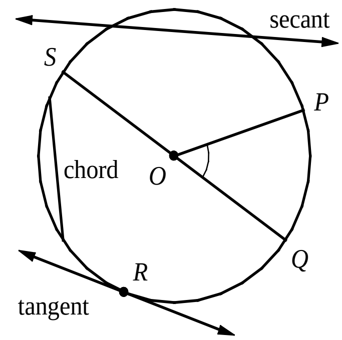
CHAPTER 19
Circles

19-1. Arcs, Angles, and Tangents

In a plane, a **circle** is the set of all points equidistant from a given point called the **center**. It follows from the definition of a circle that **all radii are equal in measure**.

A circle is usually named by its center. The circle at the right is called circle O . (symbolized as $\odot O$)



A **chord** is a segment whose endpoints lie on a circle.

A **secant** is a line that contains a chord.

A **tangent** is a line in the plane of a circle, and intersects the circle at exactly one point: the **point of tangency**.

A **central angle** is an angle whose vertex is the center of the circle.

An arc is a part of a circle. The measure of a **minor arc** is the measure of its central angle. The measure of a minor arc is less than 180.

The measure of a **semicircle** is 180.

The measure of a **major arc** is 360 minus the measure of its minor arc.

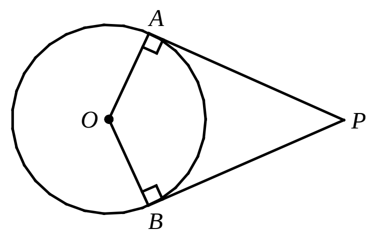
$\angle POQ$ and $\angle POS$ are central angles.
$\overset{\frown}{PQ}$, $\overset{\frown}{QR}$, $\overset{\frown}{RS}$, and $\overset{\frown}{SP}$ are minor arcs.
$\overset{\frown}{QPS}$ and $\overset{\frown}{QRS}$ are semicircles.
$\overset{\frown}{PQS}$ and $\overset{\frown}{SPR}$ are major arcs.
$m\overset{\frown}{QPS} = m\overset{\frown}{QRS} = 180$
$m\overset{\frown}{PQS} = 360 - m\overset{\frown}{SP}$

Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measure of the two arcs. $m\overset{\frown}{PQR} = m\overset{\frown}{PQ} + m\overset{\frown}{QR}$

Theorems - Tangent Lines

If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency.

$\overline{PA} \perp \overline{OA}$ and $\overline{PB} \perp \overline{OB}$

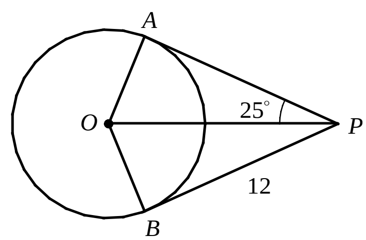


Tangents to a circle from the same exterior point are congruent.

$PA = PB$

Example 1 □ In $\odot O$ shown at the right, $\overline{PA}$ and $\overline{PB}$ are tangents, $PB = 12$, and $m\angle APO = 25$.

a. Find the measure of $\angle POA$.
b. Find the length of PA .
c. Find the radius of $\odot O$.



Solution □ a. $\overline{PA} \perp \overline{OA}$ — If a line is tangent to a circle, then the line is $\perp$ to the radius at the point of tangency.

$m\angle PAO = 90$ — Definition of $\perp$ lines
$m\angle POA + m\angle APO + m\angle PAO = 180$ — Angle Sum Theorem
$m\angle POA + 25 + 90 = 180$ — Substitution
$m\angle POA = 65$

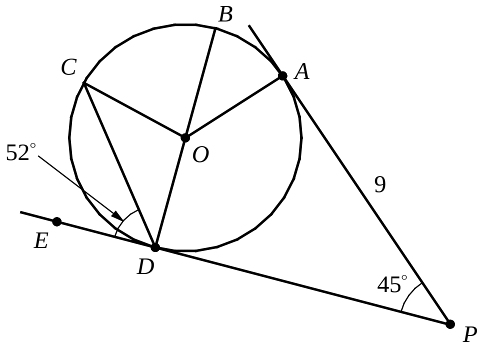
b. $PA = PB = 12$ — Tangents to a circle from the same exterior point are $\cong$.

c. $\tan 25^\circ = \dfrac{OA}{PA} = \dfrac{OA}{12}$

$OA = 12 \tan 25^\circ \approx 5.6$

Exercises - Arcs, Angles, and Tangents

Questions 1 - 4 refer to the following information.



In the figure above, $\overline{BD}$ is a diameter, and $\overline{PA}$ and $\overline{PD}$ are tangents to circle O. $m\angle CDE = 52$, $m\angle APD = 45$, and $AP = 9$.

1

What is the measure of $\angle ODC$?

2

What is the measure of $\angle OCD$?

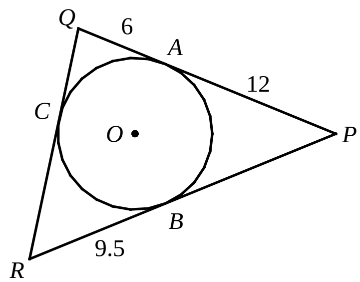
3

What is the measure of $\angle AOD$?

4

What is the length of PD?

5



In the figure above, $\odot O$ is inscribed in ΔPQR. If $PA = 12$, $QA = 6$, and $RB = 9.5$, what is the perimeter of ΔPQR?

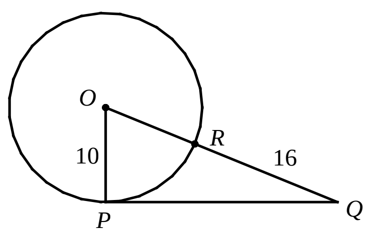
A) 46

B) 49

C) 52

D) 55

6



In the figure above, $\overline{OP}$ is a radius and $\overline{PQ}$ is tangent to circle O. If the radius of circle O is 10 and $QR = 16$, what is the length of $\overline{PQ}$?

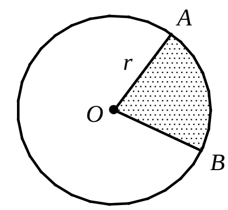
A) 16

B) 20

C) 24

D) 28

19-2. Arc Lengths and Areas of Sectors



Circumference of a circle: $C = 2\pi r$ or $C = \pi d$

Area of circle: $A = \pi r^2$

A **sector** of a circle is a region bound by two radii and an arc of the circle. The shaded region of the circle at the right is called sector AOB.

Length of $\boldsymbol{\overset{\frown}{AB} = 2\pi r \cdot \frac{m\angle AOB}{360}}$

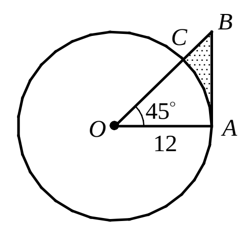
Area of sector $\boldsymbol{AOB = \pi r^2 \cdot \frac{m\angle AOB}{360}}$

The distance traveled by a wheel $\boldsymbol{= 2\pi r \times}$ **number of revolutions**

Example 1 □ In circle O shown at the right, $\overline{AB}$ is tangent to the circle.

a. Find the area of the shaded region.

b. Find the perimeter of the shaded region.



Solution □ a. $m\angle OAB = 90$ — Line tangent to a circle is $\perp$ to the radius.

$m\angle OBA = 45$ — Acute angles of a right Δ are complementary.

$OA = AB = 12$ — Legs of isosceles triangle are $\cong$.

Area of $\Delta OAB = \frac{1}{2}(12)(12) = 72$

Area of sector $AOC = \pi(12)^2 \cdot \frac{45}{360} = 18\pi$.

Area of shaded region $= 72 - 18\pi$ — Answer

b. Length of $\overset{\frown}{AC} = 2\pi(12) \cdot \frac{45}{360} = 3\pi$

Length of $BC = OB - OC = 12\sqrt{2} - 12$ — In a 45°-45°-90° Δ, the hypotenuse is $\sqrt{2}$ times as long as a leg.

Perimeter of shaded region

$= \text{length of } \overset{\frown}{AC} + BC + AB$

$= 3\pi + (12\sqrt{2} - 12) + 12 = 3\pi + 12\sqrt{2}$ — Answer

Example 2 □ The radius of a bicycle wheel is 12 inches. What is the number of revolutions the wheel makes to travel 1 mile? ($1 \text{ mile} = 5,280 \text{ ft}$)

Solution □ Let $x =$ number of revolutions.

The distance traveled by a wheel $= 2\pi r \times$ number of revolutions

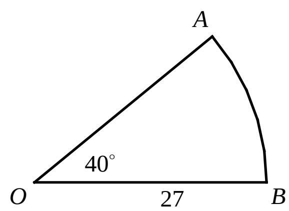
$1 \text{ mile} = 2\pi(12 \text{ in}) \times x$

$1 \times 5280 \times 12 \text{ in} = 2\pi(12 \text{ in})x$ — $1 \text{ mile} = 5280 \text{ ft} = 5280 \times 12 \text{ in}$

$x = \frac{5280 \times 12}{2\pi \times 12} = \frac{2640}{\pi} \approx 840$ — Answer

Exercises - Arc Lengths and Areas of Sectors

Questions 1 and 2 refer to the following information.



In the figure above, $\overset{\frown}{AB}$ is an arc of a circle with radius 27 cm.

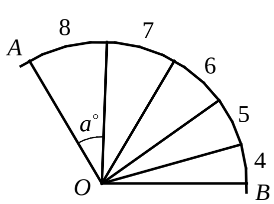
1

If the length of arc AB is $k\pi$, what is the value of k?

2

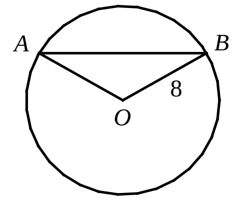
If the area of sector OAB is $n\pi$, what is the value of n?

3



The figure above shows arcs of length 8, 7, 6, 5, and 4. If $m\overset{\frown}{AB} = 120$, what is the degree measure of angle a?

4



In the figure above, the radius of the circle is 8 and $m\angle AOB = 120^\circ$. What is the length of $\overline{AB}$?

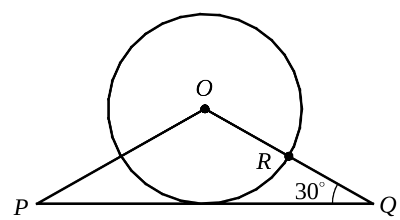
A) $8\sqrt{2}$

B) $8\sqrt{3}$

C) $12\sqrt{2}$

D) $12\sqrt{3}$

5



In the figure above, $OP = OQ$ and $\overline{PQ}$ is tangent to circle O. If the radius of circle O is 8, what is the length of $\overline{QR}$?

A) $10(\sqrt{2} - 1)$

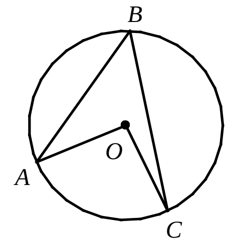
B) 6

C) $10(\sqrt{3} - 1)$

D) 8

19-3. Inscribed Angles

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle.



Theorem - Inscribed Angle

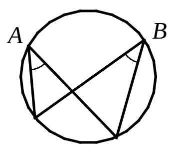
The measure of an inscribed angle is half the measure of its intercepted arc and half the measure of its central angle.

$$m\angle B = \frac{1}{2} m\overset{\frown}{AC} = \frac{1}{2} m\angle AOC$$

Corollaries to the Inscribed Angle Theorem

Corollary 1

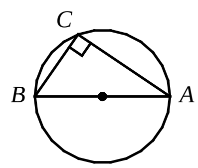
Two inscribed angles that intercept the same arc are congruent.



$\angle A \cong \angle B$

Corollary 2

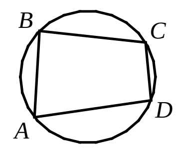
An angle inscribed in a semicircle is a right angle.



$\angle C$ is a right angle.

Corollary 3

If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.



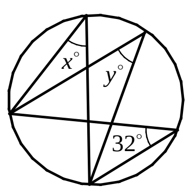
$\angle A$ is supp. to $\angle C$
$\angle B$ is supp. to $\angle D$

Example 1 □ a. In the figure below, find the values of x and y.

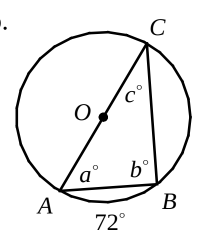
b. In the figure below, AC is a diameter and $m\overset{\frown}{AB} = 72$.
Find the values of a, b, and c.

c. In the figure below, find the values of p and q.

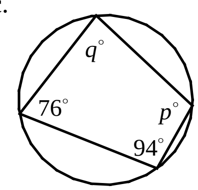
a.



b.



c.



Solution □ a. $x = y = 32$ — Inscribed $\angle s$ that intercept the same arc are $\cong$.

b. $c = 72 \div 2 = 36$ — The measure of an inscribed $\angle$ is half the measure of its intercepted arc.

$b = 90$ — An $\angle$ inscribed in a semicircle is a right $\angle$.

$a + c = 90$ — The acute $\angle s$ of a right Δ are complementary.

$a = 90 - 36 = 54$ — Substitute $c = 36$ and solve for a.

c. $p + 76 = 180$ — If a quad. is inscribed in a circle, its opposite $\angle s$ are supplementary.

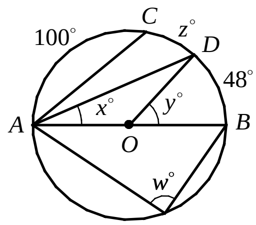
$p = 104$ — Solve for p.

$q + 94 = 180$ — If a quad. is inscribed in a circle, its opposite $\angle s$ are supplementary.

$q = 86$ — Solve for q.

Exercises - Inscribed Angles

Questions 1 - 4 refer to the following information.



In circle O above, $\overline{AB}$ is a diameter.

1

What is the value of y ?

2

What is the value of x ?

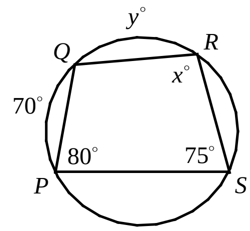
3

What is the value of w ?

4

What is the value of z ?

Questions 5 and 6 refer to the following information.



In the figure above, a quadrilateral is inscribed in a circle.

5

What is the value of x ?

A) 70

B) 80

C) 90

D) 100

6

What is the value of y ?

A) 75

B) 80

C) 85

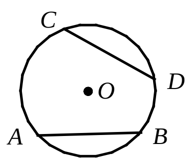
D) 90

19-4. Arcs and Chords

Theorems

Theorem 1

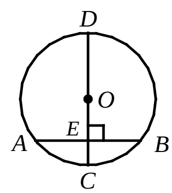
In the same circle or in congruent circles, congruent arcs have congruent chords.



If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\overline{AB} \cong \overline{CD}$.
The converse is also true.

Theorem 2

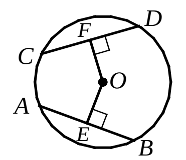
If a diameter is $\perp$ to a chord, it bisects the chord and its arc.



If diameter $\overline{CD} \perp \overline{AB}$, then $\overline{AE} \cong \overline{BE}$ and $\overset{\frown}{AC} \cong \overset{\frown}{BC}$.

Theorem 3

In the same circle or in congruent circles, chords equidistant to the center(s) are congruent.



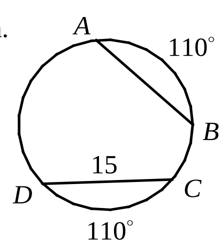
If $OE = OF$, then $\overline{AB} \cong \overline{CD}$.
The converse is also true.

Example 1 □ a. In the figure below, if $m\overset{\frown}{AB} = m\overset{\frown}{CD} = 110$ and $CD = 15$, what is the length of $\overline{AB}$?

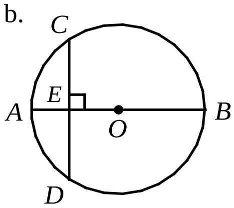
b. In the figure below, $\overline{AB} \perp \overline{CD}$. If $AB = 20$ and $CD = 16$, what is the length of $\overline{OE}$?

c. In the figure below, $OE = OF = 9$ and $BE = 12$. What is the length of $\overline{CD}$?

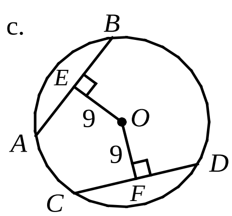
a.



b.



c.



Solution □ a. $AB = CD = 15$ — In the same circle, $\cong$ arcs have $\cong$ chords.

b. $DE = \frac{1}{2}CD = 8$ — If a diameter is $\perp$ to a chord, it bisects the chord.

$OD = OB = \frac{1}{2}AB = 10$ — In a circle, all radii are $\cong$.

$OD^2 = DE^2 + OE^2$ — Pythagorean Theorem

$10^2 = 8^2 + OE^2$ — Substitution

$OE^2 = 36$

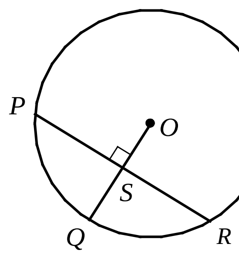
$OE = 6$ — Simplify.

c. $AB = 2BE = 2(12) = 24$ — If a diameter is $\perp$ to a chord, it bisects the chord.

$CD = AB = 24$ — In the same circle, chords equidistant to the center are $\cong$.

Exercises - Arcs and Chords

1



In circle O above, if the radius is 13 and $PR = 24$, what is the length of QS ?

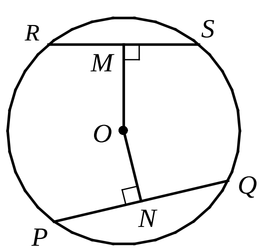
A) 6

B) 7

C) 8

D) 9

2



In the circle above, if $RS = 6$, $OM = 5$, and $ON = 4$, what is the length of PQ ?

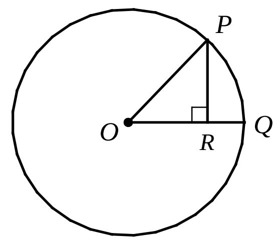
A) $4\sqrt{2}$

B) 6

C) $6\sqrt{2}$

D) $6\sqrt{3}$

3



In circle O above, the area of the circle is 9π and $PR = \sqrt{5}$. What is the length of QR ?

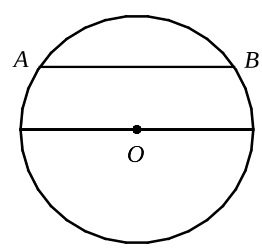
A) 1

B) $\sqrt{2}$

C) $\sqrt{3}$

D) 2

4



In the figure above, the radius of the circle is 12. If the length of chord $\overline{AB}$ is 18, what is the distance between the chord and the diameter?

A) $2\sqrt{10}$

B) $3\sqrt{7}$

C) $4\sqrt{5}$

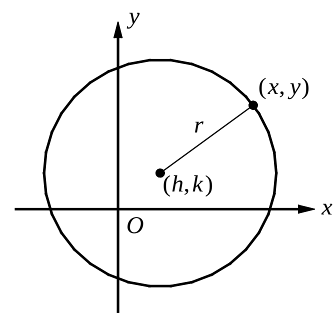
D) $6\sqrt{2}$

19-5. Circles in the Coordinate Plane

Equation of a Circle

The equation of a circle with center (h,k) and radius r is

$\boldsymbol{(x-h)^2+(y-k)^2=r^2}$.



Example 1 □ a. Write an equation of a circle with center $(-3,2)$ and $r=2$.

b. Find the center and radius of a circle with the equation $x^2+y^2-4x+6y-12=0$.

c. Write an equation of a circle that is tangent to the y- axis and has center $(4,3)$.

d. Write an equation of a circle whose endpoints of its diameter are at $(-4,8)$ and $(2,-4)$.

Solution □ a. $(x-h)^2+(y-k)^2=r^2$ Use the standard form of an equation of a circle.

$(x-(-3))^2+(y-2)^2=2^2$ Substitute $(-3,2)$ for (h,k) and 2 for r.

$(x+3)^2+(y-2)^2=4$ Simplify.

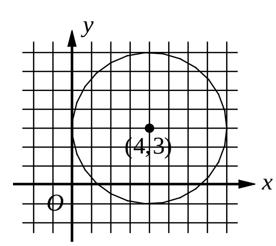
b. $x^2+y^2-4x+6y=12$ Isolate the constant onto one side.

$x^2-4x+4+y^2+6y+9=12+4+9$ Add $(-4\cdot\frac{1}{2})^2=4$ and $(6\cdot\frac{1}{2})^2=9$ to each side.

$(x-2)^2+(y+3)^2=25$ Factor.

The center is $(2,-3)$ and $r=\sqrt{25}=5$.

c. To visualize the circle, draw a sketch.
Since the circle has its center at $(4,3)$ and is tangent to the y- axis, its radius is 4 units.
The equation is $(x-4)^2+(y-3)^2=16$.



d. The center of a circle is the midpoint of its diameter.

$(h,k)=(\frac{-4+2}{2},\frac{8+(-4)}{2})=(-1,2)$

Use the distance formula to find the diameter of the circle.

$d=\sqrt{(2-(-4))^2+(-4-8)^2}=\sqrt{6^2+12^2}=\sqrt{180}=6\sqrt{5}$

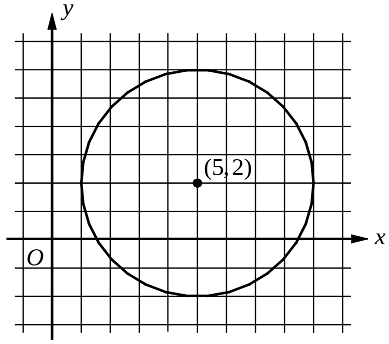
$r=\frac{1}{2}(6\sqrt{5})=3\sqrt{5}$

The equation of the circle is $(x+1)^2+(y-2)^2=(3\sqrt{5})^2$

or $(x+1)^2+(y-2)^2=45$.

Exercises - Circles in the Coordinate Plane

1



Which of the following equations represents the equation of the circle shown in the xy- plane above?

A) $(x+5)^2+(y+2)^2=4$

B) $(x-5)^2+(y-2)^2=4$

C) $(x+5)^2+(y+2)^2=16$

D) $(x-5)^2+(y-2)^2=16$

2

Which of the following is an equation of a circle in the xy- plane with center $(-2,0)$ and a radius with endpoint $(0,\frac{3}{2})$?

A) $x^2+(y-\frac{3}{2})^2=\frac{5}{2}$

B) $x^2+(y-\frac{3}{2})^2=\frac{25}{4}$

C) $(x+2)^2+y^2=\frac{25}{4}$

D) $(x-2)^2+y^2=\frac{25}{4}$

3

$$x^2+12x+y^2-4y+15=0$$

The equation of a circle in the xy- plane is shown above. Which of the following is true about the circle?

A) center $(-6,2)$, radius $=5$

B) center $(6,-2)$, radius $=5$

C) center $(-6,2)$, radius $=\sqrt{15}$

D) center $(6,-2)$, radius $=\sqrt{15}$

4

Which of the following represents an equation of a circle whose diameter has endpoints $(-8,4)$ and $(2,-6)$?

A) $(x-3)^2+(y-1)^2=50$

B) $(x+3)^2+(y+1)^2=50$

C) $(x-3)^2+(y-1)^2=25$

D) $(x+3)^2+(y+1)^2=25$

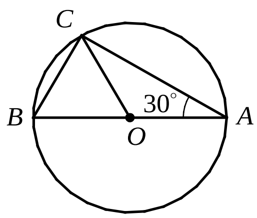
5

$$x^2+2x+y^2-4y-9=0$$

The equation of a circle in the xy- plane is shown above. If the area of the circle is $k\pi$, what is the value of k ?

Chapter 19 Practice Test

1



In the figure above, O is the center of the circle and $\overline{AB}$ is a diameter. If the length of $\overline{AC}$ is $4\sqrt{3}$ and $m\angle BAC = 30$, what is the area of circle O?

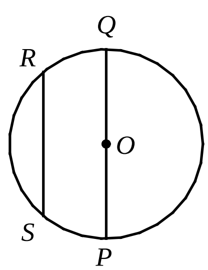
A) 12π

B) 16π

C) 18π

D) 24π

2



In the circle above, chord $\overline{RS}$ is parallel to diameter $\overline{PQ}$. If the length of $\overline{RS}$ is $\frac{3}{4}$ of the length of $\overline{PQ}$ and the distance between the chord and the diameter is $2\sqrt{7}$, what is the radius of the circle?

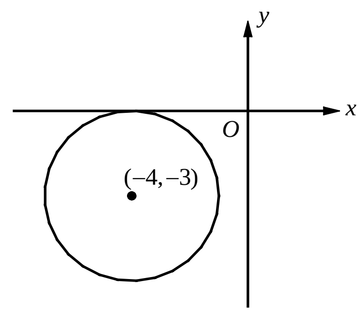
A) 6

B) $3\sqrt{7}$

C) 8

D) $4\sqrt{7}$

3



In the figure above, the circle is tangent to the x-axis and has center $(-4,-3)$. Which of the following equations represents the equation of the circle shown in the xy-plane above?

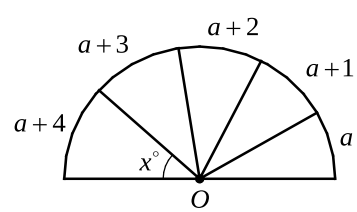
A) $(x+4)^2+(y+3)^2=9$

B) $(x-4)^2+(y-3)^2=9$

C) $(x+4)^2+(y+3)^2=3$

D) $(x-4)^2+(y-3)^2=3$

4



The figure above shows a semicircle with the lengths of the adjacent arcs a, $a+1$, $a+2$, $a+3$, and $a+4$. If the value of x is 42, what is the value of a?

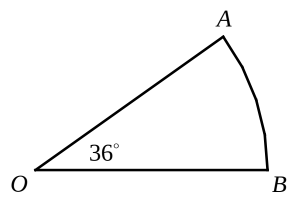
A) 7

B) 8

C) 9

D) 10

5



In the figure above, the length of arc $\overset{\frown}{AB}$ is π. What is the area of sector OAB?

A) 2π

B) $\frac{5}{2}\pi$

C) 3π

D) $\frac{7}{2}\pi$

6

$$x^2 - 4x + y^2 - 6x - 17 = 0$$

What is the area of the circle in the xy-plane above?

A) 20π

B) 24π

C) 26π

D) 30π

7

Which of the following is the equation of a circle that has a diameter of 8 units and is tangent to the graph of $y = 2$?

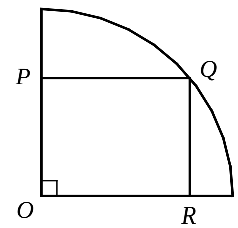
A) $(x+1)^2 + (y+2)^2 = 16$

B) $(x-1)^2 + (y-2)^2 = 16$

C) $(x+2)^2 + (y+1)^2 = 16$

D) $(x-2)^2 + (y-1)^2 = 16$

8



In the figure above, rectangle $OPQR$ is inscribed in a quarter circle that has a radius of 9. If $PQ = 7$, what is the area of rectangle $OPQR$?

A) $24\sqrt{2}$

B) $26\sqrt{2}$

C) $28\sqrt{2}$

D) $30\sqrt{2}$

9

In a circle with center O, the central angle has a measure of $\frac{2\pi}{3}$ radians. The area of the sector formed by central angle AOB is what fraction of the area of the circle?

10

A wheel with a radius of 2.2 feet is turning at a constant rate of 400 revolutions per minute on a road. If the wheel traveled $k\pi$ miles in one hour what is the value of k? (1 mile $=$ 5,280 feet)

Answer Key

Section 19-1

1. 38	2. 38	3. 135	4. 9	5. D
6. C				

Section 19-2

1. 6	2. 81	3. 32	4. B	5. D

Section 19-3

1. 48	2. 24	3. 90	4. 32	5. D
6. B				

Section 19-4

1. C	2. C	3. A	4. B

Section 19-5

1. D	2. C	3. A	4. B	5. 14

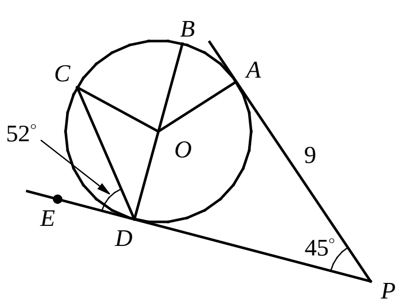
Chapter 19 Practice Test

1. B	2. C	3. A	4. D	5. B
6. D	7. A	8. C	9. $\frac{1}{3}$	10. 20

Answers and Explanations

Section 19-1

1. 38



$\overline{PD} \perp \overline{OD}$ Tangent to a $\odot$ is $\perp$ to radius.
$m\angle ODE = 90$ A right $\angle$ measures 90.
$m\angle ODC = 90 - 52$
$= 38$

2. 38

$OC = OD$ In a $\odot$ all radii are $\cong$.
$m\angle OCD = m\angle ODC$ Isosceles Triangle Theorem
$= 38$

3. 135

If a line is tangent to a circle, the line is $\perp$ to the radius at the point of tangency. Therefore, $m\angle ODP = m\angle OAP = 90$.
The sum of the measures of interior angles of quadrilateral is 360. Therefore,
$m\angle AOD + m\angle ODP + m\angle OAP + m\angle P = 360$.

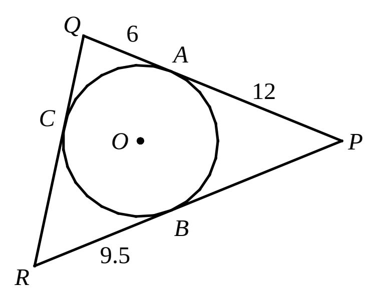
$m\angle AOD + 90 + 90 + 45 = 360$ Substitution
$m\angle AOD + 225 = 360$ Simplify.
$m\angle AOD = 135$

4. 9

Tangents to a circle from the same exterior point are congruent. Therefore,

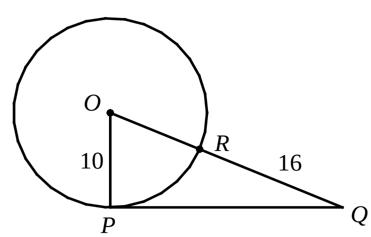
$PD = PA = 9$.

5. D



Since tangents to a circle from the same exterior point are congruent, $QA = QC = 6$, $PA = PB = 12$, and $RB = RC = 9.5$. Therefore,
Perimeter of $\Delta PQR = 2(6 + 12 + 9.5) = 55$

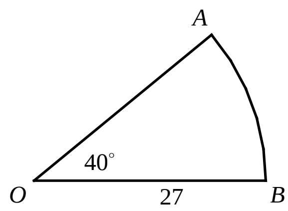
6. C



$OR = OP = 10$ In a $\odot$ all radii are $\cong$.
$OQ = OR + RQ$ Segment Addition Postulate
$= 10 + 16 = 26$
$PQ^2 + OP^2 = OQ^2$ Pythagorean Theorem
$PQ^2 + 10^2 = 26^2$ Substitution
$PQ^2 = 26^2 - 10^2 = 576$
$PQ = \sqrt{576} = 24$

Section 19-2

1. 6



Length of arc $AB = 2\pi r \cdot \dfrac{m\angle AOB}{360}$

$= 2\pi(27)\cdot\dfrac{40}{360} = 6\pi$

Thus, $k = 6$.

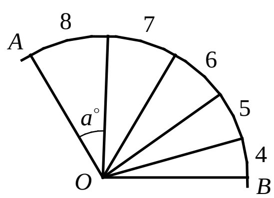
2. 81

Area of sector $OAB = \pi r^2 \cdot \dfrac{m\angle AOB}{360}$

$= \pi(27)^2 \cdot \dfrac{40}{360} = 81\pi$

Thus, $n = 81$.

3. 32



The length of arc $AB = 8+7+6+5+4 = 30$
In a circle, the lengths of the arcs are proportional to the degree measures of the corresponding arcs.

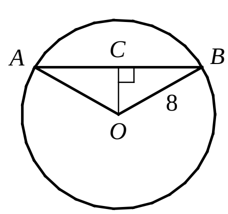
Therefore, $\dfrac{\text{length of arc } AB}{120^\circ} = \dfrac{8}{a^\circ}$.

$\dfrac{30}{120} = \dfrac{8}{a}$ Substitution

$30a = 120\times 8$ Cross Products

$a = 32$

4. B



Draw $\overline{OC}$ perpendicular to $\overline{AB}$. Since ΔAOB is an isosceles triangle, $\overline{OC}$ bisects $\angle AOB$.

$m\angle AOC = m\angle BOC = \dfrac{1}{2}m\angle AOB = \dfrac{1}{2}(120) = 60$.

ΔBOC is a 30°-60°-90° triangle.

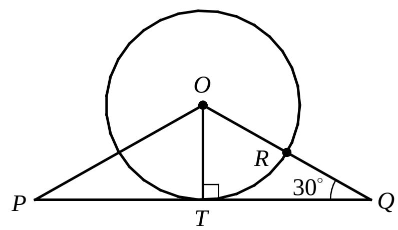
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

$OC = \dfrac{1}{2}OB = \dfrac{1}{2}(8) = 4$

$BC = \sqrt{3}\cdot OC = 4\sqrt{3}$

$AB = 2BC = 2\times 4\sqrt{3} = 8\sqrt{3}$

5. D



Let T be a point of tangency. Then $\overline{PQ}\perp\overline{OT}$, because a line tangent to a circle is $\perp$ to the radius at the point of tangency.

ΔOQT is a 30°-60°-90° triangle.

$OT = OR = 8$ In a $\odot$ all radii are $\cong$.

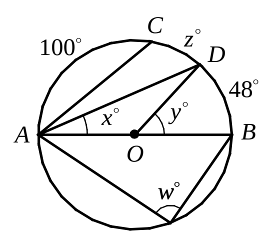
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg. Therefore,

$OQ = 2OT = 2(8) = 16$

$QR = OQ - OR = 16 - 8 = 8$

Section 19-3

1. 48



The measure of a minor arc is the measure of its central angle. Therefore, $y = 48$.

2. 24

The measure of an inscribed angle is half the measure of its intercepted arc.

Therefore, $x = \dfrac{1}{2}(48) = 24$.

3. 90

An angle inscribed in a semicircle is a right angle. Therefore, $w = 90$.

4. 32

The measure of a semicircle is 180, thus $m\overset{\frown}{ACB} = 180$.
The measure of an arc formed by two adjacent arcs is the sum of the measure of the two arcs, thus

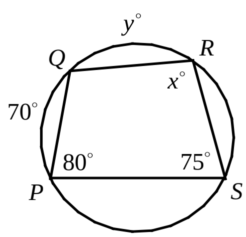
$m\overset{\frown}{ACB} = m\overset{\frown}{AC} + m\overset{\frown}{CD} + m\overset{\frown}{DB}$

$180 = 100 + z + 48$ Substitution

$180 = 148 + z$ Simplify.

$32 = z$

5. D



If a quadrilateral is inscribed in a circle, its opposite angles are supplementary. Therefore,
$x + 80 = 180$.
$x = 100$

6. B

The measure of an inscribed angle is half the measure of its intercepted arc. Therefore,

$m\angle RSP = \frac{1}{2}(m\overset{\frown}{PQ} + m\overset{\frown}{QR})$.

$75 = \frac{1}{2}(70 + y)$ Substitution

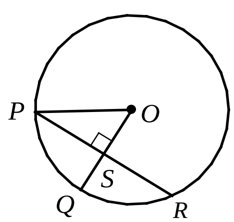
$2 \cdot 75 = 2 \cdot \frac{1}{2}(70 + y)$ Multiply each side by 2.

$150 = 70 + y$ Simplify.

$80 = y$

Section 19-4

1. C



If a diameter is $\perp$ to a chord, it bisects the chord and its arc. Therefore,

$PS = \frac{1}{2}PR = \frac{1}{2}(24) = 12$.

The radius of the circle is 13, thus $OP = OQ = 13$.

Draw $\overline{OP}$.

$OS^2 + PS^2 = OP^2$ Pythagorean Theorem

$OS^2 + 12^2 = 13^2$ Substitution

$OS^2 = 13^2 - 12^2 = 25$

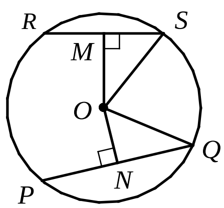
$OS = \sqrt{25} = 5$

$QS = OQ - OS$

$= 13 - 5$

$= 8$

2. C



Draw $\overline{OS}$ and $\overline{OQ}$.
If a diameter is $\perp$ to a chord, it bisects the chord and its arc. Therefore,

$MS = \frac{1}{2}RS = \frac{1}{2}(6) = 3$ and $PQ = 2NQ$.

$OS^2 = MS^2 + OM^2$ Pythagorean Theorem

$OS^2 = 3^2 + 5^2$ Substitution

$OS^2 = 34$

$OS = \sqrt{34}$

$OQ = OS = \sqrt{34}$ In a $\odot$ all radii are $\cong$.

$OQ^2 = ON^2 + NQ^2$ Pythagorean Theorem

$(\sqrt{34})^2 = 4^2 + NQ^2$ Substitution

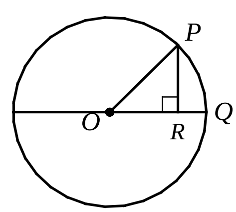
$34 = 16 + NQ^2$

$18 = NQ^2$

$NQ = \sqrt{18} = 3\sqrt{2}$

$PQ = 2NQ = 2(3\sqrt{2}) = 6\sqrt{2}$

3. A



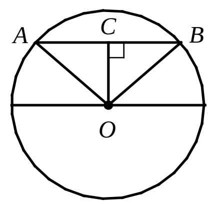
Area of the circle $= \pi r^2 = 9\pi$.

$\Rightarrow \; r^2 = 9 \; \Rightarrow \; r = 3$

Therefore, $OP = OQ = 3$.

$OR^2 + PR^2 = OP^2$	Pythagorean Theorem
$OR^2 + (\sqrt{5})^2 = 3^2$	Substitution
$OR^2 + 5 = 9$	Simplify.
$OR^2 = 9 - 5 = 4$	
$OR = \sqrt{4} = 2$	
$QR = OQ - OR = 3 - 2 = 1$	

4. B



Draw $\overline{OA}$ and $\overline{OB}$. Draw $\overline{OC} \perp$ to $\overline{AB}$.

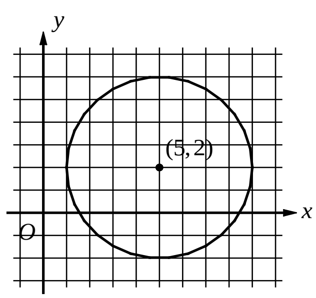
OC is the distance between the chord and the diameter.

$BC = \frac{1}{2}AB = \frac{1}{2}(18) = 9$

$OC^2 + BC^2 = OB^2$	Pythagorean Theorem
$OC^2 + 9^2 = 12^2$	Substitution
$OC^2 = 12^2 - 9^2 = 63$	
$OC = \sqrt{63}$	
$= \sqrt{9} \cdot \sqrt{7}$	
$= 3\sqrt{7}$	

Section 19-5

1. D



The equation of a circle with center (h,k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

The center of the circle shown above is $(5,2)$ and the radius is 4. Therefore, the equation of the circle is $(x-5)^2 + (y-2)^2 = 4^2$.

2. C

Use the distance formula to find the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad (x_1, y_1) = (-2, 0)$

$= \sqrt{(0-(-2))^2 + (\frac{3}{2} - 0)^2}$ $\quad (x_2, y_2) = (0, \frac{3}{2})$

$= \sqrt{4 + \frac{9}{4}}$ $\quad$ Simplify.

$= \sqrt{\frac{16}{4} + \frac{9}{4}} = \sqrt{\frac{25}{4}}$

Therefore, the equation of the circle is

$(x-(-2))^2 + (y-0)^2 = (\sqrt{\frac{25}{4}})^2$.

Choice C is correct.

3. A

$x^2 + 12x + y^2 - 4y + 15 = 0$

Isolate the constant onto one side.

$x^2 + 12x + y^2 - 4y = -15$

Add $(12 \cdot \frac{1}{2})^2 = 36$ and $(-4 \cdot \frac{1}{2})^2 = 4$ to each side.

$(x^2 + 12x + 36) + (y^2 - 4y + 4) = -15 + 36 + 4$

Complete the square.

$(x+6)^2 + (y-2)^2 = 25$

The center of the circle is $(-6, 2)$ and the radius is $\sqrt{25}$, or 5.

4. B

The center of the circle is the midpoint of the diameter. Use the midpoint formula to find the center of the circle.

$(h,k) = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

$= (\frac{-8+2}{2}, \frac{4+(-6)}{2}) = (-3, -1)$

The radius is half the distance of the diameter. Use the distance formula to find the diameter.

$d = \sqrt{(2-(-8))^2 + (-6-4)^2} = \sqrt{100+100}$

$= \sqrt{200} = \sqrt{100} \cdot \sqrt{2} = 10\sqrt{2}$

$r = \frac{1}{2}d = \frac{1}{2}(10\sqrt{2}) = 5\sqrt{2}$

Therefore, the equation of the circle is

$(x-(-3))^2 + (y-(-1))^2 = (5\sqrt{2})^2$, or

$(x+3)^2 + (y+1)^2 = 50$.

5. 14

$x^2+2x+y^2-4y-9=0$
Isolate the constant onto one side.
$x^2+2x+y^2-4y=9$
Add $(2\cdot\frac{1}{2})^2=1$ and $(-4\cdot\frac{1}{2})^2=4$ to each side.
$(x^2+2x+1)+(y^2-4y+4)=9+1+4$
Complete the square.
$(x+1)^2+(y-2)^2=14$
The radius of the circle is $\sqrt{14}$.
Area of the circle is $\pi r^2=\pi(\sqrt{14})^2=14\pi$.
Therefore, $k=14$.

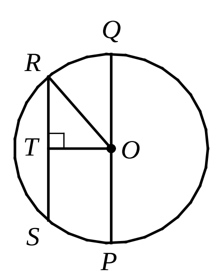
Chapter 19 Practice Test

1. B

An angle inscribed in a semicircle is a right angle. Therefore, $\angle ACB=90$.
So, ΔABC is a 30°-60°-90° triangle.
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

$AC=\sqrt{3}BC$
$4\sqrt{3}=\sqrt{3}BC$ $\quad AC=4\sqrt{3}$
$4=BC$
$AB=2BC=2(4)=8$
Therefore, the radius of circle O is 4.
Area of circle $O=\pi(4)^2=16\pi$

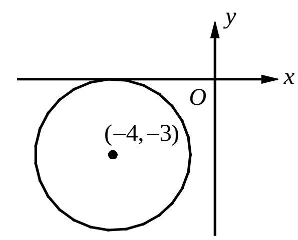
2. C



Draw $\overline{OR}$ and $\overline{OT}$ as shown above. Let the radius of the circle be r, then $OQ=OR=r$.
Since the ratio of RS to QP is 3 to 4, the ratio of RT to OQ is also 3 to 4.
Therefore, $RT=\frac{3}{4}OQ=\frac{3}{4}r$.
OT is the distance between the chord and the diameter, which is given as $2\sqrt{7}$.

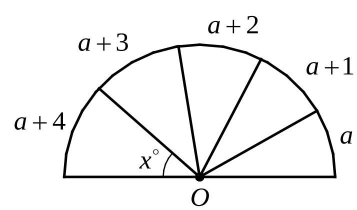
$OR^2=RT^2+OT^2$ Pythagorean Theorem
$r^2=(\frac{3}{4}r)^2+(2\sqrt{7})^2$ Substitution
$r^2=\frac{9}{16}r^2+28$ Simplify.
$r^2-\frac{9}{16}r^2=28$
$\frac{7}{16}r^2=28$
$\frac{16}{7}\cdot\frac{7}{16}r^2=\frac{16}{7}\cdot 28$
$r^2=64$
$r=\sqrt{64}=8$

3. A



If the center of the circle is $(-4,-3)$ and the circle is tangent to the x- axis, the radius is 3.
The equation is $(x-(-4))^2+(y-(-3))^2=3^2$,
or $(x+4)^2+(y+3)^2=9$.

4. D



The arc length of the semicircle is
$(a+4)+(a+3)+(a+2)+(a+1)+a=5a+10$.
In a circle, the lengths of the arcs are proportional to the degree measures of the corresponding arcs.
Therefore, $\frac{\text{arc length of semicircle}}{180^\circ}=\frac{a+4}{x^\circ}$.

$\frac{5a+10}{180}=\frac{a+4}{42}$ Substitution
$42(5a+10)=180(a+4)$ Cross Products
$210a+420=180a+720$
$30a=300$
$a=10$

5. B

Length of arc $AB = 2\pi r \cdot \frac{m\angle AOB}{360}$

$= 2\pi r \cdot \frac{36}{360} = \frac{\pi r}{5}$

Since the length of the arc is given as π,

$\frac{\pi r}{5} = \pi$. Solving the equation for r gives $r = 5$.

Area of sector $AOB = \pi r^2 \cdot \frac{m\angle AOB}{360}$

$= \pi(5)^2 \cdot \frac{36}{360} = \frac{5}{2}\pi$

6. D

$x^2 - 4x + y^2 - 6x - 17 = 0$

$x^2 - 4x + y^2 - 6x = 17$

To complete the square, add $(-4 \cdot \frac{1}{2})^2 = 4$ and

$(-6 \cdot \frac{1}{2})^2 = 9$ to each side.

$x^2 - 4x + 4 + y^2 - 6x + 9 = 17 + 4 + 9$

$(x-2)^2 + (y-3)^2 = 30$

The radius of the circle is $\sqrt{30}$, the area of the circle is $\pi(\sqrt{30})^2 = 30\pi$

7. A

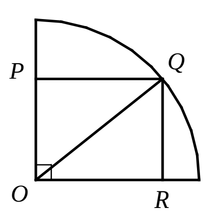
If the diameter of the circle is 8 units, the radius of the circle is 4 units. Since the radius of the circle is 4 units, the y- coordinate of the center has to be 4 units above or below $y = 2$.

The y- coordinate of the center has to be either 6 or -2. Among the answer choices, only choice A has -2 as the y- coordinate.

No other answer choice has 6 or -2 as the y- coordinate of the center.

Choice A is correct.

8. C



Draw $\overline{OQ}$. Since $\overline{OQ}$ is a radius, $OQ = 9$.

$OP^2 + PQ^2 = OQ^2$ Pythagorean Theorem

$OP^2 + 7^2 = 9^2$ Substitution

$OP^2 = 9^2 - 7^2 = 32$

$OP = \sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

Area of rectangle $OPQR = OP \times PQ$

$= 4\sqrt{2} \times 7 = 28\sqrt{2}$

9. $\frac{1}{3}$

Area of sector $AOB = \pi r^2 \cdot \frac{m\angle AOB}{360}$

The area of a sector is the fractional part of the area of a circle. The area of a sector formed by $\frac{2\pi}{3}$ radians of arc is $\frac{2\pi/3}{2\pi}$, or $\frac{1}{3}$, of the area of the circle.

10. 20

The distance the wheel travels in 1 minute is equal to the product of the circumference of the wheel and the number of revolutions per minute.

The distance the wheel travels in 1 minute

$= 2\pi r \times$ the number of revolutions per minute

$= 2\pi(2.2 \text{ ft}) \times 400 = 1{,}760\pi$ ft

Total distance traveled in 1 hour

$= 1{,}760\pi \text{ ft} \times 60 = 105{,}600\pi$ ft

$= 105{,}600\pi \text{ ft} \times \frac{1 \text{ mile}}{5{,}280 \text{ ft}} = 20\pi$ miles

Thus, $k = 20$.