1. NUMBERS

IMPORTANT FACTS AND FORMULAE

I. **Numeral :** In Hindu Arabic system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called ***digits*** to represent any number.

A group of digits, denoting a number is called a ***numeral.***

We represent a number, say 689745132 as shown below :

Ten Crores (10^8)	Crores (10^7)	Ten Lacs (Millions) (10^6)	Lacs (10^5)	Ten Thousands (10^4)	Thousands (10^3)	Hundreds (10^2)	Tens (10^1)	Units (10^0)
6	8	9	7	4	5	1	3	2

We read it as : 'Sixty-eight crores, ninety-seven lacs, forty-five thousand, one hundred and thirty-two'.

II. **Place Value or Local Value of a Digit in a Numeral :**

In the above numeral :

Place value of 2 is $(2 \times 1) = 2$; Place value of 3 is $(3 \times 10) = 30$;

Place value of 1 is $(1 \times 100) = 100$ and so on.

Place value of 6 is $6 \times 10^8 = 600000000$.

III. **Face Value :** The ***face value*** of a digit in a numeral is the value of the digit itself at whatever place it may be. In the above numeral, the face value of 2 is 2; the face value of 3 is 3 and so on.

IV. **TYPES OF NUMBERS**

1. **Natural Numbers :** Counting numbers 1, 2, 3, 4, 5, ... are called ***natural numbers.***

2. **Whole Numbers :** All counting numbers together with zero form the set of ***whole numbers.*** Thus,

   (*i*) 0 is the only whole number which is not a natural number.

   (*ii*) Every natural number is a whole number.

3. **Integers :** All natural numbers, 0 and negatives of counting numbers *i.e.*, {....., −3, − 2, −1, 0, 1, 2, 3,.....} together form the set of integers.

   (*i*) **Positive Integers :** {1, 2, 3, 4, ...} is the set of all positive integers.

   (*ii*) **Negative Integers :** {− 1, − 2, − 3, ...} is the set of all negative integers.

   (*iii*) **Non-Positive and Non-Negative Integers :** 0 is neither positive nor negative. So, {0, 1, 2, 3, ...} represents the set of non-negative integers, while {0, − 1, − 2, − 3, ...} represents the set of non-positive integers.

4. **Even Numbers :** A number divisible by 2 is called an even number. *e.g.*, 2, 4, 6, 8, 10, etc.

5. **Odd Numbers :** A number not divisible by 2 is called an odd number. *e.g.*, 1, 3, 5, 7, 9, 11, etc.

6. **Prime Numbers :** A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

   Prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

   Prime numbers Greater than 100 : Let p be a given number greater than 100. To find out whether it is prime or not, we use the following method :

   Find a whole number nearly greater than the square root of p. Let $k > \sqrt{p}$. Test whether p is divisible by any prime number less than k. If yes, then p is not prime. Otherwise, p is prime.

   e.g., We have to find whether 191 is a prime number or not. Now, $14 > \sqrt{191}$.

   Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.

   191 is not divisible by any of them. So, 191 is a prime number.

7. **Composite Numbers :** Numbers greater than 1 which are not prime, are known as composite numbers. *e.g.*, 4, 6, 8, 9, 10, 12.

   Note : (*i*) 1 is neither prime nor composite.

   (*ii*) 2 is the only even number which is prime.

   (*iii*) There are 25 prime numbers between 1 and 100.

8. **Co-primes :** Two numbers a and b are said to be co-primes, if their H.C.F. is 1. *e.g.*, (2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes.

V. TESTS OF DIVISIBILITY

1. **Divisibility By 2 :** A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8.

   Ex. 84932 is divisible by 2, while 65935 is not.

2. **Divisibility By 3 :** A number is divisible by 3, if the sum of its digits is divisible by 3.

   Ex. 592482 is divisible by 3, since sum of its digits = (5 + 9 + 2 + 4 + 8 + 2) = 30, which is divisible by 3.

   But, 864329 is not divisible by 3, since sum of its digits = (8 + 6 + 4 + 3 + 2 + 9) = 32, which is not divisible by 3.

3. **Divisibility By 4 :** A number is divisible by 4, if the number formed by the last two digits is divisible by 4.

   Ex. 892648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4.

   But, 749282 is not divisible by 4, since the number formed by the last two digits is 82, which is not divisible by 4.

4. **Divisibility By 5 :** A number is divisible by 5, if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.

5. **Divisibility By 6 :** A number is divisible by 6, if it is divisible by both 2 and 3.

   Ex. The number 35256 is clearly divisible by 2.

   Sum of its digits = (3 + 5 + 2 + 5 + 6) = 21, which is divisible by 3.

   Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.

6. **Divisibility By 8 :** A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.

   Ex. 953360 is divisible by 8, since the number formed by last three digits is 360, which is divisible by 8.

   But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.

7. **Divisibility By 9 :** A number is divisible by 9, if the sum of its digits is divisible by 9.

   Ex. 60732 is divisible by 9, since sum of digits = (6 + 0 + 7 + 3 + 2) = 18, which is divisible by 9.

   But, 68956 is not divisible by 9, since sum of digits = (6 + 8 + 9 + 5 + 6) = 34, which is not divisible by 9.

8. **Divisibility By 10 :** A number is divisible by 10, if it ends with 0.

   Ex. 96410, 10480 are divisible by 10, while 96375 is not.

9. **Divisibility By 11 :** A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

   Ex. The number 4832718 is divisible by 11, since :

   (sum of digits at odd places) – (sum of digits at even places)

   = (8 + 7 + 3 + 4) – (1 + 2 + 8) = 11, which is divisible by 11.

10. **Divisibility By 12 :** A number is divisible by 12, if it is divisible by both 4 and 3.

    Ex. Consider the number 34632.

    (*i*) The number formed by last two digits is 32, which is divisible by 4.

    (*ii*) Sum of digits = (3 + 4 + 6 + 3 + 2) = 18, which is divisible by 3.

    Thus, 34632 is divisible by 4 as well as 3. Hence, 34632 is divisible by 12.

11. **Divisibility By 14 :** A number is divisible by 14, if it is divisible by 2 as well as 7.

12. **Divisibility By 15 :** A number is divisible by 15, if it is divisible by both 3 and 5.

13. **Divisibility By 16 :** A number is divisible by 16, if the number formed by the last 4 digits is divisible by 16.

    Ex. 7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

14. **Divisibility By 24 :** A given number is divisible by 24, if it is divisible by both 3 and 8.

15. **Divisibility By 40 :** A given number is divisible by 40, if it is divisible by both 5 and 8.

16. **Divisibility By 80 :** A given number is divisible by 80, if it is divisible by both 5 and 16.

    Note : If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.

    If p and q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

    Ex. 36 is divisible by both 4 and 6, but it is not divisible by (4 × 6) = 24, since 4 and 6 are not co-primes.

VI. MULTIPLICATION BY SHORT CUT METHODS

1. Multiplication By Distributive Law :

(*i*) $a \times (b + c) = a \times b + a \times c$ (*ii*) $a \times (b - c) = a \times b - a \times c$.

Ex. (*i*) $567958 \times 99999 = 567958 \times (100000 - 1)$

$= 567958 \times 100000 - 567958 \times 1$

$= (56795800000 - 567958) = 56795232042.$

(*ii*) $978 \times 184 + 978 \times 816 = 978 \times (184 + 816) = 978 \times 1000 = 978000.$

2. Multiplication of a Number By 5^n : Put n zeros to the right of the multiplicand and divide the number so formed by 2^n.

Ex. $975436 \times 625 = 975436 \times 5^4 = \dfrac{9754360000}{16} = 609647500.$

VII. BASIC FORMULAE

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 - (a - b)^2 = 4ab$
4. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
5. $(a^2 - b^2) = (a + b)(a - b)$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
7. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
8. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
9. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

VIII. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM

If we divide a given number by another number, then :

Dividend = (Divisor × Quotient) + Remainder

IX. (*i*) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

(*ii*) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.

(*iii*) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

X. PROGRESSION

A succession of numbers formed and arranged in a definite order according to certain definite rule, is called a *progression*.

1. Arithmetic Progression (A.P.) : If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the *common difference of the A.P.* An A.P. with first term a and common difference d is given by a, $(a + d)$, $(a + 2d)$, $(a + 3d)$, ...

The nth term of this A.P. is given by $T_n = a\ (n - 1)\ d$.

The sum of n terms of this A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}\ \textbf{(first term + last term)}.$$

SOME IMPORTANT RESULTS :

(*i*) $(1 + 2 + 3 + \ldots + n) = \dfrac{n(n+1)}{2}$

(*ii*) $(1^2 + 2^2 + 3^2 + \ldots + n^2) = \dfrac{n(n+1)(2n+1)}{6}$

(*iii*) $(1^3 + 2^3 + 3^3 + \ldots + n^3) = \dfrac{n^2(n+1)^2}{4}$.

2. **Geometrical Progression** (G.P.) : A progression of numbers in which every term bears a constant ratio with its preceding term, is called a *geometrical progression*. The constant ratio is called the common ratio of the G.P.

A G.P. with first term a and common ratio r is :

$$a, ar, ar^2, ar^3, \ldots$$

In this G.P. $T_n = ar^{n-1}$.

Sum of the n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$.

OBJECTIVE GENERAL ENGLISH

FOR COMPETITIONS

— *R.S. Aggarwal*

Vikas Aggarwal

* An ideal book for Bank P.O., S.B.I.P.O., R.B.I., M.A.T., Hotel Management, C.B.I., L.I.C.A.A.O., G.I.C.A.A.O., U.T.I., Section Officers, Railways, N.D.A., C.D.S. and other competitive examinations.
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SOLVED EXAMPLES

Ex. 1. *Simplify :* (*i*) *8888 + 888 + 88 + 8* **(B.S.R.B. 1998)**

(*ii*) *11992 − 7823 − 456* **(Bank Exam, 2003)**

Sol. (*i*)

$$\begin{array}{r} 8888 \\ 888 \\ 88 \\ +\quad 8 \\ \hline 9872 \\ \hline \end{array}$$

(*ii*) 11992 − 7823 − 456 = 11992 − (7823 + 456)

= 11992 − 8279 = 3713.

$$\begin{array}{r} 7823 \\ +\ 456 \\ \hline 8279 \\ \hline \end{array} \qquad \begin{array}{r} 11992 \\ -\ 8279 \\ \hline 3713 \\ \hline \end{array}$$

Ex. 2. *What value will replace the question mark in each of the following equations ?*

(*i*) *? − 1936248 = 1635773* **(*ii*) *8597 − ? = 7429 − 4358***

(Bank P.O. 2000)

Sol. (*i*) Let x − 1936248 = 1635773. Then, x = 1635773 + 1936248 = 3572021.

(*ii*) Let 8597 − x = 7429 − 4358.

Then, x = (8597 + 4358) − 7429 = 12955 − 7429 = 5526.

Ex. 3. *What could be the maximum value of Q in the following equation ?*

5P9 + 3R7 + 2Q8 = 1114 **(Bank P.O. 1999)**

Sol. We may analyse the given equation as shown :

Clearly, 2 + P + R + Q = 11.

So, the maximum value of Q can be

(11 − 2) *i.e.*, 9 (when P = 0, R = 0).

	①	②
5	P	9
3	R	7
2	Q	8
11	1	4

Ex. 4. *Simplify :* (*i*) *5793405 × 9999* **(*ii*) *839478 × 625***

Sol. (*i*) 5793405 × 9999 = 5793405 (10000 − 1) = 57934050000 − 5793405 = 57928256595.

(*ii*) $839478 \times 625 = 839478 \times 5^4 = \dfrac{8394780000}{16} = 524673750.$

Ex. 5. *Evaluate :* (*i*) *986 × 137 + 986 × 863* **(*ii*) *983 × 207 − 983 × 107***

Sol. (*i*) 986 × 137 + 986 × 863 = 986 × (137 + 863) = 986 × 1000 = 986000.

(*ii*) 983 × 207 − 983 × 107 = 983 × (207 − 107) = 983 × 100 = 98300.

Ex. 6. *Simplify :* (*i*) *1605 × 1605* **(*ii*) *1398 × 1398***

Sol. (*i*) $1605 \times 1605 = (1605)^2 = (1600 + 5)^2 = (1600)^2 + (5)^2 + 2 \times 1600 \times 5$

$= 2560000 + 25 + 16000 = 2576025.$

(*ii*) $1398 \times 1398 = (1398)^2 = (1400 - 2)^2 = (1400)^2 + (2)^2 - 2 \times 1400 \times 2$

$= 1960000 + 4 - 5600 = 1954404.$

Ex. 7. *Evaluate :* (*313 × 313 + 287 × 287*).

Sol. $(a^2 + b^2) = \frac{1}{2}[(a+b)^2 + (a-b)^2]$

$\therefore \quad (313)^2 + (287)^2 = \frac{1}{2}[(313+287)^2 + (313-287)^2] = \frac{1}{2}[(600)^2 + (26)^2]$

$= \frac{1}{2}(360000 + 676) = 180338.$

Ex. 8. *Which of the following are prime numbers ?*

(*i*) *241* **(*ii*) *337*** **(*iii*) *391*** **(*iv*) *571***

Sol. (*i*) Clearly, $16 > \sqrt{241}$. Prime numbers less than 16 are 2, 3, 5, 7, 11, 13.

241 is not divisible by any one of them.

$\therefore$ 241 is a prime number.

(ii) Clearly, $19 > \sqrt{337}$. Prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17.
337 is not divisible by any one of them.
∴ 337 is a prime number.

(iii) Clearly, $20 > \sqrt{391}$. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
We find that 391 is divisible by 17.
∴ 391 is not prime.

(iv) Clearly, $24 > \sqrt{571}$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23.
571 is not divisible by any one of them.
∴ 571 is a prime number.

Ex. 9. ***Find the unit's digit in the product*** $(2467)^{153} \times (341)^{72}$.

Sol. Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.
Now, 7^4 gives unit digit 1.
∴ 7^{152} gives unit digit 1.
∴ 7^{153} gives unit digit $(1 \times 7) = 7$. Also, 1^{72} gives unit digit 1.
Hence, unit's digit in the product $= (7 \times 1) = 7$.

Ex. 10. ***Find the unit's digit in*** $(264)^{102} + (264)^{103}$. **(S.S.C. 1999)**

Sol. Required unit's digit = unit's digit in $(4)^{102} + (4)^{103}$.
Now, 4^2 gives unit digit 6.
∴ $(4)^{102}$ gives unit digit 6.
$(4)^{103}$ gives unit digit of the product (6×4) *i.e.*, 4.
Hence, unit's digit in $(264)^{102} + (264)^{103}$ = unit's digit in $(6 + 4) = 0$.

Ex. 11. ***Find the total number of prime factors in the expression*** $(4)^{11} \times (7)^5 \times (11)^2$.

Sol. $(4)^{11} \times (7)^5 \times (11)^2 = (2 \times 2)^{11} \times (7)^5 \times (11)^2 = 2^{11} \times 2^{11} \times 7^5 \times 11^2 = 2^{22} \times 7^5 \times 11^2$.
∴ Total number of prime factors $= (22 + 5 + 2) = 29$.

Ex. 12. ***Simplify :*** ***(i)*** $896 \times 896 - 204 \times 204$
(ii) $387 \times 387 + 114 \times 114 + 2 \times 387 \times 114$
(iii) $81 \times 81 + 68 \times 68 - 2 \times 81 \times 68$

Sol. (i) Given exp. $= (896)^2 - (204)^2 = (896 + 204)(896 - 204) = 1100 \times 692 = 761200$.

(ii) Given exp. $= (387)^2 + (114)^2 + 2 \times 387 \times 114$
$= a^2 + b^2 + 2ab$, where $a = 387$, $b = 114$
$= (a + b)^2 = (387 + 114)^2 = (501)^2 = 251001$.

(iii) Given exp. $= (81)^2 + (68)^2 - 2 \times 81 \times 68 = a^2 + b^2 - 2ab$, where $a = 81$, $b = 68$
$= (a - b)^2 = (81 - 68)^2 = (13)^2 = 169$.

Ex. 13. ***Which of the following numbers is divisible by 3 ?***
(i) 541326 ***(ii) 5967013***

Sol. (i) Sum of digits in 541326 = (5 + 4 + 1 + 3 + 2 + 6) = 21, which is divisible by 3.
Hence, 541326 is divisible by 3.

(ii) Sum of digits in 5967013 = (5 + 9 + 6 + 7 + 0 + 1 + 3) = 31, which is not divisible by 3.
Hence, 5967013 is not divisible by 3.

Ex. 14. ***What least value must be assigned to * so that the number 197*5462 is divisible by 9 ?***

Sol. Let the missing digit be x.
Sum of digits $= (1 + 9 + 7 + x + 5 + 4 + 6 + 2) = (34 + x)$.
For $(34 + x)$ to be divisible by 9, x must be replaced by 2.
Hence, the digit in place of * must be 2.

Ex. 15. ***Which of the following numbers is divisible by 4 ?***

(i) 67920594 ***(ii) 618703572***

Sol. (i) The number formed by the last two digits in the given number is 94, which is not divisible by 4.

Hence, 67920594 is not divisible by 4.

(ii) The number formed by the last two digits in the given number is 72, which is divisible by 4.

Hence, 618703572 is divisible by 4.

Ex. 16. ***Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5 ?***

Sol. Since the given number is divisible by 5, so 0 or 5 must come in place of $. But, a number ending with 5 is never divisible by 8. So, 0 will replace $.

Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.

Hence, digits in place of * and $ are 4 and 0 respectively.

Ex. 17. ***Show that 4832718 is divisible by 11.***

Sol. (Sum of digits at odd places) – (Sum of digits at even places)

$= (8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.

Hence, 4832718 is divisible by 11.

Ex. 18. ***Is 52563744 divisible by 24 ?***

Sol. $24 = 3 \times 8$, where 3 and 8 are co-primes.

The sum of the digits in the given number is 36, which is divisible by 3. So, the given number is divisible by 3.

The number formed by the last 3 digits of the given number is 744, which is divisible by 8. So, the given number is divisible by 8.

Thus, the given number is divisible by both 3 and 8, where 3 and 8 are co-primes. So, it is divisible by 3×8, *i.e.*, 24.

Ex. 19. ***What least number must be added to 3000 to obtain a number exactly divisible by 19 ?***

Sol. On dividing 3000 by 19, we get 17 as remainder.

$\therefore$ Number to be added $= (19 - 17) = 2$.

Ex. 20. ***What least number must be subtracted from 2000 to get a number exactly divisible by 17 ?***

Sol. On dividing 2000 by 17, we get 11 as remainder.

$\therefore$ Required number to be subtracted = 11.

Ex. 21. ***Find the number which is nearest to 3105 and is exactly divisible by 21.***

Sol. On dividing 3105 by 21, we get 18 as remainder.

$\therefore$ Number to be added to $3105 = (21 - 18) = 3$.

Hence, required number $= 3105 + 3 = 3108$.

Ex. 22. ***Find the smallest number of 6 digits which is exactly divisible by 111.***

Sol. Smallest number of 6 digits is 100000.

On dividing 100000 by 111, we get 100 as remainder.

$\therefore$ Number to be added $= (111 - 100) = 11$.

Hence, required number = 100011.

Ex. 23. ***On dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. Find the divisor.***

Sol. $\text{Divisor} = \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}} = \dfrac{15968 - 37}{89} = 179.$

Ex. 24. ***A number when divided by 342 gives a remainder 47. When the same number is divided by 19, what would be the remainder ?***

Sol. On dividing the given number by 342, let k be the quotient and 47 as remainder.

Then, number $= 342k + 47 = (19 \times 18k + 19 \times 2 + 9) = 19\,(18k + 2) + 9$.

$\therefore$ The given number when divided by 19, gives $(18k + 2)$ as quotient and 9 as remainder.

Ex. 25. ***A number being successively divided by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed.***

Sol.

3	x
5	$y - 1$
8	$z - 4$
	$1 - 7$

$\therefore$ $z = (8 \times 1 + 7) = 15;\ y = (5z + 4) = (5 \times 15 + 4) = 79;\ x = (3y + 1) = (3 \times 79 + 1) = 238.$

Now,

8	238
5	$29 - 6$
3	$5 - 4$
	$1 - 2$

$\therefore$ Respective remainders are 6, 4, 2.

Ex. 26. ***Find the remainder when 2^{31} is divided by 5.***

Sol. $2^{10} = 1024$. Unit digit of $2^{10} \times 2^{10} \times 2^{10}$ is 4 [as $4 \times 4 \times 4$ gives unit digit 4].

$\therefore$ Unit digit of 2^{31} is 8.

Now, 8 when divided by 5, gives 3 as remainder.

Hence, 2^{31} when divided by 5, gives 3 as remainder.

Ex. 27. ***How many numbers between 11 and 90 are divisible by 7 ?***

Sol. The required numbers are 14, 21, 28, 35, ..., 77, 84.

This is an A.P. with $a = 14$ and $d = (21 - 14) = 7$.

Let it contain n terms.

Then, $T_n = 84 \Rightarrow a + (n - 1)\,d = 84$

$\Rightarrow 14 + (n - 1) \times 7 = 84$ or $n = 11$.

$\therefore$ Required number of terms = 11.

Ex. 28. ***Find the sum of all odd numbers upto 100.***

Sol. The given numbers are 1, 3, 5, 7, ..., 99.

This is an A.P. with $a = 1$ and $d = 2$.

Let it contain n terms. Then,

$1 + (n - 1) \times 2 = 99$ or $n = 50$.

$\therefore$ Required sum $= \dfrac{n}{2}$ (first term + last term)

$= \dfrac{50}{2} \times (1 + 99) = 2500.$

Ex. 29. ***Find the sum of all 2 digit numbers divisible by 3.***

Sol. All 2 digit numbers divisible by 3 are :

12, 51, 18, 21, ..., 99.

This is an A.P. with $a = 12$ and $d = 3$.

Let it contain n terms. Then,

$12 + (n - 1) \times 3 = 99$ or $n = 30$.

$\therefore$ Required sum $= \dfrac{30}{2} \times (12 + 99) = 1665.$

Ex. 30. ***How many terms are there in 2, 4, 8, 16, ..., 1024 ?***

Sol. Clearly 2, 4, 8, 16, ..., 1024 form a G.P. with $a = 2$ and $r = \frac{4}{2} = 2$.

Let the number of terms be n. Then,

$2 \times 2^{n-1} = 1024$ or $2^{n-1} = 512 = 2^9$.

$\therefore$ $n - 1 = 9$ or $n = 10$.

Ex. 31. $2 + 2^2 + 2^3 + ... + 2^8 = ?$

Sol. Given series is a G.P. with $a = 2$, $r = 2$ and $n = 8$.

$\therefore$ $$\text{Sum} = \frac{a(r^n - 1)}{(r-1)} = \frac{2 \times (2^8 - 1)}{(2-1)} = (2 \times 255) = 510.$$

EXERCISE 1

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The difference between the local value and face value of 7 in the numeral 657903 is :
(a) 0 (b) 7896 (c) 6993 (d) 903

2. The difference between the place values of 7 and 3 in the number 527435 is :
(a) 4 (b) 5 (c) 45 (d) 6970 **(R.R.B. 2001)**

3. The sum of the smallest six-digit number and the greatest five-digit number is :
(a) 199999 (b) 201110 (c) 211110 (d) 1099999

4. If the largest three-digit number is subtracted from the smallest five-digit number, then the remainder is : **(S.S.C. 1998)**
(a) 1 (b) 9000 (c) 9001 (d) 90001

5. 5978 + 6134 + 7014 = ? **(Bank P.O. 1999)**
(a) 16226 (b) 19126 (c) 19216 (d) 19226

6. 18265 + 2736 + 41328 = ? **(Bank P.O. 2000)**
(a) 61329 (b) 62239 (c) 62319 (d) 62329

7. 39798 + 3798 + 378 = ? **(Bank P.O. 2002)**
(a) 43576 (b) 43974 (c) 43984 (d) 49532

8. 9358 − 6014 + 3127 = ? **(SIDBI, 2000)**
(a) 6381 (b) 6471 (c) 6561 (d) 6741

9. 9572 − 4018 − 2164 = ?
(a) 3300 (b) 3390 (c) 3570 (d) 7718

10. 7589 − ? = 3434 **(Bank P.O. 2003)**
(a) 721 (b) 3246 (c) 4155 (d) 11023

11. 9548 + 7314 = 8362 + ? **(S.B.I.P.O. 2000)**
(a) 8230 (b) 8410 (c) 8500 (d) 8600

12. 7845 − ? = 8461 − 3569
(a) 2593 (b) 2773 (c) 3569 (d) None of these

13. 3578 + 5729 − ?486 = 5821
(a) 1 (b) 2 (c) 3 (d) None of these

14. If $6x43 - 46y9 = 1904$, which of the following should come in place of x ?
(a) 4 (b) 6 (c) 9
(d) Cannot be determined (e) None of these

15. What should be the maximum value of B in the following equation ? **(Bank P.O. 2000)**
5A9 − 7B2 + 9C6 = 823
(*a*) 5 (*b*) 6 (*c*) 7 (*d*) 9

16. In the following sum, '?' stands for which digit ? **(M.B.A. 1998)**
? + 1? + 2? + ?3 + ?1 = 21?
(*a*) 4 (*b*) 6 (*c*) 8 (*d*) 9

17. 5358 × 51 = ?
(*a*) 273258 (*b*) 273268 (*c*) 273348 (*d*) 273358

18. 360 × 17 = ? **(R.B.I. 2003)**
(*a*) 5120 (*b*) 5320 (*c*) 6120 (*d*) 6130

19. 587 × 999 = ? **(M.B.A. 1998)**
(*a*) 586413 (*b*) 587523 (*c*) 614823 (*d*) 615173

20. 469157 × 9999 = ?
(*a*) 4586970843 (*b*) 4686970743 (*c*) 4691100843 (*d*) 584649125

21. 8756 × 99999 = ?
(*a*) 796491244 (*b*) 815491244 (*c*) 875591244 (*d*) None of these

22. The value of 112×5^4 is : **(M.B.A. 2002)**
(*a*) 6700 (*b*) 70000 (*c*) 76500 (*d*) 77200

23. 935421 × 625 = ?
(*a*) 575648125 (*b*) 584638125 (*c*) 584649125 (*d*) 585628125

24. 12846 × 593 + 12846 × 407 = ?
(*a*) 12846000 (*b*) 14203706 (*c*) 24038606 (*d*) 24064000

25. 1014 × 986 = ?
(*a*) 998804 (*b*) 998814 (*c*) 998904 (*d*) 999804

26. 1307 × 1307 = ?
(*a*) 1601249 (*b*) 1607249 (*c*) 1701249 (*d*) 1708249

27. 1399 × 1399 = ?
(*a*) 1687401 (*b*) 1901541 (*c*) 1943211 (*d*) 1957201

28. 106 × 106 + 94 × 94 = ?
(*a*) 20032 (*b*) 20072 (*c*) 21032 (*d*) 23032

29. 217 × 217 + 183 × 183 = ? **(Hotel Management, 2002)**
(*a*) 79698 (*b*) 80578 (*c*) 80698 (*d*) 81268

30. 12345679 × 72 is equal to : **(S.S.C. 2000)**
(*a*) 88888888 (*b*) 888888888 (*c*) 898989898 (*d*) 999999998

31. What number should replace *x* in this multiplication problem ?

$$\begin{array}{r} 3\,x\,4 \\ 4 \\ \hline 1216 \\ \hline \end{array}$$

(Hotel Management, 2000)
(*a*) 0 (*b*) 2 (*c*) 4 (*d*) 5

32. A positive integer, which when added to 1000, gives a sum which is greater than when it is multiplied by 1000. This positive integer is : **(M.A.T. 2003)**
(*a*) 1 (*b*) 3 (*c*) 5 (*d*) 7

33. Which of the following can be a product of two 3-digit numbers **3 and **8 ?
(*a*) 1010024 (*b*) 991014 (*c*) 9124 (*d*) None of these

34. A boy multiplies 987 by a certain number and obtains 559981 as his answer. If in the answer, both 9's are wrong but the other digits are correct, then the correct answer will be : **(C.B.I. 1997)**

(a) 553681 (b) 555181 (c) 555681 (d) 556581

35. When a certain number is multiplied by 13, the product consists entirely of fives. The smallest such number is : **(M.B.A. 2002)**

(a) 41625 (b) 42135 (c) 42515 (d) 42735

36. The number of digits of the smallest number, which when multiplied by 7 gives the result consisting entirely of nines, is :

(a) 3 (b) 5 (c) 6 (d) 8

37. $-95 \div 19 = ?$ **(Hotel Management, 2000)**

(a) -5 (b) -4 (c) 0 (d) 5

38. What should come in place of * mark in the following equation ? **(B.S.R.B. 1998)**

1*5$4 ÷ 148 = 78

(a) 1 (b) 4 (c) 6 (d) 8 (e) None of these

39. The sum of all possible two-digit numbers formed from three different one-digit natural numbers when divided by the sum of the original three numbers is equal to :

(a) 18 (b) 22 (c) 36 (d) None of these

(C.B.I. 1997)

40. If n is a negative number, then which of the following is the least ? **(M.B.A. 2002)**

(a) 0 (b) $-n$ (c) $2n$ (d) n^2

41. If x and y are negative, then which of the following statements is/are always true ?

I. $x + y$ is positive II. xy is positive III. $x - y$ is positive. **(M.A.T. 2004)**

(a) I only (b) II only (c) III only (d) I and III only

42. If $-1 \le x \le 2$ and $1 \le y \le 3$, then least possible value of $(2y - 3x)$ is :

(a) 0 (b) -3 (c) -4 (d) -5

43. If a and b are both odd numbers, which of the following is an even number ?

(a) $a + b$ (b) $a + b + 1$ (c) ab (d) $ab + 2$

44. Which of the following is always odd ?

(a) Sum of two odd numbers (b) Difference of two odd numbers

(c) Product of two odd numbers (d) None of these

45. For the integer n, if n^3 is odd, then which of the following statements are true ?

I. n is odd. II. n^2 is odd. III. n^2 is even. **(D.M.R.C. 2003)**

(a) I only (b) II only (c) I and II only (d) I and III only

46. The least prime number is :

(a) 0 (b) 1 (c) 2 (d) 3

47. What is the total number of prime numbers less than 70 ?

(a) 17 (b) 18 (c) 19 (d) 20

48. The total number of even prime numbers is :

(a) 0 (b) 1 (c) 2 (d) None of these

49. Find the sum of prime numbers lying between 60 and 75. **(R.R.B. 2000)**

(a) 199 (b) 201 (c) 211 (d) 272

50. The smallest three-digit prime number is : **(S.S.C. 2000)**

(a) 103 (b) 107 (c) 109 (d) None of these

51. Which one of the following is a prime number ?

(a) 161 (b) 221 (c) 373 (d) 437

52. The smallest value of n, for which $2n + 1$ is not a prime number, is :

(a) 3 (b) 4 (c) 5 (d) None of these

(Hotel Management, 1997)

53. The sum of three prime numbers is 100. If one of them exceeds another by 36, then one of the numbers is :

(*a*) 7 (*b*) 29 (*c*) 41 (*d*) 67

54. There are four prime numbers written in ascending order. The product of the first three is 385 and that of the last three is 1001. The last number is : **(S.S.C. 2003)**

(*a*) 11 (*b*) 13 (*c*) 17 (*d*) 19

55. How many numbers between 400 and 600 begin with or end with a digit of 5 ?

(*a*) 40 (*b*) 100 (*c*) 110 (*d*) 120

56. If we write all the whole numbers from 200 to 400, then how many of these contain the digit 7 once and only once ? **(Hotel Management, 2003)**

(*a*) 32 (*b*) 34 (*c*) 35 (*d*) 36

57. The unit's digit in the product $274 \times 318 \times 577 \times 313$ is :

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

58. The digit in unit's place of the product $81 \times 82 \times \ldots\ldots \times 89$ is :

(*a*) 0 (*b*) 2 (*c*) 6 (*d*) 8

(Section Officers', 2003)

59. If the unit digit in the product $(459 \times 46 \times 28{*} \times 484)$ is 2, the digit in place of * is :

(*a*) 3 (*b*) 5 (*c*) 7 (*d*) None of these

60. The unit's digit in the product $(3127)^{173}$ is :

(*a*) 1 (*b*) 3 (*c*) 7 (*d*) 9

61. The unit's digit in the product $(7^{71} \times 6^{59} \times 3^{65})$ is : **(L.I.C.A.A.O. 2003)**

(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 6

62. The digit in the unit's place of the number represented by $(7^{95} - 3^{58})$ is :

(*a*) 0 (*b*) 4 (*c*) 6 (*d*) 7

(A.A.O. Exam, 2003)

63. If x is an even number, then x^{4n}, where n is a positive integer, will always have :

(*a*) zero in the unit's place (*b*) 6 in the unit's place

(*c*) either 0 or 6 in the unit's place (*d*) None of these

(Hotel Management, 1997)

64. The number of prime factors of $(3 \times 5)^{12} (2 \times 7)^{10} (10)^{25}$ is :

(*a*) 47 (*b*) 60 (*c*) 72 (*d*) None of these

65. $397 \times 397 + 104 \times 104 + 2 \times 397 \times 104 = ?$

(*a*) 250001 (*b*) 251001 (*c*) 260101 (*d*) 261001

66. $186 \times 186 + 159 \times 159 - 2 \times 186 \times 159 = ?$

(*a*) 729 (*b*) 1039 (*c*) 2019 (*d*) 7029

67. $(475 + 425)^2 - 4 \times 475 \times 425$ is equal to :

(*a*) 2500 (*b*) 3160 (*c*) 3500 (*d*) 3600

68. If $(64)^2 - (36)^2 = 20z$, the value of z is :

(*a*) 70 (*b*) 120 (*c*) 180 (*d*) None of these

69. $(46)^2 - (?)^2 = 4398 - 3066$ **(B.S.R.B. 1998)**

(*a*) 16 (*b*) 28 (*c*) 36 (*d*) 42

70. $\dfrac{(856 + 167)^2 + (856 - 167)^2}{856 \times 856 + 167 \times 167}$ is equal to :

(*a*) 1 (*b*) 2 (*c*) 689 (*d*) 1023

71. $\dfrac{(469 + 174)^2 - (469 - 174)^2}{469 \times 174}$ is equal to :

(*a*) 2 (*b*) 4 (*c*) 295 (*d*) 643

72. The sum of first 45 natural numbers is :
(a) 1035 (b) 1280 (c) 2070 (d) 2140

73. The sum of even numbers between 1 and 31 is :
(a) 16 (b) 128 (c) 240 (d) 512

74. (51 + 52 + 53 + ... + 100) is equal to :
(a) 2525 (b) 2975 (c) 3225 (d) 3775

75. How many numbers between 200 and 600 are divisible by 4, 5 and 6 ?
(a) 5 (b) 6 (c) 7 (d) 8

76. How many three-digit numbers are divisible by 6 in all ?
(a) 149 (b) 150 (c) 151 (d) 166

77. If $(1^2 + 2^2 + 3^2 + ... + 10^2) = 385$, then the value of $(2^2 + 4^2 + 6^2 + ... + 20^2)$ is :
(a) 770 (b) 1155 (c) 1540 (d) (385 × 385)

78. The value of $(11^2 + 12^2 + 13^2 + 14^2 + ... + 20^2)$ is :
(a) 385 (b) 2485 (c) 2870 (d) 3255

79. If 1*548 is divisible by 3, which of the following digits can replace * ?
(a) 0 (b) 2 (c) 7 (d) 9

(S.S.C. 1999)

80. If the number 357*25* is divisible by both 3 and 5, then the missing digits in the unit's place and the thousandth place respectively are : **(Hotel Management, 1997)**
(a) 0, 6 (b) 5, 6 (c) 5, 4 (d) None of these

81. 5*2 is a three-digit number with * as a missing digit. If the number is divisible by 6, the missing digit is :
(a) 2 (b) 3 (c) 6 (d) 7

82. What least value must be assigned to * so that the number 63576*2 is divisible by 8 ?
(a) 1 (b) 2 (c) 3 (d) 4

83. What least value must be given to * so that the number 451*603 is exactly divisible by 9 ?
(a) 2 (b) 5 (c) 7 (d) 8

84. How many of the following numbers are divisible by 3 but not by 9 ?
2133, 2343, 3474, 4131, 5286, 5340, 6336, 7347, 8115, 9276
(a) 5 (b) 6 (c) 7 (d) None of these

85. Which one of the following numbers is exactly divisible by 11 ? **(C.D.S. 2003)**
(a) 235641 (b) 245642 (c) 315624 (d) 415624

86. What least value must be assigned to * so that the number 86325*6 is divisible by 11 ?
(a) 1 (b) 2 (c) 3 (d) 5

87. A number 476**0 is divisible by both 3 and 11. The non-zero digits in the hundredth and tenth place respectively are :
(a) 7, 4 (b) 7, 5 (c) 8, 5 (d) None of these

88. Which of the following numbers is divisible by 3, 7, 9 and 11 ?
(a) 639 (b) 2079 (c) 3791 (d) 37911

89. The value of P, when 4864 × 9P2 is divisible by 12, is :
(a) 2 (b) 5 (c) 8 (d) None of these

90. Which of the following numbers is exactly divisible by 24 ? **(M.B.A. 1998)**
(a) 35718 (b) 63810 (c) 537804 (d) 3125736

91. If the number 42573* is completely divisible by 72, then which of the following numbers should replace the asterisk ?

(*a*) 4 (*b*) 5 (*c*) 6 (*d*) 7

92. Which of the following numbers is exactly divisible by 99 ?

(*a*) 114345 (*b*) 135792 (*c*) 913464 (*d*) 3572404

93. The digits indicated by * and $ in 3422213*$ so that this number is divisible by 99, are respectively :

(*a*) 1, 9 (*b*) 3, 7 (*c*) 4, 6 (*d*) 5, 5

94. If x and y are the two digits of the number $653xy$ such that this number is divisible by 80, then $x + y$ is equal to :

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 6

95. How many of the following numbers are divisible by 132 ?

264, 396, 462, 792, 968, 2178, 5184, 6336 **(Hotel Management, 2002)**

(*a*) 4 (*b*) 5 (*c*) 6 (*d*) 7

96. 6897 is divisible by : **(I.A.M. 2002)**

(*a*) 11 only (*b*) 19 only

(*c*) both 11 and 19 (*d*) neither 11 nor 19

97. Which of the following numbers is exactly divisible by all prime numbers between 1 and 17 ?

(*a*) 345345 (*b*) 440440 (*c*) 510510 (*d*) 515513

98. 325325 is a six-digit number. It is divisible by : **(S.S.C. 1998)**

(*a*) 7 only (*b*) 11 only (*c*) 13 only (*d*) all 7, 11 and 13

99. The number 311311311311311311311 is : **(C.D.S. 2003)**

(*a*) divisible by 3 but not by 11 (*b*) divisible by 11 but not by 3

(*c*) divisible by both 3 and 11 (*d*) neither divisible by 3 nor by 11

100. There is one number which is formed by writing one digit 6 times (*e.g.* 111111, 444444 etc.). Such a number is always divisible by :

(*a*) 7 only (*b*) 11 only (*c*) 13 only (*d*) All of these

101. A 4-digit number is formed by repeating a 2-digit number such as 2525, 3232 etc. Any number of this form is exactly divisible by : **(S.S.C. 2000)**

(*a*) 7 (*b*) 11

(*c*) 13 (*d*) smallest 3-digit prime number

102. A six-digit number is formed by repeating a three-digit number; for example, 256256 or 678678 etc. Any number of this form is always exactly divisible by :

(*a*) 7 only (*b*) 11 only (*c*) 13 only (*d*) 1001

103. The largest natural number which exactly divides the product of any four consecutive natural numbers is :

(*a*) 6 (*b*) 12 (*c*) 24 (*d*) 120

104. The largest natural number by which the product of three consecutive even natural numbers is always divisible, is :

(*a*) 16 (*b*) 24 (*c*) 48 (*d*) 96

105. The sum of three consecutive odd numbers is always divisible by :

I. 2 II. 3 III. 5 IV. 6

(*a*) Only I (*b*) Only II (*c*) Only I and III (*d*) Only II and IV

(Hotel Management, 2003)

106. The difference between the squares of two consecutive odd integers is always divisible by : **(M.B.A. 2003)**

(*a*) 3 (*b*) 6 (*c*) 7 (*d*) 8

107. A number is multiplied by 11 and 11 is added to the product. If the resulting number is divisible by 13, the smallest original number is :
(a) 12 (b) 22 (c) 26 (d) 53

108. The sum of the digits of a 3-digit number is subtracted from the number. The resulting number is :
(a) divisible by 6 (b) divisible by 9
(c) divisible neither by 6 nor by 9 (d) divisible by both 6 and 9

109. If x and y are positive integers such that $(3x + 7y)$ is a multiple of 11, then which of the following will also be divisible by 11 ?
(a) $4x + 6y$ (b) $x + y + 4$ (c) $9x + 4y$ (d) $4x - 9y$

110. A 3-digit number $4a3$ is added to another 3-digit number 984 to give the four-digit number $13b7$, which is divisible by 11. Then, $(a + b)$ is :
(a) 10 (b) 11 (c) 12 (d) 15

111. The largest number that exactly divides each number of the sequence $(1^5 - 1), (2^5 - 2), (3^5 - 3), \ldots, (n^5 - n), \ldots$ is :
(a) 1 (b) 15 (c) 30 (d) 120

112. The greatest number by which the product of three consecutive multiples of 3 is always divisible is : **(S.S.C. 2000)**
(a) 54 (b) 81 (c) 162 (d) 243

113. The smallest number to be added to 1000 so that 45 divides the sum exactly is :
(a) 10 (b) 20 (c) 35 (d) 80

114. The smallest number that must be added to 803642 in order to obtain a multiple of 11 is : **(C.B.I. 2003)**
(a) 1 (b) 4 (c) 7 (d) 9

115. Which of the following numbers should be added to 11158 to make it exactly divisible by 77 ?
(a) 5 (b) 7 (c) 8 (d) 9

116. The least number which must be subtracted from 6709 to make it exactly divisible by 9 is : **(C.B.I. 1998)**
(a) 2 (b) 3 (c) 4 (d) 5

117. What least number must be subtracted from 427398 so that the remaining number is divisible by 15 ? **(Bank P.O. 2000)**
(a) 3 (b) 6 (c) 13 (d) 16

118. What least number must be subtracted from 13294 so that the remainder is exactly divisible by 97 ?
(a) 1 (b) 3 (c) 4 (d) 5

119. When the sum of two numbers is multiplied by 5, the product is divisible by 15. Which one of the following pairs of numbers satisfies the above condition ?
(a) 240, 335 (b) 250, 341 (c) 245, 342 (d) None of these
(Hotel Management, 1998)

120. The least number by which 72 must be multiplied in order to produce a multiple of 112, is :
(a) 6 (b) 12 (c) 14 (d) 18

121. The number of times 99 is subtracted from 1111 so that the remainder is less than 99, is : **(S.C.R.A. 1996)**
(a) 10 (b) 11 (c) 12 (d) 13

122. Find the number which is nearest to 457 and is exactly divisible by 11.
(a) 450 (b) 451 (c) 460 (d) 462
(Hotel Management, 2003)

123. The number nearest to 99547 which is exactly divisible by 687 is :

(*a*) 98928 (*b*) 99479 (*c*) 99615 (*d*) 100166

124. What largest number of five digits is divisible by 99 ?

(*a*) 99909 (*b*) 99981 (*c*) 99990 (*d*) 99999

125. The smallest number of five digits exactly divisible by 476 is : **(S.S.C. 2004)**

(*a*) 10000 (*b*) 10472 (*c*) 10476 (*d*) 47600

126. On dividing a number by 999, the quotient is 366 and the remainder is 103. The number is :

(*a*) 364724 (*b*) 365387 (*c*) 365737 (*d*) 366757

127. On dividing 4150 by a certain number, the quotient is 55 and the remainder is 25. The divisor is :

(*a*) 65 (*b*) 70 (*c*) 75 (*d*) 80

128. A number when divided by the sum of 555 and 445 gives two times their difference as quotient and 30 as the remainder. The number is : **(S.S.C. 2000)**

(*a*) 1220 (*b*) 1250 (*c*) 22030 (*d*) 220030

129. A four-digit number divisible by 7 becomes divisible by 3, when 10 is added to it. The largest such number is :

(*a*) 9947 (*b*) 9987 (*c*) 9989 (*d*) 9996

130. A number when divided by 114 leaves the remainder 21. If the same number is divided by 19, then the remainder will be : **(R.R.B. 2003)**

(*a*) 1 (*b*) 2 (*c*) 7 (*d*) 21

131. A number when divided by 296 gives a remainder 75. When the same number is divided by 37, then the remainder will be : **(C.B.I. 2003)**

(*a*) 1 (*b*) 2 (*c*) 8 (*d*) 11

132. A number when divided by 119 leaves 19 as remainder. If the same number is divided by 17, the remainder obtained is : **(Section Officers', 2001)**

(*a*) 2 (*b*) 3 (*c*) 7 (*d*) 10

133. A number when divided by 899 gives a remainder 63. If the same number is divided by 29, the remainder will be :

(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 10

134. When a number is divided by 31, the remainder is 29. When the same number is divided by 16, what will be the remainder ? **(Bank P.O. 2002)**

(*a*) 11 (*b*) 13 (*c*) 15 (*d*) Data inadequate

135. When a number is divided by 13, the remainder is 11. When the same number is divided by 17, the remainder is 9. What is the number ? **(S.B.I.P.O. 1997)**

(*a*) 339 (*b*) 349 (*c*) 369 (*d*) Data inadequate

136. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, the dividend is :

(*a*) 4236 (*b*) 4306 (*c*) 4336 (*d*) 5336

137. The difference between two numbers is 1365. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. The smaller number is : **(A.A.O. Exam, 2003)**

(*a*) 240 (*b*) 270 (*c*) 295 (*d*) 360

138. In doing a division of a question with zero remainder, a candidate took 12 as divisor instead of 21. The quotient obtained by him was 35. The correct quotient is :

(*a*) 0 (*b*) 12 (*c*) 13 (*d*) 20

(S.S.C. 2003)

139. When n is divided by 4, the remainder is 3. What is the remainder when $2n$ is divided by 4 ?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 6

140. A number when divided by 6 leaves a remainder 3. When the square of the same number is divided by 6, the remainder is : **(S.S.C. 2000)**

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

141. A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. When it is successively divided by 5 and 4, then the respective remainders will be :

(*a*) 1, 2 (*b*) 2, 3 (*c*) 3, 2 (*d*) 4, 1

(S.S.C. 2003)

142. A number was divided successively in order by 4, 5 and 6. The remainders were respectively 2, 3 and 4. The number is : **(C.B.I. 1997)**

(*a*) 214 (*b*) 476 (*c*) 954 (*d*) 1908

143. In dividing a number by 585, a student employed the method of short division. He divided the number successively by 5, 9 and 13 (factors of 585) and got the remainders 4, 8 and 12. If he had divided the number by 585, the remainder would have been :

(*a*) 24 (*b*) 144 (*c*) 292 (*d*) 584

(N.I.F.T. 1997)

144. A number when divided by 3 leaves a remainder 1. When the quotient is divided by 2, it leaves a remainder 1. What will be the remainder when the number is divided by 6 ?

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

145. $4^{61} + 4^{62} + 4^{63} + 4^{64}$ is divisible by : **(C.B.I. 2003)**

(*a*) 3 (*b*) 10 (*c*) 11 (*d*) 13

146. If x is a whole number, then $x^2 (x^2 - 1)$ is always divisible by : **(S.S.C. 1996)**

(*a*) 12 (*b*) 24 (*c*) $12 - x$ (*d*) multiple of 12

ANSWERS

1. (*c*)	**2.** (*d*)	**3.** (*a*)	**4.** (*c*)	**5.** (*b*)	**6.** (*d*)	**7.** (*b*)	**8.** (*b*)
9. (*b*)	**10.** (*c*)	**11.** (*c*)	**12.** (*d*)	**13.** (*c*)	**14.** (*c*)	**15.** (*c*)	**16.** (*c*)
17. (*a*)	**18.** (*c*)	**19.** (*a*)	**20.** (*c*)	**21.** (*c*)	**22.** (*b*)	**23.** (*b*)	**24.** (*a*)
25. (*d*)	**26.** (*d*)	**27.** (*d*)	**28.** (*b*)	**29.** (*b*)	**30.** (*b*)	**31.** (*a*)	**32.** (*a*)
33. (*b*)	**34.** (*c*)	**35.** (*d*)	**36.** (*c*)	**37.** (*a*)	**38.** (*a*)	**39.** (*b*)	**40.** (*c*)
41. (*b*)	**42.** (*c*)	**43.** (*a*)	**44.** (*c*)	**45.** (*c*)	**46.** (*c*)	**47.** (*c*)	**48.** (*b*)
49. (*d*)	**50.** (*d*)	**51.** (*c*)	**52.** (*b*)	**53.** (*d*)	**54.** (*b*)	**55.** (*c*)	**56.** (*d*)
57. (*a*)	**58.** (*a*)	**59.** (*c*)	**60.** (*c*)	**61.** (*c*)	**62.** (*b*)	**63.** (*b*)	**64.** (*d*)
65. (*b*)	**66.** (*a*)	**67.** (*a*)	**68.** (*d*)	**69.** (*b*)	**70.** (*b*)	**71.** (*b*)	**72.** (*a*)
73. (*c*)	**74.** (*d*)	**75.** (*b*)	**76.** (*b*)	**77.** (*c*)	**78.** (*b*)	**79.** (*a*)	**80.** (*b*)
81. (*a*)	**82.** (*c*)	**83.** (*d*)	**84.** (*b*)	**85.** (*d*)	**86.** (*c*)	**87.** (*c*)	**88.** (*b*)
89. (*d*)	**90.** (*d*)	**91.** (*c*)	**92.** (*a*)	**93.** (*a*)	**94.** (*a*)	**95.** (*a*)	**96.** (*c*)
97. (*c*)	**98.** (*d*)	**99.** (*d*)	**100.** (*d*)	**101.** (*d*)	**102.** (*d*)	**103.** (*c*)	**104.** (*c*)
105. (*b*)	**106.** (*d*)	**107.** (*a*)	**108.** (*b*)	**109.** (*d*)	**110.** (*a*)	**111.** (*c*)	**112.** (*c*)
113. (*c*)	**114.** (*c*)	**115.** (*b*)	**116.** (*c*)	**117.** (*a*)	**118.** (*d*)	**119.** (*b*)	**120.** (*c*)
121. (*b*)	**122.** (*d*)	**123.** (*c*)	**124.** (*c*)	**125.** (*b*)	**126.** (*c*)	**127.** (*c*)	**128.** (*d*)
129. (*c*)	**130.** (*b*)	**131.** (*a*)	**132.** (*a*)	**133.** (*c*)	**134.** (*d*)	**135.** (*b*)	**136.** (*d*)
137. (*b*)	**138.** (*d*)	**139.** (*b*)	**140.** (*d*)	**141.** (*b*)	**142.** (*a*)	**143.** (*d*)	**144.** (*c*)
145. (*b*)	**146.** (*a*)						

SOLUTIONS

1. (Local Value) − (Face Value) = (7000 − 7) = 6993.

2. (Place Value of 7) − (Place Value of 3) = (7000 − 30) = 6970.

3. Required Sum = (100000 + 99999) = 199999.

4. Required Remainder = (10000 − 999) = 9001.

5. 5978 + 6134 + 7014 = 19126.

6. 18265 + 2736 + 41328 = 62329.

7. 39798 + 3798 + 378 = 43974.

8. 9358 − 6014 + 3127 = (9358 + 3127) − 6014 = (12485 − 6014) = 6471.

9. 9572 − 4018 − 2164 = 9572 − (4018 + 2164) = (9572 − 6182) = 3390.

10. Let 7589 − x = 3434. Then, x = (7589 − 3434) = 4155.

11. Let 9548 + 7314 = 8362 + x. Then, 16862 = 8362 + x ⇔ x = (16862 − 8362) = 8500.

12. Let 7845 − x = 8461 − 3569. Then, 7845 − x = 4892 ⇔ x = (7845 − 4892) = 2953.

13. Let 3578 + 5729 − x486 = 5821.

Then, 9307 − x486 = 5821 ⇔ x486 = (9307 − 5821) ⇔ x486 = 3486 ⇔ x = 3.

14. 6x43 − 46y9 = 1904 ⇔ 6x43 = 1904 + 46y9 [1 + y = 4 ⇔ y = 3]

⇔ 6x43 = 1904 + 4639 = 6543 [∵ y = 3]

⇔ x = 5.

15. We may represent the given sum, as shown.

∴ 1 + A + C − B = 12 ⇔ A + C − B = 11.

Giving maximum values to A and C, *i.e.*,

A = 9 and C = 9, we get B = 7.

$$\begin{array}{rccc} & 1 & 1 & \\ & 5 & A & 9 \\ + & 8 & C & 6 \\ & 7 & B & 2 \\ \hline & 8 & 2 & 3 \\ \hline \end{array}$$

16. Let x + (10 + x) + (20 + x) + (10x + 8) + (10x + 1) = 200 + 10 + x.

Then, 22x = 176 ⇔ x = 8.

17. 5358 × 51 = 5358 × (50 + 1) = (5358 × 50) + (5358 × 1) = (267900 + 5358) = 273258.

18. 360 × 17 = 360 × (20 − 3) = (360 × 20) − (360 × 3) = (7200 − 1080) = 6120.

19. 587 × 999 = 587 × (1000 − 1) = (587 × 1000) − (587 × 1) = (587000 − 587) = 586413.

20. 469157 × 9999 = 469157 × (10000 − 1) = (469157 × 10000) − (469157 × 1)

= (4691570000 − 469157) = 4691100843.

21. 8756 × 99999 = 8756 × (100000 − 1) = (8756 × 100000) − (8756 × 1)

= (875600000 − 8756) = 875591244.

22. $(112 \times 5^4) = \dfrac{1120000}{2^4}$ (see the rule) $= \dfrac{1120000}{16} = 70000.$

23. $935421 \times 625 = 935421 \times 5^4 = \dfrac{9354210000}{2^4}$ (see the rule)

$= \dfrac{9354210000}{16} = 584638125.$

24. 12846 × 593 + 12846 × 407 = 12846 × (593 + 407) = 12846 × 1000 = 12846000.

25. $(1014 \times 986) = (1000 + 14) \times (1000 - 14) = (1000)^2 - (14)^2 = 1000000 - 196 = 999804.$

26. $(1307 \times 1307) = (1307)^2 = (1300 + 7)^2 = (1690000 + 49 + 18200) = 1708249.$

27. $(1399 \times 1399) = (1399)^2 = (1400 - 1)^2 = (1400)^2 + 1^2 - 2 \times 1400 \times 1$

$= 1960000 + 1 - 2800 = 1960001 - 2800 = 1957201.$

28. $(106\times106+94\times94)=\frac{1}{2}\times2(a^2+b^2)=\frac{1}{2}[(a+b)^2+(a-b)^2]$

$=\frac{1}{2}\cdot[(106+94)^2+(106-94)^2]=\frac{1}{2}\cdot[(200)^2+(12)^2]$

$=\frac{1}{2}(40000+144)=\frac{1}{2}(40144)=20072.$

29. $(217\times217+183\times183)=\frac{1}{2}\times2(a^2+b^2)=\frac{1}{2}\cdot[(a+b)^2+(a-b)^2]$

$=\frac{1}{2}\cdot[(217+183)^2+(217-183)^2]=\frac{1}{2}[(400)^2+(34)^2]$

$=\frac{1}{2}(160000+1156)=\frac{161156}{2}=80578.$

30. $12345679\times72=12345679\times(100-28)=1234567900-(12345679\times28)$

$=1234567900-[12345679\times(30-2)]$

$=1234567900-370370370+24691358=888888888.$

31. $(300+10x+4)\times4=1200+40x+16=(12\times100)+(4x+1)\times10+6$

$\therefore\ 4x+1=1\ \Leftrightarrow\ 4x=0\ \Leftrightarrow\ x=0.$

32. $(1000+N)>(1000N)$. Clearly, $N=1$.

33. When two 3-digit numbers are multiplied, the product must contain 5 or 6 digits. So, the required number is 991014.

34. $987=3\times7\times47$.

So, required number must be divisible by each one of 3, 7, 47.

None of the numbers in (*a*) and (*b*) are divisible by 3, while (*d*) is not divisible by 7.

$\therefore$ Correct answer is (*c*).

35. By hit and trial, we find that a number exactly divisible by 13 and consisting entirely of fives is 555555.

On dividing 555555 by 13, we get 42735 as quotient.

$\therefore$ Required number = 42735.

36. By hit and trial, we find that a number exactly divisible by 7 and consisting entirely of nines is 999999. Number of digits in it = 6.

37. $\frac{-95}{19}=-5.$

38. Let $\frac{x}{148}=78$. Then, $x=(148\times78)=11544$.

$\therefore$ Required digit = 1.

39. Let the one-digit numbers be x, y, z.

Sum of all possible 2-digit numbers

$=(10x+y)+(10x+z)+(10y+x)+(10y+z)+(10z+x)+(10z+y)=22(x+y+z)$

$\therefore$ Sum of all possible 2-digit numbers when divided by sum of one-digit numbers gives 22.

40. $n<0\ \Rightarrow\ 2n<0,\ -n>0$ and $n^2>0$.

$\therefore$ Least of $2n, 0, -n$ and n^2 is $2n$.

41. $x<0, y<0\ \Rightarrow\ (x+y)<0,\ xy>0$ and $x-y$ may be +ve or −ve.

$\therefore$ II is always true.

42. $\left.\begin{array}{l} y\geq1\ \Rightarrow\ 2y\geq2 \\ x\leq2\ \Rightarrow\ -3x\geq-6\end{array}\right\}\Rightarrow(2y-3x)\geq-4.$

43. Sum of two odd numbers is always even.

44. Product of two odd numbers is always odd.

45. n^3 is odd $\Rightarrow$ n is odd and n^2 is odd.

46. The least prime number is 2.

47. Prime numbers less than 70 are :
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61 and 67.
Their number is 19.

48. There is only one even prime number, namely 2.

49. Required sum = (61 + 67 + 71 + 73) = 272.

50. 100 is divisible by 2, so it is not prime.
101 is not divisible by any of the numbers 2, 3, 5, 7. So, it is prime.
Hence, the smallest 3-digit prime number is 101.

51. 161 is divisible by 7. So, it is not prime. 221 is divisible by 13. So, it is not prime.
Now, $20 > \sqrt{373}$. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
And, 373 is not divisible by any of them. So, 373 is prime.
Since 437 is divisible by 19, so it is not prime.

52. $(2 \times 1 + 1) = 3$, $(2 \times 2 + 1) = 5$, $(2 \times 3 + 1) = 7$, $(2 \times 4 + 1) = 9$, which is not prime.
$\therefore \quad n = 4$.

53. $x + (x + 36) + y = 100 \quad \Leftrightarrow \quad 2x + y = 64$.
$\therefore$ y must be even prime, which is 2.
$\therefore \quad 2x + 2 = 64 \quad \Rightarrow \quad x = 31$.
Third prime number $= (x + 36) = (31 + 36) = 67$.

54. Let the given prime numbers be a, b, c, d. Then, $abc = 385$ and $bcd = 1001$.
$\therefore \quad \frac{abc}{bcd} = \frac{385}{1001} \Leftrightarrow \frac{a}{d} = \frac{5}{13}$. So, $a = 5$, $d = 13$.

55. Numbers satisfying the given conditions are 405, 415, 425, 435, 445, 455, 465, 475, 485, 495 and 500 to 599.
Number of such numbers = (10 + 100) = 110.

56. Required numbers from 200 to 300 are 207, 217, 227, 237, 247, 257, 267, 270, 271, 272, 273, 274, 275, 276, 278, 279, 287, 297. Their number is 18.
Similarly, such numbers between 300 and 400 are also 18 in number.
$\therefore$ Total number of such numbers = 36.

57. Required digit = Unit digit in $(4 \times 8 \times 7 \times 3) = 2$.

58. Required digit = Unit digit in $(1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9) = 0$.

59. $(9 \times 6 \times 4) = 216$. In order to obtain 2 at the unit place, we must multiply 216 by 2 or 7.
$\therefore$ Of the given numbers, we have 7.

60. Unit digit in $(3127)^{173}$ = Unit digit in $(7)^{173}$. Now, 7^4 gives unit digit 1.
$\therefore \quad (7)^{173} = (7^4)^{43} \times 7^1$. Thus, $(7)^{173}$ gives unit digit 7.

61. Unit digit in 7^4 is 1.
$\therefore$ Unit digit in 7^{68} is 1.
Unit digit in 7^{71} is 3. [$1 \times 7 \times 7 \times 7$ gives unit digit 3]
Again, every power of 6 will give unit digit 6.
$\therefore$ Unit digit in 6^{59} is 6.
Unit digit in 3^4 is 1.
$\therefore$ Unit digit in 3^{64} is 1. Unit digit in 3^{65} is 3.
$\therefore$ Unit digit in $(7^{71} \times 6^{59} \times 3^{65})$ = Unit digit in $(3 \times 6 \times 3) = 4$.

62. Unit digit in 7^4 is 1. So, unit digit in 7^{92} is 1.

$\therefore$ Unit digit in 7^{95} is 3. [$\because$ Unit digit in $1 \times 7 \times 7 \times 7$ is 3]

Unit digit in 3^4 is 1.

$\therefore$ Unit digit in 3^{56} is 1.

$\therefore$ Unit digit in 3^{58} is 9.

$\therefore$ Unit digit in $(7^{95} - 3^{58}) = (13 - 9) = 4$.

63. $x^{4n} = (2^4)^n$ or $(4^4)^n$ or $(6^4)^n$ or $(8^4)n$.

Clearly, the unit digit in each case is 6.

64. $(3 \times 5)^{12} \times (2 \times 7)^{10} \times (10)^{25} = (3 \times 5)^{12} \times (2 \times 7)^{10} \times (2 \times 5)^{25}$

$= 3^{12} \times 5^{12} \times 2^{10} \times 7^{10} \times 2^{25} \times 5^{25} = 2^{35} \times 3^{12} \times 5^{37} \times 7^{10}$.

Total number of prime factors $= (35 + 12 + 37 + 10) = 94$.

65. Given Exp. $= a^2 + b^2 + 2ab$, where $a = 397$ and $b = 104$

$= (a + b)^2 = (397 + 104)^2 = (501)^2 = (500 + 1)^2 = (500)^2 + 1^2 + 2 \times 500 \times 1$

$= 250000 + 1 + 1000 = 251001$.

66. Given Exp. $= a^2 + b^2 - 2ab$, where $a = 186$ and $b = 159$

$= (a - b)^2 = (186 - 159)^2 = (27)^2$

$= (20 + 7)^2 = (20)^2 + 7^2 + 2 \times 20 \times 7 = 400 + 49 + 280 = 729$.

67. Given Exp. $= (a + b)^2 - 4ab$, where $a = 475$ and $b = 425$

$= (a - b)^2 = (475 - 425)^2 = (50)^2 = 2500$.

68. $20z = (64)^2 - (36)^2 \Leftrightarrow 20z = (64 + 36)(64 - 36)$

$\Leftrightarrow 20z = 100 \times 28 \Leftrightarrow z = \dfrac{100 \times 28}{20} = 140$.

69. Let $(46)^2 - x^2 = 4398 - 3066$.

Then, $(46)^2 - x^2 = 1332 \Leftrightarrow x^2 = (46)^2 - 1332 = (2116 - 1332)$

$\Leftrightarrow x^2 = 784 \Leftrightarrow x = \sqrt{784} = 28$.

70. Given Exp. $= \dfrac{(a + b)^2 + (a - b)^2}{(a^2 + b^2)} = \dfrac{2(a^2 + b^2)}{(a^2 + b^2)} = 2$.

71. Given Exp. $= \dfrac{(a + b)^2 - (a - b)^2}{ab} = \dfrac{4ab}{ab} = 4$.

72. We know that : $(1 + 2 + 3 + ... + n) = \dfrac{n(n + 1)}{2}$.

$\therefore (1 + 2 + 3 + ... + 45) = \left(\dfrac{45 \times 46}{2}\right) = 1035$.

73. Required numbers are 2, 4, 6, ..., 30.

This is an A.P. containing 15 terms.

$\therefore$ Required sum $= \dfrac{n}{2}$ (first term + last term) $= \dfrac{15}{2}(2 + 30) = 240$.

74. $(51 + 52 + 53 + ... + 100)$

$= (1 + 2 + 3 + ... + 100) - (1 + 2 + 3 + ... + 50)$

$= \left(\dfrac{100 \times 101}{2} - \dfrac{50 \times 51}{2}\right) = (5050 - 1275) = 3775$.

75. Every such number must be divisible by L.C.M. of 4, 5, 6, *i.e.* 60.
Such numbers are 240, 300, 360, 420, 480, 540.
Clearly, there are 6 such numbers.

76. Required numbers are 102, 108, 114, ..., 996.
This is an A.P. with $a = 102$ and $d = 6$.
Let the number of its terms be n. Then,
$a + (n - 1)\,d = 996 \Leftrightarrow 102 + (n - 1) \times 6 = 996 \Leftrightarrow n = 150.$

77. $2^2 + 4^2 + ... + 20^2 = (1 \times 2)^2 + (2 \times 2)^2 + (2 \times 3)^2 + ... + (2 \times 10)^2$
$= 2^2 \times 1^2 + 2^2 \times 2^2 + 2^2 \times 3^2 + ... + 2^2 \times 10^2$
$= 2^2\,[1^2 + 2^2 + 3^2 + ... + 10^2]$
$= 4 \times \dfrac{10 \times 11 \times 21}{6} = 4 \times 385 = 1540.$

78. $11^2 + 12^2 + 13^2 + ... + 20^2$
$= (1^2 + 2^2 + 3^2 + ... + 20^2) - (1^2 + 2^2 + 3^2 + ... + 10^2)$
$= \left[\dfrac{20\,(20+1)\,(40+1)}{6} - \dfrac{10\,(10+1)\,(20+1)}{6}\right] = 2485.$

79. $1 + x + 5 + 4 + 8 = (18 + x)$. Clearly, when $x = 0$, then sum of digits is divisible by 3.

80. Let the required number be $357y25x$.
Then, for divisibility by 5, we must have $x = 0$ or $x = 5$.
Case I. When $x = 0$.
Then, sum of digits $= (22 + y)$. For divisibility by 3, $(22 + y)$ must be divisible by 3.
$\therefore$ $y = 2$ or 5 or 8.
$\therefore$ Numbers are (0, 2) or (0, 5) or (0, 8).
Case II. When $x = 5$.
Then, sum of digits $= (27 + y)$. For divisibility by 3, we must have $y = 0$ or 3 or 6 or 9.
$\therefore$ Numbers are (5, 0) or (5, 3) or (5, 6) or (5, 9).
So, correct answer is (*b*).

81. Let the number be $5x2$. Clearly, it is divisible by 2.
Now, $5 + x + 2 = (7 + x)$ must be divisible by 3. So, $x = 2$.

82. The given number is divisible by 8, if the number $6x2$ is divisible by 8.
Clearly, the least value of x is 3.

83. $(4 + 5 + 1 + x + 6 + 0 + 3) = 19 + x$. Clearly, $x = 8$.

84. Taking the sum of the digits, we have :
$S_1 = 9, S_2 = 12, S_3 = 18, S_4 = 9, S_5 = 21, S_6 = 12, S_7 = 18, S_8 = 21, S_9 = 15, S_{10} = 24.$
Clearly, $S_2, S_5, S_6, S_8, S_9, S_{10}$ are all divisible by 3 but not by 9.
So, the number of required numbers = 6.

85. (*a*) $(1 + 6 + 3) - (2 + 5 + 4) = 1$ (No) (*b*) $(2 + 6 + 4) - (4 + 5 + 2) = 1$ (No)
(*c*) $(4 + 6 + 1) - (2 + 5 + 3) = 1$ (No) (*d*) $(4 + 6 + 1) - (2 + 5 + 4) = 0$ (Yes).

86. $(6 + 5 + 3 + 8) - (x + 2 + 6) = (14 - x)$. Now, $(14 - x)$ is divisible by 11, when $x = 3$.

87. $(4 + 7 + 6 + x + y + 0) = [17 + (x + y)]$. Also, $(0 + x + 7) - (y + 6 + 4) = (x - y - 3)$.
Now, $[17 + (x + y)]$ must be divisible by 3 and $(x - y - 3)$ is either 0 or divisible by 11.
Clearly, $x = 8$ and $y = 5$ satisfy both the conditions.

88. (*a*) 639 is not divisible by 7. (*b*) 2079 is divisible by 3, 7, 9 and 11.
(*c*) 3791 is not divisible by 3. (*d*) 37911 is not divisible by 9.
$\therefore$ Correct answer is (*b*).

89. Since 4864 is divisible by 4, so 9P2 must be divisible by 3.

∴ (11 + P) must be divisible by 3.

∴ Least value of P is 1.

90. The required number should be divisible by 3 and 8.

(a) 718 is not divisible by 8. (b) 810 is not divisible by 8.

(c) 804 is not divisible by 8.

(d) Sum of digits = 27, which is divisible by 3.

And, 736 is divisible by 8. So, given number is divisible by 3 and 8.

91. The given number should be divisible by both 9 and 8.

∴ $(4 + 2 + 5 + 7 + 3 + x) = (21 + x)$ is divisible by 9 and $(73x)$ is divisible by 8.

∴ $x = 6$.

92. The required number should be divisible by both 9 and 11.

Clearly, 114345 is divisible by both 9 and 11. So, it is divisible by 99.

93. The given number will be divisible by 99 if it is divisible by both 9 and 11.

Now, $(3 + 4 + 2 + 2 + 2 + 1 + 3 + x + y) = 17 + (x + y)$ must be divisible by 9.

Also, $(y + 3 + 2 + 2 + 3) - (x + 1 + 2 + 4) = (y - x + 3)$ must be 0 or divisible by 11.

∴ $x + y = 10$ and $y - x + 3 = 0$.

Clearly, $x = 1$, $y = 9$ satisfy both these equations.

94. Since 653xy is divisible by 5 as well as 2, so $y = 0$.

Now, 653x0 must be divisible by 8.

So, 3x0 must be divisible by 8. This happens when $x = 2$.

∴ $x + y = (2 + 0) = 2$.

95. A number is divisible by 132, if it is divisible by each one of 11, 3 and 4.

Clearly, 968 is not divisible by 3. None of 462 and 2178 is divisible by 4.

Also, 5184 is not divisible by 11.

Each one of remaining 4 is divisible by each one of 11, 3 and 4 and therefore, by 132.

96. Clearly, 6897 is divisible by both 11 and 19.

97. None of the numbers in (a) and (c) is divisible by 2.

Number in (b) is not divisible by 3.

Clearly, 510510 is divisible by each prime number between 1 and 17.

98. Clearly, 325325 is divisible by all 7, 11 and 13.

99. Sum of digits = 35 and so it is not divisible by 3.

(Sum of digits at odd places) − (Sum of digits at even places) = (19 − 16) = 3, not divisible by 11.

So, the given number is neither divisible by 3 nor by 11.

100. Since 111111 is divisible by each one of 7, 11 and 13, so each one of given type of numbers is divisible by each one of 7, 11, 13, as we may write, 222222 = 2 × 111111, 333333 = 3 × 111111, etc.

101. Smallest 3-digit prime number is 101. Clearly, 2525 = 25 × 101; 3232 = 32 × 101, etc.

∴ Each such number is divisible by 101.

102. 256256 = 256 × 1001; 678678 = 678 × 1001, etc.

So, any number of this form is divisible by 1001.

103. Required number = 1 × 2 × 3 × 4 = 24.

104. Required number = (2 × 4 × 6) = 48.

105. Let the three consecutive odd numbers be $(2x + 1)$, $(2x + 3)$ and $(2x + 5)$.

Their sum $= (6x + 9) = 3\,(2x + 3)$, which is always divisible by 3.

106. Let the two consecutive odd integers be $(2x + 1)$ and $(2x + 3)$.
Then, $(2x + 3)^2 - (2x + 1)^2 = (2x + 3 + 2x + 1)(2x + 3 - 2x - 1) = (4x + 4) \times 2$
$= 8(x + 1)$, which is always divisible by 8.

107. Let the required number be x.
Then, $(11x + 11) = 11(x + 1)$ is divisible by 13. So, $x = 12$.

108. Let the 3-digit number be xyz. Then,
$(100x + 10y + z) - (x + y + z) = 99x + 9y = 9(11x + y)$, which is divisible by 9.

109. Putting $x = 5$ and $y = 1$, we get $(3x + 7y) = (3 \times 5 + 7 \times 1) = 22$, which is divisible by 11.
$\therefore$ $4x + 5y = (4 \times 5 + 5 \times 1) = 25$, which is not divisible by 11.
$x + y + 4 = (5 + 1 + 4) = 9$, which is not divisible by 11.
$9x + 4y = (9 \times 5 + 4 \times 1) = 49$, which is not divisible by 11.
$4x - 9y = (4 \times 5 - 9 \times 1) = 11$, which is divisible by 11.

110. $\begin{array}{r} 4\,a\,3 \\ 9\,8\,4 \\ \hline 13\,b\,7 \end{array} \Rightarrow a + 8 = b \Rightarrow b - a = 8$

Also, $13b7$ is divisible by 11.
$\therefore$ $(7 + 3) - (b + 1) = (9 - b) \Rightarrow (9 - b) = 0 \Rightarrow b = 9$.
$\therefore$ $b = 9$ and $a = 1 \Rightarrow (a + b) = 10$.

111. Required number $= (2^5 - 2) = (32 - 2) = 30$.

112. Required number = Product of first three multiples of $3 = (3 \times 6 \times 9) = 162$.

113. On dividing 1000 by 45, we get remainder = 10.
$\therefore$ Required number to be added $= (45 - 10) = 35$.

114. On dividing 803642 by 11, we get remainder = 4.
$\therefore$ Required number to be added $= (11 - 4) = 7$.

115. On dividing 11158 by 77, we get remainder = 70.
$\therefore$ Required number to be added $= (77 - 70) = 7$.

116. On dividing 6709 by 9, we get remainder = 4.
$\therefore$ Required number to be subtracted = 4.

117. On dividing 427398 by 15, we get remainder = 3.
$\therefore$ Required number to be subtracted = 3.

118. On dividing 13294 by 97, we get remainder = 5.
$\therefore$ Required number to be subtracted = 5.

119. Clearly, $5 \times$ (sum of numbers) is divisible by 15.
$\therefore$ Sum of numbers must be divisible by 3.
Now, $(250 + 341) = 591$ is divisible by 3. So, required pair is 250, 341.

120. Required number is divisible by 72 as well as by 112, if it is divisible by their LCM, which is 1008.
Now, 1008 when divided by 72, gives quotient = 14.
$\therefore$ Required number = 14.

121. Let it be n times. Then, $(1111 - 99n) < 99$.
By hit and trial, we find that $n = 11$.

122. On dividing 457 by 11, remainder is 6.
$\therefore$ Required number is either 451 or 462. Nearest to 456 is 462.

123. On dividing 99547 by 687, the remainder is 619, which is more than half of 687.
So, we must add (687 − 619) = 68 to the given number.
∴ Required number = (99547 + 68) = 99615.

124. Largest number of 5 digits = 99999. On dividing 99999 by 99, we get 9 as remainder.
∴ Required number = (99999 − 9) = 99990.

125. Smallest number of 5 digits = 10000.
On dividing 10000 by 476, we get remainder = 4.
∴ Required number = [10000 + (476 − 4)] = 10472.

126. Required number $= 999 \times 366 + 103 = (1000 - 1) \times 366 + 103 = 366000 - 366 + 103$
$= 365737$.

127. $4150 = 55 \times x + 25 \Leftrightarrow 55x = 4125 \Leftrightarrow x = \frac{4125}{55} = 75$.

128. Required number $= (555 + 445) \times 2 \times 110 + 30 = 220000 + 30 = 220030$.

129. Largest number of 4 digits = 9999. On dividing 9999 by 7, we get remainder = 3.
Largest number of 4 digits divisible by 7 is (9999 − 3) = 9996.
Let $(9996 - x + 10)$ be divisible by 3. By hit and trial, we find that $x = 7$.
∴ Required number = (9996 − 7) = 9989.

130. Number $= (114 \times Q) + 21 = 19 \times 6 \times Q + 19 + 2 = 19 \times (6Q + 1) + 2$.
∴ Required remainder = 2.

131. Number $= (296 \times Q) + 75 = (37 \times 8Q) + (37 \times 2) + 1 = 37 \times (8Q + 2) + 1$.
∴ Required remainder = 1.

132. Number $= (119 \times Q) + 19 = 17 \times (7Q) + (17 + 2) = 17 \times (7Q + 1) + 2$.
∴ Required remainder = 2.

133. Number $= (899 \times Q) + 63 = (29 \times 31 \times Q) + (29 \times 2) + 5 = 29 \times (31Q + 2) + 5$.
∴ Required remainder = 5.

134. Number $= (31 \times Q) + 29$. Given data is inadequate.

135. Given number $= 13p + 11$. And, Given number $= 17q + 9$.
∴ $13p + 11 = 17q + 9 \Leftrightarrow 17q - 13p = 2$.
By hit and trial, we find that $p = 26$ and $q = 20$.
∴ Required number $= (13 \times 26 + 11) = 349$.

136. Divisor $= (5 \times 46) = 230$. Also, $10 \times Q = 230 \Rightarrow Q = 23$. And, R = 46.
∴ Dividend $= (230 \times 23 + 46) = 5336$.

137. Let the smaller number be x. Then, larger number $= (1365 + x)$.
∴ $1365 + x = 6x + 15 \Leftrightarrow 5x = 1350 \Leftrightarrow x = 270$.
Hence, the required number is 270.

138. Dividend $= (12 \times 35) = 420$. Now, dividend = 420 and divisor = 21.
∴ Correct quotient $= \frac{420}{21} = 20$.

139. Let $n = 4q + 3 \Rightarrow 2n = 8q + 6 = (8q + 4) + 2 \Rightarrow 2n = 4(2q + 1) + 2$.
So, when $2n$ is divided by 4, remainder = 2.

140. Let $x = 6q + 3$. Then, $x^2 = (6q + 3)^2 = 36q^2 + 36q + 9 = 6(6q^2 + 6q + 1) + 3$.
So, when x^2 is divided by 6, remainder = 3.

141.

4	x
5	$y - 1$
	1 − 4

∴ $y = (5 \times 1 + 4) = 9$
∴ $x = (4y + 1) = (4 \times 9 + 1) = 37$.

Now, 37 when divided successively by 5 and 4, we get :

5	37
4	7 – 2
	1 – 3

∴ Respective remainders are 2, 3.

142.

4	x
5	y – 2
6	z – 3
	1 – 4

∴

143.

5	x
9	y – 4
13	z – 8
	1 – 12

∴

Now, 1169 when divided by 585 gives remainder = 584.

144. Let $n = 3q + 1$ and let $q = 2p + 1$. Then, $n = 3(2p + 1) + 1 = 6p + 4$.

∴ The number when divided by 6, we get remainder = 4.

145. $4^{61} + 4^{62} + 4^{63} + 4^{64} = 4^{61}(1 + 4 + 4^2 + 4^3) = 4^{61} \times 85 = 4^{60} \times 340$, which is clearly divisible by 10.

146. Putting $x = 2$, we get $2^2 (2^2 - 1) = 12$. So, $x^2(x^2 - 1)$ is always divisible by 12.