

Chapter 7
Triangles

Exercise No. 7.1

Multiple Choice Questions:

In each of the following, write the correct answer:

1. Which of the following is not a criterion for congruence of triangles?

- (A) SAS
- (B) ASA
- (C) SSA
- (D) SSS

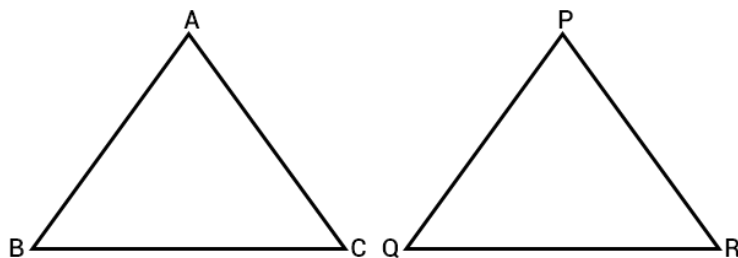
Solution:

SSA is not a criterion for congruence of triangles.
Hence, the correct option is (C).

2. If $AB = QR$, $BC = PR$ and $CA = PQ$, then

- (A) $\triangle ABC \cong \triangle PQR$
- (B) $\triangle CBA \cong \triangle PRQ$
- (C) $\triangle BAC \cong \triangle RPQ$
- (D) $\triangle PQR \cong \triangle BCA$

Solution:



Given:

$AB=QR$, $BC=PR$ and $CA=PQ$, then

The vertices are one-one corresponding that is P corresponding to C, Q to A and R to B, which is written as:

$$P \leftrightarrow C, Q \leftrightarrow A, R \leftrightarrow B$$

Under that correspondence, we have:

$$\triangle CBQ \cong \triangle PRQ$$

Hence, the correct option is (B).

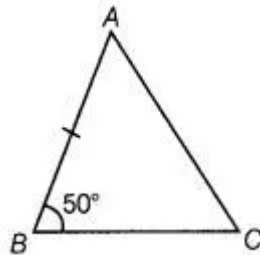
3. In $\triangle ABC$, $AB = AC$ and $\angle B = 50^\circ$. Then $\angle C$ is equal to

- (A) 40°

- (B) 50°
 (C) 80°
 (D) 130°

Solution:

According to the question, triangle ABC is:



$AB=AC$ [Given]

So, $\angle C = \angle B$ [Angles opposite to equal sides are equal]

Given: $\angle B = 50^\circ$. So, $\angle C = 50^\circ$

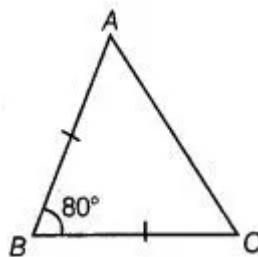
Hence, the correct option is (B).

4. In $\triangle ABC$, $BC = AB$ and $\angle B = 80^\circ$. Then $\angle A$ is equal to

- (A) 80°
 (B) 40°
 (C) 50°
 (D) 100°

Solution:

In triangle ABC:



$BC=AB$ [given]

$\angle A = \angle C$ [Since, angles opposite to equal sides are equal]

$\angle B = 80^\circ$

Therefore, $\angle A + \angle B + \angle C = 180^\circ$

$\angle A + 80^\circ - \angle A = 180^\circ$

$2\angle A = 100^\circ$

$\angle A = \frac{100^\circ}{2}$

$\angle A = 50^\circ$

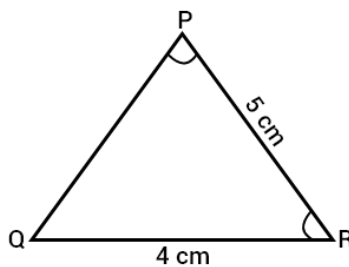
Hence, the correct option is (C).

5. In $\triangle PQR$, $\angle R = \angle P$ and $QR = 4$ cm and $PR = 5$ cm. Then the length of PQ is

- (A) 4 cm**
- (B) 5 cm**
- (C) 2 cm**
- (D) 2.5 cm**

Solution:

In triangle PQR :



$$\angle R = \angle P \text{ [Given]}$$

$$PQ = QR \text{ [Sides opposite to equal angles are equal]}$$

Now, $QR = 4$ cm, therefore, $PQ = 4$ cm.

Therefore, the length of the PQ is 4 cm

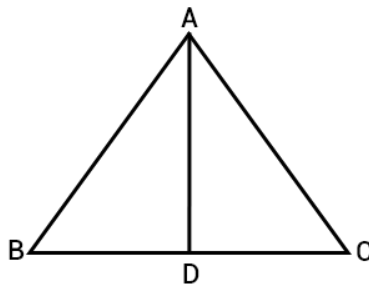
Hence, the correct option is (A).

6. D is a point on the side BC of a $\triangle ABC$ such that AD bisects $\angle BAC$. Then

- (A) $BD = CD$**
- (B) $BA > BD$**
- (C) $BD > BA$**
- (D) $CD > CA$**

Solution:

In triangle ADC ,



$$\text{Ext. } \angle ADB > \text{Int. opp } \angle DAC$$

$$\angle ADB > \angle BAD \quad [\text{Because: } \angle BAD = \angle DAC]$$

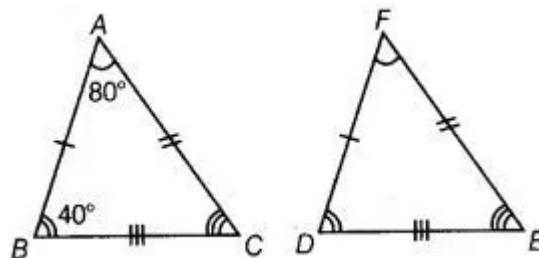
$AB > BD$ [Side opposite to greater angle is longer]
Hence, the correct option is (B).

7. It is given that $\triangle ABC \cong \triangle FDE$ and $AB = 5$ cm, $\angle B = 40^\circ$ and $\angle A = 80^\circ$. Then which of the following is true?

- (A) $DF = 5$ cm, $\angle F = 60^\circ$
- (B) $DF = 5$ cm, $\angle E = 60^\circ$
- (C) $DE = 5$ cm, $\angle E = 60^\circ$
- (D) $DE = 5$ cm, $\angle D = 60^\circ$

Solution:

Given: $\triangle ABC \cong \triangle FDE$ and $AB = 5$ cm, $\angle B = 40^\circ$ and $\angle A = 80^\circ$



$$DF = AB$$

[By CPCT]

$$DF = 5\text{cm}$$

$$\angle E = \angle C$$

[By CPCT]

$$\angle E = \angle C = 180^\circ - (\angle A + \angle B)$$

[By angle sum property of a triangle ABC]

$$\angle E = 180^\circ - (80^\circ + 40^\circ)$$

$$\angle E = 60^\circ$$

8. Two sides of a triangle are of lengths 5 cm and 1.5 cm. The length of the third side of the triangle cannot be

- (A) 3.6 cm
- (B) 4.1 cm
- (C) 3.8 cm
- (D) 3.4 cm

Solution:

Sum of any two sides of a triangle is greater than third side. So, third side of the triangle cannot be 3.4 cm because then,

$$1.5\text{cm} + 3.4\text{cm} = 4.9\text{ cm} < \text{third side [5cm]}$$

Hence, the correct option is (D).

9. In $\triangle PQR$, if $\angle R > \angle Q$, then

- (A) $QR > PR$

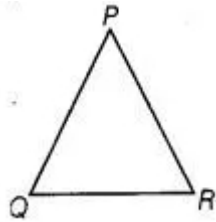
(B) $PQ > PR$

(C) $PQ < PR$

(D) $QR < PR$

Solution:

Given: In triangle PQR,
 $\angle R > \angle Q$



$PQ > PR$ [side opposite to greater angle is longer]
Hence, the correct option is (B).

10. In triangles ABC and PQR, $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$. The two triangles are

(A) isosceles but not congruent

(B) isosceles and congruent

(C) congruent but not isosceles

(D) neither congruent nor isosceles

Solution:

In triangle ABC,

$AB = AC$ [Given]

$\angle C = \angle B$ [Angles opposite to equal sides are equal]

So, in triangle ABC is an isosceles triangle.

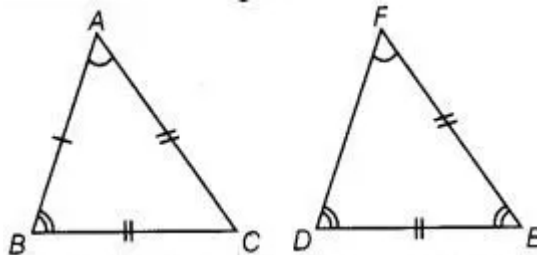
$\angle B = \angle Q$ [Given]

$\angle C = \angle P$

$\angle P = \angle Q$ [Since, $\angle C = \angle B$]

$QR = PR$ [Sides opposite to equal angles are equal]

So, in triangle PQR is also an isosceles triangle.



Hence, both triangle are isosceles but not congruent.

Hence, the correct option is (A).

11. In triangles ABC and DEF, $AB = FD$ and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom if

(A) $BC = EF$

(B) $AC = DE$

(C) $AC = EF$

(D) $BC = DE$

Solution:

Given, in $\triangle ABC$ and $\triangle DEF$, $AB = DF$ and $\angle A = \angle D$

As we know that, two triangles will be congruent by ASA rule, if two angles and the included side of one triangle are equal to the two angles and the included side of other triangle.

Since, $AC = DE$

Hence, the correct option is (B).

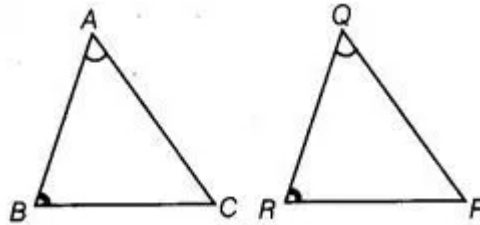
Exercise No. 7.2

Short Answer Questions with Reasoning:

1. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of $\triangle PQR$ should be equal to side AB of $\triangle ABC$ so that the two triangles are congruent? Give reason for your answer.

Solution:

Given: in $\triangle ABC$ and $\triangle PQR$, $\angle A = \angle Q$ and $\angle B = \angle R$

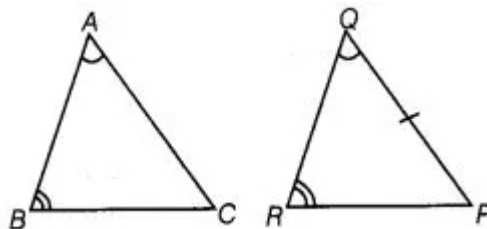


Now, the triangle ABC and PQR will be congruent if $AB = QR$ by ASA congruence rule.

2. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of $\triangle PQR$ should be equal to side BC of $\triangle ABC$ so that the two triangles are congruent? Give reason for your answer.

Solution:

Given: In triangle ABC and PQR,



$\angle A = \angle Q$ and $\angle B = \angle R$ [given]

$BC = RP$ [For the triangle to be congruent]

Hence, it will be congruent by AAS congruence rule.

3. “If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent.” Is the statement true? Why?

Solution:

Angle must be the included angles. Hence, this statement is not true.

4. “If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent.” Is the statement true? Why?

Solution:

As we know that, the sum of any two sides of the triangle is always greater than the third side.

5. Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer.

Solution:

As we know that, the sum of any two sides of the triangle is always greater than the third side.
So,

4 cm and 3cm = 4 cm + 3 cm = 7cm that is equal to the length of third side that is 7 cm.

Therefore, this is not possible to construct a triangle with length of sides 4cm , 3 cm and 7 cm.

6. It is given that $\triangle ABC \cong \triangle RPQ$. Is it true to say that $BC = QR$? Why?

Solution:

It is false that $BC = QR$ because $BC = PQ$ as $\triangle ABC \cong \triangle RPQ$.

7. If $\triangle PQR \cong \triangle EDF$, then is it true to say that $PR = EF$? Give reason for your answer.

Solution:

It is true, $PR=EF$ because this is the corresponding sides of triangle PQR and triangle EDF.

8. In $\triangle PQR$, $\angle P = 70^\circ$ and $\angle R = 30^\circ$. Which side of this triangle is the longest? Give reason for your answer.

Solution:

In triangle PQR,

$$\angle Q = 180^\circ - (\angle P + \angle R)$$

$$= 180^\circ - (70^\circ + 30^\circ)$$

$$= 180^\circ - 100^\circ$$

$$= 80^\circ$$

Now, in triangle PQR, angle Q is larger and side opposite to greater angle is longer.

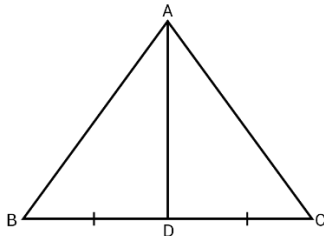
Therefore, PR is the longer side.

9. AD is a median of the triangle ABC. Is it true that $AB + BC + CA > 2 AD$? Give reason for your answer.

Solution:

In triangle ABD,

$$AB + BD > AD \dots (I)$$



$AC + CD > AD \dots (II)$ [Sum of the lengths of any two sides of a triangle must be greater than the third side]

Adding (I) and (II), get:

$$AB + BD + CD + AC > 2AD$$

$$AB + BC + CA > 2AD \quad [BD = CD \text{ as } AD \text{ is median of triangle } ABC]$$

10. M is a point on side BC of a triangle ABC such that AM is the bisector of $\angle BAC$. Is it true to say that perimeter of the triangle is greater than 2 AM? Give reason for your answer.

Solution:

To prove: $AB + BC + AC > 2AM$

Proof: We know that sum of any two side of a triangle is greater than the third side,

Now, in triangle ABM,

$$AB + BM > AM \dots (I)$$

And, in triangle ACM,

$$AC + CM > AM \dots (II)$$

Adding (I) and (II), get:

$$AB + BM + AC + CM > 2AM$$

$$AB + (BM + CM) + AC > 2AM$$

$$AB + BC + AC > 2AM$$

Hence, it is true that the perimeter of the triangle is greater than 2AM.

11. Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give reason for your answer.

Solution:

We know that sum of any two side of a triangle is greater than the third side. So,

$$9 \text{ cm} + 7 \text{ cm} = 16 \text{ cm} < 17 \text{ cm}$$

Hence, it is not possible to construct a triangle.

12. Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer.

Solution:

Yes, that is possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm because the sum of any two side of a triangle is greater than the third side.

Exercise No. 7.3

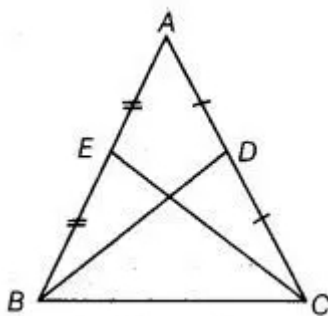
Short Answer Questions:

1. ABC is an isosceles triangle with $AB = AC$ and BD and CE are its two medians. Show that $BD = CE$.

Solution:

Given:

ABC is an isosceles triangle with $AB = AC$ and BD and CE are its two medians.



To prove: $BD = CE$

Proof: in triangle ABC,

$AB = AC$ [Given]

$$\frac{1}{2} AB = \frac{1}{2} AC$$

$AE = AD$ [D is the mid-point of AC and E is the mid-point of AB]

Now, in triangle ABD and triangle ACE,

$AB = AC$ [Given]

$\angle A = \angle A$ [Common angle]

$AE = AD$ [above proved]

Now, by SAS criterion of congruence, get:

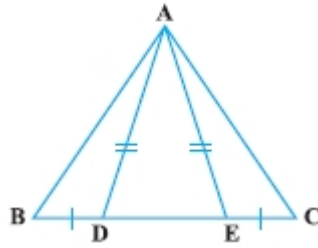
$$\triangle ABD \cong \triangle ACE$$

$BD = CE$ [CPCT]

Hence, proved.

2. In Fig., D and E are points on side BC of a $\triangle ABC$ such that $BD = CE$ and $AD = AE$.

Show that $\triangle ABD \cong \triangle ACE$.



Solution:

Given in triangle ABD,

$BD = CE$ and $AD = AE$

To prove that $\triangle ABD \cong \triangle ACE$

Proof:

$AD = AE$

[Given]

$\angle ADE = \angle AED$

[Since, angle opposite to equal sides are equal] ... (I)

$\angle ADB + \angle ADE = 180^\circ$

[Linear pair axiom]

$\angle ADB = 180^\circ - \angle ADE$

$\angle ADB = 180^\circ - \angle AED$ [From equation (i)]

In triangle ABD and triangle ACE,

$\angle ADB = \angle AEC$

[Since, $\angle AEC + \angle AED = 180^\circ$, linear pair axiom]

$BD = CE$

[Given]

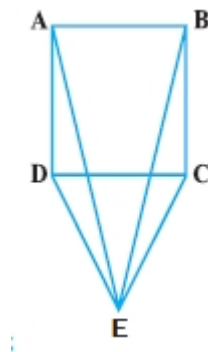
$AD = AE$

[Given]

$\triangle ABD \cong \triangle ACE$

[By SAS congruence rule]

3. CDE is an equilateral triangle formed on a side CD of a square ABCD as shown in fig. Show that $\triangle ADE \cong \triangle BCE$.



Solution:

Given in figure triangle CDE is an equilateral triangle formed on a side CD of a square ABCD.

To proof that $\triangle ADE \cong \triangle BCE$

Proof: In triangle ADE and triangle BCE,

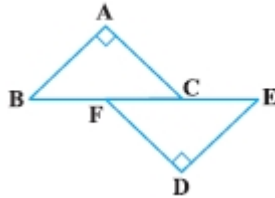
$DE = CE$ [Sides of an equilateral triangle]

$\angle ADE = \angle BCE$

$\angle ADC = \angle BCD = 90^\circ$ and $\angle EDC = \angle ECD = 60^\circ$

$$\begin{aligned}\angle ADE &= 90^\circ + 60^\circ = 150^\circ \text{ and } \angle BCE = 90^\circ + 60^\circ = 150^\circ \\ AD &= BC && [\text{Sides of a square}] \\ \triangle ADE &\cong \triangle BCE && [\text{By SAS congruence rule}]\end{aligned}$$

4. In Fig., $BA \perp AC$, $DE \perp DF$ **such that** $BA = DE$ **and** $BF = EC$. **Show that** $\triangle ADC \cong \triangle DEF$.



Solution:

See in the figure,

$BA \perp AC$, $DE \perp DF$ such that $BA = DE$ and $BF = EC$

To prove that $\triangle ADC \cong \triangle DEF$

Proof:

$BF = EC$ [Given]

Now, adding CF both sides, get:

$$BF + CF = EC + CF$$

$$BC = EF \quad \dots(I)$$

In triangle ABC and triangle DEF ,

$$\angle A = \angle D = 90^\circ \quad [BA \perp AC \text{ and } DE \perp DF]$$

$$BC = EF \quad [\text{from eq. (I)}]$$

$$BA = DE \quad [\text{Given}]$$

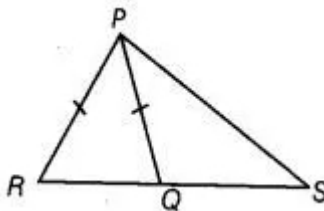
$$\triangle ABC \cong \triangle DEF \quad [\text{By RHS congruence rule}]$$

5. Q is a point on the side SR of a $\triangle PSR$ such that $PQ = PR$. Prove that $PS > PQ$.

Solution:

In triangle PSR , Q is a point on the side SR such that $PQ = PR$.

To prove that $PS > PQ$



Proof: In triangle PRQ ,

$$PQ = PR \quad [\text{Given}]$$

$$\angle R = \angle PQR \quad \dots(I) \quad [\text{Angle opposite to equal sides are equal}]$$

$\angle PQR > \angle S$... (II) [Exterior angle of a triangle is greater than each of the opposite interior angle]

Now, from equation (I) and (II), get:

$$\angle R > \angle S$$

$$PS > PR$$

$$PS > PQ$$

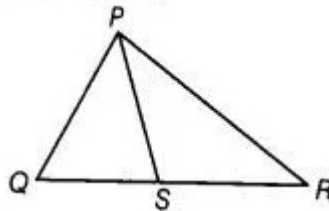
[side opposite to greater angle is longer]

$$[PQ = PR]$$

6. S is any point on side QR of a $\triangle PQR$. Show that: $PQ + QR + RP > 2 PS$.

Solution:

Given in triangle PQR, S is any point on side QR.



To prove that $PQ + QR + RP > 2PS$

Proof: In triangle PQS,

$$PQ + QS > PS \text{ (i) [Sum of two side of a triangle is greater than the third side]}$$

Now, similarly in triangle PRS,

$$SR + RP > PS \text{ (ii) [Sum of two side of a triangle is greater than the third side]}$$

Adding equation (I) and (II), get:

$$PQ + QS + SR + RP > 2PS$$

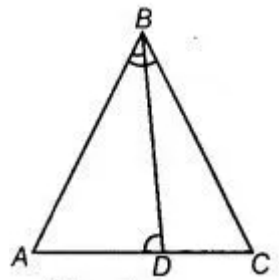
$$PQ + (QS + SR) + RP > 2PS$$

$$PQ + QR + RP > 2PS \quad [QR = QS + SR]$$

7. D is any point on side AC of a $\triangle ABC$ with $AB = AC$. Show that $CD < BD$.

Solution:

Given in triangle ABC, D is any point on side AC such that $AB = AC$.



To prove that $CD < BD$ or $BD > CD$

To prove:

$$AC = AB$$

[Given]

$$\angle ABC = \angle ACB \quad \text{(i)[Angle opposite to equal sides are equal]}$$

In triangle ABC and triangle DBC,

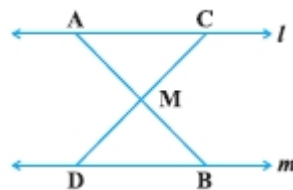
$$\angle ABC > \angle DBC \quad [\angle DBC \text{ is a internal angle of } \angle B]$$

$$\angle ACB > \angle DBC \quad [\text{From equation (I)}]$$

$$BD > CD \quad [\text{Side opposite to greater angle is longer}]$$

$$CD < BD$$

8. In Fig., $l \parallel m$ and M is the mid-point of a line segment AB. Show that M is also the mid-point of any line segment CD, having its end points on l and m , respectively.



Solution:

See in the figure, $l \parallel m$ and M is the mid-point of a line segment AB.

To prove that $MC = MD$

Proof: $l \parallel m$ [Given]

$$\angle BAC = \angle ABD \quad [\text{Alternate interior angles}]$$

$$\angle AMC = \angle BMD \quad [\text{Vertical opposite angle}]$$

In triangle AMC and triangle BMD,

$$\angle BAC = \angle ABD \quad [\text{Proved above}]$$

$$AM = BM \quad [\text{Given}]$$

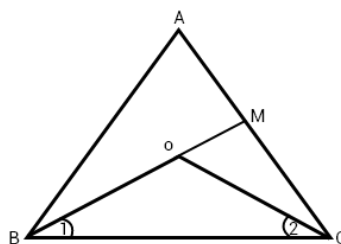
$$\angle AMC = \angle BMD \quad [\text{By ASA congruence rule}]$$

$$MC = MD \quad [\text{By CPCT}]$$

9. Bisectors of the angles B and C of an isosceles triangle with $AB = AC$ intersect each other at O. BO is produced to a point M. Prove that $\angle MOC = \angle ABC$.

Solution:

Given in the question, bisectors of the angles B and C of an isosceles triangle ABC with $AB = AC$ intersect each other at O. Now BO is produced to a point M.



In triangle ABC,

$$AB = AC$$

$$\angle ABC = \angle ACB \quad [\text{Angle opposite to equal sides of a triangle are equal}]$$

$$\frac{1}{2} \angle ABC = \frac{1}{2} \angle ACB$$

That is $\angle 1 = \angle 2$ [Since, BO and CO are bisectors of $\angle B$ and $\angle C$]

In triangle OBC, Ext. $\angle MOC = \angle 1 + \angle 2$ [Exterior angle of a triangle is equal to the sum of interior opposite angles]

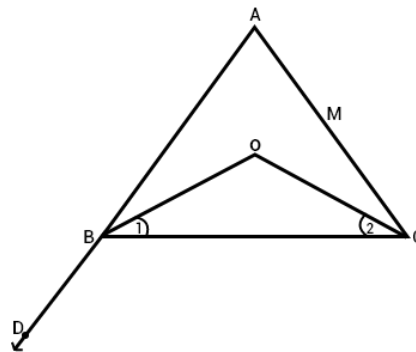
$$\text{Ext. } \angle MOC = 2\angle 1 \quad [\angle 1 = \angle 2]$$

$$\text{Hence, } \angle MOC = \angle ABC.$$

10. Bisectors of the angles B and C of an isosceles triangle ABC with $AB = AC$ intersect each other at O. Show that external angle adjacent to $\angle ABC$ is equal to $\angle BOC$.

Solution:

In triangle ABC,



$$AB = AC$$

So, $\angle B = \angle C$ [Angle opposite to equal sides of a triangle are equal]

$$\frac{1}{2} \angle B = \frac{1}{2} \angle C \quad \dots(I)$$

In triangle OBC,

$$\angle 1 = \frac{1}{2} \angle B$$

$$\text{And, } \angle 2 = \frac{1}{2} \angle C \quad [\text{By (I)}]$$

$$\angle DBC + \angle 1 + \angle OBA = 180^\circ \quad [\text{ABD is a straight line}]$$

In triangle OBC,

$$\angle 1 + \angle 2 + \angle BOC = 180^\circ$$

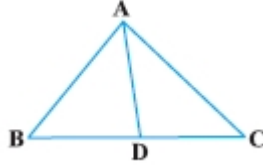
$$2\angle 1 + \angle BOC = 180^\circ \quad [\angle 1 = \angle 2] \dots (II)$$

From equation (I) and (II), get:

$$\angle DBA + 2\angle 1 = 2\angle 1 + \angle BOC$$

$$\angle DBC = \angle BOC$$

11. In Fig. 7.8, AD is the bisector of $\angle BAC$. Prove that $AB > BD$.



Solution:

In triangle ACD,

Ext. $\angle ADB > \angle DAC$ [Exterior angle of a triangle is greater than either of the interior opposite angle]

$$\angle ADB > \angle BAD$$

Since, in triangle ABD,

$$\angle ADB > \angle BAD$$

Hence, $AB > BD$. [In a triangle, side opposite to greater angle is longer]

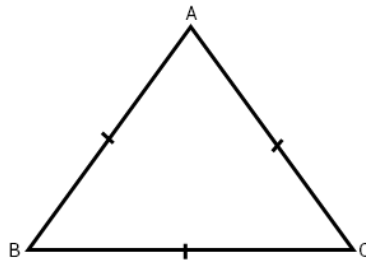
Exercise No. 7.4

Long Answer Questions:

1. Find all the angles of an equilateral triangle.

Solution:

In triangle ABC,



$$AB = AC$$

$$\angle C = \angle B \quad \dots \text{(I) [Angles opposite to equal sides of a triangle are equal]}$$

$$BC = AC$$

$$\angle A = \angle B \quad \dots \text{(II)}$$

$$\angle A + \angle B + \angle C = 180^\circ \text{ [Angle sum property of a triangle]}$$

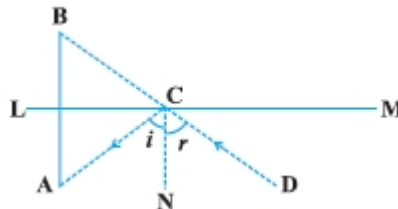
$$\angle A + \angle A + \angle A = 180^\circ \text{ [From equation (I) and (II)]}$$

$$\angle A = \frac{180^\circ}{3}$$

$$\angle A = 60^\circ$$

2. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in Fig. Prove that the image is as far behind the mirror as the object is in front of the mirror.

[Hint: CN is normal to the mirror. Also, angle of incidence = angle of reflection].



Solution:

Let AB intersect LM at O.

To prove: $AO = BO$.

$$\text{Proof: } \angle i = \angle r \quad \dots \text{(I) [Angle of incidence = Angle of reflection]}$$

$$\angle B = \angle r \text{ [Corresponding angle]} \quad \dots \text{(II)}$$

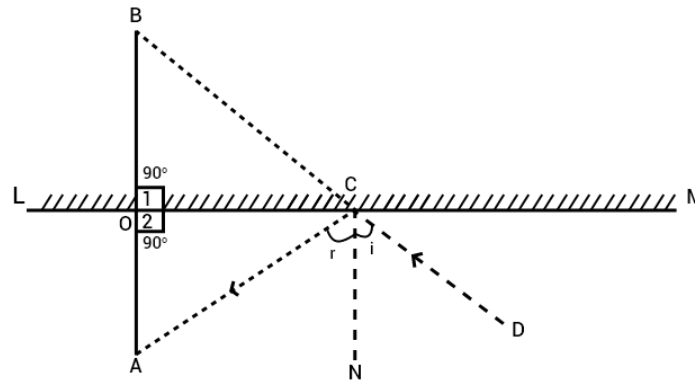
Now,

$$\angle A = \angle i \quad [\text{Alternate int. } \angle s] \dots(\text{III})$$

Since, from equation (I), (II) and (III), get:

$$\angle B = \angle A$$

$$\angle BCO = \angle ACO$$



In triangle BOC and triangle AOC, get:

$$\angle 1 = \angle 2 \quad [\text{Each} = 90^\circ]$$

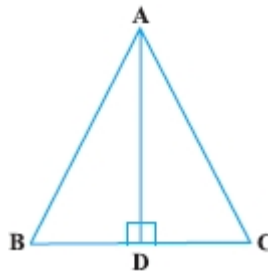
$$OC = OC \quad [\text{Common side}]$$

$$\angle BCO = \angle ACO \quad [\text{Prove above}]$$

$$\triangle BOC \cong \triangle AOC \quad [\text{ASA congruence rule}]$$

$$\text{Hence, } AO = BO \quad [\text{CPCT}]$$

3. ABC is an isosceles triangle with $AB = AC$ and D is a point on BC such that $AD \perp BC$ (as shown in Fig.). To prove that $\angle BAD = \angle CAD$, a student proceeded as follows:



In $\triangle ABD$ and $\triangle ACD$,

$$AB = AC \text{ (Given)}$$

$$\angle B = \angle C \text{ (because } AB = AC)$$

$$\text{and } \angle ADB = \angle ADC$$

$$\text{Therefore, } \triangle ABD \cong \triangle ACD \text{ (AAS)}$$

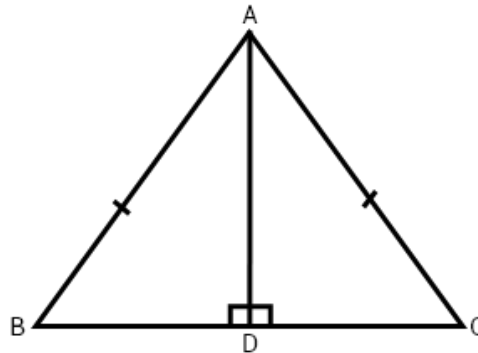
$$\text{So, } \angle BAD = \angle CAD \text{ (CPCT)}$$

What is the defect in the above arguments?

[Hint: Recall how $\angle B = \angle C$ is proved when $AB = AC$].

Solution:

In triangle ADB and triangle ADC, get:



$\angle ADB = \angle ADC$	[Each equal to 90°]
$AB = AC$	[Given]
$AD = AD$	[Common side]

Now, by RHS criterion of congruence, get:

$$\triangle ADB \cong \triangle ADC$$

So, $\angle BAD = \angle CAD$ [CPCT]

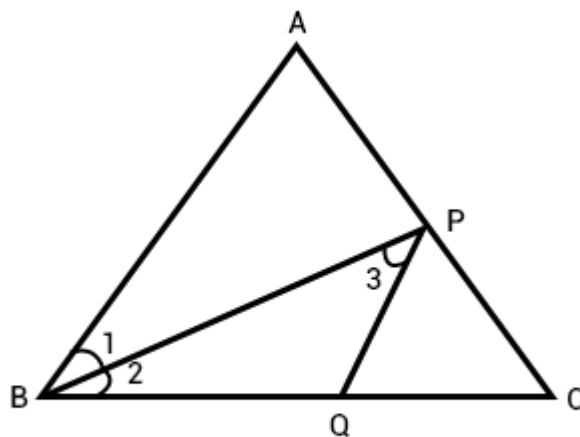
Hence, proved.

4. P is a point on the bisector of $\angle ABC$. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.

Solution:

Given in the question, P is a point on the bisector of $\angle ABC$. If the line through P, parallel to BA meet BC at Q.

To prove: BPQ is an isosceles triangle.



Proof: $\angle 1 = \angle 2$

...(I) [BP is the bisector of $\angle ABC$]

PQ is parallel to BA and BP cuts them. So,

$\angle 1 = \angle 3$ [Alternate interior angles as $PQ \parallel AB$]

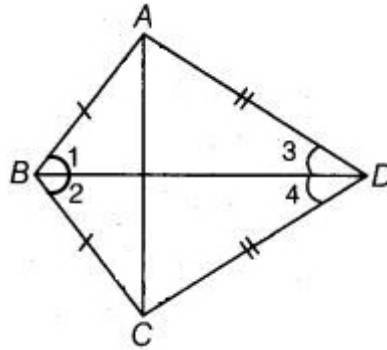
$\angle 2 = \angle 3$ [Proved above]

$PQ = BQ$ [Sides opposite to equal angle are equal]
Hence, BPQ is an isosceles triangle.

5. ABCD is a quadrilateral in which $AB = BC$ and $AD = CD$. Show that BD bisects both the angles ABC and ADC .

Solution:

In triangle ABC and triangle CBD ,



$AB = BC$ [Given]
 $AD = CD$ [Given]
 $BD = BD$ [Common side]

So, $\triangle ABC \cong \triangle CBD$ [By SSS congruence rule]

$\angle 1 = \angle 2$ [CPCT]

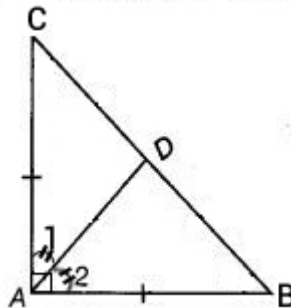
And, $\angle 3 = \angle 4$

Hence, BD bisects both the angle ABC and ADC .

6. ABC is a right triangle with $AB = AC$. Bisector of $\angle A$ meets BC at D . Prove that $BC = 2 AD$.

Solution:

Given in the question, ABC is a right triangle with $AB = AC$. Bisector of $\angle A$ meets BC at D .



To prove that $BC = 2AD$

Proof: In right triangle ABC ,

$AB = AC$ [Given]

BC is hypotenuse. So,

$$\angle BAC = 90^\circ$$

Now, in triangle CAD and triangle BAD, get:

$$\begin{array}{ll} AC = AB & \text{[Given]} \\ \angle 1 = \angle 2 & \text{[AD is the bisector of } \angle A \text{]} \\ AD = AD & \text{[Common side]} \end{array}$$

Now, by SAS criterion of congruence, get:

$$\triangle CAD \cong \triangle BAD$$

$$CD = BD \quad \text{[CPCT]}$$

$AD = BD = CD \dots (I)$ [Mid-point of hypotenuse of a rt. Triangle is equidistant from the three vertices of a triangle]

$$\text{Now, } BC = BD + CD$$

$$BC = AD + AD \quad \text{[Using (I)]}$$

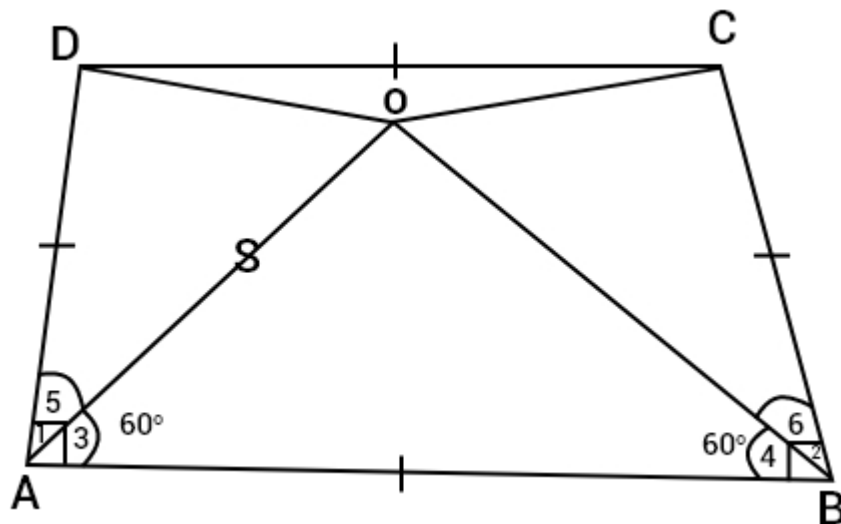
$$BC = 2AD$$

Hence, proved.

7. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that $\triangle OCD$ is an isosceles triangle.

Solution:

Given in the question, A square of ABCD and $OA = OB = AB$.



To prove that triangle OCD is an isosceles triangle.

Proof: In triangle ABCD,

$$\angle 1 = \angle 2 \quad \dots (I) \quad \text{[Each equal to } 90^\circ \text{]}$$

In triangle OAB,

$$\angle 3 = \angle 4 \quad \dots (II) \quad \text{[Each equal to } 60^\circ \text{]}$$

Now, subtracting equation (II) from equation (I), get:

$$\angle 1 - \angle 3 = \angle 2 - \angle 4$$

$$\angle 5 = \angle 6$$

In triangle DAO and triangle CBO,

$$AD = BC \quad [\text{Given}]$$

$$\angle 5 = \angle 6 \quad [\text{Proved above}]$$

$$OA = OB \quad [\text{Given}]$$

So, by SAS criterion of congruence, get:

$$\triangle DAO \cong \triangle CBO$$

$$OD = OC$$

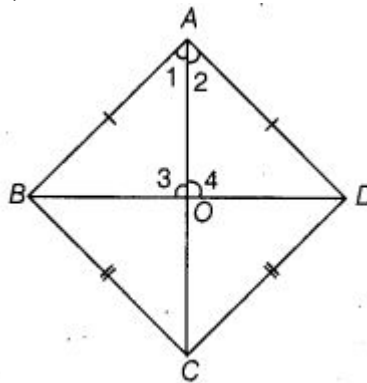
Now, in triangle OCD is an isosceles triangle.

Hence, proved.

8. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.

Solution:

Given in the question, ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC.



To prove that AD is the perpendicular bisector of BC that is $OB = OC$.

Proof: In triangle BAD and triangle CAD,

$$AB = AC \quad [\text{Given}]$$

$$BD = CD \quad [\text{Given}]$$

$$AD = AD \quad [\text{Common side}]$$

Now, by SSS criterion of congruence,

$$\triangle BAD \cong \triangle CAD$$

$$\text{So, } \angle 1 = \angle 2 \quad [\text{CPCT}]$$

Now, in triangle BAO and triangle CAO,

$$AB = AC \quad [\text{Given}]$$

$$\angle 1 = \angle 2 \quad [\text{Proved above}]$$

$$AO = AO \quad [\text{Common side}]$$

So, by SAS criterion of congruence,

$$\triangle BAO \cong \triangle CAO$$

Since, $BO = CO$ [CPCT]

And, $\angle 3 = \angle 4$ [CPCT]

$$\angle 3 + \angle 4 = 180^\circ \quad [\text{Linear pair axiom}]$$

$$\angle 3 + \angle 3 = 180^\circ$$

$$2\angle 3 = 180^\circ$$

$$\angle 3 = \frac{180^\circ}{2}$$

$$\angle 3 = 90^\circ$$

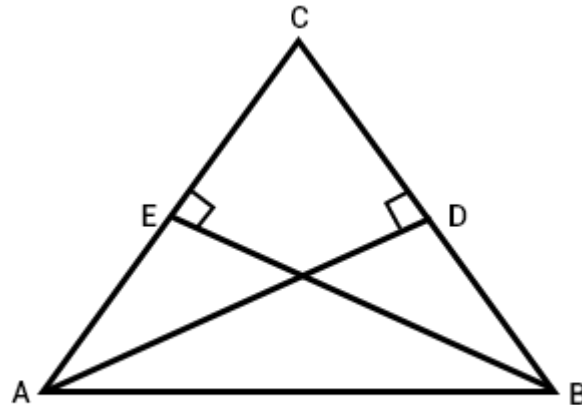
Therefore, AD is perpendicular to bisector of BC.

Hence, proved.

9. ABC is an isosceles triangle in which $AC = BC$. AD and BE are respectively two altitudes to sides BC and AC. Prove that $AE = BD$.

Solution:

In triangle ADC and triangle BEC,



$$AC = BC \quad [\text{Given}] \dots (I)$$

$$\angle ADC = \angle BEC \quad [\text{Each is } 90^\circ]$$

$$\angle ACD = \angle BCE \quad [\text{Common angle}]$$

So, $\triangle ADC \cong \triangle BEC$ [By SSS congruence rule]

$$CE = CD \quad \dots (II) \quad [\text{CPCT}]$$

Now, Subtracting equation (II) from (I), get:

$$AC - AE = BC - CD$$

$$AE = BD$$

Hence, proved.

10. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Solution:

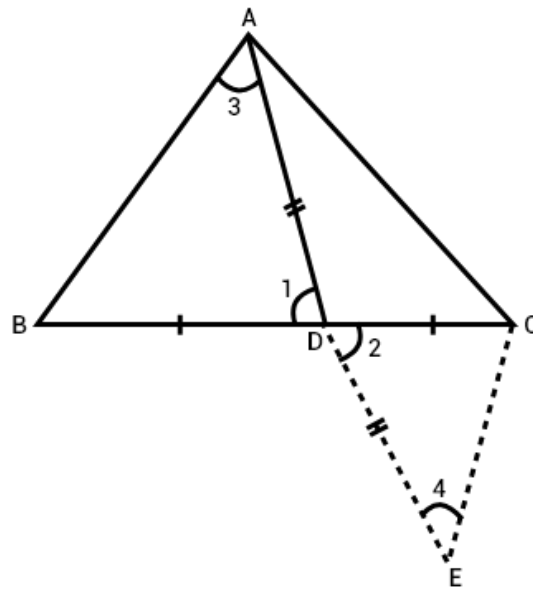
Given in triangle ABC with median AD,

To proof:

$$AB + AC > 2AD$$

$$AB + BC > 2AD$$

$$BC + AC > 2AD$$



Producing AD to E such that $DE = AD$ and join EC.

Proof: In triangle ADB and triangle EDC,

$$AD = ED \quad [\text{By construction}]$$

$$\angle 1 = \angle 2 \quad [\text{Vertically opposite angles are equal}]$$

$$DB = DC \quad [\text{Given}]$$

So, by SAS criterion of congruence,

$$\triangle ADB \cong \triangle EDC$$

$$AB = EC \quad [\text{CPCT}]$$

$$\text{And, } \angle 3 = \angle 4 \quad [\text{CPCT}]$$

Again, in triangle AEC,

$AC + CE > AE$ [Sum of the lengths of any two sides of a triangle must be greater than the third side]

$$AC + CE > AD + DE$$

$$AC + CE > AD + AD \quad [AD = DE]$$

$$AC + CE > 2AD$$

$$AC + AB > 2AD \quad [\text{Because } AB = CE]$$

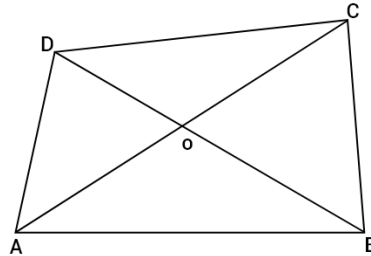
Hence proved.

Similarly, $AB + BC > 2AD$ and $BC + AC > 2AD$.

11. Show that in a quadrilateral $AB + BC + CD + DA < 2(BD + AC)$

Solution:

Given in the question, A quadrilateral,



To prove that $AB + BC + CD + DA < 2(BD + AC)$

Proof: In triangle AOB,

$OA + OB > AB$... (I) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

In triangle BOC,

$OB + OC > BC$... (II) [Same reason]

In triangle COD,

$OC + OD > CD$... (III) [Same reason]

In triangle DOA,

$OD + OA > DA$... (IV) [Same reason]

Now, adding equation (I), (II), (III) and (IV), get:

$$OA + OB + OB + OC + OD + OD + OA > AB + BC + CD + DA$$

$$2(OA + OB + OC + OD) > AB + BC + CD + DA$$

$$2\{(OA + OC) + (OB + OD)\} > AB + BC + CD + DA$$

$$2(AC + BD) > AB + BC + CD + DA$$

$$AB + BC + CD + DA < 2(BD + AC)$$

Hence, proved.

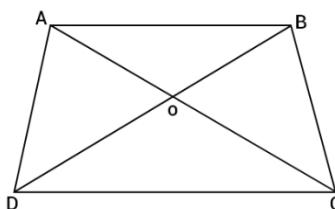
**12. Show that in a quadrilateral ABCD,
 $AB + BC + CD + DA > AC + BD$**

Solution:

Given in the question, a quadrilateral ABCD.

To prove that $AB + BC + CD + DA > AC + BD$.

Proof: In triangle ABC,



$AB + BC > AC$... (I) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

In triangle BCD,
 $BC + CD > BD$... (II) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

In triangle CDA,
 $AD + DA > AC$... (III) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

Similarly, in triangle DAB,
 $AD + AB > BD$... (IV) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

Now, adding equation (I), (II), (III) and (IV), get:

$$AB + BC + BC + CD + CD + DA + AD + AB > AC + BD + AC + BD$$

$$2AB + 2BC + 2CD > 2AC + 2BD$$

$$2(AB + BC + CD + DA) > 2(AC + BD)$$

$$AB + BC + CD + DA > AC + BD$$

Hence, proved.

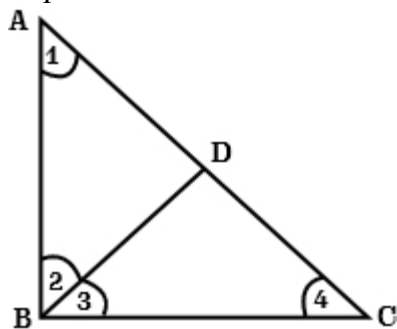
13. In a triangle ABC, D is the mid-point of side AC such that $BD = \frac{1}{2} AC$.

Show that $\angle ABC$ is a right angle.

Solution:

Given: D is the mid-point of side AC.

To prove: $\angle ABC = 90^\circ$



Proof: $AD = DC$

$$\text{And, } BD = \frac{1}{2} AC = AD \quad [\text{D is the mid-point of side AC}]$$

$$BD = AD = DC$$

In triangle ABD,

$$BD = AD$$

$$\angle 1 = \angle 2 \quad \dots \text{ (I) [Angles opposite to equal sides are equal]}$$