Progressions

INTRODUCTION

In this chapter, we will be concerned with the study of sequences, i.e., special types of functions whose domain is the set N of natural numbers. We will study particular types of sequences called *arithmetic* sequences, *geometric* sequences and *harmonic* sequences and also their corresponding series.

Premiums on life insurance, fixed deposits in a bank, loan instalments payments, disintegration or decay of radioactive materials and the like are some of the examples where the concept of sequence and series is used.

SEQUENCE

A *sequence* is a function whose domain is the set N of natural numbers and range, a subset of real numbers or complex numbers.

A sequence whose range is a subset of real numbers is called a *real sequence*. Since we will be dealing with real sequences only, we will use the term 'sequence' to denote a 'real sequence'.

Notation

The different terms of a sequence are usually denoted by $a_1, a_2, a_3, \ldots$ or $t_1, t_2, t_3, \ldots$ The subscript (always a natural number) denotes the position of the term in the sequence. The number occurring at the nth place of a sequence, i.e., t_n is called the *general term* of the sequence.

Note :

A sequence is said to be *finite* or *infinite* (accordingly as finite or infinite number of terms it has.)

PROGRESSIONS

If the terms of a sequence follow certain pattern, then the sequence is called a *progression*.

Illustration 1 Consider the following sequences:

(i) 3, 5, 7, 9, ..., 21

(ii) 8, 5, 2, –1, –4, ...

(iii) 2, 6, 18, 54, ..., 1458

(iv) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

(v) 1, 4, 9, 16, ...

We observe that each term (except the first) in (i) is formed by adding 2 to the preceding term; each term in (ii) is formed by subtracting 3 from the preceding term; each term in (iii) is formed by multiplying the preceding term by 3; each term in (iv) is formed by dividing the preceding term by 2; each term in (v) is formed by squaring the next natural number. Thus, each of (i) to (v) is a progression. Moreover, (i) and (iii) are finite sequences, whereas (ii), (iv) and (v) are infinite sequences.

However, to define a sequence we need not always have an explicit formula for the nth term. For example, for the infinite sequence 2, 3, 5, 7, 11, 13, 17, ... of all positive prime numbers, we may not be able to give an explicit formula for the nth term.

SERIES

By adding or subtracting the terms of a sequence, we obtain a *series*. A series is finite or infinite according as the number of terms in the corresponding sequence is finite or infinite.

Illustration 2 The following

(i) $3 + 5 + 7 + 9 + \ldots + 21$

(ii) $8 + 5 + 2 + (-1) + \ldots$

(iii) $2 + 6 + 18 + 54 + \ldots + 1458$

(iv) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

(v) $1 + 4 + 9 + 16 + \ldots$

are the series corresponding to the above sequences, (i) to (v).

ARITHMETIC PROGRESSION (A.P.)

A sequence whose terms increase or decrease by a fixed number is called an *arithmetic progression*. The fixed number is called the *common difference* of the A.P.

In an A.P., we usually denote the first term by a, the common difference by d and the nth term by t_n. Clearly, $d = t_n - t_{n-1}$. Thus, an A.P. can be written as

$$a, a + d, a + 2d, ..., a + (n - 1) d, ...$$

Illustration 3 Consider the series: 1, 3, 5, 7, 9, ...

Here 2nd term – 1st term = 3rd term – 2nd term = 4th term – 3rd term = ... = 2

Hence, 1, 3, 5, 7, ... are in A.P. whose first term is 1 and common difference is 2

Illustration 4 The series: 5, 3, 1, –1, –3, –5, –7, ... is in A.P. whose first term is 5 and common difference is –2.

Notes:

- A sequence $t_1, t_2, t_3, t_4, ...$ will be in A.P. if $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = ...$, i.e., $t_n - t_{n-1}$ = constant, for $n \geq 2$.
- Three numbers a, b, c are in A.P. if and only if $b - a = c - b$, i.e., if and only if $a + c = 2b$.
- Any three numbers in an A.P. can be taken as $a - d, a, a + d$. Any four numbers in an A.P. can be taken as $a - 3d, a - d, a + d$ and $a + 3d$.

  Similarly, five numbers in an A.P. can be taken as $a - 2d, a - d, a, a + d$ and $a + 2d$.

GENERAL TERM OF AN A.P.

Let a be the first term and d be the common difference of an A.P. Then, the A.P. is $a, a + d, a + 2d, a + 3d, ...$.

We also observe that

t_1, the first term, is $a = a + (1 - 1) d$;
t_2, the second term, is $a + d = a + (2 - 1) d$;
t_3, the third term, is $a + 2d = a + (3 - 1) d$;
t_4, the fourth term, is $a + 3d = a + (4 - 1) d$;
t_n, the nth term, is $a + (n - 1) d$.

Thus, the formula, $t_n = a + (n - 1) d$ gives the general term of an A.P.

Notes:

- If an A.P. has n terms, then the nth term is called the *last term* of A.P. and it is denoted by l. Therefore, $l = a + (n - 1) d$.
- If a is the first term and d the common difference of an A.P. having m terms, then nth term from the end is $(m - n + 1)$th term from the beginning.

  $\therefore$ nth term from the end $= a + (m - n) d$.

Illustration 5 A sequence $< t_n >$ is given by the formula $t_n = 10 - 3n$. Prove that it is an A.P.

Solution: We have

$$t_n = 10 - 3n \Rightarrow t_{n+1} = 10 - 3(n + 1) = 7 - 3n.$$

$$\therefore \quad t_{n+1} - t_n = (7 - 3n) - (10 - 3n) = -3,$$

which is independent of n and hence a constant. Therefore, the given sequence $< t_n >$ is an A.P.

Illustration 6 Find the nth term and 19th term of the sequence 5, 2, –1 , –4, ...

Solution: Clearly, the given sequence is an A.P. with $a = 5$ and $d = -3$

$$\therefore t_n = a + (n - 1) d = 5 + (n - 1) (-3) = -3n + 8$$

For the 19th term, putting $n = 19$, we get $t_{19} = -3.19 + 8 = -49$

Sum of n terms of an A.P.

The sum of n terms of an A.P. with first term 'a' and common difference 'd' is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Notes:

- If S_n is the sum of n terms of an A.P. whose first term is 'a' and last term is l, then

$$S_n = \frac{n}{2}(a + l).$$

- If common difference d, number of terms n and the last term l, are given then

$$S_n = \frac{n}{2}[2l - (n - 1) d]$$

- $t_n = S_n - S_{n-1}$.

Illustration 7 Find the sum of the series

$$.5 + .51 +. 52 + ... \text{ to } 100 \text{ terms}$$

Solution: The given series is an A.P. with first term, $a = .5$ and common difference, $d = .51 - .5 = .01$

$\therefore$ Sum of 100 terms

$$= \frac{100}{2}[2 \times .5 + (100 - 1) \times .01]$$

$$= 50 (1 + 99 \times .01) = 50 (1 +. 99)$$

$$= 50 \times 1.99 = 99.5$$

Illustration 8 Find the sum of 20 terms of an A.P., whose first term is 3 and the last term is 57.

Solution: We have, $a = 3, l = 57, n = 20$

$$\therefore \quad S_n = \frac{n}{2}(a+l),$$

$$\therefore S_{20} = \frac{20}{2}(3+57) = 600.$$

Hence, the sum of 20 terms is 600

GEOMETRIC PROGRESSION

A sequence (finite or infinite) of non-zero numbers in which every term, except the first one, bears a constant ratio with its preceding term, is called a *geometric* progression, abbreviated as G.P.

Illustration 9 The sequences given below:

(i) 2, 4, ,8, 16, 32, ...

(ii) 3, –6, 12, –24, 48, ...

(iii) $\frac{1}{4}, \frac{1}{12}, \frac{1}{36}, \frac{1}{108}, \frac{1}{324}, \ldots$

(iv) $\frac{1}{5}, \frac{1}{30}, \frac{1}{180}, \frac{1}{1080}, \frac{1}{6480}, \ldots$

(v) $x, x^2, x^3, x^4, x^5, \ldots$ (where x is any fixed real number)

are all geometric progressions. The ratio of any term in (i) to the preceding term is 2. The corresponding ratios in (ii), (iii), (iv) and (v) are $-2, \frac{1}{3}, \frac{1}{6}$, and x, respectively. The ratio of any term of a G.P. to the preceding term is called the *common ratio* of the G.P. Thus, in the above examples, the common ratios are $2, -2, \frac{1}{3}, \frac{1}{6}$ and x, respectively

Note:

In a G.P., any term may be obtained by multiplying the preceding term by the common ratio of the G.P. Therefore, if any one term and the common ratio of a G.P. be known, any term can be written out, i.e., the G.P. is then completely known.

In particular, if the first term and the common ratio are known, the G.P. is completely known. The first term and the common ratio of a G.P. are generally denoted by a and r, respectively.

GENERAL TERM OF G.P.

Let a be the first term and r ($\neq 0$) be the common ratio of a G.P. Let $t_1, t_2, t_3, \ldots, t_n$ denote Ist, 2nd, 3rd, ... , nth terms, respectively. Then, we have

$$t_2 = t_1 r, t_3 = t_2 r, t_4 = t_3 r, \ldots, t_n = t_{n-1} r$$

On multiplying these, we get

$$t_2 t_3 t_4 \ldots t_n = t_1 t_2 t_3 \ldots t_{n-1} r^{n-1} \Rightarrow t_n = t_1 r^{n-1}; \text{ but } t_1 = a.$$

$\therefore$ General term $= t_n = ar^{n-1}$.

Thus, if a is the first term and r the common ratio of a G.P. then the G.P. is $a, ar, ar^2, \ldots, ar^{n-1}$ or $a, ar, ar^2, \ldots$ according as it is finite or infinite.

Cor. If the last term of a G.P. consisting of n terms is denoted by l, then $l = ar^{n-1}$.

Notes:

- If a is the first term and r the common ratio of a finite G.P. consisting of m terms, then the nth term from the end is given by ar^{m-n}.
- The nth term from the end of a G.P. with the last term l and common ratio r is l/r^{n-1}.
- Three numbers in G.P. can be taken as $a/r, a, ar$; four numbers in G.P. can be taken as $a/r^3, a/r, ar, ar^3$; five numbers in G.P. can be taken as $a/r^2, a/r, a, ar, ar^2$, and so on...
- Three numbers a, b, c are in G.P. if and only if $b/a = c/b$, i.e., if and only if $b^2 = ac$.

Illustration 10 Find the nth term and 12th term of the sequence –6, 18, –54, ...

Solution: The given sequence is a G.P. with $a = -6$ and $r = -3$

$$\therefore \; t_n = ar^{n-1} = (-6)(-3)^{n-1} = (-1)^n \times 6 \times 3^{n-1}$$

For the 12th term, putting $n = 12$, we get

$$t_{12} = (-1)^{12} \times 6 \times 3^{11} = 2 \times 3^{12}.$$

Sum of *n* terms of a G.P.

The sum of first n terms of a G.P. with first term a and common ratio r is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Notes:

(i) When $r = 1$

$$S_n = a + a + \ldots \text{ up to } n \text{ terms} = na$$

(ii) It l is the last term of the G.P., then

$$S_n = \frac{lr - a}{r - a}, r \neq 1$$

Sum of An Infinite G.P. When $|r| < 1$

The sum of an infinite G.P. with first term a and common ratio r is $S_\infty = \frac{a}{1-r}$; when $|r| < 1$, i.e., $-1 < r < 1$

Illustration 11 Find the sum of 8 terms and n terms of the sequence 9, –3, 1, –1/3, ...

Solution: The given sequence is a G.P. with $a = 9$ and $r = -1/3$

We know that

$$S_8 = 9\frac{1-(-1/3)^8}{1-(-1/3)} = 9\frac{1-1/3^8}{4/3} = \frac{27}{4}\left(1-\frac{1}{3^8}\right)$$

$$= \frac{27}{4}\frac{3^8-1}{3^8} = \frac{1}{4}\frac{6561-1}{3^5} = \frac{6560}{4\times 243} = \frac{1640}{243}$$

Also, $$S_n = 9\frac{1-(-1/3)^n}{1-(-1/3)} = 9\frac{1-(-1)^n/3^n}{4/3}$$

$$= \frac{27}{4}\frac{3^n-(-1)^n}{3^n} = \frac{3^n-(-1)^n}{4\cdot 3^{n-3}}$$

Illustration 12 Find the sum of the infinite sequence 7, –1, $\frac{1}{7}, -\frac{1}{49}, \ldots$

Solution: The given sequence is a G.P. with $a = 7$ and $r = -\frac{1}{7}$, so $|r| = \left|-\frac{1}{7}\right| = < 1$

$$\therefore \quad S = \frac{7}{1-(-1/7)} = \frac{7}{8/7} = \frac{49}{8} \quad \left(\because S = \frac{a}{1-r}\right)$$

HARMONIC PROGRESSION

A sequence of non-zero numbers $a_1, a_2, a_3, \ldots$ is said to be a *harmonic* progression (abbreviated as H.P.) if the sequence $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$ is an A.P.

Illustration 13 The sequence $1, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \ldots$ is a H.P. The sequence obtained by taking reciprocals of its corresponding terms, i.e., 1, 4, 7, 10, ... is an A.P.

A general H.P. is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$

nth Term of An H.P.

nth term of H.P.

$$= \frac{1}{n\text{th term of the corresponding A.P.}}$$

Notes:

- Three numbers a, b, c are in H.P. if and only if $\frac{1}{a}, \frac{1}{c}$, are in A.P., i.e.,

$$\frac{1}{a} + \frac{1}{c} = 2\times\frac{1}{b} \text{ or } b = \frac{2ac}{a+c}.$$

- No term of H.P. can be zero.
- There is no general formula for finding the sum to n terms of H.P.
- Reciprocals of terms of H.P. are in A.P. and then properties of A.P. can be used.

Illustration 14 Find the 100th of the sequence

$$1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$$

Solution: The sequence $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ is an H.P.

Corresponding A.P. is 1, 3, 5, 7, ...

Now, for the corresponding A.P., first term $a = 1$, $d = 2$

$\therefore$ 100th term of the corresponding A.P.

$$= a + (100 - 1)\,d$$
$$= 1 + (100 - 1)\,2 = 199$$

Hence, the 100th term of the given sequence $= \frac{1}{199}$.

Some Special Sequences

1. The sum of first n natural numbers

$$\Sigma n = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

2. The sum of squares of first n natural numbers $\Sigma n^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

3. The sum of cubes of first n natural numbers $\Sigma n^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$.

Notes:

If nth term of a sequences is

$$T_n = an^3 + bn^2 + cn + d$$

then the sum of n terms is given by

$$S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d,$$

which can be evaluated using the above results.

Illustration 15 Find $2^2 + 4^2 + 6^2 + \ldots + (2n)^2$

Solution: nth term of the given series is $(2n)^2$. Then, $T_n = 4n^2$

$$\therefore \quad S_n = 4\,\Sigma\, n^2 = \frac{4n(n+1)(2n+1)}{6}$$

$$\therefore \quad S_n = \frac{2n(n+1)(2n+1)}{3}$$

Illustration 16 Sum the series $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots$ to n terms

Solution: Here, $T_n = (1^2 + 2^2 + 3^2 + \ldots n^2)$

$$= \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(2n^2+3n+1)}{6}$$

$$= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$\therefore \quad S_n = \Sigma\left(\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n\right)$$

$$= \frac{1}{3}\Sigma n^3 + \frac{1}{2}\Sigma n^2 + \frac{1}{6}\Sigma n$$

$$= \frac{1}{3}\frac{n^2(n+1)^2}{4} + \frac{1}{2}\frac{n(n+1)(2n+1)}{6}$$

$$+ \frac{1}{6}\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{12}[n(n+1) + 2n + 1 + 1]$$

$$= \frac{n(n+1)}{12}(n^2 + 3n + 2)$$

$$= \frac{n(n+1)}{12}(n+1)(n+2)$$

$$= \frac{n}{12}(n+1)^2(n+2).$$

Practice Exercises

Difficulty Level-1
(Based on Memory)

1. If the sum of the 6th and the 15th elements of an arithmetic progression is equal to the sum of the 7th, 10th and 12th elements of the same progression, then which element of the series should necessarily be equal to zero?

(*a*) 10th (*b*) 8th

(*c*) 1st (*d*) None of these

[Based on MAT, 2003]

2. The sum of the 6th and 15th elements of an arithmetic progression is equal to the sum of 7th, 10th and 12th elements of the same progression. Which element of the series should necessarily be equal to zero?

(*a*) 10th (*b*) 8th

(*c*) 1st (*d*) None of these

[Based on MAT, 2003]

3. If p, q, r, s are in harmonic progression and $p > s$, then:

(*a*) $\frac{1}{ps} \quad \frac{1}{qr}$ (*b*) $q + r = p + s$

(*c*) $\frac{1}{q} + \frac{1}{r} = \frac{1}{p} + \frac{1}{s}$ (*d*) None of these

[Based on MAT, 2003]

4. Mohan ate half a pizza on Monday. He ate half of what was left on Tuesday and so on. He followed this pattern for one week. How much of the pizza would he have eaten during the week?

(*a*) 99.22% (*b*) 95%

(*c*) 98.22% (*d*) 100%

[Based on MAT, 2003]

5. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then x is:

(*a*) $\log_a(\log_b a)$ (*b*) $\log_a(\log_e a) + \log_a(\log_e b)$

(*c*) $-\log_a(\log_a b)$ (*d*) $\log_a(\log_e b) - \log_a(\log_e a)$

[Based on MAT, 2002]

6. A person pays ₹975 in monthly instalments, each monthly instalment being less than the former by ₹5. The amount of the first instalment is ₹100. In what tune, will the entire amount be paid?

(*a*) 12 months (*b*) 26 months

(*c*) 15 months (*d*) 18 months

[Based on MAT, 2002]

7. Let S_n denote the sum of the first 'n' terms of an A.P. $S_{2n} = 3S_n$. Then, the ratio S_{3n}/S_n is equal to:

(*a*) 4 (*b*) 6

(*c*) 8 (*d*) 10

[Based on MAT, 2002]

8. Three numbers are in G.P. Their sum is 28 and product is 512. The numbers are:

(*a*) 6, 9 and 13 (*b*) 4, 8 and 16

(*c*) 2, 8 and 18 (*d*) 2, 6 and 18

[Based on MAT, 1999]

9. The sum of the series

$$1^2 + 2^2 + 3^2 + 4^2 + ... + 15^2 \text{ is:}$$

(*a*) 1080 (*b*) 1240

(*c*) 1460 (*d*) 1620

[Based on MAT, 1999]

10. If the nth term of an A.P. is $4n + 1$, then the common difference is:

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

[Based on MAT, 1999]

11. If $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$, then a, b, c form a/an:

(*a*) Arithmetic progression (*b*) Geometric progression

(*c*) Harmonic progression (*d*) None of these.

[Based on MAT, 1999]

12. In G.P., the first term is 5 and the common ratio is 2. The eighth term is:

(*a*) 640 (*b*) 1280

(*c*) 256 (*d*) 160

[Based on MAT, 2000]

13. If the arithmetic mean of two numbers is 5 and geometric mean is 4, then the numbers are

(*a*) 4, 6 (*b*) 4, 7

(*c*) 3, 8 (*d*) 2, 8

[Based on MAT, 2000]

14. The series of positive integers is divided in the following way

$$1 + (2 + 3) + (4 + 5 + 6) + (7 + 8 + 9 + 10) + ...$$

What will be the first term in the $(n + 1)$th group?

(*a*) $\frac{(n^2 - n + 2)}{2}$ (*b*) $\frac{(n^2 + n + 2)}{2}$

(*c*) $\frac{(2n^2 + 3)}{4n}$ (*d*) $\frac{n(n+1)^2}{4}$

15. What is the least value of n such that

$$(1 + 3 + 3^2 + ... + 3^n) \text{ exceeds } 2000?$$

(*a*) 7 (*b*) 5

(*c*) 8 (*d*) 6

[Based on I.P. Univ., 2002]

16. The sum of 12 terms of an A.P., whose first term is 4, is 256. What is the last term?

(*a*) 35 (*b*) 36

(*c*) 37 (*d*) 116/3

[Based on SCMHRD, 2002]

17. The harmonic mean between two numbers is 4, their arithmetic mean is A and geometric mean is G. If $2A + G^2 = 27$, then the numbers are:

(*a*) 8, 2 (*b*) 8, 6

(*c*) 6, 3 (*d*) 6, 4

18. Consider a geometric progression in which all terms are positive. If in it, any term is equal to the sum of the next two following terms, then what is the value of the common ratio?

(*a*) $\frac{1-\sqrt{5}}{2}$ (*b*) 1

(*c*) $\frac{\sqrt{5}-1}{2}$ (*d*) None of these

19. The first, sixth and 14th term of an A.P. are consecutive terms of a GP. The common ratio of the G.P. will be

(*a*) 1.5 (*b*) 2

(*c*) 3 (*d*) 2.5

20. The sum of all terms of an infinite geometric series is 6 and the sum of all terms of the infinite series formed by squaring the terms of the previous series is 24. Find the second term of the first series.

(*a*) $\frac{6}{25}$ (*b*) $\frac{8}{25}$

(*c*) $\frac{24}{25}$ (*d*) $\frac{576}{25}$

21. The average of 49th, 50th and 51st term of an arithmetic progression is equal to 49. What is the sum of the first 99 terms of this arithmetic progression?

(*a*) 4,851 (*b*) 4,950
(*c*) 5,049 (*d*) Cannot be determined

22. The number of two-digit numbers exactly divisible by 3 is:

(*a*) 33 (*b*) 32
(*c*) 31 (*d*) 30

[Based on MAT (Feb), 2008]

23. The natural numbers are divided into groups as (1), (2, 3), (4, 5, 6), (7, 8, 9, 10) and so on. The sum of the numbers in the 50th group is:

(*a*) 1225 (*b*) 24505
(*c*) 62525 (*d*) 52650

[Based on MAT (Sept), 2007]

24. A man arranges to pay off a debt of ₹3600 by 40 annual instalments which are in AP. When 30 of the instalments are paid he dies leaving one-third of the debt unpaid. The value of the 8th instalment is:

(*a*) ₹35 (*b*) ₹50
(*c*) ₹65 (*d*) None of these

[Based on MAT (Dec), 2006]

25. A club consists of members whose ages are in AP, the common difference being 3 months. If the youngest member of the club is just 7 years old and the sum of the ages of all the members is 250 year, then the number of members in the club are:

(*a*) 15 (*b*) 20
(*c*) 25 (*d*) 30

[Based on MAT (Feb), 2006]

26. How many terms are there is an AP whose first and fifth terms are – 14 and 2 respectively and the sum of terms is 40?

(*a*) 15 (*b*) 10
(*c*) 5 (*d*) 20

[Based on MAT (Dec), 2007]

27. In a geometric progression, the sum of the first and the last term is 66 and the product of the second and the last but one term is 128. Determine the first term of the series.

(*a*) 64 (*b*) 64 or 2
(*c*) 2 or 32 (*d*) 32

[Based on MAT (Feb), 2005]

28. A sequence is generated by the rule that the xth term is $x^2 + 1$ for each positive integer x. In this sequence, for any value $x > 1$, the value of $(x + 1)$th term less the value of xth term is

(*a*) $2x^2 + 1$ (*b*) $x^2 + 1$
(*c*) $2x + 1$ (*d*) $x + 2$

[Based on MAT (Feb), 2005]

29. What is the eighth term of the sequence 1, 4, 9, 16, 25, …?

(*a*) 8 (*b*) 64
(*c*) 128 (*d*) 200

[Based on MAT (Sept), 2003]

30. If the arithmetic mean of two numbers is 5 and geometric mean is 4, then the numbers are:

(*a*) 4, 6 (*b*) 4, 7
(*c*) 3, 8 (*d*) 2, 8

[Based on MAT (Dec), 2000]

31. Two numbers A and B are such that their G.M. is 20 per cent less than their A.M. Find the ratio between the numbers.

(*a*) 3:2 (*b*) 4:1
(*c*) 2:1 (*d*) 3:1

32. If $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in A.P., then the value of x is:

(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 5

33. The interior angles of a polygon are in AP, the smallest angle is 120° and the common difference is 5. Then, the number of sides of the polygon are:

(*a*) 16 (*b*) 9
(*c*) 8 (*d*) 12

[Based on MAT (May), 1999]

34. A man arranges to pay off a debt of ₹3600 in 40 annual instalments which form an AP. When 30 of the instalments are paid, he dies leaving one-third of the debt unpaid. Find the value of the first instalment.

(*a*) 55 (*b*) 53
(*c*) 51 (*d*) 49

[Based on MAT (May), 1999]

35. A five-digit number divisible by 3 is to be formed using numerical 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways this can be done is:

(*a*) 122 (*b*) 210
(*c*) 216 (*d*) 217

[Based on SNAP, 2010]

36. The sum of all even natural numbers less than 100 is:

(*a*) 2450 (*b*) 2272
(*c*) 2352 (*d*) 2468

[Based on FMS, 2005]

37. The sum of all odd numbers between 100 and 200 is:

(*a*) 6200 (*b*) 6500
(*c*) 7500 (*d*) 3750

[Based on FMS, 2006]

38. If a, b, c are in G.P. and $a^x = b^y = c^z$, then:

(*a*) $\frac{1}{x}+\frac{1}{z}=\frac{2}{y}$ (*b*) $\frac{1}{x}+\frac{1}{z}=-\frac{2}{y}$

(*c*) $\frac{1}{x}+\frac{1}{y}=\frac{2}{z}$ (*d*) $\frac{1}{x}+\frac{1}{y}=-\frac{2}{z}$

[Based on FMS, 2009]

39. The angles of a pentagon are in arithmetic progression. One of the angles, in degrees, must be

(*a*) 108 (*b*) 90

(*c*) 72 (*d*) 54

[Based on FMS, 2010]

40. If $x_{k+1} = x_k + \frac{1}{2}$ for $k = 1, 2, \ldots, n-1$ and $x_1 = 1$, find $x_1 + x_2 + \ldots + x_n$.

(*a*) $\frac{n+3}{3}$ (*b*) $\frac{n^2-1}{2}$

(*c*) $\frac{n^2-n}{4}$ (*d*) $\frac{n^2+3n}{4}$

[Based on FMS, 2010]

41. Three numbers a, b, c non-zero, form an arithmetic progression. Increasing a by 1 or increasing c by 2 results in a geometric progression. Then b equals:

(*a*) 16 (*b*) 14

(*c*) 12 (*d*) 10

[Based on FMS, 2010]

42. If the sum of the first 10 terms of an arithmetic progression is four times the sum of the first five terms, then the ratio of the first term to the common difference is:

(*a*) 1:2 (*b*) 2:1

(*c*) 1:4 (*d*) 4:1

[Based on FMS, 2011]

43. The sum to infinity of $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+\ldots$

(*a*) $\frac{1}{24}$ (*b*) $\frac{5}{48}$

(*c*) $\frac{1}{16}$ (*d*) None of these

[Based on FMS, 2011]

44. How many two digit odd numbers can be formed from the digits 1, 2, 3, 4, 5 and 8, if repetition of digit is allowed?

(*a*) 5 (*b*) 15

(*c*) 35 (*d*) 18

[Based on IIFT, 2005]

45. The positive numbers x, y, z are in arithmetic progression. They are also in harmonic progression. Then:

(*a*) They cannot be in geometric progression

(*b*) x and 2 cannot be equal

(*c*) y, z, x cannot be in arithmetic progression

(*d*) None of the above

[Based on IIFT, 2005]

46. The second term in a geometric infinite series is 2, whose sum is 25/2. Then the fourth term of the series is:

(*a*) 2/25 (*b*) 2/5

(*c*) 4/25 (*d*) 4/5

[Based on IIFT, 2005]

47. If three positive real numbers a, b and c ($c > a$) are in Harmonic Progression, then log ($a + c$) + log ($a - 2b + c$) is equal to:

(*a*) 2 log ($c - b$) (*b*) 2 log ($a - c$)

(*c*) 2 log ($c - a$) (*d*) log a + log b + log c

[Based on IIFT, 2008]

48. If the positive real numbers a, b and c are in Arithmetic Progression, such that $abc = 4$, then minimum possible value of b is:

(*a*) $2^{\frac{3}{2}}$ (*b*) $2^{\frac{2}{3}}$

(*c*) $2^{\frac{1}{3}}$ (*d*) None of these

[Based on IIFT, 2008]

49. Find the sum of the following series.

$$\frac{2}{1!}+\frac{3}{2!}+\frac{6}{3!}+\frac{11}{4!}+\frac{18}{5!}+\ldots$$

(*a*) $3e - 1$ (*b*) $3(e - 1)$

(*c*) $3(e + 1)$ (*d*) $3e + 1$

[Based on IIFT, 2010]

50. How many positive integers 'n' can we form using the digits 3, 4, 4, 5, 6, 6, 7, if we want 'n' to exceed 60,00,000?

(*a*) 320 (*b*) 360

(*c*) 540 (*d*) 720

[Based on IIFT, 2010]

51. If $\frac{1}{x}+\frac{1}{z}+\frac{1}{x-y}+\frac{1}{z-y} = 0$, which of the following statements is true?

(*a*) x, y, z are in HP of x, $\frac{y}{2}$, z are in A.P.

(*b*) x, y, z are in AP or x, y, z are in H.P.

(*c*) x, $\frac{y}{2}$, z are in HP or x, y, z are in G.P.

(*d*) x, y, z are in GP or x, y, z are in A.P.

[Based on JMET, 2006]

52. The angles of a convex hexagon in degrees are integers and in arithmetic progression. $\angle M$ denote the largest of these 6 angles. Then the maximum value that M can take is:

(*a*) 125° (*b*) 150°

(*c*) 175° (*d*) 179°

[Based on JMET, 2006]

53. How many multiples of 7 are there between 33 and 329?

(*a*) 43

(*b*) 35

(*c*) 329

(*d*) 77

[Based on ATMA, 2008]

54. In a regular polygon, each interior angle is 140°. The number of sides in the polygon will be:

(*a*) 8 (*b*) 9

(*c*) 10 (*d*) 11

[Based on ATMA, 2005]

55. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\ldots$ equals

(*a*) 2 (*b*) 3

(*c*) 5 (*d*) ∞

[Based on JMET, 2006]

Difficulty Level-2 (Based on Memory)

1. The sum of 3rd and 15th elements of an arithmetic progression is equal to the sum of 6th, 11th and 13th elements of the same progression. Then which element of the series should necessarily be equal to zero?

(*a*) 1st (*b*) 9th

(*c*) 12th (*d*) None of these

[Based on CAT, 2003]

2. If $\log_3 2$, $\log_3 (2^x - 5)$, $\log_3 (2^x - 7/2)$ are in arithmetic progression, then the value of x is equal to:

(*a*) 5 (*b*) 4

(*c*) 2 (*d*) 3

[Based on CAT, 2003]

3. The are 8436 steel balls, each with a radius of 1 centimetre, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is:

(*a*) 34 (*b*) 38

(*c*) 36 (*d*) 32

[Based on CAT, 2003]

4. Let T be the set of integers {3, 11, 19, 27, ... 451, 459, 467} and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is:

(*a*) 32 (*b*) 28

(*c*) 29 (*d*) 30

[Based on CAT, 2003]

5. In a certain examination paper, there are n questions. For $j = 1, 2, \ldots n$, there are 2^{n-j} students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is:

(*a*) 12 (*b*) 11

(*c*) 10 (*d*) 9

[Based on CAT, 2003]

6. If three positive real numbers x, y, z satisfy $y - x = z - y$ and $xyz = 4$, then what is the minimum possible value of y?

(*a*) $2/^{13}$ (*b*) $2^{2/3}$

(*c*) $2^{1/4}$ (*d*) $2^{3/4}$

[Based on CAT, 2004]

7. What is the sum of the series $1 + 2x + 4x^2 + 7x^3 + 10x^4 + \ldots$ up to ∞? (Given $0 < x < 1$)

(*a*) $\frac{1-x(1-x)}{(1-x)^3}$ (*b*) $\frac{x^2+1}{(1-x)^2}$

(*c*) $\frac{1+x(1-x)}{(1-x)^3}$ (*d*) None of these

8. A square S_1 had dimensions 6 cm × 6 cm. Another square S_2 is drawn by joining the mid-points of the sides of S_1. Square S_3 is drawn joining the mid-points of S_2 and so on. What is the sum of area (S_1) + area (S_2) + ... ∞?

(*a*) 72 cm^2 (*b*) $36\sqrt{2}(\sqrt{2}-1)$ cm^2

(*c*) 14.4 cm^2 (*d*) $36\sqrt{2}$ cm^2

9. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?

(*a*) 666 (*b*) 676

(*c*) 683 (*d*) 777

[Based on CAT, 2004]

10. The infinite sum $1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}+\ldots$ equals:

(*a*) $\frac{27}{14}$ (*b*) $\frac{21}{13}$

(*c*) $\frac{49}{27}$ (*d*) $\frac{256}{147}$

[Based on CAT, 2004]

11. The sum of the series $1 + 2 \times 2 + 3 \times 2^2 + 4 \times 2^3 + 5 \times 2^4 + \ldots + 100 \times 2^{99}$ is:

(*a*) 99×2^{100} (*b*) $99 \times 2^{100} + 1$

(*c*) 100×2^{100} (*d*) $100 \times 2^{100} + 1$

12. The value of

$$(1^3 + 2^3 + 3^3 + \dots 15^3) - (1 + 2 + 3 + \dots 15) \text{ is:}$$

(*a*) 14280 (*b*) 14400
(*c*) 12280 (*d*) 13280

13. $f(a, b)$ is a series of which the first three terms are $(a+b)^2$, (a^2+b^2) and $(a-b)^2$. We add the first n terms of the series $f(a, b)$ and call it $S(a, b)$. If $a = 7$, $b = 3$, then find $S(7, 3)$ for $n = 20$.

(*a*) –5980 (*b*) 6000
(*c*) 6960 (*d*) None of these

[Based on FMS (Delhi), 2004]

14. 30 trees are planted in a straight line at intervals of 5 m. To water them, the gardener needs to bring water for each tree, separately from a well, which is 10 m from the first tree in line with the trees. How far will he have to walk in order to water all the trees beginning with the first tree? Assume that he starts from the well.

(*a*) 4785 m (*b*) 4795 m
(*c*) 4800 m (*d*) None of these

[Based on FMS (Delhi), 2004]

15. $f(x) + 2x$; where x is an integer. If we arrange the value of $f(x)$, for $x = 25, 24, 23\dots$ (continuously decreasing value of x), we get an Arithmetic Progression (A.P.) whose first term is 50. Find the maximum value of the sum of all the terms of the A.P.

(*a*) 600 (*b*) 625
(*c*) 650 (*d*) None of these

[Based on FMS (Delhi), 2004]

16. If one of the roots of the equation

$$3x^3 + 11x^2 + 12x + 4 = 0:$$

is (–1), then all the three roots are in

(*a*) Arithmetic progression
(*b*) Geometric progression
(*c*) Harmonic progression
(*d*) None of the above

[Based on IITTM, Gwalior, 2003]

17. A man has an apple orchard and he sells to his first customer half of all the apples plus half an apple; to the 2nd customer he sells half of the rest plus half an apple, and so on. To the seventh customer, he sells half of what remains and another half an apple. And that is all he had. How many apples did the man start out with?

(*a*) 47 (*b*) 97
(*c*) 127 (*d*) 137

18. The value of $2^{\frac{1}{4}} . 2^{\frac{2}{8}} . 2^{\frac{3}{16}} . 2^{\frac{4}{32}} \dots$ is equal to:

(*a*) 1 (*b*) 2
(*c*) $\frac{3}{2}$ (*d*) $\frac{5}{2}$

19. If $\dfrac{3+5+7+\dots+n \text{ terms}}{5+8+11+\dots+10 \text{ terms}} = 7$, then the value of n is:

(*a*) 35 (*b*) 36
(*c*) 37 (*d*) 40

20. If the sum of first n natural numbers is one-fifth of the sum of their squares, then n is:

(*a*) 5 (*b*) 6
(*c*) 7 (*d*) 8

[Based on FMS (Delhi), 2002]

21. The number of ordered triplets of positive integers which are solutions of the equation:

$$x + y + z = 100 \text{ is}$$

(*a*) 4851 (*b*) 5081
(*c*) 6871 (*d*) 7081

[Based on FMS (Delhi), 2002]

22. Three distinct numbers x, y, z form a geometric progression in that order, and $x + y$, $y + z$, $z + x$ form an arithmetic progression in that order. Find the common ratio of the geometric progression.

(*a*) –2 (*b*) 2
(*c*) 0.5 (*d*) –0.5

[Based on SCMHRD, 2002]

23. The digits of a three-digit number form G.P. If 400 is subtracted from it, then we get another three-digit number whose digits form an arithmetic series. What is the sum of these two numbers?

(*a*) 1356 (*b*) 1648
(*c*) 1462 (*d*) 1000

[Based on SCMHRD, 2002]

24. Find the sum of all natural numbers not exceeding 1000, which are divisible by 4 but not by 8.

(*a*) 62500 (*b*) 62800
(*c*) 64000 (*d*) 65600

[Based on SCMHRD, 2002]

25. If a, b, c are in A.P. and x, y, z are in G.P., then the value of $x^{b-c} y^{c-a} z^{a-b}$ is:

(*a*) 0 (*b*) 1
(*c*) xyz (*d*) $x^a y^b z^c$

26. If log 2, log $(2^x - 1)$ and log $(2^x + 3)$ (all to the base 10) be three consecutive terms of an Arithmetic Progression, then the value of x is equal to:

(*a*) 0 (*b*) 1
(*c*) $\log_2 5$ (*d*) $\log_{10} 2$

[Based on REC Tiruchirapalli, 2002]

27. Consider a sequence where the nth term, $t_n = n/(n + 2)$, $n = 1, 2, \dots$ the value of $t_3 \times t_4 \times t_5 \times \dots \times t_{53}$ equals.

(*a*) 4/495 (*b*) 2/495
(*c*) 12/55 (*d*) 1/1485

[Based on CAT, 2007]

28. I open a book store with a certain number of books. On the first day, I sell 1 book; on the second day, I sell 2 books; on the third day, I sell 3 books and so on. At the end of the month (30 days). I realise that I sold the same number of books with which I started. Find the number of books in the beginning.

(*a*) 365 (*b*) 420
(*c*) 465 (*d*) 501

29. A contractor, who got the contract for building the flyover, failed to construct the flyover in the specified time and was supposed to pay ₹50,000 for the first day of extra time. This amount increased by ₹4,000 each day. If he completes the flyover after one month of stipulated time, he suffers a loss of 10% in the business. What is the amount he received for making the flyover in crores of rupee? (one month = 30 days):

(*a*) 3.1 (*b*) 3.24
(*c*) 3.46 (*d*) 3.68

Directions (Questions 30 to 34): Refer to the data below and answer the questions that follow.

The starting term of an A.P. is equal to the starting term of a G.P. The difference between the fifth term of the A.P. and the list term of the A.P. is equal to the difference between the first term of the G.P. and the 2nd term of the G.P.

30. The first term of the progression is:

(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

31. The common difference of the A.P. is:

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

32. The common ratio of the G.P. is:

(*a*) 1 (*b*) 3
(*c*) 5 (*d*) 7

33. The sum of the first four terms of the A.P. is:

(*a*) 5 (*b*) 10
(*c*) 15 (*d*) 20

34. The sum of the first four terms of G.P. is:

(*a*) 120 (*b*) 126
(*c*) 150 (*d*) 156

35. A boy throws a ball to the ground with force from a height of 10 m. After hitting the ground for the first time it rises to a height of 20 m. There after it rises only upto half the prior height on hitting the ground.

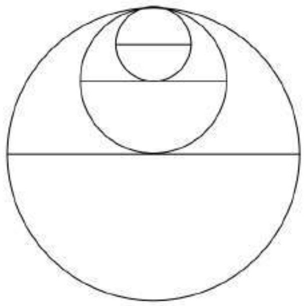
What is the total distance travelled by the ball till it comes to rest?

(*a*) 80 m (*b*) 40 m
(*c*) 50 m (*d*) 90 m

36. There are two arithmetic progressions, A_1 and A_2, whose first terms are 3 and 5 respectively and whose common differences are 6 and 8 respectively. How many terms of the series are common in the first n terms of A_1 and A_2, if the sum of the nth terms of A_1 and A_2 is equal to 6,000?

(*a*) 103 (*b*) 107
(*c*) 109 (*d*) 113

37. Infinite circles are inscribed successively inside the upper half of circles, as shown in the figure below. If the radius of the largest circle is $\frac{1}{\pi}$ units, find the sum of area of all the circles formed in square units.



(*a*) 2 (*b*) 1.5
(*c*) 1.33 (*d*) 1

38. Given $x = 1/0\, y + y^2 - y^2 - y^3 + y^4 + \ldots \infty$ and $z = 1 + y + y^2 + y^3 + y^4 + \ldots \infty$ $(|y| < 1)$. Which of the following is true?

(*a*) Harmonic mean of x and y is 1
(*b*) Arithmetic mean of x and z is 1
(*c*) Harmonic mean of x and z is 1
(*d*) None of these

39. Three numbers form an increasing geometric progression. When the second number is doubled, the numbers form an arithmetic progression. What is the ratio of the first number and the third number?

(*a*) $1{:}7 - 4\sqrt{3}$ (*b*) $1{:}7 + 4\sqrt{3}$
(*c*) $1{:}2 - \sqrt{3}$ (*d*) Either (*a*) or (*b*)

40. Twenty six men –*A*, *B*, *C* . . . *Y* and *Z* – running at the respective speeds of *a*, *b*, *c*, . . . *y* and *z* are participating in a 10 Km running race on a circular track of length 100 m. Their speeds are in arithmetic progression from *a* to *z*, in that order. If the time taken by *Z* to meet *A*, for the first time after the start, is 20 seconds and the time taken by *M* to complete the race is 52 minutes and 5 seconds, then find the time taken for all the twenty six men to meet for the first time at the starting point. (All of them started the race at same time and from the same point).

(*a*) 1,000 seconds (*b*) 500 seconds
(*c*) 225 seconds (*d*) 125 seconds

41. The pth and the $(p + 3)$th term of an arithmetic progression are in the ratio $p:p + 3$. The sum of the first $3p$ terms of the arithmetic progression and the sum of its first $4p$ terms are in the ratio 61:108. Find the value of p.

(*a*) 10 (*b*) 15

(*c*) 20 (*d*) 25

42. If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. where $a_i > 0$ for all i, then the value of $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$ is:

(*a*) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ (*b*) $\frac{n+1}{\sqrt{a_1}+\sqrt{a_n}}$

(*c*) $\frac{n}{\sqrt{a_1}-\sqrt{a_n}}$ (*d*) None of these

43. A retailer has n stones by which he can measure (or weigh) all the quantities from 1 kg to 121 kg (in integers only. e.g., 1 kg, 2 kg, 3 kg,) keeping these stones on either side of the balance. What is the minimum value of n?

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 11

44. In a certain examination paper, there are n questions. For $i = 1, 2, \ldots, n$, there are 2^{n-j} students who answered j or more questions wrongly. If the total number of wrong answers is 4,095, then the value of n is:

(*a*) 12 (*b*) 11

(*c*) 10 (*d*) 9

45. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, ..., 9 such that the first digit of the code is non zero. The code, handwritten on a slip, can however potentially create confusion when read upside down — for example, the code 91 may appear as 16. How many codes are there for which no such confusion can arise?

(*a*) 80 (*b*) 78

(*c*) 71 (*d*) 69

46. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3, \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$, respectively, then the ratio $\frac{P_1 + P_2 + P_3 + \ldots}{A_1 + A_2 + A_3 + \ldots}$ equals:

(*a*) $\frac{2(1+\sqrt{2})}{a}$ (*b*) $\frac{2(2-\sqrt{2})}{a}$

(*c*) $\frac{2(2+\sqrt{2})}{a}$ (*d*) $\frac{2(1+2\sqrt{2})}{a}$

47. The 288th term of the sequence $a, b, b, c, c, c, d, d, d, d, \ldots$ is:

(*a*) u (*b*) v

(*c*) w (*d*) x

[Based on SNAP, 2008]

48. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is:

(*a*) –2 (*b*) –4

(*c*) –12 (*d*) 8

[Based on SNAP, 2009, 2010]

49. Given that $(1^2 + 2^2 + 3^2 + \ldots + 10^2) = 385$, then the value of $(2^2 + 4^2 + 6^2 + \ldots + 20^2)$ is equal to:

(*a*) 770 (*b*) 1540

(*c*) 1155 (*d*) $(385)^2$

[Based on FMS (MS), 2006]

50. From a group of boys and girls, 15 girls leave. There are then left two boys for each girl. After this 45 boys leave. There are then 5 girls for each boy. The number of girls in the beginning was:

(*a*) 40 (*b*) 43

(*c*) 29 (*d*) None of these

[Based on FMS, 2011]

51. The ratio between the number of sides of two regular polygons is 1:2 and the ratio between their interior angles is 2:3. The number of sides of these polygons are respectively:

(*a*) 4, 8 (*b*) 5, 10

(*c*) 6, 12 (*d*) 8, 16

[Based on IIFT, 2005]

52. The inverse of the sum of the following series up to n terms can be written as $\frac{3}{4} + \frac{3}{36} + \frac{7}{144} + \ldots$:

(*a*) $\frac{(n-1)^2}{n^2+2n}$ (*b*) $\frac{n^2+2n}{(n-1)^2}$

(*c*) $\frac{n^2+2n}{(n+1)^2}$ (*d*) $\frac{(n+1)^2}{n^2+2n}$

[Based on IIFT, 2006]

53. If $H_1, H_2, H_3, \ldots, H_n$, are n Harmonic means between 'a' and 'b' ($\neq a$), then value of $\frac{H_1 + a}{H_1 - a} + \frac{H_n + b}{H_n - b}$ is equal to:

(*a*) $n + 1$ (*b*) $2n$

(*c*) $2n + 3$ (*d*) $n - 1$

[Based on IIFT, 2008]

54. Suppose a, b and c are in Arithmetic Progression and a^2, b^2 and c^2 are in Geometric Progression. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is equal to:

(*a*) $\frac{1}{2\sqrt{2}}$ (*b*) $\frac{1}{2\sqrt{3}}$

(*c*) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (*d*) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

[Based on IIFT, 2008]

55. Sum of the series $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 2001^2 - 2002^2 + 2003^2$ is:

(*a*) 2007006 (*b*) 1005004

(*c*) 200506 (*d*) None of these

[Based on IIFT, 2008]

56. A man arranged to pay off a debt of ₹3600 by 40 annual instalments which are in Arithmetical Progression when 30 of the instalments have been paid, he dies leaving one-third of the debt unpaid. The value of the 8th instalment is:

(*a*) ₹35 (*b*) ₹50

(*c*) ₹65 (*d*) None of these

57. Because of economic slowdown, a multinational company curtailed some of the allowances of its employees. Rashid, the marketing manager of the company whose monthly salary has been reduced to ₹42000 is unable to cut down on his expenditure. He finds that there is a deficit of ₹2000 between his earnings and expenses in the first month. This deficit, because of inflationary pressure, will keep on increasing by ₹500 every month. Rashid has a saving of ₹60000 which will be used to fill this deficit. After his savings get exhausted, Rashid would start borrowing from his friends. How soon will he start borrowing?

(*a*) 10th months (*b*) 11th months

(*c*) 12th months (*d*) 13th months

[Based on IIFT, 2009]

58. In a green view apartment, the houses of a row are numbered consecutively from 1 to 49. Assuming that there is a value of 'x' such that the sum of the numbers of the houses preceding the house numbered 'x' is equal to the sum of the numbers of the houses following it. Then, what will be the value of 'x'?

(*a*) 21 (*b*) 30

(*c*) 35 (*d*) 42

[Based on IIFT, 2010]

59. The operation (x) is defined by

I. $(1) = 2$

II. $(x + y) = (x)(y)$

for all positive integers x and y

If $\sum_{x=1}^{n} (x) = 1022$, then n is equal to

(*a*) 8 (*b*) 9

(*c*) 10 (*d*) 11

[Based on XAT, 2010]

60. In a list of 7 integers, one integer, denoted as x is unknown. The other six integers are 20, 4, 10, 4, 8 and 4. If the mean, median and mode of these seven integers are arranged in increasing order, they form an arithmetic progression. The sum of all possible wayes of x is:

(*a*) 26 (*b*) 32

(*c*) 34 (*d*) 40

[Based on XAT, 2011]

61. A saint has a magic pot. He puts one gold ball of radius 1 mm daily inside it for 10 days. If the weight of the first bail is 1 g and if the radius of a ball inside the pot doubles every day, how much gold has the saint made due to his magic pot?

(*a*) $\frac{(2^{30} - 69)}{7}$ g (*b*) $\frac{(2^{30} + 69)}{7}$ g

(*c*) $\frac{(2^{30} - 71)}{7}$ g (*d*) $\frac{(2^{30} + 71)}{7}$ g

[Based on JMET, 2011]

62. If x, y and z are in harmonic progression, which of the following statement(s) is/are true?

I. $x = \frac{y(x+z)}{2z}$ II. $x = \frac{z(x-y)}{y-z}$

III. $x = \frac{y-z}{x-z}$

(*a*) I only (*b*) I and II

(*c*) II only (*d*) II and III

[Based on CAT, 2009]

63. The set of natural numbers N is divided into subsets $A_1 = (1)$, $A_2 = (2, 3)$, $A_3 = (4, 5, 6)$, $A_4 = (7, 8, 9, 10)$ and so on. What is the sum of the elements of the subset A_{50}?

(*a*) 42455 (*b*) 61250

(*c*) 62525 (*d*) 65525

[Based on CAT, 2009]

64. The value with which Ram Kumar buys Bank's cash certificates every year exceeds the previous year's purchase by ₹300. After 20 years, he finds that the total value of the certificates purchased by him is ₹83000. Find the value of the certificate purchased by him in the 13th year.

(*a*) ₹4900 (*b*) ₹6900

(*c*) ₹1300 (*d*) None of these

[Based on CAT, 2010]

65. The sum of 3rd and 15th elements of an arithmetic progression is equal to the sum of 6th, 11th, and 13th elements of the same progression. Then which element of the series should necessarily be equal to zero?

(*a*) 1st (*b*) 9th

(*c*) 12th (*d*) None of these

[Based on CAT, 2010]

66. Let S_n denote the sum of the squares of the first n odd natural numbers. If $S_n = 533n$, find the value of n.

(*a*) 18 (*b*) 20

(*c*) 24 (*d*) 30

[Based on CAT, 2011]

67. Consider a sequence S whose nth term T_n is defined as $1 + 3/n$, where $n = 1, 2, \ldots$. Find the product of all the consecutive terms of S starting from the 4th term to the 60th term.

(*a*) 1980.55 (*b*) 1985.55

(*c*) 1990.55 (*d*) 1975.55

[Based on CAT, 2012]

68. If $(a^2+b^2), (b^2+c^2)$ and (a^2+c^2) are in geometric progression (G.P.), which of the following holds true?

(*a*) $b^2-c^2=\frac{a^4-c^4}{b^2+a^2}$ (*b*) $b^2-a^2=\frac{a^4-c^4}{b^2+c^2}$

(*c*) $b^2-c^2=\frac{b^4-a^4}{b^2+a^2}$ (*d*) $b^2-a^2=\frac{b^4-c^4}{b^2+a^2}$

[Based on CAT, 2012]

69. If $ax^2+bx+c=0$ and $2a$, b and $2c$ are in arithmetic progression (A.P.), which of the following are the roots of the equation?

(*a*) a, c (*b*) $-a, -c$

(*c*) $-\frac{a}{2}, -\frac{c}{2}$ (*d*) $-\frac{c}{a}, -1$

[Based on CAT, 2012]

70. The sum of first ten terms of an AP is 155 and the sum of first terms of a GP is 9. The first term of the AP is equal to the common ratio of the GP and the first term of the GP is equal to the common difference of the AP. Which can be the AP as per the given conditions?

(*a*) 2, 4, 6, 8 (*b*) 2, 5, 8, 11,

(*c*) $\frac{25}{2}, \frac{79}{6}, \frac{83}{6},$ (*d*) Both (b) and (c)

[Based on CAT, 2013]

71. If $a_1, a_2, a_3, \ldots, a_n$ be an AP and s_1, s_2 and s_3 be the sum of first n, $2n$ and $3n$ terms respectively, then $S_3-S_2-S_1$ is equal to (where a is the first term and d is the common difference):

(*a*) $3a-2n-d$ (*b*) $a(n+2d)$

(*c*) $3a+2nd$ (*d*) $2n^2d$

[Based on CAT, 2013]

72. What is the value of the following expression?

$$\left(\frac{1}{(2^2-1)}\right)+\left(\frac{1}{(4^2-1)}\right)+\left(\frac{1}{(6^2-1)}\right)+\ldots+\left(\frac{1}{(20^2-1)}\right)$$

(*a*) $\frac{9}{19}$ (*b*) $\frac{10}{19}$

(*c*) $\frac{10}{21}$ (*d*) $\frac{11}{21}$

[Based on CAT, 2000]

73. Let $u_{n+1}=2u_n+1$, $(n=0, 1, 2, \ldots)$ and $u_0=0$. Then, u_{10} would be nearest to:

(*a*) 1023 (*b*) 2047

(*c*) 4095 (*d*) 8195

[Based on CAT, 1993]

74. A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, and little finger 5, then reversed the direction, calling the ring finger 6, middle finger 7, index finger 8, thumb 9 and then back to the index finger for 10, middle finger for 11 and so on. She counted up to 1994. She ended on her:

(*a*) thumb (*b*) index finger

(*c*) middle finger (*d*) ring finger

[Based on CAT, 1993]

75. Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of the sides contains 8 balls less than each side of the triangle had. The initial number of balls is:

(*a*) 1600 (*b*) 1500

(*c*) 1540 (*d*) 1690

76. $x, 17, 3x-y^2-2$, and $3x+y^2-30$, are four consecutive terms of an increasing arithmetic sequence. The sum of the four numbers is divisible by:

(*a*) 2 (*b*) 3

(*c*) 5 (*d*) 7

[Based on XAT, 2014]

Answer Keys

Difficulty Level-1

1. (*b*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)	**6.** (*c*)	**7.** (*b*)	**8.** (*b*)	**9.** (*b*)	**10.** (*b*)	**11.** (*c*)	**12.** (*a*)	**13.** (*d*)
14. (*b*)	**15.** (*c*)	**16.** (*d*)	**17.** (*c*)	**18.** (*c*)	**19.** (*b*)	**20.** (*c*)	**21.** (*a*)	**22.** (*d*)	**23.** (*c*)	**24.** (*c*)	**25.** (*c*)	**26.** (*b*)
27. (*b*)	**28.** (*c*)	**29.** (*b*)	**30.** (*d*)	**31.** (*b*)	**32.** (*b*)	**33.** (*b*)	**34.** (*c*)	**35.** (*c*)	**36.** (*a*)	**37.** (*c*)	**38.** (*a*)	**39.** (*a*)
40. (*d*)	**41.** (*c*)	**42.** (*a*)	**43.** (*d*)	**44.** (*d*)	**45.** (*c*)	**46.** (*a*)	**47.** (*c*)	**48.** (*b*)	**49.** (*b*)	**50.** (*c*)	**51.** (*a*)	**52.** (*c*)
53. (*a*)	**54.** (*b*)	**55.** (*b*)										

Difficulty Level-2

1. (*c*)	**2.** (*d*)	**3.** (*c*)	**4.** (*d*)	**5.** (*a*)	**6.** (*b*)	**7.** (*a*)	**8.** (*a*)	**9.** (*b*)	**10.** (*c*)	**11.** (*b*)	**12.** (*a*)	**13.** (*a*)
14. (*b*)	**15.** (*c*)	**16.** (*c*)	**17.** (*c*)	**18.** (*b*)	**19.** (*a*)	**20.** (*c*)	**21.** (*a*)	**22.** (*a*)	**23.** (*c*)	**24.** (*a*)	**25.** (*b*)	**26.** (*c*)
27. (*b*)	**28.** (*c*)	**29.** (*b*)	**30.** (*b*)	**31.** (*a*)	**32.** (*c*)	**33.** (*b*)	**34.** (*d*)	**35.** (*d*)	**36.** (*b*)	**37.** (*c*)	**38.** (*c*)	**39.** (*a*)
40. (*b*)	**41.** (*c*)	**42.** (*a*)	**43.** (*c*)	**44.** (*a*)	**45.** (*c*)	**46.** (*c*)	**47.** (*d*)	**48.** (*c*)	**49.** (*b*)	**50.** (*a*)	**51.** (*a*)	**52.** (*d*)
53. (*b*)	**54.** (*d*)	**55.** (*a*)	**56.** (*c*)	**57.** (*d*)	**58.** (*c*)	**59.** (*b*)	**60.** (*d*)	**61.** (*c*)	**62.** (*b*)	**63.** (*c*)	**64.** (*a*)	**65.** (*c*)
66. (*b*)	**67.** (*b*)	**68.** (*b*)	**69.** (*d*)	**70.** (*d*)	**71.** (*d*)	**72.** (*c*)	**73.** (*a*)	**74.** (*c*)	**75.** (*c*)	**76.** (*a*)		

Explanatory Answers

Difficulty Level-1

1. (*b*) Let a be the first term and d be the common ratio of an A.P.

$\therefore\ (a + 5d) + (a + 14d)$

$$= (a + 6d) + (a + 9d) + (a + 11d)$$

$\Rightarrow\quad a + 7d = 0$

$\Rightarrow\quad$ 8th term = 0.

2. (*b*) Let a be the first term of the series in A.P.

Let d be its common difference.

$\therefore\ (a + 5d) + (a + 14d)$

$$= (a + 6d) + (a + 9d) + (a + 11d)$$

$\Rightarrow\quad 2a + 19d = 3a + 26d \Rightarrow a = -7d$

$\Rightarrow\quad$ 8th term $= a + 7d \Rightarrow -7d + 7d = 0$.

3. (*c*) p, q, r, s are in harmonical progression

$\Rightarrow\ \frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ and $\frac{1}{s}$ are in A.P.

$$\Rightarrow\quad \frac{1}{q} - \frac{1}{p} = \frac{1}{s} - \frac{1}{r}$$

$$\Rightarrow\quad \frac{1}{q} + \frac{1}{r} = \frac{1}{p} + \frac{1}{s}.$$

4. (*a*) Mohan ate $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}$ of the pizza during the week. That is,

$$\frac{\frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^7\right]}{1 - \frac{1}{2}} = 1 - \frac{1}{128} = \frac{127}{128} = 99.22\%$$

$$\left[\text{Here } a = \frac{1}{2}, r = \frac{1}{2} \text{ for the given G.P. of 7 terms}\right]$$

5. (*a*) Given statement

$\Rightarrow\quad (a^{x/2})^2 = (\log_b x) \times (\log_x a) \Rightarrow a^x = \log_b a$

$\Rightarrow\quad x \log a = \log_a [\log_b a] \Rightarrow x = \log_a [\log_b a]$.

6. (*c*) Let n be the number of months in which all the instalments can be paid

First instalment = ₹100

Last instalment = ₹5

Common Difference = –5

$\Rightarrow$ Sum of the series with n terms whose first term is 100 or common difference is (–5) = 975 i.e., $\frac{n}{2}[2a+(n-1)d]=975$

i.e., $\frac{n}{2}[2\times 100+(n-1)(-5)]=975$

i.e., $n^2-41n+390=0$

i.e., $n=26$ or $n=15$

For $n=15$, total amount paid

$$=\frac{15}{2}[2\times 100+(15-1)(-5)]$$

$$=\frac{15}{2}[200-70]=975.$$

7. (*b*) $S_n=\frac{n}{2}[a+(n-1)d]$

[where a is the first term and d is the common difference]

$S_{2n}=\frac{n}{2}[a+(n-1)d]$

$S_{3n}=\frac{3n}{2}[a+(3n-1)d]$

Given, $S_{2n}=3S_n$

$\Rightarrow\quad n[a+2nd-d]=3\left[\frac{n}{2}(a+nd-d)\right]$

$\Rightarrow\quad d=\frac{a}{1+n}$

$$\therefore\quad \frac{S_{3n}}{S_n}=\frac{\frac{3n}{2}[a+3nd-d]}{\frac{n}{2}[a+nd-d]}$$

$$=\frac{3\left[a+\frac{3na}{1+n}-\frac{a}{1+n}\right]}{a+\frac{na}{1+n}-\frac{a}{1+n}}=6.$$

8. (*b*) Let the three numbers be a, ar, ar^2, where r is the common ratio.

$\therefore\quad a+ar+ar^2=28$ and $a^3r^3=512$

$\therefore\quad ar=8\Rightarrow a+ar^2=20$

$\Rightarrow\quad 8r^2-20r+8=0$

$\Rightarrow\quad r=2,\ r=\frac{1}{2}$

If $r=2$, $a=4$. Therefore, the three numbers are 4, 8, 16.

9. (*b*) The sum of the squares of the first n natural numbers is

$$\frac{n(n+1)(2n+1)}{6}$$

Put $n=15$, we have, $1^2+2^2+3^2+4^2+...+15^2$

$$=\frac{15(15+1)(30+1)}{6}=1240.$$

10. (*b*) nth term$=a+(n-1)d=4n+1$

where a = first term and d = common difference

$\therefore\quad (a-d)+nd=1+4n\Rightarrow a-d=1,\ d=4$

$\Rightarrow\quad a=5.$

11. (*c*) $\frac{1}{b-a}-\frac{1}{c}=\frac{1}{a}-\frac{1}{b-c}$

$\Rightarrow\quad b=\frac{2ac}{a+c}\Rightarrow\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

$\Rightarrow a, b, c$ are in H.P.

12. (*a*) nth term of a G.P. $=ar^{n-1}$

where a = first term and r is the common ratio

$\therefore$ 8th term $=5\times(2)^7=5\times 128=640.$

13. (*d*) Let x, y be the numbers

$\therefore\quad \frac{x+y}{2}=5$ and $\sqrt{xy}=4\Rightarrow xy=16$

$\therefore\quad x+y=10,\ xy=16$

$\Rightarrow\quad (x-y)^2=(x+y)^2-4xy=100-64=36$

$\Rightarrow\quad x-y=6$

$\therefore\quad x=8,\ y=2.$

14. (*b*) Number of terms in the first n groups $=1+2+3+...$

$$n=\frac{n(n+1)}{2}=\frac{n^2+n}{2}$$

So, first term is the $(n+1)$th group

$$=\frac{n^2+n}{2}+1$$

$$=\frac{n^2+n+2}{2}.$$

15. (*c*) $\frac{3^n-1}{3-1}>2000\Rightarrow 3^n>4001\Rightarrow n=8.$

16. (d) $S_n=\frac{n}{2}[2a+(n-1)d]$

where $a=4$, $d=?$, $n=12$

$\therefore\quad S_{12}=256=\frac{12}{2}[2\times 4+(12-1)\times d]$

$\Rightarrow\quad d=\frac{104}{33}$

$\therefore$ Last term = 12th term $=T_{12}$

$$T_{12}=a+(n-1)d$$

$$=4+(12-1)\times\frac{104}{33}$$

$$=4+\frac{104}{3}=\frac{116}{3}.$$

17. (*c*) Let the numbers be a and b, then

$$\frac{2ab}{a+b} = 4 \text{ and } a+b+ab = 27$$

$$\Rightarrow \quad 2(a+b) - ab = 0 \text{ and } a+b+ab = 27$$

$$\Rightarrow \quad ab = 8 \text{ and } a+b = 9$$

$$\Rightarrow \quad a = 6, b = 3$$

18. (*c*) Let the 3 numbers in the geometric progression be a, ar, ar^2.

$$\therefore \quad a = ar + ar^2$$

$$\therefore \quad r^2 + r = 1$$

$$\Rightarrow \quad r^2 + r - 1 = 0$$

$$\therefore \quad r = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

$\because$ All the numbers are positive, the ratio cannot be negative

$\therefore$ $\frac{-1-\sqrt{5}}{2}$ is not possible

$$\therefore \quad r = \frac{\sqrt{5}-1}{2}.$$

19. (*b*) As per the given information: $(a + d) \times (a + 13d) = (a + 5d)^2$ Solving, $a = 3d$

Now common ratio of G.P., $(a + 5d)/(a + d) = 2$.

20. (*c*)

$$\frac{a}{1-r} = 6 \text{ and } \frac{a^2}{1-r^2} = 24$$

Dividing, $\frac{a}{1+r} = 4$

$$\Rightarrow \quad r = \frac{1}{5} \text{ and } a = \frac{24}{5}$$

Hence, second term of the first series $= a \times r = \frac{24}{25}$.

21. (*a*) Average of 49th, 50th and 51st term = 50th term = 49

Hence, $(a + 49d) = 49$

Sum of the first 99 terms = 49 terms before 50th term + 50th term + 49 terms after 50th term = 99 × 50th term = 99 × 49 = 4,851.

22. (*d*) Required numbers are 12,15,18, ..., 99

This is an AP with $a = 12$ and $d = 3$

$$\therefore \quad T_n = a + (n-1)d$$

$$99 = 12 + (n-1) \times 3$$

$$\Rightarrow \quad n - 1 = \frac{99-12}{3}$$

$$n = 29 + 1 = 30.$$

23. (*c*) Let, $\quad S = 1 + 2 + 4 + 7 + \ldots + T_n$

or, $\quad S = 1 + 2 + 4 + \ldots + T_{n-1} + T_n$

Subtracting, we get

$$0 = 1 + [1 + 2 + 3 + \ldots (n-1)] - T_n$$

$$\Rightarrow \quad T_n = 1 + 2 + 3 + \ldots + (n-1) + 1$$

$$= \frac{n(n-1)}{2} + 1$$

$\therefore$ First number of 50th term,

$$= \frac{50 \times 49}{2} + 1 = 1226$$

$\therefore$ Sum of numbers of 50th term,

$$= 1226 + 1227 + \ldots \text{ upto 50th term}$$

$$= \frac{50}{2}[2 \times 1226 + (50-1) \times 1]$$

$$= 25 \times 2501 = 62525.$$

24. (*c*) Let the first instalment be 'a' and the common difference between any two consecutive instalments be 'd'.

Using the formula for the sum of an A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

We have,

$$3600 = \frac{40}{2}[2a + (40-1)d] = 20(2a + 39d)$$

$$\Rightarrow \quad 180 = 2a + 39d \qquad (1)$$

Again, $\quad 2400 = \frac{30}{2}[2a + (30-1)d]$

$$= 15(2a + 29d)$$

$$\Rightarrow \quad 160 = 2a + 29d \qquad (2)$$

Solving Eqs. (1) and (2),

Therefore, $\quad 180 = 2a + 39 \times 2$

$$\Rightarrow \quad 2a = 102 \Rightarrow a = 51$$

Value of 8th instalment

$$= 51 + (8-1) \times 2 = 51 + 14$$

$$= ₹65.$$

25. (*c*) Let n be the number of members in the club.

Then, $\quad 250 = \frac{n}{2}\left[2 \times 7 + (n-1)\frac{3}{12}\right]$

$$\Rightarrow \quad 250 = \frac{n}{2}\left[14 + \frac{1}{4}n - \frac{1}{4}\right]$$

$$\Rightarrow \quad 250 = 7n + \frac{n^2}{8} - \frac{n}{8}$$

$$\Rightarrow \quad n = 25.$$

26. (*b*)
$$T_5 = a + (n-1) \times d$$
$$2 = -14 + 4d$$
$$d = \frac{16}{4} = 4$$
$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1) \times d]$$
$$\Rightarrow \quad 40 = \frac{n}{2}[-28 + (n-1) \times 4]$$
$$\Rightarrow \quad 80 = -28n + 4n^2 - 4n$$
$$\Rightarrow \quad 4n^2 - 32n - 80 = 0$$
$$\Rightarrow \quad n^2 - 8n - 20 = 0$$
$$\Rightarrow \quad (n-10)(n+2) = 0$$
$$\therefore \quad n = 10. \qquad (\because n \neq -2)$$

27. (*b*) Let the last term be n, then
$$a + ar^{n-1} = 66 \qquad (1)$$
and, $$ar \times ar^{n-2} = 128$$
$$\Rightarrow \quad a^2r^{n-1} = 128 \qquad (2)$$
From Eqs. (1) and (2),
$$a(66-a) = 128$$
$$\Rightarrow \quad a^2 - 66a + 128 = 0$$
$$\Rightarrow \quad a = 64, 2.$$

28. (*c*) $(x+1)^{\text{th}}$ term $- x^{\text{th}}$ term
$$= (x+1)^2 + 1 - (x^2 + 1)$$
$$= x^2 + 2x + 1 + 1 - x^2 - 1 = 2x + 1.$$

29. (*b*) 1, 4, 9, 16, 25,

$(1)^2\ (2)^2\ (3)^2\ (4)^2\ (5)^2$

Each term of the progression is the square of a natural number

Hence, the eighth term of the sequence will be $(8)^2 = 64$.

30. (*d*) Let the two numbers be x and y

Then, A.M.
$$\frac{x+y}{2} = 5$$
$$\Rightarrow \quad x + y = 10$$
and, G.M. $$\sqrt{xy} = 4 \qquad (1)$$
$$xy = 16$$
$$\Rightarrow \quad (x-y)^2 = (x+y)^2 - 4xy$$
$$= 100 - 64 = 36$$
or, $$x - y = 6 \qquad (2)$$

Solving Eqs. (1) and (2),
$$x = 8 \text{ and } y = 2.$$

31. (*b*)
$$\sqrt{AB} = 0.8 \times \frac{(A+B)}{2}$$
or, $$AB = 0.16\,(A+B)^2$$
Using option (*b*),

we find that $4 = 0.16\,(4+1)^2 \Rightarrow 4 = 4.$

32. (*b*) Given $2\log_3(2^x - 5) = \log_3 2 + \log_3(2^x - 7/2)$
$$\Rightarrow \quad (2^x - 5)^2 = 2[2^x - 7/2]$$
$$= 2^{2x} - 10.2^x + 25 = 2.2^x - 7$$
$$\Rightarrow \quad 2^{2x} - 12.2^x + 32 = 0$$
$$\Rightarrow \quad y^2 - 12y + 32 = 0$$
$$\Rightarrow \quad (y-8)(y-4) = 0$$
or, $$y = 4, 8$$
$$\therefore \quad 2^x = 3^2, 2^2 \text{ or, } x = 3, 2$$
But, $x = 2$, gives $2^x - 5 = -1$

$\therefore$ $x = 2$ is impossible, so $x = 3$.

33. (*b*) Let the polygon has n sides.

Given the smallest interior angle is 120°, hence the greatest exterior angle will be (180° – 120°) = 60°. We know sum of exterior angles of a polygon = 360°
$$60 + 55 + 50 + \ldots = 360$$
{Common difference = –5}
$$\therefore \quad \frac{n}{2}[2a + (n-1)d] = 360$$
$$\frac{n}{2}[120 + (n-1) \times -5] = 360$$
$$\Rightarrow \quad n^2 - 25n + 144 = 0$$
$$\Rightarrow \quad n = 9, 16$$
Number of sides cannot be 16

Hence, $n = 9$.

34. (*c*) Sum of 40 instalments $S_{40} = 3600$
$$= 20\,(2a + 39d)$$
$$\Rightarrow \quad 2a + 39d = 180 \qquad (1)$$
Sum of 30 instalments $S_{30} = 2400 = 15(2a + 29d)$
$$\Rightarrow \quad 2a + 29d = 160 \qquad (2)$$
Solving Eqs. (1) and (2), we get
$$a = 51 \text{ and } d = 2$$
$\therefore$ The value of first instalment = ₹51.

35. (*c*) Using the digits 0, 1, 2, 3, 4 and 5, five-digit numbers divisible by 3, can be formed using the following combinations.

Case (i): 1, 2, 3, 4, 5

Total number of numbers formed using these digits

$= 5! = 120$

Case (ii): 0, 1, 2, 4, 5

Total number of numbers formed using these digits

$= 4 \times 4 \times 3 \times 2 = 96$

Thus, total numbers $= 120 + 96 = 216$

Hence, option (*c*).

36. (*a*) We know that

Sum of even number $= n\,(n + 1)$

Here, $n = 49$

$\therefore$ Sum $= 49 \times 50 = 2450.$

37. (*c*) Sum between 100 to 200,

$$S_n = \frac{50}{2}(101 + 199)$$

$$= 25 \times 300 = 7500.$$

38. (*a*) Let $a^x = b^y = c^z = k$

$$\Rightarrow \quad a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}$$

and, $$c = k^{\frac{1}{z}}$$

$$\because \quad b^2 = ac \Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{z}} \cdot k^{\frac{1}{x}}$$

$$\Rightarrow \quad k^{\frac{2}{y}} = k^{\frac{1}{z}+\frac{1}{x}}$$

$$\Rightarrow \quad \frac{2}{y} = \frac{1}{z} + \frac{1}{x}.$$

39. (*a*) The sum of interior angles of a pentagon = 540°

Let the angles of the pentagon be $a - 2d$, $a - d$, a, $a + d$, $a + 2d$

$\therefore$ $a - 2d + a - d + a + a + d + a + 2d = 540$

$\therefore$ $5a = 5400$

$\therefore$ $a = 108°$

$\therefore$ One of the angles must be 108°.

40. (*d*) $$x_{k+1} = x_k + \frac{1}{2}$$

$\therefore$ $x_1, x_2, x_3, \ldots x_n$ form an arithmetic progression with common difference $d = 1/2$

$\because$ $x_1 = 1$, first term $a = 1$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}\left[2(1) + (n-1)\frac{1}{2}\right]$$

$$= \frac{n}{2}\left[2 + \frac{n}{2} - \frac{1}{2}\right] = \frac{n}{2}\left[\frac{n+3}{2}\right]$$

$$= \frac{n^2 + 3n}{4}.$$

41. (*c*) a, b, c form an A.P.

$$2b = a + c$$

Increasing a by 1 or c by 2 results in a G.P.

$\therefore$ $b^2 = (a + 1)c$ (1)

and, $b^2 = a(c + 2)$ (2)

$\therefore$ $(a + 1)c = a(c + 2)$

$\therefore$ $ac + c = ac + 2a$

$\therefore$ $c = 2a$

Now, $2b = a + c$

$\therefore$ $2b = a + 2a$

$\therefore$ $b = \dfrac{3a}{2}$

Putting this in Eq. (1), we get

$$\frac{9a^2}{4} = (a + 1)\,2a$$

$\therefore$ $\dfrac{9a}{4} = 2a + 2$

$\therefore$ $9a = 8a + 8$

$\therefore$ $a = 8$

$\therefore$ $b = \dfrac{3a}{2}$

$$= \frac{3 \times 8}{2} = 12.$$

42. (*a*) Let the first term and the common difference of the arithmetic progression be a and d, respectively. It is given the sum of the first ten terms is equal to four times the sum of the first five terms.

$$\text{Sum of the first five terms} = \frac{5}{2}[2a + 4d]$$

$$\text{Sum of the first ten terms} = \frac{10}{2}[2a + 9d]$$

Given, $$\frac{10}{2}[2a + 9d] = 4\left[\frac{5}{3}(2a + 49d)\right]$$

$$2a + 9d = 2[2a + 4d]$$

$$d = 2a$$

Thus, the ratio of the first term to the common difference

$$= \frac{a}{d} = \frac{1}{2}.$$

43. (*d*) $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+...$

$$=\left[\frac{1}{7}+\frac{1}{7^3}+...\right]+\left[\frac{2}{7^2}+\frac{2}{7^2}+...\right]$$

$$=\frac{\frac{1}{7}}{1-\frac{1}{7^2}}+\frac{\frac{2}{7^2}}{1-\frac{1}{7^2}}$$

$$=\frac{7}{48}+\frac{2}{48}$$

$$=\frac{9}{48}=\frac{3}{16}.$$

44. (*d*) Number are 1, 2, 3, 4, 5, 8

$\therefore$ Total digit = 6

Here repetition of digits is allowed

$\therefore$ First place can be filled by any 6 ways,

and second place can be filled by 3 ways

Total number of ways = ${}^6P_1 \times {}^3P_1 = 6 \times 3 = 18$.

45. (*c*) Since, *x*, *y*, *z* are in A.P. and also in H.P.

In A.P. $\quad y=\frac{x+z}{2}$

In H.P. $\quad y=\frac{2xz}{x+z}$

$\therefore \quad \frac{2xz}{x+z}=\frac{x+z}{2}$

$\Rightarrow \quad (x+z)^2-4xz=0$

$\therefore \quad x=z$

Hence *y*, *z* and *x* connot be in A.P.

46. (*a*) $\quad ar=2$

$\Rightarrow \quad a=\frac{2}{r}$

Also, $\quad \frac{2}{r(1-r)}=\frac{25}{2}$

$\therefore \quad r=\frac{4}{5},\frac{1}{5}$

$\therefore \quad T_4=ar^3=\frac{2}{r}\times r^3$

$$=2r^2=2\times\frac{1}{25}\text{ or, }2\times\frac{16}{25}$$

$$=\frac{2}{25}\text{ or, }\frac{32}{25}.$$

47. (*c*) *a*, *b*, *c* are in H.P.

$\therefore \quad b=\frac{2ac}{a+c} \qquad (1)$

Now, log (*a* + *c*) + log (*a* – 2*b* + *c*)

$= \log[(a+c)\times(a-2b+c)]$

$= \log[a^2-2ab+ac+ac-2bc+c^2]$

$= \log[(a^2+c^2+2ac)-2ab-2bc]$

$= \log[(a+c)^2-2b(a+c)]$

$= \log\left[(a+c)^2-2\times\frac{2ac}{a+c}\times(a+c)\right]$

$= \log[(a+c)^2-4ac]$

$= \log[a-c]^2$

We know that $c > a$

and, $\quad (a-c)^2=(c-a)^2$

$\therefore \quad \log(c-a)^2 \Rightarrow 2\log(c-a)$.

48. (*b*) $\quad a\,b\,c = 4$ and *a*, *b*, *c* are in AP.

Then, value of *b* will be minimum when all three are equal.

$\therefore \quad b^3=4$

$\Rightarrow \quad b=4^{\frac{1}{3}}=2^{\frac{2}{3}}$

49. (*b*) Given series is

$$\frac{2}{1!}+\frac{3}{2!}+\frac{6}{3!}+\frac{11}{4!}+\frac{18}{5!}+...$$

So, the *n*th term of the given series is

$$t_n=\frac{2+(n-1)^2}{n!}$$

and, $\quad S_n=\sum_{n=1}^{\infty}\left[\frac{2}{n!}+\frac{(n-1)^2}{n!}\right]$

$$=\sum_{n=1}^{\infty}\left[\frac{2}{n!}+\frac{n^2-2n+1}{n!}\right]$$

$$=\sum_{n=1}^{\infty}\left[\frac{2}{n!}+\frac{n}{(n-1)!}-\frac{2}{(n-1)!}+\frac{1}{n!}\right]$$

$$=\sum_{n=1}^{\infty}\left[\frac{3}{n!}+\frac{n-1+1}{(n-1)!}-\frac{2}{(n-1)!}\right]$$

$$=\sum_{n=1}^{\infty}\left[\frac{3}{n!}\right]+\sum_{n=2}^{\infty}\left[\frac{1}{(n-2)!}\right]+\sum_{n=1}^{\infty}\left[\frac{1}{(n-1)!}-\frac{2}{(n-1)!}\right]$$

$$=\sum_{n=1}^{\infty}\frac{3}{n!}+\sum_{n=2}^{\infty}\frac{1}{(n-2)!}-\sum_{n=1}^{\infty}\frac{1}{(n-1)!}$$

$$=3\left[1+\frac{1}{2!}+\frac{1}{3!}+...\right]+e-e$$

$$=3(e-1)$$

Remember that, $e=1+\frac{1}{2!}+\frac{1}{3!}+...$

50. (*c*) As per the given condition, number in the highest position should be either 6 or 7, which can be done in two ways.

If the first digit is 6, the other digits can be arranged in

$$= \frac{6!}{2!} = 360 \text{ ways}$$

If the first digit is 7, the other digits can be arranged in

$$= \frac{6!}{2! \times 2!} = 180 \text{ ways}$$

Thus, the required possibilities for

$$n = 360 + 180 = 540 \text{ ways.}$$

51. (*a*) $\frac{1}{x} + \frac{1}{z-y} + \frac{1}{z} + \frac{1}{x-y} = 0$

$\Rightarrow \quad \frac{x+z-y}{x(z-y)} + \frac{x+z-y}{z(x-y)} = 0$

$\Rightarrow \quad xz - xy + zx - zy = 0$

$2xz = y(x+z)$

$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

Hence, x, y, z are in H.P. and $x, \frac{y}{2}, z$ are in A.P.

52. (*c*) Let the angle be

$(a - 5d) + (a - 3d) + (a - d)$

$+ (a + d) + (a + 3d) + (a + 5d) = 720$

[Sum of angles of hexagon = 720°]

$\Rightarrow \quad 6a = 720° \Rightarrow a = 120°$

The largest angle should be a multiple of one of the convex angles and less than 180°, hence it can be 175°.

53. (*a*) Given series be 35, 42, 49, ..., 329

Here, $a = 35$

$d = 7$

and, $t_n = 329$, than $n = ?$

$\therefore \quad t_n = a + (n-1)d$

$\Rightarrow \quad 329 = 35 + (n-1) \times 7$

$\Rightarrow \quad 294 = (n-1)7$

$\Rightarrow \quad n - 1 = \frac{294}{7} = 42$

$\therefore \quad n = 42 + 1 = 43.$

54. (*b*) $\frac{(2n-4) \times 90}{n} = 140$

$18n - 36 = 14n$

$\therefore \quad n = 9.$

55. (*b*) $\frac{1}{1} + \left(\frac{1}{3} + \frac{1}{6}\right) + \left(\frac{1}{10} + \frac{1}{15}\right) + \ldots$

$= 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots$

$= \frac{3}{2}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \ldots \infty\right]$

$= \frac{3}{2} \frac{1}{\left[1 - \frac{1}{2}\right]} = \frac{3}{2} \times 2 = 3.$

Difficulty Level-2

1. (*c*) Let the first term be a and common difference d.

As we know, nth term of an AP

$= a + (n-1)\, d$

According to the question,

$a + (3-1)\, d + a + (15-1)\, d$

$= a + (6-1)\, d + a + (11-1)\, d + a + (13-1)\, d$

$\Rightarrow \quad 2a + 16\, d = 3a + 27\, d$

$\Rightarrow \quad a = -11\, d$

we have to find the value of n such that $a + (n-1)\, d = 0$.

$\therefore \quad 0 = -11d + (n-1)\, d \Rightarrow n = 12.$

2. (*d*) $\log_3 2$, $\log_3 (2^x - 5)$, $\log_3 (2^x - 7/2)$ are in arithmetic progression,

$\therefore \quad 2\log_3 (2^x - 5) = \log_3 2 + \log_3 (2^x - 7/2)$

$\Rightarrow \quad (2^x - 5)^2 = 2 \times (x^2 - 7/2)$

Putting $2x = t$

$\Rightarrow \quad (t-5)^2 = 2 \times (t - 7/2)$

$\Rightarrow \quad t^2 - 10t + 25 = 2t - 7$

$\Rightarrow \quad t^2 - 12t + 32 = 0 \Rightarrow t = 4, 8$

Now, $2^x = 4$

$\Rightarrow x = 2$ which is not possible because $2^x - 5$ is negative.

If, $2^x = 8 \Rightarrow x = 3$

$\therefore \quad x = 3.$

3. (*c*) Let there be total n layers of balls.

1st layer $\rightarrow$ 1 ball

2nd layer $\rightarrow$ 3 balls

3rd layer $\rightarrow$ 6 balls

4th layer $\rightarrow$ 10 balls

... ...

nth layer $\rightarrow \frac{n(n+1)}{2}$ balls

According to the question,

$$\sum \frac{n(n+1)}{2} = 8436$$

$$\Rightarrow \sum \frac{n^2}{2} + \frac{n}{2} = 8436$$

$$\Rightarrow \sum \frac{n^2}{2} + \sum \frac{n}{2} = 8436$$

$$\Rightarrow \frac{1}{2} \times \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \times \frac{n(n+1)}{2} = 8436$$

$$\Rightarrow \frac{1}{2}\left\{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right\} = 8436$$

Now, go through the options:

$n = 36$ satisfies

$\therefore$ Number of layers = 36.

4. (*d*) $T = \{3, 11, 19, 27, ..., 451, 459, 467\}$

Terms in set T are in A.P.

We have to find the number of elements in S (subset of T) such that no two elements add up to 470. As we know, in a finite A.P. the sum of the terms equidistant from beginning and end is always the same. Sum of first and last term = 3 + 467 = 470.

$$\text{Number of terms} = \frac{467 - 3}{8} + 1 = 59$$

So, there are 29 pairs which give 470 as sum.

3, 11, 19, 27, ..., 227, 235, 243, ?? 443, 451, 459, 467

So, the number of element such that sum of no two elements is 470 = 59 – 29 = 30.

5. (*a*) No. of students who answered one or more questions wrongly = 2^{n-j}

The value of j lies between 1 and n (including 1 and n)

$\therefore$ Total number of wrong answers

$$= 2^{n-1} + 2^{n-2} + 2^{n-3} + ... + 2^1 + 2^0$$

According to the question,

$$2^0 + 2^1 + ... + 2^{n-3} + 2^{n-2} - 2^{n-1} = 4095$$

$$\Rightarrow \frac{1(2^n - 1)}{2 - 1} = 4095 \Rightarrow 2n - 1 = 4095$$

$$\Rightarrow 2^n = 4095 + 1 = 40996 \Rightarrow n = 12.$$

6. (*b*) Since $y - x = z - y$

$\therefore$ x, y and z are in AP.

Let x, y and z are $(a - d), (a)$ and $(a + d)$

Again, $xyz = 4$

$$\Rightarrow (a - d)\, a\, (a + d) = 4$$

$$\Rightarrow a(a^2 - d^2) = 4$$

$$\Rightarrow a^2 - d^2 = \frac{4}{a} \Rightarrow d^2 = a^2 - \frac{4}{a}$$

For minimum possible value of y, i.e., a, d should be equal to zero. That is,

$$a^2 - \frac{4}{a} = 0 \Rightarrow a^2 = \frac{4}{a} \Rightarrow a^3 = 2^2$$

$$\Rightarrow a = (2^2)^{1/3} = 2^{2/3}.$$

7. (*a*) Let, $S = 1 + 2x + 4x^2 + 7x^3 + 10x^4 + ...$ (1)

Then,

$$xS = x + 2x^2 + 4x^3 + 7x^4 + ... \quad (2)$$

From (1) and (2),

$$S(1 - x) = 1 + x + 2x^2 + 3x^3 + 4x^4 + ... \quad (3)$$

$$xS(1 - x) = x + x^2 + 2x^3 + 3x^4 + ... \quad (4)$$

From (3) and (4),

$$S(1 - x)^2 = 1 + x^2 + x^3 + x^4 + ...$$

$$\Rightarrow S(1 - x)^2 = 1 + x^2(1 + x + x^2 + x^3 + ...)$$

$$\Rightarrow S(1 - x)^2 = 1 + \frac{x^2}{1 - x}$$

$$\therefore S = \frac{[1 - x(1 - x)]}{(1 - x)^3}$$

8. (*a*)

$$\text{Area} = 36 + 36\frac{1}{2} + 36\frac{1}{4} + 36\frac{1}{8} ...$$

$$= \frac{36}{1 - \frac{1}{2}} = 36 \times 2 = 72$$

$$= 72 \text{ cm}^2.$$

9. (*b*) The two-digit number is of the form $7n + 3$

First two-digit number will be for $n = 1$

i.e., $7 \times 1 + 3 = 10$

Last two-digit number will be for $n = 13$

i.e., $7 \times 13 + 3 = 94$

No. of teems = 13

$$\text{Sum of all 13 terms} = \frac{13}{2}(10 + 94)$$

$$= 13 \times 52 = 676.$$

10. (*c*) We have to find the sum of the series

$$1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}+...$$

Putting $\frac{1}{7}=x$ we get

$1+2^2x+3^2x^2+4^2x^3+5^2x^4+...$

Let, $S=1+4x+9x^2+16x^3+25x^4+...$

$$S.x=x+4x^2+9x^3+16x^4+...$$

$$S-S.x=1+3x+5x^2+7x^3+9x^4+...$$

$$x(S-Sx)=x+3x^2+5x^3+7x^4+...$$

$(S-Sx)-\{x(S-Sx)\}$

$$=1+2x+2x^2+2x^3+...+\text{to }\infty$$

$$\Rightarrow \quad (1-x)^2S=1+\frac{2x}{1-x}\text{; Since } |x|<1$$

$$\Rightarrow \quad S=\frac{1+x}{(1-x)^3}$$

We may use it as direct formula for solving this type of problem.

Substituting $x=\frac{1}{7}$ we get

$$S=\frac{1+\frac{1}{7}}{\left(1-\frac{1}{7}\right)^3}=\frac{8\times 243}{7\times 216}=\frac{49}{27}.$$

11. (*b*) Let $S=1+2\times 2+3\times 2^2+4\times 2^3+...+100\times 2^{99}$

$\therefore\ 2S=1\times 2+2\times 2^2+3\times 2^3+...+99\times 2^{99}$
$+100\times 2^{100}$

Subtracting, we get

$-S=1+1\times 2+1\times 2^2+1\times 2^3+...+1\times 2^{99}$
-100×2^{100}

$$=\frac{1(2^{100}-1)}{2-1}-100\times 2^{100}$$

$$=2^{100}-1-100\times 2^{100}$$

$\therefore\ S=100\times 2^{100}-2^{100}+1=99\times 2^{100}+1$

12. (*a*) Given expression

$$=\left[\frac{15\times 16}{2}\right]^2-\frac{15\times 16}{2}=(120)^2-120$$

$=120\times 119=14280.$

13. (*a*) $(a+b)^2, (a^2+b^2), (a-b)^2, ...$

This is a series in A.P. with common difference $(-2ab)$.

Given, $n=20$

$$\therefore \quad S(a,b)=\frac{20}{2}[2(a+b)^2+(20-1)(-2ab)]$$

$$=10\,[2a^2+2b^2+4ab-38ab]$$

$$=20\,[a^2+b^2-17ab]$$

$$\therefore \quad S(7,3)=20\,[49+9-357]$$

$$=20\times(-299)=-5980.$$

14. (*b*) To find the sum of the series:

$10+10+15+15+(20+20)+...+(150+150)+155$

$=2\,(10+15+...+150)+155$

$$=2\left[\frac{29}{2}(2\times 10+(29-1)\times 5)\right]+155$$

$=29\,(20+140)+155$

$=29\times 160+155$

$=4640+155=4795.$

15. (*c*) Max. sum $=S$

$$=50+48+46+...+6+4+2$$

$$=\frac{25}{2}\,[2\times 50+(25-1)(-2)]$$

$$=\frac{25}{2}\,[100-48]=25\times 26=650.$$

16. (*c*) The roots of the given equation are –2, –1, –2/3. These roots are neither in A.P. nor in G.P. These roots are in H.P. because $-\frac{1}{2},-1,-\frac{3}{2}$ are in A.P. with common difference $-\frac{1}{2}$.

$\therefore\ -2,-1,-\frac{2}{3}$ are in H.P.

17. (*c*) Let x be the original number of apples, then first customer bought $\frac{x}{2}+\frac{1}{2}=\frac{x+1}{2}$, the 2nd customer bought $\frac{1}{2}\left(x-\frac{x+1}{2}\right)+\frac{1}{2}=\frac{x+1}{2^2}$, the third customer bought $t\ \frac{1}{2}\left(x-\frac{x+1}{2}-\frac{x+1}{4}\right)+\frac{1}{2}=\frac{x+1}{2^3}$ and the 7th customer bought $\frac{x+1}{2^7}$, we thus have the following equation: $\frac{x+1}{2}+\frac{x+1}{2^2}+\frac{x+1}{2^3}+...+\frac{x+1}{2^2}=x$

or, $(x+1)$

$$\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^7}\right)=x$$

Computing the sum of the terms of the G.P. in the parentheses, we get

$$\frac{x}{x+1} = 1-\frac{1}{2^7} \Rightarrow x = 2^7 - 1 = 127.$$

18. (*b*) The given product = $2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots}$

Let, $S = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots$ (1)

$\therefore \quad \frac{1}{2}S = \frac{1}{8}+\frac{2}{16}+\frac{3}{32}+\ldots$ (2)

(1) – (2) $\frac{1}{2}S = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots = \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{1}{2}$

$\therefore$ The given product $= 2^1 = 2.$

19. (*a*) S_n = Sum of n terms of an A.P.

$$= \frac{n}{2}[2a+(n-1)d]$$

where a = first term, d = common difference

$$\therefore \quad \frac{3+5+7+\ldots+n\text{ terms}}{5+8+11+\ldots+10\text{ terms}} = 7$$

$$\Rightarrow \quad \frac{\frac{n}{2}[2\times3+(n-1)\times2]}{\frac{10}{2}[2\times5+(10-1)\times3]} = 7$$

$$\Rightarrow \quad \frac{n(2n+4)}{370} = 7$$

$\Rightarrow \quad 2n^2+4n-2590 = 0$

$\Rightarrow \quad n^2+2n-1295 = 0$

$\Rightarrow \quad n^2+37n-35n-1295 = 0$

$\Rightarrow \quad n(n+37)-35(n+37) = 0$

$\Rightarrow \quad (n-35)(n+37) = 0$

$\Rightarrow \quad n = 35.$

20. (*c*) Sum of the first n natural numbers $= \frac{n(n+1)}{2}$

Sum of the squares of the first n natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

$$\therefore \quad \frac{n(n+1)}{2} = \frac{1}{5}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$\Rightarrow \quad 2n+1 = 15 \Rightarrow n = 7.$

21. (*a*) x can take the values 1, 2, ... 98.

For $x = 1$, y can take the values 1, 2, ..., 98

For $x = 2$, y can take the values 1, 2, ..., 97

For $x = 98$, y can take the value 1

Since z is dependent on x and y, therefore the required number of solutions

$$= 98+97+96+\ldots+1$$

$$= \frac{98(98+1)}{2} = 4851.$$

22. (*a*) x, y, z are in G.P.

$x+y, y+z, z+x$ are in A.P. (1)

$\therefore$ Common ratio of the G.P. $= \frac{y}{x} = \frac{z}{y} = r$, say

Also (1)

$$\Rightarrow \quad y+z = \frac{(x+y)+(z+x)}{2} \Rightarrow 2x = y+z$$

$$\therefore \quad r = \frac{y}{x} = \frac{y}{(y+z)/2} = \frac{2y}{y+z} = \frac{2}{1+\frac{z}{y}}$$

$$\Rightarrow \quad r = \frac{2}{1+r}$$

$\Rightarrow \quad r^2+r-2 = 0$

$\Rightarrow \quad (r-1)(r+2) = 0$

$\Rightarrow \quad r = -2 \qquad [r = 1 \Rightarrow x = y = z].$

23. (*c*) Let the digits of a three-digit number be x, y and z and the number be $100z + 10y + x$, where x, y, z are in G.P.

$\therefore \quad y^2 = xz$

$\Rightarrow xz$ must be a square number

i.e., $xz = 9$, i.e., $x = 1, z = 9$

$\therefore \quad y = 3$, so that x, y, z are in A.P.

$\therefore$ The number is 931.

$\Rightarrow$ The other number will be 531 so that 1, 3 and 5 are in A.P. Their sum = 931 + 531 = 1462.

24. (*a*) $4+12+20+28+\ldots+996$

$T_n = a+(n-1)d$

Here, $a = 4, d = 8,$

$T_n = 996$

$\therefore \quad 996 = 4+(n-1)\times8$

$\Rightarrow \quad 8n-8+4 = 996 \Rightarrow n = 125$

$$\therefore \quad S_{125} = \frac{125}{2}[2\times4+(125-1)\times8]$$

$$= \frac{125}{2}[8\times125] = 62500.$$

25. (*b*) x, y, z are in G.P. $\Rightarrow y = \sqrt{xz}$

a, b, c are in A.P. $\Rightarrow b = \frac{a+c}{2}$

$\therefore$ The expression $= x^{\frac{a+c}{2}-c}(xz).z^{\frac{c-a}{2}a-\frac{a+c}{2}}$

$$= x^{\frac{a+c}{2} - c + \frac{c-a}{2}} \cdot z^{\frac{c-a}{2} + a - \frac{a+c}{2}}$$

$$= x^0 z^0 = 1.$$

26. (*c*) log2, log $(2^x - 1)$,

log $(2^x + 3)$ are in A.P.

$\Rightarrow \quad 2 [\log (2^x - 1)] = \log 2 + \log (2^x + 3)$

$= \log [2 \times (2^x + 3)]$

$\Rightarrow \quad \log (2^x - 1)^2 = \log [2^{x+}1 + 6)]$

$\Rightarrow \quad (2^x - 1)^2 = 2^{x+1} + 6 = 2^x \times 2 + 6$

Let, $2^x = y$

$\therefore \quad (y - 1)^2 = 2y + 6$

$\Rightarrow \quad y^2 - 2y + 1 = 2y + 6$

$\Rightarrow \quad y^2 - 4y - 5 = 0$

$\Rightarrow \quad (y - 5)(y + 1) = 0$

$\Rightarrow \quad y = 5, -1.$

If $\quad y = 5 \Rightarrow 2^x = 5$

$\Rightarrow \quad x \log 2 = \log 5$

$\Rightarrow \quad x = \frac{\log 5}{\log 2} \Rightarrow x = \log_2{}^5.$

27. (*b*) $\quad t_n = \frac{n}{n+2}$

$$t_3 = \frac{3}{3+2} = \frac{5}{3}, t_4 = \frac{4}{6}, t_5 \frac{5}{7}, \ldots t_{53} = \frac{53}{55}$$

$$t_3 \times t_4 \times t_5 \times \ldots \times t_{53} = \frac{3}{5} \times \frac{4}{6} \times \frac{5}{7} \times \ldots \times \frac{53}{55}$$

$$= \frac{2}{495}.$$

28. (*c*) The first day, I sell 1 book, on the second day, I sell 2 books and so on. This is an A.P. and for one month (i.e., 30 days), the number of books sold is same as the sum of first n natural numbers

$$= \frac{n(n+1)}{2} = \frac{30 \times 31}{2} = 465$$

The numbers of books in the beginning is 465.

29. (*b*) The sum of money that the contractor was supposed to pay for the period of one month over the stipulated time is

$$S_{30} = \frac{30}{2} \quad [2 \times 50000 + (30 - 1)\, 4000]$$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right],$$

where, $\quad a = 50{,}000,\ n = 30,\ d = 4000$

$\therefore \quad S_{30} = 10[100000 + 29 \times 4{,}000]$

$\therefore \quad S_{30} =$ ₹3240000 = ₹32.4 lakhs

Loss in the business = 10%

$\therefore$ Amount he received for making the flyover

$$= \frac{3240000}{0.1} = ₹3,24,00,000 = ₹3.24 \text{ crores.}$$

For answers to Questions 30 to 34:

Let the first term of the progression be a.

Also, the common ratio be r and common difference be d.

Thus, $\quad ar - a = (a + 4d) - a$

or, $\quad a(r - 1) = 4d \quad (1)$

Also, $\quad a(r^2 - 1) = 24d \quad (2)$

Dividing (2) by (1) we get: $r + 1 = 6$

Thus, $\quad r = 5.$

Putting in (1), $\quad 4a = 4d\ a = d.$

Also, $\quad a + 4d = 5$, we get $a = 1, d = 1$

Thus, we have the following answers.

30. (*b*)

31. (*a*)

32. (*c*)

33. (*b*)

34. (*d*)

35. (*d*) Distance travelled by the ball ground till it rises 20 m and then comes back to the ground = 40 m

Next it rises 10 m

$\therefore$ Distance from ground to top to ground = 20 m, this continues

So, the series = 40 + 20 + 10 + ...

$$40 \div \left(1 - \frac{1}{2}\right) = 80 \text{ m}$$

But the ball was first thrown from a height of 10 m.

$\therefore$ Total distance = 80 + 10 = 90 m.

36. (*b*) Given the arithmetic progressions

$A_1 \rightarrow 3, 9, 15, 21 \ldots.$ and

$A_2 \rightarrow 5, 13, 21, 29$

We can see that the first term common between the two series is 21. Since the common difference of A_1 and A_2 are 6 and 8 respectively, any two consecutive terms common between A_1 and A_2 differ by L.C.M. (6, 8) i.e., 24. So, the series of common terms also form an arithmetic progression. The series is 21, 45, 69, . . . say let us call it series A_3.

Given that

$$(t_n \text{ of } A_1) + (t_n \text{ of } A_2) = 6{,}000$$

$\Rightarrow (3 + (n - 1)\, 6) + (5 + (n - 1)\, 8) = 6{,}000$

$\Rightarrow \quad 14n - 6 = 6{,}000$

$\Rightarrow \quad n = \dfrac{6000}{14} = 429$

t_{429} of $A_1 = (3 + (429 - 1)\,6 = 2{,}571$

t_{429} of $A_2 = 5 + (429 - 1)\,8 = 3{,}427$

So, all the terms common to A_1 and A_2 will be less than 2571

$\therefore$ The number of terms common to A_1 and A_2 is same as the number of terms in A_3 less than or equal to 2571

i.e., $\left[\dfrac{2571-21}{24}\right]+1 = \left[\dfrac{2550}{24}\right]+1 = 107.$

37. (*c*) Radius of the largest circle $= \dfrac{1}{\sqrt{\pi}}$

$\Rightarrow$ Area of largest circle = 1 square unit

Now, each subsequent circle's radius is half the radius of previous circle. Therefore, areas would be circle fourth.

$\therefore$ Sum of areas of all the circles is S, where

$$S = 1 + \frac{1}{4} + \frac{1}{16} \text{ ...infinite terms}$$

$\therefore \quad S = \dfrac{4}{3}$ square units.

38. (*c*) $1 - y + y^2 - y^3 + y^4 \ldots \infty \; (|y| < 1)$

$\Rightarrow \quad x = \dfrac{1}{1+y}$

$\Rightarrow \quad y = \dfrac{1}{x} - 1 \qquad (1)$

and, $\quad z = 1 + y + y^2 + y^3 + \ldots \infty \; (|y| < 1)$

$\Rightarrow \quad z = \dfrac{1}{1-y}$

$\Rightarrow \quad y = 1 - \dfrac{1}{z} \qquad (2)$

From (1) and (2)

$$\frac{1}{x} + \frac{1}{2} = \frac{2}{1}$$

$\Rightarrow x, 1, z$ are in H.P.

or, 1 is the H.M. of x and z.

39. (*a*) Three numbers $= \dfrac{a}{r}, a, ar$

Double of second number $= 2a$

Hence, $\dfrac{a}{r}$, $2a$ and ar are in A.P.

$\sigma \Rightarrow 4a = \dfrac{a}{r} + ar \Rightarrow 4 = \dfrac{1}{r} + r$

$\Rightarrow \quad r = 2 \pm \sqrt{3}$

Since the G.P. is increasing, $r = 2 + \sqrt{3}$

$\Rightarrow \quad \dfrac{1}{r} = 2 - \sqrt{3}$

Hence, ratio of the first number and third number $= 1{:}r^2 = 1{:}7 - 4\sqrt{3}$.

40. (*b*) Given that the speeds $a, b, c \ldots z$ are in A.P

$\therefore$ Let $a, b, c \ldots$ be $a, a + \Delta, a + 2\Delta, \ldots a + 25\Delta$. Given time taken by Z to meet A, for the first time is 20 sec, i.e.,

$$\frac{100}{25\Delta + a - a} = 20 \Rightarrow \Delta = 0.2 \text{ minutes/second}$$

Time taken by M to complete the race at a speed of $a + 12\Delta$, is = 52 minutes and 5 second

$\Rightarrow \quad \dfrac{100 \times 100}{a + 12\Delta} = 3125$

$\Rightarrow \quad 10{,}000 = (a + 2.4)\,(3125)$

$\Rightarrow \quad a = 0.8$ minutes/second

$\therefore$ Time taken by all of them to meet for first time at the starting point is

$$\text{LCM}\left(\frac{100}{a}, \frac{100}{b}, \frac{100}{c}, \ldots, \frac{100}{z}\right)$$

$$= \text{LCM}\left(\frac{100}{0.8}, \frac{100}{1.0}, \frac{100}{1.2}, \ldots, \frac{100}{0.8 + 25(0.2)}\right)$$

$$= \frac{\text{LCM}(100, 100, \ldots 100)}{\text{HCF}(0.8, 1.0, 1.2 \ldots, 5.8)} = \frac{100}{0.2} = 500 \text{ seconds.}$$

41. (*c*) Let the first term and the common ratio of the arithmetic progression be a and d respectively.

Given, $\quad \dfrac{a + (p-1)d}{a + (p+2)d} = \dfrac{p}{p+3}$

$\Rightarrow \quad pa + 3a + d\,(p-1)\,(p+3) = pa + dp\,(p+2)$

$\Rightarrow \quad 3a = 3d \Rightarrow a = d$

Also given, $\dfrac{\left(\dfrac{3p}{2}\right)[2d + (3p-1)d]}{\left(\dfrac{4p}{2}\right)[2d + (4p-1)d]} = \dfrac{61}{108}$

$\Rightarrow \quad \dfrac{3}{4}\left(\dfrac{3p+1}{4p+1}\right) = \dfrac{61}{108} \Rightarrow p = 20.$

42. (*a*) Go through options

Let 1, 2, 3, 4, 5, ... be an A.P. then

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \frac{1}{\sqrt{a_3} + \sqrt{a_4}} + \frac{1}{\sqrt{a_4} + \sqrt{a_5}}$$

$$= \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}}$$

$$= -\left(\sqrt{1}-\sqrt{2}\right) - \left(\sqrt{2}-\sqrt{3}\right) - \left(\sqrt{3}-\sqrt{4}\right) - \left(\sqrt{4}-\sqrt{5}\right)$$

$$= \sqrt{5}-\sqrt{1} = \sqrt{5}-\sqrt{1} \times \frac{\sqrt{5}+\sqrt{1}}{\sqrt{5}+\sqrt{1}}$$

$$= \frac{5-1}{\sqrt{5}+\sqrt{1}} = \frac{4}{\sqrt{5}+\sqrt{1}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}.$$

43. (*c*) $n = 5$

The weight of each stone (in kg) is 1, 3, 9, 27, 81

1 kg = 1 kg

2 kg = 3 – 1 = 2 kg

3 kg = 3 kg

4 kg = (3 + 1) = 4 kg

5 kg = 9 – (3 + 1) = 5 kg

6 kg = (9 – 3) = 6 kg

7 kg = (9 + 1) – 3 = 7 kg

8 kg = (9 – 1) = 8 kg

9 kg = 9 kg

10 kg = (9 + 1) = 10 kg and so on

Remember he is allowed to put the stones on either side of the balance.

44. (*a*) There are 2^{n-j} students who answer wrongly. by 5 nor by 2. For $j = 1, 2, 3, ..., n$, the number of students will be a GP with base 2. Hence, $1 + 2 + 2^2 + ... + 2^{n-1} =$ 4,095. Using the formula, we get $2^n = 4095 + 1$

$\Rightarrow n = 12.$

45. (*c*) The digits which create confusion are 0, 1, 6, 8, 9.

The total number of two-digit codes having distinct digits and first digit non-zero.

1st place	2nd place
9 options	9 options
(1 to 9)	(one digit is already used out of 1 to 9 and 0 is included)

$= 9 \times 9 = 81$ such codes

Total number of two digit codes which can create confusion

1st digit	2nd digit
4 options	4 options
(1, 6, 8 or 9)	(one digit is already used out of 1, 6, 8 and 9 and 0 is also included)
$= 4 \times 4 = 16$	

But these 16 two-digit codes include 69 and 96, which create no confusion. Apart from these, 10, 60, 80 and 90 are such two digit codes as create no confusion because these codes are no expected. Hence total number of two-digit codes which create no confusion

$= 81 - 16 + 6 = 71.$

46. (*c*) By the given condition in the problem,

Area and perimeter of $S_1 = a^2, 4a$

Area and perimeter of $S_2 = \frac{a^2}{2}, \frac{4a}{\sqrt{2}}$

Area and perimeter of $S_3 = \frac{a^2}{4}, \frac{4a}{(\sqrt{2})^2}$

Area and perimeter of $S_4 = \frac{a^2}{8}, \frac{4a}{(\sqrt{2})^3}$

Then, required ratio

$$= \frac{4a + \frac{4a}{\sqrt{2}} + \frac{4a}{(\sqrt{2})^2} + \frac{4a}{(\sqrt{2})^3} + ...}{a^2 + \frac{a^2}{2} + \frac{a^2}{4} + \frac{a^2}{8} + ...}$$

$$= \frac{4a\left|1 + \frac{1}{\sqrt{2}} + \frac{1}{(\sqrt{2})^2} + \frac{1}{(\sqrt{2})^3} ...\right|}{a^2\left|1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} ...\right|}$$

$$= \frac{4a\left|\frac{1}{1-\frac{1}{\sqrt{2}}}\right|}{a^2\left|\frac{1}{1-\frac{1}{2}}\right|} = \frac{4a\left|\frac{\sqrt{2}}{\sqrt{2}-1}\right|}{a^2 \times 2}$$

$$\Rightarrow \frac{4a\left|\frac{\sqrt{2}}{\sqrt{2}-1}\right|}{a^2 \times 2} = \frac{2\sqrt{2}\,(\sqrt{2}+1)}{a} = \frac{2(2+\sqrt{2})}{a}.$$

47. (*d*) The last instance of nth letter is $\frac{n(n+1)\text{th}}{2}$ letter of series is $S_{23} = 276$ th and $S_{24} = 300$ th. All terms from 276 to 300 are 24th letters of the alphabet i.e., *x*.

48. (*c*) Let '*a*' be the first term and '*r*' be the common difference

$$a + ar = 12 \qquad (1)$$

$$ar^2 + ar^3 = 48 \qquad (2)$$

On dividing Eq. (2) by Eq. (1), we get

$$= \frac{ar^2(1+r)}{a(1+r)}$$

$$= \frac{48}{12} \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$$

and since terms arealternately positive and negative common ratio is '–2'

$\therefore$ First term is $a - 2a = 12$

$\Rightarrow \quad a = -12.$

49. (*b*) $385 \times 4 = 1540.$

50. (*a*) Let the initial number of boys and girls in the group $= B$ and G, respectively.

After 15 girls leave, there are two boys for each girl.

$$B = 2(G - 15) \qquad (1)$$

Then, 45 boys leave after which there are 5 girls for each boy,

$$5(B - 45) = (G - 15)$$

$$5B - 225 = G - 15$$

From Eq. (1),

$$B = 2G - 30$$

$$5(2G - 30) - 225 = G - 15$$

$$\Rightarrow \quad 9G = 225 - 15 + 150$$

$$G = \frac{360}{9} = 40.$$

51. (*a*) $$\frac{\frac{(2n_1 - 4) \times 90°}{n_1}}{\frac{(2 \times 2n_1 - 4) \times 90°}{2n_1}} = \frac{2}{3} \qquad [\because n_2 = 2n_1]$$

$$\therefore \quad n_1 = 4 \text{ and } n_2 = 8.$$

52. (*d*) The inverse of the sum of the series

$$\frac{3}{4} + \frac{3}{36} + \frac{7}{144} + \ldots \text{ is } \frac{(n+1)^2}{n^2 + 2n}.$$

53. (*b*) $a, H_1, H_2 \ldots. H_n, b$ are in H.P.

$$\therefore \quad H_1 = \frac{(n+1)ab}{a + nb}$$

and, $$H_n = \frac{(n+1)ab}{an + b}$$

$$\therefore \quad H_1 + a = \frac{(n+1)ab}{a + nb} + a$$

$$= \frac{nab + ab + a^2 + nab}{a + nb}$$

and, $$H_1 - a = \frac{n\,ab + ab - a^2 - nab}{a + nb}$$

$$\therefore \quad \frac{H_1 + a}{H_1 - a} = \frac{2\,nb + ab + a^2}{ab - a^2}$$

$$= \frac{2\,nb + b + a}{b - a}$$

Similarly, $$\frac{H_n + b}{H_n - b} = \frac{2na + a + b}{a - b}$$

$$\therefore \quad \frac{H_1 + a}{H_1 - a} + \frac{H_n + b}{H_n - b} = \frac{2nb - 2na}{b - a}$$

$$= 2n.$$

54. (*d*) a, b, c are in A.P.

$$\therefore \quad 2b = a + c$$

but it is given that

$$b + a + c = \frac{3}{2}$$

$$\Rightarrow \quad 2b + b = \frac{3}{2}$$

$$\Rightarrow \quad b = \frac{1}{2}$$

$$\therefore \quad a + c = 1 \qquad (1)$$

Now, a^2, b^2, c^2 are in G.P.

$$\therefore \quad b^4 = a^2 c^2$$

$$\Rightarrow \quad b^2 = -a\,c \qquad (\therefore a < b < c)$$

$$\Rightarrow \quad \frac{1}{4} = -a\,c$$

$$\Rightarrow \quad c = -\frac{1}{4a} \qquad (2)$$

From Eqs. (1) and (2),

$$1 - a = -\frac{1}{4a}$$

$$\Rightarrow \quad 4a - 4a^2 = -1$$

$$\Rightarrow \quad 4a^2 - 4a - 1 = 0$$

$$\Rightarrow \quad a = \frac{+4 \pm \sqrt{16 + 16}}{2 \times 4}$$

$$\Rightarrow \quad a = \frac{+4 \pm \sqrt{32}}{2 \times 4}$$

$$\Rightarrow \quad a = \frac{4}{8} - \frac{4\sqrt{2}}{8}$$

$$\Rightarrow \quad a = \frac{1}{2} - \frac{1}{\sqrt{2}}.$$

55. (*a*) $1^2 - 2^2 + 3^2 - 4^2 + \ldots 2001^2 - 2002^2 + 2003^2$

$= (1^2 + 3^2 + \ldots 2001^2 + 2003^2) - (2^2 + 4^2 + 6^2 + \ldots 2002^2)$

$= (1^2 + 3^2 + \ldots 2003^2) - 2^2\,[1^2 + 2^2 + 3^2 + \ldots 1001^2]$

$$= (1^2 + 3^2 + \ldots 2003^2) - 4\left[\frac{1001 \times (1001+1)\,(2002+1)}{6}\right]$$

$$= (1^2 + 3^2 + \ldots 2003^2) - 4 \times \left[\frac{1001 \times 1002 \times 2003}{6}\right]$$

We know that

$= [1^2 + 3^2 + 5^2 \ldots 2003^2]$

$= [1^2 + 2^2 + 3^2 + \ldots 2003^2] - [2^2 + 4^2 + 6^2 \ldots 2002^2]$

$$= \frac{2003 \times 2004 \times 2007}{6} - \frac{4 \times 1001 \times 1002 \times 2003}{6}$$

$$\therefore \quad = \frac{2003 \times 2004 \times 4007}{6} - \frac{4 \times 1001 \times 1002 \times 2003}{6}$$

$$- \frac{4 \times 1001 \times 1002 \times 2003}{6}$$

$$= \frac{2003 \times 2004 \times 4007}{6} - \frac{4 \times 2 \times 1001 \times 1002 \times 2003}{6}$$

$$= \frac{2004}{6}\,[2003 \times 4007 - 4 \times 1001 \times 2003]$$

$= 334\,[8026021 — 8020012]$

$= 334 \times 6009$

$= 2007006.$

56. (*c*) As per question, let first term $= a$ and common difference $= d$

$$2400 = \frac{30}{2}[2a + (30-1)\,d] \qquad (1)$$

$$3600 = \frac{40}{2}[2a + (40-1)\,d] \qquad (2)$$

To solve Eqs. (1) and (2)

$a = 51$ and $d = 2$

Hence, the value of 8th instalment

$= ₹[51 + (7) \times 2]$

$= ₹65.$

57. (*d*) Let Rashid's savings will last till '*n*' months

$$\therefore \quad \frac{n}{2}[2 \times 2000 + (n-1)\,500] = 60000$$

[$\because$ Expenditure every month increases by 500 rupees]

$\Rightarrow \quad n^2 + 7n - 240 = 0$

$\Rightarrow \quad n = 12.38$

$\therefore$ Rashid after 13 months will start borrowing money from his friends.

58. (*c*) From the given information, sum of the first $(x-1)$ natural numbers

= sum of the natural numbers from $(x+1)$ to 49

$$\text{So,} \quad \frac{(x-1)\,(x)}{2} = \frac{49 \times 50}{2} - \frac{(x)\,(x+1)}{2}$$

$$2x^2 = 49 \times 50 \Rightarrow x = 35.$$

59. (*b*) $\sum_{x=1}^{n}(x) = (1) + (2) + (3) + (4) + \ldots + (n)$

$= (1) + (1+1) + (1+2) + (1+3) + \ldots + (n)$

$= (1) + (1)(1) + (1)(2) + (1)(3) + \ldots + (n)$

$= 2 + 2 \times 2 + 2 \times 2 \times 2 + 2 \times 2 \times 2 \times 2 + \ldots + (n)$

$= 2 + 4 + 8 + 16 + \ldots (n)$

Now, this is a GP with common ratio = 2.

On equating this with 1022, we get $n = 9$.

60. (*d*) The integers are 4, 4, 4, 8, 10, 20 and x

Consider (A) $x < 4$

$$\text{Mean} = \frac{50+x}{?}, \text{ Median 4, Mode} = 4$$

If these are in AP, mean 4.

$$\text{So,} \quad \frac{50+x}{7} = 4 \Rightarrow x = -22$$

Consider (B) $4 < x < 8$

$$\frac{50+x}{7} \text{ i.e., } \frac{54}{7} < \text{Mean} < \frac{58}{7}$$

Mean $= x$, Mode = 4

as these are in AP, $x = 6$, Mean = 8

Consider (C) $8 < x$

$$\text{Mean} = \frac{50+x}{7} > \frac{58}{7}$$

Median = 8, Mode = 4

As these are in AP, Mean = 12 i.e., $x = 34$

So, x can be -22, 6 or 34

The sum of these is 18, which is not there among the option. In exam, we would have to make a decision. the negative value of -22 seems to have been ignored.

In this case, the mean, median and mode are all equal to 4. As 18 is not there among the options, we have to select 40 (6 + 34).

61. (*c*) Weight of a solid spherical ball is proportional to the cube of its radius. The radius and weights of the 10 bails on the day 10 are tabulated below.

Ball put on day	Radius (mm)	Weight (gm)
1	2^9	8^9
2	2^8	8^8
3	2^7	8^7
⋮	⋮	⋮
9	2^1	8
10	2^0	1

The total weight of the 10 balls on day 10 is

$$1+8+8^2+8^9=\frac{8^{10}-1}{7}=\frac{2^{30}-1}{3}$$

The weight of the 10 balls before they were put in the pot = 10 g

∴ The weight of the gold 'made' by the saint (g).

$$\frac{2^{30}}{7}-10=\frac{2^{30}-71}{7}.$$

62. (*b*) As x, y and z are in harmonic progression, $\frac{1}{x},\frac{1}{y}$ and $\frac{1}{z}$ are in arithmetic progession.

$$\therefore \frac{1}{z}-\frac{1}{y}=\frac{1}{y}-\frac{1}{x}$$

$$\Rightarrow \frac{y-z}{yz}=\frac{x-y}{xy}.$$

Multiplying both the sides by xyz, we get

$$x(y-z)=z(x-y)$$

$$x=\frac{z(x-y)}{y-z}.$$

It implies II is true.

And,

$$\frac{1}{z}-\frac{1}{y}=\frac{1}{y}-\frac{1}{x}$$

$$\Rightarrow \frac{2}{y}=\frac{1}{z}+\frac{1}{x}$$

$$\Rightarrow \frac{2}{y}=\frac{z+x}{xz}$$

$$\Rightarrow x=\frac{y(x+z)}{2z}$$

It implies I is true.

63. (*c*) A_1 has 1 element, A_2 has 2 elements, A_3 has 3 elements,…, A_{49} has 49 elements.

Number of elements in A_1, A_2, A_3,…A_{49} are all combined.

Therefore,

$$1+2+3+\cdots+49=\frac{49\times 50}{2}$$

$$=49\times 25=1225.$$

Thus, $A_{50}=(1226,1227,\ldots,1275)$.

Thus, sum of elements in A_{50},

$$\frac{50}{2}(1226+1275)=25\times 2501$$

$$=62525.$$

64. (*a*) Let the value of certificates purchased in the first year be ₹a.

The difference between the values of the certificates is ₹300 ($d=300$).

As it follows arithmetic progression, the total value of certificates after 20 years is given by

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow 83000=\frac{20}{2}[2a+(20-1)300]$$

$$\Rightarrow 83000=10(2a+5700)$$

$$\Rightarrow \quad 2a+5700=8300.$$

On simplifying, we get

$a=$ ₹1300

The value of the certificates purchased by him in the 13th year is

$$a+(n-1)d$$

$$=1300+(13-1)\times 300$$

$=$ ₹4900.

65. (*c*) If we consider the third term to be x

The 15th term will be $(x+12d)$

6th term will be $(x+3d)$

11th term will be $(x+8d)$ and 13th term will be $(x+10d)$

We are given

$2x+12d=3x+21d$ or $x+9d=0$

$x+9d$ will be the 12th term.

66. (*b*) The sum of the squares of the first n odd natural numbers = Sum of the squares of the $(2n-1)$ natural numbers − Sum of the squares of the first $(n-1)$ even natural numbers

$$\therefore S_n = \frac{(2n-1)(2c)(4n-1)}{6} - 4\left[\frac{(n-1)n(2n-1)}{6}\right]$$

$$= \frac{n(2n-1)(2n+1)}{3}$$

As $S_n = 533n$,

$$\frac{n(2n-1)(2n+1)}{3} = 533n$$

$\Rightarrow 4n^2 = 1600$

$\Rightarrow n = 20.$

67. (*b*) [29(2)]

We have

$$T_n = \frac{n+3}{n}.$$

Therefore, $T_4T_5T_6 \ldots T_{58}T_{59}T_{60}$

So,

$$\left(\frac{7}{4}\right)\left(\frac{8}{5}\right)\left(\frac{9}{6}\right)\left(\frac{10}{7}\right)\cdots\left(\frac{61}{58}\right)\left(\frac{62}{59}\right)\left(\frac{63}{60}\right)$$

$$= \frac{61.62.63}{4.5.6} = 1985.55.$$

68. (b)

As (a^2+b^2), (b^2+c^2) and (a^2+c^2) are in G.P.,

$(b^2+c^2)^2 = (a^2+b^2)(a^2+c^2)$

$\Rightarrow \quad b^4+c^4+2b^2c^2 = a^4+a^2b^2+a^2c^2+b^2c^2$

$\Rightarrow \quad b^4+c^4+b^2c^2 = a^4+a^2b^2+a^2c^2$

$\Rightarrow \quad b^2(b^2+c^2)+c^4 = a^2(b^2+c^2)+a^4$

$\Rightarrow \quad (b^2-a^2)(b^2+c^2) = a^4-c^4$

$$\therefore b^2-a^2 = \frac{a^4-c^4}{b^2+c^2}.$$

69. (*d*) As $2a$, b and $2c$ are in A.P.,

$2b = 2a+2c$

$\Rightarrow b = a+c$

$$x = \frac{-b \pm \sqrt{(a+c)^2-4ac}}{2a}$$

$$= \frac{-b \pm (a-c)}{2a}$$

$$= \frac{-(a+c)\pm(a-c)}{2a} = -\frac{c}{a} \text{ or}, -1.$$

70. (*d*) Let the A.P. be a, $a+d$, $a+2d$,........

And the G.P. be b, br, br^2, br^3,.....

Given : sum of 10 terms of A.P. = 155

So,

$$\frac{n}{2}[2a+(n-1)d] = 155$$

$$\Rightarrow \quad \frac{10}{2}(2a+9d) = 155$$

$\Rightarrow \quad 2a+9d = 31$ (1)

Also, given that $b + br = 9$

Since b = d and $r = a$

So, $d + ad = 9$ (2)

Solving (1) and (2), we get

$a = 2, \frac{25}{2}$ and $d = 3, \frac{2}{3}$

So, the AP can be 2, 5, 8, 11,…

Or, $\frac{25}{2}, \frac{79}{6}, \frac{83}{6}$,........

71. (*d*) Let us consider an AP of 6 terms, say, 1, 2, 3, 4, 5 and 6.

Then, $3n = 6$, $2n = 4$, $n = 2$

Now $S_3 - S_2 - S_1$

$= (1+2+3+4+5+6) - (1+2+3+4) - (1+2)$

$= 21 - 10 - 3 = 8$

In the above AP, $n = 2$, $d = 1$, $a = 1$

From the options, we get

$2\,n^2 d = 2 \times 2^2 \times d = 8$

that is equal to $S_3 - S_2 - S_1$,

Therefore, (*d*) is the correct answer.

72. (*c*) Given expression

$$= \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{19.21}$$

$$= \frac{1}{2}\left(1-\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{5}-\frac{1}{7}\right)$$

$$+ \ldots. + \frac{1}{2}\left(\frac{1}{19}-\frac{1}{21}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{21}\right) = \frac{1}{2} \times \frac{20}{21} = \frac{20}{42} = \frac{10}{21}.$$

73. (*a*) $U_{n+1} = 2U_n + 1 (n = 0, 1, 2, \ldots.)$

Put $n = 0, U_1 = 1$

$n = 1, U_2 = 3$

$n = 2, U_3 = 7$

$n = 3, U_4 = 15$

$n = 4, U_5 = 31$

Seeing the pattern it is clear that $U_n = 2^n - 1$

Hence, $U_{10} = (2)^{10} - 1 = 1023.$

74. (*c*) If the girl counts the way as given in the question, then counting serial for the thumb will be 1, 8, 17, 25, ….. Hence, number 1992 will also fall on thumb. Hence, number 1994 will end on her middle finger.

75. (*c*) 1600 + 669 = 2269 = Not a perfect square

1500 + 669 = 2169 = Not a perfect square

1540 + 669 = 2209 = Square of 47

1690 + 669 = 2359 = Not a perfect square

Thus, by adding 669 to each of the alternatives, we note that 1540 + 669 = 2209, and this is the only square number.

So each side of the square contains $\sqrt{2209} = 47$ balls.

$\therefore$ Each side of the triangle will contain 55 balls

Thus the triangle will contain

$$1+2+3+\ldots\ldots\ldots\ldots+55 = \frac{55(55+1)}{2} = 1540 \text{ balls.}$$

76. (*a*) Since

$x, 17, 3x - y^2 - 2$ and $3x + y^2 - 30$ are in A.P.

$\therefore 17 - x = 3x + y^2 - 30 - 3x + y^2 + 2$

$\Rightarrow \quad x + 2y^2 = 45 \qquad (1)$

Also, $17 - x = 3x - y^2 - 2 - 17$

$\Rightarrow \quad 4x - y^2 = 36 \qquad (2)$

Solving equations (1) and (2), we get $x = 13$

Now, $x + 17 + 3x - y^2 - 2 + 3x + y^2 - 30$

$= 7x - 15 = 7(13) - 15 = 76$

Out of the given options, 76 is only divisible by 2.