R$

$\Rightarrow N_\circ = \frac{R}{\lambda}$

$$N = N_\circ(1 - e^{-\lambda t}) = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

Answer.40

n= 1mole = 6×10^{23} atoms=$N_\circ$

Given, $\lambda = \frac{ln2}{t^1/_2} = \frac{ln2}{14.3} per\ day = \frac{ln2}{(14.3 \times 24)} per\ hour$

According to Law of Radioactivity,

No. of radioactive sample at time t, $N = N_{\circ}e^{-\lambda t} = 6.023 \times 10^{23} e^{\frac{-ln2 \times 70}{14.3 \times 24}}$

N= 5.2×10^{23} atoms

According to law of Radioactivity, activity of a radioactive isotope at time t is the no. of active nuclei at that time times disintegration constant of the decay process.

As $\frac{dN}{dt} = \lambda N$

$= 2.9 \times 10^{17} \frac{dis}{s}$

Activity transmitted $= \frac{1 \times 3.7 \times 10^4}{2.9 \times 10^{17}} \times 100\%$

$= 1.27 \times 10^{-11}\%$

Answer.41

Given: Pressure P= 500000 Pa= 5 atm

Volume V= 0.125L

Temperature T= 300K

Assuming the gas to be ideal, according to ideal gas equation,

$PV = nRT$ (R be universal gas constant equal to 0.082atmLmol^{-1}K^{-1}, P is the pressure, V is the volume, T is the temperature)

So, $n = \frac{PV}{RT} = \frac{5 \times 0.125}{0.082 \times 300} = 0.0254$

$N = n \times 6 \times 10^{23} = 1.5 \times 10^{22}\ atoms$

$\lambda = \lambda = \frac{ln2}{t^1/_2} = \frac{ln2}{12.3} y^{-1}$

Activity, $A = \lambda N$

$= \frac{ln2}{12.3} \times 1.5 \times 10^{22} \frac{dis}{y}$

$= 8.4 \times 10^{20} \dfrac{dis}{y}$

Answer.42

(a) $^{212}_{83}Bi \rightarrow {}^{208}_{81}Ti + \alpha$

$^{212}_{83}Bi \rightarrow {}^{212}_{84}Bi + \beta^-$

(b) $t_{1/2} = 1h$

After 1h, $^{212}_{83}Bi$ will be half decayed

So, $^{212}_{83}Bi$ is 0.5g present

$^{208}_{81}Ti$ and $^{212}_{84}Bi$ will be formed in the ratio 7/13 and total mass of the sample should be 1g. So, total mass of $^{208}_{81}Ti$ and $^{212}_{84}Bi$ is 0.5g.

⇒ Mass of $^{208}_{81}Ti$ will be ratio of mass present × total mass= $\frac{7\times0.5}{7+13} = 0.175g$

And mass of $^{212}_{84}Bi$ will be of mass present × total mass= $\frac{13\times0.5}{7+13} = 0.325g$

^{110}Ag. The activity A is measured as a function of time and the following data are obtained.

Time (s)	Activity (A) (10^8 disintegrations s^{-1})	Time (s)	Activity (A) (10^8 disintegrations s^{-1})
20	11.799	200	3.0828
40	9.1680	300	1.8899
60	7.4492	400	1.1671
80	6.2684	500	0.7212
100	5.4115		

(a) Plot $\ln(A/A_0)$ versus time.

(b) See that for large values of time, the plot is nearly linear. Deduce the half-life of ^{108}Ag from this portion of the plot.

(c) Use the half-life of ^{108}Ag to calculate the activity corresponding to ^{110}Ag in the first 50 s.

(d) Plot $\ln(A/A_0)$ versus time for ^{110}Ag for the first 50 s.

(e) Find the half-life of ^{108}Ag.

Answer.43

At time t=0, $A_\circ = 8.0 \times 10^8$ dis/s

(a) $\ln\left(\frac{A1}{A0}\right) = \ln\left(\frac{11.799}{8}\right) = 0.389) = 0.389$

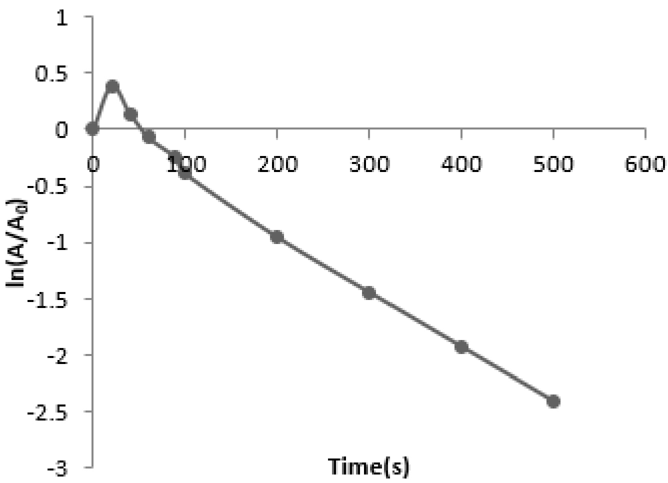
$$\ln\left(\frac{A2}{A0}\right) = \ln\left(\frac{9.168}{8}\right) = 0.136$$

$$\ln\left(\frac{A3}{A0}\right) = \ln\left(\frac{7.4492}{8}\right) = -0.0713$$

$$\ln\left(\frac{A4}{A0}\right) = \ln\left(\frac{6.2684}{8}\right) = -0.244$$

$$\ln\left(\frac{A5}{A0}\right) = \ln\left(\frac{5.4115}{8}\right) = -0.391$$

$$\ln\left(\frac{A6}{A0}\right) = \ln\left(\frac{3.0828}{8}\right) = -0.954$$



$$\ln\left(\frac{A7}{A0}\right) = \ln\left(\frac{1.8899}{8}\right) = -1.443$$

$$\ln\left(\frac{A8}{A0}\right) = \ln\left(\frac{1.1671}{8}\right) = -1.925$$

$$\ln\left(\frac{A9}{A0}\right) = \ln\left(\frac{0.7212}{8}\right) = -2.406$$

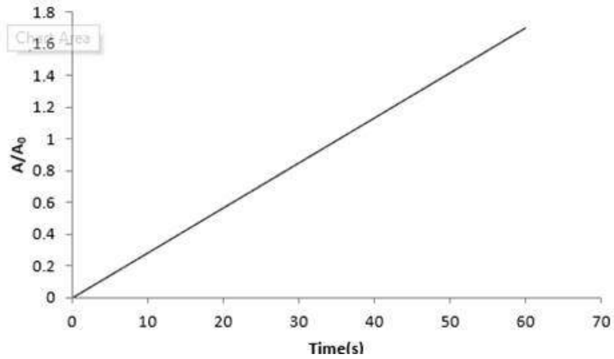
(b) For large values of time, the value of λ will be the slope negative of the slope of the curve.

∴ $\lambda = 0.028 s^{-1}$

So, $t_{1/2}$= 24.4 s

(c) $A = A_o e^{-\lambda t}$

(d) $8.0 \times 10^8 e^{-0.028\times 50} = 1.93 \times 10^8 dis/s$



(e) The half-life of ^{108}Ag from the graph is 144s.

Answer.44

Given, t_1=24h and t_2=6h.

As both reactions are occurring parallel,

So, $\frac{1}{(t^1/2)net} = \frac{1}{t1} + \frac{1}{t2}$

$$(t\,^1/_2)_{net}\,\frac{24 \times 6}{24 + 6} = 4.8h$$

$A_\circ$= 6 μCi and A=3 μCi

$\therefore ln2 = \lambda t$

$$ln2 = \frac{ln2}{4.8}\,t$$

$$t = 4.8h$$

Answer.45

Let charge at time t be Q and initial charge be q. Q is given by Q= $q_{e^{\frac{-t}{CR}}}$

And according to Law of Radioactivity,

$A = A_{\circ}e^{-\lambda t}$

So, $\frac{Energy}{Activity} = \frac{q^2 e^{\frac{-2t}{CR}}}{2C \times A_{\circ} e^{-\lambda t}}$

(where C is the capacitance, R is the resistance, t is the time, q is the charge, λ is the decay constant)

As the ratio should be independent of time,

$\therefore \frac{2}{CR} = \lambda$

$R = \frac{2\tau}{C}$

Answer.46

Given, Resistance R= 100Ω

Inductance L= 100mH

Let initial current be $i_{\circ}$

After time t,

Current at time t is given by $i = i_{\circ}(1 - e^{-\frac{tR}{L}})$

$N = N_{\circ}e^{-\lambda t}$

(where R is the resistance, L is the inductance)

So, $\frac{i}{N} = \frac{i_{\circ}(1 - e^{-\frac{tR}{L}})}{N_{\circ}e^{-\lambda t}}$

As the ratio should be independent of time,

$\therefore \frac{R}{L} = \lambda$

$10^3 = \frac{ln2}{t_{\frac{1}{2}}}$

$t_{1/2} = 6.93 \times 10^{-4}\ s$

Answer.47

Given, 1g of sample contains 0.007g ^{235}U

And 235g of ^{235}U contains 6.023× 10^{23} atoms

0.007g contains $\frac{6.023\times10^{23}}{235}$× 0.007 atoms

1 atom releases 200MeV energy

So, total energy= Energy released per atom× total no. of atoms

$$= \frac{6.023 \times 10^{23} \times 0.007 \times 200 \times 10^{6} \times 1.6 \times 10^{-19}}{235} J = 5.74 \times 10^{-8} J$$

Answer.48

Let n atoms consume per sec.

Total energy released per sec= No. of atoms consumed per sec × Energy released per atom

$= (n \times 200 \times 10^{6} \times 1.6 \times 10^{-19})$J

$300 \times 10^{3} = 3.2 \times n \times 10^{-11}$

n= 9.375× 10^{15}

235g of ^{235}U contains 6.023× 10^{23} atoms

9.375× 10^{15} atoms are present in $\frac{235\times9.375\times 10^{15}}{6.023\times10^{23}}$= 3.65× 10^{-6}g

Answer.49

(a) Energy radiated per fission$= 2 \times 10^{8} \times 1.6 \times 10^{(-19)} V$

$= 3.2 \times 10^{(-11)} \frac{J}{s}$

Usable energy or efficient energy per fission$= 3.2 \times 10^{-11} \times \frac{25}{100} = 8 \times 10^{-12} \frac{J}{s}$

Total energy needed$= 3 \times 10^{8} \frac{J}{s}$

No. of fission per sec $= \frac{total\ energy}{efficient\ energy\ per\ fission} = \frac{3\times10^{8}}{8\times10^{-12}} = 0.375 \times 10^{20}$

No. of fissions per day$= 0.375 \times 10^{20} \times 3600 \times 24 = 324 \times 10^{22}$

(b) No. of atoms disintegrated per day$= 324 \times 10^{22}$

235g of ^{235}U contains 6.023× 10^{23} atoms

324×10^{22} contains $\frac{235\times324\times10^{22}}{6.023\times10^{23}} = 1.264 \frac{kg}{day}$

(c) ^{235}U needed= 1.264kg/day that is 3% of uranium sample.

So, uranium needed per day$= \frac{126.4}{3} \frac{kg}{day}$

Uranium needed per month$= 1264\ kg$

Answer.50

(a) ${}_1^2H + {}_1^2H \rightarrow {}_1^3H + {}_1^1H$

Q value= $[2m({}_1^2H) - (m({}_1^3H) + m({}_1^1H)]u$

$= [2 \times 2.014102 - (3.016049 + 1.007835)] \times 931MeV$

$= 4.05MeV$

(b) ${}_1^2H + {}_1^2H \rightarrow {}_2^3He + n$

Q value= $[2m({}_1^2H) - (m({}_2^3He) + m(n]u$

$= [2 \times 2.014102 - (3.016029 + 1.008665)] \times 931MeV$

$= 3.25MeV$

(c) ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$

Q value= $[m({}_1^2H) + m({}_1^3H) - (m({}_2^4He) + m(n)]u$

$= [2.014102 + 3.016049 - (4.002603 + 1.008665)] \times 931MeV$

$= 17.57MeV$

Answer.51

According to electrostatic potential energy,

$PE = \frac{kz \times z}{r}$ (where k is 9×10^9 N m^2 C^{-2}, r is the distance, Z is the charge)

$$= \frac{9 \times 10^9 \times (2 \times 1.6 \times 10^{-19}) \times (2 \times 1.6 \times 10^{-19})}{2 \times 10^{-15}}$$

$= 4.6 \times 10^{-13} J$

$1.5KT = 1.5 \times 1.38 \times 10 - 23 \times T = 4.6 \times 10^{-13}$

$\Rightarrow T = 2.23 \times 10^{10} K$

Answer.52

$^{4}He + {}^{4}He = {}^{8}Be$.

Q value= $[2m({}^{4}He) - m({}^{8}Be)]u$

$= [2 \times 4.0026 - 8.0053] \times 931 MeV$

$= -93.1 KeV$

Negative sign indicates that the energy has to be provided to proceed this reaction. So, this fusion is not favorable.

Answer.53

Given, 18g of molecules contain 6.023×10^{23}atoms

1kg of molecules contain $\frac{6.023 \times 10^{23} \times 1000}{18} = 3.346 \times 10^{25}$ atoms

% of deuterium atoms=No. of atoms in 1kg of water× % of deuterium

$= 3.346 \times 10^{25} \times 99.985$

Energy of deuterium= $3.346 \times 10^{25} = (4.028204 - 3.016044) \times 931 eV$

$= 942.32 \times 1.6 \times 10^{-19}\, J$

= 1507mJ