Chapter 4

Viscous Flow of Incompressible Fluids

CHAPTER HIGHLIGHTS

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- *Buckingham's ∏-Theorem*
- *Dimensionless Numbers*
- *Average Velocity (V_{avg})*
- *Laminar, Transitional and Turbulent Flows*
- *Critical Reynold's Numbers*
- *Entrance Length*
- *Laminar Flow in Horizontal Pipes*
- *Kinetic Energy Correction Factor (α)*
- *Momentum Correction Factor*
- *Flow of a Viscous Fluid Between Two Parallel Plates*
- *Flow of Lubricant in a Journal Bearing*
- *Turbulent Flow in Pipes*
- *Shear Stress in a Turbulent Flow*
- *Boussinesq Approximation or Hypothesis*
- *Relative Roughness*
- *Turbulent Velocity Profile*
- *Friction Factor in Turbulent Flow*
- *Intensity of Turbulence in a Flow*
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- *Pipe Network*
- *Power Transmission Through Pipes*
- *Water Hammer in Pipes*
- *Mass Flow*
- *Drag Force on the Plate*
- *Summary of Fluid Frictional Resistance*

Dimensional Analysis

It is a mathematical technique which involves the study of dimensions for solving engineering problems. Each physical phenomenon can be expressed by an equation which relates several dimensions and non-dimensional quantities.

Buckingham's ∏–Theorem

If there are n variables in a dimensionally homogenous (i.e., each additive term has the same dimensions) equation and if these variables contain m fundamentals (basic or primary) dimensions, then the variables can be arranged into (n–m) dimensionless terms (or parameters) called –terms and the equation can be written in terms of these (n–m) terms.

Dimensionless Numbers

Reynolds Number (Re)

It is defined as the ratio of inertia force to the viscous force.

$$R_e = \frac{\rho VL}{\mu}$$

Where and are the density and viscosity of the fluid respectively. V is a characteristic velocity and L is a characteristic length.

For pipe flow, characteristic length is equal to the diameter of the pipe (D) and hence

$$R_e \text{ pipe flow} = \frac{\rho VD}{\mu}$$

Froude Number (Fr)

It is defined as the square root of the ratio of inertia force to the gravity force

$$F_r = \frac{V}{\sqrt{Lg}}$$

Euler Number (Eu)

It is defined as the square root of the ratio of inertia force to the pressure force.

$$Eu = \frac{V}{\sqrt{\frac{P}{\rho}}}$$

Where p is the pressure difference

Weber Number (we)

It is defined as the square root of the ratio of inertia force to the surface tension force.

$$W_e = \sqrt{\frac{\rho V^2 L}{\sigma}} \qquad \text{i.e., } W_e = \frac{V}{\sqrt{\dfrac{\sigma}{\rho L}}}$$

Where   is the surface tension

Match Number (ma)

It is defined as the square root of the ratio of inertia force to the elastic force.

$$Ma = \frac{V}{C}$$

Where C is the velocity of sound in the fluid.

Average Velocity (V_{avg})

It is defined as the average speed through a cross-section and is defined as

$$V_{\text{avg}} = \frac{\int_A \rho u(r) dA}{\rho A}$$

Where   is the fluid density, A is the cross-sectional area, $u(r)$ is the velocity at any radius 'r' (referred to the pipe centre) the distance from the pipe centerline.

For incompressible flow in a circular pipe of radius R,

$$V_{\text{avg}} = \frac{2}{R^2}\int_O^R U(r) r dr$$

Laminar, Transitional and Turbulent Flows

The three flow regimes that can be identified are: (1) laminar (2) transitional and (3) turbulent. Laminar flow is characterized by smooth streamlines and it is a highly ordered motion. Turbulent flow is characterized by velocity fluctuations and it is a highly disordered motion. In transitional flow, the flow fluctuates between laminar and turbulent in a highly disordered manner.

Critical Reynold's Numbers

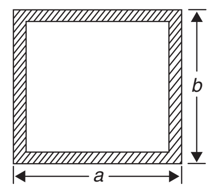
1. For flow in a circular pipe, Reynold's number is given by $R_e = \dfrac{\rho V_{AV} D}{\mu}$,

   where,
   = density of fluid flowing inside the pipe
   V_{AV} = average velocity of flow inside the pipe
   D = diameter of the pipe and
   = dynamic viscosity of the fluid inside the pipe.
2. For flow through ducts (or non-circular cross-section pipes), Reynold's number is based on the hydraulic mean diameter (D_m) instead of D.

   For non-circular pipes, $R_e = \dfrac{\rho V_{AV} D_m}{\mu}$, where

   $$D_m = \text{hydraulic diameter of duct} = \frac{4A}{P_w}$$

   Here, A = Area of flow and
   P_w = wetted perimeter of duct
   For example, for a rectangular duct of width a and height b shown in figure,

   Hydraulic mean diameter,

   $$D_m = \frac{4A}{P_w} = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$$
3. For flow over flat plate, Reynold's number is given by

   $$R_e = \frac{\rho V x}{\mu},$$

   where x = distance of the point on the plate from where the solid surface starts (measured in the direction of flow),

   The Reynolds's number at and below which the flow remains laminar (i.e., all turbulences are damped down), is called lower critical Reynolds's number.

   The Reynolds's number at and above which the flow is turbulent (i.e., flow cannot remain laminar) is called the upper critical reynolds's number. In between these two critical values of Reynolds's number, flow is transitory.

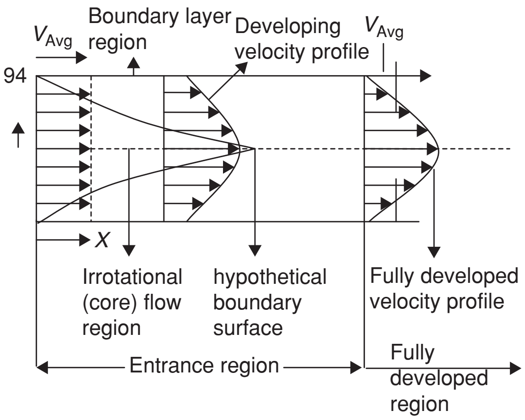
   The lower critical Reynolds's number and upper critical Reynolds's number for various types of flows are tabulated below.

Sl. No	Flow Condition	Laminar	Transitional	Turbulent
1	Flow in circular pipes	Re 2000	2000 < Re < 4000	Re 4000
2	Open channel flow	Re 500	500 < Re < 1000	Re 1000
3	Flow over plate	Re < 5 10^5		Re > 5 10^5

Entrance Region and Fully Developed Flow

When a fluid enters a circular pipe at a nearly uniform velocity, a velocity gradient develops along the pipe. A boundary layer (flow region in which effects of the viscous shearing forces caused by the fluid viscosity are significant) is produced which grows in thickness to completely fill the pipe. At a point further down stream, the velocity becomes fully developed and the region from the pipe inlet to this point is called the *entrance region* whose length is called the *entrance length* 'Le'.

The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged in the flow direction is called the 'fully developed region'



At the irrotational (core) flow region, viscous effects are negligible and velocity remains essentially constant in the radial direction. This region is separated from the boundary layer region by a hypothetical boundary surface.

The effect of the entrance region is to increase the average friction factor for the entire pipe. the increase being significant for short pipes but negligible for long pipes.

Entrance Length

In laminar flow = $\boxed{\dfrac{Le}{D} \approx 0.06R_e}$

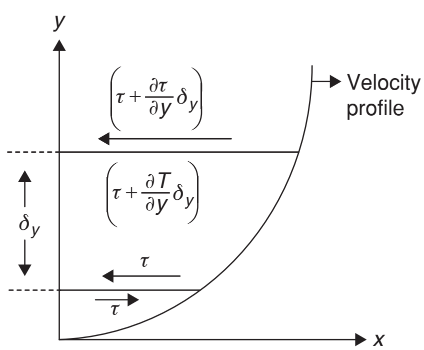
In turbulent flow, $\boxed{\dfrac{Le}{D} \approx 4.4R_e^{\frac{1}{6}}}$

$\dfrac{Le}{D}$ is sometimes referred to as the dimensionless entrance length.

Entrance length for turbulent flow is much shorter than for laminar flow

Relationship Between Shear Stress and Pressure Gradient

Let us consider a fluid element whose velocity distribution is shown in the following figure, where the shear stresses ( ) acting on the two fluid layers is also shown.



The motion of the fluid element will be resisted by shearing or frictional forces which must be overcome by maintaining a pressure gradient in the direction of flow. Here,

$$\boxed{\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x}}$$

i.e., the pressure gradient $\left(\dfrac{\partial p}{\partial x}\right)$ in the direction of flow (steady and uniform) is equal to the shear gradient $\left(\dfrac{\partial \tau}{\partial y}\right)$ in the direction normal to the direction of flow. The above equation holds for all flow conditions and geometries

Laminar Flow in Horizontal Pipes

The following discussion is based on the steady laminar incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe unless stated other wise.

1. A fully developed laminar pipe flow is merely a balance between pressure and viscous forces. For the steady fully developed laminar flow of a fluid through a horizontal circular pipe of radius R, the shear stress distribution is given by

$$\boxed{\tau = \frac{-\partial p}{\partial x} \cdot \frac{r}{2}} \quad (1)$$

Here x is the distance along the pipe.

The pressure gradient in the x-direction $\dfrac{\partial p}{\partial x}$ is larger in the entrance region than in the fully developed region where it is a constant, $\dfrac{\partial p}{\partial x} = \dfrac{-\Delta p}{L}$ where p is the pressure drop over a flow section of length L.

$$\boxed{\tau = \frac{\Delta p}{L}\frac{r}{2}}$$

Few highlighting points that can be deciphered from equation (1) are

(i) Flow will occur only if a pressure gradient exists in the flow.

(ii) Pressure decreases in the direction of flow due to viscous effects

(iii) Shear stress varies linearly across the flow section with a value of zero at the centre of the pipe ($r = 0$) and with a maximum value $\left(= \dfrac{-\partial p}{\partial x}\dfrac{R}{2} = \dfrac{\Delta p}{L}\dfrac{R}{2}\right)$ at the pipe wall.

The shear stress at the pipe wall is called the *wall shear stress* .

$$\boxed{\tau_\omega = \frac{\Delta p}{L}\frac{R}{2}}$$

$$\boxed{\tau = \frac{\tau_\omega r}{R}}$$

The wall shear stress is highest at the pipe inlet and it decreases gradually to the fully developed value. In a steady fully developed flow, wall shear stress remains constant. The above four equation are valid for turbulent flow also.

The equations stated in the following section rest on the following two assumptions:

(i) Fluid is Newtonian
(ii) No slip of fluid particles occurs at the boundary (no-slip condition), i.e., fluid particles adjacent to the pipe will have zero velocity.

2. Velocity profile (u (r)): In a fully developed laminar flow, there is no motion in the radial direction and thus the velocity component in the direction normal to the pipe axis is every where zero.

For a steady fully developed pipe flow,

(i) $\frac{\partial u}{\partial x}(r, x) = 0 \Rightarrow u = u(r)$. Velocity contains only an axial component, which is a function of only the radial component.
(ii) Acceleration experienced by the fluid is zero Local acceleration is zero as the flow is steady and convective acceleration is zero as the flow is fully developed.

The velocity profile is given by

$$\boxed{u(r) = \frac{R^2}{4\mu}\left(\frac{\Delta p}{L}\right)\left(1 - \frac{r^2}{R^2}\right)} \qquad (2)$$

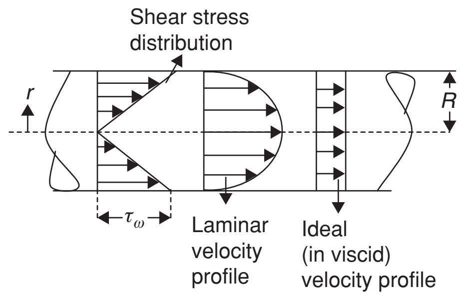
Velocity profile of a fully developed laminar flow in a pipe is parabolic while for a fully developed turbulent flow, it is much flatter.

The velocity profile has a maximum value $\left(u_{max} = \frac{R^2}{4\mu}\left(\frac{\Delta p}{L}\right)\right)$ at the pipe centerline and a minimum value (= zero) at the pipe wall.

$$\boxed{u(r) = u_{max}\left(1 - \left(\frac{r}{R}\right)^2\right)}$$

In a fully developed laminar pipe flow, the average velocity is one half of the maximum velocity i.e., $u_{max} = 2V_{avg}$

$$\boxed{u(r) = 2V_{avg}\left(1 - \left(\frac{r}{R}\right)^2\right)}$$



Kinetic Energy Correction Factor ( )

Kinetic energy correction factor is defined as the ratio of the kinetic energy of flow per second based on actual velocity across a section to the kinetic energy of flow per second based on average velocity across the same section. This factor is introduced to account for the non-uniformity of the velocity profile across an inlet or outlet due to viscosity of fluid.

Kinetic energy correction factor,

$$\alpha = \frac{1}{A}\int\left(\frac{V}{V_{avg}}\right)^3 dA,$$

where
V = local velocity at any point in the cross-section
V_{avg} = average velocity across the cross-section.
For uniform velocity distribution, = 1.0
For laminar flow through a pipe, = 2.0
For turbulent flow through a pipe, = 1.01 to 1.33

Momentum Correction Factor

The momentum correction factor is defined as the ratio of momentum of the flow per second based on the actual velocity to the momentum of the flow per second based on the average velocity across a cross-sectional area. This factor is introduced to account for the non-uniformity of the velocity across an inlet or outlet and it is defined by

$$\boxed{\beta = \frac{\int \rho\vec{v}(\vec{v}\cdot\vec{n})dA}{\mathring{m}\,\vec{v}_{avg}}}$$

If the density is uniform over the inlet or outlet and $\vec{v}$ is the same direction as $\vec{v}_{avg}$, then

$$\beta = \frac{\int \rho v(\vec{v}\cdot\vec{n})dA}{\mathring{m}V_{avg}}$$

$$= \frac{\int v(\vec{v}\cdot\vec{n})dA}{v_{avg}^2 A}$$

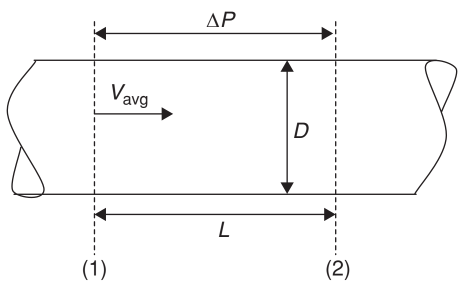
If the control surface slices normal to the inlet or outlet area, i.e., $(\vec{v}\cdot\vec{n})dA = vdA$ then

$$\beta = \frac{1}{A}\int\left(\frac{v}{v_{avg}}\right)^2 dA$$

The factor is always greater than or equal to one. For a fully developed laminar pipe flow, $\beta = \frac{4}{3}$ and for a fully developed turbulent pipe flow, 1.01   1.2.

At an inlet or outlet, if the flow is uniform then $\beta = 1$ and $\vec{v} = \vec{v}_{avg}$

Pressure Drop (P)



The pressure drop (between sections1 and 2) across a length L of a flow section in a horizontal circular pipe of diameter D,

$$\Delta P = \frac{32\mu L V_{avg}}{D^2}$$

The pressure drop for all type of flow, developed pipe flow (laminar or turbulent flows, circular or non-circular pipes, smooth or rough surfaces, horizontal or inclined pipes),

$$\Delta P = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

Where f is the *Darcy friction factor* or *Darcy weisbach friction factor* or simply the *friction factor*

$$f = \frac{8\tau_\omega}{\rho V_{avg}^2}$$

The skin friction coefficient or the *coefficient of friction* or the *Fanning friction factor*. (C_f) is defined as:

$$C_f = \frac{2\tau_\omega}{\rho V_{avg}^2}$$

$$C_f = \frac{f}{4}$$

For a fully developed laminar flow Darcy's friction factor $f = \frac{64}{R_e}$ and hence the friction factor for the flow is a function of only the Reynolds number and is independent of the roughness of the pipe surface

Friction factor is maximum for a fully developed turbulent flow

Head Loss (h_L)

The pressure drop (P) due to viscous effects or friction represents an irreversible pressure loss and is generally called as pressure loss due to friction (P_L). Head loss (h_L) in general refers to any energy loss associated with the flow but here it is stated loss to refer to the pressure losses expressed in terms of an equivalent fluid column height.

$$h_L = \frac{\Delta P_L}{\rho_g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

$$h_L = \frac{2\tau L}{\rho g r} = \frac{4L\tau_\omega}{\rho g D}$$

Equation (3) called the Darcy–Weisbach equation, is valid for laminar and turbulent flows in both circular and non-circular pipes. The head loss represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe. In equation (3), pressure drop is taken to be equivalent to the pressure loss and this is valid only under the assumptions by which the equivalency can be derived from Bernoulli's equation. The variable h_L is generally referred to as the head loss due to friction. It is to be noted that P_L and h_L both represent losses over the length of the pipe.

For the flow of an ideal (in viscid)fluid, $h_L = 0$

Required Pumping Power ($\mathring{W}_{pump,}$ L)

The required pumping power to overcome the pressure loss,

$$\mathring{W}_{pump,L} = QDP_L = \mathring{m}\, g\, h_L$$

Volumetric Flow Rate (Q)

The average velocity for laminar flow in a horizontal circular pipe,

$$V_{avg} = \frac{\Delta P D^2}{32\,\mu\, L}$$

Volumetric flow rate for laminar flow through a horizontal pipe of diameter D and length L,

$$Q = V_{avg}A = \frac{\Delta P\pi D^4}{128\mu L}$$

The above equation is called the *Poiseuille's law* or the *Hagen–Poiseuille equation*. The steady laminar viscous flow in a channel or tube from a region of high pressure to a region of low pressure is called *Poiseuille flow.*

$$Q = \frac{\pi}{2}U_{max}R^2$$

Solved Example

Example 1: In a horizontal circular pipe of length 20 m, a fluid (density = 850 Kg/m^3, viscosity = 9 poise) flows in a steady fully developed laminar manner. If the head loss and the wall shear stress associated with the flow are 5 m and 104 N/m^2 respectively, then the Darcy friction factor for the flow is

(A) 0.5 (B) 0.0073
(C) 0.1167 (D) 1.868

Solution:

Given $L = 20$ m

= 850 Kg/m^3

= 0.9 Pa. sec

$h_L = 5$ m

= 104 N/m^2

It is presumed here that all the assumptions for the pressure loss to be equal to the pressure drop are valid.

$P = h_1 \quad g = 5 \quad 850 \quad 9.81 = 41692.5\ P_a$

Now $\tau_\omega = \frac{\Delta P}{L}\times\frac{R}{2}$

Radius of the pipe, $R = \frac{2L\tau_\omega}{\Delta P}$

$$= \frac{2\times 20\times 104}{41692.5} = 0.1\text{m}$$

Now $\Delta P = \frac{32\mu V_{avg}L}{D^2}$

$$= \frac{8\mu V_{avg}L}{R^2}$$

$$\therefore\ V_{avg} = \frac{41692.5\times(0.1)^2}{8\times 0.9\times 20} = 2.895 \text{ m/s}$$

Now, $f = \frac{8\tau_\omega}{\rho V_{avg}^2}$

$$= \frac{8\times 104}{850\times 2.895^2}$$

$= 0.1167$

Example 2: The mass flow rate of the steady fully developed laminar flow of a fluid (density = 900 Kg/m^3, viscosity = 9 poise) in a horizontal pipe of diameter 0.5 m is 212.06 Kg/s. The perpendicular distance from the pipe wall at which the velocity is 0.432 m/s is

(A) 0.0236 m (B) 0.4527 m
(C) 0.0472 m (D) 0.2264 m

Solution:

Given $\dot{m} = 212.03$ Kg/s

= 900 Kg/m^3

$D = 0.5$ m

$V = 0.432$ m/s

$\mathring{m} = \rho A V_{avg}$

$$V_{avg} = \frac{212.06\times 4}{900\times\pi\times 0.5^2}$$

= 1.2 m/s

It is known that for the flow condition given, the velocity profile of the flow is given by:

$$V = V_{max}\left(1-\frac{r^2}{R^2}\right)$$

Here maximum velocity

$V_{max} = 2\quad V_{avg}$

$$\therefore\quad V = 2\times V_{avg}\left(1-\frac{r^2}{R^2}\right)$$

$$0.432 = 2\times 1.2\times\left(1-\frac{r^2}{0.25^2}\right)$$

$r = 0.2264$ m

The perpendicular distance from the pipe wall at which the velocity is 0.432 m/s is

$= R \quad r = 0.25 \quad 0.2264$

$= 0.0236$ m

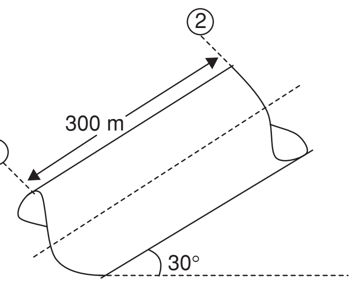
Example 3: A circular pipe of diameter 0.07 m and length 300 m is inclined at an angle 30° with the horizontal. The volumetric flow rate of the steady fully developed laminar flow of the fluid (viscosity = 8 poise, density = 800 Kg/m^3) in the pipe is 7 litre/sec. The minimum power of a pump with efficiency 70% that can maintain this flow is:

(A) 40.28 W (B) 40.28 kW
(C) 28.196 kW (D) 48.89 kW

Solution:

The energy equation is given by:

$$\frac{P_1}{\rho g}+\frac{\alpha_1 v_1^2}{2g}+Z_1+h_p = \frac{P_L}{\rho g}+\alpha_2\frac{V_2^2}{2g}+Z_2+h_t+h_L \quad (1)$$



Since no pump and turbine is involved in the flow section considered, $h_p = h = 0$.

The level Z_1 is considered as datum i.e., $Z_1 = 0$. Hence,

$$Z_2 = L \quad \sin 30 = 300 \times \frac{1}{2} = 150\,\text{m}$$

Although the velocity is not uniform across a pipe cross-section, the velocity profile does not change from section 1 to section 2 due to the fully developed flow

$_1 = {}_2$

Now $V_1 = V_2$ (from continuity equation)

Therefore, equation (1) becomes

$$\frac{P_1 - P_2}{\rho g} = 150 + h_L = 150 + \frac{32\mu L V_{avg}}{D^2 \rho g}$$ (it is assumed that $P_L \quad P$)

$$= 150 + \frac{32\mu L}{D^2 \rho g} \times \frac{Q}{\frac{\pi D^2}{4}}$$

$$= 150 + \frac{32 \times 0.8 \times 300}{0.07^2 \times 800 \times 9.81} \times \frac{0.007}{\frac{\pi \times 0.07^2}{4}}$$

$$= 513.26$$

Or $P_1 \quad P_2 = 513.26 \quad 800 \quad 9.81$

$$= 4.028\ \text{MN/m}^2$$

Power of the pump $= \dfrac{Q \times (P_1 - P_2)}{\eta}$

$$= \frac{0.007 \times 4.025 \times 10^6}{0.7}$$

$$= 40280\ \text{w} = 40.28\ \text{kw}.$$

Example 4: The velocity distribution in a pipe is given as $u = u_{max}\left(1 - \left(\frac{r}{R}\right)^3\right)$ when u_{max} is the maximum velocity at the centre of the pipe, u is the velocity at a diameter r from the pipe centerline and R is the pipe radius. The ratio of the average velocity to the maximum velocity is

(A) 1 : 2 (B) 3 : 10
(C) 1 : 1 (D) 3 : 5

Solution:

Given $u = u_{max}\left(1 - \left(\frac{r}{R}\right)^3\right)$. Consider an elementary ring of thickness dr and at a distance r from the pipe centre. The discharge through this elementary ring is given by

$$dQ = u \quad 2 \quad rdr$$

$$= u_{max}\left(1 - \left(\frac{r}{R}\right)^3\right) 2\pi r\, dr$$

The discharge through the pipe is

$$Q = \int_O^R dQ = \int_O^R u_{max}\left(1 - \left(\frac{r}{R}\right)^3\right) 2\pi r dr$$

$$= \pi R^2 u_{max} \times \frac{3}{5}$$

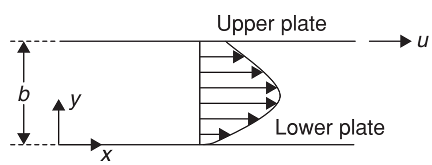
Now $Q = \quad R^2 u_{avg}$

$$\Rightarrow \quad \pi R^2 U_{avg} = \pi R^2 u_{max} \times \frac{3}{5}.$$

Flow of a Viscous Fluid Between Two Parallel Plates

Couette Flow

The laminar flow of a viscous fluid between two parallel plates, one of which is moving relative to the other, is called a *couette flow*. Consider a couette flow where the lower plate is at rest and the upper plate moves uniformity with a constant velocity u as shown in the figure below.



The velocity distribution is given by

$$u(y) = \frac{u}{b} y - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(by - y^2)$$

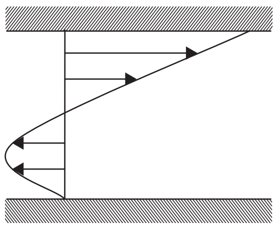
Case 1: $\frac{\partial p}{\partial x} = 0$, i.e., zero pressure gradients in the direction of motion. Then in this case, $u(y) = \frac{u_y}{b}$ which is a linear velocity distribution. This particular case is known as simple or plain couette flow or simple shear flow. This type of flow is usually used to model the lubricant motion in a journal bearing with a rotating shaft. Where the velocity of the lubricant is assumed to be linear.

Case 2: $\frac{\partial p}{\partial x} < 0$, i.e., negative pressure gradient in the direction of motion. In this case, velocity is positive over the whole gap between the plates

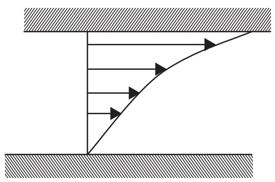
Case 3: $\frac{\partial p}{\partial x} > 0$, i.e., positive pressure gradient in the direction of motion. In this case, velocity over a portion of the gap between the plates can be negative.

Let $K = \frac{-b^2}{2\mu u}\left(\frac{\partial p}{\partial x}\right)$

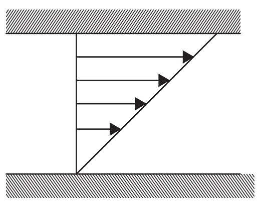
$K < -1$



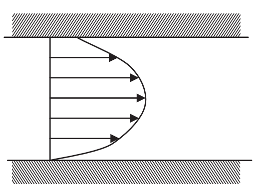
$K < 0$



$K = 0$



$K > 0$



The discharge per unit width of the plates is given by

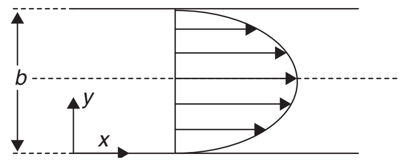
$$q = \frac{ub}{2} - \frac{b^3}{12\mu}\left(\frac{\partial p}{\partial x}\right)$$

The shear stress distribution where the fluids is Newtonian is given by

$$\tau = \mu\frac{-1}{2}\left(\frac{\partial p}{\partial x}\right)(b-2y)$$

Plane Poiseuille Flow

The laminar flow of a viscous fluid between two parallel plates, both of which are stationery, is called a *plane Poiseuille flow*. Consider a plane Poiseuille flow as shown in the following figure.



The velocity distribution is given by

$$u(y) = \frac{-1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(by - y^2)$$

The discharge per unit with is given by

$$q = -\frac{b^3}{12\mu}\frac{\partial p}{\partial x}$$

The shear stress distribution (where the fluid is a Newtonian fluid) is given by

$$\tau = \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)(b-2y)$$

The velocity profile for a plane Poiseuille flow will be a symmetric parabolic velocity profile

For a plane Poiseuille flow, the ratio of the average fluid velocity to the maximum fluid velocity is 2 : 3

Direction for questions 5 and 6: A Newtonian fluid of viscosity 1 poise flows in a steady and laminar manner between two stationery parallel horizontal plates separated by a perpendicular distance of 5 mm. The pressure gradient in the horizontal direction (x direction) is determined to be –5 KN/m^2.

Example 5: The maximum shear associated with the flow is

(A) 0 N/m^2 (B) 25 N/m^2
(C) 12.5 N/m^2 (D) 12.5 10^3 N/m^2

Solution:

The shear stress distribution is

$$\tau = \frac{-1}{2}\left(\frac{\partial p}{\partial x}\right)(b-2y)$$

The maximum shear stress occurs at y = 0

At $y = 0$, $\tau = \frac{-1}{2}\left(\frac{\partial p}{\partial x}\right)b$

Given $b = 5\ \ 10^{3}$ m

$$\frac{\partial p}{\partial x} = -5\times10^3 \text{ N/m}^2$$

$$\therefore \tau = \frac{-1}{2}\times(-5\times10^3)\times5\times10^{-3}$$

$= 12.5$ N/m^2

Example 6: The maximum velocity of the fluid is

(A) 0.1563 m/s (B) 0.1042 m/s
(C) 0.0782 m/s (D) 0.1172 m/s

Solution:

$$u(y) = \frac{-1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(by - y^2)$$

Since the velocity profile of this plane Poiseuille flow is a symmetric parabolic one, the maximum velocity will occur at $y = \frac{b}{2}$

$$\therefore U_{\max} = U\left(\frac{b}{2}\right) = \frac{-b^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

$$= \frac{-(5\times10^{-3})^2}{8\times0.1}\times(-5\times10^3)$$

$$= 0.1563 \text{ m/s}$$

> For the plane Poiseuille flow,
> $R_e = \frac{\rho V_{\text{avg}} b}{\mu}$ and friction factor
> $f = \frac{48}{R_e}$

Example 7: A laminar flow of an oil (viscosity = 20 poise) takes place between two parallel plates which are 150 mm apart. If the average velocity of flow is 1.5 m/s, then the shear stress at vertical distance of 37.5 mm from the lower plate is:

(A) 40 N/m^2 (B) 160 N/m^2
(C) 90 N/m^2 (D) 60 N/m^2

Solution:

Given = 2 Pa.sec, $b = 0.15$ m, $V_{\text{avg}} = 1.5$ m/s

The maximum velocity for a plane Poiseuille flow is given by:

$$V_{\max} = \frac{-b^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

Also for this flow

$$\frac{V_{\text{avg}}}{V_{\max}} = \frac{2}{3}$$

$$\therefore V_{\text{avg}} = \frac{-b^2}{12\mu}\left(\frac{\partial p}{\partial x}\right)$$

$$\therefore \left(\frac{\partial p}{\partial x}\right) = \frac{-1.5\times12\times2}{(0.15)^2} = -1600 \text{ N/m}^3$$

Now $\tau = \frac{-1}{2}\left(\frac{\partial p}{\partial x}\right)(b - 2y)$

At $y = 0.0375$ m

$$\tau = \frac{-1}{2}\times(-1600)\times(0.15 - 2\times0.0375)$$

$$= 60 \text{ N/m}^2.$$

Example 8: In a Couette flow, the two parallel identical plates are at a distance b meters apart and the upper plate moves with a constant velocity of u m/s with the lower plate stationary. The fluid flows between the plates such that the shear stress at the lower plate is zero and the discharge for this flow per unit width of the plates is given by q. If the viscosity of the fluid and the pressure gradient in the horizontal direction (x direction) are doubled, then the discharge per unit width becomes:

(A) q (B) 0.5 q
(C) 0.75 q (D) 2 q

Solution:

Here $\tau = \mu\frac{u}{b} - \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)(b - 2y)$

Given at $y = 0$, = 0

$$\Rightarrow 0 = \mu\frac{u}{b} - \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)b \quad \text{or} \quad \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) = \frac{u}{b^2}$$

Now $q = \frac{ub}{2} - \frac{b^3}{12\mu}\left(\frac{\partial p}{\partial x}\right)$

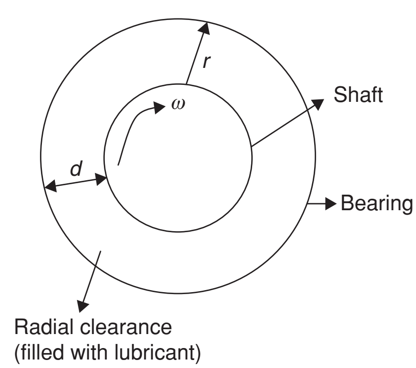
$$= \frac{ub}{2} - \frac{b^3}{6}\times\frac{u}{b^2}$$

$$= \frac{ub}{2} - \frac{ub}{6} = \frac{ub}{3}.$$

Flow of Lubricant in a Journal Bearing

The flow of the lubricant in a journal bearing is usually modeled as a simple (or plain) couette flow. The velocity,

$$\boxed{u(r) = \frac{ur}{d}}$$



Where r is the radial distance from the outer surface of the shaft to the bearing, d is the radial clearance and u is the surface speed of the shaft. If the shaft is rotating at N r.p.m. then:

$$U(r) = \frac{r\omega R}{d} = \frac{r2\pi NR}{60d}$$

Where and R are the angular velocity and radius of the shaft respectively. The Reynolds number for the lubricant flow is defined as:

$$R_e = \frac{\rho ud}{\mu}$$

The flow condition in the bearing is said to be laminar if $R_e < 500$ and turbulent if $R_e > 500$

TURBULENT FLOW IN PIPES

Turbulent flow is characterized by swirling regions of fluid called eddies which greatly enhance mass, momentum and heat transfer compared to laminar flow. Turbulence in a flow can be generated by

1. Frictional forces at the boundary solid walls
2. Flow of fluid layers, with different velocities, adjacent to one another.

Turbulence can be classified as:

1. Wall turbulence: turbulence generated and continuity impacted by the boundary walls.
2. Free turbulence: turbulence generated by two adjacent fluid layers in the absence of walls.
3. Convective turbulence: turbulence generated at regions where there is conversion of potential energy to kinetic energy by the process of mixing.

Property Values in a Turbulent Flow

At a specified location in a turbulent flow field, properties such as velocity, pressure, temperature, etc; Fluctuate with time about an average value. For a property P, the instantaneous value of the property $(\hat{P}(s,t))$ at the specified location s $(= f(x, y, z)$ in Cartesion coordinates) is given by:

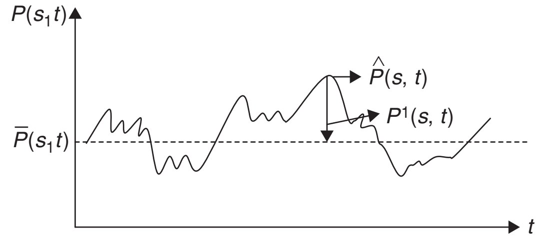
$$\hat{P}(s,t) = \bar{P}(s) + P^1(s,t)$$

where $\bar{P}(s)$ the time average or temporal mean value and P^1 (s, t) is the fluctuating component. The term $\bar{P}(s)$ is a constant with respect to time.

$$\bar{P}(s) = \lim_{T \to \alpha} \frac{1}{T}\int_0^T \hat{P}(s_1 t)dt$$

Where T is the integration time over which the indicated time averaging takes place. The time average of the turbulent fluctuating component is zero i.e.,

$$\lim_{T \to \alpha} \frac{1}{T}\int_0^T P^1(s,t)dt = \overline{P^1(s_1 t)} = 0$$



Shear Stress in a Turbulent Flow

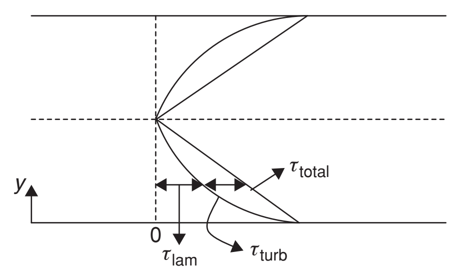
The total shear (total) in a turbulent flow is given by:

$$\tau_{\text{total}} = \tau_{\text{lam}} + \tau_{\text{turb}}$$

Where lam is the *laminar shear stress* and turb is the *turbulent shear stress.*

$$\tau_{\text{lam}} = \mu \frac{d\bar{u}}{dy}$$

Where u is the x-component of the instantaneous velocity $\hat{V}$ and $\bar{u}$ is the time average (or time mean) value of u.



$$\tau_{\text{turb}} = -\rho \overline{u^1 v^1}$$

Where $\overline{u^1 v^1}$ is the time average of the product of the fluctuating velocity component u^1 and V^1. The term $\overline{u^1 v^1}$ can be non-zero even if $\overline{u^1} = 0$ and $\overline{v^1} = 0$

The term $\overline{u^1 v^1}$ is usually found to be a negative quantity and hence shear stress is greater in turbulent flow than in laminar flow.

In laminar flow, $u^1 = v^1 = 0$ such that $\overline{u^1 v^1} = 0$

Terms such as $-\rho\overline{u^1V^1}$ *or* $-\rho\overline{(u^1)^2}$ or $-\rho\overline{V^1\omega^1}$ are called as *Reynolds stress* or *turbulent stresses*. Here, V and are the y and z components of the instantaneous velocity $\hat{V}$

Boussinesq Approximation or Hypothesis

$$\tau_{\text{turb}} = -\rho\overline{u^1V^1} = \mu_t \frac{d\bar{u}}{dy}$$

Where $_t$ is the *eddy viscosity* or *turbulent viscosity.*

$$\tau_{\text{total}} = (\mu + \mu_t)\frac{d\bar{u}}{dy}$$

$$\tau_{\text{total}} = \rho(\infty + \infty_t)\frac{d\bar{u}}{dy}$$

Where $\infty_t = \dfrac{\mu_t}{\rho}$ is the kinematic eddy viscosity or kinematic turbulent viscosity or eddy diffusivity of momentum. Kinematic eddy viscosity depends on flow conditions and it decreases towards the wall where it becomes zero.

Prandtl's Mixing Length Theory

In this theory, the eddy viscosity is $\mu_t = \rho l_m^2 \left|\dfrac{d\bar{u}}{dy}\right|$

$$\tau_{\text{turb}} = -\rho\overline{U^1V^1} = \rho{\ell_m}^2\left(\frac{d\bar{u}}{dy}\right)^2$$

Where ℓ_m is the mixing length defined as the average lateral distance through which a small mass of fluid particles would move from one larger to the adjacent layer before acquiring the velocity of the new layer.

For the steady fully developed turbulent flow of a fluid in a horizontal pipe, R_e total shear stress varies linearly with the pipe radius.

$$\tau_{\text{total}} = \tau_\omega \frac{r}{R}$$

Where $o \le r \le R$

At the wall, velocity gradients and thus wall shear stress are much larger for turbulent flow than for laminar flow.

Relative Roughness

The variable , referred to as absolute roughness, denotes the mean height of irregularities of the surface of a boundary. A boundary is generally said to be rough if the value of is high and smooth if is low. For a pipe, relative roughness $= \dfrac{\in}{D}$, where D is the diameter of the pipe.

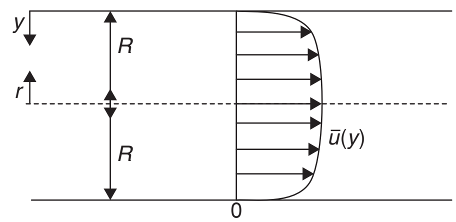
Turbulent Velocity Profile

Several empirical velocity profile units for turbulent pipe flow and among these the best known is the power – low. Velocity profile defined as follows:

$$\frac{u}{u_{\max}} = \left(\frac{y}{R}\right)^{\frac{1}{n}} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$$

Where n is a constant and whole value increases as Reynolds number increases. Many turbulent flows in practice is approximated using the one–seventh power low velocity profile where $n = 7$. Note that the power–low velocity profile cannot be used to calculate the wall shear stress, as a velocity gradient obtained will be infinity. This law is applicable to smooth pipes.

Velocity distribution are more uniform in turbulent flow than in laminar flow.

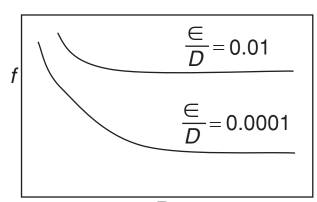


Friction Factor in Turbulent Flow

The friction factor in a fully developed turbulent pipe flow depends on the Reynolds number and the relative roughness (/D). The friction factor is a minimum for a smooth pipe and increases with roughness. For laminar flow, the friction factor decreases as Reynolds number increases and is independent of surface roughness.

Moody Chart

It presents the Darcy friction factor for pipe flow as a function of Reynolds number and relative roughness. This chart can be used for circular pipes and non-circular (taking into consideration the hydraulic diameter) pipes.



At very large Reynolds number, R_e friction factor curves in the moody chart are nearly horizontal and thus the friction factor are independent of the Reynolds number.

Intensity of Turbulence in a Flow

It is also called as degree of turbulence in a flow which is described by the relative magnitude of the root mean square value of the fluctuating components (u', v', and $'$) with respect to the time averaged velocity ($\bar{V}$)

$$I = \frac{\sqrt{\frac{1}{3}(\overline{(u')^2} + \overline{(v')^2} + \overline{(\omega')^2})}}{\bar{V}}$$

If the turbulence is Isotropic, then $u' = v' = \ '$.

Example 9: A liquid flows turbulently in a horizontal pipe with a pressure gradient of 3 kPa/m. The wall shear stress developed is 112.5 N/m^2. If the laminar shear stress is 10 N/m^2 at a radius of 35 mm, then the turbulent shear stress at this radius would be

(A) 52.5 N/m^2 (B) 10 N/m^2
(C) 42.5 N/m^2 (D) 95 N/m^2

Solution:

Given $\frac{\Delta P}{L} = 3 \times 10^3 \frac{P_a}{m}$

$= 112.5$ N/m^2

The following equations are applicable for turbulent flows.

$$\tau_\omega = \frac{\Delta P}{L}\frac{R}{2} \quad (1)$$

$$\tau_\omega = \frac{\tau_\omega r}{R} \quad (2)$$

From equation (1), we get $112.5 = 3 \times 10^3 \times \frac{R}{2}$

Radius of the pipe, $R = 0.075$ m

Now at radius, $r = 0.035$ m

$$\tau = 112.5 \times \frac{0.035}{0.075}$$

$= 52.5$ N/m^2

Here is the total shear stress i.e., $_{total} = 52.5$ N/m^2

At this radius,

$_{lam} = 10$ N/m^2

$_{total} = \ _{lam} + \ _{turb}$

$_{total} = 52.5 \quad 10 = 42.5$ N/m^2

Example 10: A fluid (density = 950 Kg/m^3, viscosity = 0.1 poise) flows with an average velocity of 1^m/s in a 100 m long horizontal pipe having an absolute roughness of 0.175 mm. The magnitude of the pressure loss due to friction is obtained by multiplying the friction factor with 19 10^5. A set of friction factor (f) values for some given combination of Reynolds number (R_e) and relative roughness (RR) values are given in the following table. The friction factor associated with the flow is

R_e	RR	f
9800	0.00175	0.0338
9500	0.0035	0.0361
19000	0.00175	0.0296
19000	0.0035	0.0325

(A) 0.0338 (B) 0.0361
(C) 0.0296 (D) 0.0325

Solution:

For turbulent or laminar flow, we have

$$\Delta P_L = \frac{2 f \rho \bar{V}^2 L}{D}$$

Gives $\frac{2\rho \bar{V}^2 L}{D} = 19 \times 10^5$

or $D = \frac{2 \times 950 \times 1^2 \times 100}{19 \times 10^5} = 0.1$ m

Gives $t = 0.175$ mm

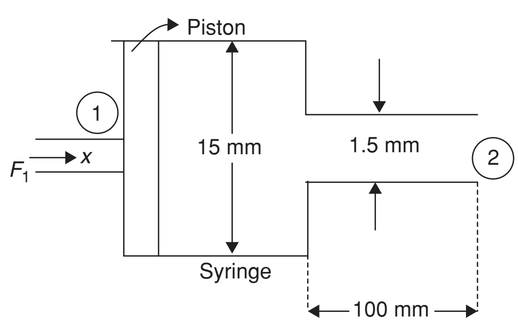
Relative roughness $= \frac{t}{D} = \frac{0.175}{100} = 0.00175$

$$R_e = \frac{\rho \bar{V} D}{\mu} = \frac{950 \times 1 \times 0.1}{0.01} = 9500$$

For $R_e = 9500$ and $RR = 0.00175$ friction factor $f = 0.0338$

Example 11: A force F_1 Newtons is required as the frictionless piston in a syringe to discharge 1944 mm^3/s of water through a needle as shown in the following figure. The force is determined by assuming fully developed laminar viscous flow through the needle. If ideal flow is assumed, then the force required on the piston to achieve the same discharge would be F_2 Newtons. The difference F_1 F_2 neglecting losses in the syringe is equal to:

(A) 0.0251 N (B) 0.2765 N
(C) 0.7856 N (D) 0.4836 N



Solution:

Consider two points 1 and 2 such that both points are in the same horizontal plane and point 1 lies in the centre of the piston cross-section while point 2 lies in the centre of the needle exit cross-section.

The energy balance equation with suitable assumption can be reduced to:

$$\frac{P_1}{\rho g}+\frac{\alpha_1 V_1^2}{2g}+Z_1=\frac{P_2^2}{\rho g}+\frac{\alpha_2 V_2^2}{2g}+z_2+h_L \qquad (1)$$

Here, $Z_1 = Z_2$

$$P_1 = P_{atm} + \frac{F_1}{A_1}$$

$P_2 = P_{atm}$

$_1 = \ _2$ (uniform velocity assumed across any cross-section)

Equation (1) becomes:

$$F_1 = \left(\frac{V_2^2 - V_1^2}{2}\right)\rho A_1 + h_L A_1 \rho g$$

When ideal flow is assumed, $h_L = 0$

$$F_1 \quad F_2 = h_L A_1 \quad g$$

$$= f\frac{L}{D_2}\times\frac{V_2^2}{2g}\times A_1\rho g$$

$$= f\frac{L}{D_2}\times\frac{V_2^2}{2}\times A_1\rho$$

$$Q = 1944 \text{ mm}^3/\text{s}$$

$$= 1944 \quad 10^{9} \text{ m}^3/\text{s}$$

$$A_1 = \frac{\pi}{4}\times D_1^2 = \frac{\pi}{4}\times(0.015)^2$$

$$= 1.767 \quad 10^{4} \text{ m}^2$$

$$A_2 = \frac{\pi}{4}\times D_2^2 = \frac{\pi}{4}\times(0.0015)^2$$

$$= 1.767 \quad 10^{6} \text{ m}^2$$

$$U_2 = \frac{Q}{A_2} = \frac{1944\times10^{-9}}{1.767\times10^{-6}} = 1.1 \text{ m/s}$$

Reynolds number of flow in the needle

$$R_e = \frac{\rho D_2 \times V_2}{\mu}$$

$$= \frac{1000\times0.0015\times1.1}{0.001}$$

$$= 1650$$

$$f = \frac{64}{R_e} = \frac{64}{1650} = 0.0388$$

$$F_1 \ F_2 = \frac{0.0388\times0.1}{0.0015}\times\frac{1.1^2}{2}\times1.767\times10^{-4}\times1000$$

$$= 0.2765 \text{ N}$$

Example 12: Water is flowing at a volumetric flow rate of 0.08 m^3/s in a horizontal pipe of length 15 m and diameter (D) varies along its length (ℓ) according to the linear relationship: $D = 0.25 \quad 0.01$ l. If the friction factor is taken to be constant for the whole pipe and equal to 0.02, then the head loss due to friction in the pipe is

(A) 0.6441 m (B) 2.0611 m
(C) 10.3059 m (D) 2.5764 m

Solution:

Head loss due to friction,

$$h_L = f\frac{L}{D}\frac{\bar{V}^2}{2g}$$

Where L is the whole length of the pipe. For a differential length of the pipe, the differential head loss due to friction can be written as:

$$dh_L = \frac{f}{D}\frac{\bar{V}^2}{2g}d\ell$$

$$= \frac{f}{D}\times\frac{Q^2\times16\times d\ell}{\pi^2\times10^4\times2g}$$

$$= \frac{.8fQ^2}{\pi^2 g}\times\frac{d\ell}{D^5}$$

$$= 0.08263 fQ^2\frac{d\ell}{(0.25-0.01\ell)^5}$$

Integrating the above equation we have

$$\int_o^{h_L} dh_L = 0.08263 fQ^2\int_o^{15}\frac{d\ell}{(0.25-0.01\ell)^5}$$

i.e., $h_L = 2.5764$ m

Loss of Energy (or Head) in Pipes

When a fluid flows in a pipe, its motion experiences some resistance due to which the available head reduces. This loss of energy or head is classified as

Major Energy Losses

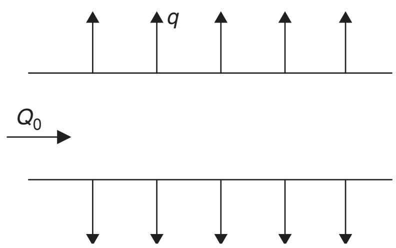
These are energy losses due to friction and the loss of head due to friction (h_L) is calculated using Darcy Weisbach equation given earlier.

In terms of the flow and resistance R, h_L can be written as

$$h_L = RQ^2$$

Flow Through Pipes with Side Tapings

Consider the flow through a pipe when a fluid is withdrawn from closely spaced side tappings along the length of the pipe as shown in the following figure.



Let the fluid be removed at a uniform rate q per unit length of the pipe. Let the volume flow rate into the pipe be Q_0 and let L and D be the length and diameter of the pipe. If f is the friction factor assumed to be constant over the length of the pipe, then

$$h_f = \frac{8Q_0^2 fL}{\pi^2 D^5 g}\left[1 - \frac{qL}{Q_0} + \frac{1}{3}\frac{q^2L^2}{Q_0^2}\right]$$

If the entire flow is drained off over the length L, then

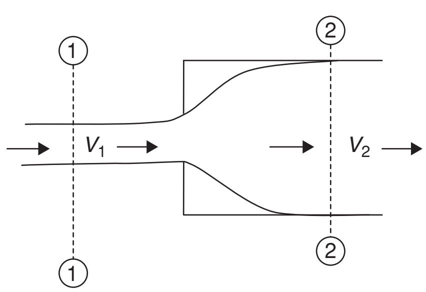
$$h_f = \frac{1}{3} f \frac{L}{D} V_0^2 \frac{1}{2g}$$

Where $V_0 = \frac{Q_0}{\frac{\pi}{4}D^2}$. The above equation indicates that the loss of head due to friction over a length L of a pipe, where the entire flow is drained off uniformly from the side tappings, becomes one third of that in a pipe of same length and diameter but without side tappings

Minor Energy Loses

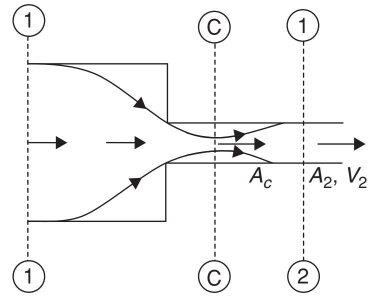
The minor energy losses include the following cases:

Loss of head due to sudden enlargement (h_e)



$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

Loss of head due to sudden contraction (h_c)



$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

Where $C_c = \frac{A_c}{A_2}$ is the coefficient of contraction. If the value of C_c is not known, then loss of head due to contraction may be taken as $0.5\frac{V_2^2}{2g}$

Loss of head due to obstruction in pipe (h_{obs})

$$h_{obs} = \left[\frac{A}{C_c(A-a)}\right]^2 \frac{V^2}{2g}$$

Where A is the area of the pipe, a is the maximum area of obstruction and V is the velocity of liquid in the pipe.

Loss of head at the entrance to pipe (h_i)

$$h_i = 0.5\frac{V^2}{2g}$$

Where V is the velocity of liquid in pipe.

Loss of head at the exit of a pipe (h_o)

$$ho = \frac{V^2}{2g}$$

Where V is the velocity at outlet of pipe

Loss of head due to bend in the pipe (h_b)

$$h_b = \frac{KV^2}{2g}$$

Where V is the mean velocity of flow of liquid and K is the coefficient of bend

Loss of head in various pipe fittings ($h_{fittings}$)

$$h_{\text{fittings}} = \frac{KV^2}{2g}$$

Where V is the mean velocity of flow in the pipe and K is the value of the coefficient that depends on the type of pipe fitting.

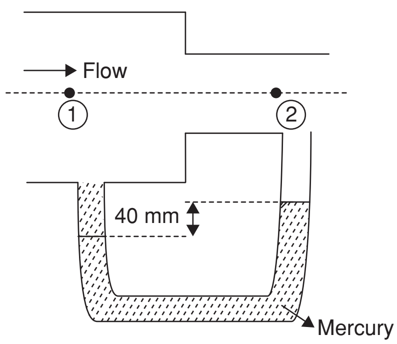
These losses (h_b and $h_{fittings}$) are sometimes expressed in terms of an equivalent length (L_e) of an unobstructed straight pipe in which an equal loss would occur for the same average flow velocity.

$$\boxed{L_e = \frac{DK}{f}}$$

NOTE

For a sudden expansion in a pipe flow, if D_1 and D_2 are the diameter of the pipe before and after the expansion respectively, the pressure rise is maximum when $\frac{D_1}{D_2} = \frac{1}{\sqrt{2}}$ and the maximum pressure rise would be $\frac{0.5\rho g V_1^2}{2g}$

Example 13: Water flows at the rate of 0.06 m^3/s in a pipe involving a sudden contraction where the pipe diameter decreases from 250 mm to 160 mm, as shown in the following figure. The coefficient of contraction is



(A) 0.655 (B) 0.543

(C) 0.792 (D) 0.125

Solution:

$$\frac{P_1 - P_2}{\rho g} = h\left(\frac{\rho m}{\rho} - 1\right)$$

Where (density of water) = 1000 Kg/m^3 and $_m$ (density of mercury) = 13600 k/m^3 and h = 40 mm

$$\therefore \frac{P_1 - P_2}{\rho g} = 0.04\left(\frac{13600}{1000} - 1\right)$$

$$= 0.504$$

The energy balance with suitable assumption can be reduced to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

Here the head loss (h_L) is equal to the head loss due to contraction:

$$h_L = h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

$Z_1 = Z_2$ (as the points 1 and 2 same horizontal plane):

$$\therefore \frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g}\left(1 + \left(\frac{1}{C_c} - 1\right)^2\right) - \frac{V_1^2}{2g}$$

Or 0.504 2 9.81 =

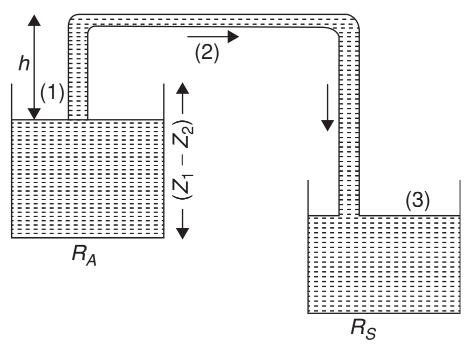
$$\left(\frac{0.06\times 4}{\pi\times(0.16)^2}\right)^2\left(1 + \left(\frac{1}{C_c} - 1\right)^2\right) - \left(\frac{0.06\times 4}{\pi\times(0.25)^2}\right)^2$$

Or $C_c = 0.655$

Flow Through Syphon

When two reservoirs, one at a higher level and another at a lower level are separated by a high level ground or hill, a long bend pipe which is used to transfer liquid from the higher altitude reservoir to the lower altitude reservoir is called a **syphon**.

Syphons are also used to (i) empty a channel not provided with any outlet orifice (ii) to take out liquid from a tank not provided with any outlet.



A syphon used for transferring liquid from a high altitude reservoir R_A to a low altitude reservoir R_B is shown in figure. The highest point of the syphon (2) is called the summit, while (1) and (2) are the free liquid surface in reservoir R_A and R_B respectively. The height difference between (1) and (2) is ($Z_1 - Z_3$). Since (1) and (3) are open to atmosphere, the corresponding pressures are $p_1 = p_3 = p_a$, where p_a = atmospheric pressure. Since (2) at a higher level than (1), pressure at (2) (i.e., p_2) is less than p_1 i.e. $p_2 < p_1$ ($p_1 = p_a$).

Atmospheric pressure p_a = 10.3 m of water column. Hence theoretically, for water flow, the pressure at summit p_2 can be 10.3 m of water but practically it must be between 7.6 m to 8.0 m. Hence the vertical height difference (h) between (2) and (1) must be restricted to (10.3 – 8.0 = 2.3 m) to (10.3   7.6 = 2.7 m), so that the pressure at summit (p_2) is in the range of 2.3 m to 2.7 m absolute. If the pressure at summit becomes less than this value, dissolved air and gases will come out of water and accumulate at the summit, hindering the flow of water.

If   is the density of liquid, V_1 = velocity of flow at (1), V_3 = velocity of flow at (3), then by applying Bernoulli's equation between points (1) and (3), we get

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{p_3}{\rho g}+\frac{V_3^2}{2g}+Z_3+h_f$$

Where h_f = head loss due to friction in syphon $=\frac{4fLV^2}{2gd}$

Here

L = length of syphon pipe,

d = diameter of siphon pipe,

V = average velocity of flow in the syphon pipe,

F = friction coefficient for syphon pipe.

We have $p_1 = p_3 = p_a$ and $V_1 = V_2 = 0$ (R_A and R_B are large tanks)

Hence,

$$(Z_1 - Z_3) = h_f = \frac{4fLV^2}{2gd} \quad (1)$$

If (Z_1 Z_3) is known, d and L are known, and then V^2 can be calculated.

Once V is known, discharge $Q=\frac{\pi}{4}d^2V$ will give the discharge through the syphon.

It must be noted that in the above calculation, we have considered all minor losses as negligible.

Now by applying Bernoulli's equation between points (1) and (2), we get

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_f^1$$

Here $V_2 = V$ (as calculated earlier)

$h_f' = \frac{4fL_1V^2}{2gd}$, where L_1 = length of siphon pipe from tank R_A to summit (2)

NOTE

$$h_f' = h_f \times \frac{L_1}{L}$$

Also $p_1 = p_a = 0$, $V_1 = 0$

$$\Rightarrow \frac{p_2}{\rho g}+\frac{V^2}{2g}+\frac{4fL_1V^2}{2gd}+(Z_2-Z_1)=0$$

i.e., $\frac{p_2}{\rho g}+\frac{V^2}{2g}+\frac{4fL_1V^2}{2gd}+h=0\ (\because Z_2-Z_1=h)$

From the above equation, minimum pressure at summit p_2 can be calculated. If minimum pressure p_2 is known, the maximum height h can be calculated.

Example 14: A large water tank empties by gravity through a syphon. The difference in levels of the high altitude and low altitude tanks is 3 m and the highest point of the siphon is 2 m above the free surface of water in the high altitude tank. The length of syphon pipe is 6 m and its bore is 25 mm. Also the length of syphon pipe from inlet to the highest point is 2.5 m. The friction coefficient for the pipe is 0.007 and all other losses a negligible. Calculate the volume flow rate of water through the syphon and the pressure head at the highest point in the pipe.

Solution:

Given Z_1 Z_3 = 3 m

L = 6 m

d = 25 mm = 25   10 ^{3} m

f = 0.007

we have $(Z_1 - Z_3) = \frac{4fLV^2}{2gd}$

$$\Rightarrow V^2 = \frac{2(Z_1-Z_3)gd}{4fL}$$

$$\Rightarrow V = \sqrt{\frac{(Z_1-Z_3)gd}{2fL}}$$

$$= \sqrt{\frac{3\times 9.81\times 25\times 10^{-3}}{2\times 0.007\times 6}}$$

= 2.96 m/s

Hence speed of flow of water in syphon is 2.96 m/s

Q = Discharge = Area   velocity

$$= \frac{\pi}{4}d^2V = \frac{\pi}{4}\times(25\times 10^{-3})^2\times 2.96$$

= 1.453   10 ^{3} m^3/s

= 1.453 litre/s (ℓ 10 ^{3} m^3 = 1 litre)

Volume flow rate through the syphon is 1.453 litre/s

Given, (Z_2 Z_1) = 2 m

L_1 = length of pipe from inlet to summit

= 2.5 m

$$\therefore\ h_f' = \frac{4fL_1V^2}{2gd}$$

$$= \frac{4\times 0.007\times 2.5\times (2.96)^2}{2\times 9.81\times 25\times 10^{-3}}$$

= 1.25 m

Applying Bernoulli's equation between inlet (1) and summit (2), we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+Z_1=\frac{p_2}{\rho g}+\frac{v^2}{2g}+Z_2+h_f^1$$

But p_1 = 0 (ℓ atmospheric pressure)

V_1 = 0 (ℓ large tank)

$$\Rightarrow \frac{p_2}{\rho g} + \frac{v^2}{2g} + h'_f + (Z_2 - Z_1) = 0$$

$$\Rightarrow \frac{p_2}{\rho g} = -(Z_2 - Z_1) - h'_f - \frac{v^2}{2g}$$

$$= -(2) - (1.25) - \frac{2.96^2}{2 \times 9.81}$$

= 2 1.25 0.45

= 3.70 m of water

Pressure head at the highest point in the syphon is 3.70 m of water (i.e., 3.7 m of water absolute)

Equivalent Pipe

An *equivalent pipe* is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe (pipe in series) consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is known as *equivalent diameter* of the pipes in series.

Consider n pipes in series where the length, diameter and friction factor associated with the i th pipe are L_i, D_i and f_I respectively. If L, D and f are the length, diameter and friction factor associated with the equivalent pipe, then neglecting minor losses we have:

$$\boxed{\frac{fL}{D^5} = \sum_{i=1}^{n} \frac{f_i L_i}{D_i^5}}$$

If the friction factor f_i is equal to f, then:

$$\boxed{\frac{L}{D^5} = \sum_{i=1}^{n} \frac{L_i}{D_i^5}}$$

The above equation is called the Dupit's equation.

Example 15: A piping system consists of a pipe of length L which can be replaced by an equivalent pipe of length L_e, diameter D_e and friction factor f_e. The length of the pipe is increased by amount L. The new pipe can be replaced by an equivalent pipe of length L_e, diameter 0.5 D_e and friction factor 0.5 f_e. If the increase in length has led to the friction factor of the new pipe being a quadruple of the old pipe, then L is equal to:

(A) 15 L (B) 3 L
(C) 4 L (D) 7 L

Solution:

Let D and f be the diameter and friction factor of the old pipe.

$$\therefore \frac{Lf}{D^5} = \frac{L_e f_e}{D_e^5} \quad (1)$$

for the new pipe,

$$\frac{(L + \Delta L) \times 4f}{D^5} = \frac{L_e \times 0.5 f_e}{(0.5 D_e)^5}$$

$$= 16 \frac{L_e f_e}{D_e^5} \quad (2)$$

Substituting equation (1) in (2), we have

$$\frac{(L + \Delta L) \times 4f}{D^5} = 16 \times \frac{Lf}{D^5}$$

$$L + \ L = 4 L$$

$$L = 3 L$$

Example 16: A piping system consists of a series of [pipes in which a 20 m long pipe of diameter 250 m (f = 0.025), containing a value (K = 1.0), suddenly expands to a 500 mm diameter pipe (f = 0.02) of length 40 m. If the velocity of flow in the 250 mm diameter pipe is 4 m/s, then the length of an equivalent pipe (f = 0.02) of diameter 500 mm for the piping system would be

(A) 1090 m (B) 865 m
(C) 1065 m (D) 1050 m

Solution:

Equivalent length for the 250 mm diameter pipe,

$$h_{e_1} = \frac{f_2 L_2}{D_2^5} \times \frac{D_e^5}{f_e}$$

$$= \frac{0.025 \times 20}{(0.25)^5} \times \frac{(0.5)^5}{0.02} = 800 \text{ m}$$

Equivalent length for the value

$$h_{e_2} = \frac{KD}{f} = \frac{1.0 \times 0.5}{0.02} = 25 \text{ m}$$

Velocity of flow in the 500 mm diameter pipe

$$= \left(\frac{0.25}{0.5}\right)^2 \times 4 = 1 \text{ m/s}$$

Head loss due to expansion

$$h_L = \frac{(V_1 - V_2)^2}{2g} = \frac{(4-1)^2}{2 \times 9.81}$$

$$= 0.4587 \text{ m}$$

Let L_{e3} be the equivalent length for the sudden expansion.

$$\text{Then } f_e \frac{L_{e_3}}{D_e} \times \frac{V_2{}^2}{2g} = h_L$$

$$\text{i.e., } L_{e_3} = 0.4587 \times 2 \times 9.81 \times \frac{0.5}{0.02 \times (1)^2}$$

$$= 225 \text{ m}$$

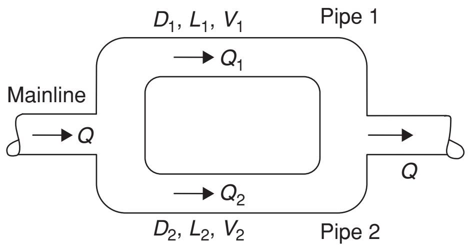
Equivalent length for the 500 mm diameter pipe of length 40 m,

L_{e4} = 40 m

Total equivalent length = L_{e1} + L_{e2} + L_{e3} + L_{e4} = 800 + 25 + 225 + 40 = 1090 m.

Pipes in Parallel

For the parallel pipe system shown below, the rate of discharge in the



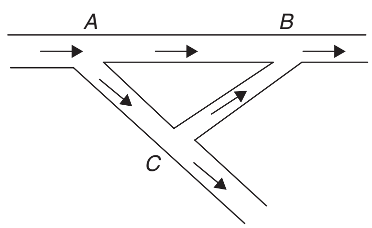
Main line is equal to the sum of the discharge in the pipes.

i.e., $\boxed{Q = Q_1 + Q_2}$

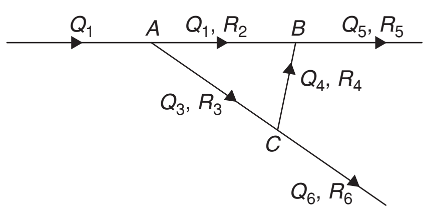
When pipes are arranged in parallel the head loss in each pipe is the same.

i.e., $\boxed{\text{Loss of head in pipe 1} = \text{Loss of head in pipe 2}}$

Pipe Network



The pipe structure shown above can be converted into a pipe network (or hydraulic circuit) with nodes (or junctions) and links. Here Q denotes the flow rate and R denotes the flow resistance.



In the above network, the algebraic sum of the flow rates at any node must be zero, i.e., the total mass flow rate towards the junction must be equal to the total mass flow rate away from it.

At a node,

$$\boxed{\sum Q \text{ in} = \Sigma Q \text{ out}}$$

For example, at node A, $Q_1 = Q_2 + Q_3$

Also in the above network, the algebraic sum of the products of the flux (Q^2) and the flow resistance (the sense being determined by the direction of flow) must be zero in any closed loop or hydraulic circuit.

In a closed loop,

$$\boxed{\sum R_i |Q_i| Q_i = 0} \quad (1)$$

For example, considering the loop ABC, we can write

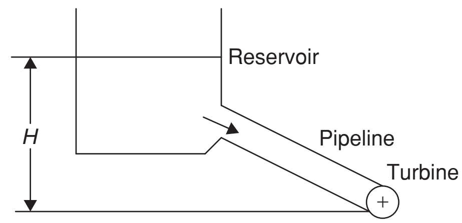
$$R_2 (Q_2)^2 \quad R_4 (Q_4)^2 \quad R_3 (Q_3)^2 = 0$$

The term $R_4 \,|\, Q_4 \,|\, Q_4$ gets converted to the negative quantity—$R_4\,(Q_4)^2$ because in the link BC, the considered loop direction (from B to C) is opposite to the flow direction (from C to B). Equation (1) is referred to as the pressure equation of the circuit. Since $hL = RQ^2$, equation (1) can be rewritten as

$$\boxed{\sum h_{L_i} = 0}$$

where the correct sign values are assigned to the h_L value.

Power Transmission Through Pipes



In the above system, hydraulic power is transmitted by a pipeline (through conveyance of the liquid) to a turbine. Here, the hydrostatic head of the liquid is transmitted by the pipeline

Potential head of liquid in the reservoir = H (difference in the liquid level in the reservoir and the turbine center).

Head available at pipe exit (or the turbine entry) = $H \;\; h_L$ (neglecting minor losses), where h_L is loss of head in the pipeline due to friction.

Power transmitted by the pipeline (or available of the exit of the pipeline),

$$\boxed{P = \rho g Q (H - h_f)}$$

Efficiency of power transmission,

$$\boxed{\eta = \frac{H - h_f}{H} \times 100}$$

$$\boxed{\text{Power transmitted will be maximum when } h_f = \frac{H}{3}}$$

$$\boxed{\text{Maximum power transmission efficiency (or efficiency of transmission at the condition of maximum power delivered) is } \frac{200}{3}\text{\% or 66.67\%}}$$

Water Hammer in Pipes

In a long pipe , when the flow velocity of water is suddenly brought to zero (by closing a value), there will be a sudden rise in pressure due to the momentum of water being destroyed. A pressure wave is transmitted along the pipe. A sudden pressure rise brings about the effect of a hammering action on the walls of the pipe. This phenomenon of sudden rise in pressure is known as *water hammer* or *hammer blow.*

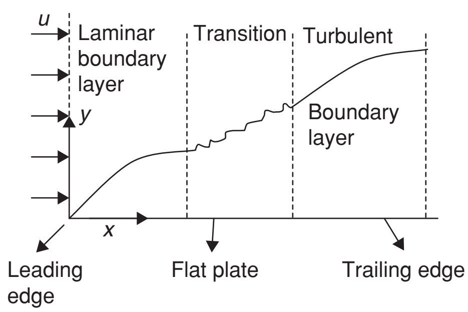
The magnitude of pressure rise depends on

1. Speed at which valve is closed
2. Velocity of flow
3. Length of pipe and
4. Elastic properties of the pipe material as well as that of the flowing fluid.

Boundary Layer Theory

When a viscous fluid flows past a stationary solid boundary, in a small layer of fluid adjacent to the boundary, the velocity of flowing fluid increase rapidly from zero at the boundary surface and approaches the velocity of the main stream. This layer is called the *boundary layer.* A boundary layer is formed when there is relative motion between a solid boundary and the fluid in contact with it.

Boundary Layer on a Flat Plate



The above figure shows a boundary layer formed on a flat plate kept parallel to the flow of fluid of velocity u. Here u is called as the free stream velocity, sometimes denoted as u . The edge of the plate facing the direction of flow is called as the *leading edge* while its rear edge is called the trailing edge.

Near the leading edge of a flat plate, the boundary layer is laminar with a parabolic velocity distribution. In the turbulent boundary layer, the velocity distribution is given by the log law or Prandtl's one–seventh power law.

Characteristics of a boundary layer are

1. The boundary layer thickness ( ) increases as the distance from the leading edge (x) decreases
2. decreases as u increases
3. increases as kinematic viscosity (v) increases
4. The wall shear stress $\tau_{\omega} = \left(= \mu \left(\frac{\partial u}{\partial y} \right) y = 0 \right)$ decreases as x increases. In the turbulent boundary layer, shows a sudden increase and then decreases with increasing x.

Boundary layer is laminar when $R_{ex}\left(= \frac{ux\rho}{\mu} \right) < 5\times10^5$ and turbulent when $> 5\times10^5$.

Boundary Layer Thickness ( )

Boundary layer thickness is defined as that distance from the boundary in which the velocity reaches 99 % of the free stream velocity ($u = 0.99\ u$)

For greater accuracy, boundary layer thickness is defined in terms of the displacement thickness (*), momentum thickness ( ) and energy thickness ($_e$)

*Displacement Thickness (*)*

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

Momentum Thickness ( )

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Energy Thickness ($_e$)

$$\delta_e = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

Note that the difference ($U - u$) is called the *velocity of defect*

Shape factor

$$S = \frac{\delta^*}{\theta}$$

Where S is called the shape factor

Energy loss The energy loss per unit width of the plate due to the boundary layer,

$$E_L = \frac{1}{2}(\rho\delta_e u) \times u^2$$

Mass flow The mass flow in the boundary layer at a position where the boundary thickness is , is given by

$$m = \int_0^{\delta} \rho u dy$$

The mass entrainment (m) between two sections where the boundary layer thickness are $_1$ and $_2$ respectively is given by

$$\Delta m = m_1 - m_2$$
$$\Delta m = \int_0^{\delta_1} \rho u dy - \int_0^{\delta_2} \rho u dy$$

Reynolds Number for the Plate

If L is the length of a plate, then Reynolds number for the whole plate $= \frac{\rho u L}{\mu}$ Reynolds number for the front half of the plate $= \frac{\rho u L}{2\mu}$.

Von Karman Momentum Equation

For a fluid flowing over a thin plate (placed at zero incidence) with a free stream velocity equal to u,

$$\boxed{\frac{\tau_\omega}{\rho v^2} = \frac{d_\theta}{dx}}$$

The above equation is called as the Von Karman momentum equation for boundary layer flow. It is used to determine the frictional drag on a smooth flat plate for both laminar and turbulent boundary layers.

Boundary Conditions for a Velocity Distribution

The following boundary conditions must be satisfied for any assumed velocity distribution in a boundary layer over a plate:

1. At the plate surface
   $y = 0, u = 0$
2. At the outer edge of boundary layer
   (i) $y = \ , u = U$
   (ii) $y = d, \dfrac{du}{dy} = 0$

Drag Force on the Plate

The drag force acting on a small distance d_x of a plate is given by

$$\boxed{\Delta F_D = \tau_\omega \times B \times dx}$$

where B is the width of the plate.

Total drag force acting on a plate of length L on one side,

$$\boxed{F_D = \int_0^L \Delta F_D = \int_0^L \tau_\omega \times B \times dx}$$

Local co-efficient of drag (C_D)*

$$\boxed{C_D{}^* = \frac{\tau_\omega}{\frac{1}{2}\rho u^2}}$$

This coefficient is also sometimes called as co-efficient of skin friction

Average Co-efficient of Drag (C_D)

$$\boxed{C_D = \frac{F_D}{\frac{1}{2}\rho A u^2}}$$

Laminar Boundary Layer Over a Flat Plate

From the solution of the Blasius equation for the laminar boundary layer on a flat plate, the following results are obtained.

$$\boxed{\delta = \frac{5x}{\sqrt{R_{e_x}}}}$$

$$\boxed{C_D^* = \frac{0.664}{\sqrt{R_{e_x}}}}$$

$$\boxed{C_D = \frac{1.328}{\sqrt{R_{e_L}}}}$$

Where $\boxed{R_{e_L} = \dfrac{uL\rho}{\mu}}$, L being the length of the plate.

Summary of Fluid Frictional Resistance

Fluid frictional resistance is the opposition force (or resistance) experienced by a fluid in motion. It exists both in streamline flow and in turbulent flow.

Fluid Friction in Streamline Flow (Laminar Flow)

1. The viscous forces predominate the inertial force in this type of flow, which occurs at low velocities.
2. Frictional resistance is proportional to the velocity of flow, contact surface area and temperature.
3. The entrance length (L_e), which is the length of pipe from its entrance to the point where flow attains fully developed profile and remains unaltered beyond that point is given by $\boxed{L_e = 0.07\, Re\, D,}$ where Re = Reynolds's number for flow and D = diameter of pipe
4. The Darcy's friction factor in smooth pipes (as per Blassius) is given by $f = \dfrac{64}{R_e}$

Fluid Friction in Turbulent Flow

1. As per Darcy–Weisbach equation, the head loss due to friction is $h_f = \dfrac{fLV^2}{2gD}$,
   where,
   L = length of pipe
   D = diameter of pipe
   V = mean velocity of flow
   f = friction factor (0.02 to 0.04 for metals)
   Hence frictional resistance is proportional to square of velocity.
2. The frictional resistance does not depend upon the pressure but it varies slightly with temperature.
3. The frictional resistance is proportional to the density of the fluid.
4. The entrance length (L_e) $50D$. Also $\boxed{L_e = 0.7\, ReD.}$
   Where,
   R_e = Reynold's number of flow and
   D = diameter of pipe.

5. Darcy's friction factor in smooth pipes (as per Blassius) is $f = \dfrac{0.3164}{\left(R_e^{\frac{1}{4}}\right)}$ for turbulent flow.

Variation of Pipe Roughness with Aging

The **relative smoothness** of a pipe $= \dfrac{R}{k}$, where R = radius of pipe and k = average height of irregularities. For rough pipes, friction factor depends only on $\left(\dfrac{R}{k}\right)$ and not on Reynold's number (R_e). The **relative roughness**, of pipe is $\dfrac{k}{R}$ (which is the reciprocal of the relative smoothness).

The average height of irregularities (i.e., k), which is a measure of the roughness of pipe, depends upon the age of pipe. The relation is $k = k_0 + \ t$,

where,

k_0 = value of pipe roughness for new pipe

t = age of pipe (in year)

= a constant

k = value of pipe roughness after t years.

Exercises

Practice Problems I

Direction for questions 1 to 20: Select the correct alternative from the given choices.

Direction for questions 1 and 2: The volumetric flow rate of the steady fully developed laminar flow of a fluid in a horizontal circular pipe of radius 0.02 m and length 50 m is 2.64 litre/sec. The pressure drop across the ends of the pipe is 2000 kN/m^2

1. The frictional drag over the entire length of the pipe is:
(A) 1256.64 N (B) 0.5 N
(C) 5 N (D) 2513.27 N

2. The power required to maintain the flow is:
(A) 5277.87 W
(B) 10057.13 W
(C) 21119.97 W
(D) 2513.27 W

Direction for questions 3 and 4: The velocity gradient at the wall of a horizontal circular pipe, in which a steady fully developed laminar flow of a Newtonian fluid (viscosity = 8 poise, density = 900 Kg/m^3) occurs, is 250 S^{-1}. The shear stress is 80 N/m^2 at a perpendicular distance of 0.01 m from pipe's centerline.

3. The velocity of flow at a perpendicular distance of 0.01 m from the pipe wall is:
(A) 2.625 m/s (B) 2 m/s
(C) 1.33 m/s (D) 1.6 m/s

4. If L is the length of the pipe, then the head loss associated with the flow is:
(A) 1.812 L (B) 1.208 L
(C) 0.725 L (D) 17.78 L

5. For a couette flow, the velocity distribution is: given by $u(y) = \dfrac{U}{b}y + \dfrac{-1}{2\mu}\left(\dfrac{\partial p}{\partial x}\right)(by - y^2)$ where U is the velocity with which the upper plate moves and b is the distance between the plates. A variable K is defined such that $K = \dfrac{-b^2}{2\mu U}\left(\dfrac{\partial \rho}{\partial x}\right)$. If $K = 1$, then, the maximum velocity of the fluid in the couette flow is:
(A) 2 U (B) 0.5 U
(C) U (D) zero

6. A fluid (density = 900 Kg/m^3 and viscosity = 3×10^{-3} Kg/ms) flows upwards between two inclined parallel identical plate at a volumetric rate of 3 ℓ/s per unit width in meters of the plates. The plates are inclined at an angle of 30° with the horizontal and the plates are 20 mm wide apart. The pressure difference between two sections that are 15 meters apart is:
(A) 66218 N/m^2
(B) 66420 N/m^2
(C) 203 N/m^2
(D) 132638 N/m^2

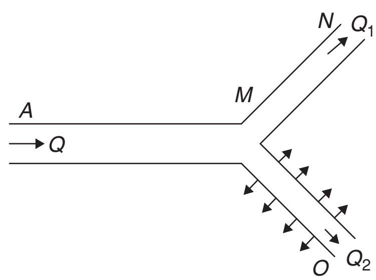
7. A jet of water discharges through a pipe of length 500 m and diameter120 mm. In order to obtain the maximum power at out let, considering a coeff of friction of 0.02, the diameter of the nozzle to attach at the end of the pipe is:
(A) 48 mm (B) 40 mm
(C) 32 mm (D) 24 mm

8. Water is flowing in a penstock pipe 2500 m long, with a flow velocity of 5 m/s. Due to the sudden closure of a valve in the line a pressure wave is generated at it fluid with a velocity of 1500 m/s. Then the maximum pressure rise in the pipe is:
(A) 7.5 MN/m^2 (B) 9.5 MN/m^2
(C) 10.2 MN/m^2 (D) 12 MN/m^2

9. Through a galvanized steel horizontal pipe of length 250 m and diameter 500 mm, water flows at the rate of 0.03 m^3/s. The friction factor (ρ) values for a set of Reynolds number (R_e) and relative roughness (RR) values are given in the following table. If the average surface roughness for galvanized steel is 0.2 mm, then the pumping power required to maintain the flow is:
(A) 27.51 W (B) 27.25 W
(C) 3.62 W (D) 5.806 W

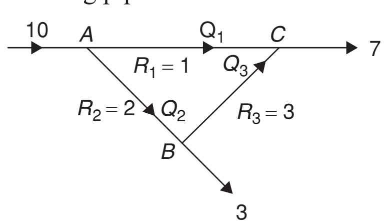
Re	RR	f
7635	0.0004	0.0332
7635	0.2	0.1573
76350	0.0004	0.0207
76350	0.2	0.1558

10. In a horizontal plane, water flows through a pipe of 200 mm diameter and 20 km length. At a point M, as shown in the following figure, the pipe is branched off into two identical parallel pipes of diameter 100 mm and length 10 km. The friction factor for all pipes is to be taken to be equal to 0.015. If in the pipe MQ, water is completely drained off from closely spaced side tappings at a constant rate of 0.01 liter/s per meter length of the pipe, then the discharge in MN (Q_1) is;
(A) 0.1577 m^3/s (B) 0.0577 m^3/s
(C) 0.1 m^3/s (D) 0.0264 m^3/s



11. A main line branches into two equal length (= 150 m) pipes A and B pipe A (f = 0.02) has a diameter of 300 mm while pipe B (f = 0.015) has a diameter of 277 mm. A valve present in pipe A ensures that the discharge in pipe A is one–half the discharges in the main line. If the k values for a full, three fourth, half and one fourth open valve are 0.2, 1.15, 5.6 and 24 respectively, then which one of the following statements is only connect?
(A) Valve is fully open
(B) Valve is almost one fourth open
(C) Valve is almost half open
(D) Valve is almost three fourth open

12. In the following pipe network



Q and R denote the flow rates and flow resistances respectively. For the closed loop ABC the equation $16\,R_2 + R_3 - 36\,R_1 = 0$ could be written. The flow resistances are in the ratio such as $R_3 : R_2 : R_1 = 3 : 2 : 1$. If h_f denotes the head loss due to friction, then the ratio $h_{f1} : h_{f2} : h_{f3}$ is equal to
(A) 3 : 32 : 36 (B) 36 : 32 : 3
(C) 32 : 36 : 3 (D) 36 : 3 : 32

Direction for questions 13 and 14: In a boundary layer, the velocity distribution is given by: $\frac{u}{U} = \frac{y}{\delta}$, where u is the velocity of a perpendicular distance y from the plate, U is the free stream velocity and δ is the boundary layer thickness.

13. The ratio of the displacement thickness to the energy thickness is:
(A) 0.5 (B) 2/3
(C) 1.5 (D) 2

14. The ratio of the momentum thickness to the energy thickness is:
(A) 2 (B) 1.5
(C) 2/3 (D) 0.5

15. A second order polynomial such as $u = a + by + cy^2$ is claimed to represent the velocity distribution in a laminar boundary layer over a flat plate. If δ represents the boundary layer thickness, then the term b/c would be equal to:
(A) –2 (B) -2δ
(C) 0 (D) 2δ

16. A person is walking over a long plate over which a laminar boundary layer has developed. On walking a certain distance form the leading edge of the plate, he observes the boundary layer to be 1 mm thick. If the walks the same distance further downstream, he will observe the boundary layer thickness to be:
(A) $\sqrt{2}$ mm (B) 2 mm
(C) 4 mm (D) 1 mm

17. The velocity (u) and boundary layer thickness (δ) for the flow by a Newtonian fluid over a flat plate is expressed as:
$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$; $\delta = \frac{4.795\pi}{\sqrt{R_{ex}}}$. If the wall shear stress is 1.5 N/m^2 at x = 1.5 m, then at x = 3.5 m, the wall shear stress will be:
(A) 1.5 N/m^2 (B) 0.75 N/m^2
(C) 0.982 N/m^2 (D) 3 N/m^2

18. For the flow of a fluid (viscosity = μ, density = ρ) over a plate of length L and width B, the wall shear stress $\tau_w = 0.327\frac{\mu U}{x}\sqrt{R_{ex}}$, when U is the free stream velocity. For this flow, the average coefficient of drag would be:

(A) $\frac{1.46}{\sqrt{R_{e_L}}}$ (B) $\frac{1.372}{\sqrt{R_{e_L}}}$
(C) $\frac{1.272}{\sqrt{R_{e_L}}}$ (D) $\frac{1.31}{\sqrt{R_{e_L}}}$

19. A 7 m long and 4 m wide plate is at zero incidence to a stream of air flowing with a velocity of 5 m/s. If the density of air is 1.21 Kg/m^3 and the viscosity is 1.45×10^{-5} m^2/s, then the total drag force on both sides of the portion of the plate where the boundary layer is laminar is:

(A) 1.59 N (B) 0.0544 N
(C) 0.1361 N (D) 0.2722 N

20. A flat plate is kept at zero incidence in a stream of fluid having uniform velocity. If the boundary layer developed over the whole plate is laminar, then the ratio of the drag force on the front half to the drag force on the rear half of the plate is:

(A) 2.414 (B) 0.414
(C) 0.707 (D) 0.293

Practice Problems 2

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. A phenomenon or process is modeled using m dimensional variables (or physical quantities) with k primary or fundamental dimensions. The number of non-dimensional parameters or variables is:

(A) $m - k$ (B) $m + k$
(C) $m \times k$ (D) m/k

2. Match the following:

A: Pipe flow	P: Froude number
B: Free surface Flow	Q: Weber number
C: Inertia force/Gravity Force	R: Euler number
D: Compressible Flow	S: Reynolds number
E: Pressure differ-ence/ dynamic pressure	T: Mach number

(A) A : S, B : T, C : P, D : Q, E : R
(B) A : S, B : Q, C : R, D : T, E : P
(C) A : P, B : Q, C : S, D : T, E : R
(D) A : S, B : Q, C : P, D : T, E : R

3. The Reynolds number for the flow of a fluid in a horizontal circular tube of constant diameter is 1200. If the diameter of the tube and the kinematic viscosity of the fluid are doubled and that the discharge at the pipe exit is unchanged, then the new Reynolds number for the flow in the tube will be

(A) 4800 (B) 300 (C) 1200 (D) 600

4. The Darcy friction factor for a fully developed laminar flow in a horizontal circular pipe is 0.032. If the inertia force acting on a fluid particle is 4 kN, then the viscous force acting on the same fluid particle is:

(A) 4000 N (B) 2 N
(C) 8×10^6 N (D) 4 N

5. The velocity profile for a steady fully developed laminar flow in a horizontal pipe of diameter D is given by $u(r) = u_m\left(0.5 - \frac{2r^2}{D^2}\right)$, where r is the radial distance from the centerline of the pipe. If the fluid viscosity is μ, then the wall shear stress is:

(A) $\frac{8\mu u_m}{D}$ (B) $\frac{4\mu u_m}{D}$
(C) $\frac{16\mu u_m}{D}$ (D) $\frac{2\mu u_m}{D}$

6. Horizontal circular pipes A and B have the respective length of 10 m and 20 m. In both the pipes, the same Newtonian fluid flows in a steady fully developed laminar manner. Even though the pressure difference across the ends of Pipe A is four times of that across the ends of pipe B, the maximum shear stress remains the same in both the pipes. The ratio of the Reynolds numbers of the flow in pipe A to the flow in pipe B is:

(A) 64 : 1 (B) 8 : 1
(C) 1 : 64 (D) 1 : 8

7. The volumetric flow rate of the steady fully developed laminar flow of a fluid (density = 900 Kg/m^3, viscosity = 1 poise) in a horizontal circular pipe is 16.493 litre/sec. The Reynolds number for the flow is determined to be 1890. If the pressure difference across the ends of the pipe is 1344 Pa, then the length of the pipe is:

(A) 1 m (B) 2 m
(C) 3 m (D) 4 m

8. For a couette flow, the velocity distribution is given by $u(y) = \frac{uy}{b} - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(by - y^2)$. Where u is the velocity with which the upper plate moves and b is the distance between the plates. A variable K is defined such that $K = \frac{-b^2}{2\mu u}\left(\frac{\partial p}{\partial x}\right)$.

If $K = -2$, then the minimum velocity of the fluid in the couette flow is:

(A) 0 (B) $\frac{-2u}{3}$
(C) $\frac{-u}{4}$ (D) $\frac{-u}{8}$

9. An oil (viscosity = 0.8 Kg/ms and density = 1400 Kg/m^3) flows in a laminar manner between two parallel inclined plates 15 mm apart and inclined at 45° to the horizontal. The pressure at two points 1.5 m vertically

apart is 100 kN/m^2 and 300 kN/m^2. If the upper plate moves at a velocity of 2.5 m/s but in a direction opposite to the flow, then the velocity of the flow at a distance of 5 mm from the lower plate is:
(A) 2.51 m/s (B) 1.23 m/s
(C) 2.42 m/s (D) 1.58 m/s

10. A shaft of radius 0.05 m rotates at 955 r.p.m in a journal bearing of radial clearance 5 mm. If the viscosity and density of the lubricant used in the bearing are 0.01 Pa. sec and 750 Kg/m^3 respectively, then which combination of the following statement about the bearing is ONLY correct?
P: Flow condition in the bearing is turbulent.
Q: Reynolds number of the lubricant flow is 1875.
R: Surface speed of the shaft is 6 m/s.
S: Flow condition in the bearing is laminar.
(A) *P*, *R* (B) *P*, *Q*
(C) *R*, *S* (D) *Q*, *S*

11. Assuming no-slip condition of the inner wall of a pipe in which the fully developed turbulent flow of a liquid occurs, which one of the following statements is ONLY correct about the conditions at the wall?
(A) Total shear stress is zero.
(B) Friction factor is zero.
(C) Reynolds stresses are non-zeros.
(D) Turbulent shear stress is zero.

12. A liquid flows turbulently in a horizontal pipe of diameter 150 mm. The wall shear stress developed is 70 N/m^2 and the maximum fluid velocity in the flow is 3 m/s. If it is assumed that the velocity profile follows the one–seventh power-law, then the ratio of the turbulent shear stress to the laminar shear stress at radius of 35 mm is:
(A) 0.428 (B) 2.335
(C) 3.336 (D) 1.428

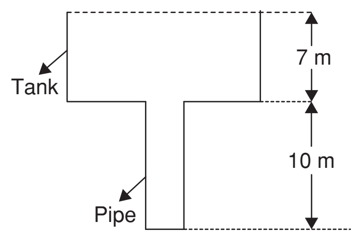
13. In a rectangular duct, a fluid of density 900 Kg/m^3 is flowing in a turbulent manner with an average velocity of 10 m/s. The width of the duct is two times the height while the length of the duct is eight times the height. If the pressure loss due to friction is 1485 Pa, then the friction factor associated with the flow is:
(A) 0.0055 (B) 0.011
(C) 0.0092 (D) 0.055

14. A fluid flows in the converging section of a circular horizontal pipe where the diameter narrows down from 40 cm to 20 cm along a length of 4 meter. If the friction factor for the section is 0.02, then the frictional head loss (neglecting entrance and exit head losses and inertia effects) for a flow of 0.1 m^3/s is:
(A) 0.0484 m (B) 0.1936
(C) 0.0968 m (D) 0.0272 m

15. A centrifugal pump draws a liquid of SG 1.6 at the rate 0.001 m^3/s from a tank, by means of a horizontal pipe of diameter 40 mm. The delivery pipe of the pump is also horizontal with diameter 25 mm. If the pump surpasses an energy equivalent to 15 m of liquid head, the pressure differences across the pump is:
(A) 163.5 kPa (B) 187.6 kPa
(C) 216.9 k Pa (D) 245.2 kPa

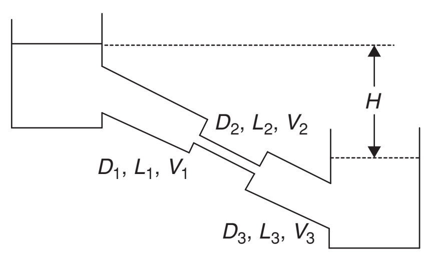
16. A pipe of diameter 150 mm and length 1500 m connects two reservoirs having differences of water level 20 m. If coefficient of friction is 0.015, the discharge through the pipe is:
(A) 0.0485 m^3/s (B) 0.0411 m^3/s
(C) 0.0281 m^3/s (D) 0.0143 m^3/sec

17. Water in a tank of length 20 m in and width 10 m is drained using a pipe of diameter 200 mm and length 10 m as shown in the figure below. The friction factor associated with the pipe is 0.02 if the height of water in the tank is 7 m, then the time required to empty the tank is:



(A) 8738 secs (B) 4369 secs
(C) 2184 secs (D) 6553 secs

18. Two reservoirs are connected by a series of pipes as shown in the following figure. The coefficient of friction is the same for all the three pipes and is equal to *f*. In the figure, *D* and *L* denote the pipe diameter and length respectively and *V* denotes the average flow velocity in the pipe. If $D_1 = D_3$ and $D_2 = 0.5\, D_1$, then the difference in level of the liquid in the two tanks (*H*) neglecting minor losses is:



(A) $\frac{f}{D_1}\frac{V_1^2}{2g}(L_1 + 32L_2 + L_3)$

(B) $\frac{f}{D_1}\frac{V_1^2}{2g}(L_1 + 8L_2 + L_3)$

(C) $\frac{4f}{D_1}\frac{V_1^2}{2g}(L_1 + 32L_2 + L_3)$

(D) $\frac{4f}{D_1}\frac{V_1^2}{2g}(L_1 + 8L_2 + L_3)$

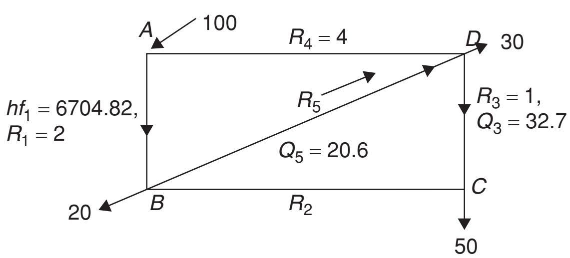
19. A piping system consists of a series of pipes in which a 500 mm diameter pipe ($f = 0.021$) of length 30 m suddenly contracts to a 200 mm diameter pipe ($f = 0.021$) of length 50 m. A 90ϒ′bend ($k = 0.5$) is present in the 500 mm diameter pipe. If the velocity of fluid in the 500 mm diameter pipe is 2 m/s, then the length of an equivalent pipe ($f = 0.021$) of diameter 500 mm for the piping system would be:

(A) 4883 m (B) 4853 m
(C) 4304 m (D) 4895 m

20. Two pipes A and B are connected parallel to a main line that is supported with 9×10^{-3} m^3/s of water from a pump. Pipe A is 110 m long and has a diameter of 55 cm. Pipe B is 850 m long. The volumetric flow rates in both the pipes are same. If the friction coefficient for both the pipes is 0.025, then the diameter of pipe B, assuming that the pipes are on the same level ground, is:

(A) 425 cm (B) 83 cm
(C) 109 cm (D) 92 cm.

21. In the following pipe network



Q, R, hf denote the flow rates, flow resistance and head losses due to friction respectively. The value of the flow resistance R_2 respectively is:

(A) 5.91 (B) 0.907
(C) 4.93 (D) 1.89

22. Two pipes having a set of diameter, length and friction factor values such as (D_1, L_1, and f_1) and (D_2, L_2, f_2) are connected in parallel between two points in a pipeline. If an equivalent pipe of diameter, length and friction factor values such as D_e, L_e and f_e respectively can replace the parallel pipes, then which one of the following relations would definitely hold TRUE?

(A) $\frac{f_1 L_1}{D_1^5} + \frac{f_2 L_2}{D_2^5} = \frac{f_e L_e}{D_e^5}$

(B) $\frac{D_1^5}{f_1 L_1} + \frac{D_2^5}{f_2 L_2} = \frac{D_e^5}{f_e L_e}$

(C) $\left(\frac{f_1 L_1}{D_1^5}\right)^{\frac{1}{2}} + \left(\frac{f_2 L_2}{D_2^5}\right)^{\frac{1}{2}} = \left(\frac{f_e L_e}{D_e^5}\right)^{\frac{1}{2}}$

(D) $\left(\frac{D_1^5}{f_1 L_1}\right)^{\frac{1}{2}} + \left(\frac{D_2^5}{f_2 L_2}\right)^{\frac{1}{2}} = \left(\frac{D_e^5}{f_e L_e}\right)^{\frac{1}{2}}$

23. For a boundary layer, a relation between the shape factor (s) and the layer thickness (δ) is written in the form: $s = k\delta^n$. If the boundary layer has a velocity distribution given by: $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{2}}$, then the values of k and n respectively are:

(A) 7/9 and 1 (B) 9/7 and 1
(C) 7/9 and 0 (D) 9/7 and 0

24. The velocity distribution in the boundary layer over the face of a spillway was observed to have the form: $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{0.22}$. At a certain section AA', the boundary layer thickness was estimated to be 70 mm. If the energy loss per meter length of the spillway is 325.64 kNm/s, then the free stream velocity of the section AA' is:

(A) 28 m/s (B) 35 m/s
(C) 21 m/s (D) 207 m/s

25. If the cubic polynomial $u = a + by + cy^3$ is claimed to represent the velocity distribution in laminar boundary layer over a flat plate, then it would have the form (with U being the free stream velocity and δ the boundary layer thickness) such as:

(A) $u = \frac{3U}{28} y - \frac{U}{28^3} y^3$ (B) $u = \frac{U}{\delta} y$

(C) $u = \frac{2}{\delta} y - \frac{1}{\delta^3} y^3$ (D) $u = \frac{U}{\delta} + \frac{y^3}{\delta^3}$

26. An incompressible fluid flows over a flat plate at a zero incidence angle. The boundary layer thickness, at a location where the Reynolds number is 1000, is 2 mm. At a location where the Reynolds number is 4000, the boundary layer thickness will be:

(A) 1 mm (B) 2 mm
(C) 4 mm (C) 8 mm

27. The velocity (u) and boundary layer thickness (δ) for the flow of a fluid over a flat plate is expressed as:

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2;\ \delta = \frac{5.48x}{\sqrt{\mathrm{Re}_x}}.$$

The free stream velocity of the fluid (viscosity = 0.01 Pa.sec) is 1.5 m/s. If the wall shear stress at $x = 1.5$ m is 1.644 N/m^2 then the density of the Newtonian fluid is:

(A) 800 Kg/m^3 (B) 700 Kg/m^3
(C) 1000 Kg/m^3 (D) 900 Kg/m^3

28. Air flowing over a smooth flat plate forms a boundary layer over the plate where the maximum thickness of the laminar boundary layer is 2.652 mm. If the kinematic viscosity of air is 0.15 stokes, then the free stream air velocity is:

(A) 1.31 mm/s (B) 20 m/s
(C) 3.2 cm/s (D) 1.8 mm/s

29. At location x_1, the thickness of the laminar boundary layer, formed by air flowing at a velocity of 2 m/s over a flat plate, is 6.45 mm. At another location x_2, the laminar boundary layer thickness is 8.17 mm. The velocity distribution for the boundary layer is: $\frac{u}{U} = \frac{3y}{2\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

If the density of air is 1.19 Kg/m^3, then the mass entrainment between the locations x_2 and x_1 is:

(A) 2.56×10^{-3} Kg/s (B) 12.15×10^{-3} Kg/s
(C) 9.59×10^{-3} Kg/s (D) 21.74×10^{-3} Kg/s

30. A flat plate kept at zero incidence in a stream of fluid with uniform velocity develops a turbulent boundary layer over the whole of the plate. If the average coefficient of drag for the whole plate having a turbulent boundary layer is given by $C_D = \frac{0.072}{(R_{e_L})^{0.2}}$, then the ratio of the drag force on the rear half of the plate to the drag force on the front half of the plate is:

(A) 1.349 (B) 0.4256
(C) 0.7411 (D) 0.5743

Previous Years' Questions

1. For air flow over a flat plate, velocity (U) and boundary layer thickness (δ) can be expressed respectively, as $\frac{U}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2$; $\delta = \frac{4.64x}{\sqrt{\text{Re}_x}}$.

If the free stream velocity is 2 m/s,and air has kinematic viscosity of 1.5×10^{-5} m^2/s and density of 1.23 Kg/m^3, the wall shear stress at $X = 1$ m, is **[2004]**

(A) 2.36×10^2 N/m^2
(B) 43.6×10^{-3} N/m^2
(C) 4.36×10^{-3} N/m^2
(D) 2.18×10^{-3} N/m^2

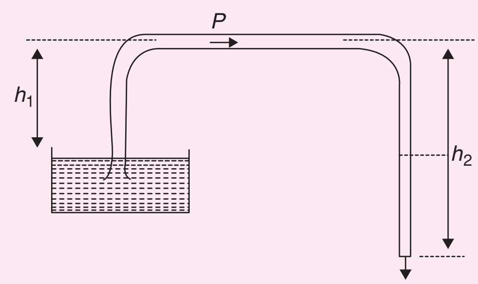
2. A centrifugal pump is required to pump water to an open tank situated 4 km away from the location of the pump through a pipe of diameter 0.2 m having Darcy's friction factor of 0.01. The average speed of water in the pipe is 2 m/s. If it is to maintain a constant head of 5 m in the tank, neglecting other minor/losses, the absolute discharge pressure at the pump exit is: **[2004]**

(A) 0.449 bar (B) 5.503 bar
(C) 44.911 bar (D) 55.203 bar

3. The velocity profile in fully developed laminar flow in a pipe of diameter D is given by $u = u_0 (1 - 4r^2/D^2)$, where r is the radial distance from the center. If the viscosity of the fluid is μ, the pressure drop across a length L of the pipe is: **[2004]**

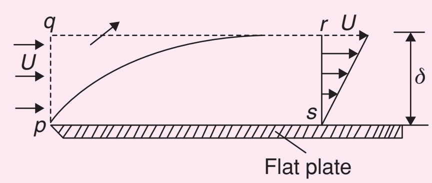
(A) $\frac{\mu u_0 L}{D}$ (B) $\frac{4\mu u_0 L}{D}$
(C) $\frac{8\mu u_0 L}{D}$ (D) $\frac{16\mu u_0 L}{D}$

4. A siphon draws water from a reservoir and discharges it out at atmospheric pressure. Assuming ideal fluid and the reservoir is large, the velocity at point P in the siphon tube is: **[2005]**



(A) $\sqrt{2gh_1}$ (B) $\sqrt{2gh_2}$
(C) $\sqrt{2g(h_2 - h_1)}$ (D) $\sqrt{2g(h_2 + h_1)}$

Direction for questions 5 and 6: A smooth flat plate with a sharp leading edge is placed along a gas stream flowing at $U = 10$ m/s. The thickness of the boundary layer at section $r - s$ is 10 mm, the breadth of the plate is 1 m (into the paper) and the density of the gas $P = 1.0$ Kg/m^3. Assume that the boundary layer is thin, two-dimensional, and follows a linear velocity distribution, $u = U(y/\delta)$, at the section $r - s$, where y is the height from plate.



5. The mass flow rate (in Kg/s) across the section $q - r$ is: **[2005]**

(A) zero (B) 0.05
(C) 0.10 (D) 0.15

6. The integrated drag force (in N) on the plate, between $p - s$, is: **[2005]**

(A) 0.67 (B) 0.33
(C) 0.17 (D) zero

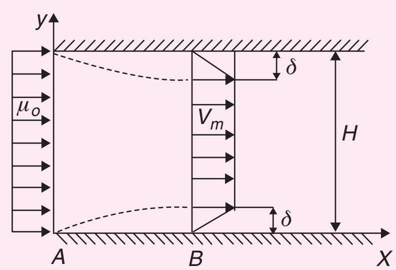
7. Consider an incompressible laminar boundary layer flow over a flat plate of length L, aligned with the direction of an oncoming uniform free stream. If F is the ratio of the drag force on the front half of the plate to the drag force on the rear half, then **[2006]**

(A) $F < 1/2$ (B) $F = 1/2$
(C) $F = 1$ (D) $F > 1$

8. Consider steady laminar incompressible axisymmetric fully developed viscous flow through a straight circular pipe of constant cross-sectional area at a Reynolds number of 5. The ratio of inertia force to viscous force on a fluid particle is: **[2006]**

(A) 5 (B) 1/5
(C) 0 (D) ∞

Direction for questions 9 and 10: Consider a steady incompressible flow through a channel as shown below.



The velocity profile is uniform with a value of u_o at the inlet section A. The velocity profile at section B downstream is

$$u = \begin{cases} V_m \dfrac{y}{\delta}, & 0 \le y \le \delta \\ V_m, & \delta \le y \le H - \delta \\ V_m \dfrac{H-y}{\delta}, & H - \delta \le y \le H \end{cases}$$

9. The ratio V_m/u_o is **[2007]**

(A) $\dfrac{1}{1-2\left(\dfrac{\delta}{H}\right)}$ (B) 1

(C) $\dfrac{1}{1-\left(\dfrac{\delta}{H}\right)}$ (D) $\dfrac{1}{1+\left(\frac{\delta}{H}\right)}$

10. The ratio $\dfrac{p_A - p_B}{\dfrac{1}{2}pu_o^2}$ (where p_A and p_B are the pressures at section A and B, respectively, and ρ is the density of the fluid) is **[2007]**

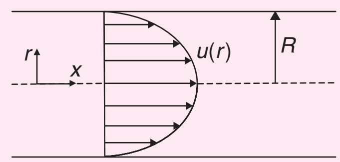
(A) $\dfrac{1}{\left(1-\left(\frac{2\delta}{H}\right)\right)^2} - 1$ (B) $\dfrac{1}{\left[1-\left(\dfrac{\delta}{H}\right)\right]^2}$

(C) $\dfrac{1}{\left(1-\left(\dfrac{\delta}{H}\right)\right)^2} - 1$ (D) $\dfrac{1}{1+\left(\dfrac{\delta}{H}\right)}$

11. Water at 25ϒC is flowing through a 1.0 km long G.I pipe of 200 mm diameter at the rate of 0.07 m^3/s. If value of Darcy friction factor for this pipe is 0.02 and density of water is 1000 Kg/m^3, the pumping power (in kW) required to maintain the flow is **[2008]**

(A) 1.8 (B) 17.4
(C) 20.5 (D) 41.0

12. The velocity profile of a fully developed laminar flow in a straight circular pipe, as shown in the figure, is given by the expression $u(r) = -\dfrac{R^2}{4\mu}\left(\dfrac{dp}{dx}\right)\left(1-\dfrac{r^2}{R^2}\right)$ where $\dfrac{dp}{dx}$ is a constant. The average velocity of fluid in the pipe is **[2008]**



(A) $-\dfrac{R^2}{8\mu}\left(\dfrac{dp}{dx}\right)$ (B) $-\dfrac{R^2}{4\mu}\left(\dfrac{dp}{dx}\right)$

(C) $-\dfrac{R^2}{2\mu}\left(\dfrac{dp}{dx}\right)$ (D) $-\dfrac{R^2}{\mu}\left(\dfrac{dp}{dx}\right)$

13. The maximum velocity of a one-dimensional incompressible fully developed viscous flow, between two fixed parallel plates, is 6 ms^{-1}. The mean velocity (in ms^{-1}) of the flow is **[2008]**

(A) 2 (B) 3
(C) 4 (D) 5

14. Oil flows through a 200 mm diameter horizontal cast iron pipe (friction factor, $f = 0.0225$) of length 500 m. The volumetric rate is 0.2 m^3/s. The head loss (in m) due to friction is (assume $g = 9.81$ m/s^2) **[2009]**

(A) 116.18 (B) 0.116
(C) 18.22 (D) 232.36

15. An incompressible fluid flows over flat plate with zero pressure gradient. The boundary layer thickness is 1 mm at a location where the Reynolds number is 1000. If the velocity of the fluid alone is increased by a factor of 4, then the boundary layer thickness at the same location, in mm will be **[2009]**

(A) 4 (B) 2
(C) 0.5 (D) 0.25

16. For steady, fully developed flow inside a straight pipe of diameter D, neglecting gravity effects, the pressure drop Δp over a length L and the wall shear stress τ_w are related by **[2010]**

(A) $\tau_w = \dfrac{\Delta pD}{4L}$ (B) $\tau_w = \dfrac{\Delta pD^2}{4L^2}$

(C) $\tau_w = \dfrac{\Delta pD}{2L}$ (D) $\tau_w = \dfrac{4\Delta pD}{4L}$

17. Water flows through a pipe having an inner radius of 10 mm at the rate of 36 Kg/hr at 25°C. The viscosity of water at 25°C is 0.001 Kg/m.s. The Reynolds number of the flow is ______ **[2011]**

18. For a fully developed flow of water in a pipe having diameter 10 cm, velocity 0.1 m/s and kinematic viscosity 10^{-5} m^2/s, the value of Darcy friction factor is ______ **[2011]**

19. Water flows through a 10 mm diameter and 250 m long smooth pipe at an average velocity of 0.1 m/s. The density and the viscosity of water are 997 Kg/m^3 and 855×10^{-6} N·s/m^2, respectively. Assuming fully–developed flow, the pressure drop (in Pa) in the pipe is ______ **[2012]**

20. Consider laminar flow of water over a flat plate of length 1 m. If the boundary layer thickness at a distance of 0.25 m from the leading edge of the plate is 8 mm, the boundary layer thickness (in mm), at a distance of 0.75 m, is ______ **[2013]**

21. Consider the turbulent flow of a fluid through a circular pipe of diameter, *D*. Identify the correct pair of statements.

I. The fluid is well-mixed

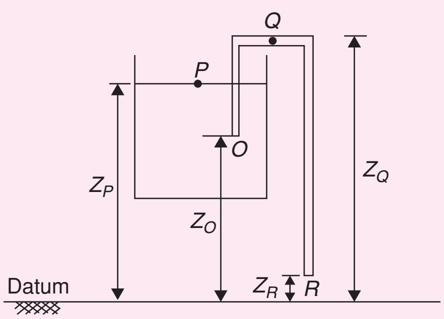
II. The fluid is unmixed

III. $R_{e_D} < 2300$

IV. $R_{e_D} > 2300$ **[2014]**

(A) I, III (B) II, IV
(C) II, III (D) I, IV

22. A siphon is used to drain water from a large tank as shown in the figure below. Assume that the level of water is maintained constant. Ignore frictional effect due to viscosity and losses at entry and exist. At the exit of the siphon, the velocity of water is: **[2014]**



(A) $\sqrt{2g(Z_Q - Z_R)}$ (B) $\sqrt{2g(Z_p - Z_R)}$

(C) $\sqrt{2g(Z_o - Z_R)}$ (D) $\sqrt{2gZ_Q}$

23. A fluid of dynamic viscosity 2×10^{-5} Kg/ m.s and density 1 Kg/m^3 flows with an average velocity of 1 m/s through a long duct of rectangular (25 mm × 15 mm) cross-section. Assuming laminar flow, the pressure drop (in Pa) in the fully developed region per meter length of the duct is: ______ **[2014]**

24. Consider fully developed flow in a circular pipe with negligible entrance length effects. Assuming the mass flow rate, density and friction factor to be constant, if the length of the pipe is doubled and the diameter is halved, the head loss due to friction will increase by a factor of **[2015]**

(A) 4 (B) 16
(C) 32 (D) 64

25. The Blausius equation related to boundary layer theory is a **[2015]**

(A) third-order linear partial differential equation
(B) third-order nonlinear partial differential equation
(C) second-order nonlinear ordinary differential equation
(D) third-order nonlinear ordinary differential equation

26. For flow through a pipe of radius *R*, the velocity and temperature distribution are as follows: $u(r, z) = C_1$, and $T(r, x) = C_2\left[1-\left(\frac{r}{R}\right)^3\right]$, where C_1 and C_2 are constants.

The bulk mean temperature is given by $T_m = \frac{2}{U_m R^2}\int_0^R u(r, x)T(r, x)\, rdr$, with U_m being the mean velocity of flow. The value of T_m is: **[2015]**

(A) $\frac{0.5C_2}{U_m}$ (B) $0.5C_2$

(C) $0.6C_2$ (D) $\frac{0.6C_2}{U_m}$

27. Within a boundary layer for a steady incompressible flow, the Bernoulli equation: **[2015]**

(A) holds because the flow is steady
(B) holds because the flow is incompressible
(C) holds because the flow is transitional
(D) does not hold because the flow is frictional

28. The total emissive power of a surface is 5000 W/m^2 at a temperature T_1 and 1200 W/m^2 at a temperature T_2, where the temperature are in Kelvin. Assuming the emissivity of the surface to be constant, the ratio of the temperatures $\frac{T_1}{T_2}$ is: **[2015]**

(A) 0.308 (B) 0.416
(C) 0.803 (D) 0.875

29. The head loss for a laminar incompressible flow through a horizontal circular pipe is h_1. Pipe length and fluid remaining the same, if the average flow velocity doubles and the pipe diameter reduces to half its previous value, the head loss is h_2. The ratio h_2/h_1 is: **[2015]**

(A) 1 (B) 4
(C) 8 (D) 16

30. For a fully developed laminar flow of water (dynamic viscosity 0.001 Pa-s) through a pipe of radius 5 cm, the axial pressure gradient is –10 Pa/m. The magnitude of axial velocity (in m/s) at a radial location of 0.2 cm is ______. **[2015]**

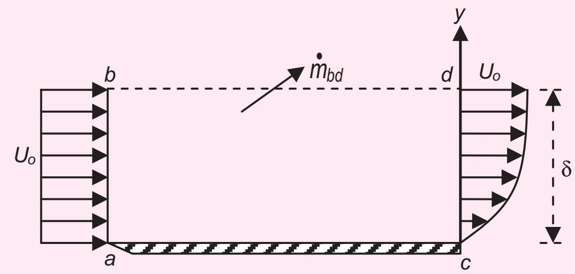
31. Three parallel pipes connected at the two ends have flow-rates Q_1, Q_2 and Q_3 respectively, and the corresponding frictional head losses are h_{L1}, h_{L2}, and h_{L3} respectively. The correct expressions for total flow rate (Q) and frictional head loss across the two ends (h_L) are: **[2015]**
(A) $Q = Q_1 + Q_2 + Q_3$; $h_L = h_{L1} + h_{L2} + h_{L3}$
(B) $Q = Q_1 + Q_2 + Q_3$; $h_L = h_{L1} = h_{L2} = h_{L3}$
(C) $Q = Q_1 = Q_2 = Q_3$; $h_L = h_{L1} + h_{L2} + h_{L3}$
(D) $Q = Q_1 = Q_2 = Q_3$; $h_L = h_{L1} = h_{L2} = h_{L3}$

32. Oil (kinematic viscosity, $\upsilon_{oil} = 1.0 \times 10^{-5}$ m^2/s) flows through a pipe of 0.5 m diameter with a velocity of 10 m/s. Water (kinematic viscosity, $\upsilon_w = 0.89 \times 10^{-6}$ m^2/s) is flowing through a model pipe of diameter 20 mm. For satisfying the dynamic similarity, the velocity of water (in m/s) is _______. **[2016]**

33. A steady laminar boundary layer is formed over a flat plate as shown in the figure as given on next page. The free stream velocity of the fluid is U_o. The velocity profile at the inlet *a*-*b* is uniform, while that at the downstream location *c*-*d* given by

$$u = U_o\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]$$

The ratio of the mass flow rate, $\dot{m}_{bd}$, leaving through the horizontal section *b*-*d* to that entering through the vertical section *a*-*b* is ______. **[2016]**



34. Consider fluid flow between two infinite horizontal plates which are parallel (the gap between them being 50 mm). The top plate is sliding parallel to the stationary bottom plate at a speed of 3 m/s. The flow between the plates is solely due to the motion of the top plate. The force per unit area (magnitude) required to maintain the bottom plate stationary is ______ N/m^2. **[2016]**

Viscosity of the fluid
$\mu = 0.44$ kg/m-s and density
$\rho = 88$ kg/m^3.

35. Consider a fully developed steady laminar flow of an incompressible fluid with viscosity μ through a circular pipe of radius R. Given that the velocity at a radial location of $R/2$ from the centerline of the pipe is U_1, the shear stress at the wall is $K\mu\, U_1/R$, where K is ________. **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. D	**2.** A	**3.** B	**4.** A	**5.** C	**6.** B	**7.** D	**8.** A	**9.** C	**10.** B
11. D	**12.** B	**13.** D	**14.** C	**15.** B	**16.** A	**17.** C	**18.** D	**19.** D	**20.** A

Practice Problems 2

1. A	**2.** D	**3.** B	**4.** B	**5.** D	**6.** C	**7.** B	**8.** D	**9.** C	**10.** B
11. D	**12.** B	**13.** A	**14.** A	**15.** C	**16.** D	**17.** B	**18.** C	**19.** A	**20.** B
21. A	**22.** D	**23.** D	**24.** B	**25.** A	**26.** C	**27.** D	**28.** B	**29.** A	**30.** C

Previous Years' Questions

1. C **2.** B **3.** D **4.** C **5.** B **6.** C **7.** D **8.** A **9.** A **10.** C
11. B **12.** A **13.** C **14.** A **15.** C **16.** A **17.** 635 to 638 **18.** 0.06 to 0.07
19. 6800 to 6900 **20.** 13.5 to 14.2 **21.** D **22.** B **23.** 1.7 to 2 **24.** D **25.** D
26. C **27.** D **28.** C **29.** C **30.** 6.2 to 6.3 **31.** B **32.** 22 to 22.5
33. 0.32 – 0.34 **34.** 26.4 **35.** 2.6 – 2.7