MATRICES AND DETERMINANTS

1m	2m	3m	4m	5m	6m	Total
1(K)	1(U)	2(K)	–	1(A)	1(A)	–

1 MARK QUESTIONS

(Knowledge)

1. If $A\begin{bmatrix}1 & 3 & 8\end{bmatrix}$ and $B = \begin{bmatrix}7\\3\\1\end{bmatrix}$ find AB.

2. Find AB if $A = \begin{bmatrix}1\\4\\2\end{bmatrix}$ and B = [1 8 1]

3. Find x if $[2\ x\ 2]\begin{bmatrix}1\\3\\5\end{bmatrix} = [9]$

4. Find x if $\begin{vmatrix}1 & 2 & 3\\2 & x & 3\\3 & 4 & 3\end{vmatrix} = 0$

5. Evaluate $\begin{vmatrix}4785 & 4787\\4789 & 4791\end{vmatrix}$

6. Evaluate $\begin{vmatrix}40 & 41 & 42\\43 & 44 & 45\\46 & 47 & 48\end{vmatrix}$

7. Find the value of $\begin{vmatrix}2 & 3\\-1 & 4\end{vmatrix}$

8. Solve for x if $\begin{vmatrix} 2 & -x \\ x & -2 \end{vmatrix} = 0$

9. If $A = [5\ 0\ -1]$ $B = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ find AB.

10. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 4 \\ 3 & 2 \\ 1 & 0 \end{bmatrix}$ find 2A + 3B.

11. Find x if $[2\ x\ 2] \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} = [18]$

12. Evaluate $\begin{vmatrix} 4423 & 4425 \\ 4428 & 4430 \end{vmatrix}$

13. If $A = \begin{bmatrix} 2 & -x \\ x & -7 \end{bmatrix}$ find $A + A^1$.

14. Solve for x : $\begin{vmatrix} x & 0 & 1 \\ 0 & x & 0 \\ 0 & -1 & x \end{vmatrix} = 8$

15. Evaluate $\begin{vmatrix} 4423 & 4425 \\ 4428 & 4430 \end{vmatrix}$

16. If $A = \begin{bmatrix} 1 & -3 & 5 \\ 6 & 2 & 4 \end{bmatrix}$ find $5A^1$.

17. If $A = \begin{bmatrix} 3 & -1 & 2 \\ 3 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 & 6 \\ 1 & 3 & -1 \end{bmatrix}$ find 2A – 3B.

18. If $[2\ x\ 2] \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = [9]$, find the value of x.

19. Evaluate $\begin{vmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$

20. If $\begin{vmatrix} x & 12 \\ 12 & x \end{vmatrix} = 0$ find x.

21. Evaluate $\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{vmatrix}$

22. Evaluate $\begin{vmatrix} 8 & 2 & 1 \\ 12 & 3 & -5 \\ 16 & 4 & 2 \end{vmatrix}$

23. Find x if $\begin{bmatrix} 3 & x \\ 4 & 7 \end{bmatrix}$ is a symmetric matrix

24. Find x if $\begin{bmatrix} 5x & 2 \\ -10 & 1 \end{bmatrix}$ is a singular matrix.

2 MARK QUESTIONS

(Understanding)

1. Solve for x : $\begin{bmatrix} x^2 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 2x & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 3 & 7 \end{bmatrix}$

2. Find the values of a and b if the matrices A and B are equal where

$$A = \begin{bmatrix} a+b & b^2+2 \\ 0 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 2a+1 & 3b \\ 0 & b^2-5b \end{bmatrix}$$

3. If $\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} a & 1 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -3 & 5 \end{bmatrix}$ find the values of a and b.

4. If $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ find the values of x and y.

5. Solve for x and y if $x\begin{pmatrix}2\\1\end{pmatrix}+y\begin{pmatrix}3\\5\end{pmatrix}+\begin{bmatrix}4\\6\end{bmatrix}=\begin{bmatrix}12\\17\end{bmatrix}$

6. Find A and B if $A+B=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}$ and $A-B=\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}$

7. Find x and y if $\begin{bmatrix}x+y & 3\\-1 & x-y\end{bmatrix}=\begin{bmatrix}4 & 3\\-1 & 8\end{bmatrix}$

8. Without expansion, show that $\begin{vmatrix}x & 4 & y+z\\y & 4 & x+z\\z & 4 & x+y\end{vmatrix}=0$.

9. Without expansion show that $\begin{vmatrix}4 & 4^2 & 4^3\\4^2 & 4^3 & 4^4\\4^3 & 4^4 & 4^5\end{vmatrix}=0$

10. Prove that if any two rows (or columns) of a determinant are interchanged then the value of the determinant changes only in sign.
11. If in a determinant, scalar multiple of the elements of any row or column is added to any other row or column then prove that the value of the determinant remains unchanged.

12. If $A=\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$ show tha $A^2-4A+3I=0$ where I is the identity matrix of second order.

13. Find the value of x and y if $\begin{bmatrix}x & y\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 1\end{bmatrix}=\begin{bmatrix}-1 & 4\end{bmatrix}$

14. If $A=\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$ show that $A^2 = 2A$.

15. Without actual expansion show that $\begin{vmatrix}43 & 1 & 6\\35 & 7 & 4\\17 & 3 & 2\end{vmatrix}=0$

16. If $A=\begin{bmatrix}5\\4\\3\end{bmatrix}$ and $B=\begin{bmatrix}3\\-1\\2\end{bmatrix}$ find AB^1.

17. If is a determinant, the elements of any row (or column) are multiplied by the same scalar, say k then the value of the new determiant is k times the value of the given determinant.

18. Solve for x : $\begin{vmatrix} 7 & 4 & 11 \\ -3 & 5 & x \\ -x & 3 & 1 \end{vmatrix} = 0$

19. Without actual expansion show that $\begin{vmatrix} x & 3 & y+z \\ y & 3 & 3+x \\ z & 3 & x+y \end{vmatrix} = 0$

20. Solve by Cramer's rule :
$2x + y = 1$, $x - 3y = 4$

21. Find x if the matrix $\begin{bmatrix} 3 & 2 & x \\ 4 & 1 & -1 \\ 0 & 3 & 4 \end{bmatrix}$ is singular.

22. Solve by Cramer's Rule : $2x + 3y = 11$, $x - y - 3 = 0$

23. Solve by Cramer's Rule : $3x + 2y = 8$, $4x - 3y = 5$

24. Solve by Cramer's Rule : $3x + 4y = 7$, $7x - y = 6$

3 MARK QUESTIONS

(Knowledge)

1. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify : $(AB)^{-1} = B^{-1} A^{-1}$.

2. Using the properties of determinants prove that $\begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix} = 1 + a + b + c$

3. Find the adjoint of the matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Verify $(AB)^{-1} = B^{-1} A^{-1}$ for the matrices

$A = \begin{bmatrix} 2 & -3 \\ 1 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}$

5. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$ without expansion.

6. Solve $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ using the properties of determinant

7. Prove that $\begin{vmatrix} a-2b-2c & 3b & 3c \\ 3a & b-2c-2a & 3c \\ 3a & 3b & c-2a-2b \end{vmatrix} = 4(a+b+c)^3$ without expansion.

8. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ b^2 & c^2 & a^2 \\ b^3 & c^3 & a^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$ without expansion.

9. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & -4 & 1 \\ 0 & 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 & -3 \\ 2 & -4 & -1 \\ 0 & 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 1 \\ -1 & -3 & 4 \end{bmatrix}$ verify $(A + B)C = AC + BC$.

10. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $A + 2B = A^2$ find B.

11. Find A and B if

$$2A - 3B = \begin{bmatrix} 2 & -4 \\ -12 & 1 \end{bmatrix} \text{ and } A + 5B = \begin{bmatrix} 1 & 24 \\ 33 & 7 \end{bmatrix}$$

12. Find x if $\begin{bmatrix} x^3 & 1 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} -2x^2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -x & 4 \\ 4 & 7 \end{bmatrix}$

13. If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ prove that $A^2 - 4A + 5I$ is a null matrix.

14. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ b^2 & c^2 & a^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

15. Prove that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

16. Prove that $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

17. Show that $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} = (x-p)(x-q)(x+p+q)$

18. Solve for x $\begin{vmatrix} x+2 & 3 & 4 \\ 2 & x+3 & 4 \\ 2 & 3 & x+4 \end{vmatrix} = 0$

19. Solve for x if $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$

20. Solve for x if $\begin{vmatrix} 1+x & 1-x & 1-x \\ 1-x & 1+x & 1-x \\ 1-x & 1-x & 1+x \end{vmatrix} = 0$

21. If $A=\begin{pmatrix}2 & 3\\ 1 & 0\end{pmatrix} B=\begin{pmatrix}2 & 1 & 4\\ 0 & 1 & 3\end{pmatrix}$ and $C=\begin{pmatrix}1 & -3 & -1\\ -2 & 4 & 5\\ 1 & 3 & -2\end{pmatrix}$ show that (AB)C = A(BC).

22. If $A=\begin{pmatrix}2 & -1\\ 1 & 4\end{pmatrix} B=\begin{pmatrix}-3 & 2\\ -1 & 4\end{pmatrix}$ show that $(AB)^1 = B^1A^1$.

23. Solve for x, y and f given $\begin{pmatrix}x & 2 & -3\\ 5 & y & 2\\ 1 & -1 & z\end{pmatrix}\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}=\begin{pmatrix}5 & 3 & 3\\ 19 & -5 & 16\\ 1 & -3 & 0\end{pmatrix}$

24. If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix} B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & 4\\ 2 & -1 & 5\end{bmatrix}$ verify BA = 6I where I is the identity matrix.

25. If $A=\begin{bmatrix}3 & 2\\ 1 & 0\end{bmatrix}$, $B=\begin{bmatrix}4 & 5 & 6\\ 0 & 1 & 2\end{bmatrix}$, $C=\begin{bmatrix}1 & -4 & 1\\ -2 & 5 & -3\\ 3 & 6 & 3\end{bmatrix}$ verify : (AB) C = A(BC)

5 MARK QUESTIONS

(Application)

1. If $A=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}$ verify A(adj A) = (adj A) A = |A| I.

   Where I is the identify matrix of 3rd order.

2. Solve by matrix method : $3x - y + 2z = 13,\ 2x + y - z = 3,\ x + 3y - 5z = -8$

3. Solve by matrix method : $x - y + 2z = 3,\ 2x + z = 1,\ 3x + 2y + z = 4$

4. Solve by matrix method : $x + y + z = 5,\ 2x + y - z = 2,\ 2x - y + z = 2$

5. Solve by matrix method : $2x - z = 0,\ 5x + y = 4,\ y + 3z = 5$
6. Solve by matrix method : $x - y - 2z = 3,\ 2x + y + z = 5,\ 4x - y - 2z = 1$
7. Verify A. adjA = adj A.A = |A| I given $A = \begin{bmatrix} 1 & 5 & 7 \\ 2 & 3 & 0 \\ -2 & 1 & 0 \end{bmatrix}$
8. Verify A. adjA = adj A.A = |A| I given $A = \begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$
9. Find A^{-1} B where $A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$

6 MARK QUESTIONS

(Skill)

1. Two factories decided to award their employees for three good values, at the rate of Rs *x*, Rs y and Rs z per person respectively. The first factory decided to honour 2, 4 and 3 employees respectively with a total prize money of Rs 29000. The second factory decided to honour respectively 5, 2 and 3 employees with a prize money of R8 30500. If the three prizes per person together cost Rs 9500, then represent the above situation by a matrix equation and solve these equations using matrix method.
2. The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers with three times the first number we get 12. Find the numbers using matrix method.
3. A school wants to award its students for the values of honesty, requlority and hardwork with a total cash award of Rs 6000.Three times the award money for hardwork added to that given for honesty amounts to Rs. 11000. The award money gives for honesty and hardwork together is double the one given for requlority represent the above situation algebraically and find the award for each value, using matrix method.
4. A mixture is to be made of three foods A, B and C. The three foods A, B, C contain nutrients P, Q, R as shown below.

Ounces per Pound of Nutrient			
Food	P	Q	R
A	1	2	5
B	3	1	1
C	4	2	1

How to form a mixture which will have 8 ounces of P, 5 ounces of Q and 7 ounces of R?

5. An amount of Rs 5000 is put into three investments of the rate of interest of 6% 7% and 8% per annum respectively the total annual income is Rs. 358. If the combined income from the first two investments is Rs 70 more than the income from the third, find the amount of each investment by matrix method.
6. Anemployee with code 2F2017 made the following record of sales during 3 months of July, August and September for three products A, B and C which have different rates of commision. Using matrix method, find out the rate of commission on items A, B and C.

Month	Sales in units			Total commission (in Rs)
	A	B	C	
July	100	100	100	700
August	200	300	800	1700
September	400	900	100	3700

7. A Sales person Samarth has the following record of sales for the month of January, February and March 1996 for three products A, B and C. He is paid a commission at fixed rate per unit but at varying rates for products A, B and C.

Months	Sales in units			Commission
	A	B	C	in ₹
January	9	10	2	800
February	15	5	4	900
March	6	10	3	850

Find the rate of commission payable on A,B & C per unit sold using matrix method.

8. The monthly expenditure in an office for 3 months is given in the table.

Month	No. of			Total
	Clerks	Peons	Typists	Monthly Salary (₹)
July	8	4	6	3750
August	9	9	6	5000
September	12	9	12	8850

Assuming that the salary for the different categories of the staff did not vary from month to month. Calculate the salary for each type of staff per month using matrix method.

9. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹80, ₹60 and ₹40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.
10. A manufacturer Nandhini produces three products x, y, z which she sells in two markets. Annual sales are indicated below.

Market	Products		
I	10000	2000	18000
II	6000	20000	8000

(a) If unit sale prices of x, y and z are ₹ 2.50, ₹ 1.50 and ₹ 1.00, respectively. Find the total revenue in each market with the help of matrix algebra. The unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and 50 paise respectively.

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