Chapter – 7

Financial Mathematics

Ex 7.1

Question 1.
Find the amount of an ordinary annuity of ₹ 3,200 per annum for 12 years at the rate of interest of 10% per year, $[(1.1)^{12} = 3.1384]$

Solution:

Here a = 3,200, n = 12, and i = $\frac{10}{100}$ = 0.1

A = $\frac{a}{i}$ $[(1 + i)^n - 1]$

= $\frac{3200}{0.1}$ $[(1 + 0.1)^{12} - 1]$

= 32000 $[(1.1)^{12} - 1]$

= 32000 $[3.1384 - 1]$ $[\because (1.1)^{12} = 3.1384]$

= 32000 [2.1384]

= ₹ 68,428.8

Question 2.
If the payment of ₹ 2,000 is made at the end of every quarter for 10 years at the rate of 8% per year, then find the amount of annuity. $[(1.02)^{40} = 2.2080]$

Solution:

Here a = 2,000, n = 10 years, and $\frac{i}{k} = \frac{\frac{8}{100}}{4} = \frac{2}{100} = 0.02$

$$A = \frac{a}{\frac{i}{k}}\left[\left(1+\frac{i}{k}\right)^{nk} - 1\right]$$

$$A = \frac{2000}{0.02}[(1 + 0.02)^{10\times4} - 1]$$

$$= \frac{200000}{2}[(1 + 0.02)^{40} - 1]$$

= 100000 [2.2080 – 1] [∵ $(1.02)^{40}$ = 2.2080]

= 100000 [1.2080]

= ₹ 1,20,800

Question 3.

Find the amount of an ordinary annuity of 12 monthly payments of ₹ 1,500 that earns interest at 12% per annum compounded monthly. [$(1.01)^{12}$ = 1.1262]

Solution:

Here a = 1,500, n = 1 year, and i = $\frac{12}{100}$

$$\frac{i}{k} = \frac{\frac{12}{100}}{12} = \frac{1}{100} = 0.01$$

$$A = \frac{a}{\frac{i}{k}}\left[\left(1 + \frac{i}{k}\right)^{nk} - 1\right]$$

$$= \frac{1500}{0.01}[(1+0.01)^{1\times12} - 1]$$

$$= [(1 + 0.01)^{1\times12} - 1]$$

= 150000 [$(1.01)^{12}$ – 1]

= 150000 [1.1262 – 1] (∵ $(1.01)^{12}$ = 1.1262)

= 150000 [0.1262]

= ₹ 18,930

Question 4.

A bank pays 8% per annum interest compounded quarterly. Find equal deposits to be made at the end of each quarter for 10 years to have ₹ 30,200? [$(1.02)^{40}$ = 2.2080]

Solution:

Here A = ₹ 30200, i = $\frac{8}{100}$

$$\frac{i}{k} = \frac{\frac{8}{100}}{4} = \frac{2}{100} = 0.02, n = 10, k = 4$$

$$A = \frac{a}{\frac{i}{k}}\left[\left(1 + \frac{i}{k}\right)^{nk} - 1\right]$$

$$30200 = \frac{a}{0.02}[(1 + 0.02)^{40} - 1]$$

$$30200 = \frac{a \times 100}{2}[(1.02)^{40} - 1]$$

$$30200 = \frac{a \times 100}{2}[2.2080 - 1]$$

$$\frac{30200 \times 2}{100} = a[1.2080]$$

$$302 \times 2 = a[1.2080]$$

$$a = \frac{302 \times 2}{1.2080}$$

$$= ₹\ 500$$

Question 5.

A person deposits ₹ 2,000 from his salary towards his contributory pension scheme. The same amount is credited by his employer also. If an 8% rate of compound interest is paid, then find the maturity amount at end of 20 years of service. $[(1.0067)^{240} = 4.966]$

Solution:

A person deposit ₹ 2,000.

The employer also credited the same amount.

a = ₹ 2,000 + ₹ 2,000 = ₹ 4,000

$$A = \frac{a}{\frac{i}{k}}\left[\left(1 + \frac{i}{k}\right)^{nk} - 1\right]$$

$$= \frac{4000}{\frac{8/100}{12}}\left[\left(1+\frac{\frac{8}{100}}{12}\right)^{240} - 1\right]$$

$$A = \frac{4000 \times 12 \times 100}{8}\left[(1+0.067)^{240} - 1\right]$$

Note:
If (1.0067) = 4.966 (Original value)
Then A = 600000 (4.966 – 1)
= 600000(3.966)
= ₹ 23,79,600

Question 6.
Find the present value of ₹ 2,000 per annum for 14 years at the rate of interest of 10% per annum. $[(1.04)^{-14} = 0.6252]$

Solution:

Here a = 2000, n = 14, and i = $\frac{10}{100}$ = 0.1

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$= \frac{2000}{0.1}\left[1 - \frac{1}{(1+0.1)^{14}}\right]$$

$$= \frac{2000}{0.1}\left[1 - (1.1)^{-14}\right]$$

= 20000 [1 – 0.2632]

= 20000 × 0.73678

= ₹ 14,735.60

Question 7.
Find the present value of an annuity of ₹ 900 payable at the end of 6 months for 6 years. The money compounded at 8% per annum. $[(1.04)^{-12} = 0.6252]$

Solution:

$$P = \frac{a}{\frac{i}{2}}\left[1 - \frac{1}{\left(1+\frac{i}{2}\right)^{2\times 6}}\right]$$

$$= \frac{900\times 2}{\frac{8}{100}}\left[1 - \frac{1}{\left(1+\frac{8/100}{2}\right)^{12}}\right]$$

$$= \frac{900\times 2\times 100}{8}[1-(1+0.04)^{-12}]$$

$$= 900 \times 25\,[1-(1.04)^{-12}]$$

$$= 22500\,[1-0.6252]$$

$$= 22500 \times 0.3748$$

$$= ₹\ 8{,}433$$

Question 8.
Find the amount at the end of 12 years of an annuity of ₹ 5,000 payable at the beginning of each year, if the money is compounded at 10% per annum.

Solution:

Here a = 5000, i = 10% = $\frac{10}{100}$ = 0.1, n = 12

Amount A = (1 + i) $\frac{a}{i}$ $[(1+i)^n - 1]$

= (1 + 0.1) $\frac{5000}{\frac{10}{100}}$ $[(1+0.1)^{12} - 1]$

= (1.1) 50000 $[(1.1)^{12} - 1]$

= 55000 [3.1384 – 1]

= 55000 [2.1384]

= ₹ 1,17,612

Question 9.
What is the present value of an annuity due of ₹ 1,500 for 16 years at 8% per annum? [$(1.08)^{15} = 3.172$]

Solution:

Present value of annuity due, $P = \frac{a(1+i)}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

Here a = 1500, n = 16, i = $\frac{8}{100}$ = 0.08

$$P = \frac{1500(1+0.08)}{\frac{8}{100}}\left[1 - \frac{1}{(1.08)^{16}}\right]$$

$$= \frac{1500 \times 100(1.08)}{8}\left[1 - \frac{1}{(1.08)^{16}}\right]$$

$$= \frac{150000}{8}\left[1.08 - \frac{1.08}{(1.08)^{16}}\right]$$

$$= \frac{150000}{8}\left[1.08 - \frac{1}{(1.08)^{15}}\right]$$

= $18750[1.08 - \frac{1}{3.1721}]$ [$\because (1.08)^{15} = 3.1721$]

= 18750[1.08 – 0.31524]

= 18750[0.7648]

= ₹ 14340

Question 10.
What is the amount of perpetual annuity of ₹ 50 at 5% compound interest per year?

Solution:

$$P = \frac{a}{i} = \frac{50}{\left(\frac{5}{100}\right)} = \frac{50 \times 100}{5} = ₹1,000$$

Ex 7.2

Question 1.
Find the market value of 62 shares available at ₹ 132 having the par value of ₹ 100.

Solution:
Market value = ₹ Number of shares × Market value of a share
= ₹ 132 × 62
= ₹ 8,184

Question 2.
How much will be required to buy 125 of ₹ 25 shares at a discount of ₹ 7.

Solution:
Face value of a share = ₹ 25
Market value of a share = ₹ 25 – 7 = ₹ 18
Amount of money required to buy 125 shares = Number of shares × Market value of a share
= ₹ 125 × 18
= ₹ 2,250

Question 3.
If the dividend received from 9% of ₹ 20 shares is ₹ 1,620, find the number of shares.

Solution:
Income = Number of shares × Face value of a share × Rate of dividend
1620 = Number of shares × 20 × 9/100
$\text{Number of shares} = \frac{1620 \times 100}{20 \times 9} = 900 \text{ shares}$

Question 4.
Mohan invested ₹ 29,040 in 15% of ₹ 100 shares of a company quoted at a premium of 20%. Calculate
(i) the number of shares bought by Mohan
(ii) his annual income from shares
(iii) the percentage return on his investment

Solution:

Investment = ₹ 29,040
Rate of dividend = 15%
Number of shares = 100
Premium = 20%

(i) Number of shares

$$= \frac{\text{Investment}}{\text{Market value of a share}}$$

$$= \frac{29040}{100+20} = \frac{29040}{120} = 242$$

(ii) Annual income from shares = (Number of shares) × (Face value of a share) × (Rate of dividend)
= 242 × 100 × 15/100
= ₹ 3630

(iii) The percentage return on his investment

$$= \frac{\text{Income}}{\text{Investment}} \times 100$$

$$= \frac{3630}{29040} \times 100 = \frac{100}{8}$$

$$= 12.5\% = 12\frac{1}{2}\%$$

Question 5.
A man buys 400 of ₹ 10 shares at a premium of ₹ 2.50 on each share. If the rate of dividend is 12% find
(i) his investment
(ii) annual dividend received by him
(iii) rate of interest received by him on his money

Solution:
(i) Given Number of shares = 400
Face value of a share ₹ 10 market values of a share = 10 + 2.50 = ₹ 12.50
Investment = Number of shares × Market value of a share = ₹ 400 × 12.50 = ₹ 5000

(ii) Annual dividend = Number of shares × Face value × Rate of dividend
= 400 × 10 × 12/100
= ₹ 480

(iii) Rate of dividend = $\frac{\text{Dividend}}{\text{Investment}} \times 100$

= $\frac{480}{5000} \times 100$

= $\frac{48}{5}$

= 9.6%

Question 6.
Sundar bought 4,500 of ₹ 10 shares, paying 2% per annum. He sold them when the price rose to ₹ 23 and invested the proceeds in ₹ 25 shares paying 10% per annum at ₹ 18. Find the change in his income.

Solution:
Number of shares = 4500/10 = 450
Income from 2% stock = Number of shares × face value × Rate of dividend
= 450 × 10 × 2/100
= ₹ 90
Selling price of450 shares = 450 × 23 = ₹ 10,350

Number of shares bought in 10% stock = $\frac{\text{Selling price of 450 shares at ₹23}}{\text{Market value}}$

= $\frac{10350}{18}$

= ₹ 575

Income, from 10% stock = No of shares × face value × Rate of dividend

= 575 × 25 × $\frac{10}{100}$

= 575 × $\frac{10}{4}$

= ₹ 1437.5

= ₹ 1437.50

Charge in his income = ₹ 1437.50 – ₹ 90 = ₹ 1347.50

Question 7.
A man invests ₹ 13,500 partly in 6% of ₹ 100 shares at ₹ 140 and partly in 5% of ₹ 100 shares at ₹ 125. If his total income is ₹ 560, how much has he invested

in each?

Solution:
Let the amount invested in 6% of ₹ 100 shares at ₹ 140 be x.
Then the amount invested in 5% of ₹ 100 shares at ₹ 125 is ₹ 13500 – x.
Income from 6% shares = Number of shares × Face value of a share × Rate of dividend

$$= \frac{x}{140} \times 100 \times \frac{6}{100}$$
$$= \frac{3x}{70}$$

Income from 5% shares = Number of shares × Face value of a share × Rate of dividend

$$= \frac{13500-x}{125} \times 100 \times \frac{5}{100}$$
$$= \frac{13500-x}{25}$$

Given that the total income = ₹ 560

$$\frac{3x}{70}+\frac{13500-x}{25}=560$$
$$\frac{(3x)\times 5+(13500-x)\times 14}{350}=560$$
$$\frac{15x+13500\times 14-14x}{350}=560$$

x + 13500 × 14 = 560 × 350
x = 196000 – 189000 = 7000
Amount invested at 6% stock = ₹ 7,000
Amount invested at 5% stock = ₹ 13500 – ₹ 7000 = ₹ 6500

Question 8.
Babu sold some ₹ 100 Shares at a 10% discount and invested his sales proceeds in 15% of ₹ 50 shares at ₹ 33. Had he sold his shares at a 10% premium instead of a 10% discount, he would have earned ₹ 450 more. Find the number of shares sold by him.

Solution:
Let the number of shares sold by Babu be x.
The face value of a share is ₹ 100.

He sold a 10% discount; the selling price of one share ₹ 90.
The selling price of x shares = ₹ 90x
He bought a share of face value ₹ 50 and the number of shares 33.
∴ Number of shares bought for the amount ₹ 90x i.e., 90x/33
∴ Face value of 50 shares = Cost of one share × Number of share

$= 50 \times \frac{90x}{33}$

The dividend is 15%.

$\therefore$ Income $= \frac{15}{100} \times$ Face value of 50 shares

$= \frac{15}{100} \times 50 \times \frac{90x}{33}$

$= \frac{225x}{11}$

Suppose he sold his shares at 10% premium instead of 10% discount, the market value of one share is ₹ 110.
Selling price of x shares = 110 × x = 110x

Number of shares bought for ₹ 33 $= \frac{110x}{33}$

Face value of 50 shares $= \frac{110x}{33} \times 50$

Income $= \frac{15}{100} \times \frac{110x}{33} \times 50 = 25x$

Change Income = ₹ 450

$$25x - \frac{225}{11}x = 450$$

$$\frac{25 \times 11x - 225x}{11} = 450$$

$$\frac{275x - 225x}{11} = 450$$

$$\frac{50x}{11} = 450$$

$x = \frac{450}{50} \times 11 = 9 \times 11 = 99$ shares

Question 9.
Which is better investment? 7% of ₹ 100 shares at ₹ 120 (or) 8% of ₹ 100 shares at ₹ 135.

Solution:
Let the investment in each case be ₹ (120 × 135)
Case (i): Income from 7% of ₹ 100 shares at ₹ 120 = 7/120 × 120 × 135
= 7 × 135
= ₹ 945

Case (ii): Income from 8% of ₹ 100 shares at ₹ 135 = 8/135 × (120 × 135)
= 8 × 120
= ₹ 960
∴ 8% of 100 shares at ₹ 135 is better investment.

Question 10.
Which is better investment? 20% stock at 140 (or) 10% stock at 70.

Solution:
Let the investment in case be ₹ 140 × 70
Income from 20% stock at ₹ 140 is = 20/140 × 140 × 70
= 20 × 70
= ₹ 1400
Income from 10% stock at 70 = 10/70 × 140 × 70 = ₹ 1400
For the same investment both stocks fetch the same income. Therefore they are equivalent shares.

Ex 7.3

Question 1.
The dividend received on 200 shares of face value ₹ 100 at 8% dividend value is:
(a) 1600
(b) 1000
(c) 1500
(d) 800

Answer:
(a) 1600
Hint:
Dividend = 200 × 100 × 8/100 = 1600

Question 2.
What is the amount related is selling 8% stacking 200 shares of face value 100 at 50?
(a) 16,000
(b) 10,000
(c) 7,000
(d) 9,000

Answer:
(b) 10,000
Hint:
Amount $= 200 \times 50 = 10000$

Question 3.
A man purchases a stock of ₹ 20,000 of face value 100 at a premium of 20%, then investment is:
(a) ₹ 20,000
(b) ₹ 24,000
(c) ₹ 22,000
(d) ₹ 30,000

Answer:
(b) ₹ 24,000
Hint:
Investment = Number of shares × Market value
$= 20000/100 \times 120$
$= 24000$

Question 4.
A man received a total dividend of ₹ 25,000 at a 10% dividend rate on a stock of face value ₹ 100, then the number of shares purchased.
(a) 3500
(b) 4500
(c) 2500
(d) 300

Answer:
(c) 2500

Question 5.
The brokerage paid by a person on this sale of 400 shares of face value ₹ 100 at 1% brokerage:
(a) ₹ 600
(b) ₹ 500
(c) ₹ 200
(d) ₹ 400

Answer:
(d) ₹ 400
Hint:
Brokerage $= 400 \times 100 \times 1/100 =$ ₹ 400

Question 6.
Market price of one share of face value 100 available at a discount of 9½ % with brokerage ½% is:
(a) ₹ 89
(b) ₹ 90
(c) ₹ 91
(d) ₹ 95

Answer:
(c) ₹ 91
Hint:
Market price = Face value – Discount + Brokerage
= 100 – 9½
= 100 – 18/2
= 100 – 9
= ₹ 91

Question 7.
A person brought a 9% stock of face value ₹ 100, for 100 shares at a discount of 10%, then the stock purchased is:
(a) ₹ 9000
(b) ₹ 6000
(c) ₹ 5000
(d) ₹ 4000

Answer:
(a) ₹ 9000
Hint:
Face value = ₹ 100
Discount = 10%
Market price of a share = 100 – 10 = 90
Number of share = 100
Stock purchased = 100 × 90 = ₹ 9000

Question 8.
The Income on 7 % stock at 80 is:
(a) 9%
(b) 8.75%
(c) 8%
(d) 7%

Answer:
(b) 8.75%
Hint:
Income = 7/80 × 100
= 0.0875 × 100
= 8.75%

Question 9.
The annual income on 500 shares of face value 100 at 15% is:
(a) ₹ 7500
(b) ₹ 5000
(c) ₹ 8000
(d) ₹ 8500

Answer:
(a) ₹ 7500
Hint:

$$\text{Income} = \frac{n \times r \times F.V}{100}$$
$$= 500 \times \frac{15}{100} \times 100$$
$$= ₹\ 7500$$

Question 10.
₹ 5000 is paid as perpetual annuity every year and the rate of C.I. 10%. Then the present value P of an immediate annuity is:
(a) ₹ 60,000
(b) ₹ 50,000
(c) ₹ 10,000
(d) ₹ 80,000

Answer:
(b) ₹ 50,000
Hint:

$$P = \frac{a}{i} = \frac{5000}{10\%} = \frac{5000 \times 100}{10} = ₹\ 50{,}000$$

Question 11.
If 'a' is the annual payment, 'n' is the number of periods and 'i' is compound interest for ₹ 1 then future amount of the annuity is:
(a) $A = \frac{a}{i}\ (1 + i)\ [(1 + i)^n - 1]$
(b) $A = \frac{a}{i}\ [(1 + i)^n - 1]$
(c) $P = \frac{a}{i}$
(d) $P = \frac{a}{i}\ (1 + i)\ [1 - (1 + i)^{-n}]$

Answer:
(b) $A = \frac{a}{i}\ [(1 + i)^n - 1]$

Question 12.
A invested some money in 10% stock at 96. If B wants to invest in an equally good 12% stock, he must purchase a stock worth of:
(a) ₹ 80
(b) ₹ 115.20
(c) ₹ 120
(d) ₹ 125.40

Answer:
(a) ₹ 80

Hint:
Let x be B stock worth.
Then $x \times \frac{12}{100} = \frac{10}{100} \times 96$
$x \times 12 = 10 \times 96$
$x = 80$

Question 13.
An annuity in which payments are made at the beginning of each payment period is called:
(a) Annuity due
(b) An immediate annuity
(c) perpetual annuity
(d) none of these

Answer:
Annuity due

Question 14.
The present value of the perpetual annuity of ₹ 2000 paid monthly at 10 % compound interest is:
(a) ₹ 2,40,000
(b) ₹ 6,00,000
(c) ₹ 20,40,000
(d) ₹ 2,00,400

Answer:
(a) ₹ 2,40,000
Hint:
$$P = \frac{a}{i} = \frac{2000 \times 12}{10\%} = \frac{2000 \times 12 \times 100}{10} = ₹\ 2,40,000$$

Question 15.
An example of a contingent annuity is:
(a) Life insurance premium
(b) An endowment fund to give scholarships to a student
(c) Personal loan from a bank
(d) All the above

Answer:
(b) An endowment fund to give scholarships to a student