



whose average distance from the sun is  $L_2$  will be:

a)  $D\left(\frac{L_2}{L_1}\right)^{\frac{1}{2}}$  days

b)  $D\left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$

c)  $D\left(\frac{L_2}{L_1}\right)^{\frac{3}{2}}$  days

d)  $D\left(\frac{L_2}{L_1}\right)$

6. Two sources of sound placed close to each other, are emitting progressive waves given by  $y_1 = 4 \sin 600 \pi t$  and  $y_2 = 5 \sin 608 \pi t$ . An observer located near these two sources of sound will hear: [1]

- i. 8 beats per second with intensity ratio 81:1 between waxing and waning
- ii. 4 beats per second with intensity ratio 81:1 between waxing and waning
- iii. 4 beats per second with intensity ratio 25:16 between waxing and waning
- iv. 8 beats per second with intensity ratio 25:16 between waxing and waning

a) i and ii

b) only iv

c) only ii

d) iii and iv

7. A truck has a velocity of 2 m/s at time  $t=0$ . It accelerates at  $2 \text{ m/s}^2$  on seeing police. What is its velocity in m/s at a time of 2 sec? [1]

a) 6

b) 3

c) 4

d) 7

8. The wavelength of ultrasonic waves in air is of the order of: [1]

a) 0.00000001 cm

b) 1 cm

c) 0.1 cm

d) 0.0001 cm

9. A solid sphere of volume  $V$  and density  $\rho$  floats at the interface of two immiscible liquids of densities  $\rho_1$  and  $\rho_2$  respectively. If  $\rho_1 < \rho < \rho_2$ , then the ratio of the volume of the parts of the sphere in upper and lower liquids is [1]

a)  $\frac{\rho + \rho_2}{\rho + \rho_1}$

b)  $\frac{\rho - \rho_1}{\rho_2 - \rho}$

c)  $\frac{\rho_2 - \rho}{\rho - \rho_1}$

d)  $\frac{\rho + \rho_1}{\rho + \rho_2}$

10. The time period of a satellite is related to the density of earth ( $\rho$ ) as: [1]

a)  $\rho$

b)  $\rho^{\frac{1}{2}}$

c)  $\rho^{-\frac{1}{2}}$

d)  $\rho^{-\frac{3}{2}}$

11. A rod has length 3 m and its mass acting per unit length is directly proportional to distance  $x$  from one of its end, then its centre of gravity from that end will be at [1]

a) 1.5 m

b) 2.5 m

c) 2 m

d) 3.0 m

12. A body A of mass 0.5 kg and specific heat 0.85 is at a temperature of  $60^\circ\text{C}$ . Another body B of mass 0.3 kg and specific heat 0.9 is at a temperature of  $90^\circ\text{C}$ . When they are connected to a conducting rod, heat will flow from [1]

a) B to A

b) A to B

c) first A to B then B to A

d) heat can't flow

13. **Assertion:** When percentage errors in the measurement of mass and velocity are 1% and 2% respectively, the percentage error in KE is 5%. [1]

**Reason:**  $\frac{\Delta E}{E} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$

- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
- b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement.
- d) Assertion is wrong statement but reason is correct statement.

14. **Assertion (A):** State variables are required to specify the equilibrium state of the system. [1]

**Reason (R):** Pressure is an intensive state variable.

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false but R is true.

15. **Assertion (A):** A planet moves faster, when it is closer to the sun in its orbit and vice versa. [1]

**Reason (R):** Orbital velocity in orbital of planet is constant.

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false but R is true.

16. **Assertion (A):** If  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C}$ , then  $\vec{A}$  may not always be equal to  $\vec{C}$ . [1]

**Reason (R):** The dot product of two vectors involves cosine of the angle between the two vectors.

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false but R is true.

**Section B**

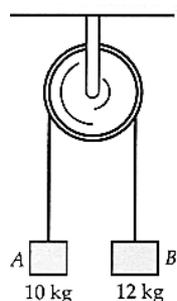
17. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C = 343 ms<sup>-1</sup>? [2]

18. Convert: [2]

- i. 3.0m/s<sup>2</sup> = ..... km/hr<sup>2</sup>
- ii. 6.67 10<sup>-11</sup>Nm<sup>2</sup>/kg<sup>2</sup> = ..... g<sup>-1</sup>cm<sup>3</sup>s<sup>-2</sup>

19. If force F, length L and time T are taken as fundamental units then what will be the dimensions of mass? [2]

20. In the Atwood's machine (figure), the system starts from rest. What is the speed and distance moved by each mass at t = 3s? [2]



21. Derive an expression for work done against gravity. [2]

OR

Assuming the earth to be a uniform sphere of radius 6400 km and density  $5.5 \text{ gcm}^{-3}$ , find the value of  $g$  on its surface. Given  $G = 6.66 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$ .

### Section C

22. Show that the Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area. [3]
23. Two vessels A and B of different materials but having identical shape, size and wall thickness are filled with ice and kept at the same place. Ice melts at the rate of  $100 \text{ g min}^{-1}$  and  $150 \text{ g min}^{-1}$  in A and B, respectively. Assuming that heat enters the vessels through the walls only, calculate the ratio of thermal conductivities of their materials. [3]
24. On a foggy day two drivers spot each other when they are just 80 metres apart. They are travelling at  $72 \text{ km h}^{-1}$  and  $60 \text{ km h}^{-1}$ , respectively. Both of them applied brakes retarding their cars at the rate of  $5 \text{ ms}^{-2}$ . Determine whether they avert collision or not. [3]
25. A railway car of mass 20 tonnes moves with an initial speed of  $54 \text{ km/hr}$ . On applying brakes, a constant negative acceleration of  $0.3 \text{ m/s}^2$  is produced. [3]
- What is the breaking force acting on the car?
  - In what time it will stop?
  - What distance will be covered by the car before it finally stops?
26. What is a refrigerator? Draw a schematic representation of a refrigerator. [3]
27. A monkey of mass 40 kg climbs on a rope which can stand a maximum tension 600 N. In which of the following cases will the rope break? [3]
- The monkey
- climbs up with an acceleration of  $6 \text{ m/s}^2$
  - climbs down with an acceleration of  $4 \text{ m/s}^2$
  - climbs up with a uniform speed of  $5 \text{ m/s}$
  - falls down the rope freely under gravity. Take  $g = 10 \text{ m/s}^2$  and ignore the mass of the rope.
28. The flow rate of water is  $0.58 \text{ L/min}$  from a tap of diameter of  $1.30 \text{ cm}$ . After some time, the flow rate is increased to  $4 \text{ L/min}$ . Determine the nature of the flow for both the flow rates. The coefficient of viscosity of water is  $10^{-3} \text{ Pa} \cdot \text{s}$  and the density of water is  $10^3 \text{ kg/m}^3$ . [3]

OR

Calculate the radius of new bubble formed when two bubbles of radius  $r_1$  and  $r_2$  coalesce?

### Section D

29. **Read the text carefully and answer the questions:** [4]
- Potential energy is the energy stored within an object, due to the object's position, arrangement or state. Potential energy is one of the two main forms of energy, along with kinetic energy. Potential energy depends on the force acting on the two objects.



- a) the escape velocity of the moon's surface is more than the r.m.s velocity of all molecules
- b) it is far away from the surface of the earth
- c) the r.m.s. velocity of all the gas molecules is more than the escape velocity of the moon's surface
- d) its surface temperature is 10°C
- (b) For an ideal gas,  $\frac{C_P}{C_V}$  is
- a)  $\leq 1$
- b) none of these
- c)  $> 1$
- d)  $< 1$
- (c) The root means square velocity of hydrogen is  $\sqrt{5}$  times that of nitrogen. If T is the temperature of the gas then:
- a)  $T(H_2) = T(N_2)$
- b)  $T(H_2) < T(N_2)$
- c)  $T(H_2) \neq T(N_2)$
- d)  $T(H_2) > T(N_2)$
- (d) Suppose the temperature of the gas is tripled and  $N_2$  molecules dissociate into an atom. Then what will be the rms speed of atom:
- a)  $v_0\sqrt{2}$
- b)  $v_0\sqrt{6}$
- c)  $v_0\sqrt{3}$
- d)  $v_0$

**OR**

The velocities of the molecules are  $v, 2v, 3v, 4v$  &  $5v$ . The RMS speed will be:

- a)  $11v$
- b)  $v(12)^{11}$
- c)  $v$
- d)  $v(11)^{12}$

**Section E**

31. A cylindrical log of wood of height  $h$  and area of cross-section  $A$  floats in a liquid. It is pressed and then released. Show that the log would execute S.H.M. with a time period. [5]

$$T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

Where  $m$  is mass of the body and  $\rho$  is the density of the liquid.

**OR**

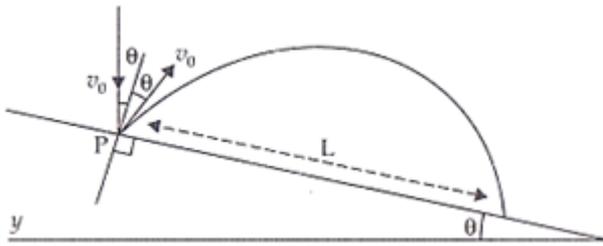
Explain the total energy in simple harmonic motion and show the graphical representation of energy in SHM.

32. A projectile is projected horizontally with a velocity  $u$ . Show that its trajectory is parabolic. And obtain expressions for: [5]

- i. Time of flight
- ii. Horizontal range
- iii. Velocity at any instant  $t$ .

**OR**

A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle with speed  $v_0$  and rebounds elastically as shown in the figure. Find the distance along the plane where it will hit the second time.



Hint:

- i. After rebound, particle still has speed  $V_0$  to start.
  - ii. Work out angle particle speed has with horizontal after it rebounds.
  - iii. Rest is similar to if particle is projected up the incline.]
33. From a uniform disk of radius  $R$ , a circular hole of radius  $\frac{R}{2}$  is cut out. The centre of the hole is at  $\frac{R}{2}$  from the centre of the original disc. Locate the centre of gravity of the resulting flat body. [5]

OR

Derive an expression for the moment of inertia of a thin uniform rod about an axis passing through its one end and perpendicular to its length. Also determine the radius of gyration about the same axis.

# Solution

## Section A

1.

(c)  $10^3$  dyne

**Explanation:** As, dimensional formula of force =  $[MLT^{-2}]$

$$n_1 = 36, M_1 = 1 \text{ kg}, L_1 = 1 \text{ m}, T_1 = 1 \text{ min} = 60 \text{ s}$$

$$n_2 = ?, M_2 = 1 \text{ g}, L_2 = 1 \text{ cm}, T_2 = 1 \text{ s}$$

So, conversion of 36 units into CGS system

$$\text{i.e., } n_2 = n_1 \left[ \frac{M_1}{M_2} \right]^a \left[ \frac{L_1}{L_2} \right]^b \left[ \frac{T_1}{T_2} \right]^c$$

$$n_2 = n_1 \left[ \frac{1 \text{ kg}}{1 \text{ g}} \right]^1 \left[ \frac{1 \text{ m}}{1 \text{ cm}} \right]^1 \left[ \frac{1 \text{ min}}{1 \text{ s}} \right]^{-2}$$

$$= 36 \left[ \frac{1000 \text{ g}}{1 \text{ g}} \right] \left[ \frac{100 \text{ cm}}{1 \text{ cm}} \right]^1 \left[ \frac{60 \text{ s}}{1 \text{ s}} \right]^{-2} = 10^3 \text{ dyne}$$

2.

(b) 36 Hz

**Explanation:** the beat frequency is given by

$$f_{\text{beat}} = f_1 - f_2$$

$$4 = f_1 - f_2 \rightarrow (1)$$

also frequency

$$f \propto \frac{1}{L}$$

$$\frac{f_1}{f_2} = \frac{L_2}{L_1}$$

$$\frac{f_1}{f_2} = \frac{40}{50} \rightarrow (2)$$

on solving equation 1 and 2

$$f_1 = 16 \text{ Hz}$$

$$f_2 = -20 \text{ Hz}$$

$$|f_1 - f_2| = 36 \text{ Hz}$$

3. (a) 3.54 cm

**Explanation:** M.I. of a disc about the central perpendicular axis,  $Mk^2 = \frac{1}{2}MR^2$

$$\therefore k = \frac{R}{\sqrt{2}} = 0.707 \quad R = 0.707 \times 5 \text{ cm}$$

$$= 3.54 \text{ cm.}$$

4.

(b)  $p_{\text{av}} = \frac{F}{A}$

**Explanation:** If F is the magnitude of the normal force acting over an area A, then the average pressure p is defined as the normal force acting per unit area.

$$p_{\text{av}} = \frac{F}{A}$$

5.

(c)  $D \left( \frac{L_2}{L_1} \right)^{\frac{3}{2}}$  days

**Explanation:** According to Kepler's law of periods,

$$T^2 \propto r^3 \Rightarrow T \propto r^{\frac{3}{2}}$$

$$\frac{D}{D'} = \left( \frac{L_2}{L_1} \right)^{\frac{3}{2}}$$

$$\Rightarrow D' = D \left( \frac{L_2}{L_1} \right)^{\frac{3}{2}} \text{ days}$$

6.

(c) only ii

**Explanation:**  $\omega_1 = 600\pi$  or  $n_1 = \frac{600\pi}{2\pi} = 300 \text{ s}^{-1}$

$\omega_2 = 608\pi$  or  $n_2 = \frac{608\pi}{2\pi} = 304 \text{ s}^{-1}$

$\therefore$  Number of beats =  $n_2 - n_1 = 304 - 300 = 4\text{s}^{-1}$

Intensity ratio =  $\frac{I_{\max}}{I_{\min}} = \left(\frac{a_2 + a_1}{a_2 - a_1}\right)^2 = \left(\frac{5 + 4}{5 - 4}\right)^2 = \frac{81}{1}$

7. (a) 6

**Explanation:** Initial velocity is given by,  $u = 2 \text{ m/s}$

Final velocity is given by,  $v = \text{m/s}$

Time duration is = final time - initial time =  $2 - 0 = 2 \text{ s}$

Acceleration,  $a = 2 \text{ m/s}^2$

We know,  $v = u + at$

$\Rightarrow v = 2 + 2 \times 2$

$\Rightarrow v = 6 \text{ m/s}$

8.

(b) 1 cm

**Explanation:** The frequency of ultrasonic waves is above 20000Hz and the speed of waves in air is 330 m/s.

$\lambda = \frac{v}{f}$

$\lambda = \frac{330}{20000}$

$\lambda = 1 \text{ cm}$

9.

(c)  $\frac{\rho_2 - \rho}{\rho - \rho_1}$

**Explanation:** Let  $V_1$  and  $V_2$  be the volumes of the parts of the sphere immersed in liquids of densities  $\rho_1$  and  $\rho_2$  respectively.

According to the law of floatation,

Weight of sphere = Weight of liquid 1 displaced + Weight of liquid 2 displaced

$V\rho g = V_1\rho_1 g + V_2\rho_2 g$

$\Rightarrow (V_1 + V_2)\rho g = V_1\rho_1 g + V_2\rho_2 g$

$\Rightarrow V_1(\rho - \rho_1) = V_2(\rho_2 - \rho)$

$\therefore \frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$

10.

(c)  $\rho^{-\frac{1}{2}}$

**Explanation:** The time period T of the artificial satellite of earth depends on average density  $\rho$  of earth.

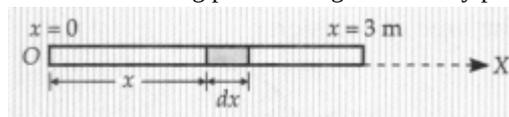
11.

(c) 2 m

**Explanation:**

Suppose the rod is placed along .x-axis. Consider a small element of thickness dx at distance x from its left end.

As the mass acting per unit length is directly proportional to distance x from one end, mass of the small element is  $dm = kx \text{ dx}$



The position of CM of the rod will be

$$x_{CM} = \frac{\int_0^3 x dm}{\int_0^3 dm}$$

$$= \frac{\int_0^3 kx^2 dx}{\int_0^3 kx dx}$$

$$= \frac{\left[\frac{x^3}{3}\right]_0^3}{\left[\frac{x^2}{2}\right]_0^3} = \frac{27}{3} \times \frac{2}{9} = 2\text{m}$$

12. (a) B to A

**Explanation:** Heat always flows from a body at higher temperature to a body at lower temperature.

13. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

**Explanation:** Kinetic energy,  $E = \frac{1}{2}mv^2$

Differentiating both sides,

$$\frac{\Delta E}{E} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$$

$$\frac{\Delta E}{E} = \frac{1}{100} + 2 \times \frac{2}{100} = \frac{5}{100} = 5\%$$

14.

- (b) Both A and R are true but R is not the correct explanation of A.

**Explanation:** Both A and R are true but R is not the correct explanation of A.

15.

- (c) A is true but R is false.

**Explanation:** According to Kepler's law of planetary motion, a planet revolves around the sun in such a way that its areal velocity is constant. i.e. it move faster, when it is closer the sun and vice-versa.

16. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:** As  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C} \Rightarrow AB \cos\theta_1 = BC \cos\theta_2$

$\Rightarrow A = C$ , only when  $\theta_1 = \theta_2$

so when angle between  $\vec{A}$  and  $\vec{B}$  is equal to angle between  $\vec{B}$  and  $\vec{C}$ , only then  $\vec{A}$  equal to  $\vec{C}$ .

### Section B

17. Length of the steel wire,  $l = 12$  m

Mass of the steel wire,  $m = 2.10$  kg

Velocity of the transverse wave,  $v = 343$  m/s

Mass per unit length,  $\mu = \frac{m}{l} = \frac{2.10}{12} = 0.175 \text{ kg m}^{-1}$

For tension T, the velocity of the transverse wave can be obtained using the relation:

$v = \sqrt{\frac{T}{\mu}}$  here T is tension in the wire

$$\therefore T = v^2 \mu$$

$$= (343)^2 \times 0.175 = 20588.575 \approx 206 \times 10^4 \text{ N}$$

18. i. 1 hour = 3600 sec so that 1 sec = 1/3600 hour

1 km = 1000 m so that 1 m = 1/1000 km

$$3.0 \text{ m s}^{-2} = 3.0 (1/1000 \text{ km})(1/3600 \text{ hour})^{-2}$$

$$= 3.9 \times 10^4 \text{ km/hr}^2$$

$$\text{ii. } 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 = \text{g}^{-1} \text{ cm}^3 \text{ s}^{-2}$$

$$= 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$$

$$= 6.67 \times 10^{-11} \times 10^3 \times (10^2)^3$$

$$= 6.67 \times 10^{-8} \text{ g}^{-1} \text{ cm}^3 \text{ s}^{-2}$$

19. Let  $m = KF^a L^b T^c$

Substituting the dimension of,  $[F] = [MLT^{-2}]$ ,  $[L] = [L]$  and  $[T] = [T]$ , we have

$$[M] = [MLT^{-2}]^a [L]^b [T]^c$$

$$[M] = M^a L^{a+b} T^{-2a+c}$$

On equating the powers on both sides, we get

$$a = 1, a + b = 0, -2a + c = 0$$

Solving these equations, we get

$$a = 1, b = -1 \text{ and } c = 2$$

Hence, dimensions of mass M are  $[F^1 L^{-1} T^2]$ .

20. The speed will be same for both block

so let us consider the block 1

Here the force on the block is

$$F = (m_1 - m_2)g = (12 - 10) \times 9.8 = 19.6 \text{ N}$$

So the acceleration of the system is  $a = \frac{F}{m_1 - m_2} = 0.6125$

So the speed after 3 s will be  $0.6125 \times 3 = 1.84 \text{ ms}^{-1}$

21. Potential energy of the body on the surface of the earth =  $-\frac{GMm}{R}$   
 Potential energy at a height  $h$  from the surface of the earth =  $-\frac{GMm}{(R+h)}$   

$$\text{Work done} = \left(-\frac{GMm}{R+h}\right) - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMm}{R} - \frac{GMm}{R+h}$$

$$= GMm \left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= \frac{GMmh}{R(R+h)} = \frac{MgR^2h}{R(R+h)} \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{(Mgh)R}{(R+h)} = \frac{Mgh}{1 + \frac{h}{R}}$$

OR

Given  $\rho = 5.5 \text{ g/cc}$   

$$= \frac{5.5 \times 10^{-3} \text{ Kg}}{(10^{-2} \text{ m})^3} = 5.5 \times 10^3 \text{ kg/m}^3$$

$$R = 6400 \text{ Km} = 6.4 \times 10^6 \text{ m}$$

$$\therefore G = \frac{3 \times 9.8}{4 \times 3.14 \times (6.4 \times 10^6) \times 5.5 \times 10^3}$$

$$= 6.6 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

### Section C

22. The physical significance of Reynold's number. Consider a narrow tube having a cross-sectional area  $A$  Suppose a fluid flows through it with a velocity  $v$  for a time interval  $\Delta t$ .

Length of the fluid = Velocity  $\times$  time =  $v \Delta t$

The volume of the fluid flowing through the tube in time  $\Delta t = Av \Delta t$

Mass of the fluid,

$$\Delta m = \text{Volume} \times \text{density} = Av \Delta t \times \rho$$

The inertial force acting per unit area of the fluid

$$= \frac{F}{A} = \frac{\text{Rate of change of momentum}}{A}$$

$$= \frac{\Delta m \times v}{\Delta t \times A} = \frac{Av \Delta t \rho \times v}{\Delta t \times A} = \rho v^2$$

Viscous force per unit area of the fluid

$$= \eta \times \text{velocity gradient} = \eta \frac{v}{D}$$

$$\frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{\rho v^2}{\eta v / D} = \frac{\rho v D}{\eta} = \text{Re}$$

Thus Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.

23. Suppose  $m_1$  and  $m_2$  be the masses of ice melted at the same time ( $t = 1 \text{ min}$ ) in vessels A and B, respectively.

The amounts of heat flowed into the two vessels will be

$$Q_1 = \frac{K_1 A (T_1 - T_2) t}{x} = m_1 L$$

$$Q_2 = \frac{K_2 A (T_1 - T_2) t}{x} = m_2 L$$

where  $L$  is latent heat of ice.

Dividing Equation (i) by Equation (ii)

$$\Rightarrow \frac{K_1}{K_2} = \frac{m_1}{m_2} = \frac{100 \text{ g}}{150 \text{ g}} = \frac{2}{3} = 2 : 3$$

24. For the first car:

$$u = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}, v = 0, a = -5 \text{ ms}^{-2}$$

$$\text{As } v^2 - u^2 = 2as$$

$$\therefore 0^2 - 20^2 = 2(-5) s_1$$

Distance covered by first car,  $s_1 = 40 \text{ m}$

For the second car:

$$u = 60 \text{ kmh}^{-1} = \frac{60 \times 5}{18}$$

$$= \frac{50}{3} \text{ ms}^{-1}, v = 0, a = -5 \text{ ms}^{-2}$$

$$\text{As } v^2 - u^2 = 2as$$

$$\therefore 0^2 - \left(\frac{50}{3}\right)^2 = 2(-5) s_2$$

Distance covered by second car,

$$s_2 = \frac{2500}{9 \times 10} = 27.78 \text{ m}$$

Total distance covered by the two cars

$$= s_1 + s_2 = 40 + 27.78 = 67.78 \text{ m}$$

As this distance is less than the initial distance (= 80 m) between the two cars, so the collision will be averted.

25. Mass of the railway car,  $m = 20 \text{ tonnes} = 20 \times 1000 \text{ kg} = 20 \times 10^4 \text{ kg}$ , Initial speed,  $u = 54 \text{ km/hr} = 54 \times \frac{5}{18} = 15 \text{ m/s}$

Negative acceleration,  $a = -0.3 \text{ m/s}^2$

a. Breaking force acting on the car,  $F = -ma$

$$F = -(2 \times 10^4 \text{ kg}) \times (-0.3 \text{ m/s}^2)$$

$$F = 6000 \text{ N}$$

b. When the railway car stops, its final velocity is zero.

i.e.  $v = 0$

Using the relation:  $v - u = at$

$$\Rightarrow 0 = 15 + (-0.3)t$$

$$\Rightarrow t = 50 \text{ s}$$

c. Using the relation:  $v^2 - u^2 = 2as$

$$\Rightarrow 0 - (15)^2 = 2(-0.3)s$$

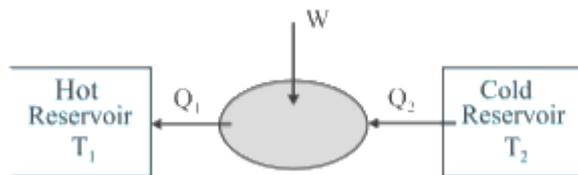
$$\Rightarrow s = 375 \text{ m}$$

26. A refrigerator or a heat pump is a heat engine working in reverse direction.

In the refrigerator, we have 2 bodies, lower temperature (cold) body which is freezer and higher temperature (hot) body which is surroundings. It takes heat from the cold reservoir and then some work is done on the refrigerator and then the amount of heat is transferred to the hot reservoir. Let  $Q_2$  be the heat takes from the cold reservoir,  $W$  is the work done on the system and then releases  $Q_1$  amount of heat to the hot reservoir.

Mathematically,  $Q_2 + W = Q_1$

The schematic representation of a refrigerator has been shown in the following Figure. Here the refrigerator extracts heat  $Q_2$  from a cold reservoir at temperature  $T_2$ , work  $W$  is done on it and finally, it rejects  $Q_1 (= Q_2 + W)$  heat to surroundings (hot reservoir) maintained at a higher temperature  $T_1$ .



The efficiency of the refrigerator can be calculated from the coefficient of performance of the refrigerator,

$$\alpha = \frac{Q_2}{Q_1 - Q_2}$$

27. mass of the monkey,  $m = 40 \text{ kg}$ ,

Tensile strength of the rope,  $T = 600 \text{ N}$  (max tension rope can hold without breaking)

Here, the rope will break if reaction ( $R$ ) exceeds the tension ( $T$ ) applied, i.e.  $R > T$

a.  $a = 6 \text{ m/s}^2$

For upward accelerated motion the net acceleration is  $(g + a)$  instead of  $g$ , hence  $R = m(g + a) = 40(10 + 6) = 640 \text{ N}$ .

Therefore the rope will break, as  $R > T$

b.  $a = 4 \text{ m/s}^2$

For downward accelerated motion the net acceleration is  $(g - a)$  instead of  $g$ , hence  $R = m(g - a) = 40(10 - 6) = 240 \text{ N}$ .

Therefore the rope will not break as  $R < T$

c.  $v = 5 \text{ m/s}$  (constant)  $a = 0$

$R = mg = 40 \times 10 = 400 \text{ N}$ . Therefore the rope will not break as  $R < T$

d. For freefall, net acceleration on the body is zero,  $a = g$ ;  $R = m(g - a) = m(g - g)$ . Therefore  $R = \text{zero}$  (Rope will not break)

28. Given, diameter,  $D = 1.30 \text{ cm} = 1.3 \times 10^{-2} \text{ m}$

Coefficient of viscosity of water,  $\eta = 10^{-3} \text{ Pa} \cdot \text{s}$

Density of water,  $\rho = 10^3 \text{ kg/m}^3$

The volume of the water flowing out per second is

$$V = vA = v \times \pi r^2 = v\pi \frac{D^2}{4}$$

$$\text{Reynold's number, } R_e = \frac{\rho v D}{\eta} = \frac{4\rho v}{\eta\pi D}$$

$$\text{Case I When } V = 0.58 \text{ L/min} = \frac{0.58 \times 10^{-3} \text{ m}^3}{1 \times 60 \text{ s}}$$

$$= 9.67 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$$

$$R_e = \frac{4 \times 10^3 \times 9.67 \times 10^{-6}}{10^{-3} \times 3.14 \times 1.3 \times 10^{-2}} = 948$$

$\therefore R_e < 1000$ , so the flow is steady or streamline

**Case II** When  $V = 4 \text{ L/min}$

$$= \frac{4 \times 10^{-3}}{60} \text{ m}^3 \text{ s}^{-1} = 6.67 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}$$

$$R_e = \frac{4 \times 10^3 \times 6.67 \times 10^{-5}}{10^{-3} \times 3.14 \times 1.3 \times 10^{-2}} = 6536$$

$\therefore R_e > 3000$ , so the flow will be turbulent.

OR

Consider two soap bubbles of radii  $r_1$  and  $r_2$  and volumes as  $V_1$  and  $V_2$ . Thus  $V_1 = \frac{4\pi r_1^3}{3}$  and  $V_2 = \frac{4\pi r_2^3}{3}$ . Let  $S$  be the surface tension of the soap solution. If  $P_1$  and  $P_2$  are excess pressure inside the two soap bubbles then  $P_1 = \frac{4S}{r_1}$ ;  $P_2 = \frac{4S}{r_2}$ . Let  $r$  be the radius of the new soap bubble formed when the two soap bubble coalesces under isothermal conditions. If  $V$  and  $P$  are volume and excess of pressure inside the new soap bubble then  $V = \frac{4}{3}\pi r^3$   $P = \frac{4S}{r}$ . As the new bubble is formed under isothermal condition, so Boyle's law holds good and hence

$$P_1 V_1 + P_2 V_2 = PV$$

$$\left(\frac{4S}{r_1} \times \frac{4}{3}\pi r_1^3\right) + \left(\frac{4S}{r_2} \times \frac{4}{3}\pi r_2^3\right) = \frac{4S}{r} \times \frac{4}{3}\pi r^3$$

$$(16 \times S \times \pi \times r_1^2) + (16 \times S \times \pi \times r_2^2) = 16S\pi r$$

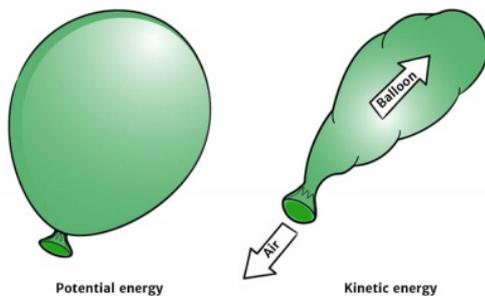
$$r = \sqrt{r_1^2 + r_2^2}$$

#### Section D

#### 29. Read the text carefully and answer the questions:

Potential energy is the energy stored within an object, due to the object's position, arrangement or state. Potential energy is one of the two main forms of energy, along with kinetic energy. Potential energy depends on the force acting on the two objects.

#### Potential and Kinetic Energy



(i) **(a)** mechanical energy

**Explanation:** mechanical energy

(ii) **(a)** potential energy decreases

**Explanation:** potential energy decreases

(iii) **(c)** both only when spring is stretched and compressed

**Explanation:** both only when spring is stretched and compressed

OR

**(b)**  $5 \times 10^5 \text{ J}$

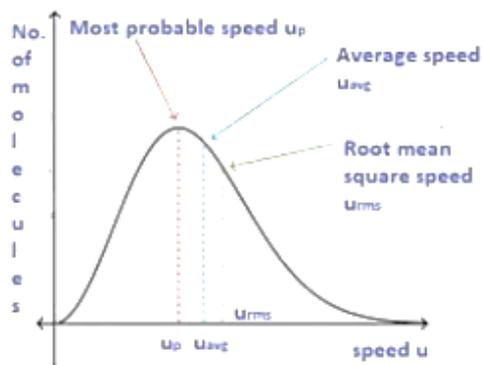
**Explanation:**  $5 \times 10^5 \text{ J}$

(iv) **(b)**  $\text{T}^{-2}$

**Explanation:**  $\text{T}^{-2}$

#### 30. Read the text carefully and answer the questions:

Root mean square velocity (RMS value) is the square root of the mean of squares of the velocity of individual gas molecules and the Average velocity is the arithmetic mean of the velocities of different molecules of a gas at a given temperature.



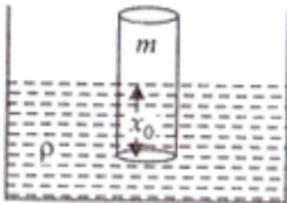
- (i) (c) the r.m.s. velocity of all the gas molecules is more than the escape velocity of the moon's surface  
**Explanation:** The r.m.s. velocity of all the gas molecules is more than the escape velocity of the moon's surface.
- (ii) (c) > 1  
**Explanation:** > 1
- (iii) (b)  $T(H_2) < T(N_2)$   
**Explanation:**  $T(H_2) < T(N_2)$
- (iv) (b)  $v_0\sqrt{6}$   
**Explanation:**  $v_0\sqrt{6}$

OR

- (d)  $v(11)^{12}$   
**Explanation:**  $v(11)^{12}$

### Section E

31. When the block is pressed downward into the liquid then an upward Buoyant force (B.F.) acts on it which moves the block upward and it moves upward from its mean position due to inertia and then again come down due to gravity. So the net restoring force on the block is given by = Buoyant force on the log by the liquid – weight of the log of wood



Say,  $V$  is = the volume of liquid displaced by the block

When the block floats then,

Weight of the block is given by,  $mg = \text{buoyant force by the liquid}$  or  $mg = V\rho g$ , [ $V\rho g$  is the weight of the displaced liquid by the block]

$mg = Ax_0\rho g$  ... (i) [ $x_0$  = is the depth of the block into the liquid just before the block is pressed and volume displaced by the liquid,  $V = Ax_0$ ]

$A$  is the area of cross-section

$x_0$  = is the depth of the block into the liquid due to its own weight

Let  $x$  height again dip in liquid when pressed into it. Hence total height of block into the liquid =  $(x + x_0)$

So net force acting upward on the block is given by =  $[A(x + x_0)]\rho \cdot g - mg$

$$F_{\text{net}} = Ax_0\rho g + Ax\rho g - Ax_0\rho g$$

$$F_{\text{restoring}} = -F_{\text{net}} = -Ax\rho g$$

(as Buoyant force is upward and displacement of the block,  $x$  is directed downwards)

$$\therefore F_{\text{restoring}} \propto -x$$

So motion is SHM with proportional constant  $k = A\rho g$

Again from SHM equation,  $a = -\omega^2 x$  ... (i)

$$F_{\text{restoring}} = -A\rho gx$$

$$\Rightarrow ma = -A\rho gx$$

$$\Rightarrow a = \frac{-A\rho gx}{m} \Rightarrow -\omega^2 x = \frac{-A\rho gx}{m} \quad [\text{putting the value of } a \text{ from equation (i)}]$$

$$\therefore \omega^2 = \frac{A\rho g}{m}$$

with  $k = A\rho g$  and  $\omega = \frac{2\pi}{T}$

Hence,  $\left(\frac{2\pi}{T}\right)^2 = \frac{A\rho g}{m} \Rightarrow \frac{T}{2\pi} = \sqrt{\frac{m}{A\rho g}} \Rightarrow T = 2\pi \sqrt{\frac{m}{A\rho g}}$

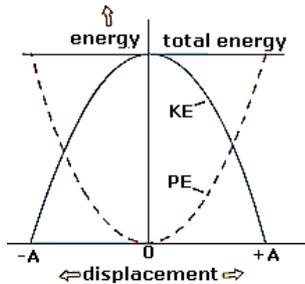
OR

The total energy of the system of a block and a spring is equal to the sum of the potential energy stored in the spring plus the kinetic energy of the block and is proportional to the square of the amplitude.

$$\frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2x^2$$

$$E = \frac{1}{2}m\omega^2A^2$$

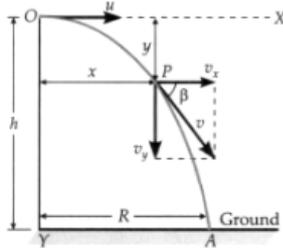
Hence, the total energy of the particle in SHM is constant and it is independent of the instantaneous displacement. Relationship between potential energy, kinetic energy, and time in Simple Harmonic Motion at  $t = 0$ , when  $x = \pm A$ .



32. **Projectile fired parallel to horizontal.** As shown in figure, suppose a body is projected horizontally with velocity  $u$  from a point  $O$  at a certain height  $h$  above the ground level. The body is under the influence of two simultaneous independent motions:

- i. Uniform horizontal velocity  $u$ .
- ii. Vertically downward accelerated motion with constant acceleration  $g$ .

Under the combined effect of the above two motions, the body moves along the path  $OPA$ .



**Horizontal projection of a projectile.**

Trajectory of the projectile. After the time  $t$ , suppose the body reaches the point  $P(x, y)$ .

The horizontal distance covered by the body in time  $t$  is

$$x = ut \therefore t = \frac{x}{u}$$

The vertical distance travelled by the body in time  $t$  is given by

$$s = ut + \frac{1}{2}at^2$$

$$\text{or } y = 0 \times t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \quad [\text{For vertical motion, } u = 0]$$

$$\text{or } y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \left(\frac{g}{2u^2}\right)x^2 \quad [\therefore t = \frac{x}{u}]$$

$$\text{or } y = kx^2 \quad [\text{Here } k = \frac{g}{2u^2} = \text{a constant}]$$

As  $y$  is a quadratic function of  $x$ , so the trajectory of the projectile is a parabola.

**Time of flight.** It is the total time for which the projectile remains in its flight (from  $O$  to  $A$ ). Let  $T$  be its time of flight.

For the vertical downward motion of the body, we use

$$s = ut + \frac{1}{2}at^2 \text{ or } h = 0 \times T + \frac{1}{2}gT^2$$

$$\text{or } T = \sqrt{\frac{2h}{g}}$$

**Horizontal range.** It is the horizontal distance covered by the projectile during its time of flight. It is equal to  $OA = R$ . Thus

$$R = \text{Horizontal velocity} \times \text{time of flight} = u \times T$$

$$\text{or } R = u \sqrt{\frac{2h}{g}}$$

**Velocity of the projectile at any instant.** At the instant  $t$  (when the body is at point  $P$ ), let the velocity of the projectile be  $v$ . The velocity  $v$  has two rectangular components:

Horizontal component of velocity,  $v_x = u$

Vertical component of velocity,  $v_y = 0 + gt = gt$

∴ The resultant velocity at point P is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2 t^2}$$

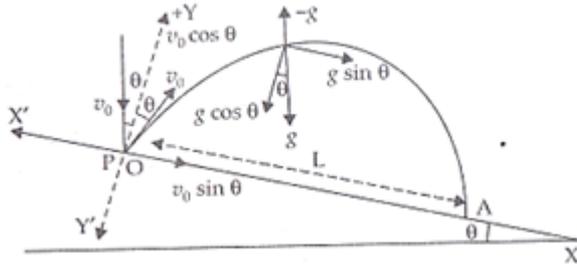
If the velocity  $v$  makes an angle  $\beta$  with the horizontal, then

$$\tan \beta = \frac{v_y}{v_x} = \frac{gt}{u}$$

$$\text{or } \beta = \tan^{-1}\left(\frac{gt}{u}\right)$$

OR

From the figure resolving the components of  $v_0$  and  $g$ , we get



$$v_x = v_0 \sin \theta \text{ and } v_y = v_0 \cos \theta$$

$$g_x = g \cos \theta, g_y = g \sin \theta \text{ acting vertically downwards}$$

Consider the motion of particle from O to A in new YOY' axis.

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\text{Where, } z = 0, \quad v_y = v_0 \cos \theta, \quad a_y = -g \sin \theta$$

$$\therefore t = T \quad (\text{time of flight}), \quad y = 0$$

$$\Rightarrow 0 = v_0 \cos \theta T - \frac{1}{2} g \sin \theta T^2$$

$$\Rightarrow T = \frac{2v_0 \cos \theta}{g \sin \theta}$$

$$T = \frac{2v_0}{g}$$

Now consider the motion along OX axis.

$$x = L, u_x = v_0 \sin \theta, a_x = g \sin \theta, t = T = \frac{2v_0}{g}$$

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$L = \left[ \frac{2v_0}{g} \right] v_0 \sin \theta + \frac{1}{2} g \sin \theta \left[ \frac{2v_0}{g} \right]^2$$

$$L = \frac{2v_0^2}{g} \sin \theta + \frac{1}{2} g \sin \theta \cdot \frac{4v_0^2}{g^2}$$

$$= \frac{2v_0^2}{g} [\sin \theta + \sin \theta] = \frac{4v_0^2}{g} \sin \theta$$

$$\Rightarrow L = \frac{4v_0^2}{g} \sin \theta.$$

Hence the value of L is  $\frac{4v_0^2}{g} \sin \theta$ .

33. The centre of mass of an object is the point at which the object can be balanced. Mathematically, it is the point at which the torques from the mass elements of an object sum to zero. The centre of mass is useful because problems can often be simplified by treating a collection of masses as one mass at their common centre of mass. The weight of the object then acts through this point.

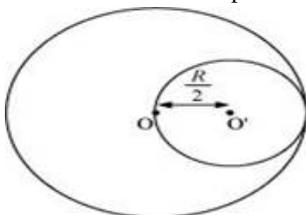
To solve this problem, first we assume that the whole disc was present whose centre of mass lies at the origin from which a small disc was cut out. So CM of remaining portion and cut out disc will lie exactly at the origin i.e Centre of Mass of the original disc at  $x = 0$

Mass per unit area of the original disc =  $\sigma$

Radius of the original disc = R

Mass of the original disc,  $M = \pi R^2 \sigma$

The disc with the cut portion is shown in the following figure:



$$\text{Radius of the smaller disc} = \frac{R}{2}$$

$$\text{Mass of the smaller disc, } M' = \pi \left(\frac{R}{2}\right)^2 \sigma = \frac{1}{4} \pi R^2 \sigma = \frac{M}{4}$$

Let O and O' be the respective centers of the original disc and the disc cut off from the original. As per the definition of the centre of mass, the centre of mass of the original disc is supposed to be concentrated at O, while that of the smaller disc is supposed to be concentrated at O'.

It is given that:

$$OO' = \frac{R}{2}$$

After the smaller disc has been cut from the original, the remaining portion is considered to be a system of two masses. The two masses are:

M (concentrated at O), and

$(-M' = \frac{M}{4})$  concentrated at O'

(The negative sign indicates that this portion has been removed from the original disc.)

Let x be the distance through which the centre of mass of the remaining portion shifts from point O.

The relation between the centers of masses of two masses is given as:

$$x = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

For the given system

$$x = \frac{M \times 0 - M' \times \left(\frac{R}{2}\right)}{M + (-M')} \quad (\text{here } M' \text{ is } M/4)$$

$$= \frac{\frac{-M}{4} \times \frac{R}{2}}{M - \frac{M}{4}} = \frac{-MR}{8} \times \frac{4}{3M} = \frac{-R}{6}$$

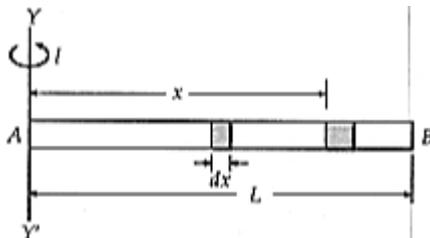
Note that shift in Centre of Mass is very less (only  $0.16 R$  or  $\frac{R}{6}$ ) as removed portion has very less mass as compared to the remaining portion.

(The negative sign indicates that the centre of mass gets shifted toward the left of point O and lies at  $\frac{R}{6}$  left towards origin.)

OR

M.I. of a thin uniform rod about a perpendicular axis through its one end.

Let a thin uniform rod AB of length L and mass M, which can rotate about an axis YY' passing through its one end A and perpendicular to its length, as shown in Fig.



$$\text{Mass per unit length of the rod} = \frac{M}{L}$$

Consider a small element of length dx of the rod at a distance x from the end A

$$\text{Mass of this small length element} = \frac{M}{L} dx$$

Moment of inertia of the small element about the axis YY',

$$dI = \text{Mass} \times (\text{distance})^2 = \frac{M}{L} dx \cdot x^2$$

The moment of inertia of the whole rod about the axis YY' can be obtained by as under

$$I = \int dI = \int_0^L \frac{M}{L} dx \cdot x^2 = \frac{M}{L} \int_0^L x^2 dx$$

$$= \frac{M}{L} \left[ \frac{x^3}{3} \right]_0^L = \frac{M}{3L} [x^3]_0^L = \frac{M}{3L} [L^3 - 0] = \frac{ML^3}{3L} \quad \text{or } I = \frac{ML^2}{3}$$

Radius of gyration. Let k be the radius of gyration of the rod about the axis YY'. Then

$$\frac{ML^2}{3} = Mk^2$$

$$\text{or } k^2 = \frac{L^2}{3}$$

$$\text{or } k = \frac{L}{\sqrt{3}}$$

Thus the radius of gyration of the rod about an axis passing through its one end and perpendicular to its length is  $\frac{L}{\sqrt{3}}$