Probability

Conditional Probability If E and F are two events associated with the same sample space of a random experiment, then the conditional probability of the event E under the condition that the event F has occurred written as $P\left(\frac{E}{F}\right)$ is given by,

$$\boxed{P\left(\frac{E}{F}\right) = \frac{P(E\cap F)}{P(F)}} \quad ; \quad P(F) \neq 0$$

Properties of Conditional Probability Let E and F be events associated with the sample space S of an experiment. Then,

(i) $\boxed{P\left(\frac{S}{F}\right) = P\left(\frac{F}{F}\right) = 1}$

(ii) $\boxed{P\left[\frac{(A\cup B)}{F}\right] = P\left(\frac{A}{F}\right) + P\left(\frac{B}{F}\right) - P\left[\frac{(A\cap B)}{F}\right]}$

(iii) $\boxed{P\left(\frac{E'}{F}\right) = 1 - P\left(\frac{E}{F}\right)}$

Multiplication Theorem on Probability

Let E and F be two events associated with a sample space of an experiment. Then,

$$P(E\cap F) = P(E)\,P\left(\frac{F}{E}\right) \quad ; \quad P(E) \neq 0$$

$$= P(F)\,P\left(\frac{E}{F}\right) \quad ; \quad P(F) \neq 0$$

If E, F and G are 3 events associated with a sample space. then,

$$\boxed{P(E\cap F\cap G) = P(E)\,P\left(\frac{F}{E}\right)\,P\left[\frac{G}{E\cap F}\right]}$$

Independent Events : Let E and F be two events associated with the same random experiment, then E and F are said to be independent if, $\boxed{P(E\cap F) = P(E).P(F)}$

Dependent Events : Two events E and F are said to be dependent if they are not independent, i.e. if $\boxed{P(E\cap F) \neq P(E).P(F)}$

Three events A, B and C are said to be independent of all the following conditions hold :

$$P(A\cap B) = P(A)\,P(B)$$
$$P(A\cap C) = P(A)\,P(C)$$
$$P(B\cap C) = P(B)\,P(C)$$
and $$P(A\cap B\cap C) = P(A)\,P(B)\,P(C)$$

Bayes' Theorem If $E_1, E_2 \ldots\ldots E_n$ are mutually exclusive and exhaustive events associated with a sample space and A as any event of non-zero probability. then ;

$$\boxed{P\left(\frac{E_i}{A}\right) = \frac{P(E_i)\,P\left(\frac{A}{E_i}\right)}{\sum_{j=1}^{n} P(E_j)\,P\left(\frac{A}{E_j}\right)}}$$

Theorem of total Probability Let $\{E_1, E_2, \ldots\ldots E_n\}$ be a partition of the sample space S. Let A be any event associated with S, then :

$$\boxed{P(A) = \sum_{j=1}^{n} P(E_j)\,P\left(\frac{A}{E_j}\right)}$$

Random variable and its probability Distribution A random variable is a real valued whose domain is the sample space of a random experiment. The Probability distribution of a random variable X is the system of numbers :

$$\begin{cases} X & : x_1 \quad x_2 \quad \ldots \quad x_n \\ P(X) & : P_1 \quad P_2 \quad \ldots \quad P_n \end{cases}$$

where $P_i > 0$; $\sum_{i=1}^{n} P_i = 1$ $\quad i = 1, 2 \ldots\ldots n$

✓ Mean of a random variable [The expectation of X OR E(X)] Let X be a random variable assume $x_1, x_2, \ldots x_n$ with probabilities $p_1, p_2, \ldots p_n$ respectively. Mean of X, denoted by μ is the number $\sum_{i=1}^{n} x_i p_i$.

$$\boxed{E(X) = \mu = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n}$$

✓ Variance of a random variable

$$\boxed{\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 p_i = \sum_{i=1}^{n} x_i^2 p_i - \mu^2}$$

or equivalently $\boxed{\sigma^2 = E(X-\mu)^2}$

Standard deviation of the random variable X is defined as :

$$\boxed{\sigma = \sqrt{Variance(X)} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p_i}}$$

✓ Bernoulli Trials Trials of a random experiment are called Bernuolli trivals, if they satisfy the following conditions :

(i) There should be finite no. of trials.
(ii) The trials should be independent.
(iii) Each trial has exactly two outcomes : success or failure.
(iv) The probability of success (or failure) remains the same in each trial.

✓ Binomial Distribution A random variable X taking values $0, 1, 2 \ldots, n$ is said to have binomial distribution with parameters n and P of its probability distribution is given by:

$\boxed{P(X = r) = {}^{n}C_r \, p^r q^{n-r}}$ where $q = 1 - p$ and $r = 0, 1, 2 \ldots n$