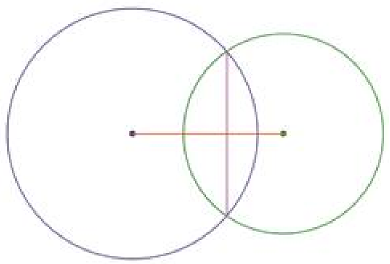
5. Circles

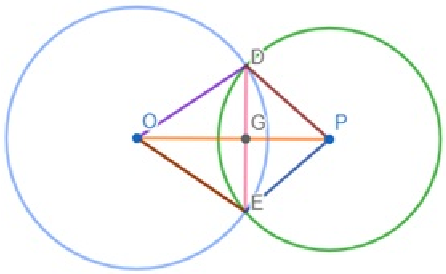
Questions Pg-82

1. Question

Prove that the line joining the centres of two intersecting circles is the perpendicular bisector of the line joining the points of intersection.



Answer



O is the centre of the bigger circle and P is the centre of the smaller circle. DE is the line joining the points of intersection. We have to prove that, OP is the perpendicular bisector of DE.

In ΔODP and ΔOEP we have,

OD = OE [radius of same circle]

PD = PE [radius of same circle]

OP is the common side.

$\therefore \Delta ODP \cong \Delta OEP$ [SSS congruency]

$\therefore \angle DOP = \angle EOP$

$\therefore \angle DOG = \angle EOG$ ……. (i)

Now in ΔODG and ΔOEG we have,

OD = OE [radius of same circle]

$\angle DOG = \angle EOG$ from (i)

OG is the common side.

$\therefore \Delta ODG \cong \Delta OEG$ [SAS congruency]

$\therefore$ DG = EG

$\therefore \angle OGD = \angle OGE$

We know, $\therefore \angle OGD + \angle OGE = 180°$

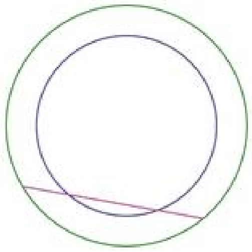
$\therefore \angle OGD = \angle OGE = 90°$

$\therefore$ DG = EG and $\angle OGD = \angle OGE = 90°$

$\therefore$ OP is perpendicular bisector of DE

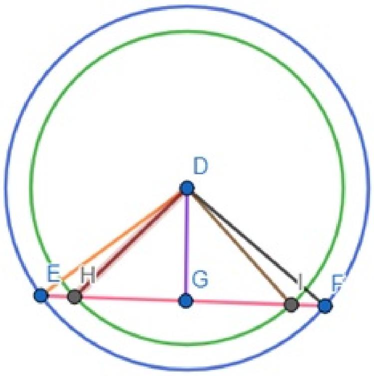
2. Question

The picture on the right shows two circles centred on the same point and a line intersecting them.



Prove that the parts of the line between the circles on either side are equal.

Answer



D is the centre of both the circles.

DG is a perpendicular drawn on EF(HI).

We have to prove that, EH = IF.

In ΔDHG and ΔDIG we have,

DH = DI [radius of the same circle]

DG is the common side.

DG is perpendicular on HI.

∴ ∠DHG = ∠DIG = 90°

∴ ΔDHG≅ΔDIG [SAS congruency]

∴ GH = GI .... (1)

In ΔDEG and ΔDFG we have,

DE = DF [radius of the same circle]

DG is the common side.

DG is perpendicular on EF.

∴ ∠DEG = ∠DFG = 90°

∴ ΔDEG≅ΔDFG [SAS congruency]

∴ GE = GF .... (2)

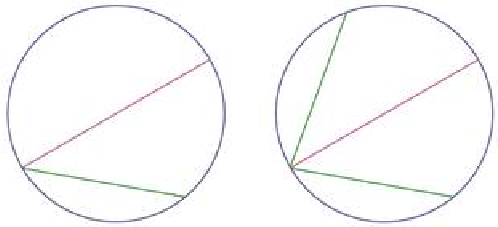
From, (2) – (1) we get,

⇒ GE – GH = GF – GI

⇒ EH = IF

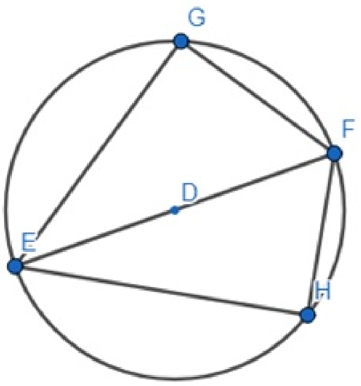
3. Question

A chord and the diameter through one of its ends are drawn in a circle. A chord of the same inclination is drawn on the other side of the diameter.



Prove that the chords are of the same length.

Answer



EF is the diameter of the circle.

Chord EG and EH both have same inclination. GF and HF are joined.

We have to prove that, EG = EH.

In ΔGEF and ΔHEF we have,

∠GEF = ∠HEF [∵ both have same inclination]

EF is the common side.

∠EGF = ∠EHF = 90° [∵ both are angle inscribed in a semi circle]

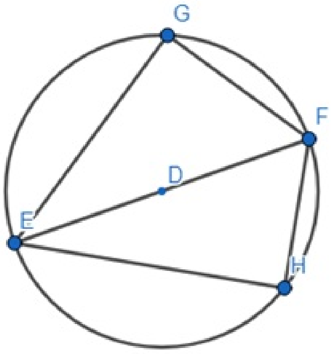
∴ΔGEF≅ΔHEF [AAS congruency]

∴ EG = EH [similar sides of congruent triangles]

4. Question

Prove that the angle made by two equal chords drawn from a point on the circle is bisected by the diameter through that point.

Answer



EG and EH are two equal chords drawn from point E.

EF is the diameter of the circle. GF and HF are joined.

Angle between two chords = ∠GEH.

We have to prove that, ∠GEF = ∠HEF.

In ΔGEF and ΔHEF we have,

EG = EH [given in the problem]

EF is the common side.

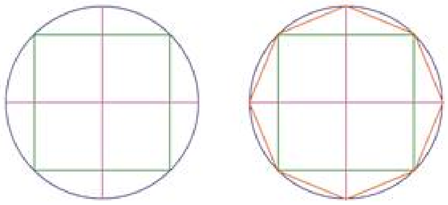
∠EGF = ∠EHF = 90° [∵ both are angle inscribed in a semi circle]

∴ΔGEF≅ΔHEF [SAS congruency]

∴ ∠GEF = ∠HEF [similar angles of congruent triangle]

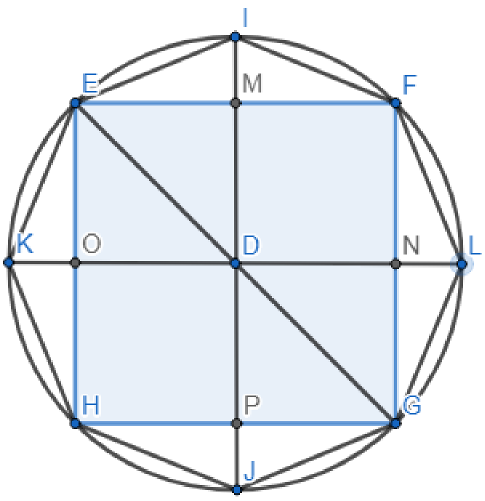
5. Question

Draw a square and a circle through all four vertices. Draw diameters parallel to the sides of the square and draw a polygon joining the end points of these diameters and the vertices of the square.



Prove that this polygon is a regular octagon.

Answer



D is the centre of the circle. EFGH is the inscribed square.

Let, side of the square = a

∴ EF = FG = GH = HE = PM = NO = a

∴ Diagonal of the square = a√2

∴ EG = a√2

Now we have, EG = IJ = KL [diameter of the circle]

∴ EG = IJ = KL = a√2

IJ = a√2 and PM = a

∴ $IM = \frac{1}{2} \times (a\sqrt{2} - a)$ [∵IM = PJ]

$$\therefore\ IM = \frac{a(\sqrt{2} - 1)}{2}$$

In ΔIME,

EM = a/2 [∵ EM = MF = EF/2]

$$IM = \frac{a(\sqrt{2}-1)}{2}$$

∠IME = 90°

∴ tan ∠MIE = EM/IM

$$\Rightarrow \tan\angle MIE = \frac{\frac{a}{2}}{\frac{a(\sqrt{2}-1)}{2}}$$

$$\Rightarrow \tan\angle MIE = \frac{1}{\sqrt{2}-1}$$

∴ tan ∠MIE = √2 + 1

∴ tan ∠MIE = tan 67.5°

∴ ∠MIE = 67.5°……… (1)

Similarly in ΔIMF,

∠MIF = 67.5°

∴ Inner angle of the polygon,

⇒ ∠EIF = ∠MIE + ∠MIF = 67.5°+67.5° = 135°

Similarly all the inner angles of the polygon = 135°

∴ All the outer angles of the Polygon = 180° - 135° = 45°

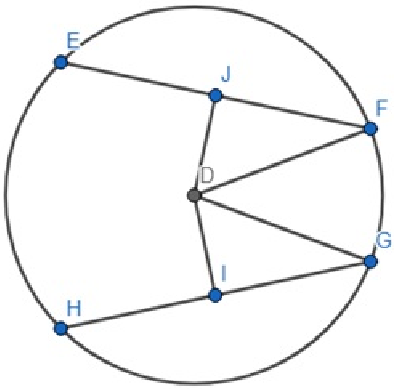
∴ It is a regular polygon.

Questions Pg-86

1. Question

Prove that chords of the same length in a circle are at the same distance from the centre.

Answer



D is the centre of the circle. EF and GH are two chords of same length.

DI and DJ are two perpendiculars drawn on GH and EF respectively.

We have to prove that, DI = DJ.

In ΔDJF and ΔDIG we have,

DF = DG [radius of the same circle]

∠DJF = ∠DIG = 90° [∵DI and DJ are perpendiculars on GH and EF]

JF = IG [∵Perpendicular drawn from the centre bisects the chords]

∴ΔDJF≅ΔDIG [RHS congruency]

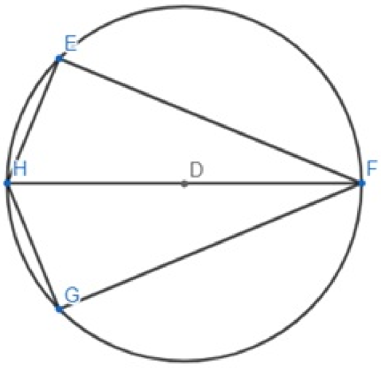
∴ DI = DJ [similar sides of congruent triangle]

∴ The chords are at same distance from the centre.

2. Question

Two chords intersect at a point on a circle and the diameter through this point bisects the angle between the chords. Prove that the chords have the same length.

Answer



EF and FG are two chord meet at point F on circle centred at D.

FH is the diameter through point F.

We have to prove that EF = FG.

We have joined EH and HG.

In ΔEHF and ΔGHF we have,

HF is the common side.

∠EFH = ∠ GFH [HF bisects the angle between two chords]

∠HEF = ∠HGF = 90° [both are angle inscribed on semicircle]

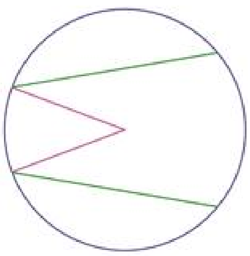
∴ΔEHF≅ΔGHF [RHS congruency]

∴ EF = FG [similar side of congruent triangle]

∴ The chords have same length.

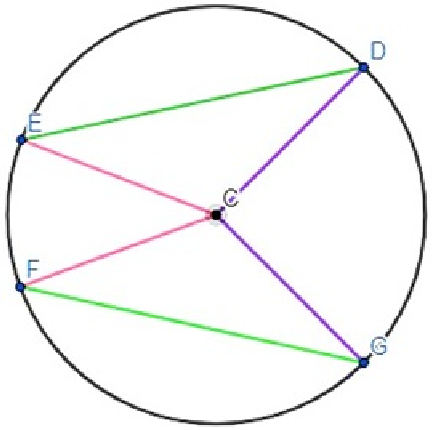
3. Question

In the picture on the right, the angles between the radii and the chords are equal.



Prove that the chords are of the same length.

Answer



C is the radius of the circle. ED and FG are two chords.

In ΔCDE we have,

EC = CD [radius of the circle]

∴ ∠CDE = ∠CED ……… (1)

In ΔCFG we have,

FC = CG [radius of the circle]

∴ ∠CGF = ∠CFG ……… (2)

In ΔCDE and ΔCFG we have,

∠CED = ∠CFG [given in the problem] ……… (3)

∠CDE = ∠CGF [from (1), (2) and (3)]

CE = CF [radius of the same circle]

∴ΔCDE≅ΔCFG [AAS congruency]

∴ ED = FG [similar sides of the triangle.

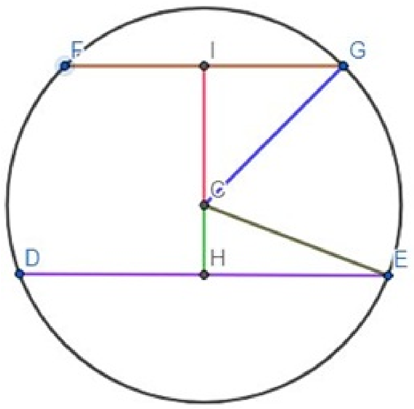
∴ The chords are of same length.

Questions Pg-87

1. Question

In a circle, a chord 1 centimetre away from the centre is 6 centimetres long. What is the length of a chord 2 centimetres away from the centre?

Answer



O is centre of the circle.

DE is the chord 1 cm away from centre. CH is perpendicular on DE.

∴ CH = 1 cm and DE = 6 cm

FG is the chord 2 cm away from centre. CI is perpendicular on FG.

∴ CI = 2 cm

From centre C, CE and CG are joined.

CE and CG both are radius of the circle.

In ΔCHE we have,

∠CHE = 90° [∵CH is perpendicular on DE]

CH = 1 cm

HE = DE/2 = 3 cm [∵perpendicular drawn from centre bisects chord]

$\therefore \text{CE} = \sqrt{(1)^2 + (3)^2} = \sqrt{10}\ \text{cm}$

In ΔCGI we have,

∠CIG = 90° [∵CI is perpendicular on FG]

CI = 2 cm

CG = CE = √10 cm

$\therefore \text{IG} = \sqrt{\left(\sqrt{10}\right)^2 - (2)^2} = \sqrt{6}\ \text{cm}$

∴ FG = 2 × IG = 2√6 cm [∵perpendicular drawn from centre bisects chord]

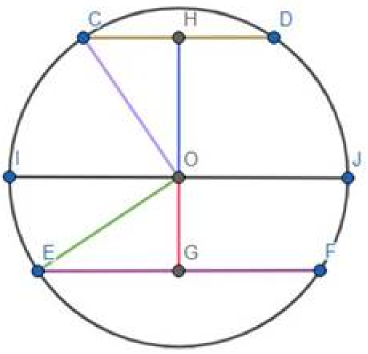
∴ The length of the chord is = 2√6 cm

2. Question

In a circle of radius 5 centimetres, two parallel chords of lengths 6 and 8 centimetres are drawn on either side of a diameter. What is the distance between them? If parallel chords of these lengths are drawn on the same side of a diameter, what would be the distance between them?

Answer

When chords are drawn on either side of diameter.



O is the centre of the circle and IJ is the diameter.

CD and EF are two parallel chord on either side of the diameter.

CD = 6 cm and EF = 8 cm

OH is the perpendicular drawn on CD from centre.

OG is the perpendicular drawn on EF from centre.

In ΔOGE we have,

∠OGE = 90° [∵ OG is perpendicular on EF]

EG = EF/2 = 4 cm [∵perpendicular drawn from centre bisects chord]

OE = 5 cm [radius]

$\therefore OG = \sqrt{(5)^2 - (4)^2} = 3 \text{ cm}$

In ΔOHC we have,

∠OHC = 90° [∵ OH is perpendicular on CD]

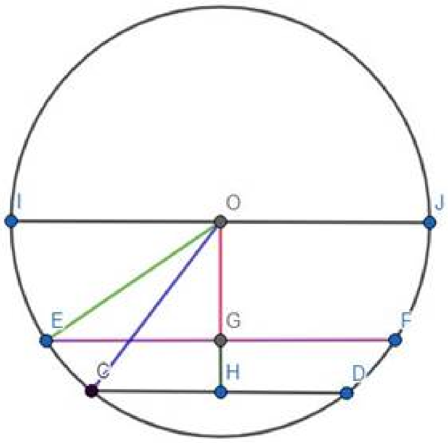
HC = CD/2 = 3 cm [∵perpendicular drawn from centre bisects chord]

OC = 5 cm [radius]

$\therefore OH = \sqrt{(5)^2 - (3)^2} = 4 \text{ cm}$

∴ Distance between the chords = HG = OH + OG = 4 + 3 = 7 cm

When the chords are drawn on same side of diameter:



O is the centre of the circle and IJ is the diameter.

CD and EF are two parallel chords on same side of the diameter.

CD = 6 cm and EF = 8 cm

OH is the perpendicular drawn on CD from centre.

OG is the perpendicular drawn on EF from centre.

In ΔOGE we have,

∠OGE = 90° [∵ OG is perpendicular on EF]

EG = EF/2 = 4 cm [∵perpendicular drawn from centre bisects chord]

OE = 5 cm [radius]

$\therefore OG = \sqrt{(5)^2 - (4)^2} = 3 \text{ cm}$

In ΔOHC we have,

∠OHC = 90° [∵ OH is perpendicular on CD]

HC = CD/2 = 3 cm [∵perpendicular drawn from centre bisects chord]

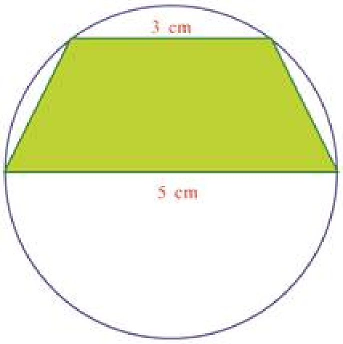
OC = 5 cm [radius]

$\therefore OH = \sqrt{(5)^2 - (3)^2} = 4 \text{ cm}$

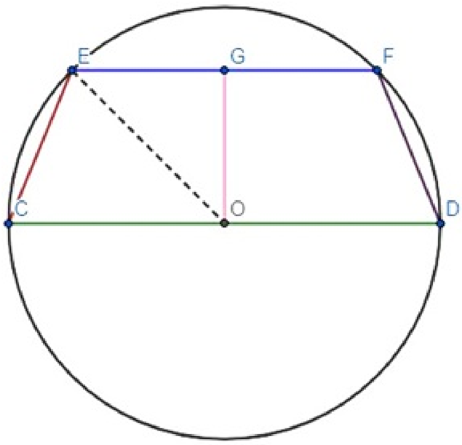
∴ Distance between the chords = HG = OH – OG = 4 – 3 = 1 cm

3. Question

The bottom side of the quadrilateral in the picture is a diameter of the circle and the top side is a chord parallel to it. Calculate the area of the quadrilateral.



Answer



O is the centre of the circle.

CD is the diameter, CD = 5 cm

EF is the chord parallel to CD, EF = 3 cm

OG is perpendicular drawn on EF from O.

In ΔOGE we have,

∠OGE = 90° [∵ OG is perpendicular on EF]

OE = 5/2 = 2.5 cm [∵ Diameter = 5 cm]

EG = 3/2 = 1.5 cm [∵perpendicular drawn from centre bisects chord]

$\therefore \text{OG} = \sqrt{(2.5)^2 - (1.5)^2} = \sqrt{6.25 - 2.25} = 2 \text{ cm}$

∴ Area of the quadrilateral,

$= \frac{\text{EF} + \text{CD}}{2} \times \text{OG}$

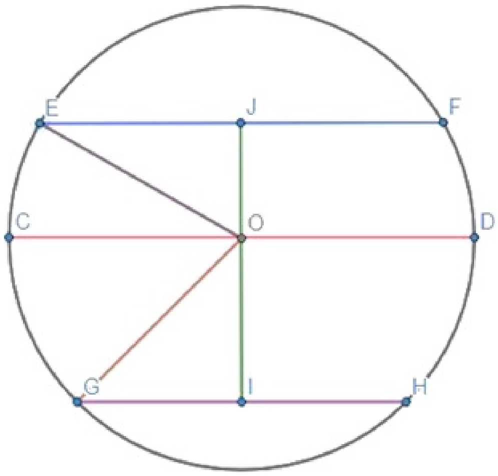
$= \frac{3 + 5}{2} \times 2$

= 8 cm^2

4. Question

In a circle, two parallel chords of lengths 4 and 6 centimetres are 5 centimetres apart. What is the radius of the circle?

Answer



O is the centre of the circle.

EF and GH are two parallel chords 5 cm apart.

EF = 6 cm and GH = 4 cm.

OJ is perpendicular on EF and OI is perpendicular drawn on GH.

IJ = 5 cm

Let, OJ = x cm and OI = (5 – x) cm

Let, radius of the circle = r cm

OE = OG = r cm

In ΔOIG we have,

∠OIG = 90° [∵OI is perpendicular drawn on GH]

IG = 4/2 = 2 cm [∵perpendicular drawn from centre bisects chord]

OI = (5 – x) cm

OG = r cm

∴ $(r)^2 = (2)^2 + (5 - x)^2$ ...... (1)

In ΔOJE we have,

∠OJE = 90° [∵ OJ is perpendicular on EF]

JE = 6/2 = 3 cm [∵perpendicular drawn from centre bisects chord]

OJ = x cm

OE = r cm

∴ $(r)^2 = (3)^2 + (x)^2$ ...... (2)

From (1) and (2) we have,

⇒ $(2)^2 + (5 - x)^2 = (3)^2 + (x)^2$

⇒ $4 + 25 - 10x + x^2 = 9 + x^2$

⇒ 29 – 10x = 9

⇒ 10x = 20

∴ x = 2

Putting the value x = 2 in (2) we get,

⇒ $(r)^2 = (3)^2 + (2)^2$

⇒ r = √13

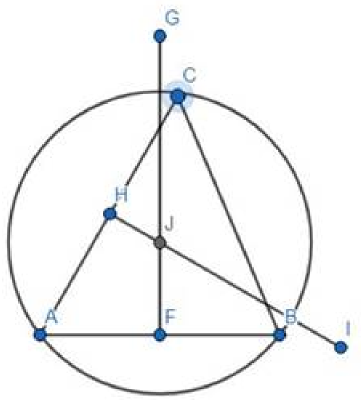
∴ radius of the circle = √13 cm

Questions Pg-92

1. Question

Draw three triangles with lengths of two sides 4 and 5 centimetres and the angle between them 60°, 90°, 120°. Draw the circumcircle of each. (Note how the position of the circumcentre changes).

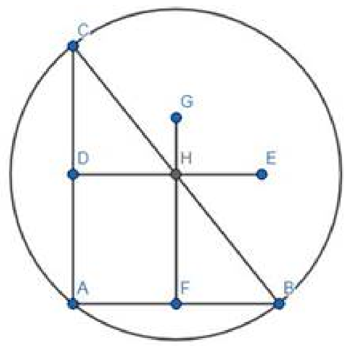
Answer



In this figure AC = 5 cm and AB = 4 cm.

∠CAB = 60°

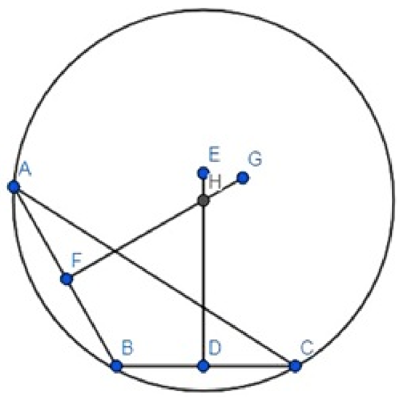
J is the circumcentre, which is inside the triangle.



AC = 5cm and AB = 4 cm.

∠CAB = 90°

H is the circumcentre, which lies on the hypotenuse of the triangle.



AB = 5 cm and BC = 4 cm.

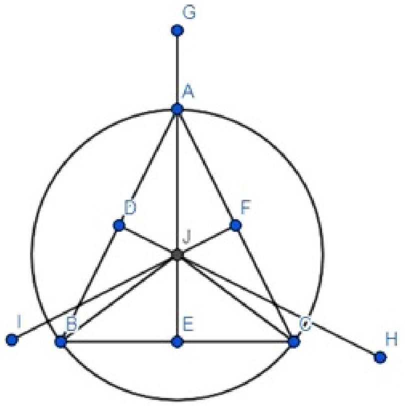
$\angle ABC = 120°$

H is the circumcentre, which is outside the triangle.

2. Question

The equal sides of an isosceles triangle are 8 centimetres long and the radius of its circumcircle is 5 centimetres. Calculate the length of its third side.

Answer



AB = AC = 8 cm

AJ = BJ = CJ = 5 cm

Let, JE = y cm and BE = x cm

In ΔJBE we have,

$\angle JEB = 90°$

BE = x cm

JE = y cm

BJ = 5 cm

$\therefore x^2 + y^2 = 5^2$

$\Rightarrow x^2 = 25 – y^2$ ……… (1)

In ΔABE we have,

$\angle AEB = 90°$

BE = x cm

AE =(AJ + JE)=(5 + y) cm

AB = 8 cm

$\therefore x^2 + (5 – y)^2 = 8^2$

$\Rightarrow x^2 = 64 – (5 – y)^2$……… (2)

From (1) and (2) we have,

$\Rightarrow 25 – y^2 = 64 – (5 – y)^2$

$\Rightarrow 25 – y^2 = 64 – 25 + 10y – y^2$

$\Rightarrow 10y = 14$

$\Rightarrow y = 1.4$

Putting the value y = 1.4 in (1) we get,

$\Rightarrow x^2 = 25 – (1.4)^2$

$\Rightarrow x^2 = 25 – 1.96$

$\Rightarrow x^2 = 23.04$

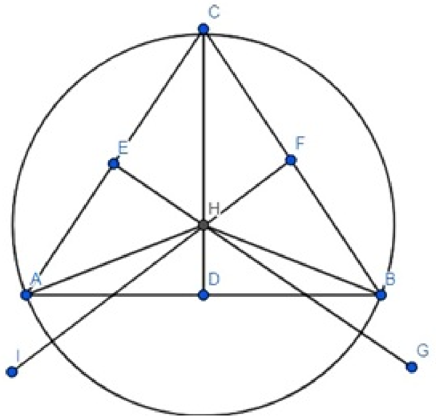
$\Rightarrow x = 4.8$

∴ Length of the third side, BC = 2x = 2 × 4.8 = 9.6 cm

3. Question

Find the relation between a side and the circumradius of an equilateral triangle.

Answer



AB = BC = CA

∠ABC = ∠ACB = ∠BAC = 60°

Let, AH = x cm

∴ AH = CH = BH = x cm

∴ DH = x/2 cm [∵H is circumcentre, ABC is equilateral]

In ΔAHD,

∠ADH = 90°

AH = x cm

HD = x/2 cm

$\therefore AD = \sqrt{(x)^2 - (\frac{x}{2})^2} = \sqrt{x^2 - \frac{x^2}{4}} = \sqrt{\frac{3x^2}{4}} = \frac{\sqrt{3}x}{2}$ cm

∴ AB = 2 × AD = 2 × x√3/2 = x√3 cm

∴ side = √3 × circumradius