

## Chapter – 8

### Heat and Thermodynamics

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#### Multiple Choice Questions

**Question 1.**

In hot summer after a bath, the body's .....

- (a) internal energy decreases
- (b) internal energy increases
- (c) heat decreases
- (d) no change in internal energy and heat

**Answer:**

- (a) internal energy decreases

**Question 2.**

The graph between volume and temperature in Charles' law is .....

- (a) an ellipse
- (b) a circle
- (c) a straight line
- (d) a parabola

**Answer:**

- (c) a straight line

**Question 3.**

When a cycle tyre suddenly bursts, the air inside the tyre expands. This process is .....

- (a) isothermal
- (b) adiabatic
- (c) isobaric
- (d) isochoric

**Answer:**

- (b) adiabatic

**Question 4.**

An ideal gas passes from one equilibrium state ( $P_1, V_1, T_1, N$ ) to another

equilibrium state

$(2P_1, 3V_1, T_2, N)$ . Then .....

- (a)  $T_1 = T_2$       (b)  $T_1 = \frac{T_2}{6}$       (c)  $T_1 = 6T_2$       (d)  $T_1 = 3T_2$

**Answer:**

(b)  $T_1 = \frac{T_2}{6}$

**Solution:**

From ideal gas equation,  $PV = NkT$

One equilibrium state  $(P_1, V_1, T_1, N)$

Another equilibrium state  $(P_2, V_2, T_2, N)$

$P_2 = 2P_1$ , and  $V_2 = 3V_1$

$$\frac{P_1 V_1}{P_2 V_2} = \frac{T_1}{T_2}$$

$$\frac{P_1 V_1}{(2P_1)(3V_1)} = \frac{T_1}{T_2} \Rightarrow \frac{1}{6} = \frac{T_1}{T_2}$$

$$\therefore \boxed{T_1 = \frac{T_2}{6}}$$

**Question 5.**

When a uniform rod is heated, which of the following quantity of the rod will increase

- (a) mass  
(b) weight  
(c) center of mass  
(d) moment of inertia

**Answer:**

- (d) moment of inertia

**Question 6.**

When food is cooked in a vessel by keeping the lid closed, after some time the steam pushes the lid outward. By considering the steam as a thermodynamic system, then in the cooking process .....

- (a)  $Q > 0, W > 0$   
(b)  $Q < 0, W > 0$

- (c)  $Q > 0, W < 0$
- (d)  $Q < 0, W < 0$

**Answer:**

- (a)  $Q > 0, W > 0$

**Question 7.**

When you exercise in the morning, by considering your body as thermodynamic system, which of the following is true?

- (a)  $\Delta U > 0, W > 0$
- (b)  $\Delta U < 0, W > 0$
- (c)  $\Delta U < 0, W < 0$
- (d)  $\Delta U = 0, W > 0$

**Answer:**

- (b)  $\Delta U < 0, W > 0$

**Question 8.**

A hot cup of coffee is kept on the table. After some time it attains a thermal equilibrium with the surroundings. By considering the air molecules in the room as a thermodynamic system, which of the following is true?

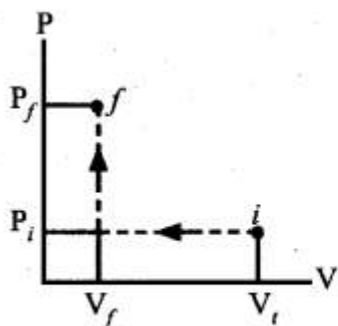
- (a)  $\Delta U > 0, Q = 0$
- (b)  $\Delta U > 0, W < 0$  (c)  $\Delta U > 0, Q > 0$
- (d)  $\Delta U = 0, Q > 0$

**Answer:**

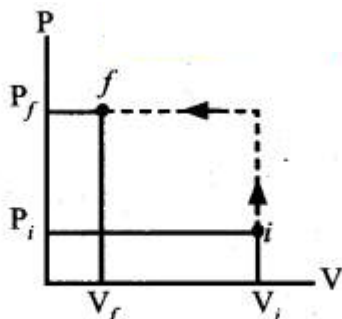
- (c)  $\Delta U > 0, Q > 0$

**Question 9.**

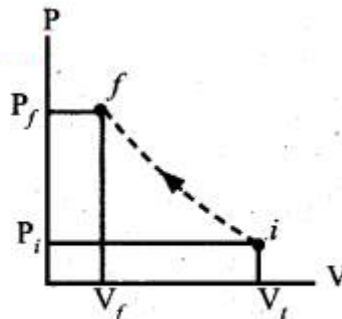
An ideal gas is taken from  $(P_i, V_i)$  to  $(P_f, V_f)$  in three different ways. Identify the process in which the work done on the gas the most.



**Process A**



**Process B**



**Process C**

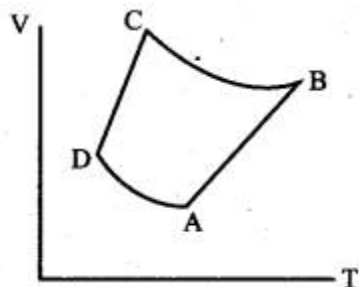
- (a) Process A
- (b) Process B
- (c) Process C
- (d) Equal work is done in Process A, B & C

**Answer:**

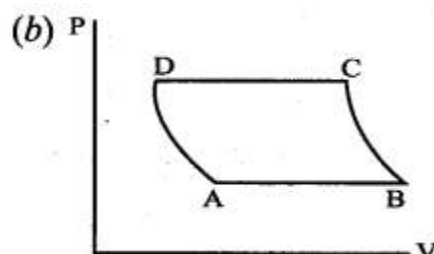
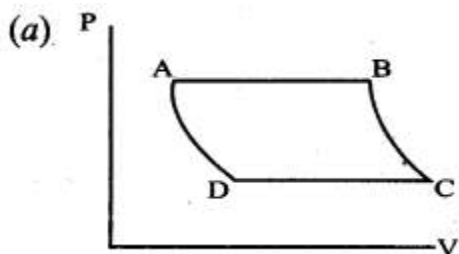
- (b) Process B

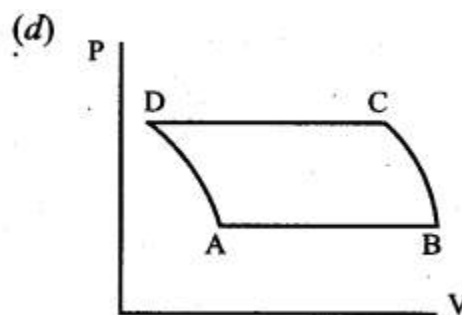
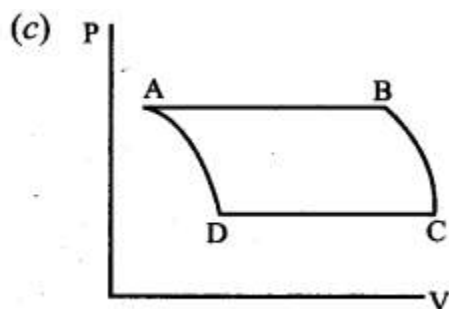
**Question 10.**

The V-T diagram of an ideal gas which goes through a reversible cycle  $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$  is shown below. (Processes  $D \rightarrow A$  and  $B \rightarrow C$  are adiabatic)

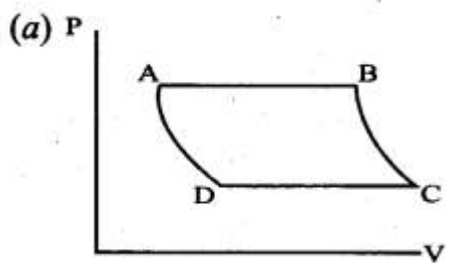


The corresponding PV diagram for the process is (all figures are schematic)





Answer:



**Question 11.**

A distant star emits radiation with maximum intensity at 350 nm. The temperature of the star is

- (a) 8280 K
- (b) 5000 K
- (c) 7260 K
- (d) 9044 K

Answer:

- (a) 8280 K

**Solution:**

According to Wien's displacement law,

$$\lambda_m = \frac{b}{T} ; \quad T = \frac{b}{\lambda_m} \quad \begin{aligned} b &= 2.898 \times 10^{-3} \text{ mK} \\ \lambda_m &= 350 \text{ nm (or) } 350 \times 10^{-9} \text{ m} \end{aligned}$$

$$= \frac{2.898 \times 10^{-3}}{350 \times 10^{-9}} = 8.28 \times 10^{-3} \times 10^6$$

$T = 8280 \text{ K}$

**Question 12.**

Identify the state variables given here?

- (a) Q, T, W
- (b) P, T, U
- (c) Q, W
- (d) P, T, Q

**Answer:**

- (b) P, T, U

**Question 13.**

In an isochoric process, we have .....

- (a)  $W = 0$
- (b)  $Q = 0$
- (c)  $\Delta U = 0$
- (d)  $\Delta T = 0$

**Answer:**

- (a)  $W = 0$

**Question 14.**

The efficiency of a heat engine working between the freezing point and boiling point of water is ..... [NEET 2018]

- (a) 6.25%
- (b) 20%
- (c) 26.8%
- (d) 12.5%

**Answer:**

- (c) 26.8%

**Solution:**

The freezing point of water =  $0^{\circ}\text{C} = 273\text{ K}$

Boiling point of water =  $100^{\circ}\text{C} = 373\text{ K}$

$$\text{Efficiency of heat engine } \eta = 1 - \frac{T_L}{T_H} = 1 - \frac{273}{373} = 0.2681 \quad \boxed{\eta = 26.8\%}$$

**Question 15.**

An ideal refrigerator has a freezer at temperature  $-12^{\circ}\text{C}$ . The coefficient of performance of the engine is 5. The temperature of the air (to which the heat ejected) is .....

- (a)  $50^{\circ}\text{C}$
- (b)  $45.2^{\circ}\text{C}$
- (c)  $40.2^{\circ}\text{C}$
- (d)  $37.5^{\circ}\text{C}$

**Answer:**

- (c)  $40.2^{\circ}\text{C}$

**Solution:**

$$\text{COP } (\beta) = \frac{T_L}{T_H - T_L} = \frac{-12^{\circ}}{T_H + 12^{\circ}}$$

$$5 = \frac{261}{T_H + 285} \Rightarrow T_H = -232.8\text{K} = 40.2^{\circ}\text{C}$$

### Short Answer Questions

**Question 1.**

'An object contains more heat'- is it a right statement? If not why?

**Answer:**

When heated, an object receives heat from the agency. Now object has more internal energy than before. Heat is the energy in transit and which flows from an object at higher temperature to an object lower temperature. Heat is not a quantity. So the statement I would prefer "an object contains more thermal energy".

**Question 2.**

Obtain an ideal gas law from Boyle's and Charles' law.

**Answer:**

Boyle's law: When the gas is kept at constant temperature, the pressure of the

gas is inversely proportional to the volume  $P \propto 1/V$

Charles' law: When the gas is kept at constant pressure, the volume of the gas is directly proportional to absolute temperature  $V \propto T$ .

**Question 3.**

Define one mole.

**Answer:**

One mole of any substance is the amount of that substance which contains Avogadro number ( $N_A$ ) of particles (such as atoms or molecules).

**Question 4.**

Define specific heat capacity and give its unit.

**Answer:**

Specific heat capacity of a substance is defined as the amount of heat energy required into raise the temperature of 1 kg of a substance by 1 Kelvin or  $1^\circ\text{C}$

$$\Delta Q = ms\Delta T$$

Therefore,

$$s = \frac{1}{m} \left( \frac{\Delta Q}{\Delta T} \right)$$

The SI unit for specific heat capacity is  $\text{J kg}^{-1} \text{K}^{-1}$

**Question 5.**

Define molar specific heat capacity.

**Answer:**

Molar specific heat capacity is defined as heat energy required to increase the

$$C = \frac{1}{\mu} \left( \frac{\Delta Q}{\Delta T} \right)$$

temperature of one mole of substance by  $1\text{K}$  or  $1^\circ\text{C}$ .

**Question 6.**

What is a thermal expansion?

**Answer:**

Thermal expansion is the tendency of matter to change in shape, area, and volume due to a change in temperature.



**Question 7.**

Give the expressions for linear, area and volume thermal expansions.

**Answer:**

**Linear Expansion:** In solids, for a small change in temperature  $\Delta T$ , the fractional change in length ( $\Delta L/L_0$ ) is directly proportional to  $\Delta T$ .

$$\frac{\Delta L}{L_0} = \alpha_L \Delta T$$

Therefore, 
$$\alpha_L = \frac{\Delta L}{L_0 \Delta T}$$

**Area Expansion:** For a small change in temperature  $\Delta T$  the fractional change in area ( $\Delta A/A_0$ ) of a substance is directly proportional to  $\Delta T$  and it can be written as

$$\frac{\Delta A}{A_0} = \alpha_A \Delta T$$

Therefore, 
$$\alpha_A = \frac{\Delta A}{A_0 \Delta T}$$

**Volume Expansion:** For a small change in temperature  $\Delta T$  the fractional change in volume ( $\Delta V/V_0$ ) of a substance is directly proportional to  $\Delta T$ .

$$\left( \frac{\Delta V}{V_0} \right) = \alpha_V \Delta T$$

Therefore, 
$$\alpha_V = \frac{\Delta V}{V_0 \Delta T}$$

**Question 8.**

Define latent heat capacity. Give its unit.

**Answer:**

Latent heat capacity of a substance is defined as the amount of heat energy required to change the state of a unit mass of the material.

**Question 9.**

State Stefan-Boltzmann law.

**Answer:**

Stefan Boltzmann law states that, the total amount of heat radiated per second per unit area of a black body is directly proportional to the fourth power of its absolute temperature.

$$E \propto T^4 \text{ (or) } E = \sigma T^4$$

**Question 10.**

What is Wien's law?

**Answer:**

Wien's law states that, the wavelength of maximum intensity of emission of a black body radiation is inversely proportional to the absolute temperature of the black body.

$$\lambda_m \propto \frac{1}{T} \text{ (or) } \lambda_m = \frac{b}{T}$$

**Question 11.**

Define thermal conductivity. Give its unit.

**Answer:**

The quantity of heat transferred through a unit length of a material in a direction normal to Unit surface area due to a unit temperature difference under steady state conditions is known as thermal conductivity of a material.

**Question 12.**

What is a black body?

**Answer:**

A black body is one which neither reflects nor transmits but absorbs whole of the heat radiation incident on it.

The absorptive power of a perfect black body is unity.

**Question 13.**

What is a thermodynamic system? Give examples.

**Answer:**

Thermodynamic system: A thermodynamic system is a finite part of the universe. It is a collection of large number of particles (atoms and molecules) specified by certain parameters called pressure (P), Volume (V) and

Temperature (T). The remaining part of the universe is called surrounding. Both are separated by a boundary.

**Examples:** A thermodynamic system can be liquid, solid, gas and radiation.

**Question 14.**

What are the different types of thermodynamic systems?

**Answer:**

1. Open system can exchange both matter and energy with the environment.
2. Closed system exchange energy but not matter with the environment.
3. Isolated system can exchange neither energy nor matter with the environment.

**Question 15.**

What is meant by 'thermal equilibrium'?

**Answer:**

Two systems are said to be in thermal equilibrium with each other if they are at the same temperature, which will not change with time.

**Question 16.**

What is mean by state variable? Give example.

**Answer:**

In thermodynamics, the state of a thermodynamic system is represented by a set of variables called thermodynamic variables.

**Examples:** Pressure, temperature, volume and internal energy etc.

**Question 17.**

What are intensive and extensive variables? Give examples.

**Answer:**

Extensive variable depends on the size or mass of the system.

**Example :** Volume, total mass, entropy, internal energy, heat capacity etc.

Intensive variables do not depend on the size or mass of the system.

**Example:** Temperature, pressure, specific heat capacity, density etc.

**Question 18.**

What is an equation of state? Give an example.

**Answer:**

Equation of state: The equation which connects the state variables in a specific manner is called equation of state. An ideal gas obeys the equation  $PV = NkT$  at thermodynamic equilibrium.

For example, if we push the piston of a gas container, the volume of the gas will decrease but pressure will increase or if heat is supplied to the gas, its temperature will increase, pressure and volume of the gas may also increase.

**Question 19.**

State Zeroth law of thermodynamics.

**Answer:**

The zeroth law of thermodynamics states that if two systems, A and B, are in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other.

**Question 20.**

Define the internal energy of the system.

**Answer:**

The internal energy of a thermodynamic system is the sum of kinetic and potential energies of all the molecules of the system with respect to the center of mass of the system.

**Question 21.**

Are internal energy and heat energy the same? Explain.

**Answer:**

Internal energy and thermal energy do not mean the same thing, but they are related. Internal energy is the energy stored in a body. It increases when the temperature of the body rises, or when the body changes from solid to liquid

or from liquid to gas.

“Heat is the energy transferred from one body to another as a result of a temperature difference.”

**Question 22.**

Define one calorie.

**Answer:**

One calorie is defined as the amount of heat energy needed to raise the temperature of one gram of water by one degree Celsius at a pressure of one atmosphere.

**Question 23.**

Did joule converted mechanical energy to heat energy? Explain.

**Answer:**

Joule essentially converted mechanical energy to internal energy. In his experiment potential energy is converted to rotational kinetic energy of paddle wheel and this rotational kinetic energy is converted to internal energy of water.

**Question 24.**

State the first law of thermodynamics.

**Answer:**

This law states that ‘Change in internal energy ( $\Delta U$ ) of the system is equal to heat supplied to the system ( $Q$ ) minus the work done by the system ( $W$ ) on the surroundings’.

**Question 25.**

Can we measure the temperature of the object by touching it?

**Answer:**

No, we can't measure the temperature of the object touching it. Because the temperature is the degree of hotness or coolness of a body. Only we can sense the hotness or coolness of the object.

**Question 26.**

Give the sign convention for Q and W.

**Answer:**

System gains heat Q is positive

System loses heat Q is negative

Work done on the system W is negative

Work done by the system W is positive

**Question 27.**

Define the quasi-static process.

**Answer:**

A quasi-static process is an infinitely slow process in which the system changes its variables (P, V, T) so slowly such that it remains in thermal, mechanical and chemical equilibrium with its surroundings throughout.

**Question 28.**

Give the expression for work done by the gas.

**Answer:**

When a gas expands against pressure, it does work on the surroundings. The work done in expansion for volume  $V_1$  to  $V_2$  is given by

$$W = \int_{V_1}^{V_2} P dV$$

If the pressure remains constant during expansion,

Then  $W = P(V_2 - V_1) = P\Delta V$

If the volume remains constant, then  $W = 0$ .

If there is no external pressure, then no work is done. For example, when a gas expands freely in vacuum, no work is done by it.

**Question 29.**

What is PV diagram?

**Answer:**

PV diagram is a graph between pressure P and volume V of the system. The P-V diagram is used to calculate the amount of work done by the gas during expansion or on the gas during compression.

**Question 30.**

Explain why the specific heat capacity at constant pressure is greater than the specific heat capacity at constant volume.

**Answer:**

It implies that to increase the temperature of the gas at constant volume requires less heat than increasing the temperature of the gas at constant pressure. In other words  $s_p$  is always greater than  $s_v$ .

**Question 31.**

Give the equation of state for an isothermal process.

**Answer:**

It is a process in which the temperature remains constant but the pressure and volume of a thermodynamic system will change. The ideal gas equation is  $PV = \mu RT$

**Question 32.**

Give an expression for work done in an isothermal process.

**Answer:**

The work done by the gas,  $W = \int_{V_i}^{V_f} P \cdot dV$

Writing pressure in terms of volume and temperature

$$P = \frac{\mu RT}{V}$$

Substituting this value, we get

$$W = \int_{V_i}^{V_f} \frac{\mu RT}{V} dV = \mu RT \int_{V_i}^{V_f} \frac{dV}{V}$$

By performing the integration in equation, we get

$$W = \mu RT \ln \left( \frac{V_f}{V_i} \right)$$

**Question 33.**

Express the change in internal energy in terms of molar specific heat capacity.

**Answer:**

When the gas is heated at constant volume the temperature increases by  $dT$ . As no work is done by the gas, the heat that flows into the system will increase only the internal energy. Let the change in internal energy be  $dU$ .  $dU = \mu \omega dT$

**Question 34.**

Apply first law for

- (a) an isothermal
- (b) adiabatic
- (c) isobaric processes.

**Answer:**

(a) For an isothermal process since temperature is constant, the internal energy is also constant. This implies that  $dU$  or  $\Delta U = 0$ .

For an isothermal process, the first law of thermodynamics can be written as follows,

$$Q = W$$

(b) This is a process in which no heat flows into or out of the system ( $Q = 0$ ). But the gas can expand by spending its internal energy or gas can be compressed through some external work. So the pressure, volume and temperature of the system may change in an adiabatic process. For an adiabatic process, the first law becomes  $\Delta U = W$ .

(c) The first law of thermodynamics for isobaric process is given by  $\Delta U = Q - P\Delta V$

$$W = P\Delta V, \Delta U = Q - \mu RT_f \left[ 1 - \frac{T_i}{T_f} \right]$$

**Question 35.**

Give the equation of state for an adiabatic process.

**Answer:**

The equation of state for an adiabatic process is given by

$$PV^\gamma = \text{constant}$$

Here  $\gamma$  is called adiabatic exponent

( $\gamma = C_p/C_v$ ) which depends on the nature of the gas



**Question 36.**

Give an equation state for an isochoric process.

**Answer:**

The equation of state for an isochoric process is given by

$$P = \left( \frac{\mu R}{V} \right) T$$

where  $\left( \frac{\mu R}{V} \right) = \text{Constant}$

We can infer that the pressure is directly proportional to temperature. This implies that the P-T graph for an isochoric process is a straight line passing through origin.

**Question 37.**

If the piston of a container is pushed fast inward. Will the ideal gas equation be valid in the intermediate stage? If not, why?

**Answer:**

When the piston is compressed so quickly that there is no time to exchange heat to the surrounding, the temperature of the gas increases rapidly. In this intermediate stage the ideal gas equation be not valid. Because this equation can be relates the pressure, volume and temperature of thermodynamic system at equilibrium.

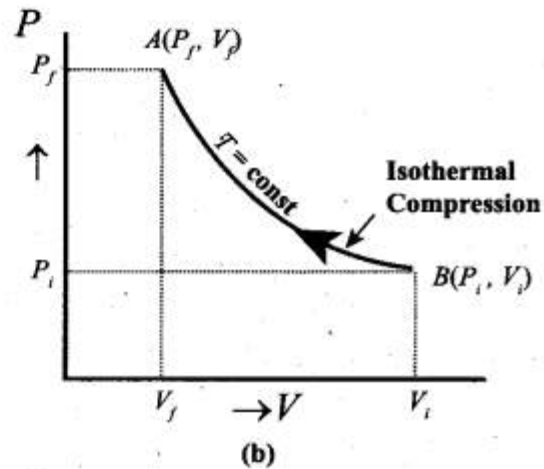
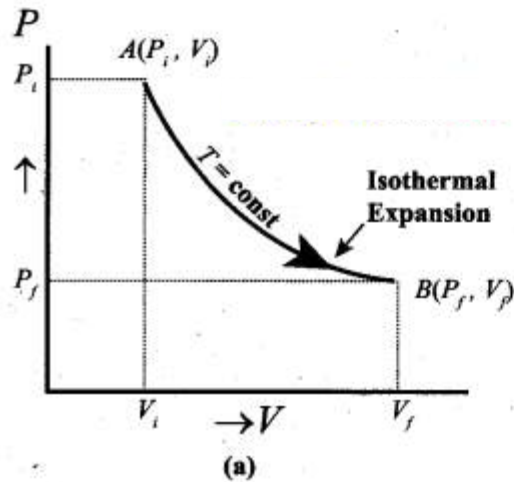
**Question 38.**

Draw the PV diagram for:

- (a) Isothermal process
- (b) Adiabatic process
- (c) isobaric process
- (d) Isochoric process
- (a) Isothermal process:

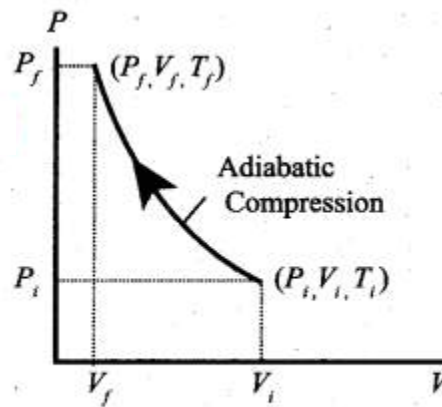
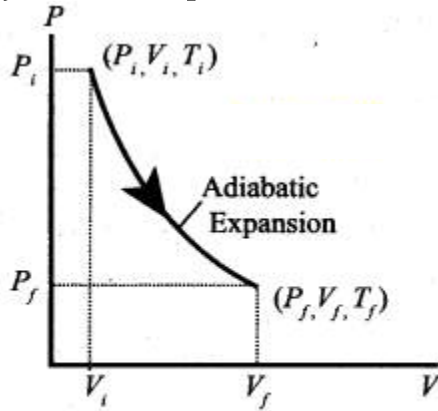
**Answer:**

- (a) Isothermal process



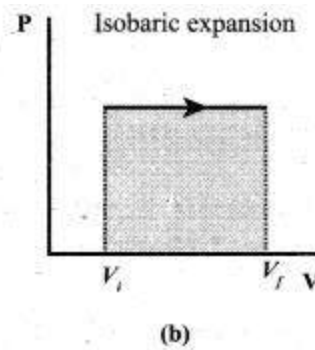
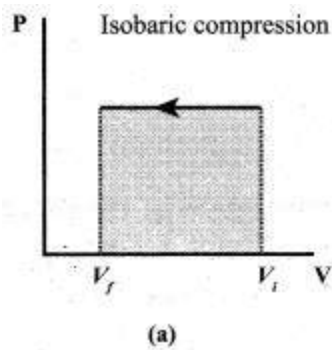
(a) Quasi-static isothermal expansion (b) Quasi-static isothermal compression

(b) Adiabatic process:



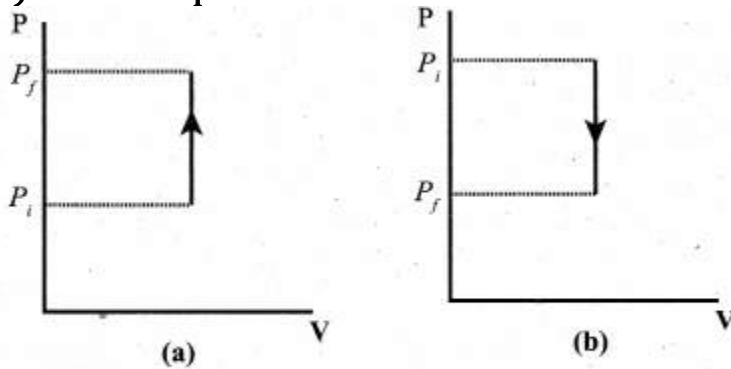
PV diagram for adiabatic expansion and adiabatic compressor

(c) isobaric process:



PV diagram for an isobaric process

(d) Isochoric process:



Isochoric process with  
(a) increased pressure and (b) decreased pressure

**Question 39.**

What is a cyclic process?

**Answer:**

This is a thermodynamic process in which the thermodynamic system returns to its initial state after undergoing a series of changes. Since the system comes back to the initial state, the change in the internal energy is zero. In cyclic process, heat can flow in to system and heat flow out of the system. From the first law of thermodynamics, the net heat transferred to the system is equal to work done by the gas.

$$Q_{\text{net}} = Q_{\text{in}} - Q_{\text{out}} = W \text{ (for a Cyclic Process)}$$

**Question 40.**

What is meant by a reversible and irreversible processes?

**Answer:**

**Reversible processes:** A thermodynamic process can be considered reversible only if it is possible to retrace the path in the opposite direction in such a way that the system and surroundings pass through the same states as in the initial, direct process.

**Irreversible processes:** All natural processes are irreversible. Irreversible process cannot be plotted in a PV diagram, because these processes cannot have unique values of pressure, temperature at every stage of the process.

**Question 41.**

State Clausius form of the second law of thermodynamics.

**Answer:**

“Heat always flows from hotter object to colder object spontaneously”. This is known as the Clausius form of second law of thermodynamics.

**Question 42.**

State Kelvin-Planck statement of second law of thermodynamics.

**Answer:**

**Kelvin-Planck statement:** It is impossible to construct a heat engine that operates in a cycle, whose sole effect is to convert the heat completely into work. This implies that no heat engine in the universe can have 100% efficiency.

**Question 43.**

Define heat engine.

**Answer:**

Heat engine is a device which takes heat as input and converts this heat into work by undergoing a cyclic process.

**Question 44.**

What are processes involved in a Carnot engine?

**Answer:**

There are four processes involved in a Carnot engine:

1. source
2. sink
3. insulating stand
4. working substance

**Question 45.**

Can the given heat energy be completely converted to work in a cyclic process? If not, when can the heat be completely converted to work?

**Answer:**

According to first law of thermodynamics work can be completely converted into heat. Since the system comes back to the initial stage, the change in the internal energy is zero. In cyclic process, heat can flow in to system and heat flow out of the system. The net heat transferred to the system is equal to work done by the gas.

$$Q_{\text{net}} = Q_{\text{in}} = Q_{\text{out}} = W \text{ (for 3 Cyclic Process)}$$

**Question 46.**

State the second law of thermodynamics in terms of entropy.

**Answer:**

“For all the processes that occur in nature (irreversible process), the entropy always increases. For reversible process entropy will not change”. Entropy determines the direction in which natural process should occur.

**Question 47.**

Why does heat flow from a hot object to a cold object?

**Answer:**

Because entropy increases when heat flows from hot object to cold object. If heat were to flow from a cold to a hot object, entropy will decrease leading to violation of second law thermodynamics.

**Question 48.**

Define the coefficient of performance.

**Answer:**

It is defined as the ratio of heat extracted from the cold body (sink) to the external work done by the compressor  $W$ .

## Long Answer Questions

### Question 1.

Explain the meaning of heat and work with suitable examples.

#### Answer:

**Meaning of heat:** When an object at higher temperature is placed in contact with another object at lower temperature, there will be a spontaneous flow of energy from the object at higher temperature to the one at lower temperature. This energy is called heat. This process of energy transfer from higher temperature object to lower temperature object is called heating. Due to flow of heat sometimes the temperature of the body will increase or sometimes it may not increase.



**Meaning of work:** When you rub your hands against each other the temperature of the hands increases. You have done some work on your hands by rubbing. The temperature of the hands increases due to this work. Now if you place your hands on the cheek, the temperature of the cheek increases. This is because the hands are at higher temperature than the cheek.

In the above example, the temperature of hands is increased due to work and temperature of the cheek is increased due to heat transfer from the hands to the chin. It is shown in the Figure. By doing work on the system, the temperature in the system will increase and sometimes may not. Like heat, work is also not a quantity and through the work energy is transferred to the system. So we cannot use the word 'the object contains more work' or 'less work'.

Either the system can transfer energy to the surrounding by doing work on surrounding or the surrounding may transfer energy to the system by doing work on the system. For the transfer of energy from one body to another body through the process of work, they need not be at different temperatures.

## Question 2.

Discuss the ideal gas laws.

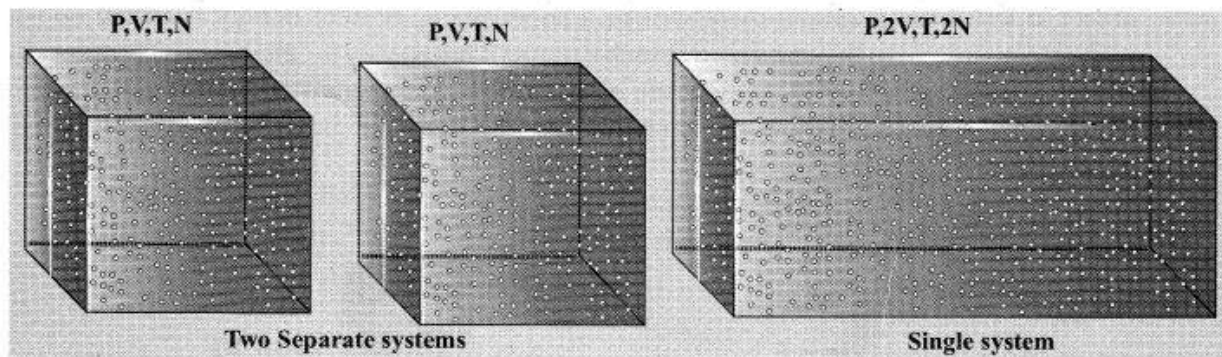
### Answer:

Boyle's law: For a given gas at low pressure (density) kept in a container of volume  $V$ , experiments revealed the following information.

When the gas is kept at constant temperature, the pressure of the gas is inversely proportional to the volume  $P \propto 1/V$  Charles' law: When the gas is kept at constant pressure, the volume of the gas is directly proportional to absolute temperature  $V \propto T$ .

By combining these two equations we have  $PV = CT$ . Here  $C$  is a positive constant.

We can infer that  $C$  is proportional to the number of particles in the gas container by considering the following argument. If we take two containers of same type of gas with same volume  $V$ , same pressure  $P$  and same temperature  $T$ , then the gas in each container obeys the above equation.  $PV = CT$ . If the two containers of gas is considered as a single system, then the pressure and temperature of this combined system will be same but volume will be twice and number of particles will also be double.



For this combined system,  $V$  becomes  $2V$ , so  $C$  should also double to match with the ideal gas equation  $\frac{P(2V)}{T} = 2C$ . It implies that  $C$  must depend on the number of particles in the gas and also should have the dimension of  $\left[\frac{PV}{T}\right] = \text{JK}^{-1}$ . So we can write the constant  $C$  as  $k$  times the number of particles  $N$ .

Here  $k$  is the Boltzmann constant ( $1.381 \times 10^{-23} \text{ JK}^{-1}$ ) and it is found to be a

universal constant. So the ideal gas law can be stated as follows

$$PV = NkT \dots(1)$$

The equation (1) can also be expressed in terms of mole.

Suppose if a gas contains  $\mu$  mole of particles then the total number of particles can be written as

$$N = \mu N_A \dots\dots (2)$$

where  $N_A$  is Avogadro number ( $6.023 \times 10^{23} \text{ mol}^{-1}$ )

Substituting for  $N$  from equation (2), the equation (1) becomes  $PV = \mu N_A kT$ .

Here  $N_A k = R$  called universal gas constant and its value is  $8.314 \text{ J /mol. K}$

So the ideal gas law can be written for  $\mu$  mole of gas as

$$PV = \mu RT \dots(3)$$

This is called the equation of state for an ideal gas. It relates the pressure, volume and temperature of thermodynamic system at equilibrium.

### Question 3.

Explain in detail the thermal expansion.

#### Answer:

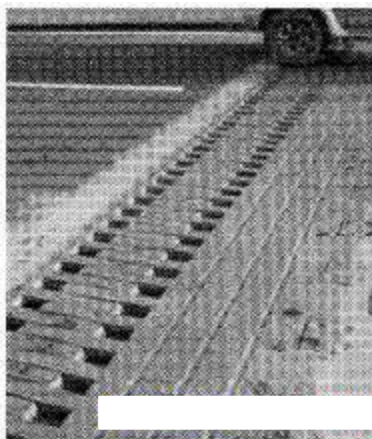
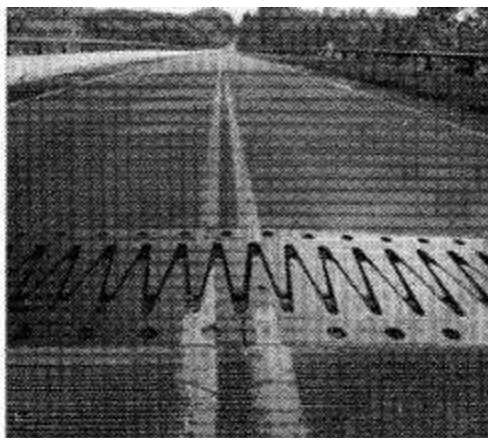
Thermal expansion is the tendency of matter to change in shape, area, and volume due to a change in temperature.

All three states of matter (solid, liquid and gas) expand when heated. When a solid is heated, its atoms vibrate with higher amplitude about their fixed points.

The relative change in the size of solids is small. Railway tracks are given small gaps so that in the summer, the tracks expand and do not buckle.

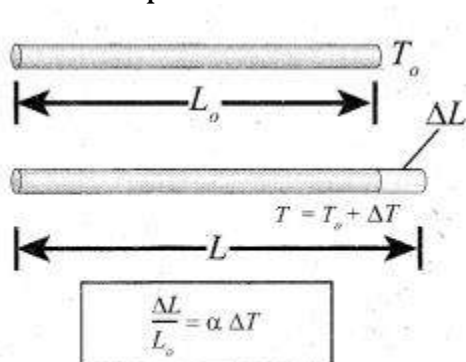
Railroad tracks and bridges have expansion joints to allow them to expand and contract freely with temperature changes.



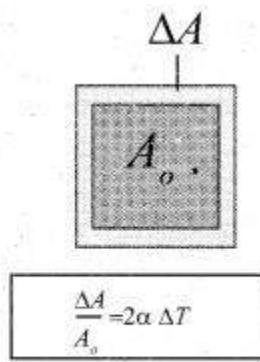


Liquids, have less intermolecular forces than solids and hence they expand more than solids. This is the principle behind the mercury thermometers. In the case of gas molecules, the intermolecular forces are almost negligible and hence they expand much more than solids. For example in hot air balloons when gas particles get heated, they expand and take up more space. The increase in dimension of a body due to the increase in its temperature is called thermal expansion.

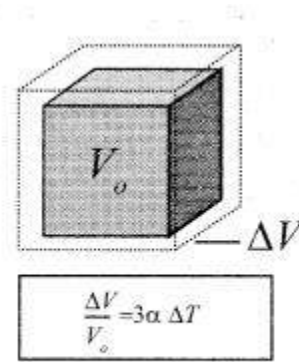
The expansion in length is called linear expansion. Similarly the expansion in area is termed as area expansion and the expansion in volume is termed as volume expansion.



(a) Linear expansion



(b) Area expansion



(c) Volume expansion

**Linear Expansion:** In solids, for a small change in temperature  $\Delta T$ , the fractional change in length  $\left(\frac{\Delta L}{L_0}\right)$  is directly proportional to  $\Delta T$

$$\frac{\Delta L}{L_0} = \alpha_L \Delta T$$

Therefore,

$$\alpha_L = \frac{\Delta L}{L_0 \Delta T}$$

Where,  $\alpha_L$  = coefficient of linear expansion,  
 $\Delta L$  = Change in length,  
 $L$  = Original length,  
 $\Delta T$  = Change in temperature.

**Area Expansion:** For a small change in temperature  $\Delta T$  the fractional change in area  $\left(\frac{\Delta A}{A_0}\right)$  of a substance is directly proportional to  $\Delta T$  and it can be written as

$$\frac{\Delta A}{A_0} = \alpha_A \Delta T$$

Therefore, 
$$\alpha_A = \frac{\Delta A}{A_0 \Delta T}$$

Where,  $\alpha_A$  = coefficient of area expansion.  
 $\Delta A$  = Change in area,  
 $A$  = Original area,  
 $\Delta T$  = Change in temperature.

**Volume Expansion:** For a small change in temperature  $\Delta T$  the fractional change in volume  $\left(\frac{\Delta V}{V_0}\right)$  of a substance is directly proportional to  $\Delta T$ .

$$\left(\frac{\Delta V}{V_0}\right) = \alpha_V \Delta T$$

Therefore, 
$$\alpha_V = \frac{\Delta V}{V_0 \Delta T}$$

Where,  $\alpha_V$  = coefficient of volume expansion,  
 $\Delta V$  = Change in volume,  
 $V$  = Original volume,  
 $\Delta T$  = Change in temperature,

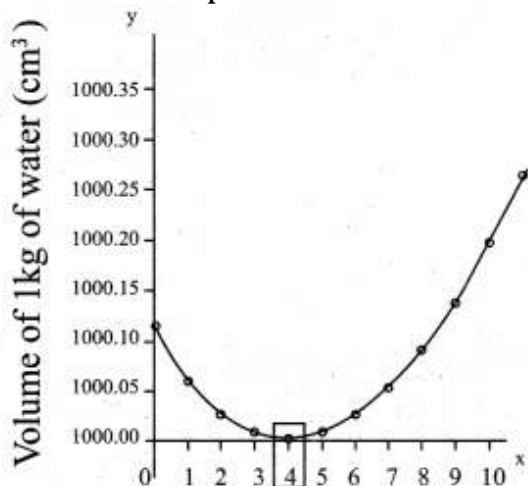
Unit of coefficient of linear, area and volumetric expansion of solids is  $^{\circ}\text{C}^{-1}$  or  $\text{K}^{-1}$

#### Question 4.

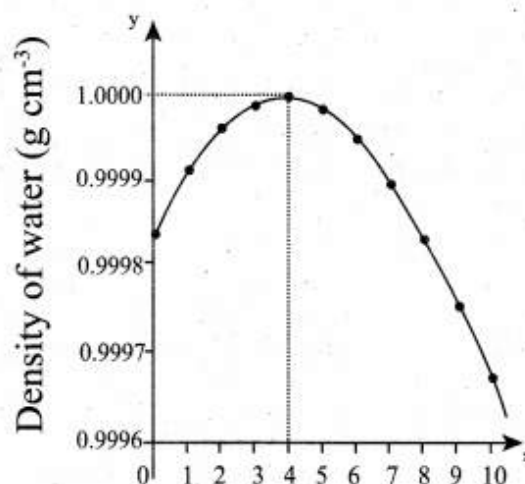
Describe the anomalous expansion of water: How is it helpful in our lives?

**Answer:**

**Anomalous expansion of water :** Liquids expand on heating and contract on cooling at moderate temperatures. But water exhibits an anomalous behavior. It contracts on heating between  $0^{\circ}\text{C}$  and  $4^{\circ}\text{C}$ . The volume of the given amount of water decreases as it is cooled from room temperature, until it reach  $4^{\circ}\text{C}$ . Below  $4^{\circ}\text{C}$  the volume increases and so the density decreases. This means that the water has a maximum density at  $4^{\circ}\text{C}$ . This behavior of water is called anomalous expansion of water.



(a) Temperature  $^{\circ}\text{C}$



(b) Temperature  $^{\circ}\text{C}$

#### Anomalous Expansion of water

In cold countries during the winter season, the surface of the lakes will be at lower temperature than the bottom as shown in the Figure. Since the solid water (ice) has lower density than its liquid form, below  $4^{\circ}\text{C}$ , the frozen water will be on the top surface above the liquid water (ice floats). This is due to the anomalous expansion of water. As the water in lakes and ponds freeze only at the top the species living in the lakes will be safe at the bottom.

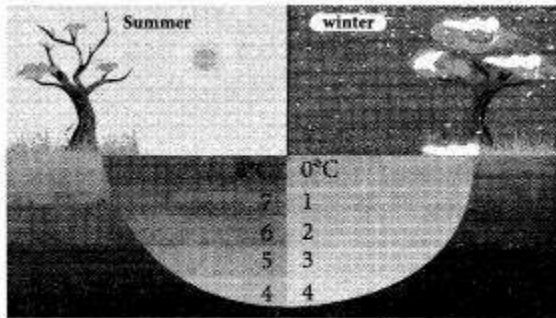
#### Question 5.

Explain Calorimetry and derive an expression for final temperature when two thermodynamic systems are mixed.

**Answer:**

**Calorimetry:** Calorimetry means the measurement of the amount of heat released or absorbed by thermodynamic system during the heating process. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by

the cold body. No heat is allowed to escape to the surroundings. It can be Anomalous expansion of water in lakes mathematically expressed as

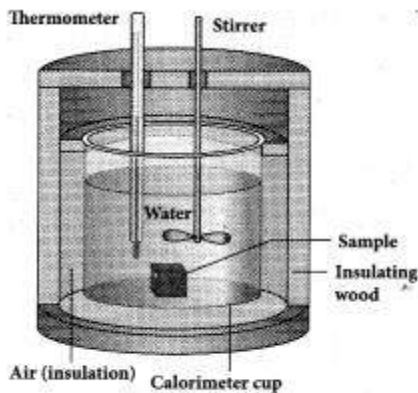


Anomalous expansion of water in lakes

$$Q_{\text{gain}} = -Q_{\text{lost}}$$

$$Q_{\text{gain}} + Q_{\text{lost}} = 0$$

Heat gained or lost is measured with a calorimeter. Usually the calorimeter is an insulated container of water. A sample is heated at high temperature ( $T_1$ ) and immersed into water at room temperature ( $T_2$ ) in the calorimeter. After some time both sample and water reach a final equilibrium temperature  $T_f$ . Since the calorimeter is insulated, heat given by the water.



Calorimeter with sample of block

$$Q_{\text{gain}} = -Q_{\text{lost}}$$

Note the sign convention. The heat lost is denoted by negative sign and heat gained is denoted as positive.

From the definition of specific heat capacity

$$Q_{\text{gain}} = m_2 s_2 (T_f - T_2)$$

$$Q_{\text{lost}} = m_1 s_1 (T_f - T_1)$$

Here  $S_1$  and  $s_2$  specific heat capacity of hot sample and water respectively.

So we can write  $m_2 s_2 (T_f - T_2) = -m_1 s_1 (T_f - T_1)$

$$m_2 s_2 T_f - m_2 s_2 T_2 = -m_1 s_1 T_f + m_1 s_1 T_1$$

$$m_2 s_2 T_f + m_1 s_1 T_f = m_2 s_2 T_2 + m_1 s_1 T_1$$

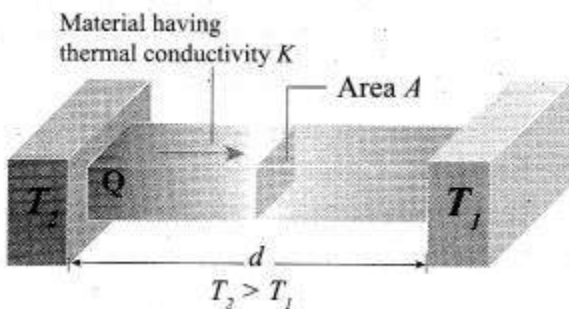
The final temperature  $T_f = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$

### Question 6.

Discuss various modes of heat transfer.

#### Answer:

There are three modes of heat transfer: Conduction, Convection and Radiation.



**Conduction:** Conduction is the process of direct transfer of heat through matter due to temperature difference. When two objects are in direct contact with one another, heat will be transferred from the hotter object to the colder one. The objects which allow heat to travel easily through them are called conductors.

**Convection:** Convection is the process in which heat transfer is by actual movement of molecules in fluids such as liquids and gases. In convection, molecules move freely from one place to another.

Boiling water in a cooking pot is an example of convection. Water at the bottom of the pot receives more heat. Due to heating, the water expands and the density of water decreases at the bottom. Due to this decrease in density, molecules rise to the top. At the same time the molecules at the top receive less heat and become denser and come to the bottom of the pot. This process goes on continuously. The back and forth movement of molecules is called convection current.

To keep the room warm, we use room heater. The air molecules near the heater will heat up and expand. As they expand, the density of air molecules will decrease and rise up while the higher density cold air will come down. This circulation of air molecules is called convection current.

**Radiation:** When we keep our hands near the hot stove we feel the heat even though our hands are not touching the hot stove. Here heat transferred from the hot stove to our hands is in the form of radiation. We receive energy from the sun in the form of radiations. These radiations travel through vacuum and reach the Earth. It is the peculiar character of radiation which requires no medium to transfer energy from one object to another. The conduction or convection requires medium to transfer the heat.

Radiation is a form of energy transfer from one body to another by electromagnetic waves.

**Example:**

1. Solar energy from the Sun.
2. Radiation from hot stove.

**Question 7.**

Explain in detail Newton's law of cooling.

**Answer:**

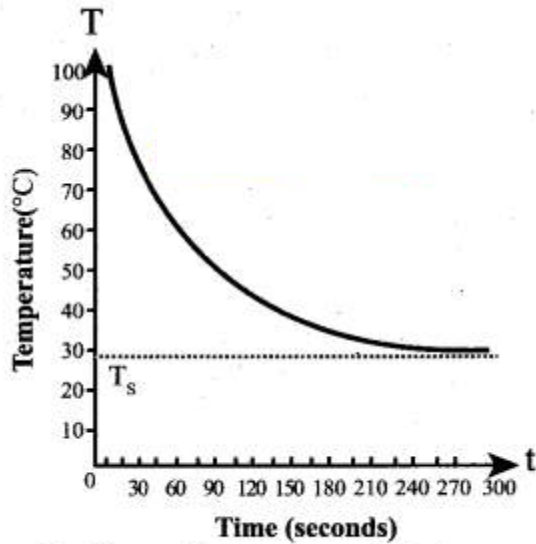
Newton's law of cooling: Newton's law of cooling states that the rate of loss of heat of a body is directly proportional to the difference in the temperature between that body and its surroundings.

$$\frac{dQ}{dt} \propto (T - T_s) \quad \dots(1)$$

The negative sign indicates that the quantity of heat lost by liquid goes on decreasing with time. Where,

T = Temperature of the object

T<sub>s</sub> = Temperature of the surrounding



Cooling of hot water with time

From the graph in figure it is clear that the rate of cooling is high initially and decreases with falling temperature.

Let us consider an object of mass  $m$  and specific heat capacity  $s$  at temperature  $T$ . Let  $T_s$  be the temperature of the surroundings. If the temperature falls by a small amount  $dT$  in time  $dt$ , then the amount of heat lost is,

$$dQ = msdT \quad \dots(2)$$

Dividing both sides of equation (2) by  $dt$

$$\frac{dQ}{dt} = \frac{msdT}{dt} \quad \dots(3)$$

From Newton's law of cooling  $\frac{dQ}{dt} \propto -(T - T_s)$

$$\frac{dQ}{dt} = -a(T - T_s) \quad \dots(4)$$

Where  $a$  is some positive constant. From equation (3) and (4)

$$-a(T - T_s) = ms \frac{dT}{dt}$$

$$\frac{dT}{T - T_s} = -\frac{a}{ms} dt \quad \dots(5)$$

Integrating equation (5) on both sides,



$$\int \frac{dT}{T - T_s} = - \int \frac{a}{ms} dt$$

$$\ln(T - T_s) = - \frac{a}{ms} t + b_1$$

Where  $b_1$ , is the constant of integration. Taking exponential both sides, we get

$$T = T_s + b_2 e^{-\frac{a}{ms} t} \quad \dots(6)$$

Here  $b_2 = e^{b_1} = \text{Constant}$

### Question 8.

Explain Wien's law and why our eyes are sensitive only to visible rays?

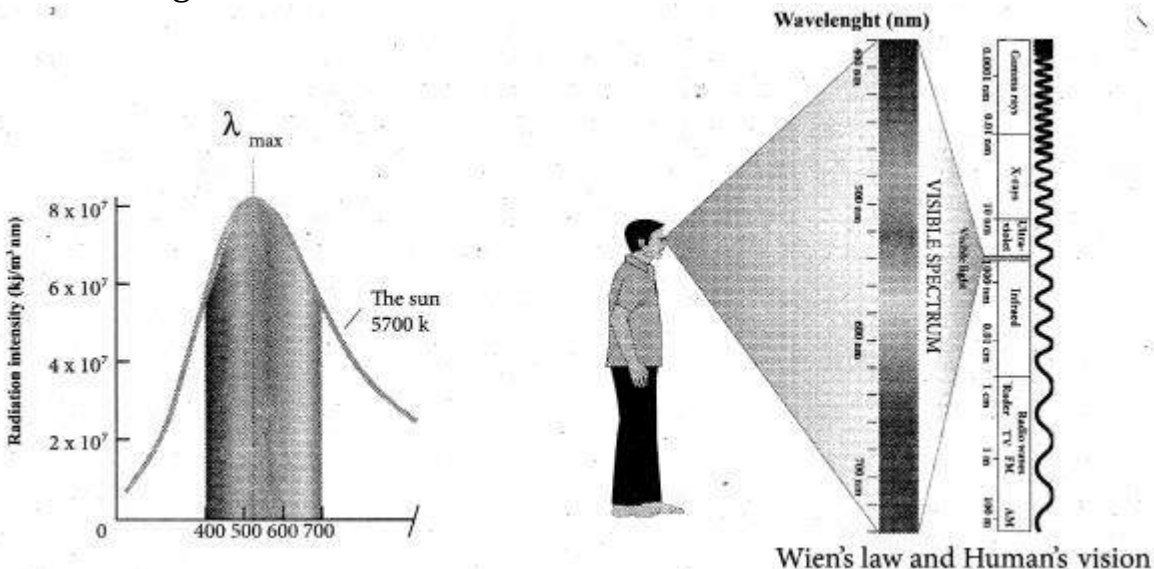
**Answer:**

#### Wien's law and Vision:

The Sun is approximately taken as a black body. Since any object above 0 K will emit radiation, Sun also emits radiation. Its surface temperature is about 5700 K. By substituting this value in the equation of Wien's law.

$$\lambda_m = \frac{b}{T} = \frac{2.898 \times 10^{-8}}{5700} \approx 508 \text{ nm}$$

It is the wavelength at which maximum intensity is 508 nm. Since the Sun's temperature is around 5700 K, the spectrum of radiations emitted by Sun lie between 400 nm to 700 nm which is the visible part, of the spectrum. It is shown in Figure.





The humans evolved under the Sun by receiving its radiations. The human eye is sensitive only in the visible not in infrared or X-ray ranges in the spectrum. Suppose if humans had evolved in a planet near the star Sirius (9940K), then they would have had the ability to see the Ultraviolet rays!

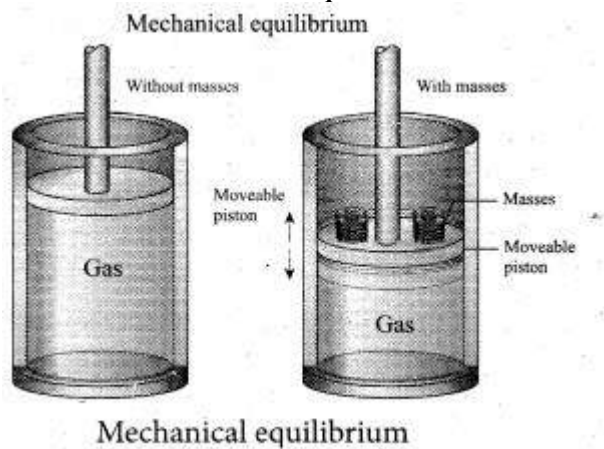
### Question 9.

Discuss the:

- (a) thermal equilibrium
- (b) mechanical equilibrium
- (c) chemical equilibrium
- (d) thermodynamic equilibrium.

**Answer:**

**(a) Thermal equilibrium:** When a hot cup of coffee is kept in the room, heat flows from coffee to the surrounding air. After sometime the coffee reaches the same temperature as the surrounding air and there will be no heat flow from coffee to air or air to coffee. It implies that the coffee and surrounding air are in thermal equilibrium with each other. Two systems are said to be in thermal equilibrium with each other if they are at the same temperature, which Mechanical equilibrium will not change with time.



**(b) Mechanical equilibrium:** Consider a gas container with piston. When some mass is placed on the piston, it will move downward due to downward gravitational force and after certain humps and jumps the piston will come to rest at a new position. When the downward gravitational force given by the piston is balanced by the upward force exerted by the gas, the system is said to be in mechanical equilibrium. A system is said to be in mechanical equilibrium if no unbalanced force acts on the thermodynamic system or on the surrounding by thermodynamic system.

(c) **Chemical equilibrium:** If there is no net chemical reaction between two thermodynamic systems in contact with each other then it is said to be in chemical equilibrium.

(d) **Thermodynamic equilibrium:** If two systems are set to be in thermodynamic equilibrium, then the systems are at thermal, mechanical and chemical equilibrium with each other. In a state of thermodynamic equilibrium the macroscopic variables such as pressure, volume and temperature will have fixed values and do not change with time.

### Question 10.

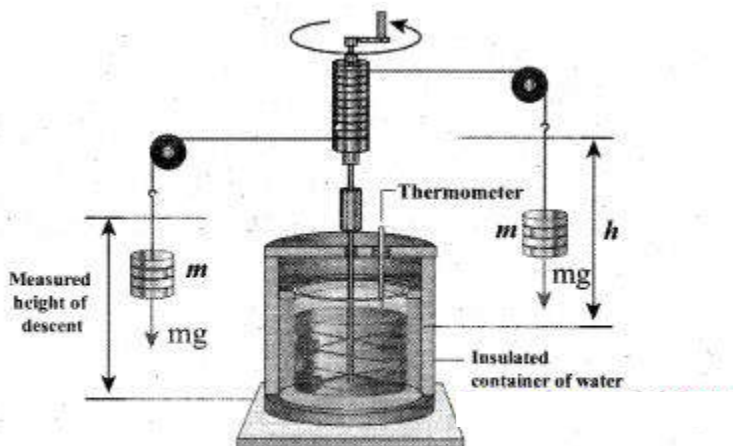
Explain Joule's Experiment of the mechanical equivalent of heat.

#### Answer:

Joule's mechanical equivalent of heat: The temperature of an object can be increased by heating it or by doing some work on it.

In the eighteenth century, James Prescott Joule showed that mechanical energy can be converted into internal energy and vice versa.

In his experiment, two masses were attached with a rope and a paddle wheel as shown in Figure. When these masses fall through a distance  $h$  due to gravity, both the masses lose potential energy equal to  $2 mgh$ . When the masses fall, the paddle wheel turns. Due to the turning of wheel inside water, frictional force comes in between the water and the paddle wheel.



Joule's experiment for determining the mechanical equivalent of heat energy.

This causes a rise in temperature of the water. This implies that gravitational

potential energy is converted to internal energy of water. The temperature of water increases due to the work done by the masses. In fact, Joule was able to show that the mechanical work has the same effect as giving heat. He found that to raise 1 g of an object by 1°C, 4.186 J of energy is required. In earlier days the heat was measured in calorie.

$$1 \text{ cal} = 4.186 \text{ J}$$

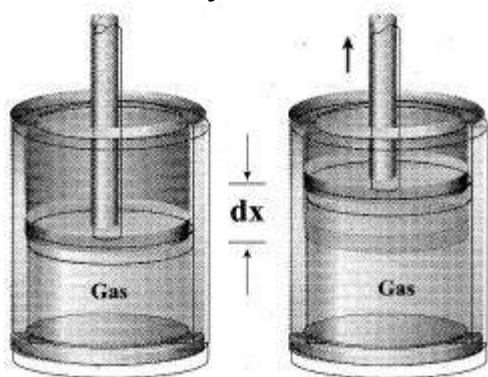
This is called Joule's mechanical equivalent of heat.

### Question 11.

Derive the expression for the work done in a volume change in a thermodynamic system.

#### Answer:

Work done in volume changes: Consider a gas contained in the cylinder fitted with a movable piston. Suppose the gas is expanded quasi-statically by pushing the piston by a small distance  $dx$ . Since the expansion occurs quasi-statically the pressure, temperature and internal energy will have unique values at every instant.



Work done by the gas

The small work done by the gas on the piston

$$dW = Fdx \dots(1)$$

The force exerted by the gas on the piston  $F = PA$ .

Here  $A$  is area of the piston and  $P$  is pressure exerted by the gas on the piston.

Equation (1) can be rewritten as Work done by the gas

$$dW = PA dx \dots(2)$$

But  $A dx = dV =$  change in volume during this expansion process.

So the small work done by the gas during the expansion is given by

$$dW = PdV \dots(3)$$

$dV$  is positive since the volume is increased. Here,  $dW$  is positive.

In general the work done by the gas by increasing the volume from  $V_i$  to  $V_f$  is given by

$$W = \int_{V_i}^{V_f} P dV \quad \dots(4)$$

Suppose if the work is done on the system, then  $V_i > V_f$ . Then,  $W$  is negative.

Note here the pressure  $P$  is inside the integral in equation (4). It implies that while the system is doing work, the pressure need not be constant. To evaluate the integration we need to first express the pressure as a function of volume and temperature using the equation of state.

### Question 12.

Derive Mayer's relation for an ideal gas.

#### Answer:

Mayer's relation: Consider  $\mu$  mole of an ideal gas in a container with volume  $V$ , pressure  $P$  and temperature  $T$ .

When the gas is heated at constant volume the temperature increases by  $dT$ .

As no work is done by the gas, the heat that flows into the system will increase only the internal energy. Let the change in internal energy be  $dU$ .

If  $C_v$  is the molar specific heat capacity at constant volume, from equation.

$$C_v = \frac{1}{\mu} \frac{dU}{dT} \quad \dots(1)$$

$$dU = \mu C_v dT \quad \dots(2)$$

Suppose the gas is heated at constant pressure so that the temperature increases by  $dT$ . If ' $Q$ ' is the heat supplied in this process and ' $dV$ ' the change in volume of the gas.

$$Q = \mu C_p dT \quad \dots(3)$$

If  $W$  is the workdone by the gas in this process, then

$$W = P dV \quad \dots(4)$$

But from the first law of thermodynamics,

$$Q = dU + W \quad \dots (5)$$

Substituting equations (2), (3) and (4) in (5), we get,

$$\mu C_p dT = \mu C_v dT + P dV$$

For mole of ideal gas, the equation of state is given by

$$PV = \mu RT \Rightarrow P dV + V dP = \mu R dT$$

Since the pressure is constant,  $dP = 0$

$$\therefore C_p dT = C_v dT + R dT$$

$$\therefore C_p = C_v + R \text{ (or) } C_p - C_v = R \quad \dots(6)$$

This relation is called Mayer's relation. It implies that the molar specific heat capacity of an ideal gas at constant pressure is greater than molar specific heat capacity at constant volume. The relation shows that specific heat at constant pressure ( $s_p$ ) is always greater than specific heat at constant volume ( $s_v$ ).

### Question 13.

Explain in detail the isothermal process.

**Answer:**

**Isothermal process:** It is a process in which the temperature remains constant but the pressure and volume of a thermodynamic system will change. The ideal gas equation is

$$PV = \mu RT$$

Here,  $T$  is constant for this process. So the equation of state for isothermal process is given by

$$PV = \text{Constant} \quad \dots(1)$$

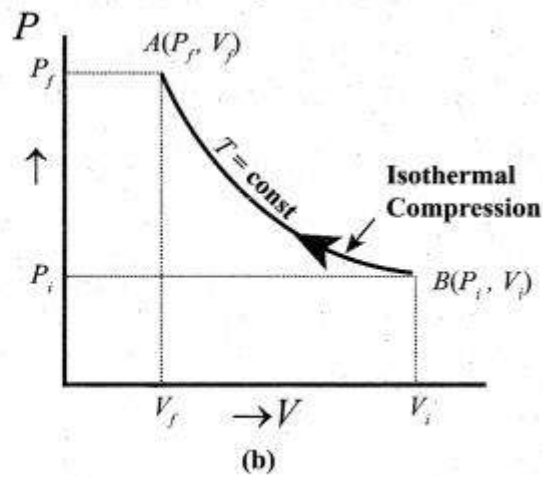
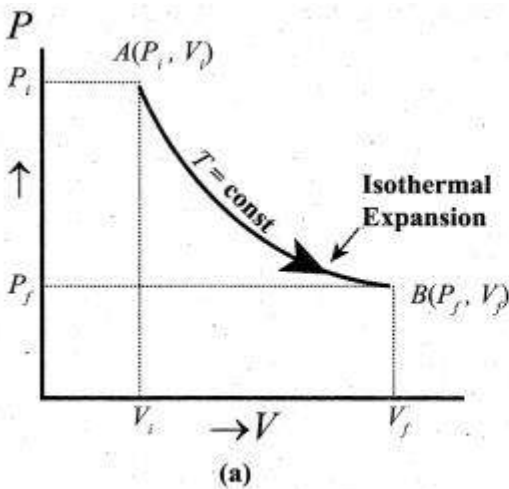
This implies that if the gas goes from one equilibrium state ( $P_1, V_1$ ) to another equilibrium state ( $P_2, V_2$ ) the following relation holds for this process

$$P_1 V_1 = P_2 V_2 \quad \dots(2)$$

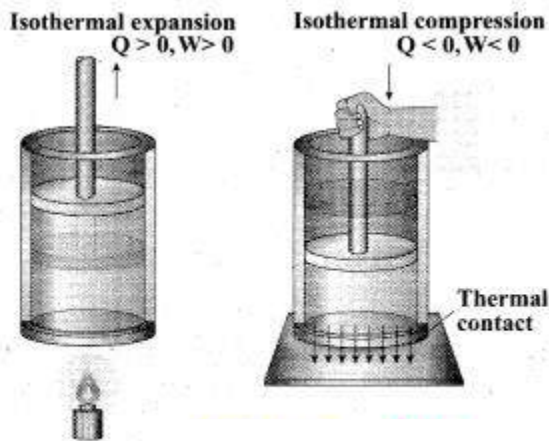
Since  $PV = \text{constant}$ ,  $P$  is inversely proportional to  $v \left( P \propto \frac{1}{V} \right)$ .

This implies that  $PV$  graph is a hyperbola. The pressure-volume graph for constant temperature is also called isotherm. We know that for an ideal gas the internal energy is a function of temperature only. For an isothermal process since temperature is constant, the internal energy is also constant. This implies that  $dU$  or  $\Delta U = 0$ .

For an isothermal process, the first law of thermodynamics can be written as,  $Q = W \quad \dots(3)$



(a) Quasi-static isothermal expansion (b) Quasi-static isothermal compression



Isothermal expansion and isothermal compression

From equation (3), we infer that the heat supplied to a gas is used to do only external work. It is a common misconception that when there is flow of heat to the system, the temperature will increase. For isothermal process this is not true. The isothermal compression takes place when the piston of the cylinder is pushed. This will increase the internal energy which will flow out of the system through thermal contact.

#### Question 14.

Derive the work done in an isothermal process.

#### Answer:

Work done in an isothermal process: Consider an ideal gas which is allowed to expand quasi-statically at constant temperature from initial state  $(P_i, V_i)$  to the final state  $(P_f, V_f)$ . We can calculate the work done by the gas during this

process. The work done by the gas,

$$W = \int_{V_i}^{V_f} P dV \quad \dots(1)$$

As the process occurs quasi-statically, at every stage the gas is at equilibrium with the surroundings. Since it is in equilibrium at every stage the ideal gas law is valid. Writing pressure in terms of volume and temperature,

$$P = \frac{\mu RT}{V} \quad \dots(2)$$

Substituting equation (2) in (1) we get

$$W = \int_{V_i}^{V_f} \frac{\mu RT}{V} dV$$
$$W = \mu RT \int_{V_i}^{V_f} \frac{dV}{V} \quad \dots(3)$$

In equation (3), we take  $\mu RT$  out of the integral, since it is constant throughout the isothermal process.

By performing the integration in equation (3), we get

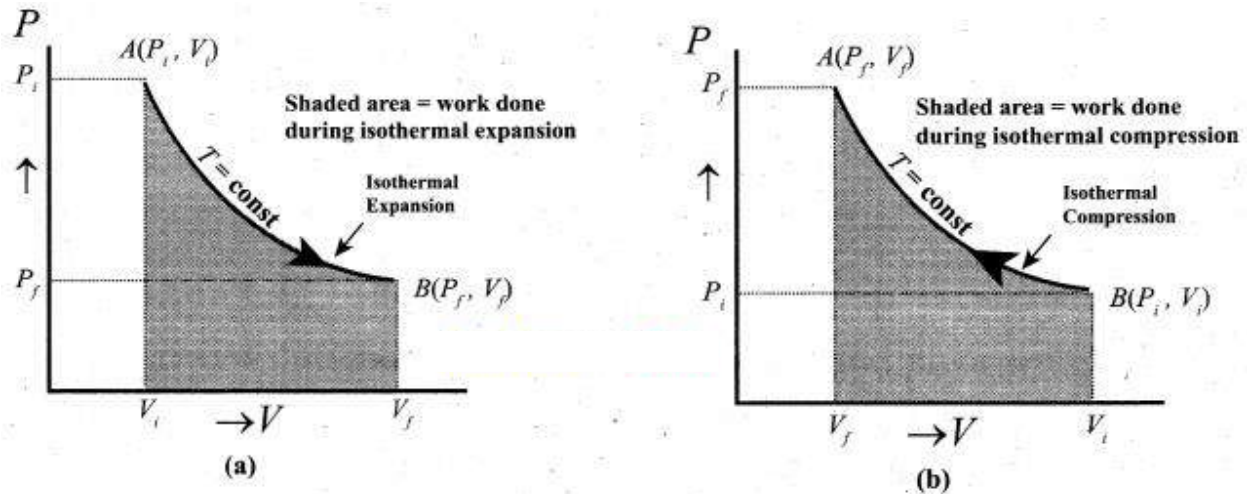
$$W = \mu RT \ln \left( \frac{V_f}{V_i} \right) \quad \dots(4)$$

Since we have an isothermal expansion,  $\frac{V_f}{V_i} > 1$ , so  $\ln \left( \frac{V_f}{V_i} \right) > 0$ . As a result the work done by the gas during an isothermal expansion is positive.

The above result in equation (4) is true for isothermal compression also. But in an isothermal compression  $\frac{V_f}{V_i} < 1$ , so  $\ln \left( \frac{V_f}{V_i} \right) < 0$ .

As a result the work done on the gas in an isothermal compression is negative. In the PV diagram the work done during the isothermal expansion is equal to the area under the graph.





Work done in an isothermal process.

Similarly for an isothermal compression, the area under the PV graph is equal to the work done on the gas which turns out to be the area with a negative sign.

### Question 15.

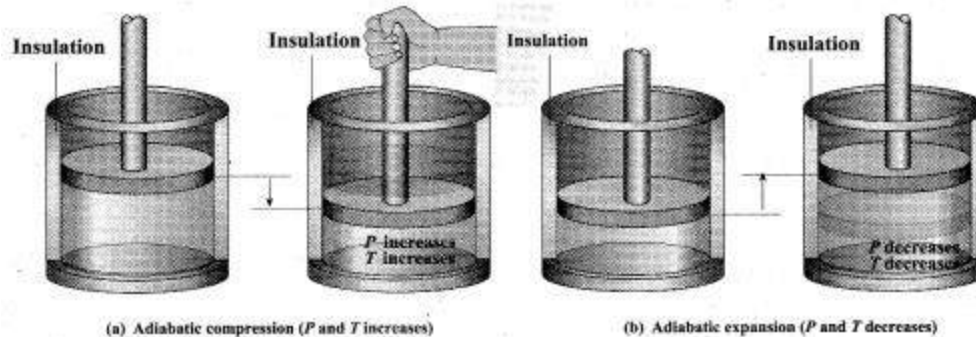
Explain in detail an adiabatic process.

#### Answer:

**Adiabatic process:** This is a process in which no heat flows into or out of the system ( $Q = 0$ ). But the gas can expand by spending its internal energy or gas can be compressed through some external work. So the pressure, volume and temperature of the system may change in an adiabatic process.

For an adiabatic process, the first law becomes  $\Delta U = W$ .

This implies that the work is done by the gas at the expense of internal energy or work is done on the system which increases its internal energy.



Adiabatic compression and expansion

The adiabatic process can be achieved by the following methods:



(i) Thermally insulating the system from surroundings so that no heat flows into or out of the system; for example, when thermally insulated cylinder of gas is compressed (adiabatic compression) or expanded (adiabatic expansion) as shown in the Figure.

(ii) If the process occurs so quickly that there is no time to exchange heat with surroundings even though there is no thermal insulation. A few examples are shown in Figure.

The equation of state for an adiabatic process is given by

$$PV^\gamma = \text{constant} \quad \dots(1)$$

Here  $\gamma$  is called adiabatic exponent ( $\gamma = C_p/C_v$ ) which depends on the nature of the gas. The equation (1) implies that if the gas goes from an equilibrium state ( $P_i, V_i$ ) to another equilibrium state ( $P_f, V_f$ ) adiabatically then it satisfies the relation

$$P_i V_i^\gamma = P_f V_f^\gamma \quad \dots(2)$$

The PV diagram for an adiabatic process is also called adiabat. But actually the adiabatic curve is steeper than isothermal curve.

We can also rewrite the equation (1) in terms of T and V. From ideal gas

equation, the pressure  $P = \frac{\mu RT}{V}$ . Substituting this equation in the equation (1), we have

$$\frac{\mu RT}{V} V^\gamma = \text{constant (or)} \quad \frac{T}{V} V^\gamma = \frac{\text{constant}}{\mu R}$$

Note here that is another constant. So it can be written as

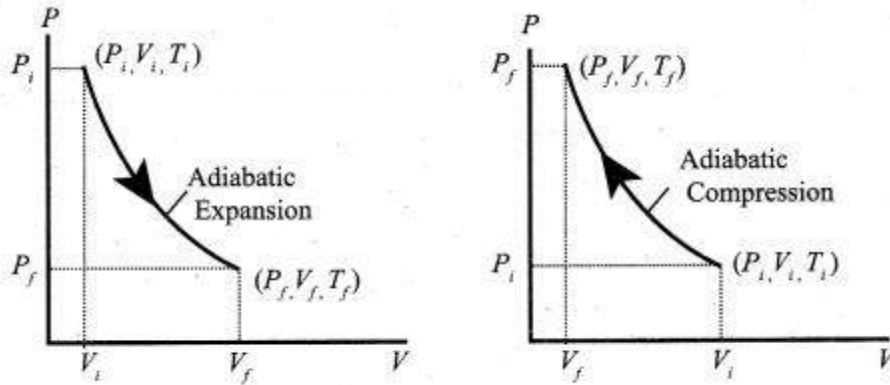
$$TV^{\gamma-1} = \text{constant.} \quad \dots(3)$$

The equation implies that if the gas goes from an initial equilibrium state ( $T_i, V_i$ ) to final equilibrium state ( $T_f, V_f$ ) adiabatically then it satisfies the relation

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \quad \dots(4)$$

The equation of state for adiabatic process can also be written in terms of T and P as

$$T^\gamma P^{1-\gamma} = \text{constant.} \quad \dots(5)$$



PV diagram for adiabatic expansion and adiabatic compression

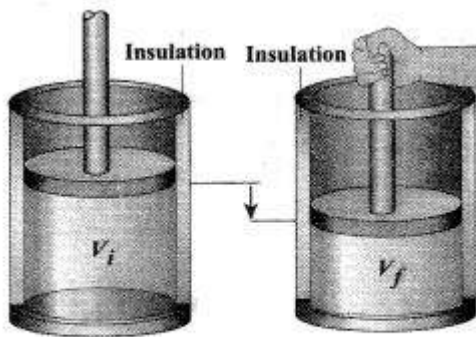
### Question 16.

Derive the work done in an adiabatic process.

#### Answer:

Work done in an adiabatic process: Consider  $\mu$  moles of an ideal gas enclosed in a cylinder having perfectly non conducting walls and base. A frictionless and insulating piston of cross sectional area  $A$  is fitted in the cylinder.

Let  $W$  be the work done when the system goes from the initial state  $(P_i, V_i, T_i)$  to the final state  $(P_f, V_f, T_f)$  adiabatically.



Work done in an adiabatic process

$$W = \int_{V_i}^{V_f} P dV \quad \dots(1)$$

By assuming that the adiabatic process occurs quasi-statically, at every stage the ideal gas law is valid. Under this condition, the adiabatic equation of state is  $PV^\gamma = \text{constant}$  (or)

$P = \frac{\text{constant}}{V^\gamma}$  can be substituted in the equation (1), we get

$$\begin{aligned} W_{adia} &= \int_{V_i}^{V_f} \frac{\text{constant}}{V^\gamma} dV = \text{constant} \int_{V_i}^{V_f} V^{-\gamma} dV \\ &= \text{constant} \left[ \frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_i}^{V_f} = \frac{\text{constant}}{1-\gamma} \left[ \frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}} \right] \\ &= \frac{1}{1-\gamma} \left[ \frac{\text{constant}}{V_f^{\gamma-1}} - \frac{\text{constant}}{V_i^{\gamma-1}} \right] \end{aligned}$$

But,

$$P_i V_i^\gamma = P_f V_f^\gamma = \text{constant.}$$

$$\therefore W_{adia} = \frac{1}{1-\gamma} \left[ \frac{P_f V_f^\gamma}{V_f^{\gamma-1}} - \frac{P_i V_i^\gamma}{V_i^{\gamma-1}} \right]$$

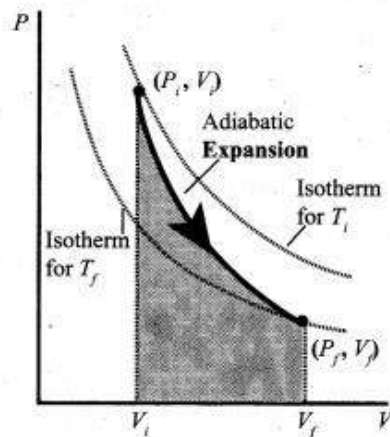
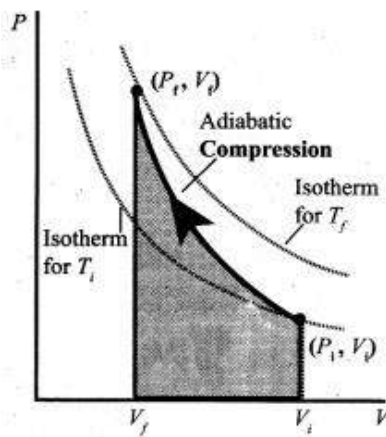
$$W_{adia} = \frac{1}{1-\gamma} [P_f V_f - P_i V_i] \quad \dots(2)$$

From ideal gas law,  $P_f V_f = \mu R T_f$  and  $P_i V_i = \mu R T_i$   
Substituting in equation (2), we get

$$\therefore W_{adia} = \frac{\mu R}{\gamma-1} [T_i - T_f] \quad \dots(3)$$

In adiabatic expansion, work is done by the gas. i.e.,  $W_{adia}$  is positive. As  $T_i > T_f$  the gas cools during adiabatic expansion.

In adiabatic compression, work is done on the gas. i.e.,  $W_{adia}$  is negative. As  $T_i < T_f$  the temperature of the gas increases during adiabatic compression.



PV diagram - Work done in the adiabatic process

To differentiate between isothermal and adiabatic curves in PV diagram, the adiabatic curve is drawn along with isothermal curve for  $T_f$  and  $T_i$ . Note that adiabatic curve is steeper than isothermal curve. This is because  $\gamma > 1$  always.

### Question 17.

Explain the isobaric process and derive the work done in this process.

**Answer:**

**Isobaric process:** This is a thermodynamic process that occurs at constant pressure. Even though pressure is constant in this process, temperature, volume and internal energy are not constant. From the ideal gas equation, we have

$$V = \left( \frac{\mu R}{P} \right) T \quad \dots(1)$$

Here  $\frac{\mu R}{P} = \text{constant}$

In an isobaric process the temperature is directly proportional to volume.

$$V \propto T \text{ (Isobaric process) } \dots (2)$$

This implies that for a isobaric process, the V-T graph is a straight line passing through the origin.

If a gas goes from a state  $(V_i, T_i)$  to  $(V_f, T_f)$  at constant pressure, then the system satisfies the following equation

$$\frac{T_f}{V_f} = \frac{T_i}{V_i} \quad \dots(3)$$

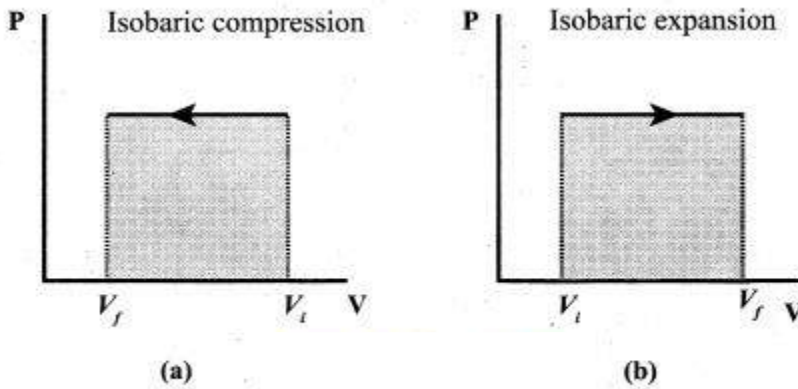
### Examples for Isobaric process:

(i) When the gas is heated and pushes the piston so that it exerts a force equivalent to atmospheric pressure plus the force due to gravity then this process is isobaric.

(ii) Most of the cooking processes in our kitchen are isobaric processes. When the food is cooked in an open vessel, the pressure above the food is always at atmospheric pressure. The PV diagram for an isobaric process is a horizontal line parallel to volume axis.

Figure (a) represents isobaric process where volume decreases figure

(b) represents isobaric process where volume increases.



PV diagram for an isobaric process

The work done in an isobaric process: Work done by the gas

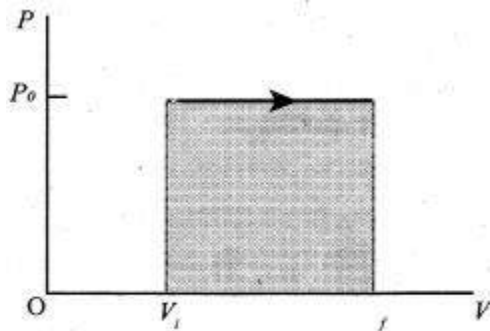
$$W = \int_{V_i}^{V_f} P dV \quad \dots(4)$$

In an isobaric process, the pressure is constant, so P comes out of the integral,

$$W = P \int_{V_i}^{V_f} dV \quad \dots(5)$$

$$W = P[V_f - V_i] = P\Delta V \quad \dots(6)$$

Where  $\Delta V$  denotes change in the volume. If  $\Delta V$  is negative, W is also negative. This implies that the work is done on the gas. If  $\Delta V$  is positive, W is also positive, implying that work is done by the gas equation.



Work done in an isobaric process

The equation (6) can also be rewritten using the ideal gas equation.

From ideal gas equation

$$PV = \mu RT \text{ and } V = \frac{\mu RT}{P}$$

Substituting this in equation (6) we get

$$W = \mu RT_f \left( 1 - \frac{T_i}{T_f} \right) \quad \dots(7)$$

In the PV diagram, area under the isobaric curve is equal to the work done in isobaric process. The shaded area in the above diagram is equal to the work done by the gas.

The first law of thermodynamics for isobaric process is given by

$$\Delta U = Q - P\Delta V \quad \dots(8)$$

$$W = P\Delta V, \Delta U = Q - \mu RT_f \left[ 1 - \frac{T_i}{T_f} \right]$$

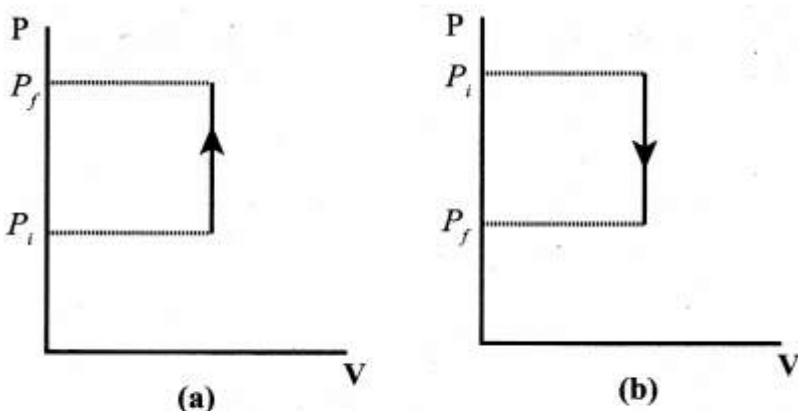
### Question 18.

Explain in detail the isochoric process.

**Answer:**

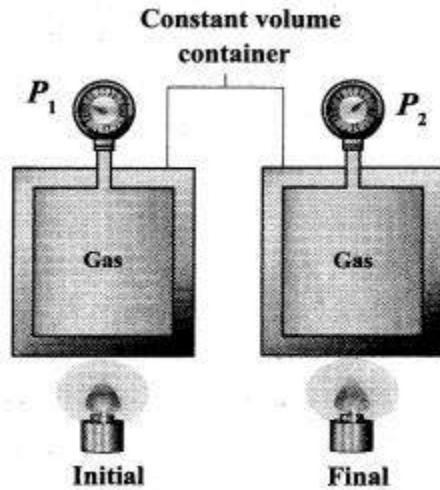
**Isochoric process:** This is a thermodynamic process in which the volume of the system is kept constant. But pressure, temperature and internal energy continue to be variables.

The pressure – volume graph for an isochoric process is a vertical line parallel to pressure axis as shown in Figure.



Isochoric process with (a) increased pressure and  
(b) decreased pressure

The equation of state for an isochoric process is given by



Isochoric process

$$P = \left( \frac{\mu R}{V} \right) T \quad \dots(1)$$

where  $\left( \frac{\mu R}{V} \right) = \text{constant}$

We can infer that the pressure is directly proportional to temperature. This implies that the P-T graph for an isochoric process is a straight line passing through origin. If a gas goes from state  $(P_i, T_i)$  to  $(P_f, T_f)$  at constant volume, then the system satisfies the following equation

$$\frac{P_i}{T_i} = \frac{P_f}{T_f} \quad \dots(2)$$

For an isochoric processes,  $\Delta V = 0$  and  $W = 0$ . Then the first law becomes  $\Delta U = Q \dots(3)$

Implying that the heat supplied is used to increase only the internal energy. As a result the temperature increases and pressure also increases.

Suppose a system loses heat to the surroundings through conducting walls by keeping the volume constant, then its internal energy decreases. As a result the temperature decreases; the pressure also decreases.

### Question 19.

What are the limitations of the first law of thermodynamics?

**Answer:**

Limitations of first law of thermodynamics: The first law of thermodynamics explains well the inter convertibility of heat and work. But it does not indicate the direction of change.

For example,

(a) When a hot object is in contact with a cold object, heat always flows from the hot object to cold object but not in the reverse direction. According to first law, it is possible for the energy to flow from hot object to cold object or from cold object to hot object. But in nature the direction of heat flow is always from higher temperature to lower temperature.

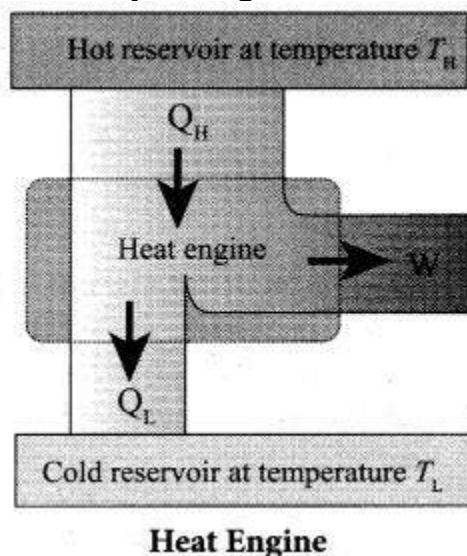
(b) When brakes are applied, a car stops due to friction and the work done against friction is converted into heat. But this heat is not reconverted to the kinetic energy of the car. So the first law is not sufficient to explain many of natural phenomena.

**Question 20.**

Explain the heat engine and obtain its efficiency.

**Answer:**

Heat Engine: In the modern technological world, the role of automobile engines plays a vital role in for transportation. In motor bikes and cars there are engines which take in petrol or diesel as input, and do work by rotating wheels. Most of these automobile engines have efficiency not greater than 40%. The second law of thermodynamics puts a fundamental restriction on efficiency of engines. Therefore understanding heat engines is very important.





**Reservoir:** It is defined as a thermodynamic system which has very large heat capacity. By taking in heat from reservoir or giving heat to reservoir, the reservoir's temperature does not change.

**Example:** Pouring a tumbler of hot water in to lake will not increase the temperature of the lake. Here the lake can be treated as a reservoir. When a hot cup of coffee attains equilibrium with the open atmosphere, the temperature of the atmosphere will not appreciably change. The atmosphere can be taken as a reservoir.

**We can define heat engine as follows:** Heat engine is a device which takes heat as input and converts this heat in to work by undergoing a cyclic process.

A heat engine has three parts:

- (a) Hot reservoir
  - (b) Working substance
  - (c) Cold reservoir
- A Schematic diagram for heat engine is given below:

**1. Hot reservoir (or) Source:** It supplies heat to the engine. It is always maintained at a high temperature  $T_H$ .

**2. Working substance:** It is a substance like gas or water, which converts the heat supplied into Work.

**3. Cold reservoir (or) Sink:** The heat engine ejects some amount of heat ( $Q_L$ ) in to cold reservoir after it doing work. It is always maintained at a low temperature  $T_L$ .

The heat engine works in a cyclic process. After a cyclic process it returns to the same state. Since the heat engine returns to the same state after it ejects heat, the change in the internal energy of the heat engine is zero.

The efficiency of the heat engine is defined as the ratio of the work done (output) to the heat absorbed (input) in one cyclic process.

Let the working substance absorb heat  $Q_H$  units from the source and reject  $Q_L$  units to the sink after doing work  $W$  units.

We can write. Input heat = Work done + ejected heat

$$Q_H = W + Q_L$$

$$W = Q_H - Q_L$$

Then the efficiency of heat engine

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Note here that  $Q_H$ ,  $Q_L$  and  $W$  all are taken as positive, a sign convention followed in this expression.

Since  $Q_L < Q_H$ , the efficiency ( $\eta$ ) always less than 1. This implies that heat absorbed is not completely converted into work. The second law of thermodynamics placed fundamental restrictions on converting heat completely into work.

### Question 21.

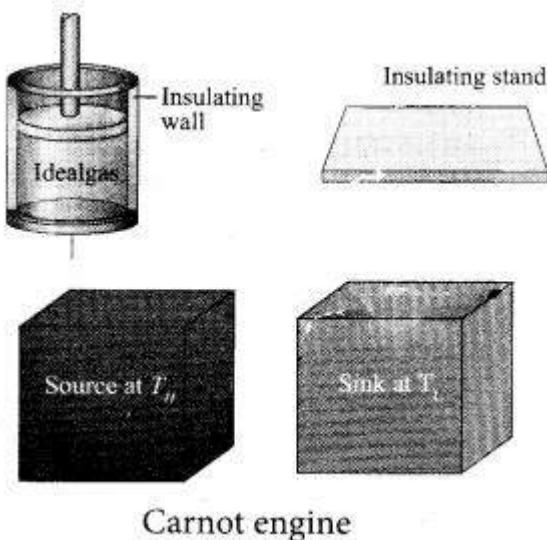
Explain in detail Carnot heat engine.

#### Answer:

In the year 1824 a young French engineer Sadi Carnot proved that a certain reversible engine operated in cycle between hot and cold reservoir can have maximum efficiency. This engine is called Carnot engine.

A reversible heat engine operating in a cycle between two temperatures in a particular way is called a Carnot Engine. The carnot engine has four parts which are given below.

(i) **Source:** It is the source of heat maintained at constant high temperature  $T_H$ . Any amount of heat can be extracted from it, without changing its temperature.



(ii) **Sink:** It is a cold body maintained at a constant low temperature  $T_L$ . It can

absorb any amount of heat.

(iii) **Insulating stand:** It is made of perfectly non-conducting material. Heat is not conducted through this stand.

(iv) **Working substance:** It is an ideal gas enclosed in a cylinder with perfectly non-conducting walls and perfectly conducting bottom. A non-conducting and frictionless piston is fitted in it.

### Question 22.

Derive the expression for Carnot engine efficiency.

#### Answer:

Efficiency of a Carnot engine: Efficiency is defined as the ratio of work done by the working substance in one cycle to the amount of heat extracted from the source.

$$\eta = \frac{\text{Work done}}{\text{Heat extracted}} = \frac{W}{Q_H} \quad \dots(1)$$

From the first law of thermodynamics,  $W = Q_H - Q_L$

$$\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \quad \dots(2)$$

Applying isothermal conditions, we get,

$$Q_H = \mu R T_H \ln \left( \frac{V_2}{V_1} \right)$$

$$Q_L = \mu R T_L \ln \left( \frac{V_3}{V_4} \right) \quad \dots(3)$$

Here we omit the negative sign. Since we are interested in only the amount of heat ( $Q_L$ ) ejected into the sink, we have

$$\therefore \frac{Q_L}{Q_H} = \frac{T_L \ln \left( \frac{V_3}{V_4} \right)}{T_H \ln \left( \frac{V_2}{V_1} \right)} \quad \dots(4)$$

By applying adiabatic conditions, we get,

$$T_H V_2^{\gamma-1} = T_L V_3^{\gamma-1}$$

$$T_H V_1^{\gamma-1} = T_L V_4^{\gamma-1}$$

By dividing the above two equations, we get

$$\left( \frac{V_2}{V_1} \right)^{\gamma-1} = \left( \frac{V_3}{V_4} \right)^{\gamma-1}$$

Which implies that  $\frac{V_2}{V_1} = \frac{V_3}{V_4} \quad \dots(5)$

Substituting equation (5) in (4), we get

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \quad \dots(6)$$

$\therefore$  The efficiency  $\eta = 1 - \frac{T_L}{T_H} \quad \dots(7)$

Note :  $T_L$  and  $T_H$  should be expressed in Kelvin scale.

Important results:

1.  $\eta$  is always less than 1 because  $T_L$  is less than  $T_H$ . This implies the efficiency cannot be 100%.

2. The efficiency of the Carnot's engine is independent of the working substance. It depends only on the temperatures of the source and the sink. The greater the difference between the two temperatures, higher the efficiency.

3. When  $T_H = T_L$  the efficiency  $\eta = 0$ . No engine can work having source and sink at the same temperature.

### Question 23.

Explain the second law of thermodynamics in terms of entropy.

**Answer:**

The quantity  $Q/T$  is called entropy. It is a very important thermodynamic property of a system.

It is also a state variable.  $Q_H/T_H$  is the entropy received by the Carnot engine from hot reservoir is entropy given out by the Carnot engine to the cold reservoir. For reversible engines (Carnot Engine) both entropies should be same, so that the change in entropy of the Carnot engine in one cycle is zero. But for all practical engines like diesel and petrol engines which are not

reversible engines, they satisfy the relation  $\frac{Q_L}{T_L} > \frac{Q_H}{T_H}$

In fact we can reformulate the second law of thermodynamics as follows “For all the processes that occur in nature (irreversible process), the entropy always increases. For reversible process entropy will not change”. Entropy determines the direction in which natural process should occur.

Entropy increases when heat flows from hot object to cold object. If heat were to flow from a cold to a hot object, entropy will decrease leading to violation of second law thermodynamics.

Entropy is also called ‘measure of disorder’. All-natural process occur such that the disorder should always increases. Consider a bottle with a gas inside. When the gas molecules are inside the bottle it has less disorder. Once it spreads into the entire room it leads to more disorder. In other words when the gas is inside the bottle the entropy is less and once the gas spreads into entire room, the entropy increases.

From the second law of thermodynamics, entropy always increases. If the air molecules go back in to the bottle, the entropy should decrease, which is not allowed by the second law of thermodynamics. The same explanation applies to a drop of ink diffusing into water. Once the drop of ink spreads, its entropy is increased. The diffused ink can never become a drop again. So the natural processes occur in such a way that entropy should increase for all irreversible process.

### Question 24.

Explain in detail the working of a refrigerator.

#### Answer:

**Refrigerator:** A refrigerator is a Carnot's engine working in the reverse order.

**Working Principle:** The working substance (gas) absorbs a quantity of heat  $Q_L$  from the cold body (sink) at a lower temperature  $T_L$ . A certain amount of work  $W$  is done on the working substance by the compressor and a quantity of heat  $Q_H$  is rejected to the hot body (source) ie, the atmosphere at  $T_H$ . When you stand beneath of refrigerator, you can feel warmth air. From the first law of thermodynamics, we have

$$Q_L + W = Q_H \dots (1)$$

As a result the cold reservoir (refrigerator) further cools down and the surroundings (kitchen or atmosphere) gets hotter.

**Coefficient of performance (COP) ( $\beta$ ):** COP is a measure of the efficiency of a refrigerator. It is defined as the ratio of heat extracted from the cold body (sink) to the external work done by the compressor  $W$ .

$$\text{COP} = \beta = \frac{Q_L}{W} \dots (2)$$

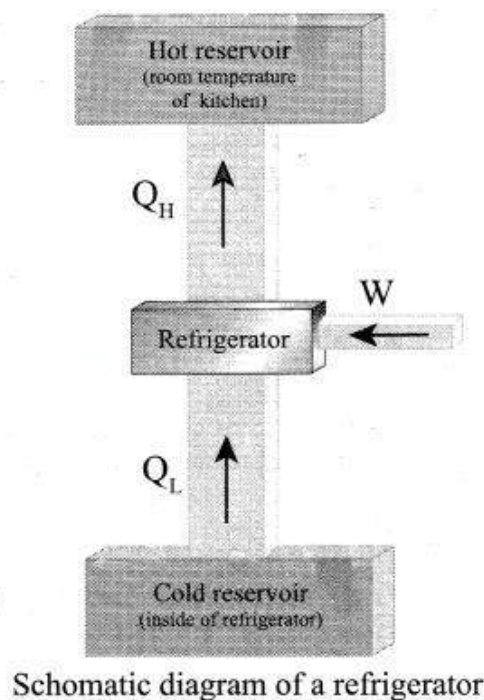
From the equation (1)  $\beta = \frac{Q_L}{Q_H - Q_L}$

$$\beta = \frac{1}{\frac{Q_H}{Q_L} - 1} \dots (3)$$

But we know that  $\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$

Substituting this equation into equation (3), we get

$$\beta = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{T_L}{T_H - T_L}$$



#### Inferences.

1. The greater the COP, the better is the condition of the refrigerator. A typical refrigerator has COP around 5 to 6.

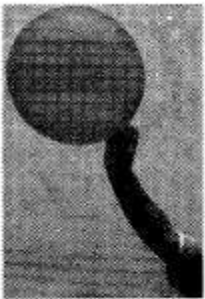
2. Lesser the difference in the temperatures of the cooling chamber and the atmosphere, higher is the COP of a refrigerator.

3. In the refrigerator the heat is taken from cold object to hot object by doing external work. Without external work heat cannot flow from cold object to hot object. It is not a violation of second law of thermodynamics, because the heat is ejected to surrounding air and total entropy of (refrigerator + surrounding) is always increased.

## Numerical Problems

### Question 1.

Calculate the number of moles of air is in the inflated balloon at room temperature as shown in the figure.



The radius of the balloon is 10 cm, and pressure inside the balloon is 180 kPa

**Answer:**

$$P = 180 \text{ kPa} = 180 \times 10^3 \text{ Pa}, V = \frac{4}{3} \pi r^3, r = 10 \text{ cm}, R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1},$$

$$T = 27^\circ \text{C} = 300 \text{ K}$$

From ideal gas equation of state

$$PV = \mu RT$$

$$\mu = \frac{PV}{RT} = \frac{180 \times 10^3 \times \frac{4}{3} \times 3.14 \times (10 \times 10^{-2})^3}{8.314 \times 300}$$

$$= \frac{0.7536 \times 10^3}{2494.2} = 3.021 \times 10^{-4} \times 10^3$$

$$\boxed{\mu \cong 0.3 \text{ mol}}$$

**Question 2.**

In the planet Mars, the average temperature is around  $-53^{\circ}\text{C}$  and atmospheric pressure is  $0.9\text{ kPa}$ . Calculate the number of moles of the molecules in unit volume in the planet Mars? Is this greater than that in earth?

**Answer:**

$$T = -53^{\circ}\text{C} = 220\text{ K}$$

$$P = 0.9 \times 10^3\text{ Pa}$$

$$V = 1\text{ m}^3$$

$$\text{Number of molecules } \mu = \frac{VP}{RT} = \frac{0.9 \times 10^3}{8.314 \times 220} = 0.00049 \times 10^3 ; \mu = 0.49\text{ mol}$$

**Question 3.**

An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume  $V_1$  and contains ideal gas at pressure  $P_1$  and temperature  $T_1$ . The other chamber has volume  $V_2$  and contains ideal gas at pressure  $P_2$  and temperature  $T_2$ . If the partition is removed without doing any work on the gases, calculate the final equilibrium temperature of the container.

**Answer:**

Let  $T$  be the equilibrium temperature and let  $n_1$  and  $n_2$  be the number of moles in vessels 1 and 2 respectively. As there is no loss of energy,

$$n_1 \left( \frac{3}{2} RT_1 \right) + n_2 \left( \frac{3}{2} RT_2 \right) = (n_1 + n_2) \left( \frac{3}{2} RT \right)$$

$$n_1 T_1 + n_2 T_2 = (n_1 + n_2) T$$

$$T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

Now,

$$n_1 = \frac{P_1 V_1}{RT_1}, n_2 = \frac{P_2 V_2}{RT_2}$$

Substituting  $n_1$  and  $n_2$  values and solving, we get

$$T = \frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$$

**Question 4.**

The temperature of a uniform rod of length  $L$  having a coefficient of linear

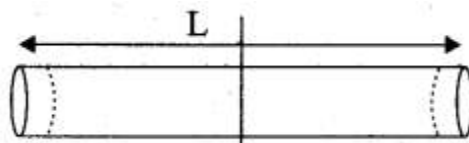


expansion  $\alpha_L$  is changed by  $\Delta T$ . Calculate the new moment of inertia of the uniform rod about axis passing through its center and perpendicular to an axis of the rod.

**Answer:**

Moment of inertia of a uniform rod of mass and length  $l$  about its perpendicular bisector. Moment of inertia of the rod

$$I = \frac{1}{12} ML^2$$



Increase in length of the rod when temperature is increased by  $\Delta T$ , is given by  $L' = L(1 + \alpha_L \Delta T)$

New moment of inertia of the rod

$$I' = \frac{ML'^2}{12} = \frac{M}{12} L^2 (1 + \alpha_L \Delta T)^2$$

$$I' = I(1 + \alpha_L \Delta T)^2$$

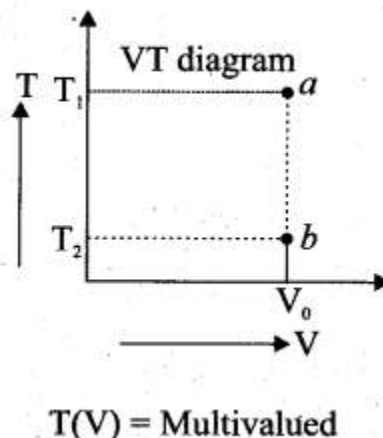
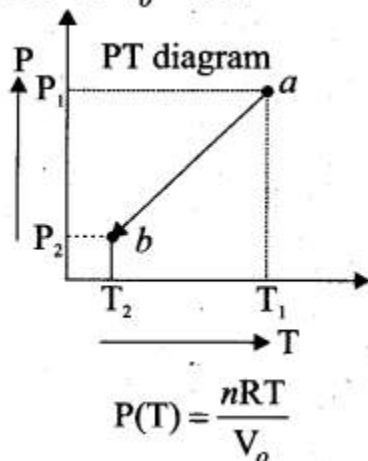
**Question 5.**

Draw the TP diagram ( $P - x$  axis,  $T - y$  axis), VT ( $T - x$  axis,  $V - y$  axis) diagram for

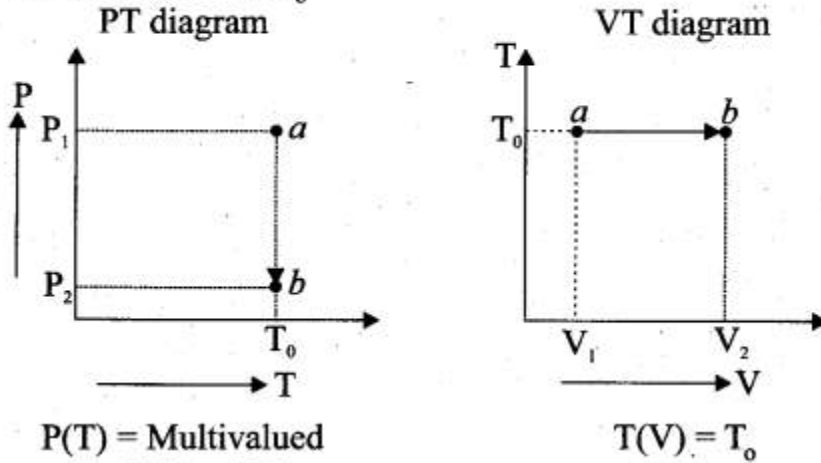
- (a) Isochoric process
- (b) Isothermal process
- (c) Isobaric process

**Answer:**

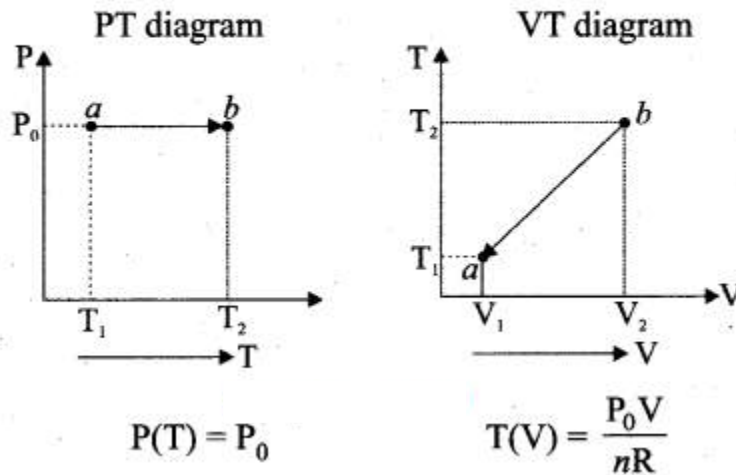
(a) **Isochoric process** :  $PV_o = nRT$



(b) **Isothermal process:**  $PV = nRT_0$



(c) **Isobaric process:**  $P_0 V = nRT$



### Question 6.

A man starts bicycling in the morning at a temperature around  $25^\circ\text{C}$ , he checked the pressure of tire which is equal to be 500 kPa. Afternoon he found that the absolute pressure in the tyre is increased to 520 kPa. By assuming the expansion of tyre is negligible, what is the temperature of tyre at afternoon?

**Answer:**

For ideal gas equation of state

$$PV = nRT$$

$$P_1 = 500 \text{ kPa}, T_1 = 25^\circ\text{C} = 25 + 273 = 298\text{K}, P_2 = 520 \text{ kPa}, T_2 = ?$$

Expansion of tyre is negligible ( $V_{\text{constant}}$ )

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore T_2 = \left( \frac{P_2}{P_1} \right) T_1 = \frac{520}{500} \times 298 = \frac{154960}{500} = 309.92 \text{ K}$$

$$T_2 = 309.92 - 273 ; T_2 = 36.9^\circ \text{C}$$

### Question 7.

Normal human body temperature is  $98.6^\circ \text{F}$ . During high fever if the temperature increases to  $104^\circ \text{F}$ , what is the change in peak wavelength that emitted by our body? (Assume human body is a black body).

### Answer:

Normal human body temperature ( $T$ ) =  $98.6^\circ \text{F}$

Convert Fahrenheit into Kelvin,

$$\frac{F - 32}{180} = \frac{K - 273}{100}$$

So,  $T = 98.6^\circ \text{F} = 310 \text{ K}$

From Wien's displacement law

$$\text{Maximum wavelength, } \lambda_{\max} = \frac{b}{T} = \frac{2.898 \times 10^{-3}}{310} = 9348 \times 10^{-9} \text{ m}$$

$$\boxed{\lambda_{\max} = 9348 \text{ nm}} \quad (\text{at } 98.6^\circ \text{F})$$

During high fever, human body temperature,  $T = 104^\circ \text{F} = 313 \text{ K}$

$$\text{Peak wavelength} \quad \lambda_{\max} = \frac{b}{T} = \frac{2.898 \times 10^{-3}}{313} = 9259 \times 10^{-9} \text{ m}$$

$$\boxed{\lambda_{\max} = 9259 \text{ nm}} \quad (\text{at } 104^\circ \text{F})$$

### Question 8.

In an adiabatic expansion of the air, the volume is increased by 4%, what is percentage change in pressure? (For air  $\gamma = 1.4$ )

### Answer:

From equation for adiabatic process,

$$PV^\gamma = \text{constant}$$

Using differentiation, we get

$$P\gamma V^{\gamma-1}dV + dP.V^\gamma = 0$$

$$\frac{dP}{P} = -\gamma \frac{dV}{V}$$

Volume 'V' is increased. by 4% and  $\gamma = 1.4$

$$\frac{dP}{P} \times 100 = -\gamma \left( \frac{dV}{V} \times 100 \right) = -1.4 \times 4 = -5.6$$

Pressure is decreased. by 5.6%

### Question 9.

In a petrol engine, (internal combustion engine) air at atmospheric pressure and temperature of  $20^\circ\text{C}$  is compressed in the cylinder by the piston to  $1/8$  of its original volume. Calculate the temperature of the compressed air. (For air  $\gamma = 1.4$ )

Answer:

$$T_1 = 20^\circ\text{C} = 20 + 273 = 293\text{K}, V_1 = 1 \text{ m}^3, V_2 = \frac{1}{8} V_1 \text{ m}^3, \gamma = 1.4$$

From equation of adiabatic process

$$TV^{\gamma-1} = \text{constant}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = \left( \frac{V_1}{V_2} \right)^{\gamma-1} T_1$$

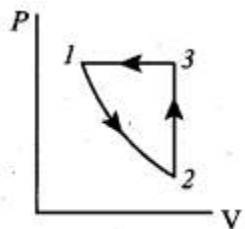
$$T_2 = \left( \frac{1}{\left( \frac{1}{8} \right)} \right)^{1.4-1} \times 293 = (8)^{0.4} \times 293$$

$$= 673.1 \text{ K} = 673.1 - 273 ; \boxed{T_2 \cong 400^\circ\text{C}}$$

### Question 10.

Consider the following cyclic process consist of isotherm, isochoric and isobar

which is given in the figure.



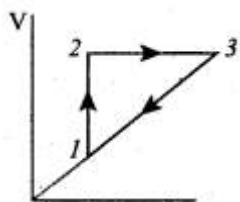
Draw the same cyclic process qualitatively in the V-T diagram where T is taken along x – direction and V is taken along y – direction. Analyze the nature of heat exchange in each process.

**Answer:**

Process 1 to 2 = increase in volume. So heat must be added.

Process 2 to 3 = Volume remains constant. Increase in temperature.

The given heat is used to increase the internal energy.



Process 3 to 1 : Pressure remains constant. Volume and Temperature are reduced. Heat flows out of the system.

It is an isobaric compression where the work is done on the system.

**Question 11.**

An ideal gas is taken in a cyclic process as shown in the figure.

Calculate

- (a) work done by the gas.
- (b) work done on the gas
- (c) Net work done in the process

**Answer:**

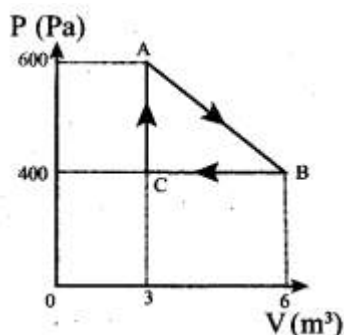
- (a) Work done by the gas (along AB)

$$W = P \times \Delta V = 600 \times 3 = 1800 \text{ J}; \quad \boxed{W = 1.8 \text{ kJ}}$$

(b) Work is done on the gas (along BC)

$$W = -P \times \Delta V = -400 \times 3 = -1200 \text{ J}$$

$$\boxed{W = -1.2 \text{ kJ}}$$



(c) Net work done in the process = Area under the curve AB  
= Rectangle area + triangle area

$$= (l \times b) + \left( \frac{1}{2} \times b \times h \right) = (400 \times 3) + \left( \frac{1}{2} \times 3 \times 200 \right)$$

$$= 1200 + 300 = 1500 \text{ J}; \quad \boxed{W = 1.5 \text{ kJ}}$$

### Question 12.

For a given ideal gas  $6 \times 10^5 \text{ J}$  heat energy is supplied and the volume of gas is increased from  $4 \text{ m}^3$  to  $6 \text{ m}^3$  at atmospheric pressure. Calculate

- the work done by the gas
- change in internal energy of the gas
- graph this process in PV and TV diagram.

### Answer:

Heat energy supplied to gas  $Q = 6 \times 10^5 \text{ J}$

Change in volume  $\Delta V = (6 - 4) = 2 \text{ m}^3$

$1 \text{ atm} = 1.013 \times 10^5 \text{ Nm}^{-2}$

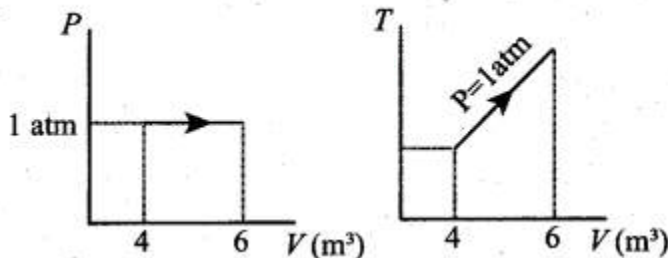
(a) Work done by the gas  $W = P \times \Delta V = 1.013 \times 10^5 \times 2 = 2.026 \times 10^5$   
 $W = 202.6 \text{ kJ}$

(b) Change in internal energy of the gas

$$\Delta U = Q - P \cdot \Delta V = 6 \times 10^5 - (202.6 \times 10^3) \\ = 6 \times 10^5 - 2.026 \times 10^5 = 3.974 \times 10^5$$

$$\Delta U = 397.4 \text{ kJ}$$

(c)



### Question 13.

Suppose a person wants to increase the efficiency of the reversible heat engine that is operating between  $100^\circ\text{C}$  and  $300^\circ\text{C}$ . He had two ways to increase the efficiency,

(a) By decreasing the cold reservoir temperature from  $100^\circ\text{C}$  to  $50^\circ\text{C}$  and keeping the hot reservoir temperature constant

(b) by increasing the temperature of the hot reservoir from  $300^\circ\text{C}$  to  $350^\circ\text{C}$  by keeping the cold reservoir temperature constant. Which is the suitable method?

### Answer:

Heat engine operates at initial temperature =  $100^\circ\text{C} + 273 = 373 \text{ K}$

Final temperature =  $300^\circ\text{C} + 273 = 573 \text{ K}$

At melting point =  $273 \text{ K}$

$$\text{Efficiency } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273}{573} = 0.3491 ; \eta = 34.9\%$$

(a) By decreasing the cold reservoir, efficiency

$$T_1 = 350^\circ\text{C} + 273 = 623 \text{ K}, \quad T_2 = 100^\circ\text{C} + 273 = 373 \text{ K}$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{373}{623} = 0.4013$$

$$\eta = 40.13\%$$

(b) By increasing the temperature of hot reservoir, efficiency,

$$T_1 = 350^\circ\text{C} + 273 = 623 \text{ K}, T_2 = 100^\circ\text{C} + 273 = 373 \text{ K}$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{373}{623} = 0.401$$

$$\eta = 40.12\%$$

Method (a) More efficiency than method (b).

#### Question 14.

A Carnot engine whose efficiency is 45% takes heat from a source maintained at a temperature of  $327^\circ\text{C}$ . To have an engine of efficiency 60% what must be the intake temperature for the same exhaust (sink) temperature?

**Answer:**

Efficiency of Carnot engine ( $\eta_1$ ) = 45% = 0.45

Initial intake temperature ( $T_1$ ) =  $327^\circ\text{C} = 600 \text{ K}$

New efficiency ( $\eta_2$ ) = 60% = 0.6

Efficiency of Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

$T_1$  is temperature of source ;  $T_2$  is temperature of sink

$$\text{1st Case:} \quad T_2 = (1 - \eta) T_1 = (1 - 0.45) \times 600 \Rightarrow T_2 = 330\text{K}$$

$$\text{2nd Case:} \quad \frac{T_2}{T_1} = 1 - \eta$$

$$T_1 = \frac{T_2}{1 - \eta} = \frac{330}{1 - 0.6} = \frac{330}{0.4}$$

$$T_1 = 825\text{K} = 825 - 273 ; \boxed{T_1 = 552^\circ\text{C}}$$

#### Question 15.

An ideal refrigerator keeps its content at  $0^\circ\text{C}$  while the room temperature is  $27^\circ\text{C}$ . Calculate its coefficient of performance.

**Answer:**

$$\text{Content placed at temperature} \quad T_L = 0^\circ\text{C} = 0 + 273$$

$$T_L = 273\text{K}$$



Refrigerator placed at temperature  $T_H = 27^\circ\text{C} = 27 + 273 = 300\text{K}$

Coefficient of performance  $(\beta) = \frac{T_L}{T_H - T_L} = \frac{273}{300 - 273} = \frac{273}{27} ; \boxed{\beta = 10.11}$