Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 8

with Solution

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. If one of the zeroes of the cubic polynomial $x^3 - 7x + 6$ is 2, then the product of the other two zeroes is **[1]**

   a) 2 b) 3

   c) -3 d) – 2

2. The line segments joining the midpoints of the sides of a triangle form four triangles, each of which is **[1]**

   a) an isosceles triangle b) an equilateral triangle

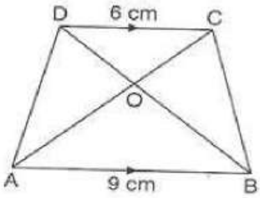
   c) similar to the original triangle d) congruent to the original triangle

3. The father's age is six times his son's age. Four years later, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively **[1]**

   a) 6 and 36 b) 4 and 24

   c) 3 and 24 d) 5 and 30

4. In trapezium ABCD, if $AB||DC$, AB = 9 cm, DC = 6 cm and BD = 12 cm, then BO is equal to **[1]**



a) 7 cm. b) 7.2 cm.

c) 7.5 cm. d) 7.4 cm.

5. The solution of 217x + 131y = 913 and 131x + 217y = 827 is **[1]**

a) x = 2 and y =2 b) x = 2 and y = 3

c) x = 3 and y = 2 d) x = 3 and y = 3

6. A number is selected from first 50 natural numbers. What is the probability that it is a multiple of 3 or 5? **[1]**

a) $\frac{21}{50}$ b) $\frac{12}{25}$

c) $\frac{23}{50}$ d) $\frac{13}{25}$

7. $\sum(x_i - \overline{x})$ is equal to **[1]**

a) 1 b) -1

c) 0 d) 2

8. If cos A = $\frac{4}{5}$, then the value of tan A is? **[1]**

a) $\frac{4}{3}$ b) $\frac{3}{4}$

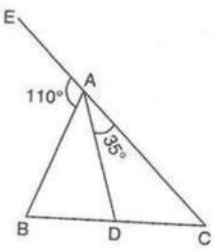
c) $\frac{3}{5}$ d) $\frac{5}{3}$

9. If $n = 2^3 \times 3^4 \times 5^4 \times 7$, then the number of consecutive zeros in n, where n is a natural number, is **[1]**

a) 2 b) 3

c) 7 d) 4

10. In the adjoining figure if exterior $\angle EAB = 110^0, \angle CAD = 35^0$, AB = 5cm, AC = 7cm and BC = 3cm, then CD is equal to **[1]**



a) 2 cm.

b) 1.9 cm.

c) 1.75 cm.

d) 2.25 cm.

11. The coordinates of the point P dividing the line segment joining the points A (1, 3) and B(4, 6) in the ratio 2: 1 are **[1]**

a) (2, 4)

b) (3, 5)

c) (4, 2)

d) (5, 3)

12. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots then k = ? **[1]**

a) 2 or 0

b) -2 or 0

c) 2 or -2

d) 0 only

13. If $\tan\theta = \frac{4}{3}$ then $(\sin\theta + \cos\theta) =$? **[1]**

a) $\frac{7}{5}$

b) $\frac{7}{3}$

c) $\frac{5}{7}$

d) $\frac{7}{4}$

14. A ramp for disabled people in a hospital must slope at not more than 30^{o}. If the height of the ramp has to be 1 m, then the length of the ramp be **[1]**

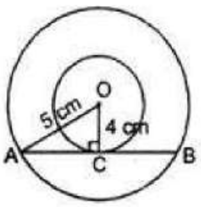
a) 3 m

b) 1 m

c) 2 m

d) $\sqrt{3}$ m

15. If radii of two concentric circles are 4 cm and 5 cm, then the length of the chord of one circle which is tangent to the other circle is: **[1]**



a) 9 cm

b) 3 cm

c) 1 cm

d) 6 cm

16. It is given that $\triangle ABC \sim \triangle DFE$, $\angle A = 30^{o}$, $\angle C = 40^{o}$, AB = 5 cm, AC = 8 cm and DF = 7.5 cm. Then, which of the following is true? **[1]**

a) $\angle F = 110^{o}$, DE = 12 cm

b) $\angle F = 40^{o}$, DE = 12 cm

c) $\angle D = 110^{o}$, EF = 12 cm

d) $\angle D = 30^{o}$, EF = 12 cm

17. If the median of the data: 6, 7, x - 2, x, 17, 20, written in ascending order, is 16. Then x = **[1]**

a) 18

b) 16

c) 15

d) 17

18. **Assertion (A):** If the product of the zeroes of the quadratic polynomial $x^2 + 3x + 5k$ is -10 then value of k is -2. **[1]**

Reason (R): Sum of zeroes of quadratic polynomial $ax^2 + bx + c$ is $-\frac{b}{a}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 300 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + lh)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. If $x^2 + k(4x + k - 1) + 2 = 0$ has equal roots then k = **[1]**

a) $-\frac{2}{3}, 1$

b) $-\frac{3}{2}, -\frac{1}{3}$

c) $\frac{3}{2}, \frac{1}{3}$

d) $\frac{2}{3}$, - 1

Section B

21. Find the ratio in which the point P (-1, y) lying on the line segment joining A (-3, 10) and B(6, -8) divides it. Also find the value of y. **[2]**

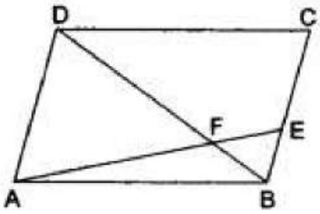
22. Determine the set of values of k for which the following quadratic equation have real roots: $2x^2 + kx - 4 = 0$ **[2]**

23. Prove that $3\sqrt{2}$ is irrational **[2]**

24. If $\triangle$ABC and $\triangle$DEF are similar triangles such that $\angle$A = 57° and $\angle$E = 83°. Find $\angle$C. **[2]**

OR

ABCD is a parallelogram and E is a point on BC. If the diagonal BD intersects AE at F, prove that $AF \times FB = EF \times FD$.



25. Prove the trigonometric identity: **[2]**

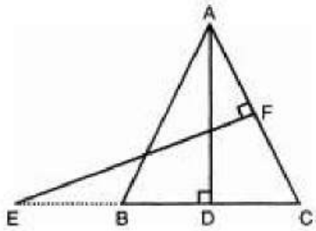
$\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}=\frac{2}{\sin^2 A-\cos^2 A}=\frac{2}{2\sin^2 A-1}=\frac{2}{1-2\cos^2 A}$

OR

Evaluate: $\frac{2}{3}\left(\cos^4 30^\circ-\sin^4 45^\circ\right)-3\left(\sin^2 60^\circ-\sec^2 45^\circ\right)+\frac{1}{4}\cot^2 30^\circ$

Section C

26. In the given figure, AB = AC. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC. Prove that $AB \times EF = AD \times EC$? **[3]**



27. A plane left 30 minutes later than the schedule time and in order to reach its destination 1500 km away in time it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed. **[3]**

28. Prove that the points (2a, 4a), (2a, 6a) and (2a + $\sqrt{3}$a, 5a) are the vertices of an equilateral triangle. **[3]**

OR

The three vertices of a parallelogram ABCD taken in order are A (-1, 0), B(3, 1) and C(2, 2). Find the height of a parallelogram with AD as its base.

29. A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point B which is at 'b' meters above A is β. **[3]**

Prove that the height of the tower is b tan α cot β.

OR

A path separates two walls. A ladder leaning against one wall rests at a point on the path. It reaches a height of 90 m on the wall and makes an angle of 60° with the ground. If while resting at the same point on the path, it were made to lean against the other wall, it would have made an angle of 45° with the ground. Find the height it would have reached on the second wall.

30. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 year. **[3]**

Age (in years)	Number of policyholders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

31. Prove that $(5 + 3\sqrt{2})$ is irrational. **[3]**

Section D

32. Solve the following system of equations graphically **[5]**
$x + 3y = 6$
$2x - 3y = 12$
and hence find the value of a, if $4x + 3y = a$.

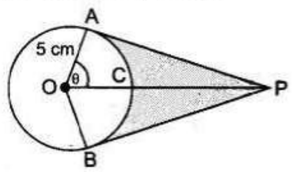
OR

Solve system of equations:
$11x + 15y + 23 = 0$
$7x - 2y - 20 = 0$.

33. A semicircular region and a square region have equal perimeters. The area of the square region exceeds that of the semicircular region by 4 cm^2. Find the perimeters and areas of the two regions. **[5]**

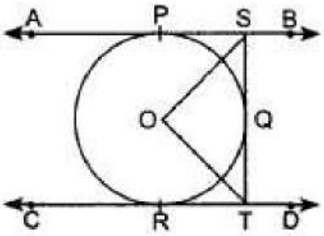
OR

An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area.



34. In figure AB and CD are two parallel tangents to a circle with centre O. ST is tangent segment between the two parallel tangents touching the circle at Q. Show **[5]**

that $\angle SOT = 90^{o}$



35. Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) consecutive days? (iii) different days? **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Suman is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 360 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) Find the total number of rows of candies.

(ii) How many candies are placed in last row?

OR

Find the number of candies in 12th row.

(iii) If Aditya decides to make 15 rows, then how many total candies will be placed by him with the same arrangement?

37. **Read the text carefully and answer the questions:** **[4]**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Find the volume of the Hermika, if the side of cubical part is 10 m.

(ii) Find the volume of cylindrical base part whose diameter and height 48 m and 14 m.

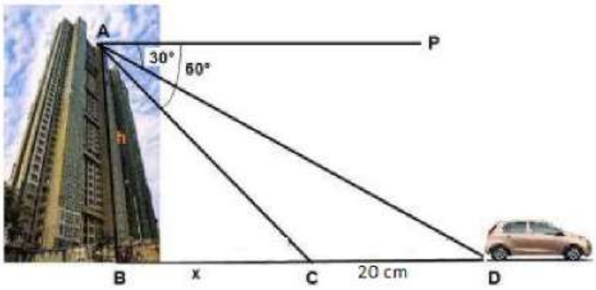
OR

If the diameter of the Anda is 42 m, then find the volume of the Anda.

(iii) If the volume of each brick used is 0.01 m^3, then find the number of bricks used to make the cylindrical base.

38. **Read the text carefully and answer the questions:** **[4]**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^o. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to 30^o.



(i) Find the value of x.

(ii) Find the height of the building AB.

(iii) Find the distance between top of the building and a car at position D?

OR

Find the distance between top of the building and a car at position C?

Solution

Section A

1. **(c)** -3

 Explanation: Let α, β, γ are the zeroes of the given polynomial.

 Given: $\alpha = 2$

 Since $\alpha\beta\gamma = \frac{constant\ term\ C}{Coefficient\ of\ x^3} = \frac{-c}{a}$

 $\Rightarrow 2 \times \beta\gamma = \frac{-6}{1}$

 $\Rightarrow \beta\gamma = \frac{-6}{2} = -3$

2. **(c)** similar to the original triangle

 Explanation: The line segments joining the midpoints of a triangle form 4 triangles which are similar to the given (original) triangle.

3. **(a)** 6 and 36

 Explanation: Let 'x' year be the present age of father and 'y' year be the present age of son.

 Four years later, given condition becomes,

 (x + 4) = 4(y + 4)

 x + 4 = 4y + 16

 x - 4y - 12 = 0 …(i)

 and initially, x = 6y …(ii)

 On putting the value of from Eq. (ii) in Eq. (i), we get

 6y - 4y - 12 = 0

 2y = 12

 Hence, y = 6

 Putting y = 6, we get x = 36.

 Hence, present age of father is 36 years and age of son is 6 years.

4. **(b)** 7.2 cm.

 Explanation: In ΔCOD and ΔAOB,

 $\angle$DOC = $\angle$AOB [Vertically opposite]

 And $\angle$DCO = $\angle$OAB [Alternate angles]

 $\Rightarrow\Delta COD \sim \Delta AOB$ [AA similarity]

 Let OB = x cm

 $\therefore \frac{AB}{CD} = \frac{OB}{OD}$

 $\Rightarrow \frac{9}{6} = \frac{x}{12-x}$

 $\Rightarrow$ 108 - 9x = 6x

 $\Rightarrow$ 15x = 108

 $\Rightarrow$ x = 7.2cm

5. **(c)** x = 3 and y = 2

 Explanation: Firstly add up both eq.

 217x + 131y = 913,

 131x + 217y = 827,

 348x + 348y = 1740

 Dividing both side by 348

 We get x + y = 5 ... (i)

 Similarly Subtract given eqn 217x + 131y = 913 - (131x + 217y = 827)

86x - 86y = 86
Dividing both side by 86
We get x - y = 1 ... (ii)equation
Now, solve equation (i) and (ii)
x + y = 5
x - y = 1
2x = 6
$\Rightarrow$ x = 3
Put x = 3 in equation (i)
x + y = 5
3 + y = 5
y = 5 - 3
$\Rightarrow$ y = 2
Hence, x = 3 y = 2

6. **(c)** $\frac{23}{50}$
Explanation: Total numbers = 1 to 50 = 50
Numbers which are multiples of 3 or 5, are 3,5, 6,9, 10, 12, 15, 18, 20, 21 , 24, 25, 27, 30, 33 35, 36, 39,
40, 42, 45, 48, 50 = 23
$\therefore P(E) = \frac{m}{n} = \frac{23}{50}$

7. **(c)** 0
Explanation: $\sum x_i - \overline{x}$ = $\sum x_i$ - $\sum(\overline{x})$
$\sum(\overline{x})$ = n $(\overline{x})$ by definition
But $\overline{x} = \frac{(\sum x_i)}{n}$ by definition
So, $\sum x_i$ - $\sum(\overline{x})$ = $\sum x_i$ - n $\frac{(\sum x_i)}{n}$
which is equal to = $(\sum x_i)$ - $(\sum x_i)$ = 0

8. **(b)** $\frac{3}{4}$
Explanation: Given: cos A = $\frac{4}{5}$ ...(i)
we know that tan A = $\frac{\sin A}{\cos A}$
Also we know that, sin A = $\sqrt{(1 - \cos^2 A)}$ ...(ii)
Thus,
Substituting eq. (i) in eq. (ii), we get
Sin A = $\sqrt{1 - \frac{16}{25}}$
$= \sqrt{(9/25)} = \frac{3}{5}$
Therefore, $\tan A = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$

9. **(b)** 3
Explanation: Since, it is given that
$n = 2^3 \times 3^4 \times 5^4 \times 7$
$= 2^3 \times 5^4 \times 3^4 \times 7$
$= 2^3 \times 5^3 \times 5 \times 3^4 \times 7$
$= (2 \times 5)^3 \times 5 \times 3^4 \times 7$
$= 5 \times 3^4 \times 7 \times (10)^3$
So, this means the given number n will end with 3 consecutive zeroes.

10. **(c)** 1.75 cm.

Explanation: Here, ∠BAD = 180° - (∠EAB + ∠ADC) = 180° - 110° - 35° = 35°
Since, AD bisects ∠A.
$\therefore \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{\mathrm{BD}}{\mathrm{CD}}$ [Internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle]
$\Rightarrow \frac{5}{7} = \frac{3-\mathrm{CD}}{\mathrm{CD}}$
⇒ 5CD = 21 - 7CD ⇒ 5CD + 7 CD = 21
⇒ 12CD = 21 ⇒ CD = 1.75 cm

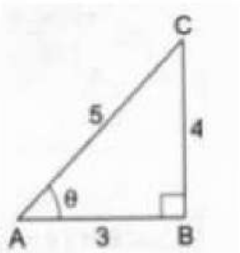
11. **(b)** (3, 5)

Explanation: Point P divides the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2: 1
Let coordinates of P be (x, y), then
$x = \frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{2\times4+1\times1}{2+1} = \frac{8+1}{3} = \frac{9}{3} = 3$
$y = \frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{2\times6+1\times3}{2+1} = \frac{12+3}{3} = \frac{15}{3} = 5$
∴ Coordinates of P are (3, 5)

12. **(c)** 2 or -2

Explanation: Since the roots are equal, we have D = 0.
∴ $36k^2 - 4 \times 9 \times 4 = 0 \Rightarrow 36k^2 = 144 \Rightarrow k^2 = 4 \Rightarrow k = 2$ or -2.

13. **(a)** $\frac{7}{5}$

Explanation: 

$\tan\theta = \frac{4}{3} = \frac{BC}{AB}$
$\therefore AC^2 = AB^2 + BC^2 = (3)^2 + (4)^2 = 25$
$\Rightarrow \quad AC = \sqrt{25} = 5$
$\therefore (\sin\theta + \cos\theta) = \left(\frac{4}{5} + \frac{3}{5}\right) = \frac{7}{5}$

14. **(c)** 2 m

Explanation: Let the height of the ramp be AB = 1 m, the slope of the ramp AC and angle of elevation = $\theta = 30^{\circ}$
In triangle ABC,
$\sin^{\circ}30 = \frac{AB}{AC}$
$\Rightarrow \frac{1}{2} = \frac{1}{\mathrm{AC}}$
⇒ AC = 2 meters
Therefore, the length of the ramp is 2 m.

15. **(d)** 6 cm

Explanation: Here OC is perpendicular to AB.
Then OC bisects AB i.e., AC = BC
Now, in triangle OAC, $\mathrm{OA}^2 = \mathrm{AC}^2 + \mathrm{OC}^2$
$\Rightarrow (5)^2 = AC^2 + (4)^2 \Rightarrow AC^2 = 25 - 16$
⇒AC = 3 Therefore, length of tangent AB = AC + BC = 3 + 3 = 6 cm

16. **(a)** $\angle F = 110^o$, DE = 12 cm

Explanation: In $\Delta ABC, \angle A + \angle B + \angle C = 180^o$

$\Rightarrow 30^o + 40^o + \angle B = 180^o$

$\Rightarrow \angle B = 110^o$

Since $\triangle ABC \sim \triangle DFE$

therefore, $\angle B = \angle F = 110^o$

Also $\frac{DF}{DE} = \frac{AB}{AC}$

$\Rightarrow \frac{7.5}{DE} = \frac{5}{8}$

$\Rightarrow$ DE = 12 cm

17. **(d)** 17

Explanation: Median of 6, 7, x - 2, x, 17, 20 is 16

Here n = 6

$\therefore$ Median = $\frac{1}{2}\left[\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)\text{th}\right]$ term

$= \frac{1}{2}\left[\frac{6}{2}th + \left(\frac{6}{2}+1\right)th\right]$ term

$= \frac{1}{2}$(3rd + 4th) term

$= \frac{1}{2}(x - 2 + x)$

$= \frac{1}{2}(2x - 2) = x - 1$

$\therefore x - 1 = 16$

$\Rightarrow x = 16 + 1 = 17$

18. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Reason is true as we know that Sum of zeroes = $-\frac{b}{a}$

Also, we know that Product of zeroes = $\frac{c}{a}$

$\Rightarrow \frac{5k}{1} = -10 \Rightarrow k = -2$

So, the Assertion is true. But Reason is not the correct explanation of assertion.

19. **(d)** A is false but R is true.

Explanation: A is false but R is true.

20. **(d)** $\frac{2}{3}$, - 1

Explanation: The given quadric equation is $x^2 + k(4x + k - 1) + 2 = 0$, and roots of the equation are equal.

We have to find the value of k.

$x^2 + k(4x + k - 1) + 2 = 0$

$x^2 + 4kx + (k^2 - k + 2) = 0$

Here, a = 1, b = 4k and, c = k^2 - k + 2

We know that, D = b^2 - 4ac

$= (4k)^2 - 4\times 1\times(k^2 - k + 2)$

$= 16k^2 - 4k^2 + 4k - 8$

$= 12k^2 + 4k - 8$

$= 4\ (3k^2 + k - 2)$

The given equation will have real and distinct roots, if D = 0

$4(3k^2 + k - 2) = 0$

$3k^2 + k - 2 = 0$

$3k^2 + 3k - 2k - 2 = 0$

$3k(k + 1) - 2(k + 1) = 0$

$(k + 1)(3k - 2) = 0$

$(k + 1) = 0$ or $(3k - 2) = 0$

$k = -1$ or $k = \frac{2}{3}$

Therefore, the value of $k = \frac{2}{3}, -1$

Section B

21. Let P divide A and B in the ratio of r:1

P(-1, y), A(-3, 10), B(6, -8)

Using the section formula for x coordinate, we get

$-1 = \frac{6r-3}{r+1} \Rightarrow -r - 1 = 6r - 3$

$7r = 2 \Rightarrow r = \frac{2}{7}$

Hence, P divides the line AB in the ratio of 2:7

Hence, using the section formula,

$y = \frac{-8r+10}{r+1}$

$\Rightarrow \therefore y = \frac{-16+70}{2+7} = \frac{54}{9} = 6 \left[\text{Substituting } r = \frac{2}{7} \right]$

22. Here we have, $2x^2 + kx - 4 = 0$

Here, $a = 2,\ b = k\ and\ c = -4$

$\therefore D = b^2 - 4ac$

$= k^2 - 4 \times 2 \times (-4)$

$= k^2 + 32$

$\Rightarrow D = k^2 + 32$

The given equation will have real roots, if

$D \geq 0$

$\Rightarrow k^2 + 32 \geq 0$ for all $k \in R$

$\therefore k \in R$

23. Let us assume, to the contrary, that $3\sqrt{2}$ is rational. Then, there exist co-prime positive integers a and b such that

$3\sqrt{2} = \frac{a}{b}$

$\Rightarrow \quad \sqrt{2} = \frac{a}{3b}$

$\Rightarrow \quad \sqrt{2}$ is rational $\left[\because 3, a \text{ and } b \text{ are integers } \therefore \frac{a}{3b} \text{ is a rational number } \right]$

This is a contradiction. Hence our assumption is wrong.

So, $3\sqrt{2}$ is an irrational number.

24. According to the question,we have,

$\triangle ABC \sim \triangle DEF$ (given)

$\Rightarrow \quad \angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$

$\angle A = 57^\circ, \angle B = 83^\circ$

But $\angle A + \angle B + \angle C = 180^\circ$ (angle sum property of a triangle)

$\Rightarrow \quad \angle C = 180^\circ - \angle A - \angle B = 180^\circ - 57^\circ - 83^\circ$

$\angle C = 180^\circ - 140^\circ = 40^\circ$

OR

Given: ABCD is a parallelogram and E is a point on BC. The diagonal BD intersects AE at F.

To prove: $AF \times FB = EF \times FD$

Proof: Since ABCD is a parallelogram, then its opposite sides must be parallel.

$\therefore$In $\triangle ADF$ and $\triangle EBF$

$\angle$FDA = $\angle$EBF and $\angle FAD = \angle FEB$ [Alternate interior angles]

$\angle AFD = \angle BFE$ [vertically opposite angles]

Therefore,by AAA criteria of similar triangles,we have,

$\triangle$ADF = $\triangle$EBF

Since the corresponding sides of similar triangles are proportional. Therefore,we have,

$\frac{AF}{FD} = \frac{EF}{FB}$

$\Rightarrow AF \times FB = EF \times FD$

25. We have,

$\mathrm{L.H.S} = \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$

$\Rightarrow \quad \mathrm{L.H.S} = \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$

$\Rightarrow \quad \mathrm{L.H.S} = \frac{(\sin^2 A + \cos^2 A + 2\sin A\cos A) + (\sin^2 A + \cos^2 A - 2\sin A\cos A)}{\sin^2 A - \cos^2 A}$ [

$\because (a \pm b)^2 = a^2 \pm 2ab + b^2$]

$\Rightarrow \quad \mathrm{L.H.S} = \frac{(1 + 2\sin A\cos A) + (1 - 2\sin A\cos A)}{\sin^2 A - \cos^2 A}$

$\Rightarrow \quad \mathrm{L.H.S} = \frac{2}{\sin^2 A - \cos^2 A}$

$\Rightarrow \quad \mathrm{L.H.S} = \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{\sin^2 A - (1 - \sin^2 A)}$ [$\because \sin^2 A + \cos^2 A = 1$]

$\Rightarrow \quad \mathrm{L.H.S} = \frac{2}{2\sin^2 A - 1} = \frac{2}{2(1 - \cos^2 A) - 1} = \frac{2}{1 - 2\cos^2 A}$ = R.H.S [$\because$

$\sin^2 A = 1 - \cos^2 A$ & $\cos^2 A = 1 - \sin^2 A$]

Hence proved.

OR

Here, we will use the values of known angles of different trigonometric ratios.

$= \frac{2}{3}\left(\cos^4 30^\circ - \sin^4 45^\circ\right) - 3\left(\sin^2 60^\circ - \sec^2 45^\circ\right) + \frac{1}{4}\cot^2 30^\circ$

$= \frac{2}{3}\left(\frac{9}{16} - \frac{1}{4}\right) - 3\left(\frac{3}{4} - 2\right) + \frac{1}{4}(3)$

$= \frac{2}{3}\left(\frac{5}{16}\right) + 3\left(\frac{5}{4}\right) + \frac{3}{4}$

$= \frac{113}{24}$

Section C

26. We have, AB=AC it means $\triangle$ABC is an isosceles triangle.

Since angles opposite to equal sides of a triangle are equal

$\therefore \quad \angle B = \angle C$

Now, in Δ's ABD and ECF, we have

$\angle$ABD = $\angle$ECF [$\because \angle$B = $\angle$C]

$\angle$ADB = $\angle$EFC = 90° [$\because AD \perp BC$ and $EF \perp AC$]

So, by AA-criterion of similarity, we have

$\Delta ABD \sim \Delta ECF$

$\Rightarrow \quad \frac{AB}{EC} = \frac{AD}{EF}$

$\Rightarrow \quad AB \times EF = AD \times EC$

27. Let usual speed = x km/hr

New speed = (x + 250)km/hr

Total distance = 1500 km

Time taken by usual speed = $\frac{1500}{x}$hr

Time taken by new speed = $\frac{1500}{x+250}$ hr

According to question,

$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$

$\Rightarrow \frac{1500x+1500\times250-1500x}{x^2+250x} = \frac{1}{2}$

$\Rightarrow x^2 + 250x = 750000$

$\Rightarrow x^2 + 250x - 750000 = 0$

$\Rightarrow x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow x(x + 1000) - 750(x + 1000) = 0$

$\Rightarrow x = 750$ or $x = -1000$

Therefore, usual speed is 750 km/hr, -1000 is neglected.

28. Let A(2a, 4a), B(2a, 6a) and C(2a + $\sqrt{3}a$, 5a) be the given point:

$AB = \sqrt{(2a-2a)^2 + (6a-4a)^2}$

$\Rightarrow \quad AB = \sqrt{(0)^2 + (2a)^2}$

$\Rightarrow \quad AB = \sqrt{4a^2}$

$\Rightarrow AB = 2a$

$BC = \sqrt{(2a + \sqrt{3a} - 2a)^2 + (5a - 6a)^2}$

$\Rightarrow \quad BC = \sqrt{(\sqrt{3}a)^2 + (-a)^2}$

$\Rightarrow \quad BC = \sqrt{3a^2 + a^2}$

$\Rightarrow \quad BC = \sqrt{4a^2}$

$\Rightarrow BC = 2a$

$AC = \sqrt{(2a + \sqrt{3}a - 2a)^2 + (5a - 4a)^2}$

$\Rightarrow \quad AC = \sqrt{(\sqrt{3}a)^2 + (a)^2}$

$\Rightarrow \quad AC = \sqrt{3a^2 + a^2}$

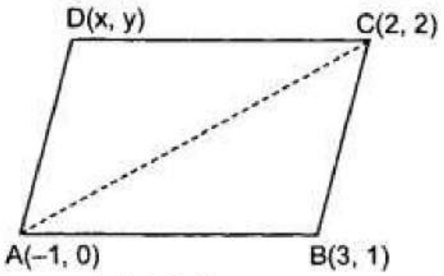
$\Rightarrow \quad AC = \sqrt{4a^2}$

$\Rightarrow AC = 2a$

Since, AB = BC = AC

$\therefore$ ABC is an equilateral triangle.

OR



Area of $\triangle$ABC

$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2} [-1(1 - 2) + 3(2 - 0) + 2(0 - 1)]$

$= \frac{1}{2} [1 + 6 - 2] = \frac{5}{2}$ sq. units

Area of $\|$gm = 2 $\times$ area of $\triangle$ABC

$\Rightarrow$ Area of $\|$gm = 2 $\times$ $\frac{5}{2}$ = 5 sq. units

Let coordinates of D are (x, y)

Mid point of AC $= \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$

Mid-point of BD $= \left(\frac{3+x}{2}, \frac{1+y}{2}\right)$

$\therefore$ Diagonals of a $\|$gm bisect each other

$\therefore$ Mid-point of BD = Mid-point of AC

$\Rightarrow \left(\frac{3+x}{2}, \frac{1+y}{2}\right) = \left(\frac{1}{2}, 1\right)$

$\Rightarrow \frac{3+x}{2} = \frac{1}{2}$ and $\frac{1+y}{2} = 1$

$\Rightarrow$ x = - 2

$\Rightarrow$ y = 1

Now AD = $\sqrt{(-1+2)^2 + (0+1)^2} = \sqrt{2}$

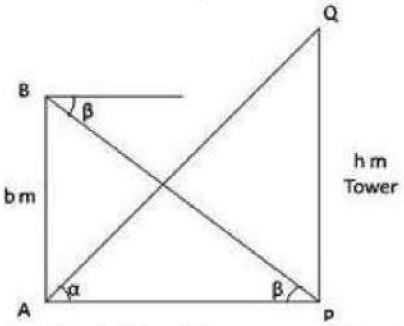
Also area of $\|$gm = base $\times$ height

$\Rightarrow$ AD $\times$ height = 5

$\Rightarrow \sqrt{2} \times$ height = 5

$\Rightarrow$ height $= \frac{5}{\sqrt{2}} = \frac{5}{2}\sqrt{2}$ units.

29.



Let height of tower = QP = h m

In ΔBAP

$\tan\beta = \frac{BA}{AP}$

$\Rightarrow \tan\beta = \frac{b}{AP}$

$\Rightarrow AP = \frac{b}{\tan\beta}$

$\Rightarrow AP = b \times \cot\beta$ .... (i)

In ΔQPA

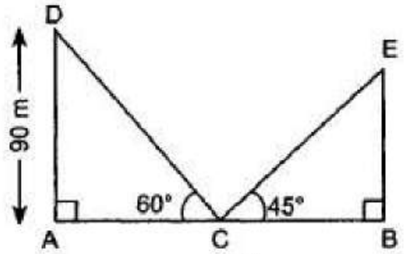
$\tan\alpha = \frac{QP}{AP}$

$\Rightarrow QP = AP \times \tan\alpha$

$\Rightarrow QP = b\cot\beta \times \tan\alpha$ From (i)

OR

Let AB is path



In rt. $\triangle$DAC, $\frac{DC}{AD} = cosec\ 60^\circ$

$\Rightarrow \quad \frac{\text{DC}}{90} = \frac{2}{\sqrt{3}}$

$\text{DC} = \frac{2}{\sqrt{3}} \times 90\text{m} = \frac{180}{\sqrt{3}}\text{m}$

Now, $DC = CE$

$\therefore \quad \text{CE} = \frac{180}{\sqrt{3}}\text{m}$

In rt. $\triangle$EBC,

$\frac{BE}{CE} = \sin 45^\circ$

$\Rightarrow \quad \mathrm{BE} = \frac{1}{\sqrt{2}} \times \frac{180}{\sqrt{3}} \mathrm{m}$

$\Rightarrow \quad BE = 73.47\mathrm{m}$

30. To calculate the median age, we need to find the class intervals and their corresponding frequencies.

It is shown below:

Class interval	Frequency	Cumulative Frequency
Below 20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100

Now, n = 100

So, $\frac{n}{2} = \frac{100}{2} = 50$

This observation lies in class 35 - 40.

So, 35 - 40 is the median class.

Therefore,

l = 35

h = 5

cf = 45

f = 33

$\therefore$ Median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 35 + \left(\frac{50-45}{33}\right) \times 5$

$= 35 + \frac{25}{33} = 35 + 0.76 = 35.76$ years

Hence, the median age is 35.76 years.

31. Let $5 + 3\sqrt{2}$ is rational. It can be written in the form $\frac{p}{q}$.

$(5 + 3\sqrt{2}) = \frac{p}{q}$

$3\sqrt{2} = \frac{p}{q} - 5$

$3\sqrt{2} = \frac{p-5q}{q}$

$\sqrt{2} = \frac{p-5q}{3q}$

As p - 5q and 3q are integers .

So, $\frac{p-5q}{3q}$ is rational number .

But $\sqrt{2}$ is not rational number .

Since a rational number cannot be equal to an irrational number. Our assumption that $5 + 3\sqrt{2}$ is rational wrong.

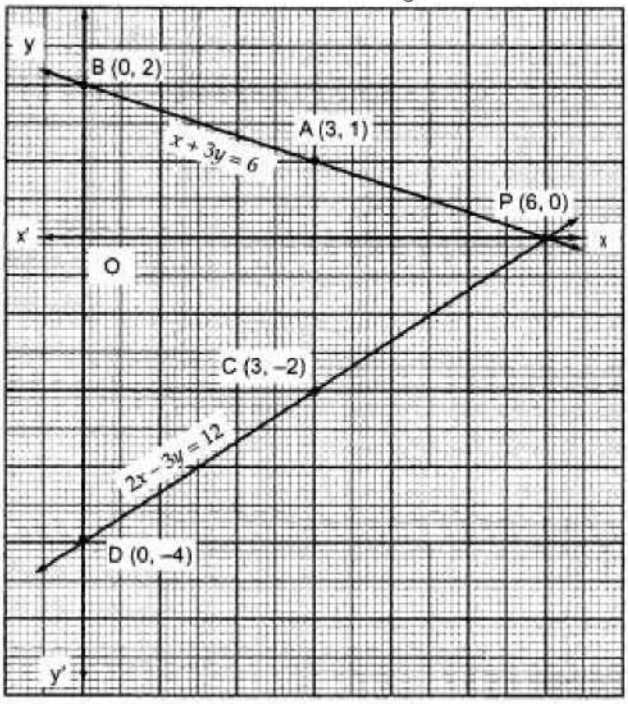
Hence, $5 + 3\sqrt{2}$ is irrational.

Section D

32. Graph of the equation $x + 3y = 6$:
We have, $x + 3y = 6 \ \Rightarrow x = 6 - 3y$
When y = 1, we have x = 6 - 3 =3
When y = 2, we have x = 6 - 6 = 0
Thus, we have the following table:

x	3	0
y	1	2

Plotting the points $A(3, 1)\ and\ B(0, 2)$ and drawing a line joining them, we get the graph of the equation x + 3y = 6 as shown in Fig.
Graph of the equation $2x - 3y = 12$:
We have, $2x - 3y = 12 \Rightarrow y = \frac{2x-12}{3}$
When x=3, we have $y = \frac{2\times3-12}{3} = -2$



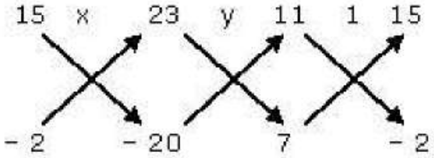
When x=0, we have $y = \frac{0-12}{3} = -4$

x	3	0
y	-2	-4

Plotting the points $C(3, -2)\ and\ D(0, -4)$ on the same graph paper and drawing a line joining them, we obtain the graph of the equation $2x - 3y = 12$ as shown in Fig.
Clearly, two lines intersect at P(6, 0).
Hence, $x = 6, y = 0$ is the solution of the given system of equations.
Putting x = 6, y = 0 in $a = 4x + 3y$, we get
a = (4 × 6) + (3 × 0) = 24

OR

The given system of equations is
11x + 15y + 23 = 0 ....(1)
7x - 2y - 20 = 0 ....(2)
To solve the equations (1) and (2) by cross multiplication method,
we draw the diagram below:



Then,

$\Rightarrow \frac{x}{(15)(-20)-(-2)(23)} = \frac{y}{(23)(7)-(-20)(11)} = \frac{1}{(11)(-2)-(7)(15)}$

$\Rightarrow \frac{x}{-300+46} = \frac{y}{161+220} = \frac{1}{-22-105}$

$\Rightarrow \frac{x}{-254} = \frac{y}{381} = \frac{1}{-127}$

$\Rightarrow x = \frac{-254}{-127} = 2$ and $y = \frac{381}{-127} = -3$

Hence, the required solution of the given pair of equations is

x = 2, y = -3

Verification: Substituting x = 2, y = -3,

We find that both the equations (1) and (2) are satisfied as shown below:

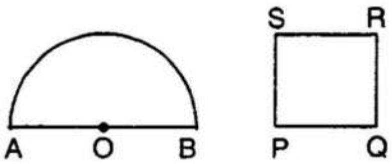
11x + 15y + 23 = 11(2) + 15(-3) + 23

= 22 - 45 + 23 = 0

7x - 2y - 20 = 7(2) - 2(-3) - 20

= 14 + 6 - 20 = 0

Hence, the solution we have got is correct.

33. 

Let radius of semicircular region be r units.

Perimeter = 2r + πr

Let side of square be x units

Perimeter = 4x units.

A.T.Q, 4x = 2r + πr $\Rightarrow x = \frac{2r+\pi r}{4}$

Area of semicircle = $\frac{1}{2}\pi r^2$

Area of square = x^2

A.T.Q, $x^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \left(\frac{2r+\pi r}{4}\right)^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \frac{1}{16}\left(4r^2 + \pi^2 r^2 + 4\pi r^2\right) = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow 4r^2 + \pi^2 r^2 + 4\pi r^2 = 8\pi r^2 + 64$

$\Rightarrow 4r^2 + \pi^2 r^2 - 4\pi r^2 = 64$

$\Rightarrow r^2\left(4 + \pi^2 - 4\pi\right) = 64$

$\Rightarrow r^2(\pi - 2)^2 = 64$

$\Rightarrow r = \sqrt{\frac{64}{(\pi-2)^2}}$

$\Rightarrow r = \frac{8}{\pi-2} = \frac{8}{\frac{22}{7}-2}$ = 7 cm

Perimeter of semicircle =$2 \times 7 + \frac{22}{7} \times 7$ = 36 cm

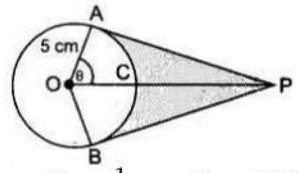
Perimeter of square = 36 cm

Side of square = $\frac{36}{4}$= 9 cm

Area of square = 9× 9 = 81 cm^2

Area of semicircle = $\frac{\pi r^2}{2} = \frac{22}{2\times7} \times 7 \times 7$ = 77 cm^2

OR



$\cos\theta = \frac{1}{2}$ or, $\theta = 60^\circ$

Reflex $\angle AOB = 120^\circ$

∴ ADB = $\frac{2\times3.14\times5\times240}{360}$ = 20.93 cm

Hence length of elastic in contact = 20.93 cm

Now, AP = $5\sqrt{3}$cm

a (△OAP) = $\frac{1}{2}\times$ base × height = $\frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$

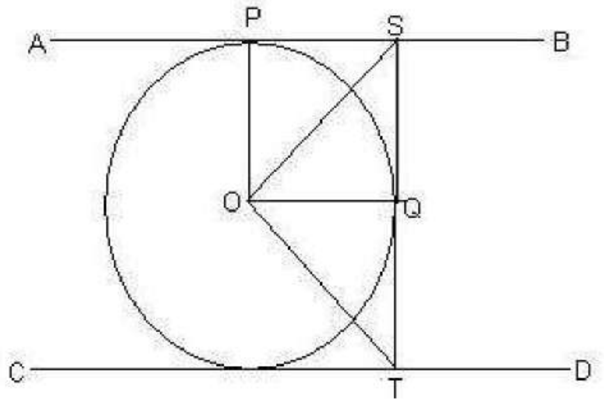
Area (△OAP + △OBP) = 2× $\frac{25\sqrt{3}}{2} = 25\sqrt{3}$ = 43.25 cm^2

Area of sector OACB = $\frac{\theta}{360} \times \pi r^2$

= $\frac{25\times3.14\times120}{360}$ = 26.16 cm^2

Shaded Area = 43.25 - 26.16 = 17.09 cm^2

34. Given, AB and CD are two parallel tangents to a circle with centre O.



From the figure we get,

AB⊥ST then ∠ASQ = 90° and

CD⊥TS then ∠CTQ = 90°

∠ASO = ∠QSO = $\frac{90^\circ}{2}$= 45°

Similarly, ∠OTQ = 45°

Consider ΔSOT,

∠OTS = 45° and ∠OST = 45°

∠SOT + ∠OTS + ∠OST = 180° (angle sum property)

∠SOT = 180° - (∠OTS + ∠OST) = 180° - (45° +45°)

= 180° - 90° = 90°

∴ ∠SOT = 90^o

35. Total favourable outcomes associated to the random experiment of visiting a particular shop in the same week (Tuesday to Saturday) by two customers Shyam and Ekta are:

(T, T) (T, W) (T, TH) (T, F) (T, S)

(W, T) (W, W) (W, TH) (W, F) (W, S)

(TH, T) (TH, W) (TH, TH) (TH, F) (TH, S)

(F, T) (F, W) (F, TH) (F, F) (F, S)
(S, T) (S, W) (S, TH) (S, F) (S, S)
∴Total number of favourable outcomes = 25

i. The favourable outcomes of visiting on the same day are (T, T), (W, W), (TH, TH), (F, F) and (S, S).
∴Number of favourable outcomes = 5
Hence required probability $= \frac{Number\ of\ favorable\ outcomes}{Number\ of\ total\ outcomes} = \frac{5}{25} = \frac{1}{5}$

ii. The favourable outcomes of visiting on consecutive days are (T, W), (W, T), (W, TH), (TH, W), (TH, F), (F, TH), (S, F) and (F, S).
∴ Number of favourable outcomes = 8
Hence required probability $= \frac{Number\ of\ favorable\ outcomes}{Number\ of\ total\ outcomes} = \frac{8}{25}$

iii. Number of favourable outcomes of visiting on different days are 25 - 5 = 20
∴Number of favourable outcomes = 20
Hence required probability $= \frac{Number\ of\ favorable\ outcomes}{Number\ of\ total\ outcomes} = \frac{20}{25} = \frac{4}{5}$

Section E

36. **Read the text carefully and answer the questions:**
Suman is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 360 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) Let there be 'n' number of rows
Given 3, 5, 7... are in AP
First term a = 3 and common difference d = 2
$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow 360 = \frac{n}{2}[2 \times 3 + (n-1) \times 2]$
$\Rightarrow 360 = n[3 + (n - 1) \times 1]$
$\Rightarrow n^2 + 2n - 360 = 0$
$\Rightarrow (n + 20)(n - 18) = 0$
$\Rightarrow n = -20$ reject
n = 18 accept

(ii) Since there are 18 rows number of candies placed in last row (18^{th} row) is
$a_n = a + (n - 1)d$
$\Rightarrow a_{18} = 3 + (18 - 1)2$
$\Rightarrow a_{18} = 3 + 17 \times 2$
$\Rightarrow a_{18} = 37$

OR

The number of candies in 12th row.
$a_n = a + (n - 1)d$
$\Rightarrow a_{12} = 3 + (12 - 1)2$
$\Rightarrow a_{12} = 3 + 11 \times 2$
$\Rightarrow a_{12} = 25$

(iii) If there are 15 rows with same arrangement
$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 3 + (15 - 1) \times 2]$

$\Rightarrow S_{15} = 15[3 + 14 \times 1]$

$\Rightarrow S_{15} = 255$

There are 255 candies in 15 rows.

37. **Read the text carefully and answer the questions:**
Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Volume of Hermika = side^3 = $10 \times 10 \times 10$ = 1000 m^3

(ii) r = radius of cylinder = 24, h = height = 16

Volume of cylinder = $\pi r^2 h$

$\Rightarrow V = \frac{22}{7} \times 24 \times 24 \times 14 = 25344$ m^3

OR

Since Anda is hespherical in shape r = radius = 21

V = Volume of Anda = $\frac{2}{3} \times \pi \times r^3$

$\Rightarrow V = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$\Rightarrow V = 44 \times 21 \times 21 = 19404$ m^3

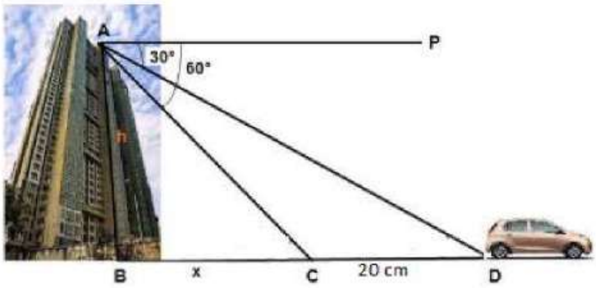
(iii) Volume of brick = 0.01 m^3

$\Rightarrow$ n = Number of bricks used for making cylindrical base = $\frac{\text{Volume of cylinder}}{\text{Volume of one brick}}$

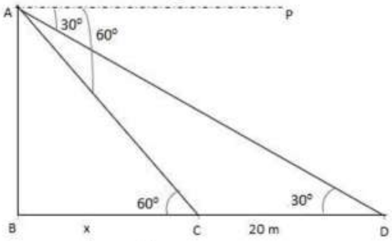
$\Rightarrow n = \frac{25344}{0.01} = 2534400$

38. **Read the text carefully and answer the questions:**
Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60°. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to

30^{o}.



(i) The above figure can be redrawn as shown below:



From the figure,

let AB = h and BC = x

In $\triangle$ABC,

$\tan 60 = \frac{\text{AB}}{\text{BC}} = \frac{\text{h}}{x}$

$\sqrt{3} = \frac{\text{h}}{x}$

h = $\sqrt{3}$x ...(i)

In $\triangle$ABD,

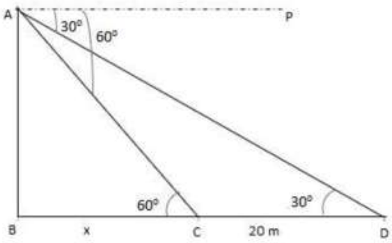
$\tan 30 = \frac{\text{AB}}{\text{BD}} = \frac{\text{h}}{x+20}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{x+20}$ [using (i)]

x + 20 = 3x

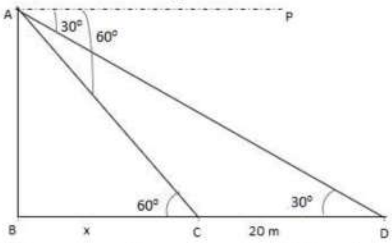
x = 10 m

(ii) The above figure can be redrawn as shown below:



Height of the building, h = $\sqrt{3}$x = $10\sqrt{3}$ = 17.32 m

(iii)The above figure can be redrawn as shown below:



Distance from top of the building to point D.

In $\triangle$ABD

$\sin 30^{o} = \frac{AB}{AD}$

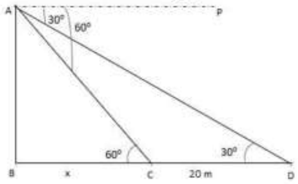
$\Rightarrow AD = \frac{AB}{\sin 30^{0}}$

$\Rightarrow AD = \frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}m$

OR

The above figure can be redrawn as shown below:



Distance from top of the building to point C is

In $\triangle$ABC

sin $60^{o} = \frac{AB}{AC}$

$\Rightarrow AC = \frac{AB}{\sin 60^{0}}$

$\Rightarrow AC = \frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AD = 20 m