# Chapter – 14
## Statistics

## Exercise 14.3

Q.1 The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them

| Monthly consumption (in units) | Number of consumers |
|---|---|
| 65-85 | 4 |
| 85-105 | 5 |
| 105-125 | 13 |
| 125-145 | 20 |
| 145-165 | 14 |
| 165-185 | 8 |
| 185-205 | 4 |

**Answer:**

| Monthly consumption (in units) | Number of consumers | Cumulative frequency |
|---|---|---|
| 65-85 | 4 | 4 |
| 85-105 | 5 | 9 |
| 105-125 | 13 | 22 |
| 125-145 | 20 | 42 |
| 145-165 | 14 | 56 |
| 165-185 | 8 | 64 |
| 185-205 | 4 | 68 |
| | N = 68 | |

N= 68

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times W$$

where, l = lower limit of the median group
n = total frequency.
f = cumulative frequency of the group before median group f = frequency of median group
W = Group Width
$\frac{N}{2} = 34$
Hence,
Median class = 125 - 145
Cumulative frequency = 42
Lower limit, l = 125

cf = 22

f = 20

h = 20

Hence,

Median can be calculated as:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times W$$

$$= 125 + \left(\frac{34 - 22}{20}\right) \times 20$$

$$= 125 + 12$$

$$= 137$$

Now, mode can be calculated as:

$$\text{Mode} = l + \left(\frac{f1 - f0}{2f1 - f0 - f2}\right) h$$

where,

l = lower limit of the modal class $f_1$ = absolute frequency of the modal class

$f_0$ = absolute frequency of the class before modal class

$f_2$ = absolute frequency of the class after modal class

h = class width

Modal class = 125-145

l = 125

h = 20

$f_1$ = 20

$f_0$ = 13

$f_2$ = 14

$$\text{Mode} = l + \left(\frac{f1 - f0}{2f1 - f0 - f2}\right) h$$

$$= 125 + \left(\frac{20-13}{2\sim20-13-14}\right) 20$$

$$= 125 + \frac{7}{13} \times 20$$

$= 125 + 10.76$

$= 135.76$

Now, mean of the following data can be calculated as:

| Class interval | fi | xi | di | ui | fiui |
|---|---|---|---|---|---|
| 065-085 | 4 | 75 | -60 | -3 | -12 |
| 085-105 | 5 | 95 | -40 | -2 | -10 |
| 105-125 | 13 | 115 | -20 | -1 | -13 |
| 125-145 | 20 | 135 | 0 | 0 | 0 |
| 145-165 | 14 | 155 | 20 | 1 | 14 |
| 165-185 | 8 | 175 | 40 | 2 | 16 |
| 185-205 | 4 | 195 | 60 | 3 | 12 |
| | $\Sigma fi = 68$ | | | | $\Sigma fiui = 7$ |

$$\bar{x} = a + \frac{\Sigma fiui}{\Sigma fi} \times h$$

where, a = assumed mean $f_i$ = frequency of ith term

$u_i$ = a - $x_i$ / h

h = class width

$$= 32.5 + \frac{-23}{35} \times 5$$

$= 137.05$

Hence,

Mean, Median and Mode are more or less equal in this distribution.

Q.2 If the median of the distribution given below is 28.5, find the values of x and y

| Class interval | Frequency |
|---|---|
| 0-10 | 5 |
| 10-20 | X |
| 20-30 | 20 |
| 30-40 | 15 |
| 40-50 | Y |
| 50-60 | 5 |
| Total | 60 |

**Answer:**

Let's make a cumulative frequency table for the above problem

| Class interval | Frequency | Cumulative frequency |
|---|---|---|
| 0-10 | 5 | 5 |
| 10-20 | X | 5+x |
| 20-30 | 20 | x+25 |
| 30-40 | 15 | 40+x |
| 40-50 | Y | 40+x+y |
| 50-60 | 5 | 45+x+y |
| Total | 60 | |

Total frequency, N= 60

$\frac{N}{2} = 30$

Now,

Given median = 28.5, lies in 20 - 30

Median class = 20-30

frequency corresponding to median class, f = 20

cumulative frequency of the class preceding the median class, cf = 5 + x

Lower limit, l = 20

class height, h = 10
Now,
Median can be calculated as:

$$\text{Median} = l + \left(\frac{\frac{n}{2}-c.f}{f}\right) \times W$$

$$28.5 = 20 + \left(\frac{30-5-x}{20}\right) \times 10$$

$$28.5 - 20 = \frac{25-x}{2}$$

$$8.5 = \frac{25-x}{2}$$

$$25 - x = 8.5 \times 2$$

$$\Rightarrow 25 - x = 17$$

$$\Rightarrow x = 25-17$$

$$\Rightarrow x = 8$$

Now,
From the cumulative frequency we can find the value of x + y as:

$$45 + x + y = 60$$

$$\Rightarrow x + y = 60 - 45$$

$$\Rightarrow x + y = 15$$

$$\Rightarrow y = 15 - x$$

as, x = 8

$$\Rightarrow y = 15 - 8$$

$$\Rightarrow y = 7$$

Hence,
Value of x = 8 and y = 7

Q.3 A life insurance agent found the following data for distribution of ages of 100 policyholders. Calculate the median age, if policies are given only to persons having age 18 years on wards but less than 60 years.

| Age ( in years) | Number of policy holders |
|---|---|
| Below 20 | 2 |
| Below 25 | 6 |
| Below 30 | 24 |
| Below 35 | 45 |
| Below 40 | 78 |
| Below 45 | 89 |
| Below 50 | 92 |
| Below 55 | 98 |
| Below 60 | 100 |

**Answer:**

In this case, we are given less than (or below) cumulative frequency distribution, we need to convert it into normal frequency distribution.

So, we need to find class intervals and corresponding frequency. Since, the difference between ages in each class is 5, we can take the first class interval as 15 - 20 and its frequency will be same as frequency of below 20 class. Also, for other class, class interval will can be found as following and corresponding frequency can be find by subtracting the previous frequency from the cumulative frequency.

| Class intervals | Frequency | Cumulative frequency |
|---|---|---|
| 15-20 | 2 | 2 |
| 20-25 | 4 | 6 |
| 25-30 | 18 | 24 |
| 30-35 | 21 | 45 |
| 35-40 | 33 | 78 |
| 40-45 | 11 | 89 |
| 45-50 | 3 | 92 |
| 50-55 | 6 | 98 |
| 55-60 | 2 | 100 |

As per the question,

N=100

$\frac{N}{2}$ =50

Hence,

Median class = 35-45

Cumulative frequency = 100

Lower limit, l = 35

cf = 45
f = 33
h = 5
Now,
Median can be calculated as:

$$Median = l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times W$$

where,

l = lower limit of median class n = total frequency of the data cf = cumulative frequency of the class before median class f = frequency of the median class

$$= 35 + \left(\frac{50-45}{33}\right) \times 5$$

$$Median = 35 + \frac{25}{33}$$

Median = 35.75 years

Q.4 The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

| Length ( in mm) | Number of leaves |
|---|---|
| 118-126 | 3 |
| 127-135 | 5 |
| 136-144 | 9 |
| 145-153 | 12 |
| 154-162 | 5 |
| 163-171 | 4 |
| 172-180 | 2 |

Find the median length of the leaves

(Hint: The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to117.5 - 126.5, 126.5 - 135.5, . . ., 171.5 - 180.5)

**Answer:**

The cumulative frequency of the data can be calculated as:

| Class interval | frequency | Cumulative frequency |
|---|---|---|
| 117.5-126.5 | 3 | 3 |
| 126.5-135.5 | 5 | 8 |
| 135.5-144.5 | 9 | 17 |
| 144.5-153.5 | 12 | 29 |
| 153.5-162.5 | 5 | 34 |
| 162.5-171.5 | 4 | 38 |
| 171.5-180.5 | 2 | 40 |

As per the question,

N= 40

$\frac{N}{2} = 20$

Hence,

Median class =144.5-153.5

Lower limit, l = 144.5

cf = 17

f = 12

h = 9

Now,

Median can be calculated as:

$$Median = l + \left( \frac{\frac{n}{2} - c.f}{f} \right) \times W$$

$= 144.5 + \left(\frac{20-17}{12}\right) \times 9$

$= 144.5 + \frac{9}{4}$

$= 146.75$

Q.5 The following table gives the distribution of the life time of 400 neon lamps:

| Life time ( in hours) | Number of lamps |
|---|---|
| 1500-2000 | 14 |
| 2000-2500 | 56 |
| 2500-3000 | 60 |
| 3000-3500 | 86 |
| 3500-4000 | 74 |
| 4000-4500 | 62 |
| 4500-5000 | 48 |

Find the median life time of a lamp

**Answer:**

The cumulative frequency of the given data can be calculated as:

| Life time ( in hours) | Number of lamps | Cumulative frequency |
|---|---|---|
| 1500-2000 | 14 | 14 |
| 2000-2500 | 56 | 70 |
| 2500-3000 | 60 | 130 |
| 3000-3500 | 86 | 216 |
| 3500-4000 | 74 | 290 |
| 4000-4500 | 62 | 352 |
| 4500-5000 | 48 | 400 |

As per the question,
N= 400

$$\frac{N}{2} = 200$$

Hence,
Median class = 3000-3500

Now,
Median class = 3000-3500
frequency corresponding to median class, f = 86
cumulative frequency of the class preceding the median class, cf =130
Lower limit, l = 3000
class height, h = 500
Now,

Median can be calculated as:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times W$$

$$= 3000 + \left(\frac{200 - 130}{86}\right) \times 500$$

$$= 3000 + 406.97$$
$$= 3406.97$$

Q.6 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows

| Number of letters | 1-4 | 4-7 | 7-10 | 10-13 | 13-16 | 16-19 |
|---|---|---|---|---|---|---|
| Number of Surnames | 6 | 30 | 40 | 16 | 4 | 4 |

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames

**Answer:**

The cumulative frequency of the given data can be calculated as

| Class interval | Frequency | cumulative frequency |
|---|---|---|
| 1-4 | 6 | 6 |
| 4-7 | 30 | 36 |
| 7-10 | 40 | 76 |
| 10-13 | 16 | 92 |
| 13-16 | 4 | 96 |
| 16-19 | 4 | 100 |

As per the question,

N= 100

$\frac{N}{2} = 50$

Hence,

Median class = 7-10

Lower limit, l = 7

cf = 36

f = 40

h = 3

Now,

Median can be calculated as:

$$\text{Median} = l + \left( \frac{\frac{h}{2} - c.f}{f} \right) \times h$$

$$= 7 + \left( \frac{50 - 36}{40} \right) \times 3$$
$$= 7 + \frac{14}{40} \times 3$$
$$= 8.05$$

Now, mode can be calculated as: class corresponding to maximum frequency.

Modal class = 7-10
l = 7
h = 3
f1 = 40
f0 = 30
f2 = 16

Mode $= l + \left(\dfrac{f1-f0}{2f1-f0-f2}\right) h$

$= 7 + \left(\dfrac{40-30}{2\times40-30-16}\right) 3$

$= 7 + \left(\dfrac{10}{34}\right) \times 3$

$= 7.88$

Now, mean of the following data can be calculated as:

| Class interval | fi | xi | fixi |
|---|---|---|---|
| 1-4 | 6 | 2.5 | 15 |
| 4-7 | 30 | 5.5 | 165 |
| 7-10 | 40 | 8.5 | 340 |
| 10-13 | 16 | 11.5 | 184 |
| 13-16 | 4 | 14.5 | 51 |
| 16-19 | 4 | 17.5 | 74 |
| | $\Sigma fi = 100$ | | $\Sigma fixi = 825$ |

$\bar{x} = \dfrac{\Sigma fixi}{\Sigma fi}$

$= \dfrac{825}{100}$

$= 8.25$

Q.7 The distribution below gives the weights of 30 students of a class. Find the median weight of the students

| Weight ( in kg). | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 |
|---|---|---|---|---|---|---|---|
| Number of students | 2 | 3 | 8 | 6 | 6 | 3 | 2 |

**Answer:**

The cumulative frequency of the given data can be calculated as:

| Class interval | frequency | Cumulative frequency |
|---|---|---|
| 40-45 | 2 | 2 |
| 45-50 | 3 | 5 |
| 50-55 | 8 | 13 |
| 55-60 | 6 | 19 |
| 60-65 | 6 | 25 |
| 65-70 | 3 | 28 |
| 70-75 | 2 | 30 |

As per the question,
N= 30

$\frac{N}{2}$ =15
Hence,
Median class = 55-60
Lower limit, l = 55
cf = 13
f = 6
h = 5
Now,
Median can be calculated as:

$$Median = l + \left(\frac{\frac{n}{2}-c.f}{f}\right) \times h$$

where, l = lower limit of median class
n = total frequency of distribution
c.f. = cumulative frequency of the class before median class
f = frequency of the median class
h = class width

$$= 55 + \left(\frac{15-13}{6}\right) \times 6$$

$$= 55 + \frac{2}{6} \times 5$$

$$= 55 + 1.67$$

Median = 56.67
Median weight is 56.57 kg