

CBSE Class 09
Mathematics
Sample Paper 10 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

- i. All the questions are compulsory.
 - ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
 - iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
 - iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
 - v. Use of calculators is not permitted.
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Section A

1. The $\frac{p}{q}$ form of the number 0.8 is
 - a. 1
 - b. $\frac{1}{8}$
 - c. $\frac{8}{10}$
 - d. $\frac{8}{100}$
2. The degree of the zero polynomial is
 - a. 0
 - b. any natural number

c. 1

d. not defined

3. If two supplementary angles are in the ratio 2 : 7, then the angles are :

a. 35^0 , 145^0

b. 70^0 , 110^0

c. 40^0 , 140^0

d. 50^0 , 130^0

4. The construction of a $\triangle ABC$, given $AB = 5.2$ cm, $\angle A = 45^0$ is possible when the sum of AC and BC is equal to ____.

a. 5 cm

b. 4.6 cm

c. 4.8 cm

d. 5.5 cm

5. The value of $\left(x - \frac{1}{x}\right) \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2}\right)$ is

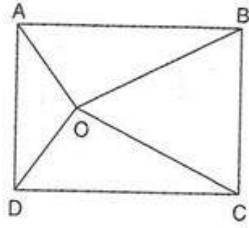
a. $x^4 + \frac{1}{x^4}$

b. $x^3 - \frac{1}{x^3} + 2$

c. $x^2 + \frac{1}{x^2} - 2$

d. $x^4 - \frac{1}{x^4}$

6. ABCD is a rectangle with O as any point in its interior. If $ar(\triangle AOD) = 3 \text{ cm}^2$ and $ar(\triangle BOC) = 6 \text{ cm}^2$, then $ar(\text{rect } ABCD)$ is



- a. 12 cm^2 .
 - b. 9 cm^2 .
 - c. 15 cm^2 .
 - d. 18 cm^2 .
7. The zero of the polynomial $(x - 2)^2 - (x + 2)^2$ is
- a. 0
 - b. 2
 - c. 1
 - d. -2
8. The sides of a triangle are x, y and z. If $x + y = 7 \text{ m}$, $y + z = 9 \text{ m}$, and $z + x = 8 \text{ m}$, then area of the triangle is :
- a. 4 m^2
 - b. 7 m^2
 - c. 5 m^2
 - d. 6 m^2
9. The diameter of a right circular cylinder is 21 cm and its height is 8 cm. The Volume of the cylinder is
- a. 1056 cu cm
 - b. 2772 cu cm

c. 1386 cu cm

d. 528 cu cm

10. If the probability of happening of an event is $\frac{3}{7}$, then the probability of not happening of this event is:

a. 1

b. $\frac{4}{7}$

c. 0

d. $\frac{2}{7}$

11. Fill in the blanks:

The sum of rational and an irrational number is always _____.

12. Fill in the blanks:

Equation of a line which passes through the origin is _____.

OR

Fill in the blanks:

The positive solutions of the equation $ax + by + c = 0$ always lies in _____ quadrant.

13. Fill in the blanks:

The point whose ordinate is 4 and which lies on Y-axis is _____.

14. Fill in the blanks:

Segment of a circle is the region between an arc and _____ of the circle.

15. Fill in the blanks:

If V is the volume of a cuboid of dimensions a, b, c and S is its surface area, then

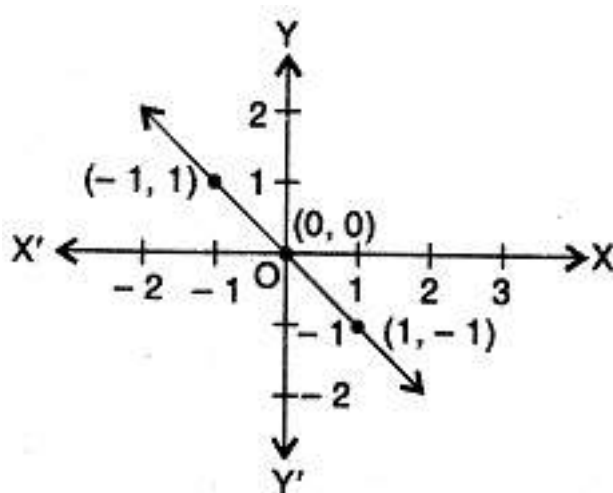
$\frac{2}{S} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$ equals to _____.

16. Simplify the following by rationalising the denominator: $\frac{1}{5+\sqrt{2}}$
17. Give an example of a polynomial which is trinomial of degree 2.
18. A rectangular container, whose base is a square of side 5 cm, stands on a horizontal table, and holds water upto 1 cm from the top. When a cube is placed in the water it is completely submerged, the water rises to the top and 2 cubic cm of water overflows. Calculate the volume of the cube and also the length of its edge.

OR

A cylinder whose height is two-thirds of its diameter, has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.

19. Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.
20. From the choices given below, choose the equation where graph are given in fig.
- $y = x$
 - $x + y = 0$
 - $y = 2x$
 - $2 + 3y = 7x$



21. If $x = \sqrt[3]{28}$ and $y = \sqrt[3]{27}$, find the value of $x + y - \frac{1}{x^2 + xy + y^2}$.
22. Find the value of the following equation for $x = 1$, $y = 1$ as a solution. $9ax + 12ay = 63$

23. Find k, if $x^{51} + 2x^{60} + 3x + k$ is divisible by $x + 1$.

OR

Find the value of $x^2 + \frac{1}{x^2}$, if $x - \frac{1}{x} = \sqrt{3}$.

24. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of two villages decided to take over some portion of his plot from one of the corners to construct a health center. Itwaari agrees to the above proposal with the condition that he should be given an equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

25. The value of π up to 50 decimal places is given below:

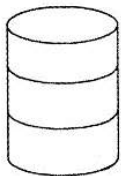
3.14159265358979323846264338327950288419716939937510

- i. Make a frequency distribution of the digits from 0 to 9 after the decimal point.
- ii. What are the most and the least frequently occurring digits?

OR

The mean of five numbers is 27. If one number is excluded, their mean is 25. Find the excluded number.

26. In fig. you see the frame of lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.



27. Evaluate $\frac{15}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}}$, given that $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

OR

Find an irrational number between $\frac{1}{7}$ and $\frac{1}{3}$.

28. Plot the following points and check whether they are collinear or not: (0, 0), (2, 2), (5, 5)
29. Find four solutions for the following equation : $12x + 5y = 0$

OR

Two batsman Rahul and Anil while playing a cricket match scored 120 runs. For this, write a linear equation in two variables and draw the graph.

30. Draw a right-angled triangle whose hypotenuse measures 6 cm and the length of one of whose sides containing the right angle is 4 cm.
31. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60° . Find the angles of the parallelogram.
32. ABC is an isosceles triangle with $AB = AC$ and AD is altitude of the triangle. Prove that AD is also a median of the triangle.

OR

If perpendiculars from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.

33. A rhombus sheet, whose perimeter is 32 m and whose one diagonal is 10 m long, is painted on both sides at the rate of Rs.5 per m^2 . Find the cost of painting.
34. A recent survey found that the ages of workers in a factory are distributed as follows:

Age (in years)	20-29	30-39	40-49	50-59	60 and above
Numbers of workers	38	27	86	46	3

If a person is selected at random, find the probability that the person is:

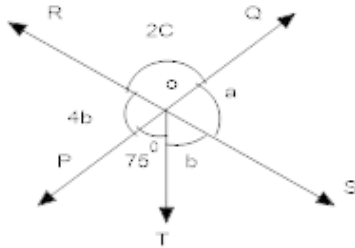
- 40 years or more.
- Under 40 years.
- under 60 but over 39 years.

35. In a cyclic quadrilateral ABCD if $AB \parallel CD$ and $\angle B = 70^\circ$, find the remaining angles.

OR

If P, Q and R are the mid-points of the sides BC, CA and AB of a triangle and AD is the perpendicular from A to BC, prove that P, Q, R and D are concyclic.

36. In fig two straight lines PQ and RS intersect each other at O, if $\angle POT = 75^\circ$ Find the values of a, b and c



37. The polynomial $3x^3 + ax^2 + 3x + 5$ and $4x^3 + x^2 - 2x + a$ leave remainder when divided by $(x - 2)$ respectively. If $R_1 - R_2 = 9$, find the value of a.

OR

If $x^4 + \frac{1}{x^4} = 47$, find the value of $x^3 + \frac{1}{x^3}$

38. The volume of the two spheres are in the ratio 64 : 27. Find the difference of their surface areas, if the sum of their radii is 7.

OR

The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost to paint 1 cm^2 of the surface is Rs. 0.05. Find the total cost to paint the vessel all over. (Use $\pi = 22/7$)

39. In a $\triangle ABC$, the internal bisectors of $\angle B$ and $\angle C$ meet at P and the external bisectors of $\angle B$ and $\angle C$ meet at Q. Prove that $\angle BPC + \angle BQC = 180^\circ$.

40. Construct a frequency polygon for the following data:

Age (in years):	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
Frequency:	2	4	6	8	9	6	5	3	1

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Solution
Section A

1. (c) $\frac{8}{10}$

Explanation:

$$\frac{8}{10}$$

Or, $\frac{4}{5}$

2. (d) not defined

Explanation:

The general form of a polynomial is $a_n x^n$, where n is a natural number.

For zero polynomial $a_n = 0$.

Since the largest value of n for which a_n is non-zero is negative infinity (all the integers are bigger than negative infinity).

Therefore, the degree of zero polynomials is not defined.

3. (c) $40^0, 140^0$

Explanation:

We know that supplementary angles are those angles whose sum is 180°

The two given supplementary angles are in the ratio 2 : 7

Let the common ratio be x

So angles are 2x and 7x respectively

$$2x + 7x = 180^0$$

$$9x = 180^0$$

$$x = \frac{180^\circ}{9} = 20^0$$

$$2x = 2 \times 40^0 = 40^0$$

$$7x = 7 \times 20^0 = 140^0$$

4. (d) 5.5 cm

Explanation: To construct a triangle whose base, base angle and sum of other two sides are given is possible when the sum of other two sides is greater than its base.

5. (d) $x^4 - \frac{1}{x^4}$

Explanation:

$$\begin{aligned} & \left(x - \frac{1}{x}\right) \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2}\right) \\ &= \left(x^2 - \frac{1}{x^2}\right) \left(x^2 + \frac{1}{x^2}\right) \text{ [Using identity } (a+b)(a-b) = a^2 - b^2 \text{]} \\ &= x^4 - \frac{1}{x^4} \text{ [Using identity } (a+b)(a-b) = a^2 - b^2 \text{]} \end{aligned}$$

6. (d) 18 cm^2 .

Explanation: According to question,

$$\begin{aligned} \text{area}(\triangle AOD) + \text{area}(\triangle BOC) &= \frac{1}{2} \times \text{area}(\parallel gm ABCD) \\ \Rightarrow \frac{1}{2} \times \text{area}(\parallel gm ABCD) &= 3 + 6 = 9 \text{ sq. cm} \\ \Rightarrow \text{area}(\parallel gm ABCD) &= 18 \text{ cm}^2 \end{aligned}$$

7. (a) 0

$$\begin{aligned} \textbf{Explanation:} & (x-2)^2 - (x+2)^2 \\ &= (x-2+x+2)(x-2-x-2) \text{ [Using identity } a^2 - b^2 = (a+b)(a-b) \text{]} \\ &= (2x)(-4) \\ &= -8x \end{aligned}$$

Then the zero is

$$-8x = 0$$

$$\Rightarrow x = 0$$

8. (d) 6 m^2

Explanation: Adding given three equaitons,

$$2x + 2y + 2z = 24 \Rightarrow x + y + z = 12$$

Therefore, $s = 12/2 = 6$ m

$$\begin{aligned} \text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{6(6-x)(6-y)(6-z)} \\ &= \sqrt{6(12-6-x)(12-6-y)(12-6-z)} \\ &= \sqrt{6(y+z-6)(x+z-6)(x+y-6)} \\ &= \sqrt{6(9-6)(8-6)(7-6)} \\ &= \sqrt{6 \times 3 \times 2 \times 1} \\ &= 6 \text{ sq. m} \end{aligned}$$

9. (b) 2772 cu cm

Explanation:

Volume of cylinder, $V = \pi r^2 h$ (where $\pi = 22/7$)

Given diameter, $d = 21$ cm, height, $h = 8$ cm

for radius, $r = d/2 = 21/2$

$$V = (22/7) \times (21/2)^2 \times 8 = 2772 \text{ cu cm}$$

10. (b) $\frac{4}{7}$

Explanation:

The probability of happening of an event = $\frac{3}{7}$

The probability of not happening of this event = $1 - \frac{3}{7} = \frac{4}{7}$

11. irrational

12. $y = x$

OR

1st quadrant

13. (0, 4)

14. chord

15. $\frac{1}{V}$

16. The given expression is : $\frac{1}{5+\sqrt{2}}$

Multiplying the numerator and denominator by $5 - \sqrt{2}$ we get

$$\frac{1}{5+\sqrt{2}} = \frac{1}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}}$$

$$= \frac{5-\sqrt{2}}{(5)^2 - (\sqrt{2})^2}$$

$$= \frac{5-\sqrt{2}}{25-2}$$

$$= \frac{5-\sqrt{2}}{23}$$

$$\text{Hence } \frac{1}{5+\sqrt{2}} = \frac{5-\sqrt{2}}{23}$$

17. Required polynomial should have three terms with highest power of the variable 2.

$\therefore x^2 + x - 1$ or $y^2 + 8y + 11$ or $y^2 - 6y - 7$ are some of the possible polynomials.

18. Let the length of each edge of the cube be x cm

Then,

Volume of the cube = Volume of water inside the tank + Volume of water that over flowed

$$x^3 = (5 \times 5 \times 1) + (2) = 25 + 2$$

$$x^3 = 27$$

$$x = 3 \text{ cm}$$

Hence, volume of cube = 27cm^3 and edge of the cube = 3 cm

OR

Height of cylinder = $\frac{2}{3}$ diameter

$$= \frac{2}{3} \times 2r$$

$$= \frac{4}{3} r$$

Volume of cylinder = Volume of sphere

$$\Rightarrow \pi r^2 \times \frac{4}{3} r = \frac{4}{3} \pi (4)^3$$

$$\Rightarrow r^3 = 4^3$$

$$\Rightarrow r = 4 \text{ cm}$$

19. No, because then the sum of four angles of the quadrilateral will be more than 360° whereas sum of four angles of a quadrilateral is always equal to 360° .

20. $x + y = 0$

put $x = 0, y = 0$

put $x = 1, y = -1$

put $x = -1, y = 1$

in the given graph all conditions are satisfied.

Hence, the correct equation is (ii) $x + y = 0$

21. We have,

$$x + y - \frac{1}{x^2 + xy + y^2}$$

$$= x + y - \frac{(x-y)}{(x-y)(x^2 + xy + y^2)}$$

$$= x + y - \frac{(x-y)}{x^3 - y^3}$$

$$= x + y - \frac{(x-y)}{28-27} [\because x = \sqrt[3]{28} \text{ and } y = \sqrt[3]{27} \therefore x^3 = 28, y^3 = 27]$$

$$= x + y - (x - y) = 2y = 2 \times 3 = 6 [\because y = \sqrt[3]{27} = (3^3)^{1/3} = 3]$$

22. $9ax + 12ay = 63$

If $x = 1, y = 1$ is a solution, then

$$9a + 12a = 63$$

$$\Rightarrow 21a = 63$$

$$\Rightarrow a = \frac{63}{21} = \frac{3}{1}$$

23. Let $p(x) = x^{51} + 2x^{60} + 3x + k$

Given that, $p(x)$ is divisible by $x + 1$.

$$\therefore p(-1) = 0$$

$$\begin{aligned} \Rightarrow (-1)^{51} + 2(-1)^{60} + 3(-1) + k &= 0 \\ \Rightarrow -1 + 2 - 3 + k &= 0 \Rightarrow k - 4 + 2 = 0 \\ \Rightarrow k - 2 &= 0 \Rightarrow k = 2 \end{aligned}$$

OR

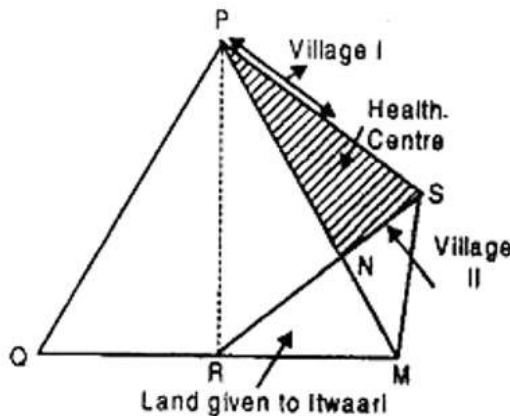
According to the question,

$$x - \frac{1}{x} = \sqrt{3}$$

Squaring both the sides,

$$\begin{aligned} \Rightarrow \left(x - \frac{1}{x}\right)^2 &= (\sqrt{3})^2 \\ \Rightarrow x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} &= 3 \\ \Rightarrow x^2 + \frac{1}{x^2} &= 3 + 2 \\ \Rightarrow x^2 + \frac{1}{x^2} &= 5 \end{aligned}$$

24.



Let Itwaari has land in shape of quadrilateral PQRS.

Draw a line through S parallel to PR, which meets QR produced at M.

Let diagonals PM and RS of new formed quadrilateral intersect each other at point N.

Now, $\triangle PRS$ and $\triangle PMR$ are on the same base PR and between the same parallels.

$\therefore \text{ar}(\triangle PRS) = \text{ar}(\triangle PMR)$ [Triangles on the same base and same parallel are equal in area]

Subtracting $\text{ar}(\triangle PNR)$ from both sides,







$$\text{ar}(\triangle PRS) - \text{ar}(\triangle PNR) = \text{ar}(\triangle PMR) - \text{ar}(\triangle PNR)$$

$$\Rightarrow \text{ar}(\triangle PSN) = \text{ar}(\triangle MNR)$$

It implies that Itwari will give corner triangular-shaped plot PSN to the

Grampanchayat for health centre and will take equal amount of land (denoted by $\triangle MNR$) adjoining his plot so as to form a triangular plot PQM.

25. i.

Digits	Tally Marks	Frequency
0	II	2
1		5
2		5
3		8
4	III	4
5		5
6	III	4
7	III	4
8		5
9		8
Total		50

ii. The most frequently occurring digits are 3 and 9. The most least frequently occurring digit is 0.

OR

The mean of five numbers is 27

Then, sum of five numbers = $5 \times 27 = 135$

If one number is excluded, then new mean is 25

\therefore Sum of four numbers = $4 \times 25 = 100$

\therefore Excluded number = $135 - 100 = 35$

26. Given $2r = 20$ cm, thus $r = 10$ cm

$h = 30$ cm

\therefore Cloth required = Curved surface area of lampshade

$$\begin{aligned}
&= 2\pi r(h + 2.5 + 2.5) \\
&= 2\pi r(h + 5) = 2 \times \frac{22}{7} \times 10 \times (30 + 5) \\
&= 2200 \text{ cm}^2
\end{aligned}$$

27. We have,

$$\begin{aligned}
&\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80} \\
&= \sqrt{10} + \sqrt{2^2 \times 5} + \sqrt{2^2 \times 10} - \sqrt{5} - \sqrt{2^4 \times 5} \\
&= \sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 2^2\sqrt{5} \\
&= \sqrt{10} + 2\sqrt{10} + 2\sqrt{5} - \sqrt{5} - 4\sqrt{5} \\
&= (1 + 2)\sqrt{10} + (2 - 1 - 4)\sqrt{5} = 3\sqrt{10} - 3\sqrt{5} = 3(\sqrt{10} - \sqrt{5}) \\
&\therefore \frac{15}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}} \\
&= \frac{15}{3(\sqrt{10} - \sqrt{5})} = \frac{5}{\sqrt{10} - \sqrt{5}} \\
&= \frac{5(\sqrt{10} + \sqrt{5})}{(\sqrt{10} - \sqrt{5})(\sqrt{10} + \sqrt{5})} \text{ [Multiplying and dividing by } \sqrt{10} + \sqrt{5}] \\
&= \frac{5(\sqrt{10} + \sqrt{5})}{10 - 5} \\
&= \sqrt{10} + \sqrt{5} = 3.162 + 2.236 = 5.398
\end{aligned}$$

OR

To find irrational number, firstly we will divide 1 by 7 and 1 by 3.

Now,

$$\begin{array}{r}
7 \overline{) 10} \quad 0.142857... \\
\underline{7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{35} \\
50 \\
\underline{49} \\
1
\end{array}$$

$$\therefore \frac{1}{7} = 0.142857... = 0.\overline{142857}$$

Now,

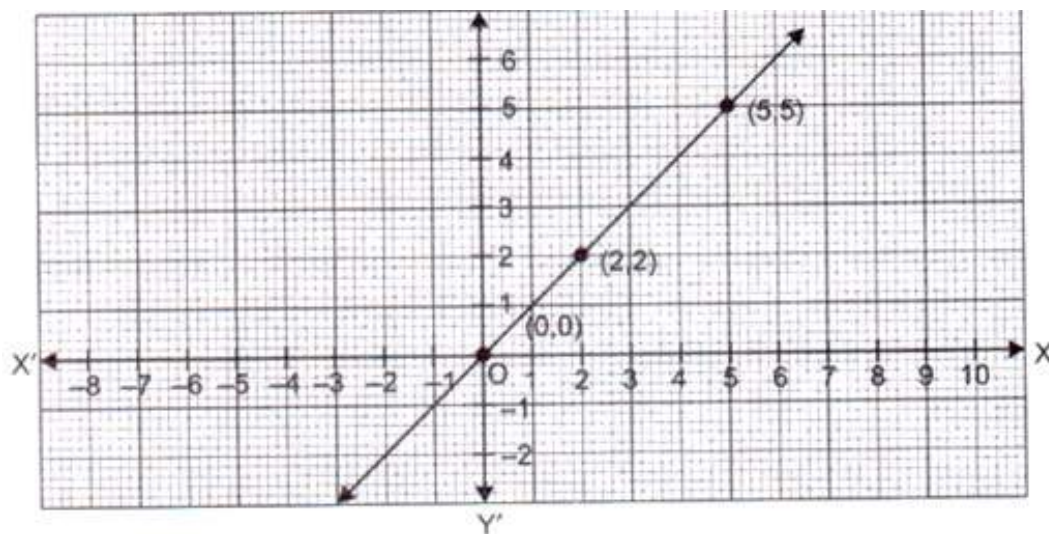
$$\begin{array}{r} 3 \overline{) 10} 0.33... \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

Thus, $\frac{1}{3} = 0.333... = 0.\overline{3}$

That means the required irrational numbers will lie between $0.\overline{142857}$ and $0.\overline{3}$.

Also, the irrational numbers have non-terminating non-repeating decimals. Hence, the required irrational number between $\frac{1}{7}$ and $\frac{1}{3}$ is $0.2101001000... .$

28.



From the graph, we find that all the three points lie on the same straight line. Hence, the given points are collinear.

29. $12x + 5y = 0$

$$\Rightarrow 5y = -12x$$

$$\Rightarrow y = \frac{-12}{5}x$$

Put $x = 0$, then $y = \frac{-12}{5}(0) = 0$

Put $x = 5$, then $y = \frac{-12}{5}(5) = -12$

Put $x = 10$, then $y = \frac{-12}{5}(10) = -24$

Put $x = 15$, then $y = \frac{-12}{5}(15) = -36$

$\therefore (0, 0), (5, -12), (10, -24)$ and $(15, -36)$ are the four solutions of the equation $12x + 5y = 0$

OR

Let the runs scored by Rahul be x and that the runs scored by Anil be y .

According to the given condition, we have

$$x + y = 120 \Rightarrow x = 120 - y \dots(i)$$

For different values of y we will get the respective values of x from the equation.

For graph, taking $y = 40$, we get

$$x = 120 - 40 = 80$$

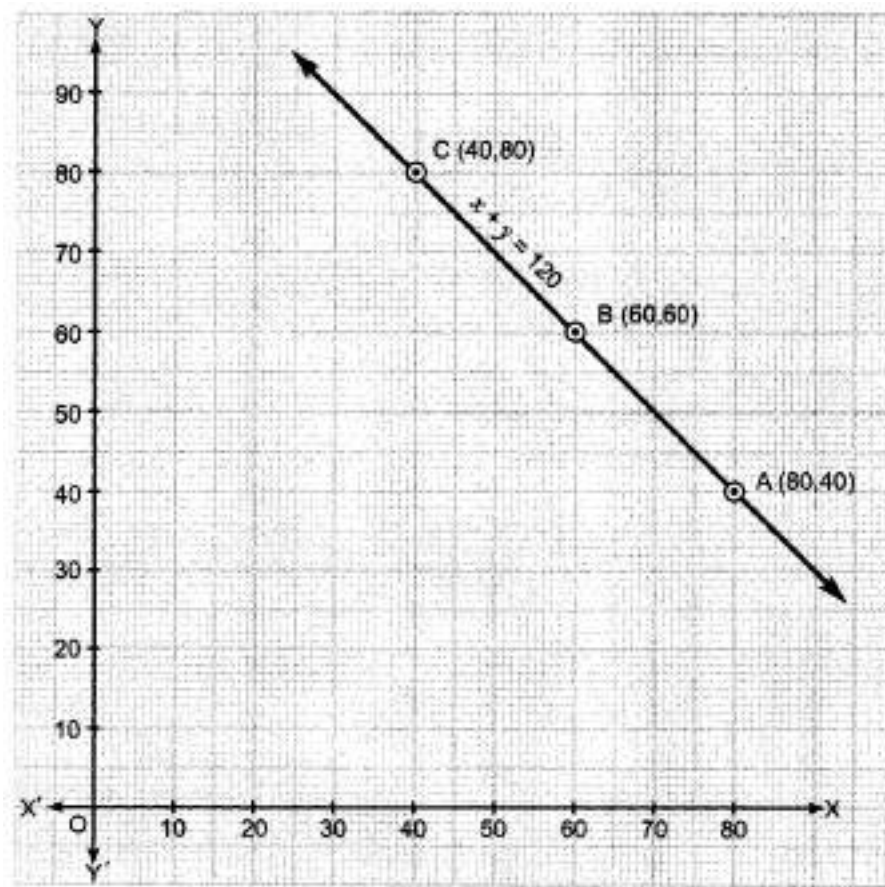
Again, taking $y = 60$, we get

$$x = 120 - 60 = 60 \text{ and}$$

taking $y = 80$, we get

$$x = 120 - 80 = 40$$

X	80	60	40
Y	40	60	80
	A	B	C

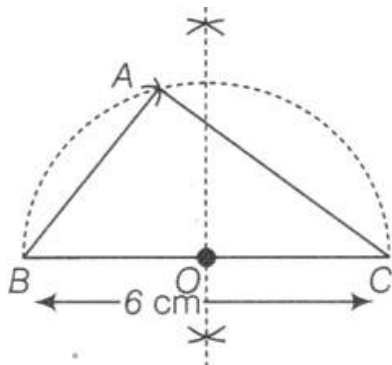


The Above figure represents the graph for the equation $x + y = 120$

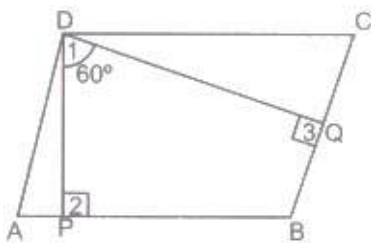
30. Steps of Construction:

- i. Draw a line segment $BC = 6$ cm.
- ii. Draw perpendicular bisector of BC which intersects BC at O .
- iii. Taking O as centre and radius OB , draw a semi-circle on BC .
- iv. Taking B as centre and radius equal to 4 cm, draw an arc, cutting the semi-circle at A . Also, here $\angle A$ would be of 90° as angle drawn in a semi-circle from diameter is always right angle.
- v. Join AB and AC .

Thus, ABC is the required right angle triangle.



31. In quadrilateral PBQD,



$$\begin{aligned}\angle 1 + \angle 2 + \angle B + \angle 3 &= 360^\circ \\ \Rightarrow 60^\circ + 90^\circ + \angle B + 90^\circ &= 360^\circ \\ \Rightarrow \angle B + 240^\circ &= 360^\circ \\ \Rightarrow \angle B &= 360^\circ - 240^\circ \\ \Rightarrow \angle B &= 120^\circ\end{aligned}$$

$$\text{Now, } \angle ADC = \angle B = 120^\circ$$

[\because Opposite angles of a parallelogram are equal]

$$\angle A + \angle B = 180^\circ$$

[\because Sum of consecutive interior angles is 180°]

$$\Rightarrow \angle A + 120^\circ = 180^\circ$$

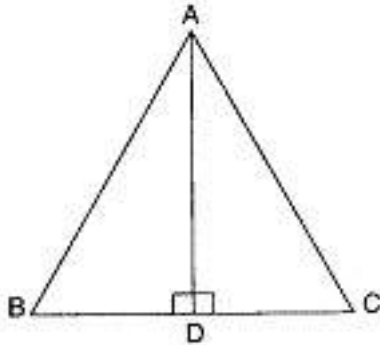
$$\Rightarrow \angle A = 180^\circ - 120^\circ$$

$$\Rightarrow \angle A = 60^\circ$$

$$\text{But, } \angle C = \angle A = 60^\circ$$

[\because Opposite angles of a parallelogram are equal]

32. In right triangles $\triangle ABD$ and $\triangle ACD$,



$$AB = AC \dots [\text{Given}]$$

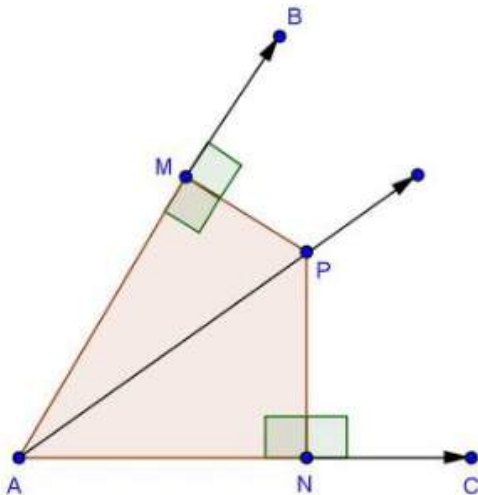
$$\text{side } AD = \text{side } AD \dots [\text{Common}]$$

$$\therefore \triangle ABC \cong \triangle ACD \dots [\text{R.H.S. Axiom}]$$

$$\therefore BD = DC \dots [\text{c.p.c.t.}]$$

AD is a median of $\triangle ABC$

OR



$$\text{Here } PM = PN$$

$$\text{and } \angle PMA = \angle PNA = 90^\circ$$

In $\triangle APM$ and $\triangle APN$,

$$AP = AP [\text{common}]$$

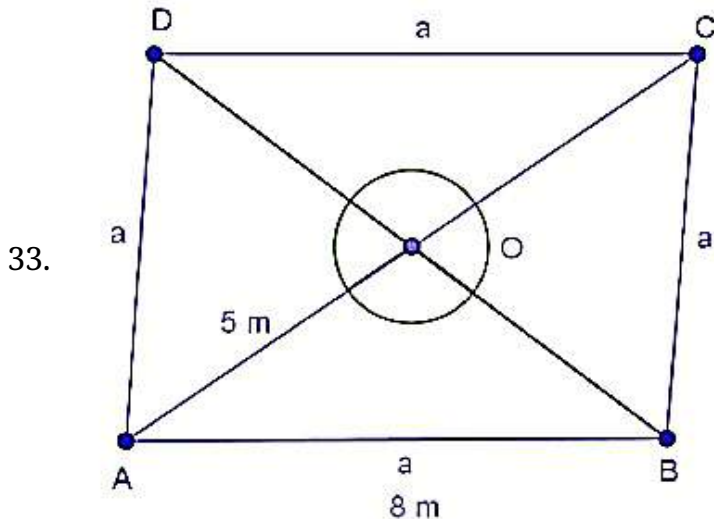
$$PN = PM [\text{given}]$$

$$\angle PMA = \angle PNA = 90^\circ \text{ [given]}$$

By RHS congruence criterion $\triangle APM \cong \triangle APN$,

$$\therefore \angle MAP = \angle NAP \text{ [c.p.c.t.]}$$

Hence, AP is the bisector of $\angle BAC$.



Since perimeter = 32 m

$$\Rightarrow 4a = 32\text{m [perimeter of rhombus} = 4 \times \text{side]}$$

$$\Rightarrow a = 8\text{m}$$

$$\text{Let, } AC = 10 \Rightarrow OA = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5\text{m}$$

$$\therefore OB^2 = AB^2 - OA^2 \text{ [by pythagoras theorem]}$$

$$\Rightarrow OB = \sqrt{8^2 - 5^2} = \sqrt{64 - 25} = \sqrt{39} \text{ m}$$

$$\text{Now, } BD = 2OB = 2\sqrt{39} \text{ m}$$

$$\therefore \text{Area of sheet} = \frac{1}{2} \times BD \times AC = \frac{1}{2} \times 2\sqrt{39} \times 10 = 10\sqrt{39} \text{ m}^2$$

$$\therefore \text{Cost of printing on both sides at the rate of Rs.5 per m}^2$$

$$= \text{Rs. } 2 \times 10\sqrt{39} \times 5$$

$$= \text{Rs. } 625.00$$

34. Total number of workers = $38 + 27 + 86 + 46 + 3 = 200$

i. P (person is 40 years or more) = P (Person having age 40 to 49 years) + P (person having age 50 to 59 years) + P (person having age 60 and above)

$$= \frac{86}{200} + \frac{46}{200} + \frac{3}{200}$$

$$= \frac{135}{200} = 0.675 = 0.68$$

ii. P (person is under 40 years) = P (person having age 20 to 29 years) + P (person

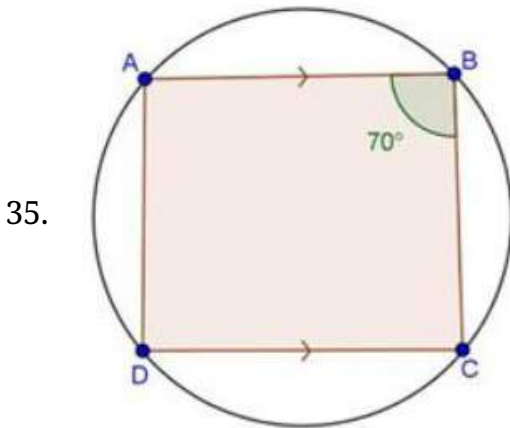
having age 30 to 39 years)

$$= \frac{38}{200} + \frac{27}{200}$$

$$= \frac{65}{200} = 0.325 = 0.33$$

iii. P (person having age under 60 but over 39 years) = P (person having age 40 to 49 years) + P (person having age 50 to 59 years)

$$= \frac{86}{200} + \frac{46}{200} = \frac{132}{200} = 0.66$$



We have, $\angle B = 70^\circ$

Since, ABCD is a cyclic quadrilateral

Then, $\angle B + \angle D = 180^\circ$

$$\Rightarrow 70^\circ + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 180^\circ - 70^\circ = 110^\circ$$

Since, AB \parallel DC and BC is transversal.

Then, $\angle B + \angle C = 180^\circ$ [Co-interior angles]

$$\Rightarrow 70^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 70^\circ = 110^\circ$$

Now, $\angle A + \angle C = 180^\circ$ [Opposite angles of cyclic quad.]

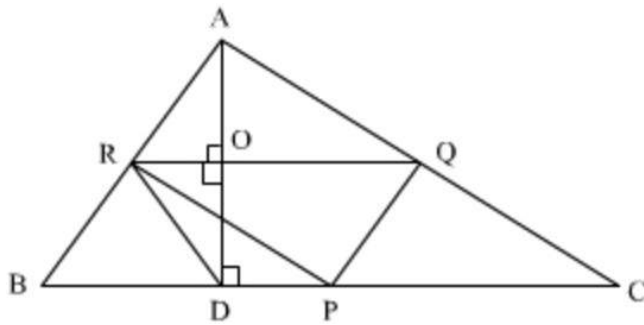
$$\Rightarrow \angle A + 110^\circ = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - 110^\circ = 70^\circ$$

OR

Given: P, Q, R and S are mid points and $AD \perp BC$

To prove : R, D, P and Q are con-cyclic.



Construction: Join RD, RP, RQ and PQ

Proof: In $\triangle ABC$, R and Q are midpoints of AB and CA respectively.

$\therefore RQ \parallel BC$ (Midpoint theorem)

Similarly, $PQ \parallel AB$ and $PR \parallel CA$

Now in quadrilateral BPQR,

$BP \parallel RQ$ and $PQ \parallel BR$

\therefore Quadrilateral BPQR is a parallelogram.

Similarly, quadrilateral ARPQ is a parallelogram.

$\therefore \angle A = \angle RPQ$ (Opposite angles of parallelogram are equal)

$PR \parallel AC$ and PC is the transversal,

$\therefore \angle BPR = \angle C$ (Corresponding angles)

$\angle BPQ = \angle DPR + \angle RPQ = \angle A + \angle C \dots(1)$

$RQ \parallel BC$ and BR is the transversal,

$\therefore \angle ARO = \angle B$ (Corresponding angles) $\dots(2)$

In $\triangle ABD$, R is the mid point of AB and $OR \parallel BD$.

\therefore O is the mid point of AD (Converse of mid point theorem)

$\Rightarrow OA = OD$

In $\triangle AOR$ and $\triangle DOR$,

$OA = OD$ (Proved)

$\angle AOR = \angle DOR (90^\circ)$ { $\angle ROD = \angle ODP$ (Alternate angles) & $\angle AOR = \angle ROD = 90^\circ$ (linear pair) }

$OR = OR$ (Common)

$\therefore \triangle AOR \cong \triangle DOR$ (SAS congruence criterion)

$\Rightarrow \angle ARO = \angle DRO$ (CPCT)

$\Rightarrow \angle DRO = \angle B$ (Using (2))

In quadrilateral PRQD,

$\angle DRO + \angle DPQ = \angle B + (\angle A + \angle C) = \angle A + \angle B + \angle C$ (Using (1))

$$\Rightarrow \angle DRO + \angle DPQ = 180^\circ (\angle A + \angle B + \angle C = 180^\circ)$$

Hence, quadrilateral PRQD is a cyclic quadrilateral.

Thus, the points P, Q, R and D are concyclic.

36. PQ intersect RS at O

$$\therefore \angle QOS = \angle POR [\text{vertically opposite angles}]$$

$$a = 4b \dots (1)$$

Also,

$$a + b + 75^\circ = 180^\circ [\because POQ \text{ is a straight line}]$$

$$\therefore a + b = 180^\circ - 75^\circ$$

$$= 105^\circ$$

Using, (1)

$$4b + b = 105^\circ$$

$$5b = 105^\circ$$

Or

$$b = \frac{105^\circ}{5} = 21^\circ$$

Now $a = 4b$

$$a = 4 \times 21^\circ$$

$$a = 84^\circ$$

Again, $\angle QOR$ and $\angle QOS$

$$\therefore a + 2c = 180^\circ$$

$$\text{Using, (2) } 84^\circ + 2c = 180^\circ$$

$$2c = 180^\circ - 84^\circ$$

$$2c = 96^\circ$$

$$c = \frac{96^\circ}{2} = 48^\circ$$

Hence,

$$a = 84^\circ, b = 21^\circ \text{ and } c = 48^\circ$$

37. Let $f(x) = 3x^3 + ax^2 + 3x + 5$

$$\text{and } g(x) = 4x^3 + x^2 - 2x + a$$

Here, the zero of $(x - 2)$ is $x = 2$ [$\because x - 2 = 0 \Rightarrow x = 2$]

Where $f(x)$ and $g(x)$ are divided by $(x - 2)$, then we get the remainders R_1 and R_2

$$\therefore f(2) = 3(2)^3 + a(2)^2 + 3(2) + 5$$

$$= 24 + 4a + 6 + 5 = 35 + 4a = R_1$$

$$\text{and } g(2) = 4(2)^3 + (2)^2 - 2(2) + a$$

$$= 32 + 4 - 4 + a = 32 + a = R_2$$

$$\text{Also, } R_1 - R_2 = 9$$

$$\therefore 35 + 4a - (32 + a) = 9$$

$$\Rightarrow 3 + 3a = 9 \Rightarrow 3a = 6 \Rightarrow a = 2$$

OR

We know that

$$(x^2 + \frac{1}{x^2})^2 = x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2}$$

$$\Rightarrow (x^2 + \frac{1}{x^2})^2 = (x^4 + \frac{1}{x^4}) + 2$$

$$\Rightarrow (x^2 + \frac{1}{x^2})^2 = 47 + 2 \text{ [Putting } x^4 + \frac{1}{x^4} = 47]$$

$$\Rightarrow (x^2 + \frac{1}{x^2})^2 = 49 = 7^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 7 \text{ [Taking square root of both sides]}$$

Now,

$$(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow (x + \frac{1}{x})^2 = 7 + 2 \text{ [Using: } x^2 + \frac{1}{x^2} = 7]$$

$$\Rightarrow (x + \frac{1}{x})^2 = 9 = 3^2$$

$$\Rightarrow x + \frac{1}{x} = 3$$

$$\Rightarrow (x + \frac{1}{x})^3 = 3^3 \text{ [Cubing both sides]}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3(x + \frac{1}{x}) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27 \text{ [Putting } x + \frac{1}{x} = 3]$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 27 - 9 \Rightarrow x^3 + \frac{1}{x^3} = 18$$

38. Let the radii of two spheres be r_1 cm and r_2 cm respectively.

Let the volumes of two spheres be V_1 and V_2 respectively. Then,

$$\frac{V_1}{V_2} = \frac{64}{27}$$

$$\Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\Rightarrow \frac{r_1^3}{r_2^3} = \frac{4^3}{3^3} \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \Rightarrow r_1 = \frac{4}{3}r_2$$

But, $r_1 + r_2 = 7$ [Given]

$$\Rightarrow \frac{4}{3}r_2 + r_2 = 7$$

$$\Rightarrow \frac{7}{3}r_2 = 7$$

$$\Rightarrow r_2 = 7 \times \frac{3}{7} = 3 \text{ cm}$$

$$\therefore r_1 = \frac{4}{3} \times 3 = 4 \text{ cm}$$

Let S_1 and S_2 be the surface areas of two spheres. Then,

$$S_1 = 4\pi r_1^2 = 4\pi \times 4 \times 4 = 64\pi \text{ cm}^2 \text{ and,}$$

$$S_2 = 4\pi r_2^2 = 4\pi \times 3 \times 3 = 36\pi \text{ cm}^2$$

$$\therefore S_1 - S_2 = 64\pi - 36\pi = 28\pi \text{ cm}^2$$

$$= 28 \times \frac{22}{7} \text{ cm}^2 = 88 \text{ cm}^2$$

OR

Let the external and internal radii of the hemispherical vessel be R cm and r cm respectively. Then, $R = 12.5$ cm and $r = 12$ cm.

Now,

$$\text{Area of outer surface} = 2\pi R^2$$

$$\text{Area of the ring at the top} = \pi R^2 - \pi r^2$$

$$\therefore \text{Total area to be painted} = (2\pi R^2 + 2\pi r^2 + \pi R^2 - \pi r^2)$$

$$\Rightarrow \text{Total area to be painted} = \pi (3R^2 + r^2)$$

$$\Rightarrow \text{Total area to be painted} = \frac{22}{7} \times \{3 \times (12.5)^2 + (12)^2\} \text{ cm}^2$$

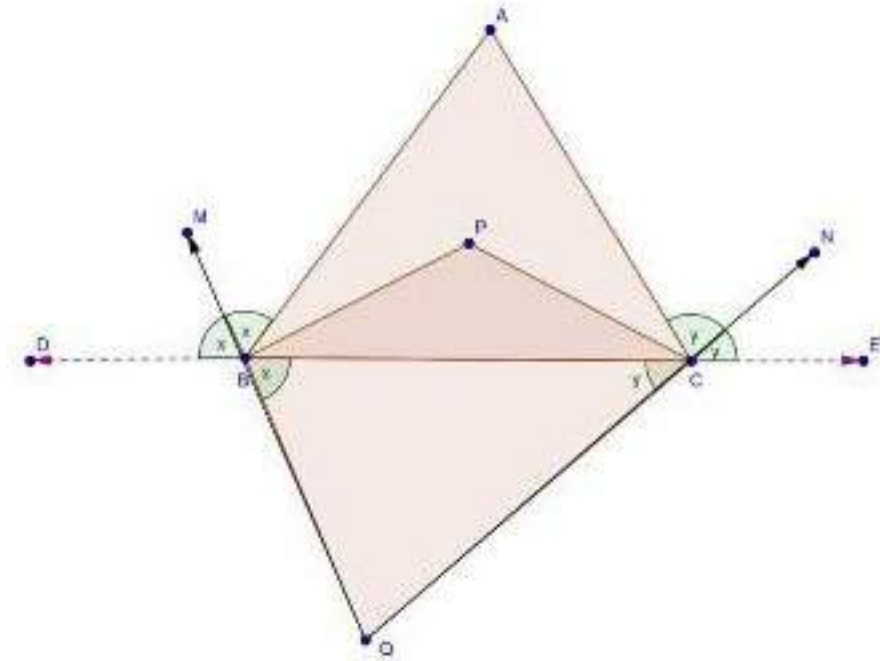
$$\Rightarrow \text{Total area to be painted} = \frac{22}{7} \times \{3 \times (\frac{25}{2})^2 + (12)^2\} \text{ cm}^2$$

$$\Rightarrow \text{Total area to be painted} = \frac{22}{7} \times [468.75 + 144] \text{ cm}^2$$

$$\Rightarrow \text{Total area to be painted} = \frac{22}{7} \times 612.75 \text{ cm}^2 = \frac{13480.5}{7} \text{ cm}^2 = 1925.78 \text{ cm}^2$$

$$\therefore \text{Cost of painting} = \text{Rs. } (1925.78 \times 0.05) = \text{Rs. } 96.28$$

39.



Given: In $\triangle ABC$, internal bisector of $\angle B$ and $\angle C$ meet at P and external bisector of $\angle B$ and $\angle C$ meet at Q

To prove: $\angle BPC + \angle BQC = 180^\circ$

Let $\angle ABD = 2x$ and $\angle ACE = 2y$

$\angle ABC = 180^\circ - 2x$ (linear pair)

$\angle ACB = 180^\circ - 2y$ (linear pair)

$\angle A + \angle ABC + \angle ACB = 180^\circ$ (sum of all angles of a triangle)

$\Rightarrow \angle A + 180^\circ - 2x + 180^\circ - 2y = 180^\circ$

$\Rightarrow 2x + 2y = 180^\circ + \angle A$

$\Rightarrow x + y = 90^\circ + \frac{1}{2} \angle A$

Now in $\triangle BQC$,

$x + y + \angle BQC = 180^\circ$

$\Rightarrow 90^\circ + \frac{1}{2} \angle A + \angle BQC = 180^\circ$

$\Rightarrow \angle BQC = 90^\circ - \frac{1}{2} \angle A \dots (i)$

and we know that $\angle BPC = 90^\circ + \frac{1}{2} \angle A \dots (ii)$

Adding (i) and (ii) we get

$\angle BPC + \angle BQC = 180^\circ$

Hence proved.

40. First we obtain the class marks as given in the following table.

Age (in years)	Class Marks	Frequency
0 - 2	1	2
2 - 4	3	4
4 - 6	5	6
6 - 8	7	8
8 - 10	9	9
10 - 12	11	6
12 - 14	13	5
14 - 16	15	3
16 - 18	17	1

Now, we plot the points (1, 2), (3, 4), (5, 6), (7, 8), (9, 9), (11, 6), (13, 5), (15, 3) and (17, 1). Now, we join the plotted points by line segments. The endpoints (1, 2) and (17, 1) are joined to the mid-points (-1, 0) and (19, 0) respectively of imagined class-intervals to obtain the frequency polygon.

