24. Trigonometric Ratios of Complementary Angle

Let us Work Out 24

1 A. Question

Let us evaluate :

$$\frac{\sin 38°}{\cos 52°}$$

Answer

Given, $\frac{\sin 38°}{\cos 52°}$

Need to evaluate the given equation

⇒ we know that cos(90 - θ) = sinθ

∴ cos 52° = cos(90 - 38)°

= sin 38°

$\Rightarrow \frac{\sin 38°}{\cos 52°} = \frac{\sin 38°}{\sin 38°}$

= 1

1 B. Question

Let us evaluate :

$$\frac{\operatorname{cosec} 79°}{\sec 11°}$$

Answer

Given, $\frac{\operatorname{cosec} 79°}{\sec 11°}$

Need to evaluate the given equation

⇒ we know that sec(90 - θ) = cosecθ

∴ sec 11° = sec(90 - 79)°

= cosec 79°

$\Rightarrow \frac{\text{cosec } 79°}{\sec 11°} = \frac{\text{cosec } 79°}{\text{cosec } 79°}$

= 1

1 C. Question

Let us evaluate :

$$\frac{\tan 27°}{\cot 63°}$$

Answer

Given, $\frac{\tan 27°}{\cot 63°}$

Need to evaluate the given equation

⇒ we know that tan(90 - θ) = cotθ

∴ tan 27° = tan(90 - 63)°

= cot 63°

$\Rightarrow \frac{\tan 27°}{\cot 63°} = \frac{\cot 63°}{\cot 63°}$

= 1

2 A. Question

Let us show that:

sin66° - cos24° = 0

Answer

Given, sin66° - cos24° = 0

Need to prove the given equation as zero

⇒ we know that cos(90 - θ) = sinθ

∴ cos24° = cos(90 - 66)°

= sin66° - - - eq (1)

⇒ sin66° - cos24° = 0

[substitute eq(1)]

⇒ sin66° - sin66° = 0

Hence, LHS = RHS

2 B. Question

Let us show that:

$\cos^2 57° + \cos^2 33° = 1$

Answer

Given, $\cos^2 57° + \cos^2 33°$

Need to prove the given equation as one

⇒ we know that $\cos(90 - \theta) = \sin\theta$

∴ $\cos 33° = \cos(90 - 57)°$

$= \sin 57°$ - - - eq (1)

⇒ Substitute eq(1) in the given equation

∴ $\cos^2 57° + \sin^2 57°$

⇒ we know that, $\sin^2\theta + \cos^2\theta = 1$

∴ $\cos^2 57° + \sin^2 57° = 1$

Hence, proved

2 C. Question

Let us show that:

$\cos^2 75° - \sin^2 15° = 0$

Answer

Given, $\cos^2 75° - \sin^2 15°$

Need to prove the given equation as zero

⇒ we know that $\cos(90 - \theta) = \sin\theta$

∴ $\cos 75° = \cos(90 - 15)°$

$= \sin 15°$ - - - eq (1)

⇒ Substitute eq(1) in the given equation

∴ $\cos^2 75° - \sin^2 15° =$

$\sin^2 15° - \sin^2 15° = 0$

Hence, proved

2 D. Question

Let us show that:

$\text{cosec}^2 48° - \tan^2 42° = 1$

Answer

Given, $\text{cosec}^2\ 48° - \tan^2\ 42°$

Need to prove the given equation as one

⇒ we know that $\tan(90 - θ) = \cot θ$

∴ $\tan 42° = \tan(90 - 48)°$

$= \cot 48°$ - - - eq (1)

⇒ Substitute eq(1) in the given equation

∴ $\text{cosec}^2\ 48° - \cot^2 48°$

⇒ we know that $\cot θ = \frac{\cos θ}{\sin θ}$

And $\text{cosec} θ = \frac{1}{\sin θ}$

∴ $\frac{1}{\sin^2 48°} - \frac{\cos^2 48}{\sin^2 48}$

$= \frac{1 - \cos^2 48}{\sin^2 48}$

[$\sin^2 θ + \cos^2 θ = 1$

∴ $1 - \cos^2 θ = \sin^2 θ$]

⇒ $\frac{\sin^2 48}{\sin^2 48}$

$= 1$

∴ $\text{cosec}^2\ 48° - \tan^2\ 42° = 1$

Hence, proved

2 E. Question

Let us show that:

$\sec 70° \sin 20° + \cos 20° \text{cosec} 70° = 2$

Answer

Given, $\sec 70° \sin 20° + \cos 20° \text{cosec} 70° = 2$

Need to prove the given equation as two

⇒ we know that sec(90 - θ) = cosecθ and cosec(90 - θ) = secθ

∴ sec70° = sec(90 - 20)°

= cosec20° - - - eq (1)

And cosec70° = cosec(90 - 20)

= sec20° - - - - - eq(2)

⇒ Substitute eq(1) and eq(2) in the given equation

∴ cosec20°sin20° + cos20°sec20° = 2

$\Rightarrow \frac{1}{\sin 20^\circ}\sin 20^\circ + \cos 20^\circ \frac{1}{\cos 20^\circ} = 2$

$[\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ and $\sec\theta = \frac{1}{\cos\theta}]$

$\Rightarrow \frac{\sin 20}{\sin 20} + \frac{\cos 20}{\cos 20} = 2$

⇒ 2 = 2

Hence, proved

3 A. Question

If two angles and are complementary angle, let us show that

$\sin^2\alpha + \sin^2\beta = 1$

Answer

Given, two angles are complementary $\alpha + \beta = 90°$

$\Rightarrow \sin^2\alpha + \sin^2\beta$

$\Rightarrow \sin^2\alpha + \sin^2(90 - \alpha)$

$\Rightarrow \sin^2\alpha + \cos^2\alpha$

= 1

$[\sin^2\theta + \cos^2\theta = 1]$

3 B. Question

If two angles and are complementary angle, let us show that

$\cot\beta + \cos\beta = \frac{\cos\beta}{\cos\alpha}(1 + \sin\beta)$

Answer

Given, cotβ + cosβ

$[\cot\beta = \frac{\cos\beta}{\sin\beta}]$

$\Rightarrow \frac{\cos\beta}{\sin\beta} + \cos\beta$

$= \cos\beta(\frac{1}{\sin\beta} + 1)$

$= \cos\beta(\frac{1 + \sin\beta}{\sin\beta})$

$= \frac{\cos\beta}{\sin\beta}(1 + \sin\beta)$

$= \frac{\cos\beta}{\sin(90-\alpha)}(1 + \sin\beta)$

$= \frac{\cos\beta}{\cos\alpha}(1 + \sin\beta)$

3 C. Question

If two angles and are complementary angle, let us show that

$$\frac{\sec\alpha}{\cos\alpha} - \cot^2\beta = 1$$

Answer

Given, $\frac{\sec\alpha}{\cos\alpha} - \cot^2\beta$

$\Rightarrow \frac{\sec\alpha}{\cos\alpha} - \cot^2\beta$

$\Rightarrow [\sec\alpha = \frac{1}{\cos\alpha}$ and $\cot\beta = \frac{\cos\beta}{\sin\beta}]$

$\Rightarrow \frac{\frac{1}{\cos\alpha}}{\cos\alpha} - \frac{\cos^2\beta}{\sin^2\beta}$

$\Rightarrow \frac{1}{\cos^2\alpha} - \frac{\cos^2\beta}{\sin^2\beta}$

[cos α = cos(90 - β)]

$\Rightarrow \frac{1}{\cos^2(90-\beta)} - \frac{\cos^2\beta}{\sin^2\beta}$

$\Rightarrow \frac{1}{\sin^2\beta} - \frac{\cos^2\beta}{\sin^2\beta}$

$\Rightarrow \frac{1-\cos^2\beta}{\sin^2\beta}$

$[\sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \frac{\sin^2\beta}{\sin^2\beta}$

= 1

4. Question

If sin 17° = $\frac{x}{y}$, let us show that sec 10° - sin73° = $\frac{x^2}{y\sqrt{y^2-x^2}}$

Answer

Given, $\sin 17° = \frac{x}{y}$

To show, $\sec 17° - \sin 73° = \frac{x^2}{y\sqrt{y^2-x^2}}$

$[\sec\theta = \frac{1}{\cos\theta}]$

$\Rightarrow \frac{1}{\cos 17°} - \sin 73°$

$\Rightarrow \sin 73° = \sin(90 - 17) = \cos 17$

$\Rightarrow \frac{1}{\cos 17°} - \cos 17°$

$= \frac{1-\cos^2 17}{\cos 17}$

$[\sin^2\theta + \cos^2\theta = 1]$

$= \frac{\sin^2 17}{\cos 17}$

$= \frac{\sin^2 17}{\sqrt{1-\sin^2 17}}$

$= \frac{\left(\frac{x}{y}\right)^2}{\sqrt{1-\left(\frac{x}{y}\right)^2}}$

$= \frac{\left(\frac{x}{y}\right)^2}{\sqrt{\left(\frac{y-x}{y}\right)^2}}$

$= \frac{(x)^2}{y\sqrt{y^2-x^2}}$

5. Question

Let us show that $\sec^2 12° - \frac{1}{\tan^2 78°} = 1$

Answer

Given, $\sec^2 12 - \frac{1}{\tan^2 78} = 1$

$[\sec = \frac{1}{\cos\theta}]$

$\Rightarrow \tan 78 = \tan(90 - 12) = \cot 12$

$\therefore \sec^2 12 - \frac{1}{\cot^2 12} = 1$

$\Rightarrow \frac{1}{\cos^2 12} - \frac{1}{\cot^2 12}$

$= \frac{1}{\cos^2 12} - \frac{\sin^2 12}{\cos^2 12}$

$= \frac{1-\sin^2 12}{\cos^2 12}$

$= \frac{\cos^2 12}{\cos^2 12}$

= 1

Hence, proved

6. Question

$\angle A + \angle B = 90°$, let us show that $1 + \frac{\tan A}{\tan B} = \sec^2 A$

Answer

Given, $\angle A + \angle B = 90°$

To show that $1 + \frac{\tan A}{\tan B} = \sec^2 A$

$\Rightarrow 1 + \frac{\tan A}{\tan B}$

$\Rightarrow \tan B = \tan(90 - A) = \cot A$

$= 1 + \frac{\tan A}{\cot A}$

$[\cot A = \frac{1}{\tan A}]$

$= 1 + \frac{\tan A}{\frac{1}{\tan A}}$

$= 1 + \tan^2 A$

$= \sec^2 A$

7. Question

Let us show that $\operatorname{cosec}^2 22° \cot^2 68° = \sin^2 22° + \sin^2 68° + \cot^2 68°$

Answer

Given, $\operatorname{cosec}^2 22° \cot^2 68° = \sin^2 22° + \sin^2 68° + \cot^2 68°$

$\Rightarrow \sin^2 22° + \sin^2 68° + \cot^2 68°$

$= \sin^2 22° + \sin^2 (90 - 22)° + \cot^2 68°$

$= \sin^2 22° + \cos^2 22° + \cot^2 68°$

$= 1 + \cot^2 68°$

$= \operatorname{cosec}^2 68°$

8. Question

If ∠A + ∠B = 90°, let us show that, $\sqrt{\frac{\sin p}{\cos Q} - \sin P \operatorname{Cos} Q} - \cos P$

Answer

Given, ∠ A + ∠ B = 90°

To show $\sqrt{\frac{\sin P}{\cos Q} - \sin P \cos Q} = \cos P$

$\Rightarrow \sqrt{\frac{\sin P}{\cos Q} - \sin P \cos Q}$

$= \sqrt{\frac{\sin(90 - Q)}{\cos Q} - \sin(90 - Q)\cos Q}$

$= \sqrt{\frac{\cos Q}{\cos Q} - \cos Q \cos Q}$

$= \sqrt{1 - \cos^2 Q}$

$= \sqrt{\sin^2 Q}$

= sinQ

= sin(90 - p)

= cosP

Hence proved

9. Question

Let us prove that cot 12°cot38° cot ° cot52° cot 78° cot 60° = $\frac{1}{\sqrt{3}}$

Answer

Given, cot12° cot38° cot52° cot78° cot60°

⇒ we know that cot60 = $\frac{1}{\sqrt{3}}$

⇒ cot12° cot38° cot52° cot78° $(\frac{1}{\sqrt{3}})$

= cot(90 - 78)cot(90 - 52)cot52° cot78° $(\frac{1}{\sqrt{3}})$

= tan78° tan52° cot52° cot78° $(\frac{1}{\sqrt{3}})$

= tan78° tan52° $\frac{1}{\tan 52°}$ $\frac{1}{\tan 78°}$ $(\frac{1}{\sqrt{3}})$

= $(\frac{1}{\sqrt{3}})$

10. Question

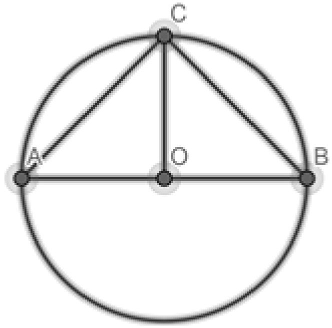
AOB is a diameter of a circle with centre O and C is any point on the circle, joining A.C; B,C; and O, C let us show that

(i) tan∠ABC = cot ∠ACO

(ii) $\sin^2$∠BCO + $\sin^2$∠ACO = 1

(iii) cosec^2∠CAB - 1 = $\tan^2$∠ABC

Answer



(i) tan∠ABC = cot ∠ACO

⇒ ∠ AOC = 90° = ∠ BOC

⇒ ∠ CAO = 60° and

∠ ACO = 30°

⇒ tan∠ ABC = tan(90 - ∠ ACO)

[tan(90 - θ) = cotθ]

= cot∠ ACO

(ii) $\sin^2$∠BCO + $\sin^2$∠ACO = 1

⇒ $\sin^2$∠BCO + $\sin^2$∠ACO = 1

⇒ $\sin^2$∠BCO + $\sin^2$∠ACO

⇒ $\sin^2$∠BCO + $\sin^2$(90 - ∠ BCO)

⇒ $\sin^2$∠BCO + $\cos^2$∠BCO

= 1

[since, $\sin^2\theta + \cos^2\theta = 1$]

(iii) cosec^2∠CAB - 1 = $\tan^2$∠ABC

⇒ cosec^2∠CAB - 1

⇒ cosec(90 - ∠CAB) = sec∠CAB

⇒ ∠ CAB = ∠ ABC

⇒ $\sec^2$∠CAB - 1 = $\tan^2$∠ ABC

⇒ $\sec^2$∠ ABC – 1 = $\tan^2$∠ ABC

⇒ $\tan^2$∠ ABC = $\tan^2$∠ ABC

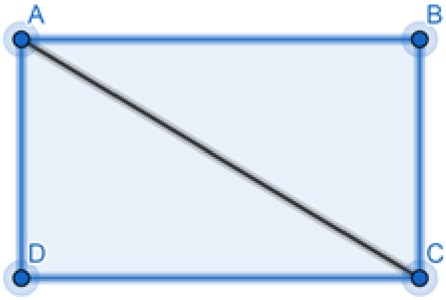
11. Question

ABCD is a rectangular figure, joining A,C let us prove that

(i) tan∠ACD = cot∠ACB

(ii) $\tan^2\angle CAD + 1 = \frac{1}{\sin^2 \angle BAC}$

Answer



AC is the diagonal line joining ABCD

(i) tan∠ACD = cot∠ACB

⇒ ∠ACD + ∠ACB = 90

⇒ tan∠ACD

= tan(90 - ∠ACB)

= cot∠ACB

(ii) $\tan^2\angle CAD + 1 = \frac{1}{\sin^2 \angle BAC}$

⇒ $\tan^2\angle CAD + 1$

= $\text{cosec}^2\angle ADC$

[∠ADC = ∠BAC, from the properties]

= $\text{cosec}^2\angle BAC$

= $\frac{1}{\sin^2 \angle BAC}$

12 A1. Question

The value of (sin 43° cos47° + cos43° sin47°)

A. 0

B. 1

C. sin4°

D. cos4°

Answer

⇒ Option B is correct as it satisfy the value

Given, sin43°cos47° + cos43°sin47°

Need to find the value

⇒ sin43°cos(90 - 43)° + cos43°sin(90 - 43)°

⇒ sin43°sin43° + cos43°cos43°

⇒ $\sin^2 43° + \cos^2 43°$

= 1

[Since, $\sin^2\theta + \cos^2\theta = 1$]

⇒ Option A is incorrect because by solving the equation we get the value as 1

⇒ Option C is incorrect because by solving the equation we get the value as 1

⇒ Option D is incorrect because by solving the equation we get the value as 1

12 A2. Question

The value of $\left(\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°}\right)$ is

A. 0

B. 1

C. 2

D. none of this

Answer

Given, $\left(\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°}\right)$

⇒ $\left(\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°}\right)$

⇒ $\left(\frac{\tan 35°}{\cot(90-35)°} + \frac{\cot 78°}{\tan(90-78)°}\right)$

⇒ $\left(\frac{\tan 35°}{\tan 35°} + \frac{\cot 78°}{\cot 78°}\right)$

= 1 + 1

= 2

⇒ Option A is incorrect as it does not match the value 2

⇒ Option B is incorrect as it does not match the value 2

⇒ Option C is correct as it match the value 2

⇒ Option D is incorrect as it does not match the value 2

12 A3. Question

The value of {cos(40° + θ) - sin(50° - θ)} is

A. 2cosθ

B. 7sinθ

C. 0

D. 1

Answer

Given, cos(40° + θ) - sin(50° - θ)

[cos(90° - θ) = sinθ]

⇒ cos(90° - (50° - θ)) - sin(50° - θ)

⇒ sin(50° - θ) - sin(50° - θ)

= 0

⇒ Option C is the correct as the value of cos(40° + θ) - sin(50° - θ) is 0

⇒ Option A is incorrect, since it does not match the value

⇒ Option B is incorrect, since it does not match the value

⇒ Option D is incorrect, since it does not match the value

12 A4. Question

ABC is triangle. Sin $\left(\frac{B+C}{2}\right) =$

A. $\sin\frac{A}{2}$

B. $\cos\frac{A}{2}$

C. sinA

D. cosA

Answer

In a triangle A+B+C =180°

(B+C)/2 = 90°-A/2

Sin $\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$ (As sin(90°-A) = cos A)

12 A5. Question

If A + B = 90° and tan A = $\frac{3}{4}$, value of cot B is

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

Answer

Given, A + B = 90°

And tan A = $\frac{3}{4}$

⇒ cotB = cot(90 - A)

= tan A

= $\frac{3}{4}$

∴ Option A is correct, the value of cot B is also $\frac{3}{4}$

⇒ Option B is incorrect, as it does not match the given value

⇒ Option C is incorrect, as it does not match the given value

⇒ Option D is incorrect, as it does not match the given value

12 B. Question

Let us write whether the following statements are true of false:

(i) The value of cos54° and sin36° are equal.

(ii) The simplified value of (sin 12° - cos78°) is 1.

Answer

(i) The statement is **true**

⇒ cos54° = cos(90 - 36)°

= sin36°

[cos(90 - θ) = sinθ]

∴ the value of cos54° and sin36° has same values

(ii) The given statement is **False**

⇒ sin12° - cos78°

⇒ sin12° - cos(90 - 12)°

⇒ sin12° - sin12°

= 0

12 C. Question

Let us fill up the blanks:

(i) The value of (tan 15° x tan 45° x tan 60° tan 75°) is______.

(ii) The value of (sin 12° x cos 18° x sec 78° cosec 72°) is ______.

(iii) If A and B are complementary to each other, siin A = ______.

Answer

(i) The value of tan15° tan45° tan60° tan75° is $\sqrt{3}$

⇒ we know that tan45° = 1 and tan60° = $\sqrt{3}$

⇒ tan15°(1)($\sqrt{3}$)tan(90 - 15)

⇒ $\sqrt{3}$ × tan15° cot15°

⇒ $\sqrt{3}$ × tan15° $\frac{1}{\tan 15^\circ}$

= $\sqrt{3}$ × 1

= $\sqrt{3}$

(ii) The value of sin12°cos18° × sec78°cosec72° is **1**

⇒ sin12°cos18° sec78°cosec72°

⇒ sin12°cos18°sec(90 - 12)°cosec(90 - 18)°

⇒ sin12°cos18°cosec12°sec18°

⇒ sin12°cos18°$\frac{1}{\sin 12^\circ}\frac{1}{\cos 18^\circ}$

= 1

(iii) sinA = cosB

⇒ If A and B are complementary angles then A + B = 90° and A = 90 - B

⇒ sinA = sin(90 - B)

= cosB

13 A. Question

If sin 10θ = cos8θ and 10θ is a positive acute angle, let us find the value of tan 9θ.

Answer

Given, sin10θ = cos8θ

And 10θ is a positive acute angle

⇒ sin10θ = cos8θ

⇒ cos(90 - 10θ) = cos8θ

⇒ 90 - 10θ = 8θ

⇒ 90 = 18θ

⇒ θ = 5

Hence, the value of θ is 5

13 B. Question

If tan 4θ × tan 6θ = 1 and 6θ is a positive acute angle, let us find the value of θ.

Answer

Given, 6θ as a positive acute angle

⇒ tan4θ tan6θ = 1

⇒ tan4θ cot(90 - 6θ) = 1

$\Rightarrow \cot(90 - 6\theta) = \frac{1}{\tan 4\theta}$

$\Rightarrow$ cot(90 - 6θ) = cot4θ

$\Rightarrow$ 90 - 6θ = 4θ

$\Rightarrow$ 10θ = 90

$\Rightarrow$ θ = 9

13 C. Question

let us find the value of

$$\frac{2\sin^2 63° + 1 + 2\sin^2 27°}{3\cos^2 17° - 2 + 3\cos^2 73°}$$

Answer

Given, $\frac{(2\sin^2 63° + 1 + 2\sin^2 27)}{(3\cos^2 17° - 2 + 3\cos^2 73°)}$

$\Rightarrow \frac{2\sin^2 63° + 1 + 2\sin^2 (90-63)°}{3\cos^2 (90-73)° - 2 + 3\cos^2 73°}$

$\Rightarrow \frac{2\sin^2 63° + 1 + 2\cos^2 63°}{3\sin^2 73° - 2 + 3\cos^2 73°}$

$\Rightarrow \frac{2(\sin^2 63° + \cos^2 63°) + 1}{3(\sin^2 73° + \cos^2 73°) - 2}$

$\Rightarrow \frac{2+1}{3-2}$

$= \frac{3}{1}$

= 3

13 D. Question

let us find the value of (tan 1° × tan2° × tan 3°……….tan89°)

Answer

$\Rightarrow$ tan1° tan2° tan3° ……tan87° tan88° tan89°

$\Rightarrow$ tan1° tan2° tan3° ……tan(90 - 3)° tan(90 - 2)° tan(90 - 1)°

$\Rightarrow$ tan1° tan2° tan3° …..cot3° cot2° cot1°

= 1

13 E. Question

If sec 5A = cosec (A + 36°) and 5A is a positive acute angle, let us find the value of A.

Answer

Given, sec5A = cosec(A + 36°)

And 5A is positive acute angle

$\Rightarrow \sec 5A = \text{cosec}(A + 36^\circ)$

$\Rightarrow \text{cosec}(90^\circ - 5A) = \text{cosec}(A + 36^\circ)$

$[\sec\theta = \text{cosec}(90 - \theta)]$

$\Rightarrow 90 - 5A = A + 36$

$\Rightarrow 90 - 36 = 6A$

$\Rightarrow 6A = 54$

$\Rightarrow A = 9$

Hence, the value of A is 9