Talent & Olympiad

Quadratic Equations

1. Identify the quadratic equation from the following.

(a)
$$p + \frac{1}{p} = 1, p \neq 0$$
 (b) $p^2 + \frac{1}{p} = 1, p \neq 0$
(c) $x^2 - \frac{1}{x} = 1, x \neq 0$ (d) $x^2 + 2\sqrt{x} - 1 = 0$

2. Find the roots of the quadratic equation

(a)
$$\frac{-1}{2}$$
,1
(b) $-1,\frac{1}{2}$
(c) $\frac{1}{2}$,1
(d) $-1,\frac{-1}{2}$

3. Which of the following statements is correct? (a) x=1 is a root of $2x^2+3x+1=0$. (b) x=2 is not a root of $6x^2+7x-5=0$. (c) x=-1 is a root of $3x^2-x-1=0$. (d) $x=-\frac{2}{5}$ is not a root of $5x^2-8x-4=0$.

4. Find the value of 'p' for which
$$m = \frac{1}{\sqrt{3}}$$
 is a

root of the equation

$$pm^{2} + (\sqrt{3} - \sqrt{2})m - 1 = 0$$

(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\sqrt{6}$ (d) $\sqrt{7}$

- 5. For what respective values of 'm' and 'n' are $x = \frac{-2}{5}$ and $x = \frac{5}{3}$ the roots of $mx^2 + nx - 10 = 0$? (a) 15,-19
 - (b) -19, 15 (c) 19,-15 (d) -15, 19
- 6. The sides of two square plots are (2x-1)mand (5x+4)m. The area of the second square plot is 9 times the area of the first square plot. Find the side of the larger plot. (a) 15m
 - (b) 13m
 - (c) 31 m
 - (d) 39m

7. What are the roots of $17a^2 - 20a + 10 = 10a^2 + 2a + 7?$ (a) $\frac{1}{7}$, 3 (b) $3, \frac{-1}{7}$ (c) $\frac{-1}{7}, -3$ (d) $-3, -\frac{1}{7}$

8. Identify the factors of $\frac{4x^2}{5} = 4x - 5$.

(a)
$$\frac{2}{5}, \frac{2}{5}$$

(b) $\frac{-5}{2}, \frac{5}{2}$
(c) $\frac{5}{2}, \frac{5}{2}$
(d) $\frac{-2}{5}, \frac{2}{5}$

- 9. The age of a man is the square of his son's age. A year ago, the man's age was eight times the age of his son. What is the present age of the man?
 (a) 47 years
 (b) 49 years
 (c) 36 years
 (d) 48 years
- 10. Find two consecutive even numbers whose product is double that of the greater number.
 (a) 1, 3
 (b) 4, 6
 (c) 2, 4
 (d) 6, 8
- 11. The length and breadth of a rectangle are (3k + 1) cm and (2k 1) cm respectively. Find the perimeter of the rectangle if its area is 144 cm^2 .
 - (a) 50 cm (b) 10 cm (c) 32 cm (d) 25 cm
- 12. The sum of squares of two consecutive positive even numbers is 340. Find them.
 (a) 12, 14
 (b) 12, 10
 (c) 10, 8
 (d) 14, 16

13. Find two consecutive positive odd numbers, the sum of whose squares is 514.

- (a) 11, 13
- (b) 15, 17
- (c) 11, 9
- (d) 13, 15

- 14. The area of a rectangular cardboard is 80 cm². If its perimeter is 36 cm, find its length.
 (a) 40 cm
 (b) 10 cm
 (c) 20 cm
 (d) 8 cm
- **15.** Find two consecutive integers whose product is 600. (a) 30, 20 (b) 50, 12 (c) 15, 40 (d) 24, 25
- 16. Find the present age of a boy whose age 12 years from now will be the square of his present age.
 (a) 5 years
 (b) 7 years
 (c) 4 years
 (d) 6 years
- 17. Identify the correct statement.(a) The roots of the quadratic equation

 $2y^2 + 9y = 0$ are 0 and $\frac{-9}{2}$.

(b) The value of 'k' for which $4m^2 + k - 15 = 0$ has a root m = 3 is 7. (c) The quadratic equation $(4x - 11)^2 = 0$ has two distinct roots.

(d) $7x^2 - 12x - 18 = 0$ is not a quadratic equation.

18. Find the roots of $3x^2 - 2\sqrt{6x} + 2 = 0$.

(a) $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$	(b) $\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$
(c) $\frac{\sqrt{2}}{3}, \frac{\sqrt{3}}{2}$	(d) $\frac{\sqrt{2}}{3}, \frac{\sqrt{3}}{3}$

- **19.** Divide 63 into two parts such that their product is 962. (a) 24, 3C (b) 28, 35 (c) 26, 37 (d) 27, 36
- **20.** Which of the following is a quadratic equation?

(a)
$$x - \frac{5}{x} = x^2$$

(b) $x^2 + \frac{2}{x^2} = 1$
(c) $2x^2 + 3\sqrt{x} + 4 = 0$
(d) $x^2 - 1 = 2x^2 + 4$

- **21.** The quadratic equation $ax^2 + bx + c = 0$ has no real root. Which of the following is true? (a) $b^2 - 4ac < 0$ (b) $b^2 - 4ac = 0$ (c) $b^2 - 4ac > 0$ (d) $b^2 + 4ac < 0$
- **22.** What is the nature of the roots of the quadratic equation $25x^2 49 = 0$? (a) Real and distinct (b) Real and equal (c) Irrational (d) No real roots
- 23. When are the roots of a quadratic equation real and equal?
 (a) When the discriminant is positive.
 (b) When the discriminant is zero.
 (c) When the discriminant is negative.
 (d) When the discriminant is non-negative.
 24. How are the roots of 3x²+7x+8=0?
 (a) Real and unequal
- (a) Real and unequal
 (b) Real and equal
 (c) Not real
 (d) Cannot be determined.
- **25.** What is the value of 'k' for which the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ are real and equal? (a) 9 only (b) -9 only (c) 9 or-9 (d) Neither 9 nor-9.
- **26.** Find the sum of the roots of $x^2 + x - 210 = 0$ (a) -2 (b) 29 (c) 20 (d) -1

27. In the quadratic equation $9x^2 + \alpha x - 2 = 0$, find the value of a for which $x = \frac{1}{3}$ is its solution. (a) -2 (b) 3 (c) -4 (d) 6

28. The ratio of the length and breadth of a rectangular photo frame is 3 : 2. Find its length if its area is 864 cm².

(a) 54 cm	(0) 20 cm
(c) 24 cm	(d) 36 cm

29. A two digit number is 4 times the sum of its digits and also 16 more than the product of digits. Find the number.(a) 48 (b) 36

(a) 1 0	(0) 50
(c) 44	(d) 32

- **30.** A quadratic equation $\alpha x^2 + 5x + \beta = 0$ has two roots $x = \frac{1}{3}$ and x = -2. Find the respective values of α and β . (a) 3, 2 (b) 2, -5 (c)-3, 5 (d) 3, -2
- **31.** Find the common root of the equations $x^2 - 7x + 10 = 0$ and $x^2 - 10x + 16 = 0$. (a) - 2 (b) 3 (c) 5 (d) 2
- **32.** If the product of the roots of $x^2 - 3x + k = 10$ is - 2, what is the value of k'? (a) -2 (b) 8 (c) 12 (d)-8
- **33.** If $2a^2 + a 2 = 1$ and a > 0, find 'a'. (a) $\frac{3}{2}$ (b) 1

(c) 3 (c	1) -1

- **34.** Find 'a' if $a 3 = \frac{10}{a}$. (a) 5,-2 (b) $-\sqrt{7}$,7 (c) $\sqrt{7}$,7 (d) -5,2
- **35.** Find the value of 'p' so that $x^2 + 5px + 16 = 0$ has no real root.
 - (a) Greater than $\frac{8}{5}$ (b) Less than $\frac{-8}{5}$ (c) Lies between $\frac{-8}{5}$ and $\frac{8}{5}$ (d) Less than $\frac{15}{8}$

- **36.** Find the value of 'k' for which $x^2 4x + k = 0$ has coincident roots. (a) 4 (b) -4 (c) 0 (d) -2
- **37.** If the roots of $x^2 + 4mx + 4m^2 + m + 1 = 0$ are real, which of the following is true? (a) m = -1 (b) $m \le -1$ (c) $m \ge -1$ (d) $m \ge 0$
- **38.** What is the ratio of the sum and the product of roots of $7x^2 12x + 18 = 0$ (a) 7:12 (b) 2:3 (c) 3:2 (d) 7:18
- **39.** Which of the following is the quadratic equation one of whose roots is $3-2\sqrt{3}$? (a) $x^2+6x-3=0$ (b) $x^2-6x-3=0$ (c) $x^2+6x+3=0$ (d) $x^2-6x+3=0$
- **40.** If a and Rare the roots of the equation $x^2 - 8x + p = 0$ such that $\alpha^2 + \beta^2 = 40$, find the value of 'p'. (a) 8 (b) 10 (c) 12 (d) 14
- **41.** Which of the following quadratic polynomials can be factorized into a product of real linear factors?
 - (a) $2x^2-5x+9$ (b) $2x^2+4x-5$ (c) $3x^2+4x+6$ (d) $5x^2-3x+2$
- **42.** If α and β are the roots of the equation $x^2 3x + 2 = 0$, which of the following is the equation whose roots are $(\alpha + 1)$ and $(\beta + 1)$?
 - (a) $x^2 + 5x + 6 = 0$
 - (b) $x^2 5x 6 = 0$
 - (c) $x^2 5x 6 = 0$
 - (d) $x^2 5x + 6 = 0$
- **43.** Which of the following equations has 2 as a root?
 - (a) $2x^2 7x + 6 = 0$
 - (b) $x^2 4x + 5 = 0$ (c) $3x^2 - 6x - 2 = 0$
 - (c) 3x 6x 2 = 0(d) $x^2 + 3x - 12 = 0$
- 4

- **44.** If the equation ax-5x+c=0 has 10 as the sum of the roots and also as the product of the roots, which of the following is true?
 - (a) a-c=5(b) a=2, c=3
 - (c) a = 5, c = 1
 - (d) a = 3, c = 2
- **45.** Find the product of the roots of the quadratic equation $9m^2 + 24m + 16 = 0$.

(a)	$\frac{4}{3}$	(b)	$\frac{9}{16}$
(c)	$\frac{16}{9}$	(d)	$\frac{3}{4}$

- **46.** What is the nature of the roots of
 - $3x^2 + x + 6 = 0?$
 - (a) Real and equal
 - (b) Real and distinct
 - (c) Not real
 - (d) Cannot be determined.
- 47. The perimeter and area of a rectangular park are 80 m and 400 m². What is its length?
 (a) 20m
 (b) 15m
 (c) 30m
 (d) 40m
- **48.** If a and P are the roots of the equation $x^2 + kx + 12 = 0$ such that $\alpha \beta = 1$, what is the value of 'k'? (a) 0 (b) ± 5

()	()
(c) ± 1	(d) ± 7

- **49.** What is the value of 'k' for which $2x^2 kx + k$ has equal roots? (a) 4 only (b) 0 only (c) 8 only (d) 0, 8
- 50. Which of the following statements is true?
 (a) x²+x+1=0 has no real roots.
 (b) x²-4x+3=0 and x²-x-2=0 have two common roots.

(c) $x^2 - 3x - 4 = 0$ have real and equal roots. (d) The roots of $ax^2 + bx + c = 0, a \neq 0$ are reciprocal to each other if $a \neq c$.

Answer - Keys											
1.	A	2.	D	3.	В	4.	С	5.	A	6.	D
7.	A	8.	С	9.	В	10.	С	11.	A	12.	A
13.	В	14.	В	15.	D	16.	С	17.	A	18.	В
19.	С	20.	D	21.	A	22.	A	23.	В	24.	С
25.	С	26.	D	27.	В	28.	D	29.	A	30.	D
31.	D	32.	В	33.	В	34.	A	35.	С	36.	A
37.	В	38.	В	39.	В	40.	С	41.	В	42.	D
43.	A	44.	A	45.	С	46.	С	47.	A	48.	D
49.	D	50.	А								

Solutions

(a) A quadratic equation has a degree 2.
 In (b) and (c), the degree of the polynomial is 3.

In (d), $x^2 + 2\sqrt{x} + 1$ is not a polynomial as $\sqrt{x} = x^{1/2}$ the power of the variable is not an integer.

In (a), $p + \frac{1}{p} = 1 \Longrightarrow p^2 - p + 1 = 0$ is a quadratic equation.

- 2. (d) $2m^2 + 3m + 1 = 0$ $\Rightarrow (m+1)(2m+1) = 0$ $\Rightarrow m = -1 \text{ or } -\frac{1}{2}$
- **3.** (b) Not available
- 4. (c) Not available
- **5.** (a) Not available
- **6.** (d) Not available
- **7.** (a) Not available
- **8.** (c) Not available
- 9. (b) Let the present age of the son be 'x' years. Then the father's age is x^2 years. One year ago, the son's age was (x-1) years and the father's age was (x^2-1) years. According to the problem, $(x^2-1)=8(x-1)$ $\Rightarrow (x-7)(x-1)=0$ $\Rightarrow x=1or7$ If $x=1, x^2=1$ \Rightarrow the father's age is 1 year is ridiculous.

If x = 7, then $x^2 = 49$

Hence, the present age of the father is 49 years.

- (c) Not available
 (a) Not available
- **12.** (a) Not available
- 13. (b) Not available
- **14.** (b) Not available
- **15.** (d) Let the two consecutive integers be 'x' and x+1. According to the problem, x(x+1) = 600 $\Rightarrow \quad x = -25 \text{ or } 24$ $\Rightarrow \quad x+1 = 24+1 = 25$ \therefore The required numbers are 24 and 25.
- 16. (c) Let the present age of the boy be 'x' years.
 12 years from now, his age will be (x+12) years.
 According to the problem, (x+12) = x²
 ⇒ (x-4)(x+3) = 0

 $\Rightarrow x = 4or - 3$

Since age cannot be negative, the required present age of the boy is 4 years.

- **17.** (a) Not available
- 18. (b) Not available
- **19.** (c) Not available
- **20.** (d) Not available
- **21.** (a) Not available
- **22.** (a) $b^2 4ac = -4(25)(-49) = 4900 > 0$

 \therefore The roots of the given quadratic equation are real and distinct.

- **23.** (b) When the value of the discriminant is zero, the roots of quadratic equation are real and equal.
- 24. (c) Not available
- **25.** (c) Not available
- **26.** (d) Not available
- **27.** (b) Not available
- **28.** (d) Not available
- **29.** (a) Let the digits in the tens place and the ones place be x and y respectively. Then, according to the problem, $10x + y = 4(x + y) \implies y = 2x$ and 10x + y = xy + 16 $\implies x = 4 \text{ or } 2$ If x = 2, then y = 2x = 4 \therefore The number is 24. If x = 4 then y = 8. \therefore The number is 48. Hence, the required number is 48.
- **30.** (d) Not available
- **31.** (d) Not available
- **32.** (b) Not available
- **33.** (b) Not available
- **34.** (a) Not available
- **35.** (c) Given $x^2 + 5px + 16 = 0$ has no real root

$$\Rightarrow b^{2} - 4ac < 0$$

$$\Rightarrow (5p^{2}) - 4(1)(16) < 0$$

$$\Rightarrow p \le \pm \frac{8}{5}$$

$$\Rightarrow$$
 p lies between $\frac{-8}{5}$ and $\frac{8}{5}$

- **36.** (a) Since the roots are coincident, $b^2 - 4ac = 0$ $\Rightarrow (-4)^2 - (1)(k) = 0 \Rightarrow k = 4$
- **37.** (b) Given, $x^2 + 4mx + 4m^2 + m + 1 = 0$ have real roots. $\Rightarrow b^2 - 4ac \ge 0$
 - $\Rightarrow (4m^2) 4(1)(4m^2 + m + 1) \ge 0$ $\Rightarrow 4m + 4 \le 0 \Rightarrow m \le -1$
- **38.** (b) Given, $7x^2 12x + 18 = 0$ Sum of roots $= \frac{-b}{a} = \frac{12}{7}$ Product of roots $= \frac{c}{a} = \frac{18}{7}$ \Rightarrow The required ratio $= \frac{12}{18} = \frac{2}{3} = 2:3$
- **39.** (b) Let $\alpha = 3 + 2\sqrt{3}$ and $\beta = 3 2\sqrt{3}$ $\alpha + \beta = 3 + 2\sqrt{3} + 3 - 2\sqrt{3} = 6$ $\alpha\beta = (3)^2 - (2\sqrt{3})^2 = 9 - 12 = -3$ \therefore The required quadratic equation is $x^2 - 6x - 3 = 0$.
- **40.** (c) The given equation is $x^2 8x + p = 0$ $\alpha + \beta = 8, \alpha\beta = p$ $\alpha^2 + \beta^2 = 40$ (Given) $\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40$ $\Rightarrow p = 12$
- **41.** (b) Except for the equation in option (b) all options have discriminant < 0 and hence cannot have real linear factors.
- **42.** (d) Given α and β are the roots of $x^2 3x + 2 = 0$. $\alpha + \beta = 3$ and $\alpha\beta = 2$

$$\therefore (\alpha + 1) + (\beta + 1) = \alpha + \beta + 2$$

= 3+2=5
$$(\alpha + \beta)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$$

= 2+3+1=6
Hence, the required equation is
$$x^{2} - 5x + 6 = 0$$

- **43.** (a) Not available
- **44.** (a) Not available
- **45.** (c) Not available
- **46.** (c) Not available
- **47.** (a) Not available
- **48.** (d) Given equation is $x^2 + kx + 12 = 0$. $\alpha + \beta = \frac{-b}{a} = -k \text{ and } \alpha\beta = 12$ $\alpha - \beta = 1$ (Given) $\alpha = 1 + \beta$ $\alpha\beta = (1 + \beta)\beta = 12$ $\Rightarrow \beta^2 + \beta - 12 = 0$ $\Rightarrow \beta = 3, -4$ If $\beta = 3, \alpha = 4$, and $\beta = -4, \alpha = -3$ $\alpha + \beta = 7 \text{ or } -7 = k$
- **49.** (d) Not available
- **50.** (a) Not available