Random variable

A random variable(x) is defined as a function that associates a real number (the probability value) to an outcome of an experiment. $|X: S \rightarrow N\{0, 1, 2, 3\}|$

Ex- Tossing two fair coin,

sample space $= \{\underset{2}{HH}, \underset{1}{HT}, \underset{1}{TH}, \underset{0}{TT}\}$

$X =$ (count the number of time Head turns)

$= 2, 1, 0$

What is the popability that X can take the vale 2,

$P(X=2) = P(HH) = 1/4$

What " " " 1,

$P(X=1) = P(TH+HT) \Rightarrow 1/4 + 1/4 \Rightarrow 1/2$

What " " " 0,

$P(X=0) = P(TT) \Rightarrow 1/4$

Ex- fliping three fair coin,

$SS = \{\underset{1/8}{HHH}, \underset{1/8}{HHT}, \underset{1/8}{HTH}, \underset{1/8}{HTT}, \underset{1/8}{THH}, \underset{1/8}{THT}, \underset{1/8}{TTH}, \underset{1/8}{TTT}\}$

$X =$ (defin no of Teil comeout)

$X = 3, 2, 1, 0$

$\rightarrow$ random variable.

What is the probability that X can take the vale 0;

$P(X=0) = P(HHH) = 1/8$.

$P(X=1) = P(HHT + HTH + THH)$

$= 3/8$.

} probability distribution for random variable.

$P(X=2) = 3/8$

$P(X=3) = 1/8$

$\boxed{Ex-3}$ Rolling two dice.

Sample space $= \{(1,1)\ (1,2)\ \ldots\ldots\ (6,6)\}$
$\qquad\qquad 1/36 \quad 1/36 \qquad\qquad 1/36$

$X = $ (sum values on two dice)
$= \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
→ Random value.

$P(X=2) = 1/36$

$P(X=3) = P\{(1,2)\ (2,1)\} = \frac{2}{36}$

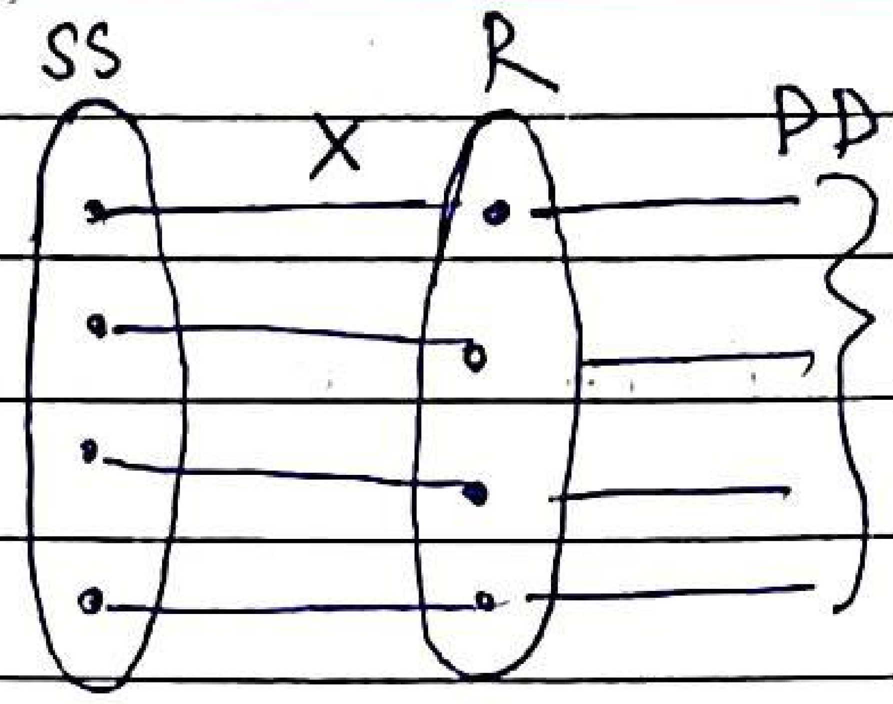
$P(X=4) = P\{(1,3)\ (2,2)\ (3,1)\} = \frac{3}{36}$

⋮

$P(X=12) = P\{(6,6)\} = 1/36$.

Random variable

- Discrete Random variable
- Continuous Random variable



PD (probability distribution)

add them all up then the sum will be '1'.

- A fair coin flip twoice - $SS = \{HH, HT, TH, TT\}$

$X =$ (define no of time head come out)

$P(X=0) = 1/4$

$P(X=1) = 2/4$

$P(X=2) = 1/4$

R

$(1/4 + 2/4 + 1/4 \Rightarrow 4/4 \Rightarrow 1)$

$\boxed{EX-4}$ Have a coin, then probability of getting head 'P'. and probability of getting Tail is '(P-1)'. Experiment is fliping a coin until I get a had and 'N' denote a random variable which Indicates the number of time I have flip the coin until I get the first head.

→ $N =$ (define no of time fliped the coin until I get the first head)

$N = (1, 2, 3, 4, \ldots\ldots)$

Probability of getting head In first attempt -

$P(N=1) = P(H) = P$

$P(N=2) = P(TH) = (1-P)P.$

$P(N=3) = P(TTH) = (1-P)(1-P)P$

$P(N=4) = P(TTTH) = (1-P)(1-P)(1-P)P$

$\vdots$

$P(N=n) = P(TT\cdots H) = (1-P)^{n-1}P$

$$\sum_{n=1}^{\infty} P(N=n) = P + (1-P)P + (1-P)^2 P + \cdots + (1-P)^{n-1} P.$$

$$= P\left(\underset{a}{1} + \underset{r}{(1-P)} + (1-P)^2 + \cdots + (1-P)^{n-1}\right) \text{ G.P}$$

$1-(1-p)^n \cong 1$

$$p\left(\frac{1-(1-p)^n}{1-(1-p)}\right) = 1$$

$$\boxed{\sum_{n=1}^{\infty} P(X=n) \Rightarrow p\left(\frac{1}{1-(1-p)}\right) = 1}$$ (sum of all are '1').

- <u>Bernoulli random variable</u> :-

→ In a experiment whose outcome can be classified as either a success or a failure, let '$x=1$' if the outcome is success and '$x=0$' if it is a failure. The probability mass function of 'x' is give by -

$P(0) = P(X=0) = (1-p)$
$P(1) = P(X=1) = P$, where 'p' is probability of success,

x is bernoulli random variable.

<u>example</u> -

1> Rolling a dice - ~~picked up '1' - if '1' comes up~~ then success

$SS = \{1, | 2, 3, 4, 5, 6\}$ if any other comes up then failure

$\frac{1}{6}$ | $\frac{5}{6}$

↓ success ↓ failure

$p(X=1) = \frac{1}{6}$ $p(X=0) = \frac{5}{6}$

In this experiement probability of getting success is $\frac{1}{6}$ and failure is $\frac{5}{6}$.

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- Binomial random Variable:

→ suppose 'n' independent experiments, each of which results in a 'success' with probability P and in a failure with probability (1-P) are to be performed. If 'x' represents the number of successes that occurs in 'n' trails, then x is called binominal random variable with parameters (n, p).

The probability mass function of a binomal random variable having parameters (n, P) is given by

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$$\boxed{P(i) = {}^{n}C_{i}\, p^{i}\,(1-P)^{n-i}, \quad i = 0, \dots n}$$

Ex-1 flipping 4 coin, $P(H) = 1/2$ and $P(T) = 1/2$ here getting a ~~success~~ Head is success and getting a tail is failure. what is the prob that you get axectly one success.

→

HTTT + THTT + TTHT + TTTH

probability of getting '1' success is-

$$P(1) = {}^{4}C_{1}\, P\,(1-P)^{4-1}$$

$$= {}^{4}C_{1}\, P\,(1-P)^{3}$$

* Ex-2

It is known that any item produced by a certain machine will be defective with probability 0.1, independently of any other item. what is the probability that in a sample of 3 items,

(a) exactly one is defective.

$$P(i) = {}^{n}C_{i}\, p^{i}\,(1-P)^{n-i}$$

$$P(X=1) \Rightarrow P(1) = {}^{3}C_{1}\,(0.1)\,(1-0.1)^{3-1} \Rightarrow {}^{3}C_{1}\,(0.1)(0.9)^{2}$$

(b) Atmost one

$P(x=0) = {}^3C_0 (0.1)^0 (0.9)^3$

$\underbrace{P(X=0)+P(X=1)} = {}^3C_1 (0.1)(0.9)^2 + {}^3C_0 (0.1)^0 (0.9)^3.$

(c) At least one ~~(P(0)+P(1)+P(2))~~

$\underbrace{P(X=1)+P(X=2)+P(X=3)}$

Poisson random variable –

→ A random variable 'X', taking on one of the values 0, 1, 2, ... is said to be poisson random variable with perameter λ, if for some $\lambda > 0$,

$$P(i) = \boxed{P(X=i) = e^{-\lambda} \frac{\lambda^i}{i!}}, \quad i = 0, 1, 2, 3, \dots$$

Note: $\lambda \longrightarrow (n\uparrow \; P\downarrow) \rightarrow$ (In Binomial r.v)

$\lambda = np$

Binomial random variable can be appoximated using possion random variable.

Ex-1 Suppose that the number of typographical errors on a single page of this ~~bag~~ book has a poisson distribution with $\lambda = 1$. Calculate the propability that there is at least one error on this page.

→ $P(X=2) = e^{-\lambda} \frac{\lambda^i}{i!}$

$= e^{-1} \frac{\lambda^2}{2!}$

prob of no errors,

$P(0) = e^{-1} \frac{\lambda^0}{0!}$

$= \frac{1}{e}.$

→ $P(X=0) + P(X=1) + P(X=2) + \dots = 1$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - e^{-1} \frac{\lambda^0}{0!}$

$= \left(1 - \frac{1}{e}\right)$

$\boxed{\text{Ex-2}}$ If the number of accidents occuring on a high-way is a poisson random variable with parameter $\lambda = 3$, what is the probability that no accidents occur today.

$\rightarrow$ $P(i) = P(X=i) = e^{-\lambda} \dfrac{\lambda^i}{i!}$

for no accident - $P(X=0) = e^{-3} \dfrac{3^0}{0!} \Rightarrow 1/e^3$.

for one accident today. $P(X=1) = e^{-3} \dfrac{3^1}{1!}$

$= \dfrac{3}{e^3}$

number of accident today atleast '1'.

$\therefore P(X \geq 1) = 1 - P(X=0)$

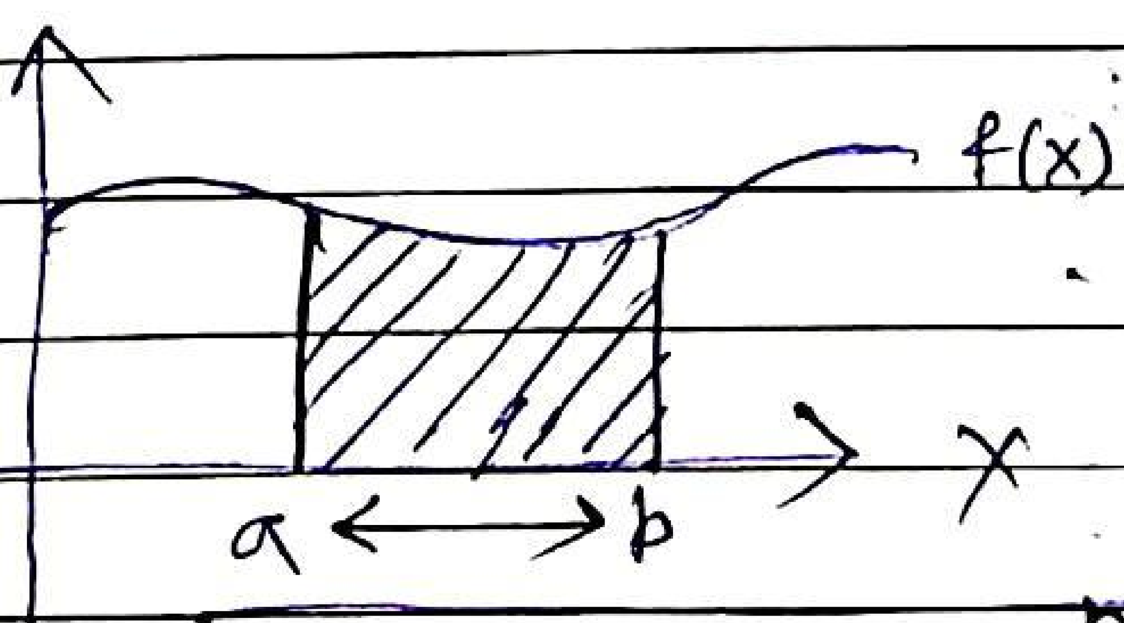
$= 1 - \dfrac{1}{e^3}$

- Continuous Random variable - (continuous)

$\rightarrow$ 'X' is a continuous random variable if the set of possible values it could take is uncountable, and there exists a non negative function $f(x)$, called probability density function, having the property that for any set 'B' of real numbers -

$$P\{x \in B\} = \int_B f(x)\, dx .$$

1890 – 1920 (take a range)



$$P(a \leq X \leq b) = \int_a^b f(x)\, dx$$

$P(X=a) = \int_a^a f(x)\, dx = 0$

$P(x \leq a) = \int_{-\infty}^{a} f(x)\, x$

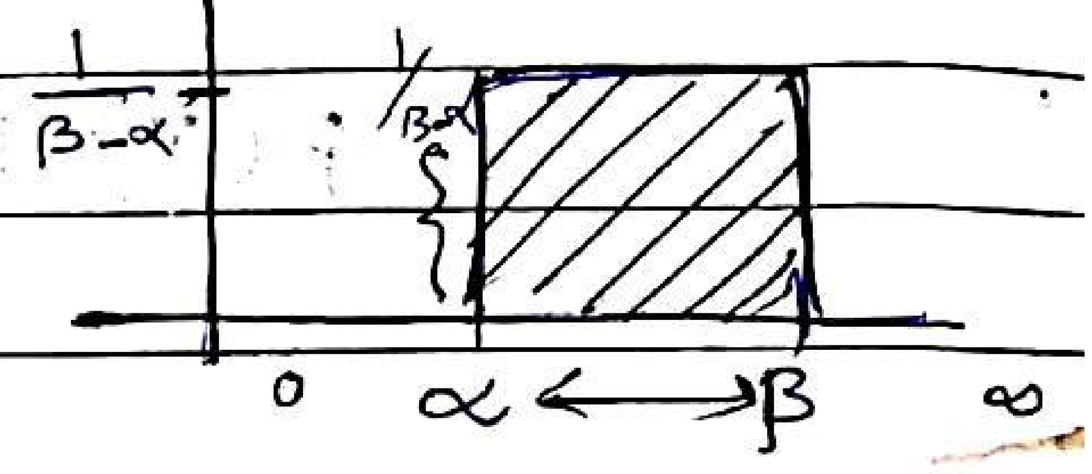
→ Total probability under the area curve always '1'.

$$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$$

- Uniform Random variable – (continuous) RV

→ 'x' is a uniform random variable on the interval (α, β) if its probability density function is given by,

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } \alpha < x < \beta \\ 0 & \text{otherwise.} \end{cases}$$



three periods $(-\infty, \alpha]$ $[\alpha, \beta]$ $[\beta, \infty)$

0 $\quad \frac{1}{\beta-\alpha} \times (\beta-\alpha)$ $\quad 0$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{\alpha} f(x)\,dx + \int_{\alpha}^{\beta} f(x)\,dx + \int_{\beta}^{\infty} f(x)\,dx$$

$$\Rightarrow 0 + \int_{\alpha}^{\beta} \frac{1}{\beta-\alpha}\,dx + 0$$

$$\Rightarrow \frac{1}{(\beta-\alpha)} [x]_{\alpha}^{\beta}$$

$$\Rightarrow \frac{1}{(\beta-\alpha)} (\beta-\alpha)$$

$$\Rightarrow 1$$

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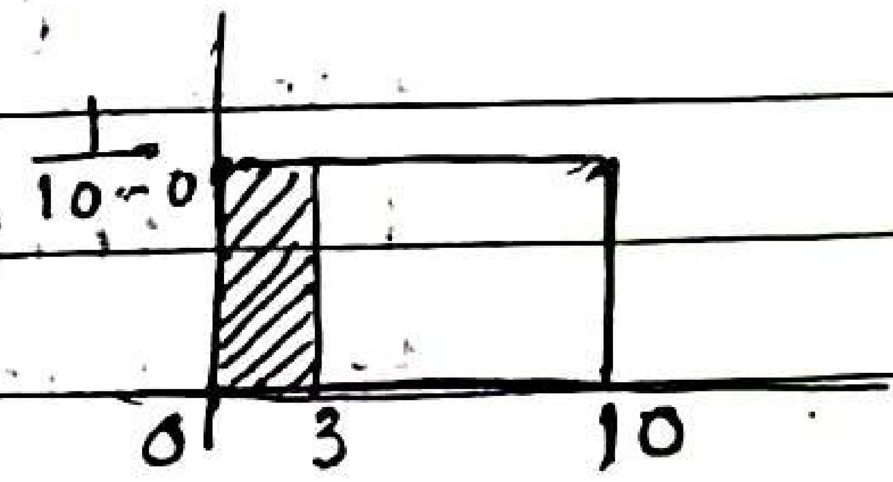
Ex-1 If 'x' is uniformly distributed over $(0, 10)$, calculate the probability that –

(a) $x < 3$ (b) $x > 7$ (c) $1 < x < 6$.

$\rightarrow$ $f(x) = \begin{cases} \frac{1}{10-0} & 0 < x \leqslant 10 \\ 0 & \text{otherwise.} \end{cases}$



(a) $\int_{-\infty}^{3} f(x)\, dx \Rightarrow \int_{0}^{3} \frac{1}{10}\, dx \Rightarrow \frac{1}{10} [x]_{0}^{3}$

$\Rightarrow \frac{1}{10}(3+0)$

$\Rightarrow 3/10.$

(b) $\int_{7}^{\infty} f(x)\, dx \Rightarrow \int_{7}^{10} \frac{1}{10}\, dx \Rightarrow \frac{1}{10}(10-7) \Rightarrow \frac{3}{10}.$

(c) $\int_{1}^{6} f(x)\, dx = \left(\frac{1}{10}\right) \times 5 \Rightarrow \frac{5}{10}\ \ 1/2.$

- Exponential Random variable –

$\rightarrow$ A continuous random variable whose probability density function is given by, for some $\lambda > 0$.

$$\boxed{f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geqslant 0 \\ 0, & \text{if } x < 0 \end{cases}}$$

$$\int_{-\infty}^{\infty} \lambda e^{-\lambda x}\, dx = 1$$

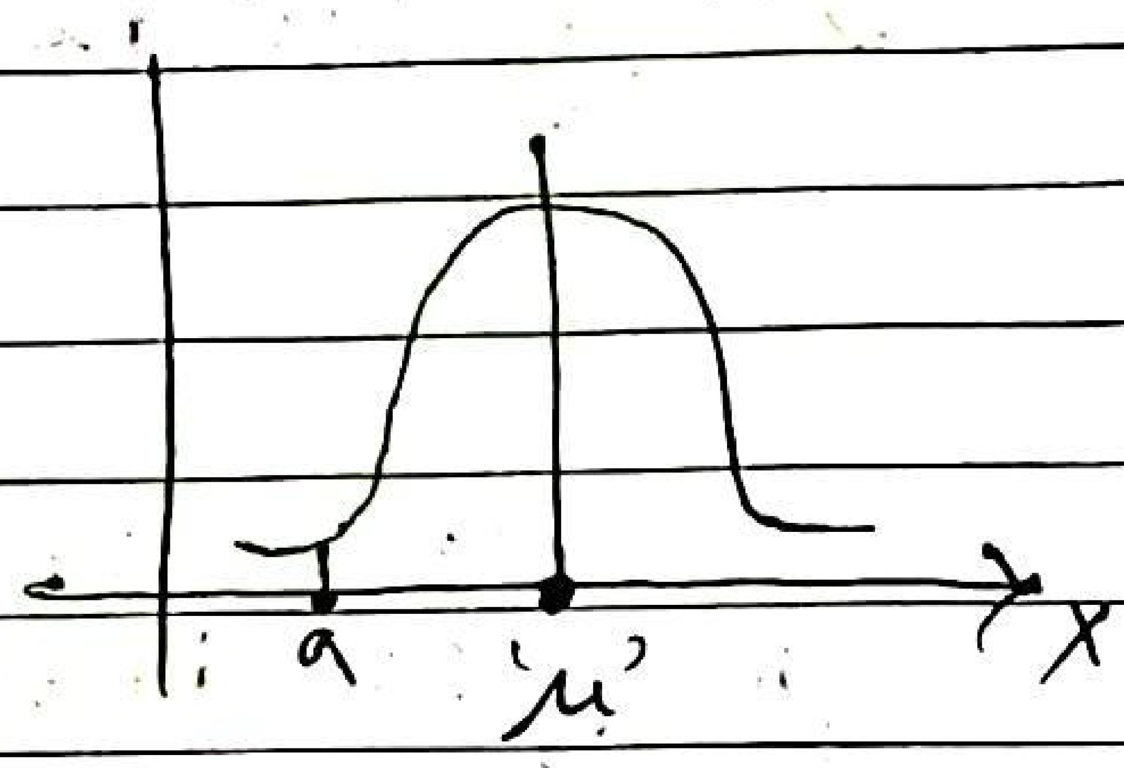
$$\boxed{\begin{aligned} &\text{Ex}- P(a \leqslant x \leqslant b) \\ &= \int_{a}^{b} \lambda e^{-\lambda x}\, dx \end{aligned}}$$

- Normal random variable –

→ We say that a continuous random variable 'x' is a normal random variable (or 'x' is normally distributed) with parameters μ and σ^2, if the ~~der~~ density of 'x' is given by- Remember it.

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}}, \quad -\infty < x < \infty .$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$



(a) $P(X \leq a)$

(b) $P(X \geq a)$

(a) $\int_{-\infty}^{a} f(x)\,dx$ (b) $\int_{a}^{\infty} f(x)\,dx$.