Class XI Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer(VSA) questions of 2 marks each.
4. Section C has 6 Short Answer(SA) questions of 3 marks each.
5. Section D has 4 Long Answer(LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternatives in all such questions.

Section A

1. A single letter is selected at random from the word **PROBABILITY**. The probability that it is a vowel is **[1]**

   a) $\frac{1}{3}$ b) $\frac{3}{11}$

   c) $\frac{4}{11}$ d) $\frac{2}{11}$

2. Mean deviation of n observations x_1, x_2, ... x_n from their mean $\bar{x}$ is **[1]**

   a) $\frac{1}{n}\sum_{i=1}^{n}|x_i-\bar{x}|$ b) $\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$

   c) $\sum_{i=1}^{n}(x_i-\bar{x})$ d) $\sum_{i=1}^{n}(x_i-\bar{x})^2$

3. The effective rate of return which is equivalent to nominal rate of 8% p.a. compounded quarterly is: [Given $(1.02)^4$ = 1.0824] **[1]**

   a) 8.5% b) 8.16%

   c) 7.95% d) 8.24%

4. If log 0.0007392 = -3.1313, then log 73.92 is **[1]**

   a) 1.8687 b) 2.8687

   c) 1.1313 d) 2.1313

5. For two sets A and B if A is a null set, then Cartesian product $A \times B$ will be **[1]**

   a) set B b) $A \cup B$

c) ϕ d) A or B

6. $\log_{10}0.0001$ is equal to **[1]**

a) -4
b) 2
c) -2
d) 4

7. If $P(A \cap B) = \frac{1}{3}$, $P(A \cup B) = \frac{5}{6}$ and $P(A) = \frac{1}{2}$ then which one of the following is correct? **[1]**

a) A and B are mutually exclusive events
b) A and B are independent events
c) A and B are mutually exclusive and equally likely events
d) A and B are equally likely events

8. The equation of the circle passing through the origin which cuts off intercept of length 6 and 8 from the axes is **[1]**

a) $x^2 + y^2 + 12x + 16y = 0$
b) $x^2 + y^2 + 6x + 8y = 0$
c) $x^2 + y^2 - 12x - 16y = 0$
d) $x^2 + y^2 \pm 6x \pm 8y = 0$

9. The average of 15 numbers is 42. The sum of these numbers is: **[1]**

a) 630
b) 620
c) 435
d) 600

10. Mode of the data 3, 2, 5, 2, 3, 5, 6, 6, 5, 3, 5, 2, 5 is **[1]**

a) 5
b) 6
c) 3
d) 4

11. The value of $\frac{3+\log 343}{2+\frac{1}{2}\log\left(\frac{49}{4}\right)+\frac{1}{3}\log\left(\frac{1}{125}\right)}$, is: **[1]**

a) $\frac{3}{2}$
b) 3
c) 2
d) 1

12. A sum of money at compound interest amounts to thrice of itself in 5 years. In how many years will it be 9 times of itself? **[1]**

a) 15 years
b) 12 years
c) 9 years
d) 10 years

13. 0! is always taken as **[1]**

a) 1
b) 2
c) ∞
d) 0

14. X and Y are independent events such that $P(X \cap \bar{Y}) = \frac{2}{5}$ and $P(X) = \frac{3}{5}$. Then P(Y) is equal to: **[1]**

a) $\frac{2}{3}$
b) $\frac{1}{3}$
c) $\frac{1}{5}$
d) $\frac{2}{5}$

15. A person write 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, then the probability that all letters are not placed in the right envelopes, is **[1]**

a) $\frac{1}{4}$
b) $\frac{23}{24}$
c) $\frac{15}{24}$
d) $\frac{11}{24}$

16. The compound interest on ₹ 30,000 at 7% per annum is ₹ 4347. This period (in years) is: **[1]**

a) 2
b) 4
c) 3
d) $2\frac{1}{2}$

17. In how many ways can we select 9 balls out of 6 red balls, 5 white balls and 5 blue balls if 3 balls of each colour are selected? **[1]**

a) 2000
b) 40
c) 400
d) 200

18. If the relation R in the set {1, 2, 3, 4} given by R = {(1, 1), (1, 2), (1, 4), (3, 1), (3, 2), (4, 3), (4, 2)} then domain of R is given by **[1]**

a) domain (R) = {1, 2, 4}
b) domain (R) = {3, 2, 4}
c) domain (R) = {1, 2, 3}
d) domain (R) = {1, 3, 4}

19. **Assertion(A):** For a distribution, if Σx_i^2 = 232, Σx_i = 16 and n = 8, then standard deviation (S.D.) is 5. **[1]**
Reason(R): Standard deviation (S.D.) $= \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If 5th term of a G.P. is 9 and 11th term is 16, then 8th term is 12. **[1]**
Reason (R): In a G.P., $a_n = \frac{a_{n-k}+a_{n+k}}{2}$, n, k ∈ N.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. A and B undertake to do a piece of work for ₹ 600, A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days find the share of each. **[2]**

22. Prove that A - B = A ∩ B'. **[2]**

OR

If A and B are two sets such that n(A) = 37, n(B) = 26 and n(A ∪ B) = 51, find n{A ∩ B).

23. In covering a distance of 40 km Sachin takes 3 hours more than Utkarsh. If Sachin doubles his speed then he would take 1 hour less than Utkarsh. Find their speeds. **[2]**

24. Find the derivative of $\frac{x}{2x+1}$, with respect to x. **[2]**

OR

Differentiate the following function w.r.t. x: $\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x}$

25. Convert the given decimal number 569 to the binary number. **[2]**

Section C

26. Find n so that $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ maybe the A.M. between a and b. **[3]**

OR

Given a G.P. with a = 729 and 7th term = 64, determine S_7.

27. Find the foot of the perpendicular from the point (3, 8) to the line x + 3y = 7. **[3]**

28. If A = [1, 2, 3] and f g, h and s are relations corresponding to the subsets of A $\times$ A indicated against them, which of f, g, h and s are functions? In case of a function, find its domain and range. **[3]**

i. f = {(2,1), (3, 3)}
ii. g = {(1, 2), (1, 3), (2, 3), (3,1)}
iii. h = {(1, 3), (2,1), (3, 2)}
iv. s = {(1, 2), (2, 2), (3, 1)}

29. Divide ₹ 21866 into two parts such that the amount of one in 3 years is same as the amount of the second in 5 years, the rate of compound interest being 5% per annum. **[3]**

30. The population of a town in the year 2014 was 150,500. If the annual increasing during three successive years he at the rate of 7%, 8% and 6% respectively, find the population at the end of 2017. **[3]**

31. Find the correlation coefficient between the heights of husbands and wives based on the following data (given in inches) and interpret the result. **[3]**

Couple	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Height of husband	76	75	75	72	72	71	71	70	68	68	68	68	67	67	62
Height of wife	71	70	70	67	71	65	65	67	64	65	65	66	63	65	61

Section D

32. Three persons A, B and C apply for a job of Manager in a private company. Chances of their selection (A, B and C are in the ratio 1:2:4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C. **[5]**

OR

60% students read Hindi newspaper, 40% students read Tamil newspaper and 20% students read both Hindi and Tamil newspaper. Find the probability that a student selected at random reads

i. Tamil newspaper given that he has already read Hindi newspaper.
ii. Hindi newspaper given that he has already read Tamil newspaper.
iii. neither Hindi nor Tamil newspaper.

33. Evaluate: $\lim\limits_{x \to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$ **[5]**

34. Calculate Karl Pearson's coefficient of skewness for the following data: **[5]**

CI	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
f	3	6	9	7	4	2

OR

Find the mean deviation about the mean for the data

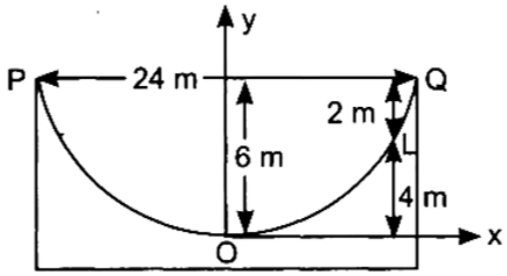
Income per day in ₹	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of persons	4	8	9	10	7	5	4	3

35. Find the equations of two straight lines passing through (1, 2) and making an angle of 60° with the line x + y = 0. Find also the area of the triangle formed by the three lines. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

From different media we see lots of beautiful structural work. In case of bridges we see the bridges supported by parabolic shape support.



A beam is supported at its ends by supports which are 24 metres apart. As the load is concentrated at the centre, there is a deflection of 6 m at the centre and deflected beam is in the shape of parabola.

(a) Find the equation of parabola.

(b) Find the Equation of directrix of parabola?

(c) How far from centre is deflection of 2m?

(d) Find the Length of latus rectum of parabola?

37. **Read the text carefully and answer the questions:** **[4]**

In a library 25 students read physics, chemistry and mathematics books. It was found that 15 students read mathematics, 12 students read physics while 11 students read chemistry. 5 students read both mathematics and chemistry, 9 students read physics and mathematics. 4 students read physics and chemistry and 3 students read all three subject books.

(a) Find the number of students who read none of the subject.

(b) Find the number of students who read atleast one of the subject.

(c) Find the number of students who read only one of the subjects.

(d) Find the number of students who read only mathematics.

38. **Read the text carefully and answer the questions:** **[4]**

Out of 7 boys and 5 girls a team of 7 students is to be made.

(a) Find the number of ways, if team contain at least 3 girls.

(b) Find the number of ways, if team contain exactly 3 girls.

(c) if exactly 3 girls are selected and are arranged in a row for photograph. Find number of ways if all girls and all the boys will stand together.

(d) The number of ways to arrange 3 girls and 4 boys if no two boys and girls will stand together.

Solution

Section A

1.

(c) $\frac{4}{11}$

Explanation: Total number of alphabet in the word probability = 11

Number of vowels in word Probability = 4 i.e. (O, A, I, I)

Required Probability $= \frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}}$

$\therefore$ P(letter is vowel) = $\frac{4}{11}$

2. **(a)** $\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$

Explanation: Mean Deviation, MD = $\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$

where, $\bar{x}$ is mean n is number of observations

3.

(b) 8.16%

Explanation: 8.16%

4. **(a)** 1.8687

Explanation: log 0.0007392 = -3.1313 = - 4 + 4 - 3.1313 = - 4 + 0.8687

$\Rightarrow$ log 0.0007392 = $\bar{4}$.8687

$\Rightarrow$ mantissa of digits 7392 = 0.8687

$\therefore$ log 73.92 = 1.8687 ($\because$ Characteristic of log 73.92 = 1)

5.

(c) ϕ

Explanation: set A is null get (Given)

So, Cartesian product A $\times$ B = $\phi \times$ B = ϕ

6. **(a)** -4

Explanation: $\log_{10}$ 0.0001

= $\log_{10} 10^{-4}$

= -4 [$\because \log_a a^x = x$]

7.

(b) A and B are independent events

Explanation: As we know P(B) = P(A $\cup$ B) - P(A) + P(A $\cap$ B)

= $\frac{5}{6} - \frac{1}{2} + \frac{1}{3} = \frac{5-3+2}{6} = \frac{2}{3}$.

Now, P(B)$\cdot$P(A) = $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ = P(A $\cap$ B)

$\Rightarrow$ A and B are independent events.

8.

(d) $x^2 + y^2 \pm 6x \pm 8y = 0$

Explanation: Given that we need to find the equation of the circle passing through the origin and cuts off intercepts 6 and 8 from x and y - axes

Since the circle is having intercept a from x-axis the circle must pass through (6, 0) and (- 6, 0) as it already passes through the origin.

Since the circle is having intercept 8 from y-axis the circle must pass through (0, 8) and (0, - 8) as it already passes through the origin.

Let us assume the circle passing through the points O(0, 0), A(6, 0) and B(0, 8).

We know that the standard form of the equation of the circle is given by:

$\Rightarrow x^2 + y^2 + 2fx + 2gy + c = 0$ ...(1)

Substituting O(0, 0) in (1), we get,

$\Rightarrow 0^2 + 0^2 + 2f(0) + 2g(0) + c = 0$

$\Rightarrow c = 0$ ...(2)

Substituting A(6, 0) in (1), we get,

$\Rightarrow 6^2 + 0^2 + 2f(6) + 2g(0) + c = 0$

$\Rightarrow 36 + 12f + c = 0$ ...(3)

Substituting B(0, 8) in (1), we get,

$\Rightarrow 0^2 + 8^2 + 2f(0) + 2g(8) + c = 0$

$\Rightarrow 64 + 16g + c = 0$ ...(4)

On solving (2), (3) and (4) we get,

$\Rightarrow f = -3, b = -4$ and $c = 0$

Substituting these values in (1), we get

$\Rightarrow x + y^2 + 2(-3)x + 2(-4)y + 0 = 0$

$\Rightarrow x^2 + y^2 - 6x - 8y = 0$

Similarly, we get the equation $x^2 + y^2 + 6x + 8y = 0$ for the circle passing through the points (0, 0), (- 6, 0), (0, - 8),

$x^2 + y^2 - 6x + 8y = 0$ for the circle passing through the points (0, 0), (6, 0), (0, - 8) .

$\therefore$ The equations of the circles are $x^2 + y^2 \pm 6x \pm 8y = 0$

9. **(a)** 630

Explanation: 630

10. **(a)** 5

Explanation: Most repeated value is the mode. Here it is 5

11.

(b) 3

Explanation: 3

12.

(d) 10 years

Explanation: In 5 years, a sum P becomes 3p.

$\therefore$ In next 5 years, a sum of 3P becomes 3(3P) = 9P i.e. in 10 years amount becomes 9 times.

13. **(a)** 1

Explanation: We have ${}^nP_r = \frac{n!}{(n-r)!}$ .....(i)

Number of ways you can arrange n thing in n available spaces = n!

$\Rightarrow {}^nP_n = n!$ ....(ii)

But from (i) we get ${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$ .....(iii)

Now from (ii) and (iii) we get $\frac{n!}{0!} = n! \Rightarrow 0! = 1$

14.

(b) $\frac{1}{3}$

Explanation: $\frac{1}{3}$

15.

(b) $\frac{23}{24}$

Explanation: Total number of ways of placing four letters in 4 envelops = 4! = 24

All the letters can be dispatched in the right envelops in only one way. Therefore, the probability that all the letters are placed in the right envelops is $\frac{1}{24}$

Hence, probability that all the letters are not placed in the right envelops = $1 - \frac{1}{24} = \frac{23}{24}$

16. **(a)** 2

Explanation: Amount = ₹(30,000 + 4347) = ₹ 34347

Let the time be n years.

Then, using formula of compound interest $A_n = P(1 + i)^n$

Here, A_n = 34347, i = $\frac{7}{100}$ = 0.04 and P = 30,000

$\therefore$ 34347 = 30,000 $(1 + 0.07)^n$

$\therefore$ $(1.07)^n = \frac{34347}{30,000}$

$\Rightarrow \left(\frac{107}{100}\right)^n = \frac{11449}{10000}$

$\Rightarrow \left(\frac{107}{100}\right)^n = \left(\frac{107}{100}\right)^2$

= n = 2 years.

17. **(a)** 2000

Explanation: Required number of ways $= \left({}^6C_3 \times {}^5C_3 \times {}^5C_3\right)$

$= \left({}^6C_3 \times {}^5C_2 \times {}^5C_2\right) = \left(\frac{6\times5\times4}{3\times2\times1} \times \frac{5\times4}{2\times1} \times \frac{5\times4}{2\times1}\right)$ = 2000

18.

(d) domain (R) = {1, 3, 4}

Explanation: The set of all first elements of the ordered pairs in R is called domain of relation.

i.e. domain (R) : {a : (a, b) $\in$ R}

$\therefore$ Domain (R) = {1, 3, 4}

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion (A): $\sum x_i^2 = 232$

$\sum x_i = 16$, x = 8

Standard deviation $(\sigma) = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$

$\sigma = \sqrt{\frac{232}{8} - \left(\frac{16}{8}\right)^2}$

$= \sqrt{29 - 4}$

$= \sqrt{25}$

σ = 5

Hence, A is correct.

Reason (R): Standard deviation (S.D)

$= \sqrt{\frac{\sum x_i^2}{x} - \left(\frac{\sum x_i}{x}\right)^2}$

R is also true.

Hence, Both A and R are true and R is the correct explanation of A.

20.

(c) A is true but R is false.

Explanation: We know that in a G.P.

$a_n = \sqrt{a_{n-k} \cdot a_{n+k}}$

$\therefore$ Reason is false.

Given a_5 = 9 and a_{11} = 16.

So, $a_8 = \sqrt{a_{8-3} \times a_{8+3}}$

$\Rightarrow a_8 = \sqrt{9 \times 16} = \sqrt{144} \Rightarrow a_8 = 12$

$\therefore$ Assertion is true.

Section B

21. C's 1 days' work = $\frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right) = \frac{1}{24}$

$\therefore$ A : B : C = Ratio of their 1 day's work

= $\frac{1}{6} : \frac{1}{8} : \frac{1}{24}$ = 4 : 3 : 1

$\therefore$ A's share = ₹ $\left(600 \times \frac{4}{8}\right)$ = ₹ 300

B's share = ₹ $\left(600 \times \frac{3}{8}\right)$ = ₹ 225

C's share = ₹ $\left(600 \times \frac{1}{8}\right)$ = ₹ 75

22. Suppose x be some element in set A - B that is x $\in$ (A - B)

Now if we prove that x $\in$ (A $\cap$ B’) then (A - B) = (A $\cap$ B’)

$x \in (A - B)$ means $x \in A$ and $x \notin B$
Now $x \notin B$ means $x \in B'$
Therefore, we can say that $x \in A$ and $x \in B'$
Thus $x \in A \cap B'$.
And as $x \in A \cap B'$ and also $x \in A - B$ we can conclude that
$A - B = A \cap B'$

OR

Given: n(A) = 37
n(B) = 26
$n(A \cup B) = 51$
To Find: $n(A \cap B)$
We know that,
$|A \cup B| = |A| + |B| - |A \cap B|$ (where A and B are two finite sets)
Thus,we have
$n(A \cup B) = n(A) + n(B) – n(A \cap B)$
$51 = 37 + 26 - n(A \cap B)$
$n(A \cap B) = 63 - 51 = 12$
Thus,
$n(A \cap B) = 12$

23. Let Sachin's speed be S_1 km/h and Utkarsh's speed be S_2 km/h, then
$\frac{40}{S_1} - \frac{40}{S_2} = 3$ and $\frac{40}{S_2} - \frac{40}{2S_1} = 1$
On adding, we get $\frac{40}{S_1} - \frac{40}{2S_1} = 4 \Rightarrow \frac{40-20}{S_1} = 4 \Rightarrow S_1 = 5$ km/h
Putting this value of S_1 in equation $\frac{40}{S_1} - \frac{40}{S_2} = 3$, we get $\frac{40}{5} - \frac{40}{S_2} = 3 \Rightarrow S_2 = 8$ km/h.
$\therefore$ Sachin speed is 5 km/h and Utkarsh speed is 8 km/h.

24. Let $y = \frac{x}{2x+1}$
Differentiating w.r.t. x, we get
$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{2x+1}\right)$
$= \frac{(2x+1)\frac{dx}{dx} - x \cdot \frac{d}{dx}(2x+1)}{(2x+1)^2}$
$= \frac{(2x+1) \cdot 1 - x \cdot 2}{(2x+1)^2}$
$= \frac{2x+1-2x}{(2x+1)^2}$
$= \frac{1}{(2x+1)^2}$

OR

Let $y = \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x} = \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x} \times \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}-x}$
$= \frac{(x^2+1)+x^2-2x\sqrt{x^2+1}}{(x^2+1)-x^2} = \frac{2x^2+1-2x\sqrt{x^2+1}}{1}$
$= 2x^2 + 1 - 2x\sqrt{x^2+1}$, differentiating w.r.t. x, we get
$\frac{dy}{dx} = 2 \cdot 2x + 0 - 2\,[x \cdot \frac{1}{2}(x^2+1)^{\frac{-1}{2}} \cdot 2x + \sqrt{x^2+1} \cdot 1]$
$= 4x - 2\left[\frac{x^2}{\sqrt{x^2+1}} + \sqrt{x^2+1}\right] = 4x - 2 \cdot \frac{x^2+(x^2+1)}{\sqrt{x^2+1}} = 2\left[2x - \frac{2x^2+1}{\sqrt{x^2+1}}\right]$

25. Given: decimal number = 569

	Quotient	Remainder
2	569	
2	284	1
2	142	0
2	71	0
2	35	1
2	17	1
2	8	1
2	4	0
2	2	0
2	1	0

$\therefore (569)_{10} = (1000111001)_2$

Section C

26. The arithmetic mean between a and b is $\frac{a+b}{2}$

According to given, $\frac{a^n+b^n}{a^{n-1}+b^{n-1}} = \frac{a+b}{2}$

$\Rightarrow 2a^n + 2b^n = a^n + ab^{n-1} + a^{n-1}b + b^n$

$\Rightarrow a^n + b^n - ab^{n-1} - a^{n-1}b = 0$

$\Rightarrow (a^n - a^{n-1}b) + (b^n - ab^{n-1}) = 0$

$\Rightarrow a^{n-1}(a - b) + b^{n-1})(b - a) = 0$

$\Rightarrow (a - b)(a^{n-1} - b^{n-1}) = 0$

$\Rightarrow a^{n-1} - b^{n-1} = 0$ $(\because a \neq b)$

$\Rightarrow a^{n-1} = b^{n-1} \Rightarrow \frac{a^{n-1}}{b^{n-1}} = 1$

$\Rightarrow \quad \left(\frac{a}{b}\right)^{n-1} = 1 = \left(\frac{a}{b}\right)^0$

$\Rightarrow n - 1 = 0 \Rightarrow n = 1$

OR

Let r be the common ratio of G.P. Then

$a_7 = 64 \Rightarrow ar^{7-1} = 64 \Rightarrow 729\, r^6 = 64$

$\Rightarrow \quad r^6 = \frac{64}{729} \Rightarrow r^6 = \left(\frac{2}{3}\right)^6$ or $\left(-\frac{2}{3}\right)^6$

$\Rightarrow \quad r = \frac{2}{3}$ or $-\frac{2}{3}$

When r = $\frac{2}{3}, S_7 = \frac{a\left(1-r^7\right)}{1-r} = \frac{729\left(1-\left(\frac{2}{3}\right)^7\right)}{1-\frac{2}{3}}$ $3 \times 729\left(1-\left(\frac{2}{3}\right)^7\right)$

= 2187 - 128 = 2059

When r = $\frac{2}{3}, S_7 = \frac{729\left(1-\left(-\frac{2}{3}\right)^7\right)}{1-\left(-\frac{2}{3}\right)}$ = $\frac{3}{5} \times 729\left(1+\frac{2^7}{3^7}\right)$

= $\frac{1}{5}$(2187 + 128) = $\frac{2315}{5}$ = 463

27. The given equation of the line is

x + 3y - 7

$\Rightarrow y = -\frac{1}{3}x + \frac{7}{3}$

$\therefore$ Slope of the line, $m_1 = -\frac{1}{3}$

Let m_2 be the slope of the perpendicular line.

$\therefore m_1 m_2 = -1$

$\Rightarrow -\frac{1}{3} \times m_2 = -1$

$\Rightarrow m_2 = 3$

$\therefore$ Equation of the perpendicular line with slope 3 and passing through (3, 8) is y - 8 = 3 (x - 3)

⇒ 3x - y - 1 = 0

∴ The foot of perpendicular is the point of intersection of the lines x + 3y - 7 = 0 and 3x - y - 1 = 0.

Solving these equations, we get

x = 1, y = 2, So (1, 2) is the foot of the perpendicular.

28. i. f is not a function because the element 1 of A does not appear as the first component of ordered pairs of f, so 1 has no image in A.

ii. g is not a function because the different pairs (1, 2) and (1, 3) of g have the same first component i.e. the element 1 of A has two different images in A.

iii. h is a function because each element of A has a unique image in A.
Domain of h = {1, 2, 3} = A and range of h = {3, 1, 2} = A.

iv. s is a function because each element of A has a unique image in A.
Domain of s = {1, 2, 3} = A and range of s = {2, 1}.

29. Divide ₹ 21866 into two parts such that the amount of one in 3 years is same as the amount of the second in 5 years, the rate of compound interest being 5% per annum.

Divide Rs. 21866 into two parts such that the amount of one part in 3 years is the same as the amount of the second part in 5 years.

Let, First part (That put for 3 years) = x

So, Second part (That put for 5 years) = 21866 - x

We know the formula for compound interest:

$A = P\left(1 + \frac{R}{100}\right)^n$

So from the given condition, we get,

$x\left(1 + \frac{5}{100}\right)^3 = (21866 - x)\left(1 + \frac{5}{100}\right)^5$

$x = (21866 - x)\left(1 + \frac{5}{100}\right)^{5-3}$

$x = (21866 - x)\left(\frac{105}{100}\right)^2$

$x = (21866 - x)\left(\frac{21}{20}\right)\left(\frac{21}{20}\right)$

$\frac{400x}{441} + x = 21866$

$\frac{400x + 441x}{441} = 21866$

$\frac{841x}{441} = 21866$

$x = \frac{21866 \times 441}{841}$

x = 11466

Therefore,

The first part (That put for 3 years) = ₹11466

So, the second part (That put for 5 years) = 21866 - 11466 = ₹10400

30. Let P be the population at the end of 2017. Here,

P_0 = 150,500, r_1 = 7, r_2 = 8 and r_3 = 6

∴ P = $P_0\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\left(1 + \frac{r_3}{100}\right)$

⇒ P = 150,500$\left(1 + \frac{7}{100}\right)\left(1 + \frac{8}{100}\right)\left(1 + \frac{6}{100}\right)$ = 150500$\left(\frac{107}{100} \times \frac{108}{100} \times \frac{106}{100}\right)$

⇒ P = $\frac{1505 \times 107 \times 108 \times 106}{10000}$

⇒ log P = log$\left(\frac{1505 \times 107 \times 108 \times 106}{10^4}\right)$

⇒ log P = log 1505 + log 107 + log 108 + log 106 - log 10^4

⇒ log P = log 1505 + log 107 + log 108 + log 106 - 4 log 10

⇒ log P = 3.4775 + 2.0294 + 2.0334 + 2.0253 - 4 = 5.2656

⇒ P = antilog (5.2656) = 184,400

31. We use assumed means A = 70, B = 66, and shall use the formula (iii)

Couple	x	u = x - A = x - 70	u^2	y	v = y - B = y - 66	v^2	uv
1	76	6	36	71	5	25	30

2	75	5	25	70	4	16	20
3	75	5	25	70	4	16	20
4	72	2	4	67	1	1	2
5	72	2	4	71	5	25	10
6	71	1	1	65	-1	1	-1
7	71	1	1	65	-1	1	-1
8	70	0	0	67	1	1	0
9	68	-2	4	64	-2	4	4
10	68	-2	4	65	-1	1	2
11	68	-2	4	65	-1	1	2
12	68	-2	4	66	0	0	0
13	67	-3	9	63	-3	9	9
14	67	-3	9	65	-1	1	3
15	62	-8	64	61	-5	25	40
Total		0	194		5	127	140

$$\therefore \quad r=\frac{\Sigma uv-\frac{1}{N}\Sigma u\Sigma v}{\sqrt{\Sigma u^2-\frac{(\Sigma u)^2}{N}}\sqrt{\Sigma v^2-\frac{(\Sigma v)^2}{N}}}=\frac{140-\frac{(0)(5)}{15}}{\sqrt{194-\frac{(0)^2}{15}}\sqrt{(127)^2-\frac{(5)^2}{15}}}$$

= 0.89, which is a strong positive correlation.

This shows that tall men usually marry tall women and short men marry short women (called assertive mating).

Section D

32. Let us define the following events

A = selecting person A

B = selecting person B

C = selecting person C

P(A) = $\frac{1}{1+2+4}$, P(B) = $\frac{2}{1+2+4}$

and P(C) = $\frac{4}{1+2+4}$

P(A) = $\frac{1}{7}$, P(B) = $\frac{2}{7}$

and P(C) = $\frac{4}{7}$

Let E = Event to introduce the changes in their profit.

Also given $P\left(\frac{E}{A}\right)$ = 0.8, $P\left(\frac{E}{B}\right)$ = 0.5 and $P\left(\frac{E}{C}\right)$ = 0.3

$\Rightarrow P\left(\frac{\overline{E}}{A}\right)$ = 1 - 0.8 = 0.2, $P\left(\frac{\overline{E}}{B}\right)$ = 1 - 0.5 = 0.5

and $P\left(\frac{\overline{E}}{C}\right)$ = 1 - 0.3 = 0.7

The probability that change does not take place by the appointment of C,

$$P\left(\frac{C}{E}\right)=\frac{P(C)\cdot P\left(\frac{\overline{E}}{C}\right)}{P(A)\times P\left(\frac{\overline{E}}{A}\right)+P(B)\times P\left(\frac{\overline{E}}{B}\right)+P(C)\times P\left(\frac{\overline{E}}{C}\right)}$$

$$=\frac{\frac{4}{7}\times 0.7}{\frac{1}{7}\times 0.2+\frac{2}{7}\times 0.5+\frac{4}{7}\times 0.7}$$

$=\frac{2.8}{0.2+1.0+2.8}=\frac{2.8}{4}$ = 0.7

OR

Let H denote the event that the student reads Hindi newspaper and T denotes the event that the student reads Tamil newspaper.

Given P(H) = $\frac{60}{100}$, P(T) = $\frac{40}{100}$ and P(H ∩ T) = $\frac{20}{100}$.

i. P(T | H) = $\frac{P(T\cap H)}{P(H)}=\frac{\frac{20}{100}}{\frac{60}{100}}=\frac{1}{3}$.

ii. P(H | T) = $\frac{P(H \cap T)}{P(T)}$ = $\frac{\frac{20}{100}}{\frac{40}{100}}$ = $\frac{1}{2}$.

iii. P(H' ∩ T') = 1 - P(H ∪ T) = 1 - (P(H) + P(T) - P(H ∩ T))

= 1 - $\left(\frac{60}{100} + \frac{40}{100} - \frac{20}{100}\right). = \frac{20}{100} = \frac{1}{5}$

33. $\lim\limits_{x \to 0} \frac{1+x-1+x}{x[\sqrt{1+x}+\sqrt{1-x}]}$ [By rationalising]

$= \lim\limits_{x \to 0} \frac{2}{\sqrt{1+x}+\sqrt{1-x}} = \frac{2}{1+1} = 1$

34. Let A = 25 and h = 10

C.I.	**x_i**	**f_i**	$d_i = \frac{x_i - 25}{10}$	**$f_i d_i$**	$f_i d_i^2$	**cf**
0 - 10	5	3	-2	-6	12	3
10 - 20	15	6	-1	-6	6	9 (c)
(l) 20 - 30	25	9 (f)	0	0	0	18 ← Median class
30 - 40	35	7	1	7	7	25
40 - 50	45	4	2	8	16	29
50 - 60	55	2	3	6	18	31
		$\sum\limits_{i=1}^{n} f_i$ = 31		$\sum\limits_{i=1}^{n} f_i d_i$ = 9	$\sum\limits_{i=1}^{n} f_i d_i^2$ = 59	

Mean $A + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i} \times h$ = 25 + $\frac{9}{31}$ $\times$ 10 = 25 + 2.90 = 27.90

For median: $\frac{N}{2} = \frac{31}{2}$ = 15.5

Median class: 20 - 30

Median = l + $\frac{\frac{N}{2} - c}{f} \times h$ = 20 + $\frac{15.5-9}{9} \times 10$ = 20 + $\frac{65}{9}$ = 20 + 7.22 = 27.22

Standard deviation (σ) = 10 $\sqrt{\frac{\sum_{i=1}^{n} f_i d_i^2}{N} - \left(\frac{\sum_{i=1}^{n} f_i d_i}{N}\right)^2}$ = 10 $\sqrt{\frac{59}{31} - \left(\frac{9}{31}\right)^2}$

σ = 10$\sqrt{1.90 - (0.29)^2}$ = 10$\sqrt{1.90 - 0.08}$ = 10$\sqrt{1.82}$

= 10 $\times$ 1.349 = 13.49

$\therefore$ S_{kp} = $\frac{\text{Mean} - \text{Mode}}{\sigma}$ = $\frac{3(\text{Mean} - \text{Median})}{\sigma}$ = $\frac{3(27.90 - 27.22)}{13.49}$

= $\frac{3 \times 0.68}{13.49}$ = $\frac{2.04}{13.49}$ = 0.15

OR

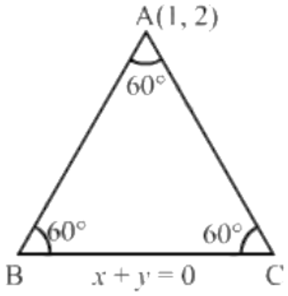
Income per day	Mid values x_i	f_i	$f_i x_i$	$\lvert x_i - 358 \rvert$	$f_i \lvert x_i - 358 \rvert$
0 - 100	50	4	200	308	1232
100 - 200	150	8	1200	208	1664
200 - 300	250	9	2250	108	972
300 - 400	350	10	3500	8	80
400 - 500	450	7	3150	92	644
500 - 600	550	5	2750	192	960
600 - 700	650	4	2600	292	1168
700 - 800	750	3	2250	392	1176
		50	17900		7896

Mean $\bar{x} = \frac{1}{N} \sum f_i x_i = \frac{1}{50} \times 17900 = 358$

Mean deviation about mean $= \frac{1}{N} \sum_{i=1}^{n} f_i |x_i - \bar{x}|$

$= \frac{1}{50} \times 7896 = 157.92$

35. Suppose A(1, 2) be the vertex of the triangle ABC and x + y = 0 be the equation of BC.



Now, we have to find the equations of sides AB and AC, each of which makes an angle 60° with the line x + y = 0

We know the equations of two lines passing through a point (x_1, y_1) and making an angle α with the line whose slope is m.

$\implies$ y - $y_1 = \frac{m \pm \tan\alpha}{1 \mp m \tan\alpha}$ (x - x_1)...(i)

Here,we have

x_1 = 1, y_1 = 2, α = 60°, m = -1,substituting in (i)

Therefore, the equations of the required sides are

$\implies$ y - 2 $= \frac{-1+\tan 60^\circ}{1+\tan 60^\circ}$ (x - 1) and y - 2 $= \frac{-1-\tan 60^\circ}{1-\tan 60^\circ}$ (x - 1)

$\Rightarrow$ y - 2 $= \frac{\sqrt{3}-1}{\sqrt{3}+1}$ (x - 1) and y - 2 $= \frac{\sqrt{3}+1}{\sqrt{3}-1}$ (x - 1)

$\Rightarrow$ y - 2 = (2 - $\sqrt{3}$) (x - 1) and y - 2 = (2 + $\sqrt{3}$) (x - 1)

Solving x + y = 0 and y - 2 = (2 - $\sqrt{3}$) (x - 1), we obtain

$x = -\frac{\sqrt{3}+1}{2}, y = \frac{\sqrt{3}+1}{2}$

$\therefore B \equiv \left(-\frac{\sqrt{3}+1}{2}, \frac{\sqrt{3}+1}{2}\right)$ or $C \equiv \left(\frac{\sqrt{3}-1}{2}, -\frac{\sqrt{3}-1}{2}\right)$

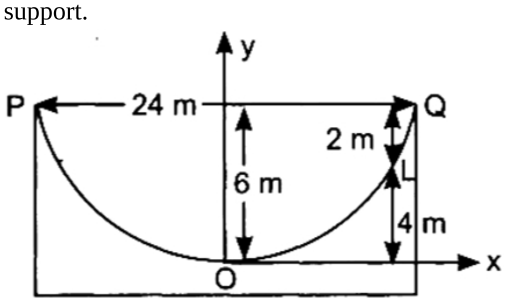
AB = BC = AD = $\sqrt{6}$ units

$\therefore$ Area of the required triangles $= \frac{\sqrt{3}\times(\sqrt{6})^2}{4} = \frac{3\sqrt{3}}{2}$ square units.

Section E

36. **Read the text carefully and answer the questions:**

From different media we see lots of beautiful structural work. In case of bridges we see the bridges supported by parabolic shape support.



A beam is supported at its ends by supports which are 24 metres apart. As the load is concentrated at the centre, there is a deflection of 6 m at the centre and deflected beam is in the shape of parabola.

(i) Point Q(12, 6) lies on parabola x^2 = 4ay

$\therefore$ $(12)^2$ = 4a $\times$ 6 $\Rightarrow$ a = $\frac{12\times12}{6\times4}$ = 6

$\therefore$ Equation is x^2 = 4 $\times$ 6 $\times$ y $\Rightarrow$ x^2 = 24y

(ii) Equation of directrix is y + 6 = 0

$\Rightarrow$ y + 6 = 0

(iii) Point (x, 4) lies on parabola

x^2 = 24 $\times$ 4 $\Rightarrow$ x^2 = 96 $\Rightarrow$ x = 4$\sqrt{6}$m

(iv) Length of latus rectum, 4a = 4 $\times$ 6 = 24m

37. **Read the text carefully and answer the questions:**

In a library 25 students read physics, chemistry and mathematics books. It was found that 15 students read mathematics, 12 students read physics while 11 students read chemistry. 5 students read both mathematics and chemistry, 9 students read physics and mathematics. 4 students read physics and chemistry and 3 students read all three subject books.

(i) Atleast one = 11 + 9 + 5 + 4 - 2(3)

= 29 − 6 = 23

⇒ None = 25 − 23 = 2

(ii) The number of students who reading atleast one of the subject is 23.

(iii) Only maths = 15 - 9 - 5 + 3 = 4

Only physics = 12 - 9 - 4 + 3 = 2

Only chemistry = 5 ⇒ Total = 11

(iv) The number of students who reading only mathematics is 4.

38. **Read the text carefully and answer the questions:**

Out of 7 boys and 5 girls a team of 7 students is to be made.

(i) 7 boys, 5 girls

ways to select at least 3 girls

= 3 girls 4 boys or 4 girls 3 boys or 5 girls 2 boys

$= {}^5C_3 \times {}^7C_4 + {}^5C_4 \times {}^7C_3 + {}^5C_5 \times {}^7C_2$

$= 10 \times 35 + 5 \times 35 + 1 \times 21$

= 350 + 175 + 21

= 546

(ii) Ways to select exactly three girls

= 3 girls 4 boys

$= {}^5C_3 \times {}^7C_4 = 350$

(iii) Ways of arranging 3 girls and 4 boys if all girls and boys stand together

$= 2! \times 3! \times 4!$

$= 2 \times 6 \times 24$

= 288

Total ways of selecting and arranging

$= 288 \times 350$

= 100800

(iv) Ways to arrange boys = 4!

B _ B _ B _ B

Ways to arrange girls = 3!

Total ways of selecting and arranging

$= 4! \times 3! \times 350$

$= 24 \times 6 \times 350$

= 50400