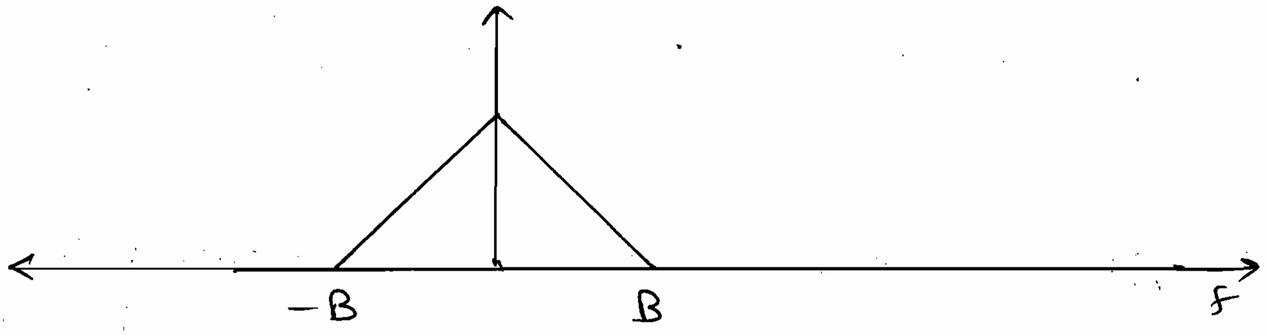
④ Vestigial Side Band Modulation: (VSB).

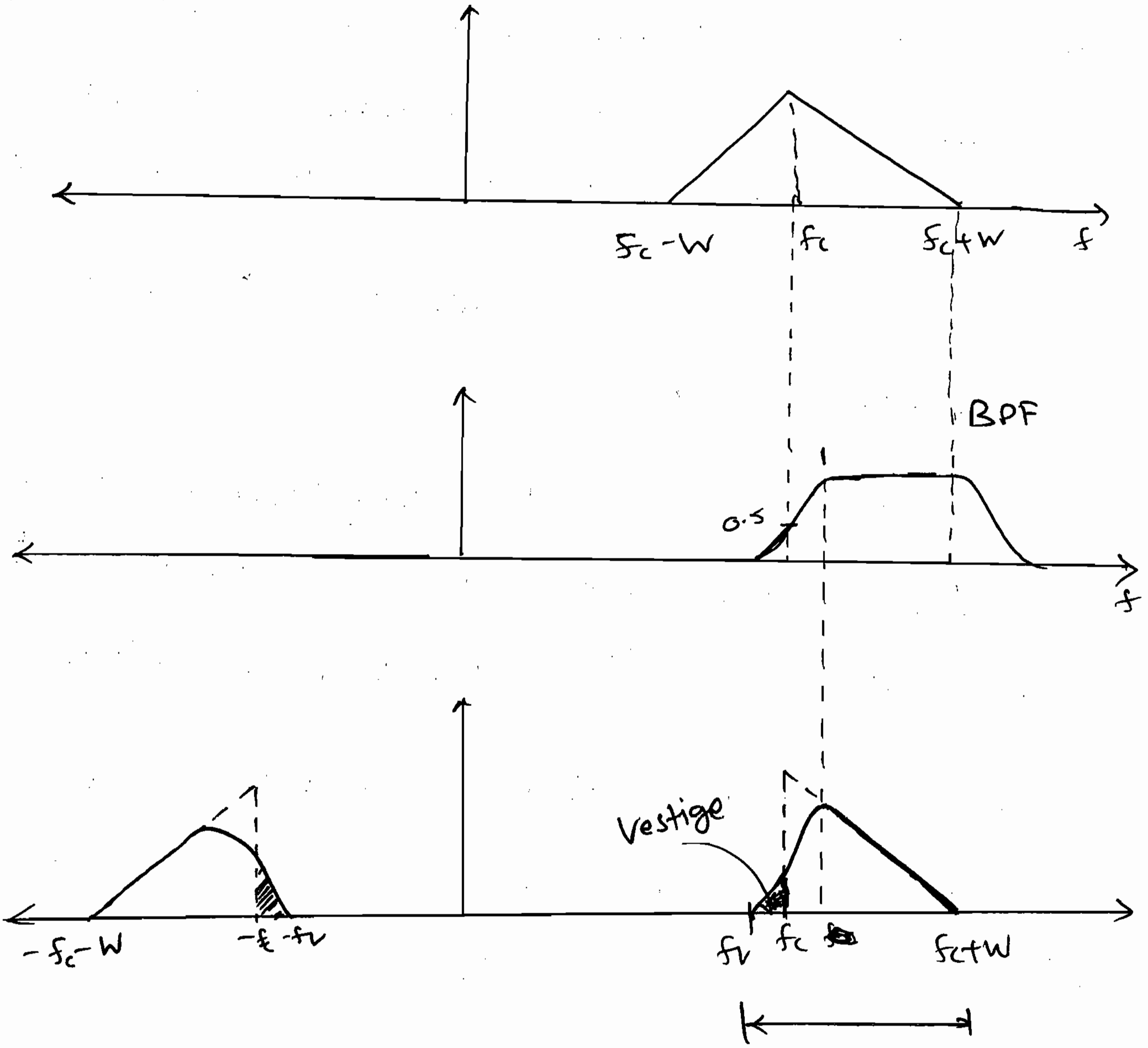
→ Video signal consist of significant freq. upto 4.5 MHz. If AM (or) DSB is used the BW is 9 MHz and it is not possible to transmit more no. of signals. So AM and DSB are not used for video signal transm.

→ SSB modulation is suitable for voice transmission only.

→ The Generation and Demodulation technique of VSB are same as SSB except few modification at the Band pass filter.

→ The Bandpass filter should be designed so that the gain is not constant in the entire upper side band.





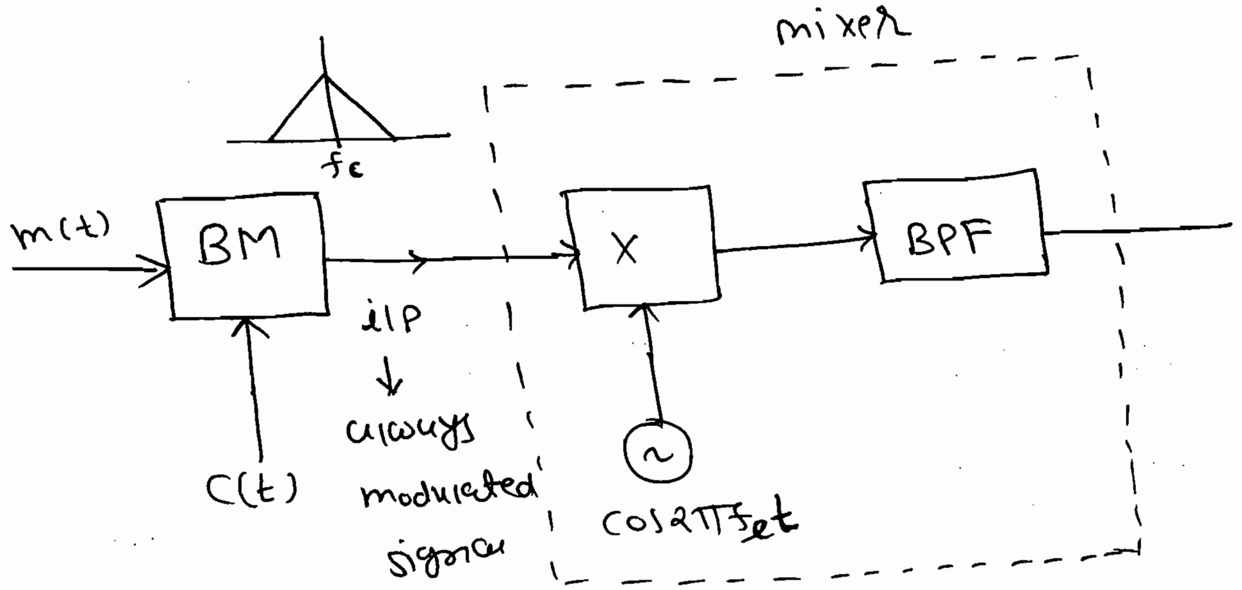
$BW = W + f_v.$

⇒ BW of Vestige depends on design of BPF.

* Mixer: $\boxed{H.B.}$

→ Mixing (or) Heterodyning.

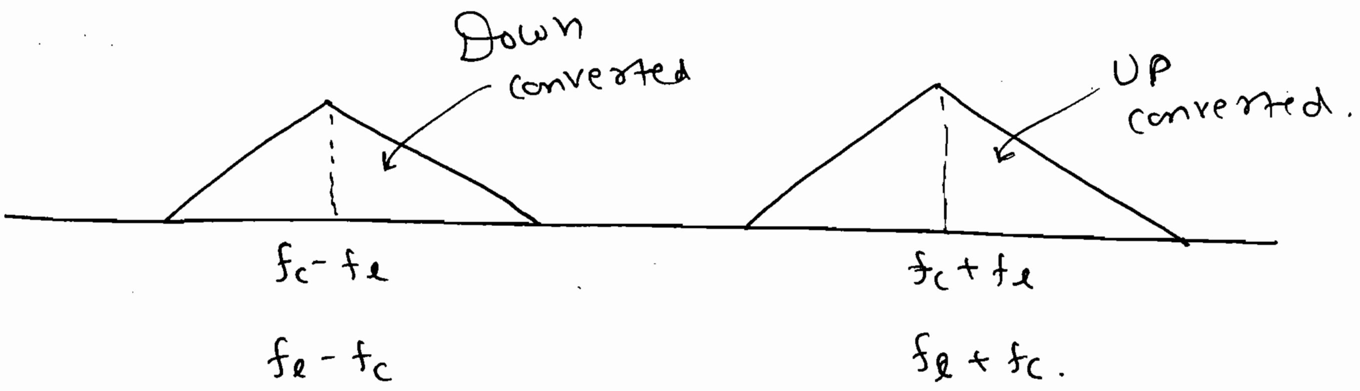
→ Mixer is used to change the carrier frequency of a modulated signal.



$\Rightarrow$ o/p of multiplier = $m(t).\cos 2\pi f_c t.\cos 2\pi f_l t.$

① Condition - ⊛ :- $f_c > f_l$ ② Condition: $f_l > f_c$

$\Rightarrow \frac{1}{2}\left[m(t)\cos 2\pi(f_c+f_l)t + m(t)\cos 2\pi(f_c-f_l)t\right].$



⊛

☆ Angle Modulation

⇒ The generalized eqn of the carrier is

$$\overset{\overset{Am}{\downarrow}}{A_c} \cos(\underbrace{2\pi \overset{\overset{FM}{\downarrow}}{f_c} t + \overset{\overset{PM}{\downarrow}}{\phi}}_{\theta(t)}).$$

↙ H.B

⇒ $A_c \cos(\theta(t))$.

→ "$\underline{2\pi f_c t + \phi}$" is called as the angle of the carrier.

⇒ Whenever the frequency (or) the phase changes angle also changes. So FM & PM are also called as Angle modulation Technique.

* Frequency Modulation:-

Definition:

⇒ FM is defined as the process in which the frequency of the carrier is varied according to the message signal.

⇒ Freq. of Carrier before modulation is f_c and after modulation f_i where

⇒ $\boxed{f_i = f_c + k_f m(t)}$ ← H.B.

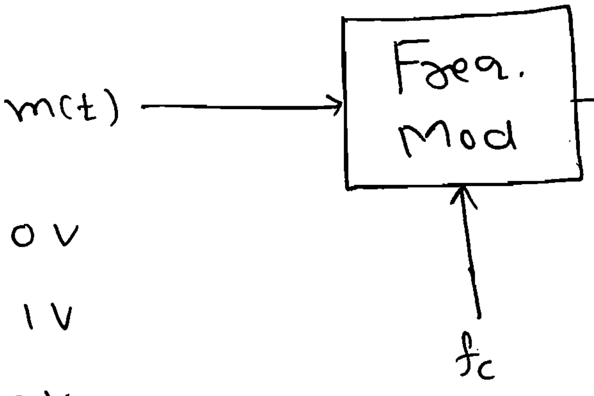
K_f = Frequency Sensitivity of modulator $\left(\frac{Hz}{Volt}\right)$.

⇒ If the Carrier is not modulated $m(t)=0$

and $\boxed{f_i = f_c}$

→ 'k_f' indicate the change in the freq. per 1V change of the message signal.

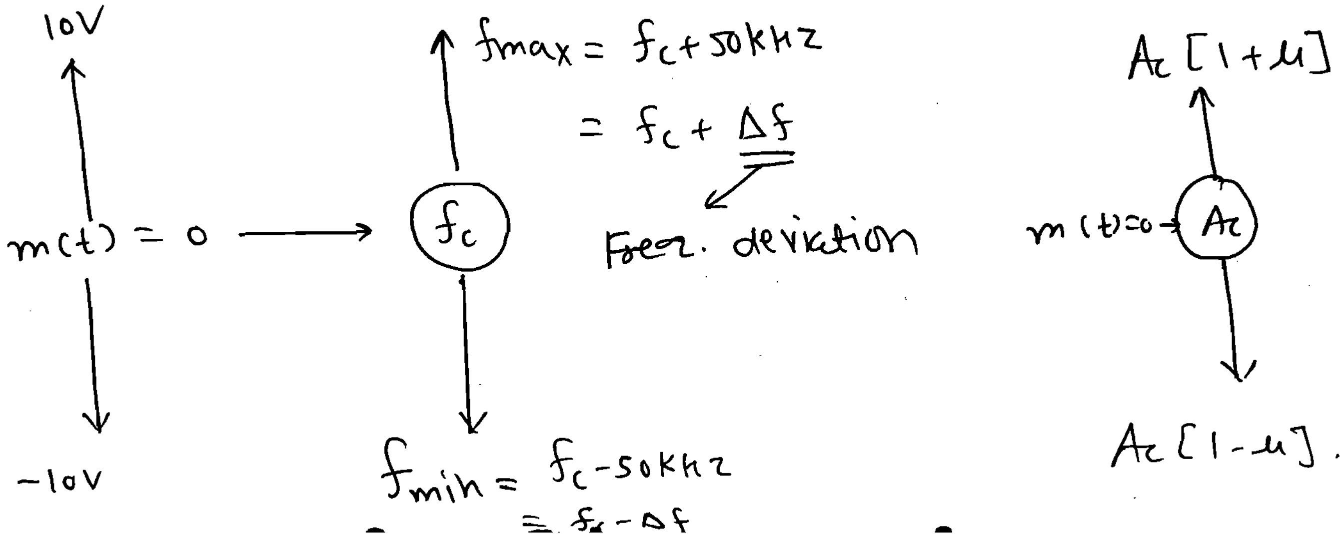
⇒ $K_f = 5\ kHz/volt$



$\rightarrow f_i = f_c + K_f m(t)$.

$= 0V$	$f_i = f_c$
$= 1V$	$f_i = f_c + 5kHz$.
$= 2V$	$f_i = f_c + 10kHz$
$= 5V$	$f_i = f_c + 25kHz$
$= 10V$	$f_i = f_c + 50kHz$
$= -10V$	$f_i = f_c - 50kHz$.

→ FM modulator is a Voltage to Frequency converter.



$f_{max} = f_c + 50kHz$
$= f_c + \underline{\Delta f}$

$f_{min} = f_c - 50kHz$
$= f_c - \Delta f$

$\Rightarrow$ In the case of single tode modulation,

$m(t) = A_m \cos 2\pi f_m t.$

$\rightarrow f_i = f_c + K_f m(t).$

$f_i = f_c + K_f A_m \cos 2\pi f_m t.$

$\rightarrow f_{max} = f_c + K_f A_m. = f_c + \Delta f.$

$\rightarrow f_{min} = f_c - K_f A_m. = f_c - \Delta f.$

$\Rightarrow \Delta f = K_f A_m.$ ✓

Generalized formula. ↙ H.B.

$$\therefore \boxed{\Delta f = K_f [m(t)]_{max}}$$

$\Rightarrow A_c \cos(\theta(t)).$

$\rightarrow \theta(t) = \omega_i t$

$\therefore \theta(t) = 2\pi f_i t.$

$\therefore f_i \ \frac{d}{dt}\theta(t) = 2\pi f_i$

↙ H.B

$$\therefore \boxed{f_i = \frac{1}{2\pi}\frac{d}{dt}(\theta(t)).}$$

$\Rightarrow \theta(t) = 2\pi \int f_i \, dt$

$= 2\pi \int [f_c + K_f m(t)] \, dt.$

$$\boxed{\theta(t) = 2\pi f_c t + 2\pi k_f \int m(t)\,dt.} \leftarrow \underline{\text{H.B.}}$$

→ The above eqn represent the angle in the case of FM.

→ So, Time domain eqn of FM for <u>multitone</u> Modulation is,

$$\therefore \boxed{S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right].} \swarrow \underline{\text{H.B.}}$$

↑ multitone

→ for single tone modulation

let, $m(t) = A_m \cos 2\pi f_m t.$

$$\therefore S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int A_m \cos 2\pi f_m t\right].$$

$$= A_c \cos\left[2\pi f_c t + \frac{2\pi k_f \cdot A_m}{2\pi f_m} \cdot \sin 2\pi f_m t\right].$$

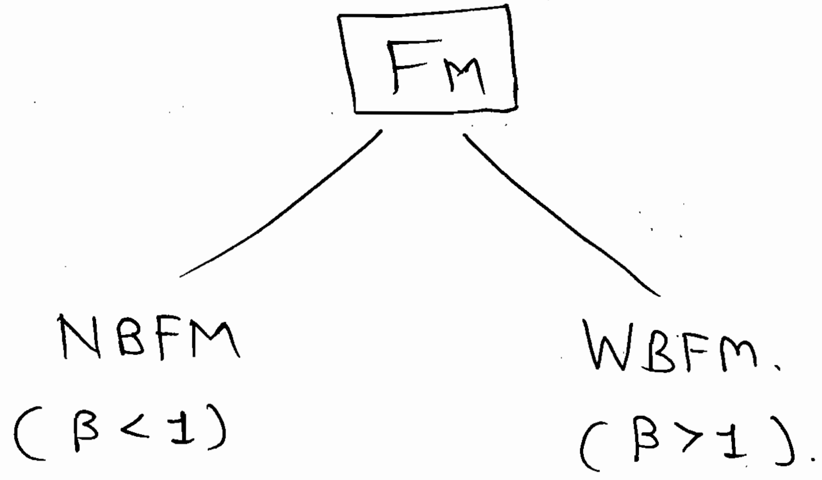
$$\therefore S(t) = A_c \cos\left[2\pi f_c t + \left(\frac{k_f A_m}{f_m}\right) \sin 2\pi f_m t\right].$$

$$\therefore \boxed{s(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right].}$$

↑ single tone.

$$\Rightarrow \boxed{\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m} = \text{Modulation index.} = \frac{\text{Freq. deviation}}{\text{message freq.}}}$$

⇒



→ In AM, modulation index concept is used for the demodulation and when envelope Detector is used as demodulator.

→ In DSB, SSB and VSB, modulation index concept is not used because the envelope detector is not used as demodulator.

→ In FM, Modulation index concept is required for anulyzing the Spectrum.

① Narrow Band FM (NBFM) : $\boxed{\beta < 1}$

$$\rightarrow \quad S(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right].$$

$$= A_c \cos 2\pi f_c t \cdot \cos\left[\beta \cdot \sin 2\pi f_m t\right]$$

$$- A_c \sin 2\pi f_c t \cdot \sin\left[\beta \sin 2\pi f_m t\right].$$

let, $\theta = \beta \sin 2\pi f_m t.$

$$\therefore S(t) = A_m \cos 2\pi f_c t \cdot \cos\theta - A_c \sin 2\pi f_c t \cdot \sin\theta .$$

$\theta < 1.$

$\cos\theta \simeq 1$

$\sin\theta \simeq \theta.$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$\Rightarrow \lim_{\theta \to 0} \sin\theta = \theta$

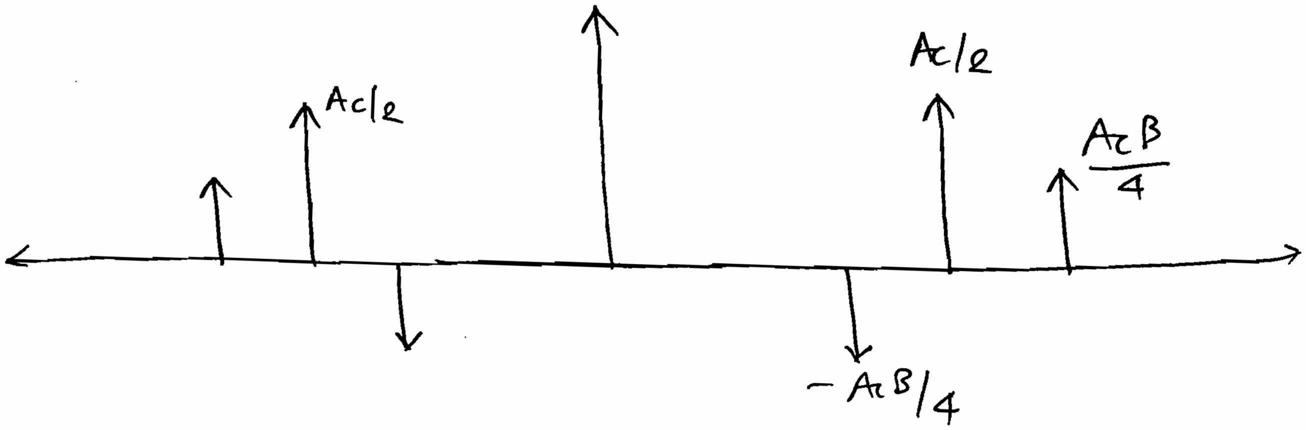
$$\therefore S(t) = A_c \overset{\cos}{} 2\pi f_c t \cdot \underline{\underline{1}} - A_c \sin 2\pi f_c t \cdot \underline{\underline{\theta}}.$$

H.B. ↓

$$\therefore \boxed{S(t) = A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t.}$$

$$\therefore \boxed{S(t) \simeq A_c \cos 2\pi f_c t + \frac{A_c\beta}{2} \cos 2\pi (f_c + f_m) t - \frac{A_c\beta}{2} \cos 2\pi (f_c - f_m) t.} \leftarrow \underline{\text{H.B}}$$

$\Rightarrow$



→ The LSB in the spetrum is out of Phase by 180°.

Ex: (1) An Am signal and NBFm signal having Same modulation index are added, the resultant signal is,

Ⓐ Am

Ⓑ DSB

Ⓒ SSB

[✓ Ⓓ SSB With Carrier.]

Ans: LSB = $\frac{A_c\mu}{4}$, LSB = $-\frac{A_c\beta}{4}$

$\mu = \beta$ so, ans Ⓓ SSB with Carrier.

⇒ The magnitude Spectrum of NBFm is Same as Am. So, BW and power are also Same as Am.

$$\rightarrow \boxed{P_t = P_c\left[1 + \frac{\beta^2}{2}\right].} \leftarrow \text{M.B}$$

NOTE:

⇒ Practically NBFm is not used because of its Similarity to Am.

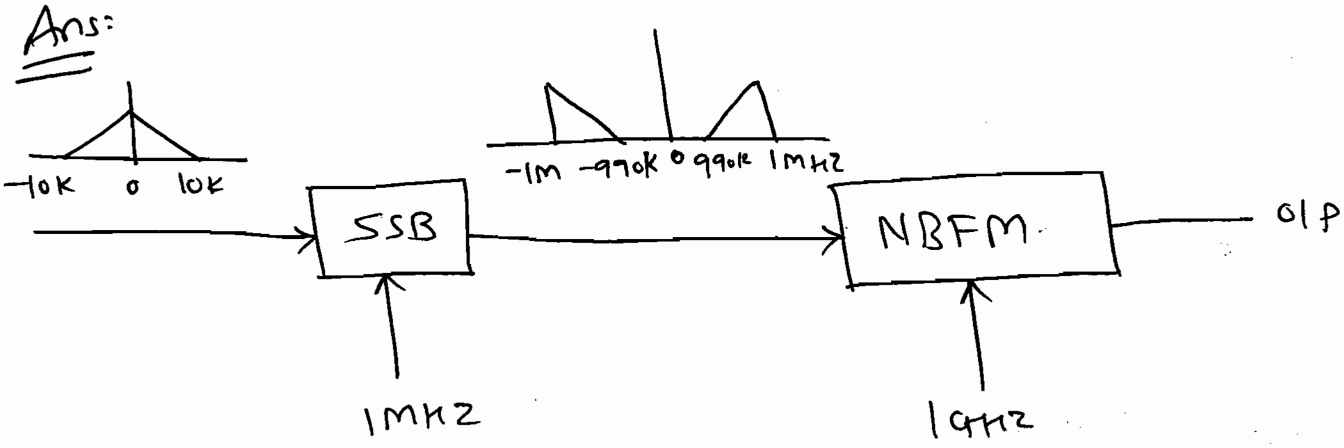
Gate: 2006:

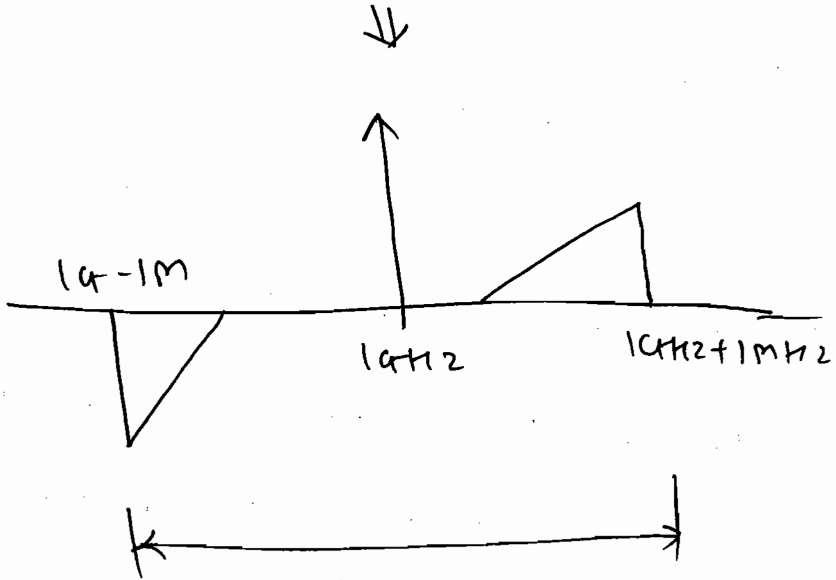
Ex- 2: A message signal bandlimited to 10 KHz is lower sideband SSB modulated with a carrier frequency of 1 MHz. The resultant signal is again passed through NBFM modulator having a carrier freq. of 1 GHz. Determine the BW of signal at O/P.

Ⓐ 10 KHz Ⓑ 20 KHz

Ⓒ 1 MHz [✓ Ⓓ] 2 MHz.

Ans:





2 MHz
= BW.

② Wide Band FM : (WBFM) : $(\beta > 1)$

⇒ Time domain eqn for the single tone modulation is

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t].$$

*** Bessel Function:**

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(x\sin\theta - n\theta)} \cdot d\theta$$

⇒ Properties:

① $J_n(x) = (-1)^n J_{-n}(x).$

② $\sum_{n=-\infty}^{\infty} J_n^2(x) = 1. \leftarrow$ H.B

⇒ Time domain eqn of WBFM is ↙ H.B

$$\boxed{s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \cos[2\pi(f_c + n f_m)t]}$$

$\therefore s(t) = A_c J_0(\beta) \cdot \cos 2\pi f_c t$

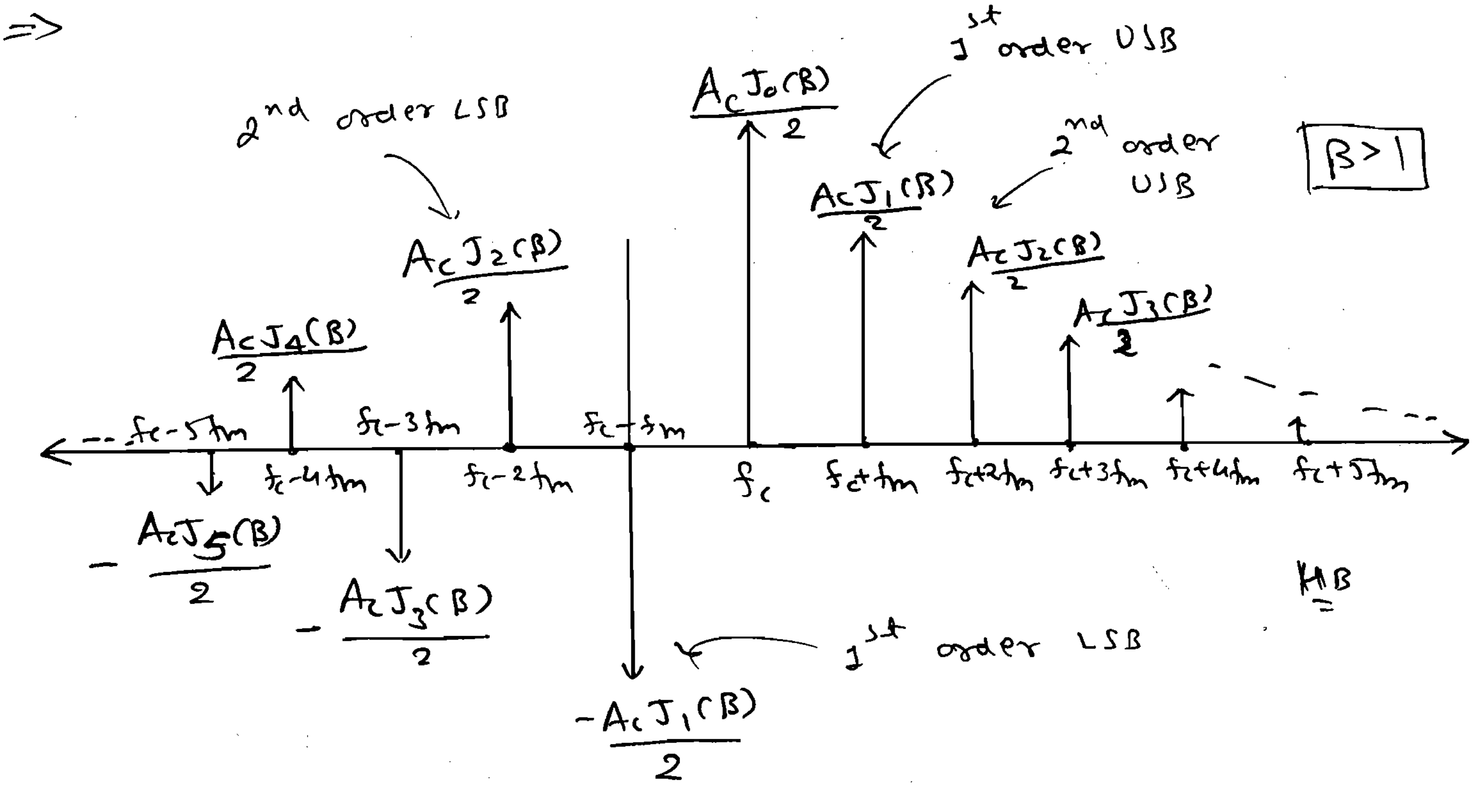
$+ A_c J_1(\beta) \cdot \cos 2\pi (f_c + f_m)t + A_c J_{-1}(\beta) \cdot \cos 2\pi (f_c - f_m)t$

$+ A_c J_2(\beta) \cdot \cos 2\pi (f_c + 2f_m)t + A_c J_{-2}(\beta) \cdot \cos 2\pi (f_c - 2f_m)t$ +

$+ A_c J_3(\beta) \cdot \cos 2\pi (f_c + 3f_m)t + A_c J_{-3}(\beta) \cdot \cos 2\pi (f_c - 3f_m)t$

$+ \dots$

=>



* Analysis of spectrum:

=> The spectrum consist of carrier and infinite no. of upper and lower sideband frequencies.

=> Theoretical BW is infinite.

=> The magnitude of the spectral component depends on the Bessel function coefficient. But Bessel function's values gradually decreases as n increases. So, the magnitude of higher order of frequencies are negligible.

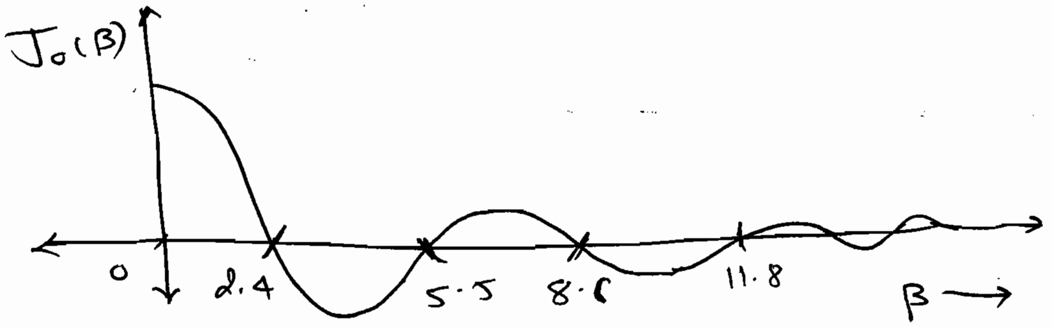
=> The carrier magnitude in the spectrum varies with modulation index.

=> The Bessel function coefficient $J_0(\beta)=0$ where $\beta = 2.4, 5.5, 8.6, \ldots$

⇒ The Bessel function coefficient

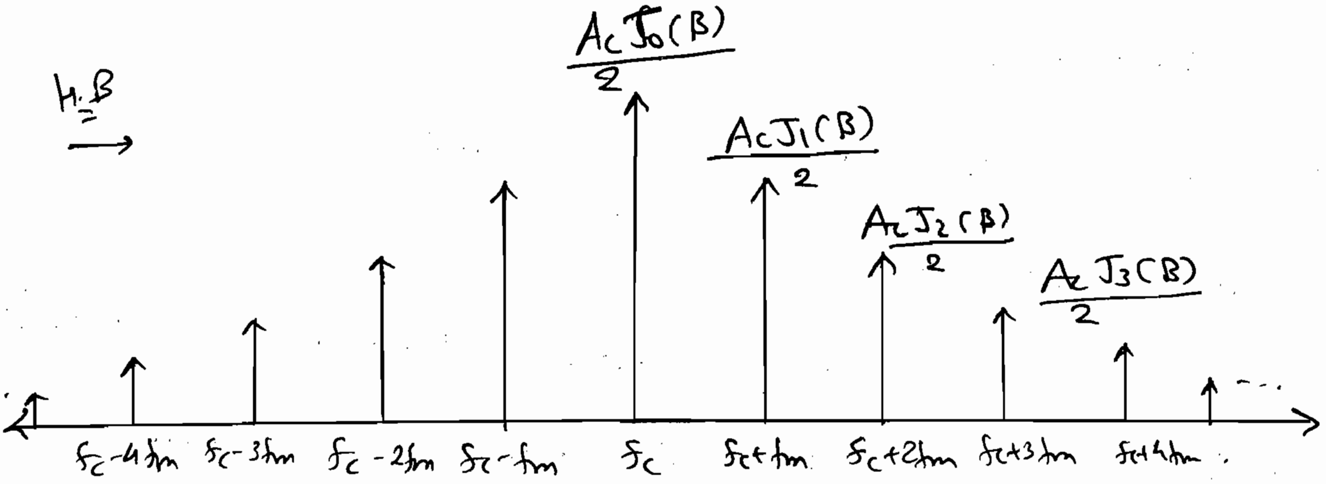
$\boxed{J_0(\beta) = 0.}$ ← H.B.

where $\beta = 2.4, 5.5, 8.6, \ldots$



⇒ For these values of β, the carrier will zero magnitude in the spectrum and modulation efficiency is 100%.

H.B →



[Magnitude Spectrum].

* Power Calculation: H.B.

⇒ $$P = \frac{V_{rms}^2}{R}$$

$$\therefore P_{f_c} = \frac{\left(\frac{A_c J_0(\beta)}{\sqrt{2}}\right)^2}{R}.$$

$$\therefore P_{f_c} = \frac{A_c^2 J_0^2(\beta)}{2R}.$$

→ $$\left.\begin{aligned} P_{f_c+f_m} &= \frac{A_c^2}{2R} J_1^2(\beta) \\ P_{f_c-f_m} &= \frac{A_c^2}{2R} J_1^2(\beta) \end{aligned}\right\}$$ first order SB power $= \frac{A_c^2 J_1^2(\beta)}{R}$

→ $$\left.\begin{aligned} P_{f_c+2f_m} &= \frac{A_c^2}{2R} J_2^2(\beta) \\ P_{f_c-2f_m} &= \frac{A_c^2}{2R} J_2^2(\beta) \end{aligned}\right\}$$ 2nd order SB power $= \frac{A_c^2 J_2^2(\beta)}{R}$.

$$\therefore P_t = \frac{A_c^2}{2R}\left[\sum_{n=-\infty}^{\infty} J_n^2(\beta)\right]$$

$$\therefore P_t = \frac{A_c^2}{2R} \cdot 1$$

✓

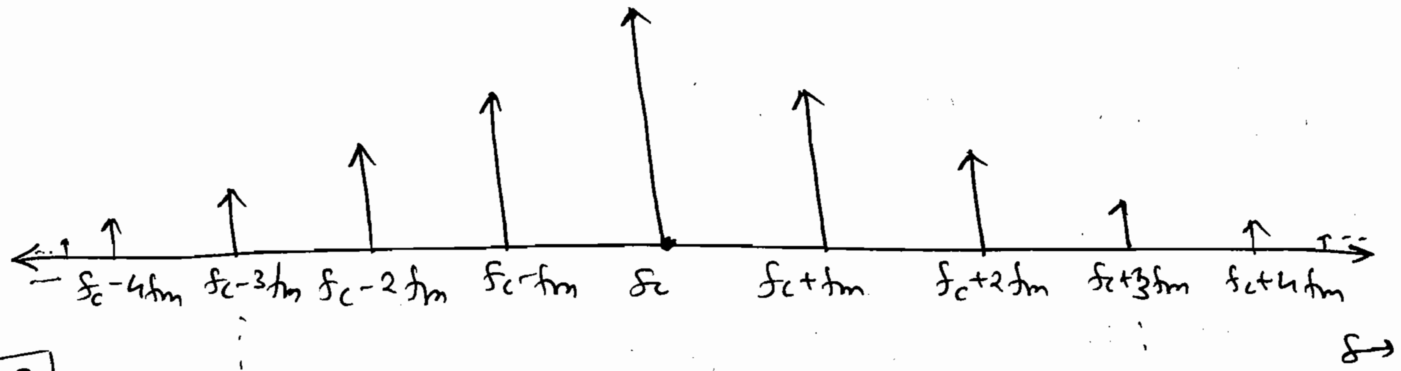
$$\therefore \boxed{P_t = \frac{A_c^2}{2R}.}$$ → same as Unmodulated carrier power [$A_c \cos 2\pi f_c t$].

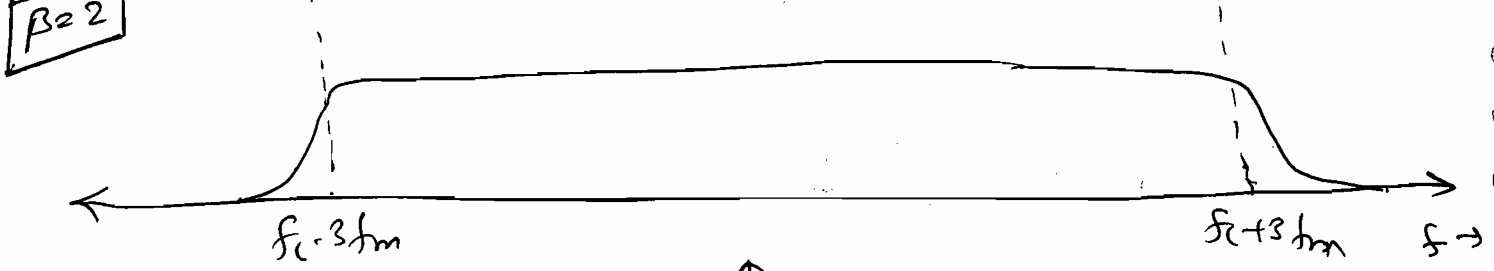
* Transmission B.W. of FM signal using Carson's Rule:

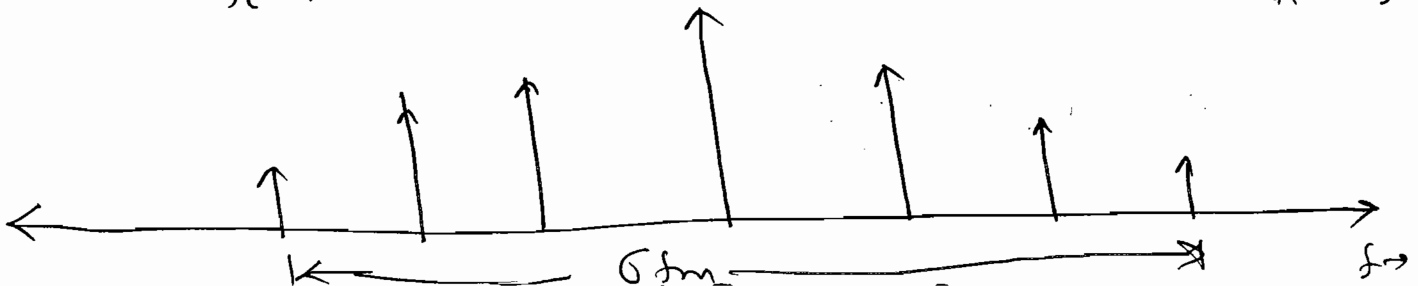
⇒ The theoretical B.W. of FM signal is ∞.

⇒ Practically BW of the signal should be as minimum as possible. So insignificant frequencies should be eliminated.

⇒ According to Carson's rule $\beta+1$ Upper and Lower sidebands will have significant magnitude and contains 99% of the total power. So, the FM signal is passed through a Band pass filter to eliminate the insignificant frequencies.







⇒ **Carson's rule:** H.B

$$\boxed{B.W. = 2(\beta+1) f_m} \quad — ①$$

$$= 2\left(\frac{\Delta f}{f_m}+1\right) f_m$$

H.B.

$$\boxed{B.W. = 2\Delta f + 2 f_m} \quad — ②$$

Ex-1 A carrier signal is frequency modulated by a sinusoidal signal of amplitude 20V and freq. 100 KHZ. The freq. sensitivity of modulator is 25 KHZ/V.

(i) Determine the freq. deviation, modulation index & BW.

(ii) Repeat the above calculation when the amplitude of the message signal is 40V.

Ans: $A_m = 20V$, $f_m = 100$ KHZ, $k_f = 25$ KHZ/V.

(i) $\Delta f = K_f \cdot A_m = 20 \times 25 = 500$ KHZ.

$$\therefore \beta = \frac{\Delta f}{f_m} = \frac{500 \text{ KHZ}}{100 \text{ KHZ}} = 5.$$

$\therefore \beta = 5.$

$BW = 2(\beta+1) f_m.$

$\therefore \quad BW = 2(6) \times 100\,kHz.$

$= 1200\,KHz.$

$\therefore \quad \boxed{BW = 1.2\,MHz}$

(ii) When $A_m = 40V$

$\Delta f = K_f \cdot A_m = 25\,KHz/V \times 40V$

$\Delta f = 1000\,KHz$

$\therefore \quad \beta = \frac{\Delta f}{f_m} = \frac{1000\,KHz}{100\,KHz} = 10.$

$\therefore \quad BW = 2(\beta+1) f_m$

$= 2 \times 11 \times 100\,KHz$

$\boxed{BW = 2.2\,MHz.}$

Ex-2 Consider the Fm signal

$s(t) = 10 \cos[2\pi 10^6 t + 8 \sin 4\pi 10^3 t]$

(i) Determine Modulation Index, Freq. deviation, BW and power.

(ii) Repeat the above calculation when the message freq. is doubled.

Ans: $A_c = 10$, $f_c = 10^6$, $\beta = 8$, $f_m = 2 \times 10^3$ Hz.

(i) $\boxed{\beta = 8}$

$\therefore \ \beta = \dfrac{\Delta f}{f_m}$

$\therefore \ \Delta f = \beta \cdot f_m$

$\Delta f = 8 \times 2 \text{KHz} = 16 \text{KHz}.$

$\therefore \ BW = 2(\beta+1) f_m = 2(9) \times 2 \text{KHz}$

$= ~~8\cdot 8~~ \ 36 \text{ KHz}.$

$\therefore \ P_t = \dfrac{A_c^2}{2R} = \dfrac{100}{2 \times 1} = 50W.$

(ii)

~~$A_m' = 2A_m$.~~

$\therefore$

$\Rightarrow$ when, $f_m' = 2 f_m$.

$\therefore \ \beta = 4$ as $\beta = \dfrac{k_f \cdot A_m}{f_m}$.

so, $\Delta f = 4 \times 4 \text{kHz} = 16 \text{KHz}.$

$BW = 2(4+1) \, 4 \text{KHz}$

$BW = 40 \text{ kHz}$

$\therefore \ \boxed{P_t = 50W.}$

Ex-3 A Carrier signal is frequency modulated by a sinusoidal signal and the frequency deviation is 8 KHz. The message freq. is 2 KHz.

i) Determine the modulation index and BW.

ii) Repeat the above calculation when the amplitude of the message signal is increased by a factor of 3 and its freq. is reduced to 1 KHz.

Ans: $\Delta f = 8\text{ KHz}$, $f_m = 2\text{ KHz}$.

$\therefore$ (i) $$\beta = \frac{\Delta f}{f_m} = \frac{8\text{ KHz}}{2\text{ KHz}} = 4.$$

$$\therefore \quad BW = 2(\beta+1) f_m = 2(5) \times 2\text{ KHz}$$
$$= 20\text{ KHz}.$$

(ii) When $A_m' = 3A_m$. and $\Rightarrow$ ~~A_m'~~

$f_m' = f_m - 1\text{KHz} = f_m/_2 = 1\text{ KHz}.$

$$\therefore \quad \Delta f = K_f . A_m'$$

$$\therefore \quad \Delta f = 3 \times 8 = 24 \text{ KHz}.$$

$$\beta = \frac{24\text{ KHz}}{1\text{ KHz}} = 24.$$

$$BW = 2(\beta+1) f_m = 2(25) \times 1K = 50\text{ KHz}.$$

Ex-4 ** * A carrier signal is freq. modulated by a sinusoidal signal and freq. deviation is 50 KHZ. Determine the modulation index and BW when message freq. is (i) $f_m = 500$ KHz (ii) $f_m = 500$ Hz.

Ans:- $\Delta f = 50$ KHZ, $f_m = 500$ KHz

(i) $\beta = \frac{\Delta f}{f_m} = \frac{50 \text{ KHz}}{500 \text{ KHz}}$

$\boxed{\beta = 0.1}$ (NBFM)

So, $BW = 2 f_m$ (same as AM)

$\boxed{BW = 1 \text{ MHz}}$

(ii) $\beta = \frac{\Delta f}{f_m} = \frac{50 \text{ KHz}}{500} = 100 > 1$ (WBFM).

$\therefore$ $BW = 2(\beta + 1) f_m$

$= 2 \times 101 \times 500$

$\therefore \boxed{BW = 101 \text{ KHz.}}$

Ex-5 A carrier signal is freq. modulated by a sinusoidal signal and freq. deviation is 50 KHz. Det. the modulation index and BW when the message freq. is

(i) $f_m = 500$ KHz (ii) $f_m = 500$ Hz.

Ex-8 ** *A sinusoidal signal of 4 KHz is used as modulating signal for an Am and FM transmitters. Both of the transmitter uses the same carrier. The freq. deviation of the FM transmitter is 4 times the BW of the Am signal. The magnitudes of the freq. at $f_c + 4\,KHz$ are same in Am and FM. Determine the modulation index of Am & FM.

$J_1(2) = 0.577$, $J_1(4) = 0.06$, $J_1(8) = 0.235$.

Ans:

$f_m = 4\,KHz$

$\Delta f = 4 \times BW_{Am}$

$\Delta f = 4 \times 2 f_m$

$\therefore \Delta f = 8 f_m$

$\therefore \beta = \frac{\Delta f}{f_m}$

$\therefore \boxed{\beta = 8}$

$$\frac{\cancel{A_c} J_1(\beta)}{2} = \frac{\cancel{A_c}\mu}{4}$$

$$J_1(8) = \frac{\mu}{2}$$

$\therefore \mu = 2 \times 0.235$

$\boxed{\mu = 0.47}$

Ex-7 A carrier signal is modulated by message signal $m(t) = A_m \cos 2\pi f_m t$. In a certain experiment conducted with $f_m = 1\,KHz$ increasing A_m (starting from 0V) it was found that the carrier magnitude in a spectrum becomes zero for the first time when Am is 2V.

① Det. frequency sensitivity of Modulator.

② Det. the Value of Am for which the carrier magnitude become zero for the second time.

Ans:

(i) $\uparrow \beta = \frac{k_f \cdot A_m \uparrow}{f_m}$

$\beta = 2.4, 5.5, 8.6, ..$

$\therefore 2.4 = \frac{k_f \cdot 2V}{1 \text{ kHz}}$

$\therefore \boxed{k_f = 1.2 \text{ kHz/volt.}}$

(ii) $\beta = \frac{k_f \cdot A_m}{f_m}$.

$\therefore 5.5 = \frac{1.2 \times A_m}{1 \text{ kHz.}}$

$\therefore \boxed{A_m = 4.6V}$

***

Ex-8 FM transmitter radiates 100 W when the carrier is not modulated. the carrier is now modulated and the modulation index is adjusted so that the magnitude of the 1st order sidebands is zero in the spectrum under this condition ① ~~ex~~ calculate the power at carrier freq. ② calculate the power in all the remaining sideband. ③ calculate the power in the second order sideband.

$J_0(0) = 1$ $\quad J_0(3.8) = -0.4$ $\quad J_1(2.4) = 0.52$

$J_0(2.4) = 0$ $\quad J_0(5.1) = -0.16$ $\quad J_1(3.8) = 0.$

$J_1(5.1) = -0.33$

$J_2(2.4) = 0.43$

$J_2(3.8) = 0.41$

$J_2(5.1) = 0.$

Ans: $P_t = \dfrac{A_c^2}{2R} = 100W.$

$\therefore A_c^2 = 200.$

$A_c = 14.14\ V$

$\therefore$ MI is adjustead so that the magnitude of the 1st order SB. is zero

$\therefore$ so, $J_1(3.8) = 0 \Rightarrow J_1(\beta) = 0$

$\therefore \boxed{\beta = 3.8}$

① $P_{fc} = \dfrac{A_c^2}{2R} \cdot J_0^2(\beta)$

$= \dfrac{A_c^2}{2R} \cdot J_0^2(3.8)$

$= 100 \times (-0.4)^2$

$P_{fc} = 16\ W$

② $P_{SB} = P_t - P_{fc}$

$= 100 - 16$

$\therefore \boxed{P_{SB} = 84\ W}$

③ $P_{2nd\ order} = \frac{A_c^2}{R} J_2^2(3.8)$

$= \frac{100}{2} \cdot (0.41)^2$

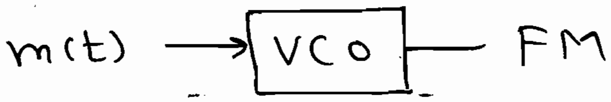
∴ $\boxed{P_{2nd\ order} = 33.3\ W}$

* Generation of WBFM signal.

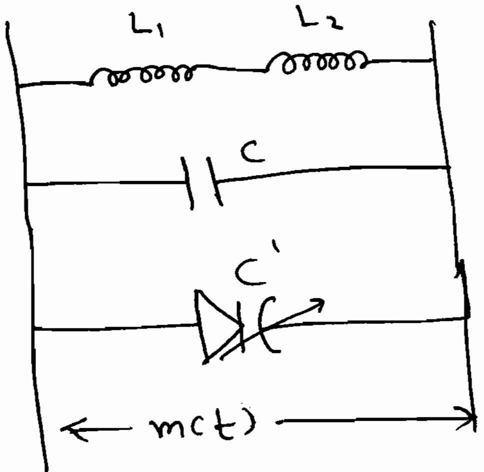
① Direct Method.

② Indirect method (OR) Arm Strong method (Practically it is not used).

① Direct Method:



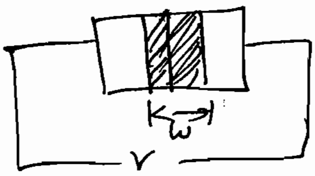
⇒ FM modulator is Voltage to frequency converter. So, VCO (voltage control oscillator) used to convert voltage variation into freq. variation.



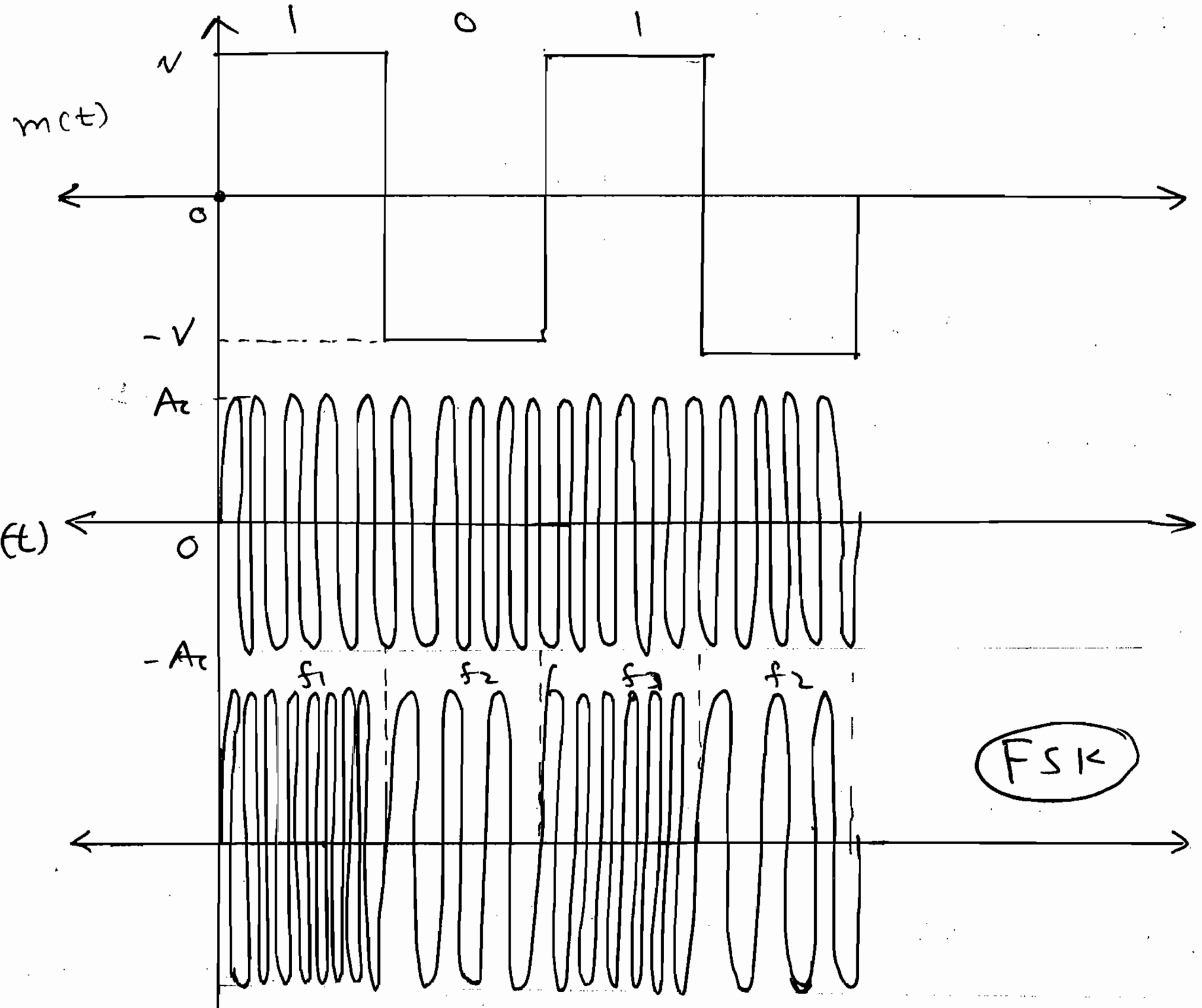
VCO

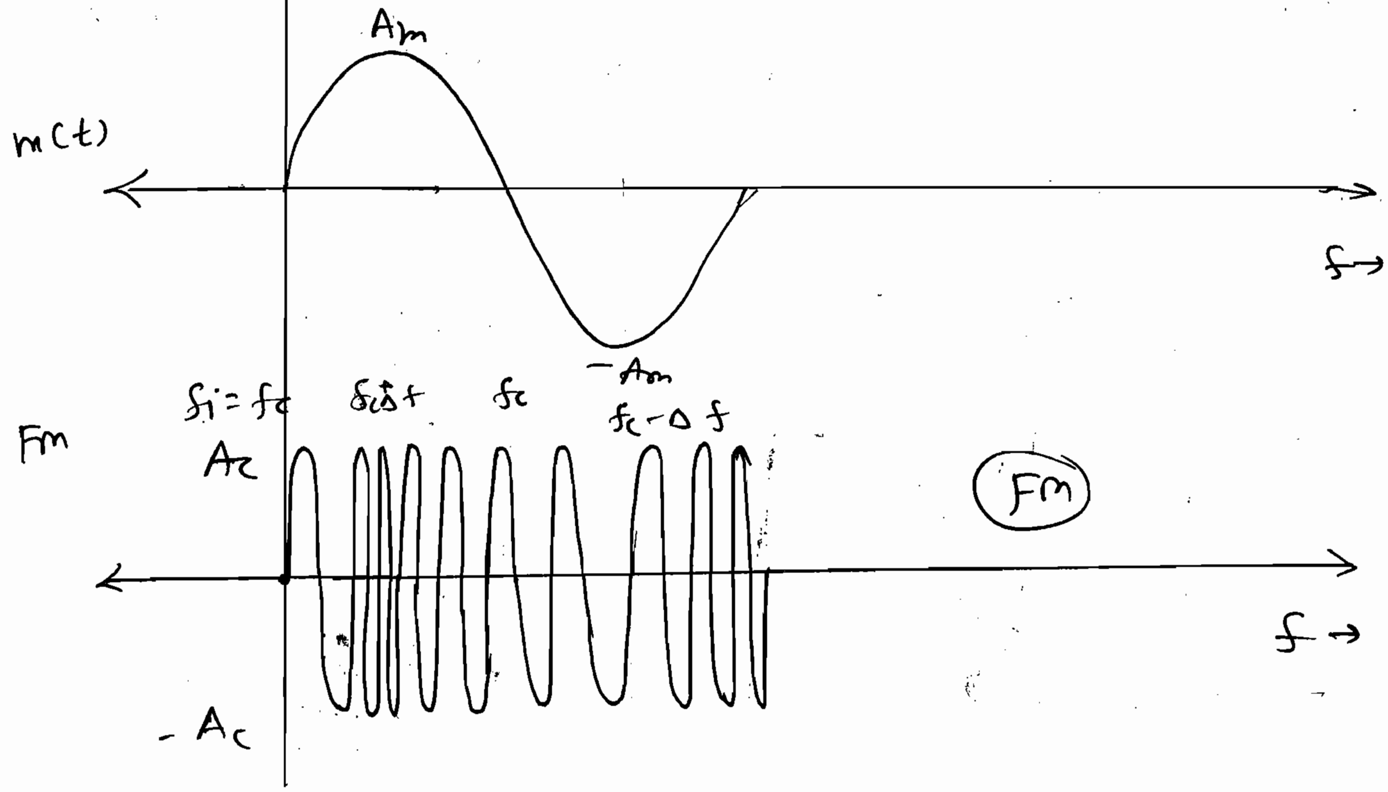
Varactor diode

$C = \frac{\epsilon A}{W}$



$$\therefore \boxed{f = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C')}}}$$





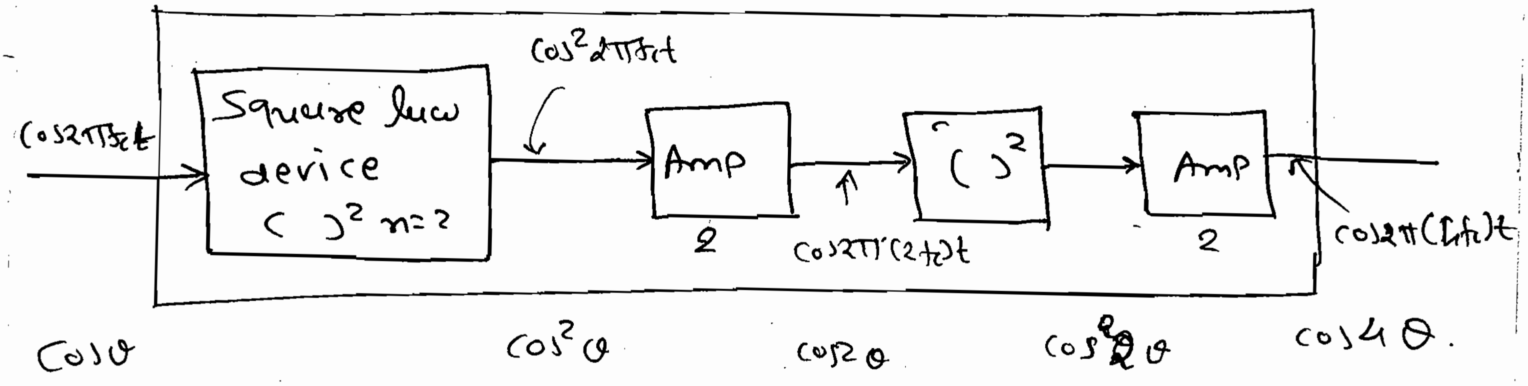
$$f_1 = f_c + k_f V = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C_1)}}$$

$$f_2 = f_c - k_f V = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C_2)}}.$$

② Armstrong Method:

⇒ Freq Multiplier 'n':

$\boxed{n=4}$



→ $\cos^2 2\pi f_c t = \frac{1}{2} + \frac{1}{2}\cos 2\pi(2f_c)t.$

→ Assume that the message signal & carrier signal are applied to NBFM modulator.

The o/p signal is:

$$A_c \cos\left[2\pi f_c t + \underset{\hookrightarrow\,\beta<1}{\beta} \sin 2\pi f_m t\right]$$

⇒ If the above signal is passed through a 'freq. multiplier by n' then o/p is:

$$A_c \cos[n(2\pi f_c t + \beta \sin 2\pi f_m t)].$$

$$= A_c \cos[2\pi(nf_c)t + (n\beta)\sin 2\pi f_m t].$$ ↙W.B

⇒ In a freq. multiplier carrier freq. f_c & β is increased by a factor of n, but the message frequency is same.

i.e.

$\rightarrow$ NBFM $\rightarrow n=10 \rightarrow$ WBFM $\longrightarrow$ mixer $f_l = 9$ MHz $\longrightarrow$ O/P

↳ $= 11$ MHz

$f_c = 1$ MHz	$f_c = 10$ MHz	$f_c = 1$ MHz
$\beta = 0.8$	$\beta = 8$	$\beta = 8$
$f_m = 2$ kHz	$f_m = 2$ kHz	$f_m = 2$ kHz
$(\beta < 1)$	$(\beta > 1)$	O/P

Ex-1 Consider a NBFM signal

$s(t) = 10\cos[2\pi 10^6 t + 0.4 \sin 2\pi 10^3 t]$. The signal is passed through to freq. multiplier connected in cascade $n_1 = 2$ & $n_2 = 5$. Determine the carrier freq., modulation index, & BW at the o/p of first multiplier & second multiplier.

Ans: $A_c = 10V$, $f_c = 1 MHz$, $f_m = 1 KHz$, $\beta = 0.4$.

$f_c = 1 MHz$

$f_m = 1 kHz$

$\beta = 0.4$

$f_c = 2 MHz$

$f_m = 1 KHz$

$\beta = 0.8 < 1$

(NBFM)

$BW = 2 f_m$

$\boxed{BW = 2 KHz}$

$f_c = 10 MHz$

$f_m = 1 KHz$

$\beta = 4$

(WBFM)

$BW = 2(\beta+1) f_m$

$BW = 2 \times 5 \times 1 KHz$

$\boxed{BW = 10 KHz}$

⇒ We know that,

modulation index $\beta = \frac{\Delta f}{f_m}$ → No change.

↓H.B.

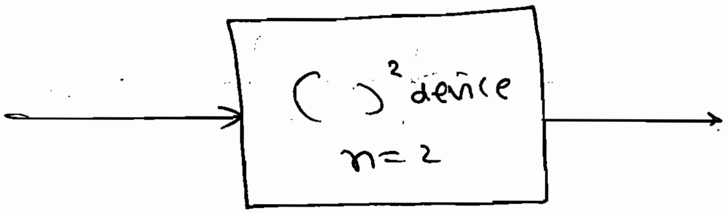
but f_m is not change

∴ Δf change, during conversion of NBFM to WBFM.

⇒ Hardware complexity of Armstrong Modulator is very high so it is not Practically used.

Ex-1 FM signal is having frequency deviation of 90 KHz is passed through a square law device the message frez. is 5 KHz. Determine β, BW, at the O/P.

Sol^n:



$\Delta f = 90$ KHz

$f_m = 5$ KHz

$\beta = 18$

$\Delta f = \beta f_m = 36 \times 5 = 180$ KHz

$\beta = 2 \times 18 = 36$

$$BW = 2(\beta+1) f_m$$

$$= 2 \times 36 \times 5 \text{ KHz}$$

$$\boxed{BW = 370 \text{ KHz}}$$

(OR)

$$BW = 2\left(\frac{\Delta f}{f_m} + 1\right) f_m$$

$$= 2\Delta f + 2f_m$$

$$= 2(180) + 10$$

$$\boxed{BW = 370 \text{ KHz}}$$

* Demodulation of FM signal:

⇒ FM demodulator is a frequecy to voltage Converter.

① Frequency discrimination method.

② phase discrimination method.

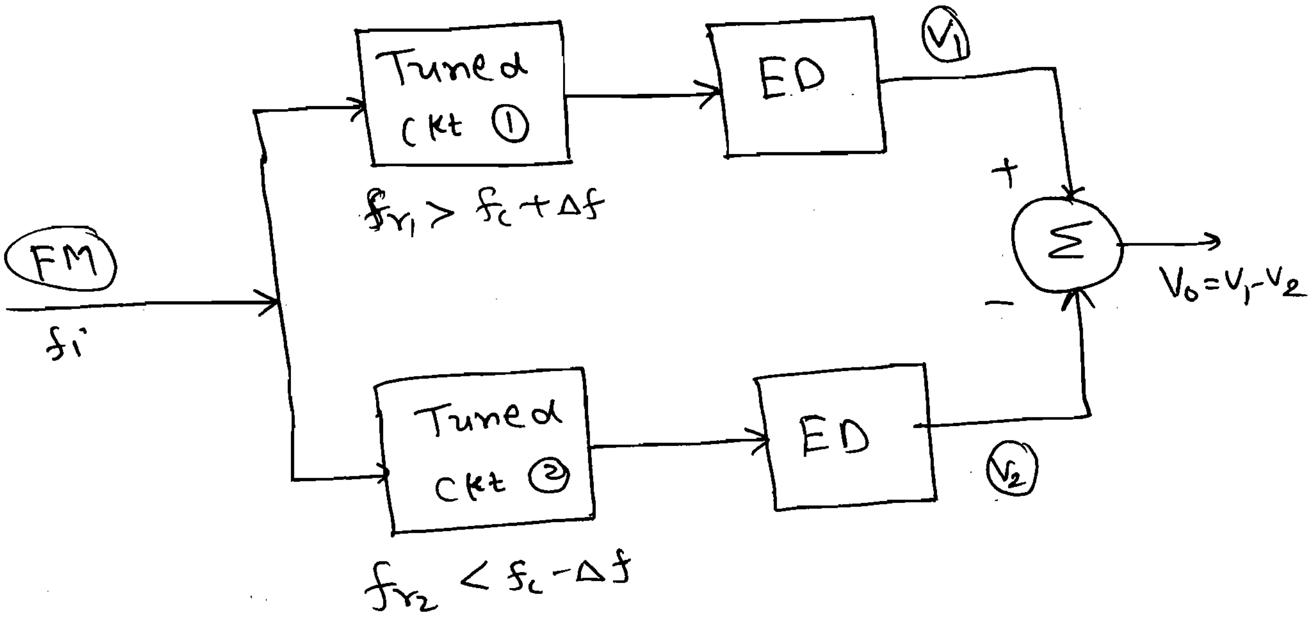
→ Foster seeley discrimination.

→ Ratio detector.

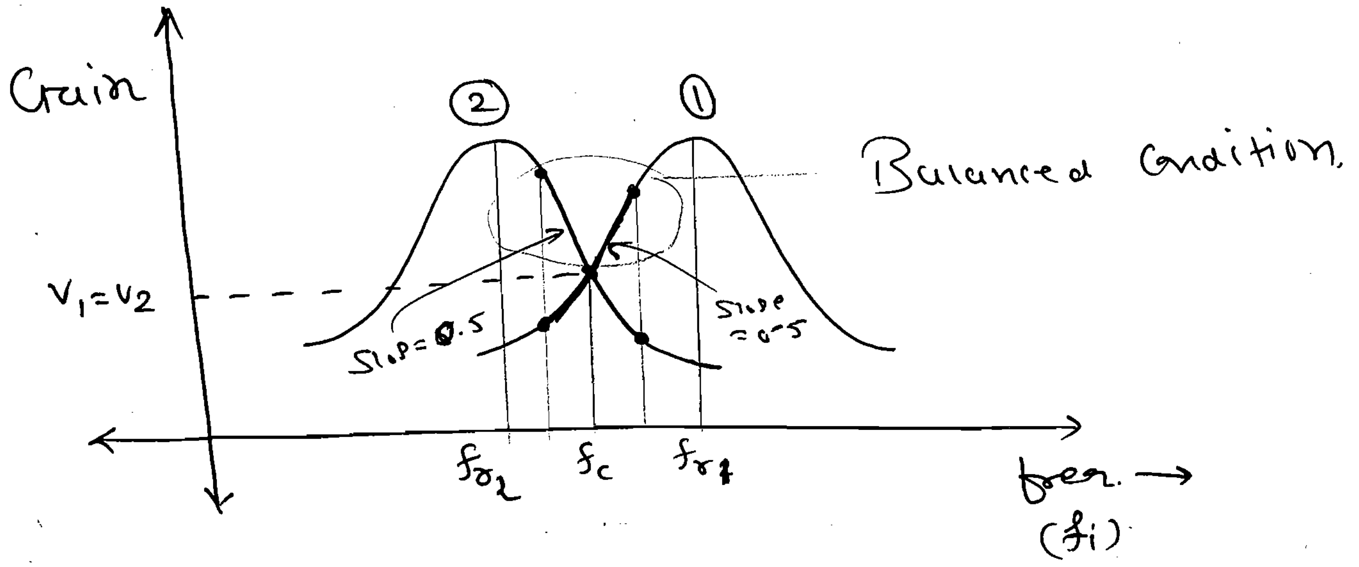
→ PLL (phase locked loop).

① Balanced Slop Detector :-

⇒



⇒ The Freq. Response of the Tuned ckt:



⇒ Condition:

① $f_i = f_c$:

⇒ The gain of the 1st tuned ckt is same as the 2nd tuned ckt. So the o/p of the EDs $\boxed{V_1 = V_2}$. So, $V_0 = 0$

② **$f_i > f_c$:**

⇒ The gain of the 1st tuned ckt is larger than gain of the 2nd tuned ckt. so, $V_1 > V_2$ and so, the output voltage $V_o = \underline{V_1 - V_2 > 0}$ (Positive).

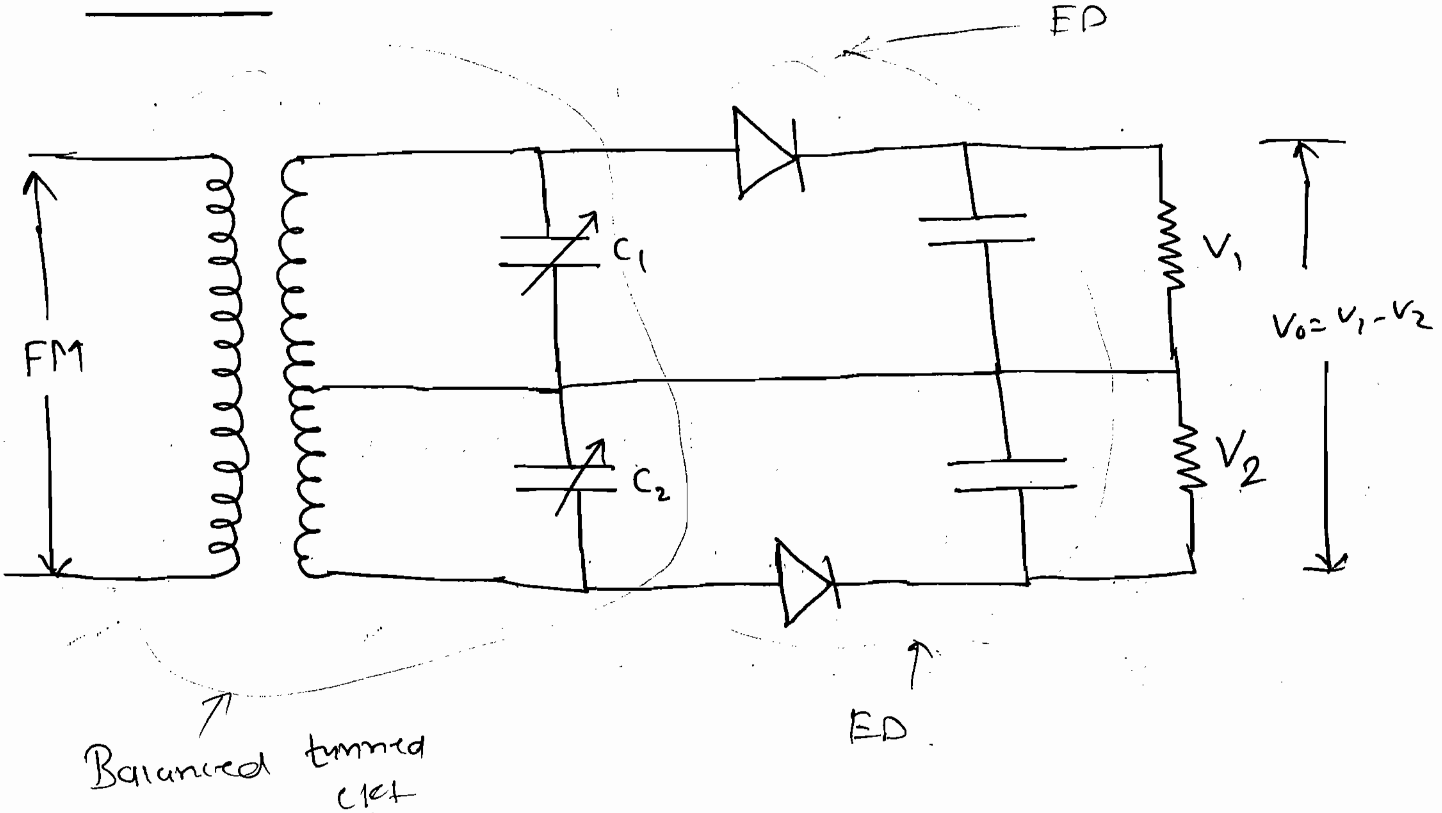
③ **$f_i < f_c$:**

⇒ The gain of the 1st tuned ckt is less than the 2nd tuned ckt.

So, $V_1 < V_2$.

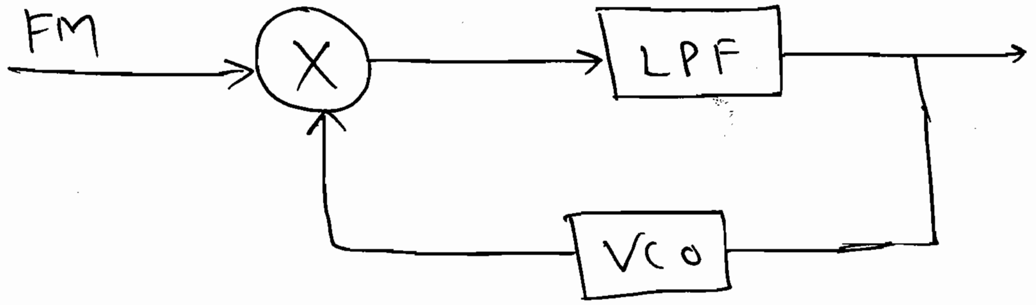
∴ o/p voltage, $V_o = V_1 - V_2$ = negative < 0.

* **Hardware implementation of Balanced slope detector:**



* FM demodulation using PLL:

=)



first order PLL

=> The working principle of PLL is same of synchronous detector.

→ In AM, DSB & SSB the carrier frequency is constant. so, local oscillator is used to generate the same carrier.

→ In fm the carrier freq. is varied according to the message signal using VCO. so, the local oscillator is replaced with VCO to generate the same carrier.

→ If the input to the PLL is Cos [$2\pi f_c t + \phi$] then o/p of PLL is:

then the o/p of PLL is:

$$\boxed{V_o \propto \frac{d[\phi]}{dt}}$$

=> Time domain eqn of Multitone modn is:

$$A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\, dt\right].$$

∴ o/p $V_o \propto \frac{d}{dt}[\phi]$.

$$\therefore V_o \propto \frac{d}{dt}\left[2\pi K_f \int m(t)\, dt\right].$$

$$\therefore V_o \propto 2\pi K_f\, m(t).$$

$$\therefore V_o = \frac{\cancel{2\pi} K_f}{\cancel{2\pi} K_v} \cdot m(t).$$

$$\therefore \boxed{V_o = \frac{K_f}{K_v} \cdot m(t).}$$ ↙ H.B.

K_f :-> freq. sensitivity of the VCO at transmitter.

K_v :-> freq. sensitivity of the VCO at Receiver.

* Phase Modulation:

⇒ Time domain eqn of the PM signal is

$$S(t) = A_c \cos\left[2\pi f_c t + K_p m(t)\right]. \leftarrow \text{H.B.}$$

$$\therefore \boxed{\phi(t) = K_p m(t)} \text{ (rad).}$$

K_p = phase sensitivity of the modulator (rad/volts).

for singletone modulation

$$m(t) = A_m \cos 2\pi f_m t.$$

$$\therefore S(t) = A_c \cos\left[2\pi f_c t + K_p A_m \cos 2\pi f_m t\right].$$

$$\text{H.B} \nearrow \boxed{\Delta\phi = K_p \cdot A_m} = \text{Phase deviation.}$$

$$\therefore S(t) = A_c \cos\left[2\pi f_c t + \Delta\phi \cos 2\pi f_m t\right].$$

$$\therefore \boxed{S(t) = A_c \cos\left[2\pi f_c t + \beta \cos 2\pi f_m t\right].} \leftarrow \text{H.B}$$

$$\Rightarrow \boxed{K_p m(t) = \Delta\phi = \beta = \text{modulation index.}} \leftarrow \text{H.B}$$

⇒ In phase modulation phase deviation and modulation index are same.

⇒ The magnitude Spectrum of the PM signal is Same as the FM signal except a phase shift of 90° at message freq. So, the magnitude Spectrum of the PM signal is Same as the FM signal.

Ex-1 Consider an angle modulated signal $s(t) = 10\cos[2\pi f_c t + 5\sin 8\pi 10^3 t]$. Determine the freq. deviation and phase deviation.

Ans: (i) FM → $\Delta f = \beta \cdot f_m = 5 \times 4\,KHz = 20\,KHz.$

(ii) PM → $\Delta\phi = \beta = 5$ rad.

Ex-2 Consider an angle ~~modulated index~~ modulated signal

$s(t) = 10\cos[2\pi 10^6 t + 8\sin 2\pi 10^3 t]$

(i) Assuming the given signal is Fm determine the modulation index, freq. deviation, BW & power.

(ii) Repeat the above calculation when the message freq. is doubled.

Sol^n:

$A_c = 10$, $f_c = 1\,\text{MHz}$, $f_m = 1\,\text{KHz}$

(i) $\beta = 8$,

$\Delta f = \beta \cdot f_m = 8 \times 1\,\text{KHz} = 8\,\text{KHz}$.

$\therefore\ BW = 2(\beta+1) f_m = 2 \times 9 \times 1\,\text{KHz} = 18\,\text{KHz}$.

$P = \dfrac{A_c^2}{2R} = \dfrac{100}{2} = 50\text{W}$.

(ii) when $f_m' = 2 f_m = 2\,\text{KHz}$.

$\therefore\ \beta = \dfrac{\Delta f}{f_m} = \dfrac{8\,\text{KHz}}{2\,\text{KHz}} = 4$. $\quad \Delta f = 8\,\text{KHz}$

$\therefore\ BW = 2(\beta+1) f_m = 2(5) \times 2 = 20\,\text{KHz}$.

Ex-3 Repeat the above numerical problem assume that the given signal is PM.

Sol^n:

(i) $\beta = 8$.

Phase deviation $\Delta\phi = \beta = 8$ rad.

$BW = 2(\beta+1) f_m$

$= 2 \times 9 \times 10\,\text{KHz}$

$BW = 18\,\text{KHz}$

$P_t = \dfrac{A_c^2}{2R} = \dfrac{100}{2} = 50\,\text{W}$

(ii) when $f_m' = 2 f_m = 2 \times 1\,\text{KHz} = 2\,\text{KHz}$.

$\beta = 8 = \Delta\phi$. $\quad P_t = 50\,\text{W}$

$BW = 2(\beta+1) f_m'$

$= 2(9) \times 2\,\text{KHz} = 36\,\text{KHz}$.

* Conclusion:

① In Am & pm, the BW of the modulated signal is double when the message freq. is double. But in FM, BW is not double.

② In pm, modulation index β is independent of modulating frequency.

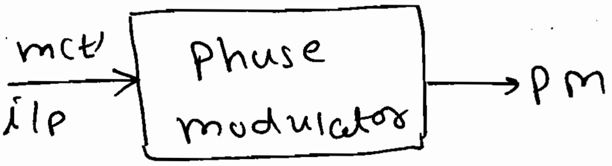
* Time domain eqn for multitone modulation:

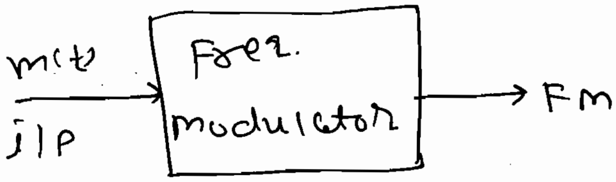
PM

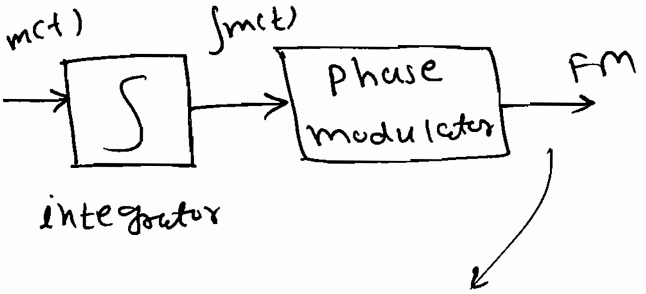
→ $S(t) = A_c \cos[2\pi f_c t + k_p m(t)]$

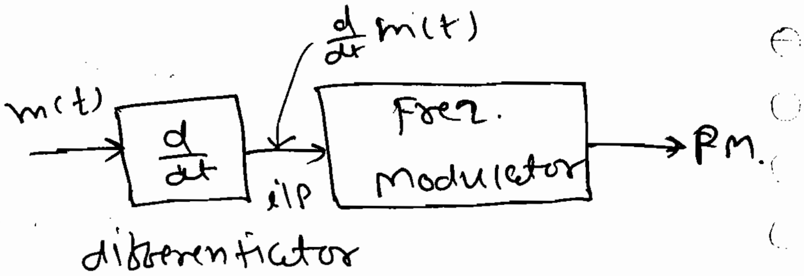
FM

→ $S(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int m(t)dt]$







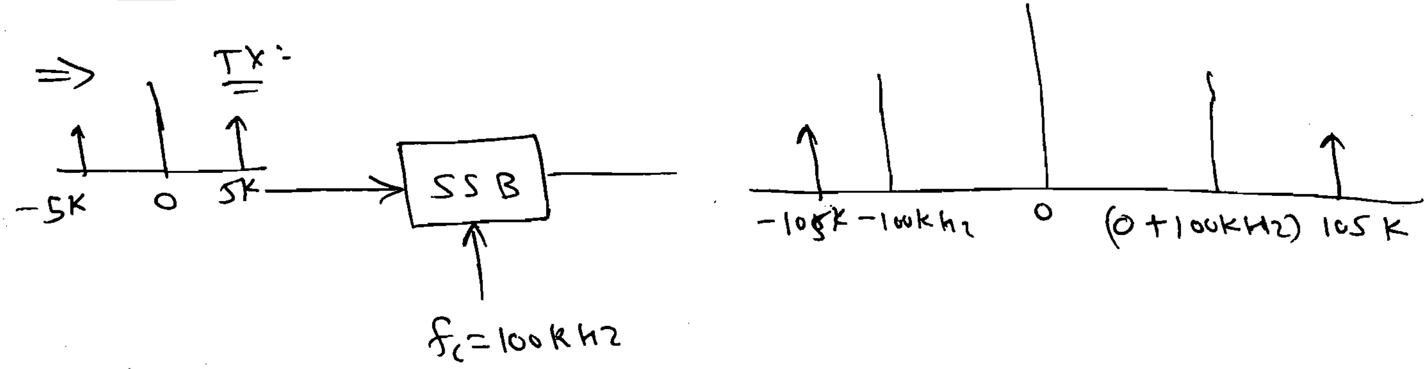


$A_c \cos[2\pi f_c t + K_p \int m(t)dt]$

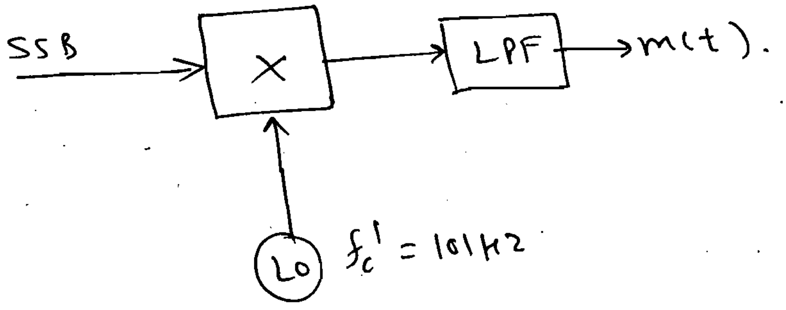
$K_p = 2\pi k_f$

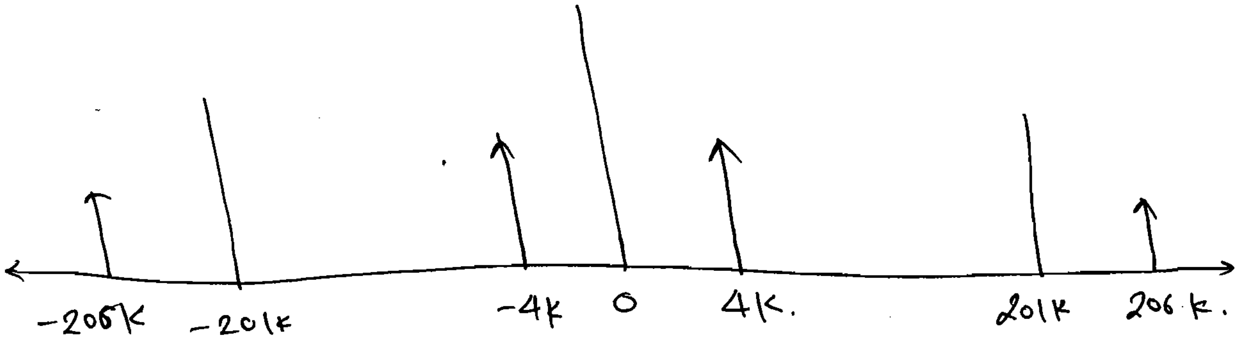
* IMP Concept: ← N.B

⇒ Tx:



⇒ Reciever: Using coherent | synchronous detection.





Conclusion:

Tx:	$f_m = 5\,kHz$	$f_c = 100\,kHz$	USB → 105 kHz	LSB.
Rx:	Lo → 100kHz	→ o/p:	$f_m = 5\,kHz$.	5kHz.
	Lo → 101 kHz	→ o/p:	$f_m = 4\,kHz$.	6kHz
	Lo → 102 kHz	→ o/p:	$f_m = 3\,kHz$.	7 kHz.
	Lo → 99 kHz	→ o/p:	$f_m = 6\,kHz$.	4 kHz
	Lo → 98 kHz	→ o/p:	$f_m = 7\,kHz$.	3 kHz

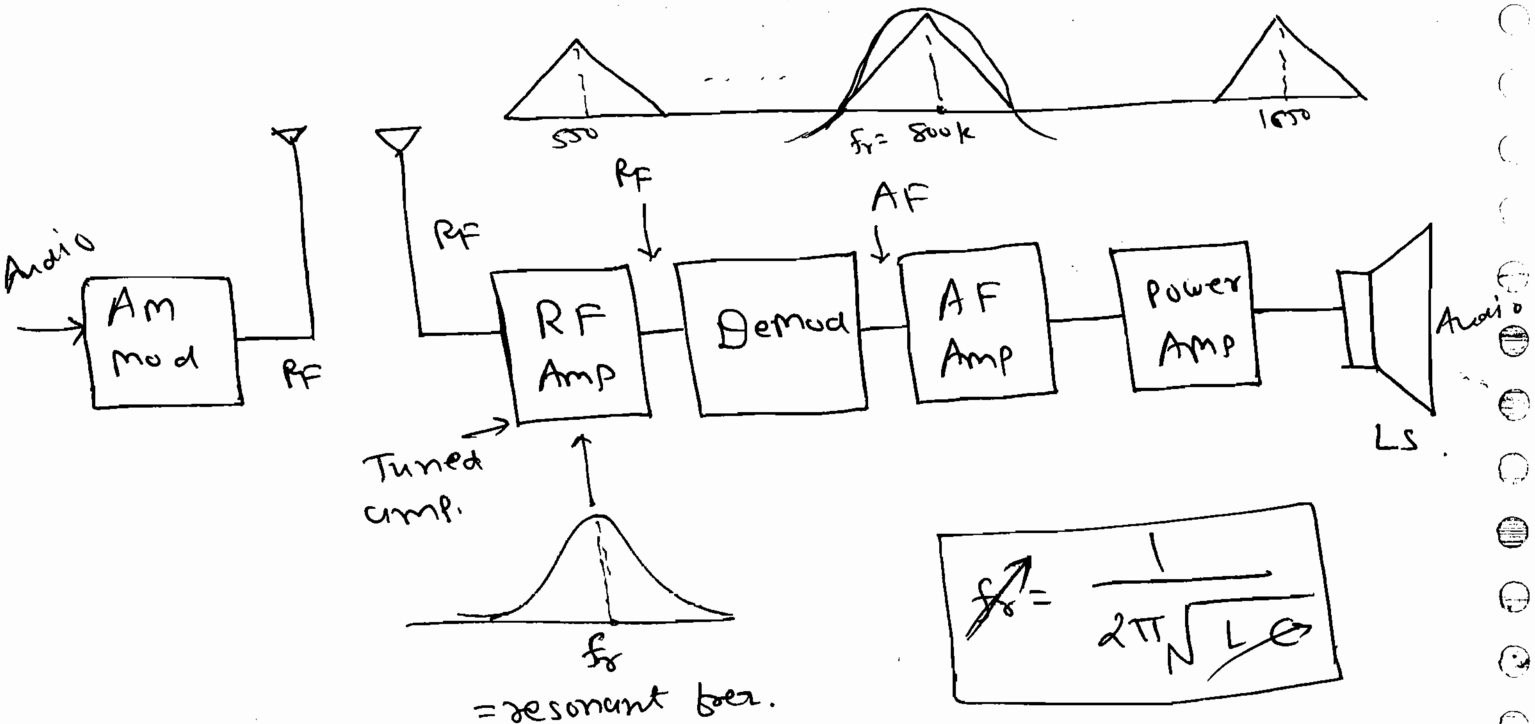
for LSB $f_m' =$ sub.

☆ AM Receivers :-

① Tuned Radio Freq. Receiver. (TRF Rx).

② Superhetrodyne Receiver.

① Tuned Radio Frequency Rx (TRF Rx) :-



$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

FM	:	88M – 108MHz
AM	:	550 K – 1050 kHz.

← H.B.

⇒ In order to select the 800kHz signal, we have to be set resonant freq. at 800 kHz. by Changing capacitor value.

⇒ So, tunning in TRF Rx is nothing but the setting a f_r in order to get desired signal.

⇒ International standard BW of signal = 10 kHz
(including Guard Band)

⇒ After Receiving the signal from antenna RF Amp. is used to increase the signal strength. The RF amp. is also a tuned amp. which is used to select the required signal by adjusting the resonant freq.

⇒ Demodulator converts RF signal into AF.

⇒ AF amp. and Power amp. is used to increase the signal strength to the required level.

* Characteristic Parameters of a Receiver:

① Sensitivity.

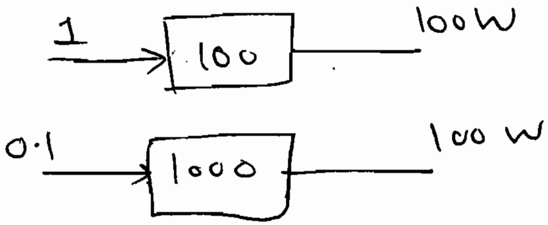
② Selectivity.

③ Fidelity.

① Sensitivity:

⇒ It is defined as the minimum signal strength that should be maintain at i/p of a receiver to get a standard o/p.

⇒ Sensitivity depends on the overall gain of the receiver.



② Selectivity:

⇒ Selectivity is defined as the ability of the receiver to select the required freqs only.

⇒ Assume that a receiver is tuned to 800 KHz the tuned ckt at the RF amplifier has to select the freq from 795 KHz to 805 KHz. This is possible only when the resonant freq. is adjusted to 800 KHz and Q should be 80. ($BW = \frac{fr}{Q}$) ⇒ $Q = 80$. But simultaneously variation of resonant freq. and Q is not possible in tuned ckt. ~~and Q is not~~

⇒ If the BW of the tuned ckt is greater than 10 KHz adjecent signal freq. are selected. If the BW is less than 10 KHz some of the required freqs are attenuated.

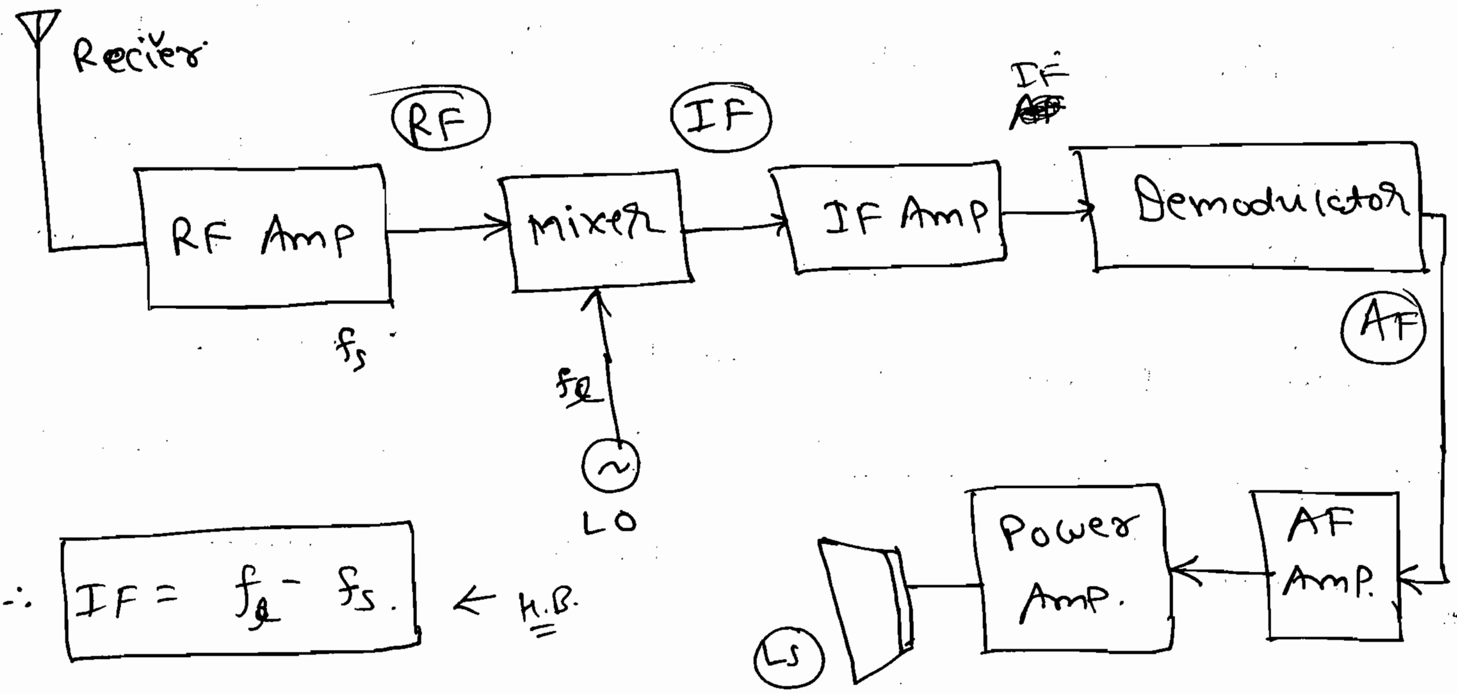
③ Fidelity:

⇒ Fidelity is defined as the ability of the receiver to reproduced all audio freq. at the o/p.

⇒ In AM ~~radio~~ the audio signal is Band limited to 5 KHz as the Bandwidth alloted to each broadcasting station is 10KHz.

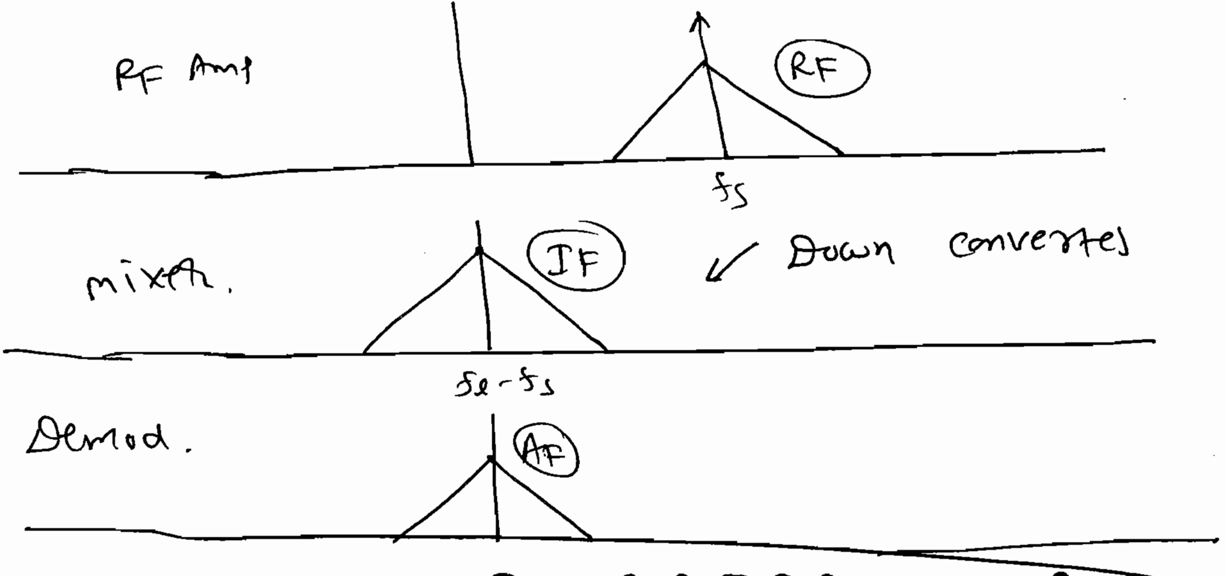
⇒ The highest audio freq. at O/P of the receiver is 5 KHz. So, the signal fidelity is very low in AM receivers.

② Superhetrodyne Receiver:



∴ $\boxed{IF = f_l - f_s.}$ ← H.B.

f_s = Carrier freq. of the tuned station.



→ After receiving signal from the antenna RF Amp is used to increase the signal strength.

→ Mixer is used to down convert the RF signal into IF.

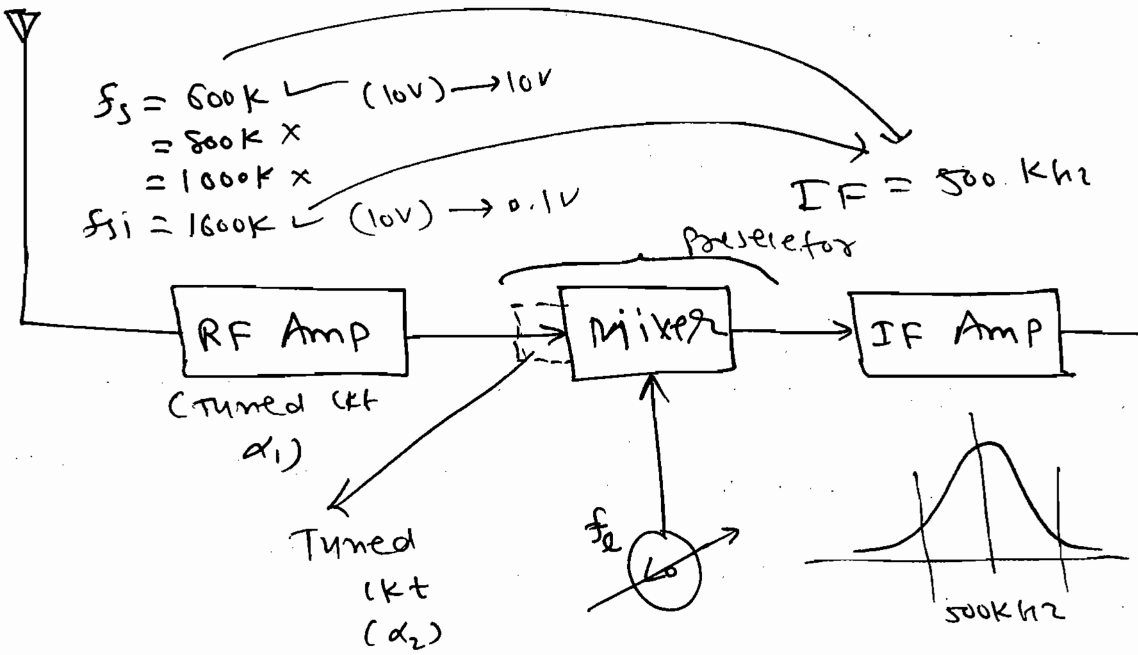
→ The IF is a constant value and depends on the Application.

→ In Am radio the IF used is 455 KHz

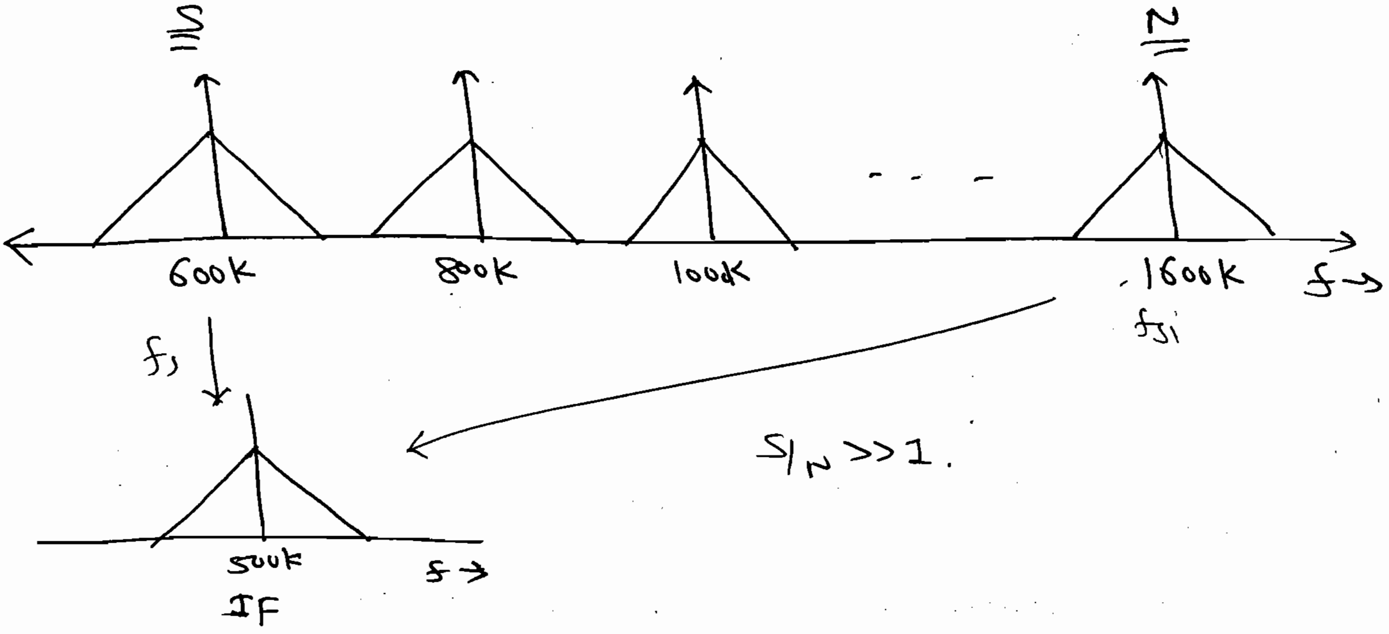
The local oscillator freq. should be adjusted so that $f_0 - f_s$ is 455 KHz.
This Process is called as tunning.

→ The IF amplifier is a tuned amplifier which is always tuned 455 KHz.

* Image freq. and its Suppression:



⇒

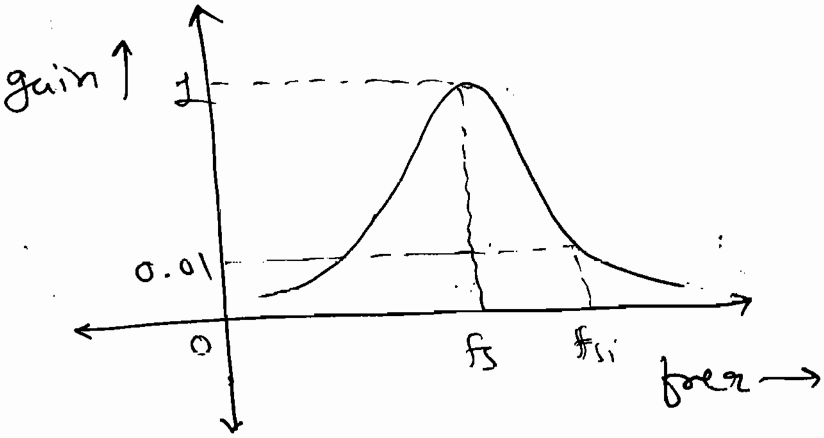


⇒ Consider a receiver with IF 500 KHz and assume that receiver is tuned at 600 KHz. The Local oscillator freq. is adjusted to 1100 KHz to down convert signal to 500KHz. Assume that another signal received from antenna having a carrier freq. of 1600 KHz. This signal is also down converted to 500KHz and causes interference to the required signal, the signal which is causing interference is called as the image freq.

$$\boxed{f_{si} = f_s + 2IF} \leftarrow \underline{\underline{\text{M.B}}}$$

$1600K = 600K + (2 \times 500K)$

→ To reduced the image frequency signal strength of a tuned ckt is used at the i/p of the mixer. the freq. response of tuned ckt as shown in fig.

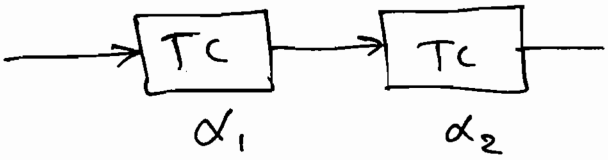


⇒ The gain of tuned ckt at fsi should be as minimum as possible.

⇒ To determine the suppression factor of the tuned ckt, Image Rejection Ratio is used.

$$\boxed{IRR = \alpha = \frac{G_{fs}}{G_{fsi}} = \sqrt{1 + Q^2 \rho^2}}$$

$$\boxed{\text{where, } \rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}}$$



$\alpha = \alpha_1 \cdot \alpha_2.$

Q-1 A Superhetrodyne receiver having no RF Amp. is tuned to 555 kHz. The Local oscillator freq. is adjusted to 1010 kHz & Q is 50. Determine the image freq. and image Rejection Ratio.

Ans:

$f_s = 555$ kHz $Q = 50$

$f_l = 1010$ kHz

$\therefore IF = f_l - f_s = 1010 - 555 = 455$ kHz.

$\therefore f_{si} = f_s + 2IF$

$\therefore f_{si} = 555 + (2 \times 455) = 555 + 910$

$\therefore \boxed{f_{si} = 1465 \text{ kHz}}$

$$\therefore \rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}.$$

$$\therefore \rho = \frac{1465}{555} - \frac{555}{1465}.$$

$\therefore \rho = 2.26.$

$$\therefore \alpha = \sqrt{1 + Q^2\rho^2} = \sqrt{1 + (2500)^2(2.26)^2}$$

$\therefore \boxed{\alpha = 113.}$

Ex-2 A Superhetrodyne receiver having RF Amp. is tuned to 1200 kHz, the IF 450 kHz. The Q of the tuned ckt at the RF Amp. and i/p of the mixer are same and is equal to 65. Calculate the image freq & IRR.

Ans:

$f_s = 1200$ kHz.

$IF = 450$ kHz.

$Q = Q_2 = 65$.

$\therefore \quad f_{si} = f_s + 2IF$

$= 1200 + 900$

$$\boxed{f_{si} = 2100 \text{ kHz}}$$

$\therefore \quad \rho = \dfrac{f_{si}}{f_s} - \dfrac{f_s}{f_{si}}$

$= \dfrac{2100}{1200} - \dfrac{1200}{2100}$

$$\therefore \boxed{\rho = 1.178}$$

$\therefore \quad \alpha = \alpha_1 \cdot \alpha_2$

$\alpha = \sqrt{1 + Q_1^2 \rho^2} \cdot \sqrt{1 + Q_2^2 \rho^2}$

$\therefore \quad \alpha = (1 + Q^2 \rho^2) \quad (\because Q_1 = Q_2 = Q)$.

$\therefore \quad \alpha = 1 + (65)^2 (1.178)^2$

$$\therefore \boxed{\alpha = 5870.}$$

Ex-3 A Superheterodyne receiver having no RF Amp. is tuned to 1000 KHz the IF is 455 KHz & Q is 100. ① Determine the Image freq. & IRR. ② Repeat the above calculation when receiver is tuned to 25 MHz.

Ans: (i) $f_s = 1000$ KHz

$IF = 455$ KHz.

$Q = 100$.

$\therefore f_{si} = f_s + 2IF$

$\therefore f_{si} = 1000 + 910 = 1910$ KHz.

$\therefore \rho = \frac{1910}{1000} - \frac{1000}{1910}$.

$\therefore \rho = 1.3864$.

$\alpha = \sqrt{1 + Q^2 \rho^2} = \sqrt{1 + 1.38^2 . (100)^2}$

$\therefore \boxed{\alpha = 138.64}$.

(ii) $f_s = 25$ MHz.

$\therefore f_{si} = 25M + 2 \times 455K$.

$f_{si} = 25.910$

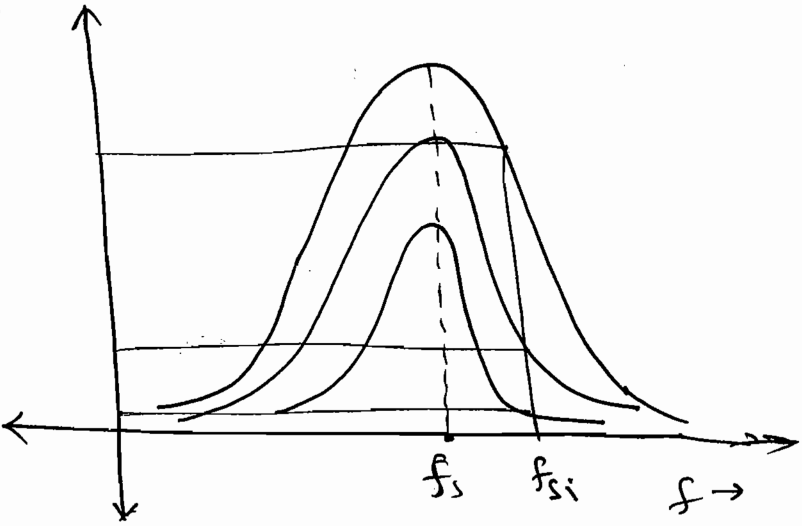
$\therefore \rho = \frac{25.910}{25} - \frac{25}{25.910}$.

$\therefore \rho = 0.036$

$\boxed{\alpha = 7.22}$

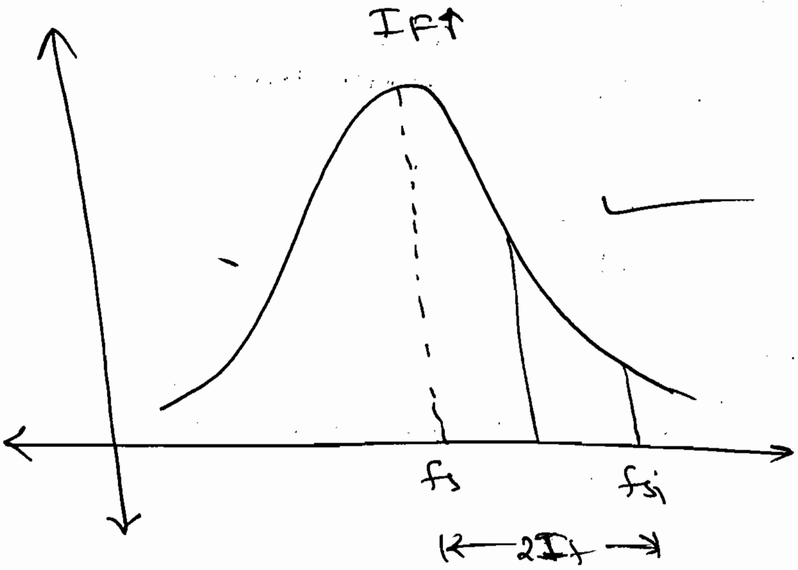
⇒ To improve the image Rejection Ratio, either Q (or) 'IF' should be increased.

⇒



Practically not possible because BW must 10kHz.

⇒



Ex-1 Determine the value of IF Required in the above numerical problem so that the IRR at 25 MHz is also equal to 138.6.

Ans: $f_s = 25$ MHz $\alpha = \sqrt{1 + (100)^2 (1.386)^2}$

$Q = 100$ $= 138.6$.

$\therefore \alpha = 138.6 = \sqrt{1 + (100)^2 \cdot \rho^2}$

$\Rightarrow \boxed{\rho = 1.38}$

$$\therefore \rho = 1.386 = \frac{25 + 2IF}{25} - \frac{25}{25+2IF}$$

$$\Rightarrow \boxed{IF = 11.4 \text{ MHz}}$$

* **In FM:**

88M - 108 M Hz

$\boxed{IF = 10.7 \text{ MHz}}$

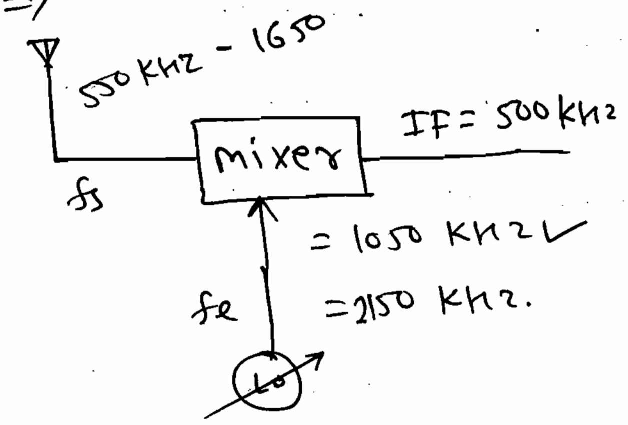
In Am

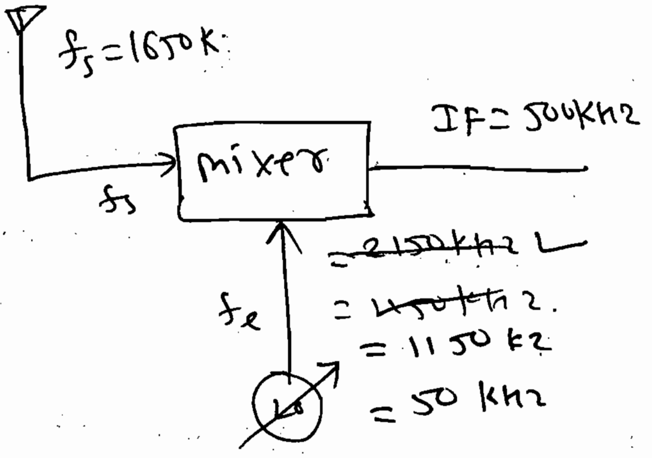
$\boxed{IF = 455 \text{ KHz}}$

IMP:

* Why should Local oscillator freq. should always larger than signal freq.?

⇒





$$\frac{1}{2\pi\sqrt{(L_1+L_2)C}}$$

① $f_e > f_s$

$$1050k = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_{max}}}$$

$$2150k = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_{min}}}$$

$$\therefore \frac{C_{max}}{C_{min}} = \left(\frac{2150}{1050}\right)^2$$

$$\simeq 4$$

② $f_e < f_s$

$$50k = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_{max}}}$$

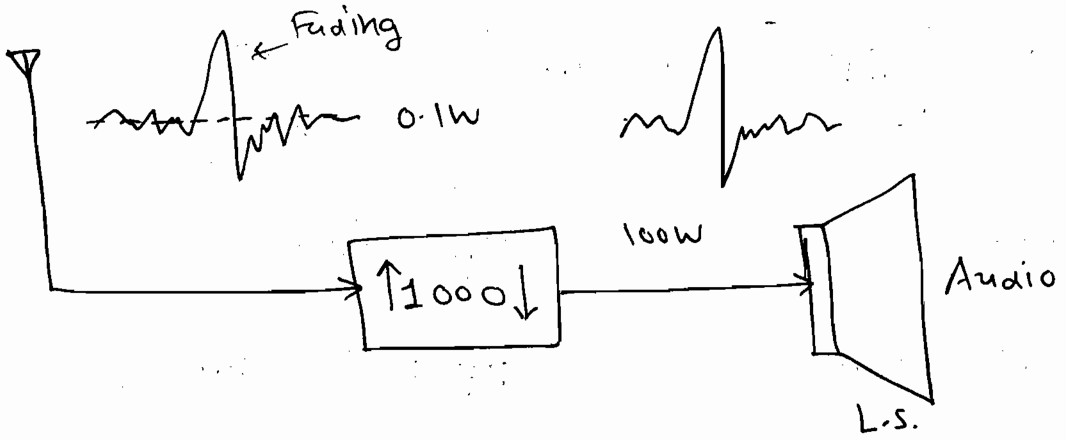
$$1150k = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_{min}}}$$

$$\therefore \frac{C_{max}}{C_{min}} = \left(\frac{1150}{50}\right)^2$$

$$= 529$$

It is impossible to get such a large ratio.

☆ AGC Circuit:



⇒ The signal strength from the antenna in any wireless communication system is not constant is called fading.

⇒ Signal fading occurs due to various propagation losses. If overall gain of the receiver is constant the o/p signal strength also changes.

⇒ AGC circuit is used to maintain a constant audio o/p irrespective of the receivers variations at the i/p of the

⇒ AGC circuit will control the overall gain of the receiver according to signal strength at variation in the the i/p.

* FM Receiver :

* International Standards:

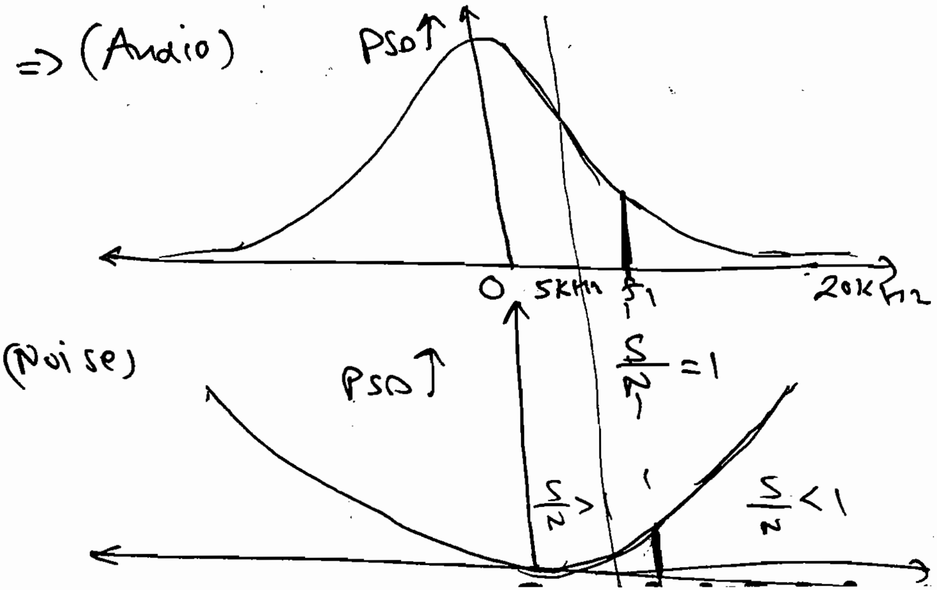
- ⊙ 88 MHz = 108 MHz ← f_c
- ⊙ IF = 10.7 MHz
- ⊙ B.W. = 200 KHz = $2\Delta f + 2f_m$ = $150K + 30KHz + \underbrace{20KHz}_{\text{Guard Band}}$
- ⊙ $\Delta f = 75$ KHz
- ⊙ $f_m = 15$ KHz
- ⊙ $\beta = 5$

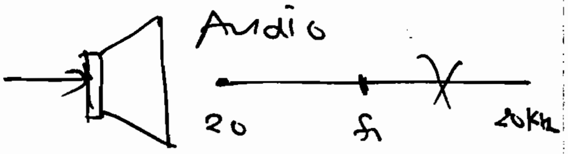
* Preemphesis (Tx) and Deemphasis (Rx):

⇒ These Concept are used in FM Radio not in AM Radio because Am signal is Band limited to 5 KHz and upto 5 KHz signal power is very more than the noise power and hence, $S/N >> 1$.

⇒ These techniques are used to improve the fideity of an Audio signals.

⇒ (Audio)

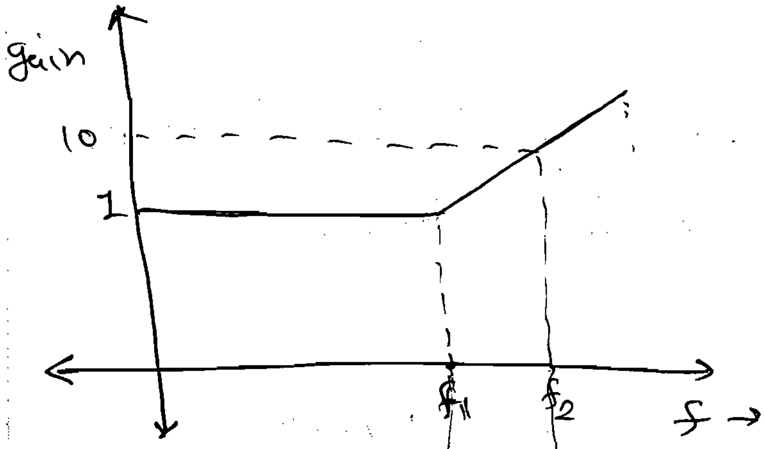




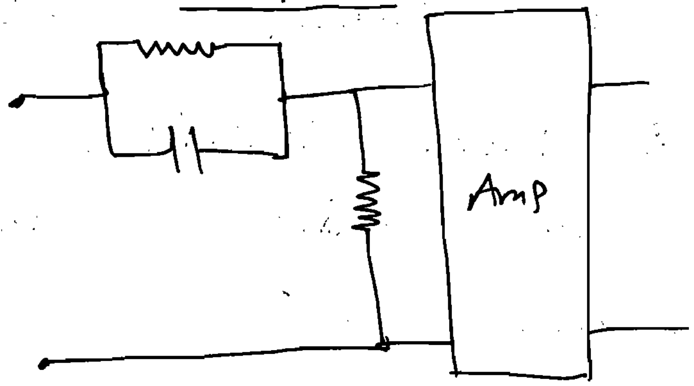
→ In Audio signal transmission $\frac{S}{N} > 1$ at low frequencies and $S/N < 1$ at high freq. So, high freq of the Audio signal will not be reproduced & signal Fidelity decreases.

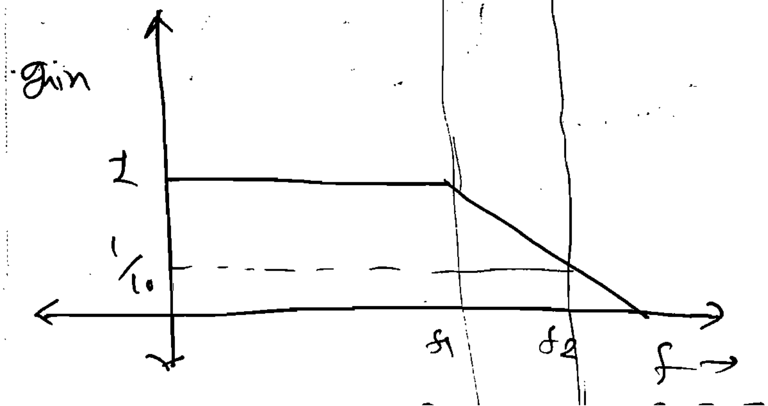
→ To improve the fidelity preemphasis technique is used ~~to~~ at Tx before modulator.

⇒ "Preemphesis is defined as the Process of boosting the high freq.s of the Audio signal so that the signal to noise ratio is greater than '1' (SNR > 1) at all Audio frequencies."

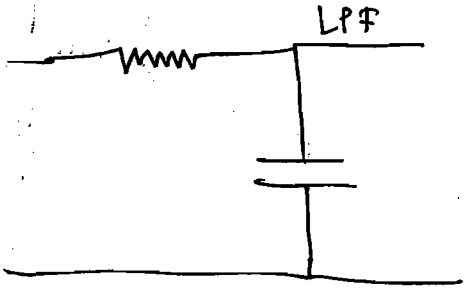


Preemphesis



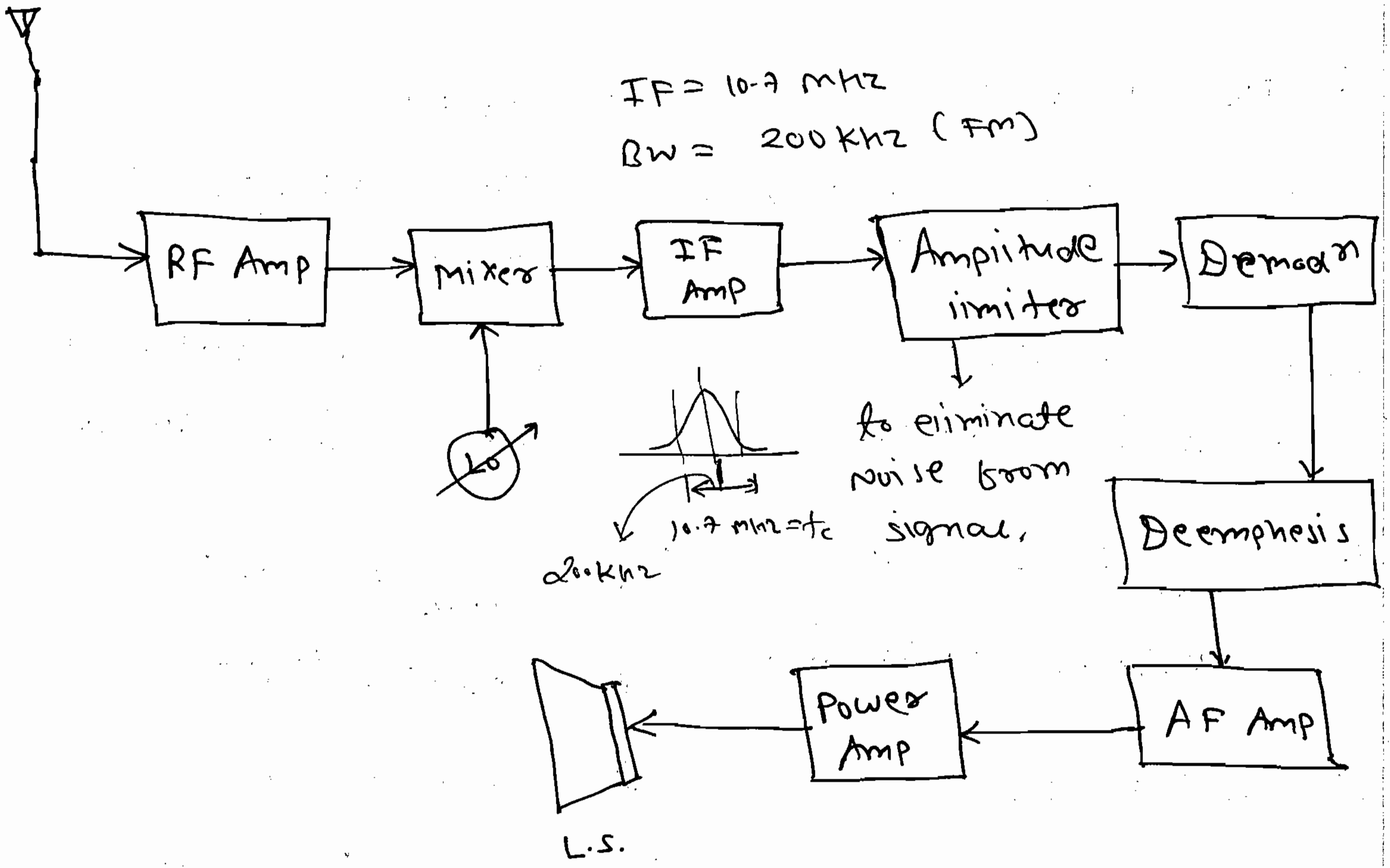


De emphesis



⇒ whenever Preemphesis and deemphesis used Practically in music system, it is called as Dolby Noise Reduction System and Dolby is one of the laboratory of USA.

* Block Diagram of FM Radio:



⇒ FM demodulator is a freq. to voltage converter

→ Frequency variation will be converted back into voltage variation only if the envelope of the FM signal is constant.

→ Due to noise, Amplitude distortion occurs and it is not possible to convert freq. into voltage. so Amplitude limiter is used to eliminate the noise or to maintain a constant envelope.

→ In Am Amplitude limiter is not used as the information is in the form of amplitude.

* Major differences in Am & Fm:

AM	FM
① $P_t = P_c + \frac{P_c \mu^2}{2}$	① $P_t = P_c$
② Am requires more power	② Fm requires less power.
③ Power varies with modulation index.	③ power is independent of modulation index.
④ $\eta_{max} = 33.33\ \%$ (singletone modulation)	④ $\eta_{max} = 100\%$; $\beta = 2.4, 5.5, 8.6, \ldots$
⑤ $BW = 2 f_m$	⑤ $2(\beta+1) f_m$
⑥ BW is very low.	⑥ BW is ~~varies~~ very high.
⑦ BW is independent of modulation index.	⑦ B.W. varies with modulation index.
⑧ Am receiver is less complex.	⑧ Fm receiver is more complex.
⑨ The effect of Noise is more.	⑨ The effect of noise is very less.

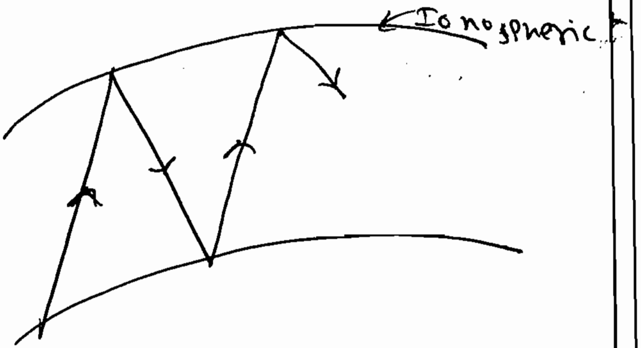
⑩ 550K – 1650 KHz

$IF = 455 KHz$

$B.W. = 10 KHz$

$\mu = 1$

⑪ Ionospheric Propogation



⑫ → Area coverage is more

⑬ Freq. reuse is not possible.

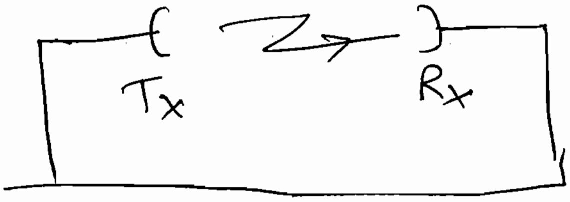
⑩ 88 M - 108 MHz

$IF = 10.7 MHz$

$B.W. = 200 KHz$

$\beta = 5$

⑪ Line of sight Propogation (LOS)



→ microwave freq. 'G' Hz

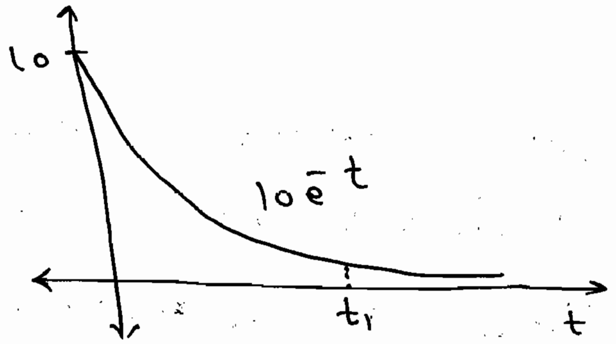
⑫ Area covered by antenna is resticated due to LOS.

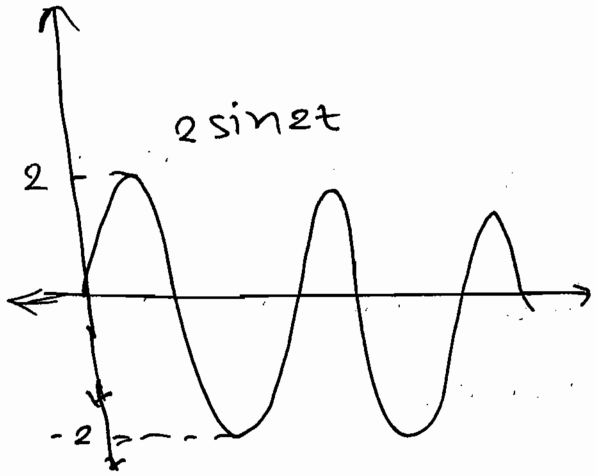
⑬ Freq. reuse is Possible.

☆ Random Signal Theory

=> The i/p to the Communication receiver is signal & noise. The signal is completely deterministic but the noise voltage changes randomly w.r.t. time. So Probability concepts are used to analyze a random signal.

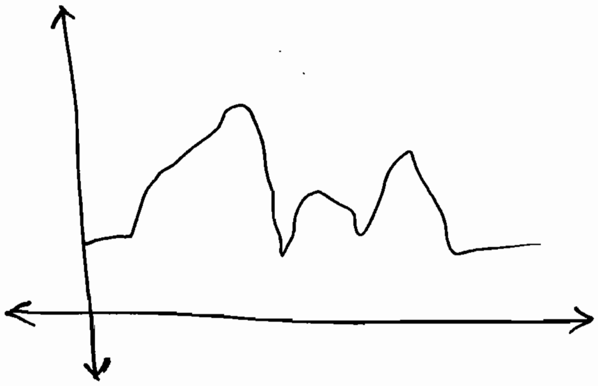
=> Deterministic signals:





-> At any instant of time it can be determine.

=> Random signal:



Impossible to represent Random ~~signal~~ Variations mathematically.

~~★ Random Signal Theory~~:

* Probability Distribution Function (PDF)

(or) Cummulative Distribution Function (CDF).

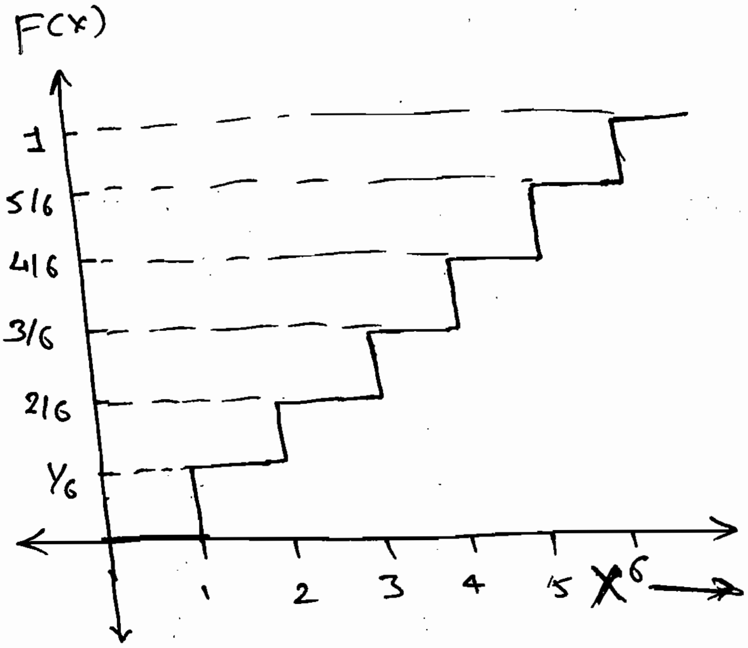
⇒ PDF mathmatically defined as,

$$F_X(x_1) = P[X \le x_1]$$

$$F_X(5) = P[X \le 5].$$

Dice:

X	P[X=x]
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6



$$F(0.99) = P[x \le 0.99] = 0$$

$$F(1) = P[x \le 1] = 1/6.$$

$$F(1.8) = P[x \le 1.8] = 1/6$$

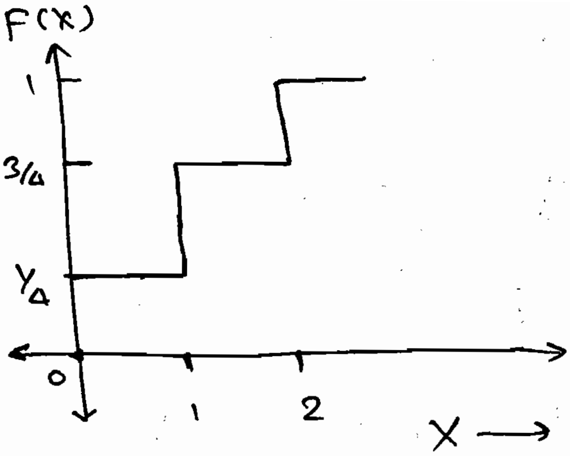
$$F[2] = P[x \le 2] = 2/6$$

Q A coin is tossed twice and the Random Variable X represent the number of Head. sketch the PDF.

Solⁿ:

		X
T	T	0
T	H	1
H	T	1
H	H	2

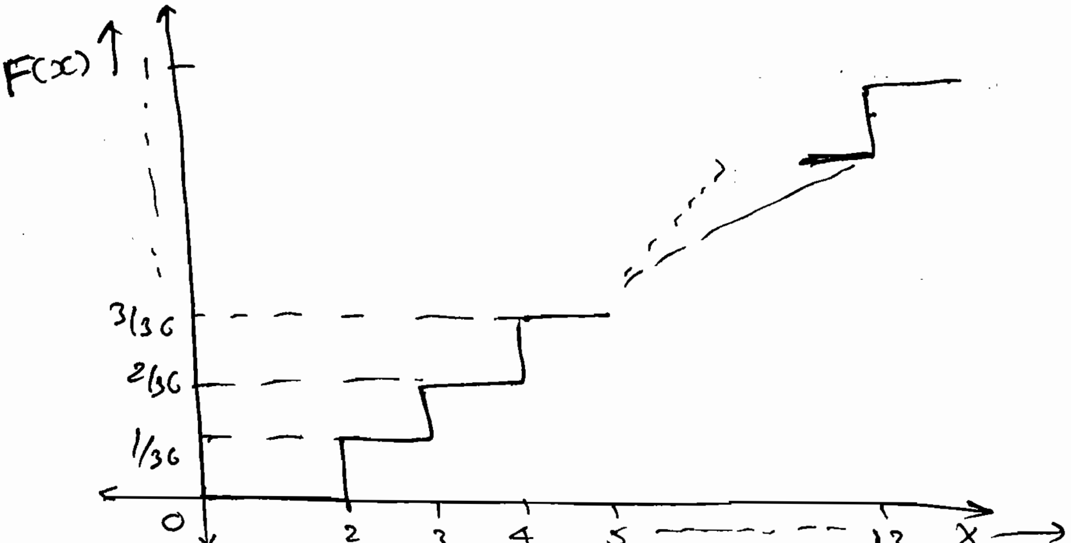
X	P[X=x]
0	1/4
1	1/2
2	1/4
	1



$\boxed{Q}$ Two dice are thrown simulteneonsly and Random Variable X represent the sum of the two outcomes. sketch the Probability Dis. fⁿ?

Ans:

X	P[X=x]	
2	1/36	(1,1)
3	2/36	(1,2), (2,1)
4	3/36	(1,3), (3,1), (2,2)
5	4/36	
6	5/36	
7		
8		
9		
10		
11		
12	1/36	(6,6).



* Properties of PDF :

① $0 \le F(x) \le 1$

② $F(x)$ is a increasing function of x.

③ $F(\infty) = 1$ & $F[-\infty] = 0$.

$P[x \le \infty] = 1$ & $P[x \le -\infty] = 0$.

④ $P[x > x_1] = 1 - P[x \le x_1] = 1 - F[x_1]$.

⑤ $P[x_1 < x \le x_2] = F[x_2] - F[x_1]$

e.g. $P[3 < x \le 5] = F[5] - F[3]$

$= 5/6 - 3/6$

$= 2/6 = 1/3$.

⑥ $P[x = x_1] = F[x_1] - F[\underset{\downarrow}{x_1^-}]$

Small decremental value

e.g. $P[x = 2] = F[2] - F[2^-]$

$= F[2] - F[1.99]$

$= 2/6 - 1/6$.

$= 1/6$.

***

$P[x = 3.4] = F[3.4] - F[3.4^-]$

$= F[3.4] - F[3.399]$

$= 3/6 - 3/6$

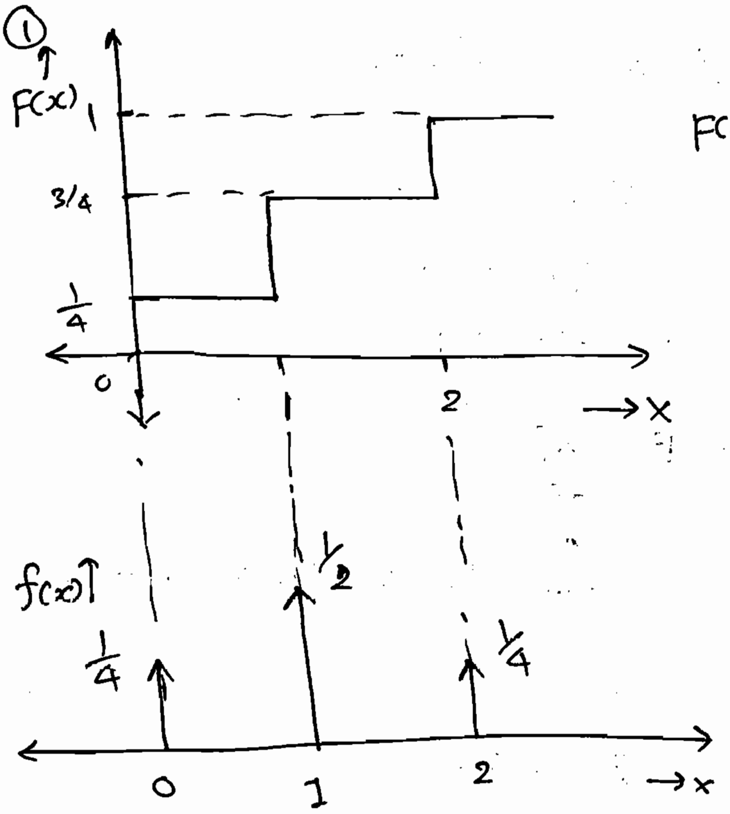
$= 0$.

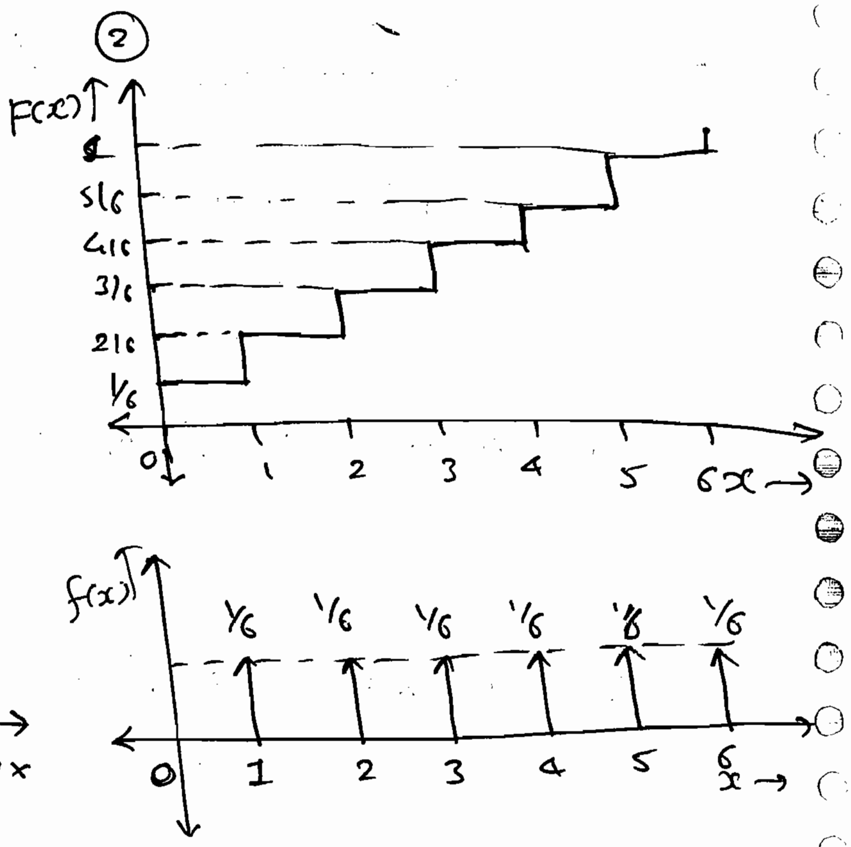
* Probability density function (pdf)

small letters

→ pdf indicates the distribution of total probability to various random variables.

$$\boxed{f(x) = \frac{d}{dx} F(x).}$$





* Properties of pdf:

① $\int_{-\infty}^{\infty} f(x).\, dx = 1$ or Total area = 1 Imp.

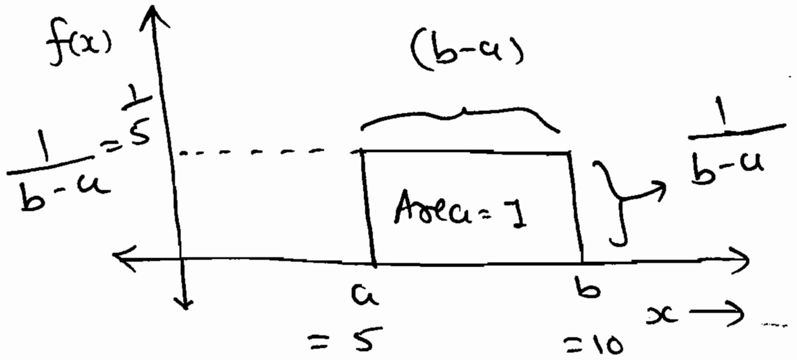
② $P[x \le x_1] = F(x_1) = \int_{-\infty}^{x_1} f(x)\, dx$

③ $P[x > x_1] = 1 - F(x_1) = \int_{x_1}^{\infty} f(x)\, dx.$

④ $P[x_1 < x \le x_2] = \int_{x_1}^{x_2} f(x)dx.$

① Uniform Density Function:

⇒ If pdf is constant within specified range, then the Random variable is said to be uniformly distributed.



Area

$A = (b-a) \times \frac{1}{(b-a)}$

$= 1$

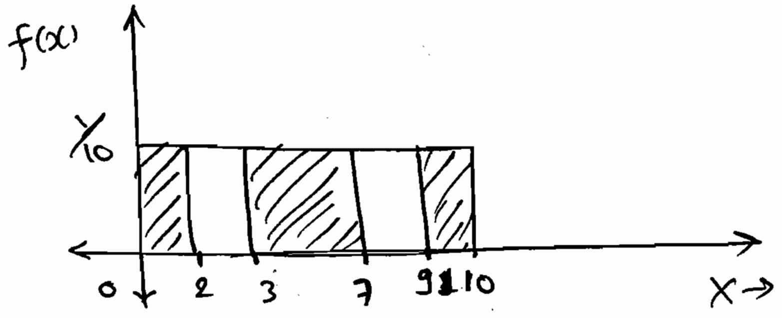
$\boxed{Q}$ A continuous R.V. is uniformly distributed in the [0,10] ① Sketch the pdf & determine the ~~p~~ following probability.

i) $P[x \le 2]$ ② Sketch the PDF.

ii) $P[x > 9]$

iii) $P[3 < x \le 7]$.

Sol^n:



① $P[x \le 2] = \int_{-\infty}^{2} f(x).dx = \frac{1}{10} \times 2 = 0.2$

(ii) $P[x > 9] = \int_{9}^{\infty} f(x).dx = (10-9) \times \frac{1}{10} = 0.1.$

(iii) $P[3 < x \leq 7] = \int_{3}^{7} f(x).dx = \frac{1}{10} \times [7-3]$

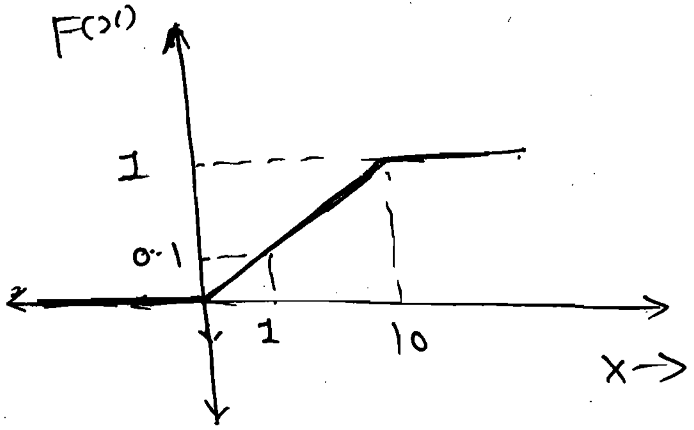
$= 0.4.$

~~Q~~ ②

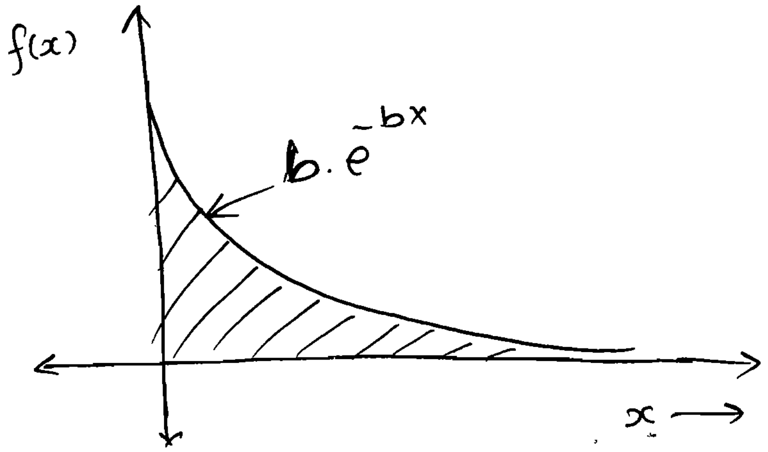
$$F(x) = \int_{-\infty}^{x} f(x).dx.$$

$$= \int_{0}^{\cancel{10}\,x} \frac{1}{10}.dx.$$

$$F(x) = \frac{x}{10}.$$



② Exponential Density Function:



$f(x) = b.e^{-bx}, \quad x \geq 0$

$= 0, \quad x < 0.$

$$\int_{0}^{\infty} a.e^{-bx}.dx = 1.$$

$$\therefore \frac{a}{b}.[0 - e^{-bx}]_{0}^{\infty} = 1.$$

$\Rightarrow \frac{a}{b} = 1 \Rightarrow \boxed{a=b}$ Cond^n for valid Pdf.

* Identity valid Pdf:

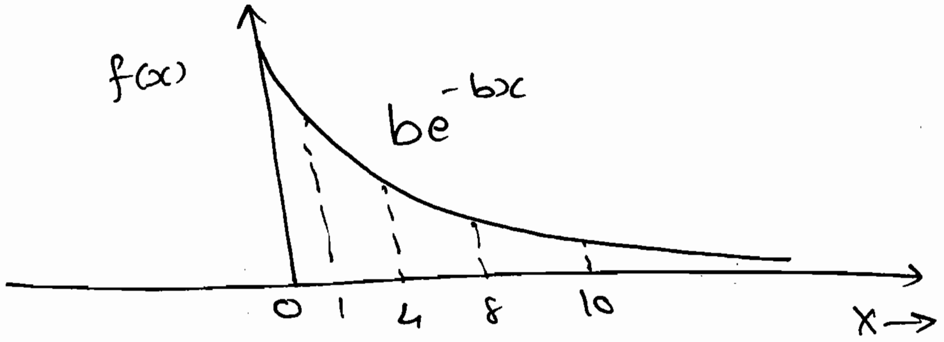
→ $f(x) = 10e^{-10x}$, $x \geq 0$ ✓ (valid). [∵ $a = b$]

→ $f(x) = 5e^{-10x}$, $x \geq 0$ ✗ (∵ $a \neq b \Rightarrow 10 \neq 5$].

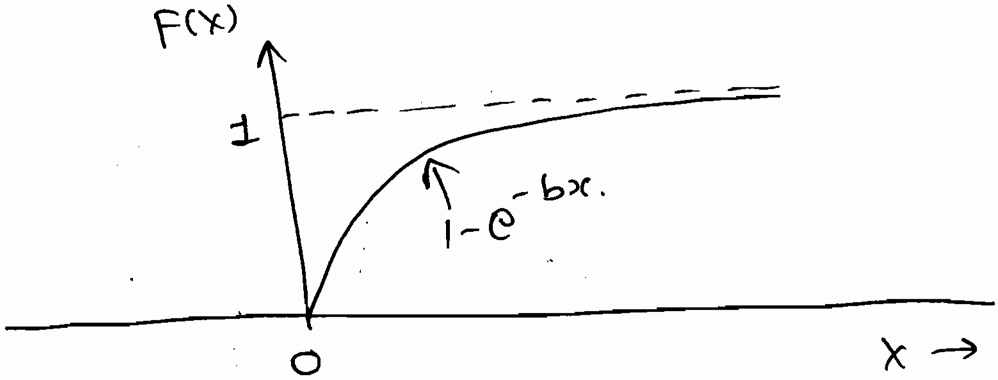
(Not valid)

⇒

pdf



PDF



$$\Rightarrow F(X) = \int_{-\infty}^{x} f(x).\,dx$$

$$= \int_{\overset{0}{\cancel{-\infty}}}^{x} be^{-bx}.\,dx$$

$$= -\frac{b}{b}\left[e^{-bx}\right]_{0}^{x}$$

$$\boxed{F(x) = [1 - e^{-bx}]}$$

③ Laplacian Density Function:

→ $f(x) = a\cdot e^{-b|x|}, \quad -\infty < x < \infty$

$= a\cdot e^{-bx}. \quad x > 0$

$= a.\ e^{bx}, \quad x < 0.$

Cond^n for valid pdf:

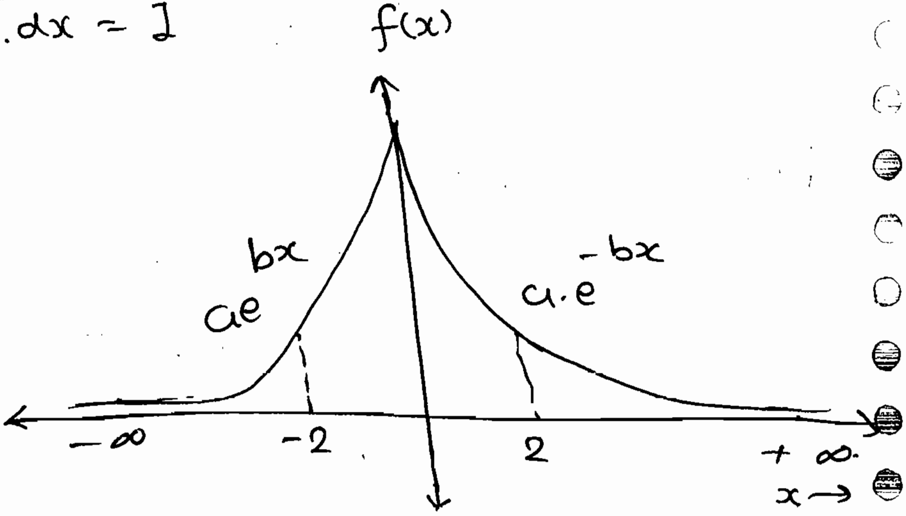
=> $\int_{-\infty}^{\infty} f(x) = 1$

$\int_{-\infty}^{0} e^{-bx}.dx + \int_{0}^{\infty} e^{bx}.dx = 1$

∴ $\frac{a}{b} + \frac{a}{b} = 1$

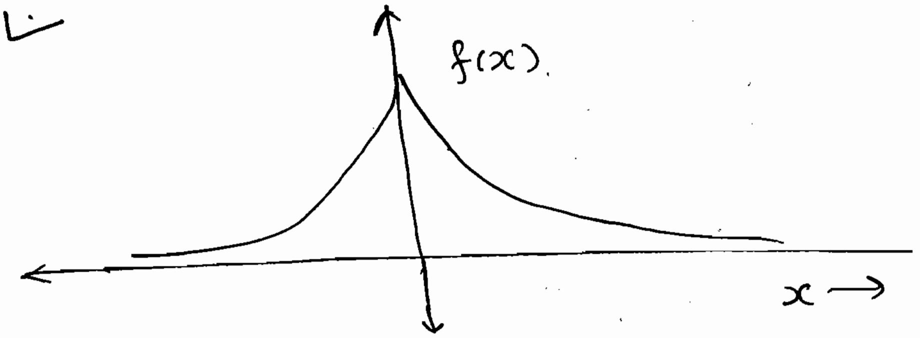
$\frac{2a}{b} = 1.$

=> $\boxed{a = b/2}$

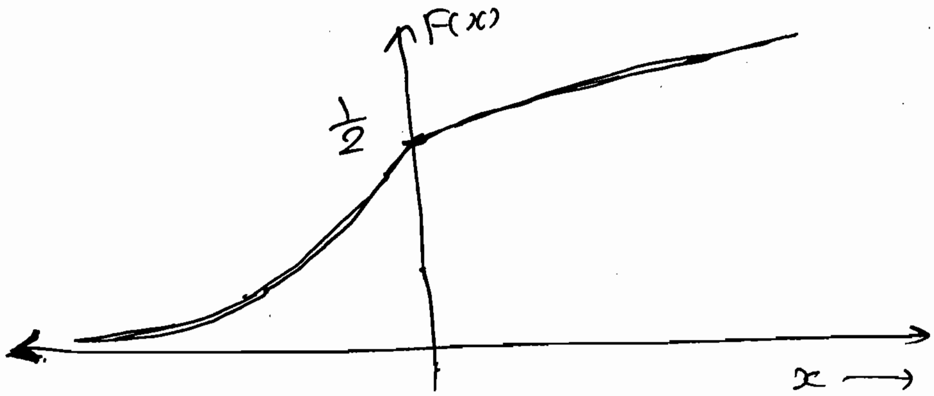


Q Sketch F(x) from f(x). given.

Gate ✓



Sol^n:



* Statistical Averages of a Random Variable.

① Mean (or) Average Value:

→ $\bar{X} = E[X] = m_1 = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$ → dc Component only.

② Mean Square Value:

→ $\overline{X^2} = E[X^2] = m_2 = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$ → Total Power.

③ Variance:

→ $\sigma^2 = E[(X-\bar{X})^2] = m_2 - m_1^2$ = ac Power

= (mean square − square of mean).

④ Standard Deviation:

→ SD = $\sqrt{\text{Variance}} = \sigma = \sqrt{m_2 - m_1^2}$

= $\sqrt{\text{ac power}}$

= rms value of ac component.

* Properties of Mean:

① $E[K] = K$, $K = \text{constant}$

② $E[Kx] = K \cdot \bar{x}$

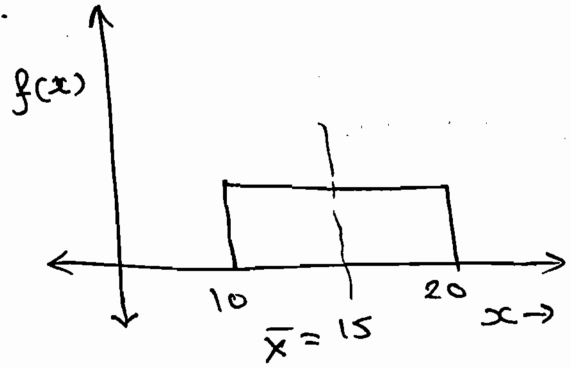
③ $E[x+K] = E[x] + E[K] = \bar{x} + K$

④ $E[x+y] = E[x] + E[y]$.

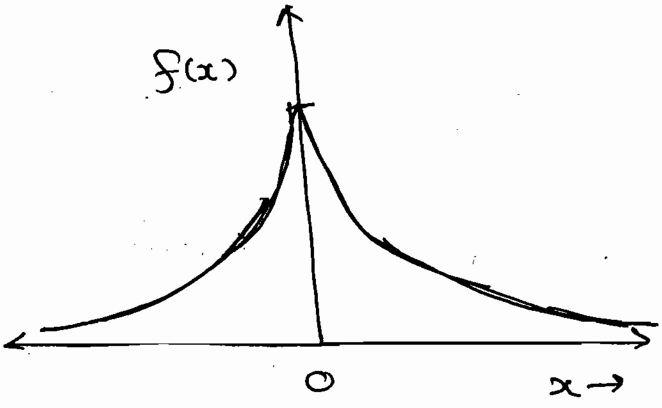
⑤ $E[xy] = E[x] \cdot E[y]$ provided that x & y are independent R.V.

⑥ If the Probability density function is summetrical w.r.t. a x-axis then the average value (mean) is equal to the summetry point.

e.g.



$\bar{x} = 15$



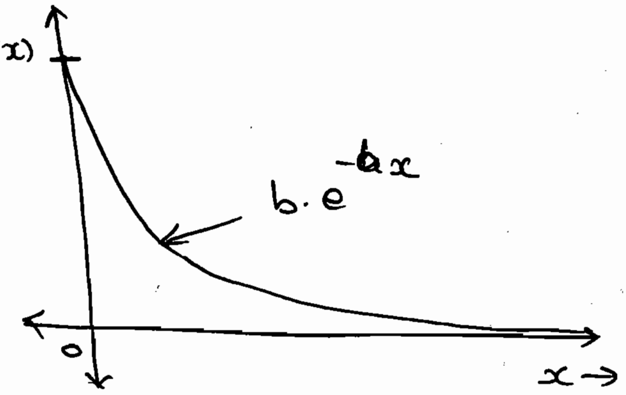
$\bar{x} = 0$.

Q-1 Determine the statistical avg. of exp. f^n.

Ans: $f(x) = b \cdot e^{-bx}$.

① mean

$$\bar{X} = E[X] = \int_0^\infty x \cdot f(x) \cdot dx$$



$$= \int_0^\infty x \cdot b \cdot e^{-bx} \cdot dx.$$

$$= b \cdot \left[(x)\left(\frac{e^{-bx}}{-b}\right) - (1)\cdot\left(\frac{e^{-bx}}{b^2}\right) \right]_0^\infty$$

$$= b\left[0 - 0 + \frac{1}{b^2} \right]$$

$$\boxed{\bar{X} = \frac{1}{b}} = m_1$$

② Mean square,

$$m_2 = \overline{X^2} = E[x^2] = \int_0^\infty x^2 \cdot b \cdot e^{-bx} \cdot dx.$$

$$= b\left[(x^2)\cdot\left(\frac{e^{-bx}}{-b}\right) - (2x)\left(\frac{e^{-bx}}{b^2}\right) + (2)\left(-\frac{e^{-bx}}{b^3}\right) \right]_0^\infty$$

$$= b\left[0 + \frac{2}{b^3} \right]$$

$$\boxed{m_2 = \frac{2}{b^2}}$$

③ Variance $= \sigma^2 = m_2 - m_1^2$

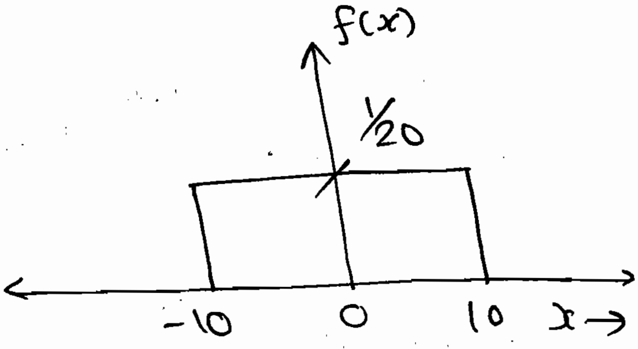
$$\sigma^2 = \frac{2}{b^2} - \frac{1}{b^2} = \frac{1}{b^2}.$$

④ Standard deviation

$$S.D. = \sqrt{m_2 - m_1^2} \Rightarrow \boxed{S.D. = \frac{1}{b}}$$

Q-2 A continous R.V. is uniformly distn. in the [-10,10]. Determine the statistical average.

Soln:



① $f(x) = \frac{1}{20}$.

① Mean

$$m_1 = \bar{X} = E[X] = \int_{-10}^{10} X \cdot \frac{1}{20} \cdot dx$$

$$\boxed{m_1 = 0}$$

② Mean square,

$$m_2 = \overline{X^2} = E[x^2] = \int_{-10}^{10} x^2 \cdot \frac{1}{20} \cdot dx.$$

$$m_2 = \frac{2}{20} \int_{0}^{10} x^2 \cdot dx = \frac{1}{10} \times \frac{1000}{3} = \frac{100}{3}.$$

$$\boxed{m_2 = \frac{100}{3}}$$

③ Variance,

$$\sigma^2 = \left(\sqrt{m_2 - m_1^2}\right)^2$$

$$\sigma^2 = m_2 - m_1^2$$

$$= \frac{100}{3} - 0$$

$$\boxed{\sigma^2 = \frac{100}{3}}$$

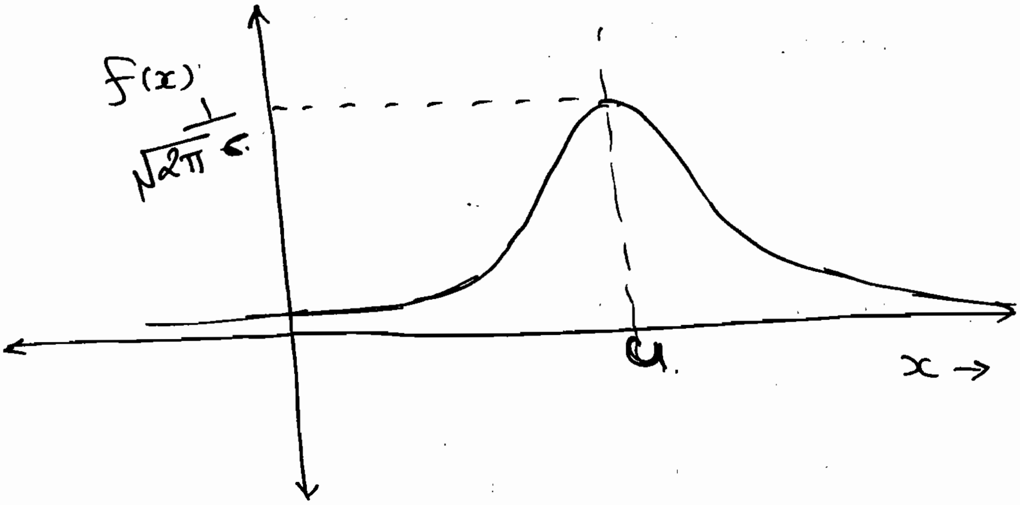
④ S.D. = $\sqrt{\text{variance}}$.

$$\boxed{S.D. = \frac{10}{\sqrt{3}}}$$

④ Gaussian Density Function:

⇒ Pdf $\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}}}$ $-\infty < x < \infty$.

①



⇒ ① Mean:

f(x) is symmetrical w.r.t. x axis at x=a.

So, mean = $m_1 = E[X] =$ 'a'. ⇒ $\boxed{m_1 = a}$

② $m_2 = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx =$ '$\sigma^2 + a^2$'

⇒ $\boxed{m_2 = \sigma^2 + a^2}$.

③ Variance $\sigma^2 = m_2 - m_1^2$

Variance = $\sigma^2 + a^2 - a^2 = \sigma^2$

∴ $\boxed{\text{Variance} = \sigma^2}$

④ $S.D. = \sigma = \sqrt{\text{Variance}}$

$$\boxed{S.D. = \sigma}$$

$\boxed{Q}$ find m_2 of $f(x) = \frac{1}{\sqrt{(\ \ )}} \cdot e^{-\frac{(x-2)^2}{100}}$.

Soln: Here, $a = 2$, $2\sigma^2 = 100$

$\sigma^2 = \frac{100}{2}$

$$\boxed{\sigma^2 = 50}$$

S.D. = $\sqrt{\sigma^2}$

$$\boxed{S.D. = 5\sqrt{2}}$$

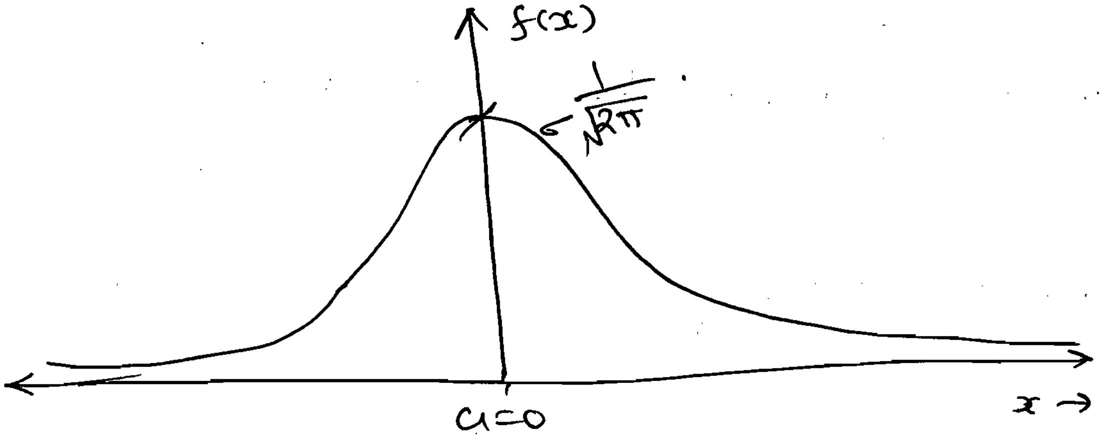
$\therefore m_2 = \sigma^2 + a^2$

$= 50 + 4$

$\boxed{m_2 = 54}$ $\boxed{m_1 = a = 2}$

$\Rightarrow$ if $a = 0$, then

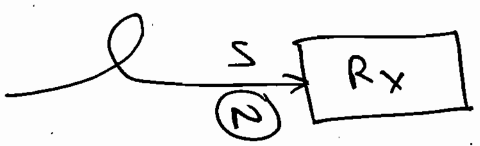
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$



★ Random Process:

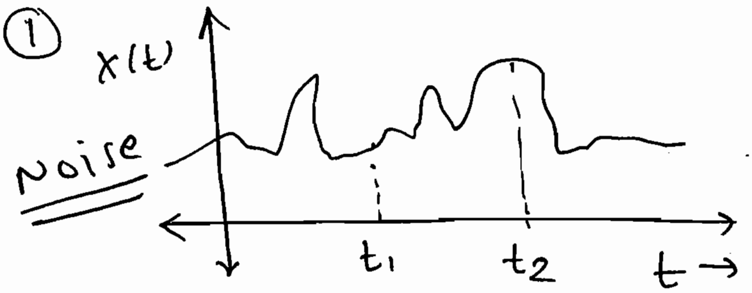
⇒ A random Variable which is function of time is called as the Random Process.

⇒ Random Process $= X(t)$.

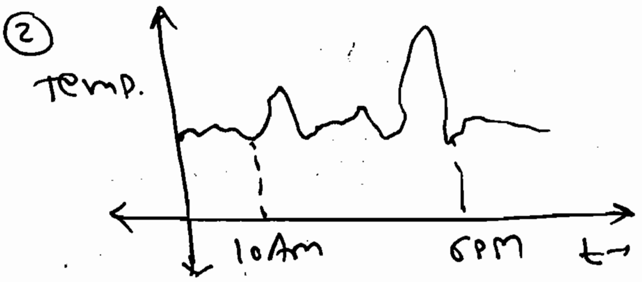


e.g.

①



②



* PDF of a Random Process:

$\rightarrow X \rightarrow F[x_1] = P[X \le x_1]$.

$\rightarrow X(t) \rightarrow F[x_1\, t_1] = P[X(t) \le x_1]$

→ 1st order PDF.

$$F[x_1, t_1] = P[X(t_1) \le 2mv]$$

$$= P[X(2PM) \le 25^\circ C].$$

$$= P[X[2Am) \le 25^\circ C]$$

$\rightarrow F[x_1 x_2 t_1 t_2] = P[X(t_1) \le x_1 \;\&\; X[t_2) \le x_2]$.

→ 2nd order PDF.

* Pdf of a Random Process:

→ $f(x) = \frac{d}{dx} F(x)$.

→ $f(x_1 t_1) = \frac{\partial}{\partial x} F(x_1 t_1)$ → 1^{st} order.

→ $f(x_1 x_2 t_1 t_2) = \frac{\partial^2}{\partial x_1 \partial x_2} F[x_1 x_2 t_1 t_2]$

→ 2^{nd} order.

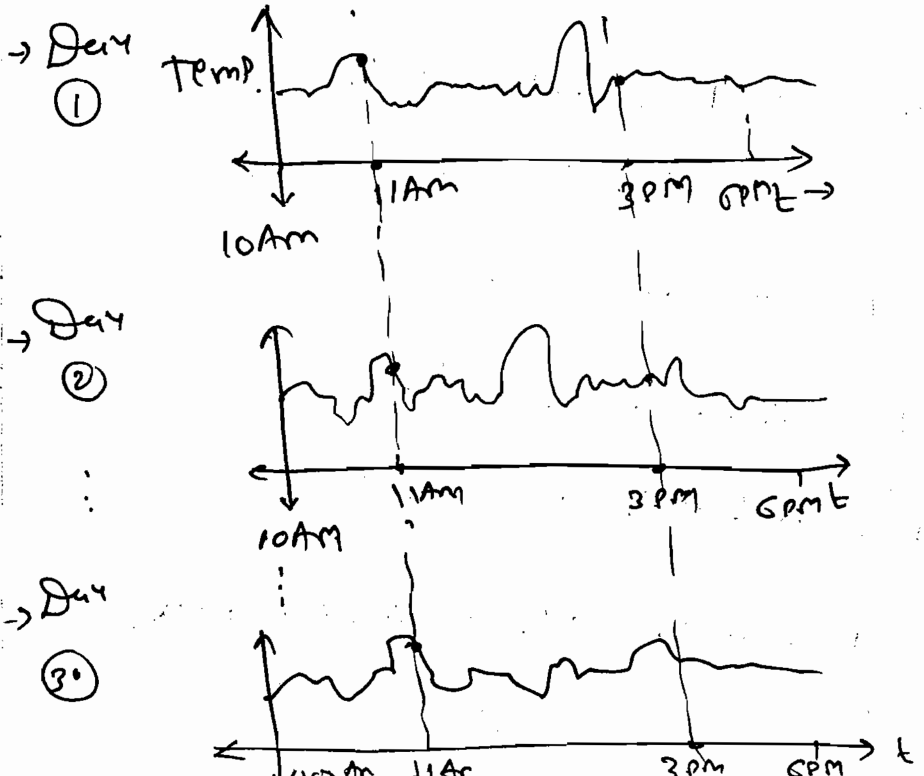
* Statistical averages of a Random Process:

⇒ ① Ensemble averages.

② Time averages.

⇒ Ensemble means collection of data.

① Ensemble averages:



① Mean,

$$m_1(t) = E[X(t)] = \int_{-\infty}^{\infty} X(t) \cdot f(xt) \cdot \underline{\underline{dx}}$$

② Mean Square,

$$m_2(t) = E[X^2(t)] = \int_{-\infty}^{\infty} X^2(t) \cdot f(xt) \, \underline{\underline{dx}}.$$

③ Variance,

$$= m_2(t) - m_1^2(t).$$

④ Standard Deviation,

$$S.D. = \sqrt{Variance}$$

$$S.D. = \sqrt{m_2(t) - m_1^2(t)}.$$

⑤ <u>Auto Corelation Function:</u>

$$ACF = E[X(t_1) \cdot X(t_2)].$$

→ ACF is used to determine the simillarity of a Random Process at two instant of time,

$$\boxed{ACF = E[X(t) \cdot X(t+\tau)]}$$

$$t_1 \to t.$$

$$t_2 \to t+\tau.$$

② Time averages:

⇒ ① mean,

$$< x(t) > = m_1 = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} x(t) \, dt$$

② Mean square,

$$< x^2(t) > = m_2 = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} x^2(t) \, dt$$

③ Variance, $= m_2 - m_1^2$

④ S.D. $= \sqrt{m_2 - m_1^2}$.

⑤ $ACF = < x(t) \cdot x(t+\tau) >$

$t_1 \to t$
$t_2 \to t+\tau$.

$$ACF = \frac{1}{\tau} \int_{t}^{t+\tau} x(t) \cdot x(t+\tau) \, dt.$$

⇒ If the Ensemble avgs and Time avgs are equal then the Random Process is called Ergodic Random process.

⇒ A random Process is said to be stationary of order one if the following condition is satisfied,

$\Rightarrow$ $\boxed{f(x_1, t_1) = f(x_1\ t_1+\Delta t).}$

↑ Stationary of order one.

$\Rightarrow$ $\boxed{f(x_1 x_2 t_1 t_2) = f(x_1 x_2\ t_1+\Delta t\ t_2+\Delta t).}$

↑ Stationary of order two.

* Wide Sense Stationary:

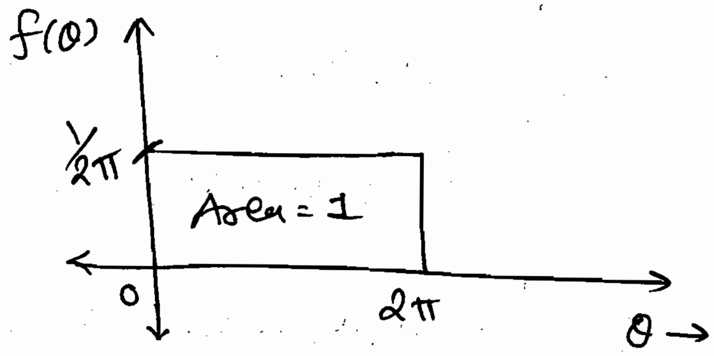
$\Rightarrow$ A random Process is said to be Wide sense stationary if the following two Conditions are satisfied:

$$\boxed{\begin{array}{l} ①\ E[X(t)] = \text{constant} \\ ②\ \text{The ACF should depends only on } t_2 - t_1 \text{ i.e. } \tau. \end{array}}$$

Q Consider a Random Process $X[t] = A\cos[\omega_0 t + \theta]$, where A & ω_0 are Constant and θ is a R.V. Uniformally distributed in $[0, 2\pi]$. Determine the mean & ACF and show that Random Process is WSS.

Ans: $X(t) = A\cos[\omega_0 t + \theta]$.

$\Rightarrow \quad f(\theta) = \frac{1}{2\pi}.$



$$\rightarrow E[X] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

$$E[x(t)] = \int_{0}^{2\pi} x(t) \cdot f(\theta) \cdot d\theta.$$

$$= \int_{0}^{2\pi} A\cos[\omega_0 t + \theta] \cdot \frac{1}{2\pi} \cdot d\theta.$$

$\Rightarrow \boxed{E[x(t)] = 0} \leftarrow$ Ensemble mean.

$$\Rightarrow \quad ACF = E[x(t_1) \cdot x(t_2)].$$

$$= E\left[A\cos[\omega_0 t_1 + \theta] \cdot A\cos[\omega_0 t_2 + \theta]\right]$$

$$= E\left[\frac{A^2}{2}\cos[\omega_0(t_1+t_2) + 2\theta\right.$$
$$\left. + \frac{A^2}{2}\cos[\omega_0(t_1-t_2)]\right].$$

$$= E\left[\frac{A^2}{2} \cdot \cos[\omega_0(t_2+t_1) + 2\theta]\right]$$
$$+ E\left[\frac{A^2}{2} \cdot \cos[\omega_0(t_2-t_1)]\right].$$

$$= 0 + \frac{A^2}{2} \cdot \cos[\omega_0(t_2-t_1)].$$

$\Rightarrow$ which is independent of R.V. i.e here θ.

So, ① $E[x(t)] = 0 =$ constant.

② ACF is depends on $(t_2 - t_1)$.

So, Random Process in WSS.

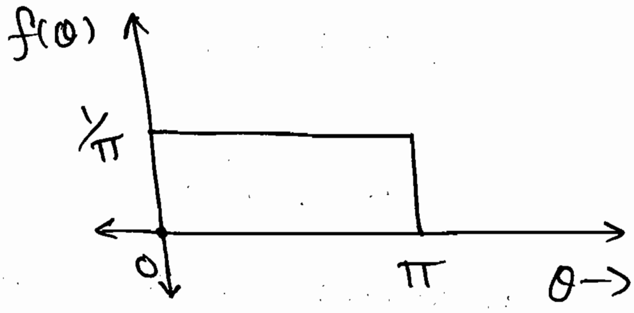
$$ACF = \frac{A^2}{2} \cos \omega_0 \tau$$

$$\boxed{ACF = \frac{A^2}{2} \cos 2\pi f_0 \tau. = R(\tau).}$$

Q-2 Consider a Random Process $x(t) = A\cos[\omega_0 t + \theta]$, A & ω_0 are Constants and θ is a R.V. uniformally distributed in the $[0, \pi]$. check weither the R.P. is WSS or not?

Soln: $x(t) = A\cos[\omega_0 t + \theta]$.

→ $f(\theta) = \frac{1}{\pi}$.



① Ensemble mean,

$$E[x(t)] = \int_0^{\pi} A\cos[\omega_0 t + \theta] . \frac{1}{\pi} \, d\theta.$$

$$= \int_0^{\frac{\pi}{2}} A\cos(\omega_0 t + \theta) \, d\theta - \int_{\pi/2}^{\pi} A\cos(\omega_0 t + \theta) \, d\theta.$$

$$= A\left[\sin(\omega_0 t + \theta)\right]_0^{\frac{\pi}{2}} - A \left[\sin(\omega_0 t + \theta)\right]_{\pi/2}^{\pi}$$

$$= A\left[\cos\omega_0 t - \sin \cancel{\omega_0 t} + \cancel{\sin}\,\omega_0 t + \cos\omega_0 t\right]$$

$$= 2A\cos\omega_0 t \qquad \boxed{ACF = \frac{A^2}{2}\cos 2\pi f_0 \tau}$$

$\Rightarrow E[x(t)] = 2A\cos\omega_0 t$

which is not constant.

So, Random Process is not WSS.

* Properties of ACF:

$\Rightarrow \quad ACF = R(\tau) = E[x(t).\,x(t+\tau)].$

① $R(0) = E[x^2(t)]$

$\Rightarrow ACF = R(0) =$ Mean square = Power.

② The Fourier Transform of ACF is nothing but the Power spectral Density (PSD).

$$ACF \xleftrightarrow{FT} PSD \quad (W/Hz)$$

$$\Rightarrow \boxed{R(\tau) \xleftrightarrow{FT} S(f).}$$

$\Rightarrow$ PSD indicates how the total Power is distributed to various freq.

$$\Rightarrow \quad \boxed{S(f) = \int_{-\infty}^{\infty} R(\tau) \cdot e^{-j2\pi f\tau} \cdot d\tau .}$$

$$\boxed{R(\tau) = \int_{-\infty}^{\infty} S(\underline{\underline{f}}) \cdot e^{j2\pi f\tau} \cdot \underline{\underline{df}} .} \quad \leftarrow \underline{\underline{\text{H.B.}}}$$

$$\underline{\omega = 2\pi f}$$

$$f = \omega / 2\pi$$

IMP

$$\therefore \quad R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\underline{\underline{\omega}}) \cdot e^{j2\pi f\tau} \cdot d\underline{\underline{\omega}} .$$

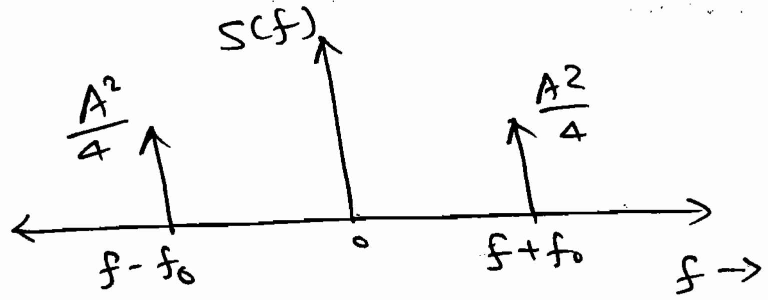
* $x(t) = A\cos(\omega_0 t + \theta)$.

$\Rightarrow$ & $ACF = R(\tau) = \frac{A^2}{2} \cos 2\pi f_0 \tau$.

$\rightarrow S(f) = \frac{A^2}{4} [\delta(f - f_0) + \delta(f + f_0)]$.

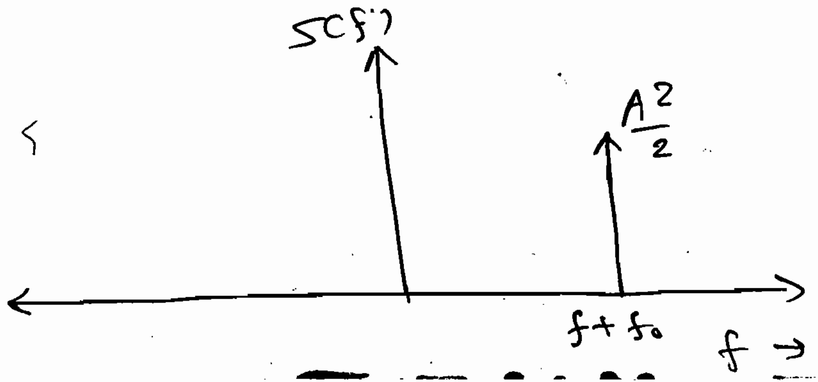
PSD

$\Longrightarrow$



(a) Two sided PSD.

$\Rightarrow$



(b) ~~Two~~ single sided PSD.

⇒ ③ $R(0) = \int_{-\infty}^{\infty} S(f).df$ = Power

→ Area under the PSD should gives Power.

→ Noise Power = Mean square

$= \int x^2 \cdot f(x)\, dx.$

$= R(0).$

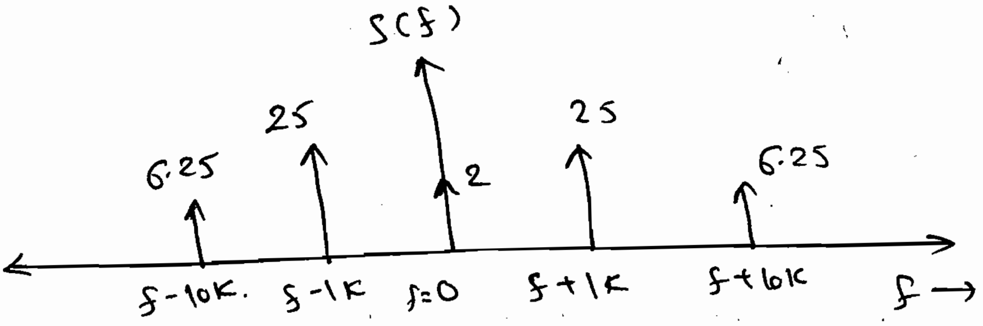
$= \int (PSD)\, \underline{\underline{df}}$

$= \frac{1}{2\pi} \int (PSD)\, \underline{\underline{d\omega}}.$

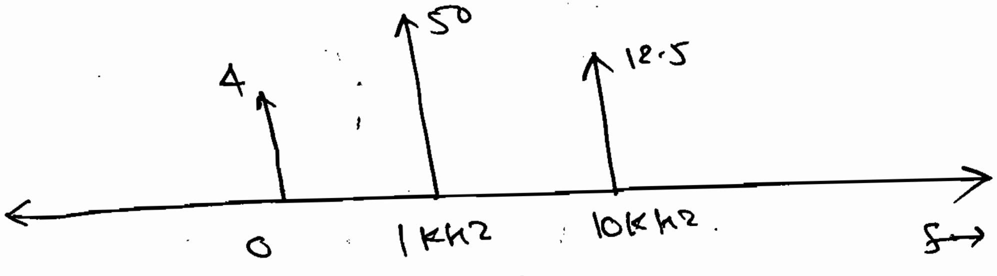
= Area under PSD.

* $\underline{\underline{2}} + 10\cos 2\pi 10^3 t + 5\cos 2\pi 10^4 t$

DC.
$f = 0Hz$

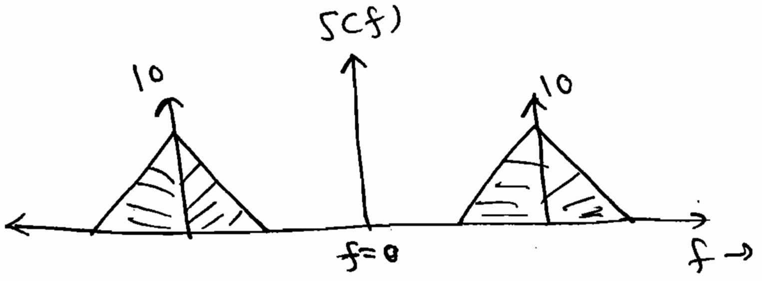


~~d~~· (Two sided PSD)



(single sided PSD)

⊛ IMP:



Mean = DC = impulse at f=0.

, otherwise mean = 0.

☆ Noise:-

* Generalized defⁿ:

⇒ Any unwanted signal interfering with the required signal is called as noise.

⇒ Noise are classified into two types:

① External Noise.

② Internal Noise.

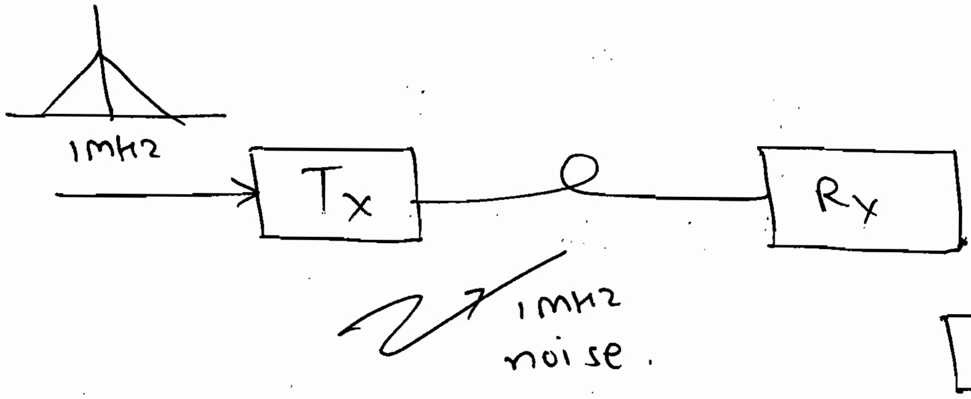
① External Noise:

⇒ Noise which is generated outside the system is called as the External Noise.

e.g. ① Atmospheric noise

→ Sudden electrical disturbutions.

⇒



$\boxed{S/N >> 1}$

② Solar noise

→ radiation from SUN.

③ Industrial Noise.

② Internal Noise:

⇒ The Noise which is generated within the System is called Internal Noise.

e.g.: Radiations from electrical appliances.

=> Whenever the noise is transmitted along with the signal intentationally to disturb the Communication system then it is called signal Jamming.

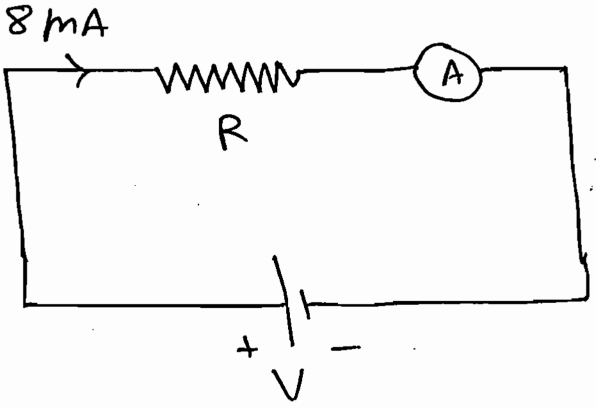
for e.g. Mobile Phone Jammer.

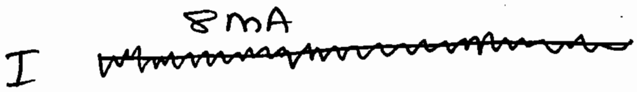
→ Transmit Noise signal at signal freq. having noise Power greater than signal Power. to Jam the desired signal.

☆ Internal Noise:

① Thermal Noise (or) White noise:

=>





=> Thermal Noise is given by following emperical formula.

$$\boxed{P = KTB \text{ watts.}}$$ ↙H.B.

where, K = Boltzman Constant

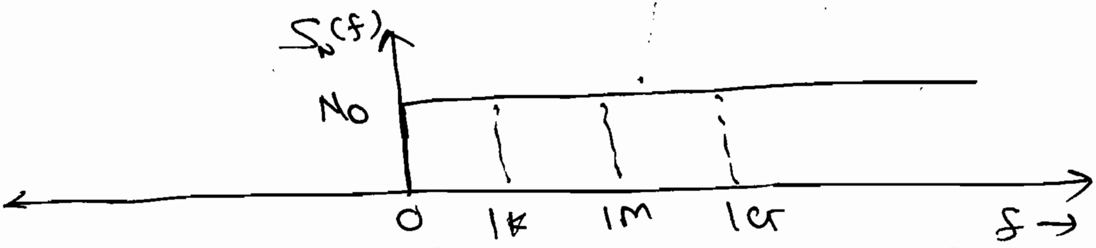
$= 1.38 \times 10^{-23}$ J/°K.

T = Temp.

B = Bandwidth.

$$\boxed{KT = \frac{\text{watts}}{\text{Hz}} = \text{PSD.}}$$ ← H.B.

→ $KT = N_o$ = PSD of Thermal Noise.



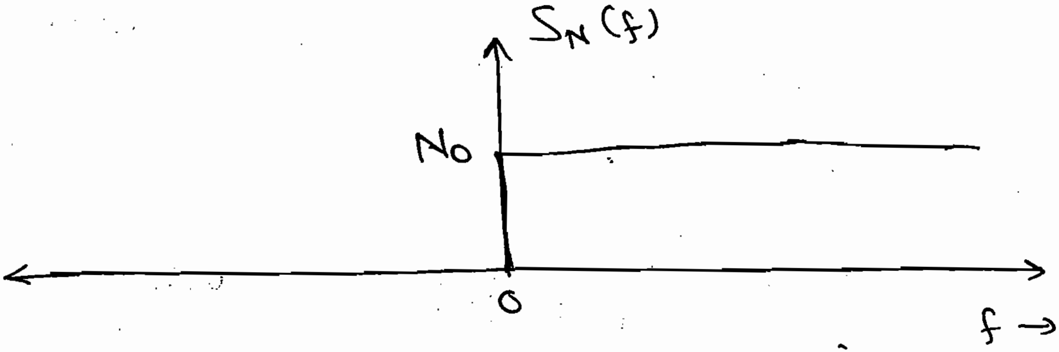
* Coloured Noise:

⇒ When noise affecting only narrow band of freqs then it is called coloured noise.

⇒ Noise is affecting all freqs. equally.

(∵ No is independent of freq.).

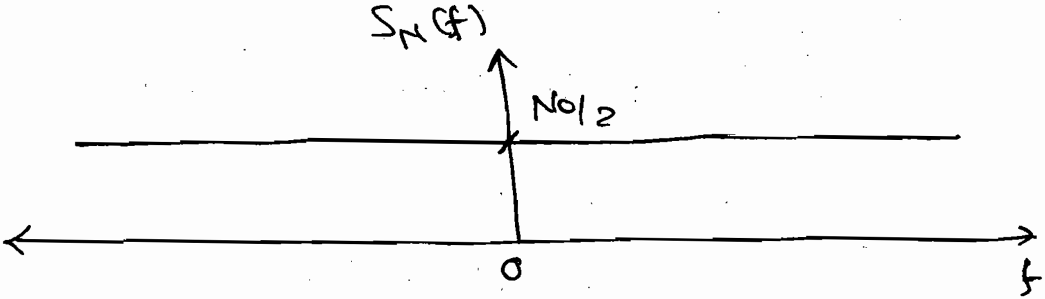
→



(single side PSD of Noise)

↙ H.B.

→

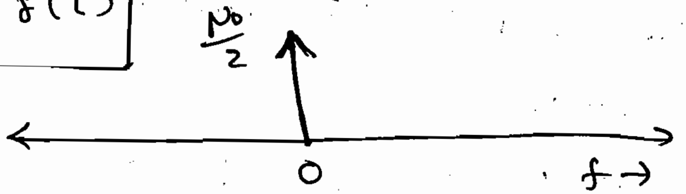


(Two Sided PSD of Noise)

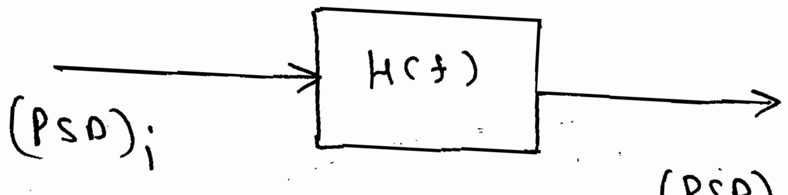
↕ IFF

H.B ↓

* * * $\boxed{R(\tau) = \frac{N_o}{2}\,\delta(\tau)}$



Most IMP topic

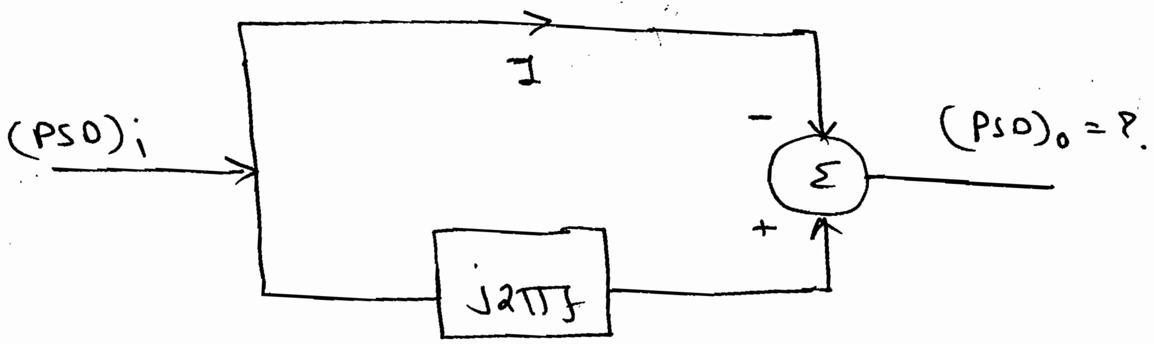


$$(PSD)_o = (PSD)_i \cdot |H(f)|^2.$$

$$\Rightarrow \boxed{PSD_o = (PSD)_i \cdot |H(f)|^2} \leftarrow \underline{\underline{H.B.}}$$

Gate:

Q



Soln:

$$H(f) = -1 + j2\pi f$$

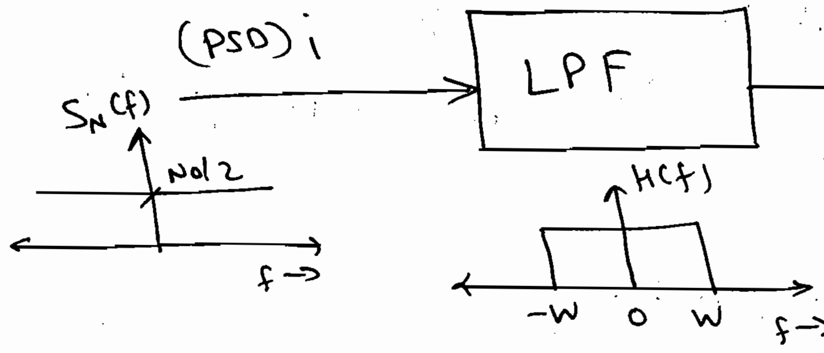
$$|H(f)| = \sqrt{1 + (2\pi f)^2}$$

$$|H(f)|^2 = 1 + (2\pi f)^2.$$

$$(PSD)_o = |H(f)|^2 \cdot (PSD)_i$$

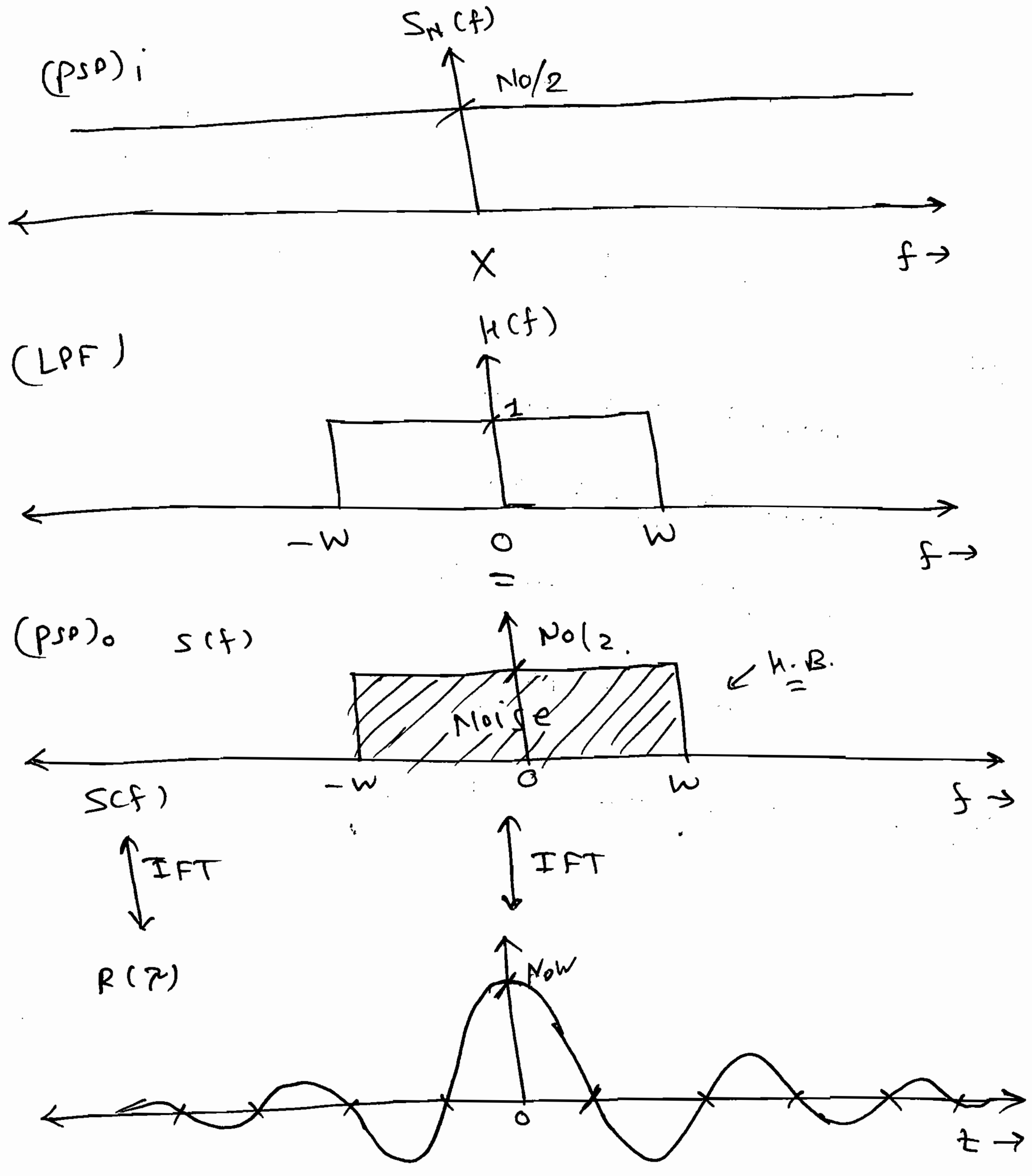
$$\therefore \boxed{(PSD)_o = (1 + 4\pi^2 f^2)(PSD)_i}$$

① When White Noise Passed through LPF:



$(PSD)_o = |H(f)|^2 \cdot (PSD)_i$

$|H(f)| = 1 \Rightarrow |H(f)|^2 = 1.$



$\Rightarrow$ Noise Power $= \int_{-W}^{W} \frac{N_0}{2} . df$

$\boxed{\text{Noise Power} = 'N_0.W' \text{ watts.}}$ ← H.B.

ACF ⇒ $R(\tau) = \frac{N_0}{2} \times 2W. \text{ sinc } [2W\tau].$

$\boxed{R(\tau) = N_0W. \text{ Sinc } (2W\tau).}$ ← H.B

$\Rightarrow$ $R(0)$ = Mean square = Power = N_0W watts.

$\boxed{Q}$ White noise having a two sided spetral density of 4×10^{-3} watts/Hz is passed through an ideal LPF having a cut off freq 2 kHz. Determine the noise power and ACF at the o/p of the filter.

Soln:

$\frac{N_0}{2} = 4 \times 10^{-3}$ watts/Hz

$\Rightarrow N_0 = 8 \times 10^{-3}$ watts/Hz.

$W = 2 \text{ kHz} = 2000 \text{ Hz.} = f$

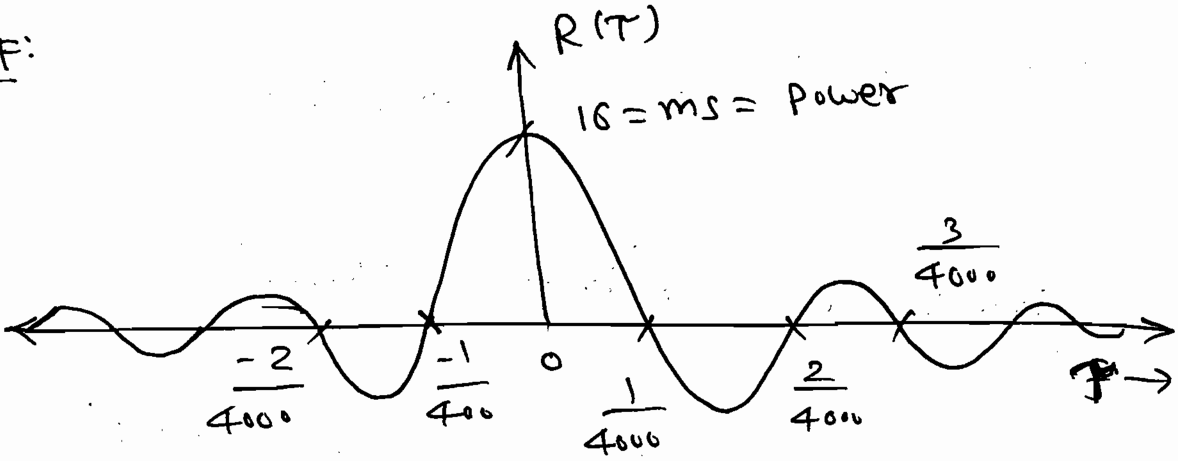
Noise Power $= N_0W$

$= 8 \times 10^{-3} \times 2 \times 10^{3}$

$\boxed{\text{Noise Power} = 16 \text{ watts.}}$

$\Rightarrow \boxed{\text{ACF} = R(\tau) = 16 \sin [4000\tau].}$

⇒ ACF:



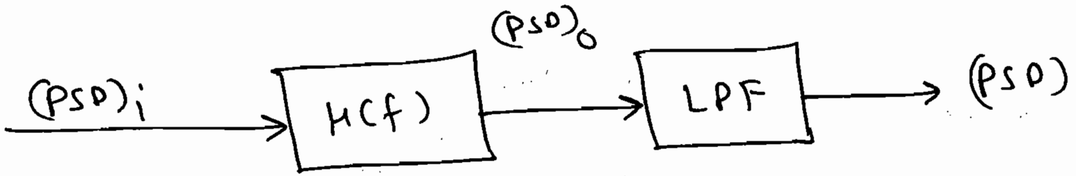
→ Spacing is $\left(\frac{2}{4000} - \frac{1}{4000}\right) = \frac{1}{4000} = \frac{1}{2W}$.

→ If the ACF is sampled at $\frac{1}{4000}, \frac{2}{4000}, \frac{3}{4000}$ at this, the samples are zero (or) this these samples are said to be uncorrelated.

$\boxed{Q}$ ** White Noise having a uniform spectrum density of N_0 watts/Hz is passed through a system having transfer function $H(f) = 2 \cdot e^{-j2\pi f t_0}$. The o/p of an ideal LPF having a cut-off freq of 'B' Hz. Determine Noise power at the o/p of the filter.

Gate-2005

Soln:



$H(f) = 2 \cdot e^{-j2\pi f t_0}$ $\quad f_c = B$.

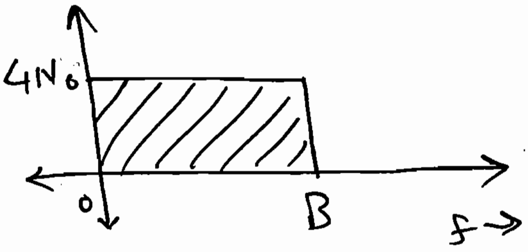
$\Rightarrow |H(f)| = 2$

$\Rightarrow |H(f)|^2 = 4$.

$\Rightarrow (PSD)_i = N_o$

$(PSD)_o = |H(f)|^2 \cdot (PSD)_i$

$\therefore (PSD)_o = 4N_o$



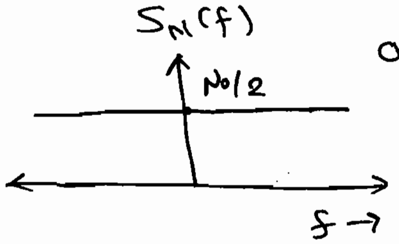
$\Rightarrow$ Noise power

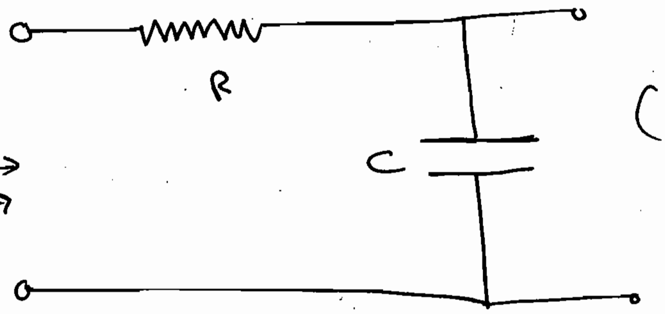
$$= \int_0^B 4N_o \cdot df$$

$$\boxed{\text{Noise power} = 4N_oB \text{ watts}}$$

* **When white noise passed through RC LPF.**

$\Rightarrow$





$(PSD)_o = |H(f)|^2 (PSD)_i$

$$V_o = \frac{\frac{1}{SC}}{R + \frac{1}{SC}} \cdot V_{in}$$

$$\therefore H(s) = \frac{V_o(S)}{V_{in}(S)} = \frac{1}{1 + RSC}.$$

$s = j\omega$

$$\therefore H(j\omega) = \frac{1}{1 + j\omega CR}.$$

$\omega = 2\pi f$

$$\therefore H(f) = \frac{1}{1 + j2\pi fRC}.$$

$$\therefore \quad |H(f)|^2 = \frac{1}{1 + (2\pi f RC)^2}.$$

$$\therefore (PSD)_0 = \frac{N_0}{2} \cdot \frac{1}{1 + (2\pi f RC)^2}.$$

$$\therefore (PSD)_0 = \frac{\frac{N_0}{2}}{1 + (\omega RC)^2} = \frac{\frac{N_0}{2(RC)^2}}{\omega^2 + \left(\frac{1}{RC}\right)^2}.$$

ACF $\quad \underline{\underline{S(\omega)}} \qquad\qquad \underline{\underline{R(\tau)}}$

$$\frac{2a}{\omega^2 + a^2} \xleftrightarrow{FT} e^{-a|t|}.$$

$$\Rightarrow \frac{N_0}{4RC} \cdot \frac{\frac{2}{RC}}{\omega^2 + \left(\frac{1}{RC}\right)^2} \xleftrightarrow{FT} \frac{N_0}{4RC} \cdot e^{-\frac{|\tau|}{RC}}.$$

$$\therefore \boxed{R(\tau) = \frac{N_0}{4RC} \cdot e^{-\frac{|\tau|}{RC}}} \leftarrow H.B.$$

Noise Power = mean square = $R(0)$

$$\boxed{\text{Noise Power} = \frac{N_0}{4RC}.} \leftarrow \underline{\underline{H.B.}}$$

$\boxed{Q}$ The white noise having a Two sided PSD of 4×10^{-3} W/Hz is passed through simple RC LPF ckt. having 3db cut off freq of ~~the~~ ~~o/p~~ ~~of~~ ~~RC~~ ~~ckt~~ ~~the~~ 2kHz. Determine the noise power at the o/p of RC ckt.

Solⁿ:

$$\frac{N_o}{2} = 4 \times 10^{-3}$$

$$\therefore \underline{N_o} = 8 \times 10^{-3} \quad \omega/Hz.$$

$\therefore$ Now, $$f_{3dB} = \frac{1}{2\pi RC} \quad \text{for RC LPF.}$$

$$\therefore \quad 2000 = \frac{1}{2\pi RC}$$

$$\Rightarrow \frac{1}{RC} = 4\pi \times 1000$$

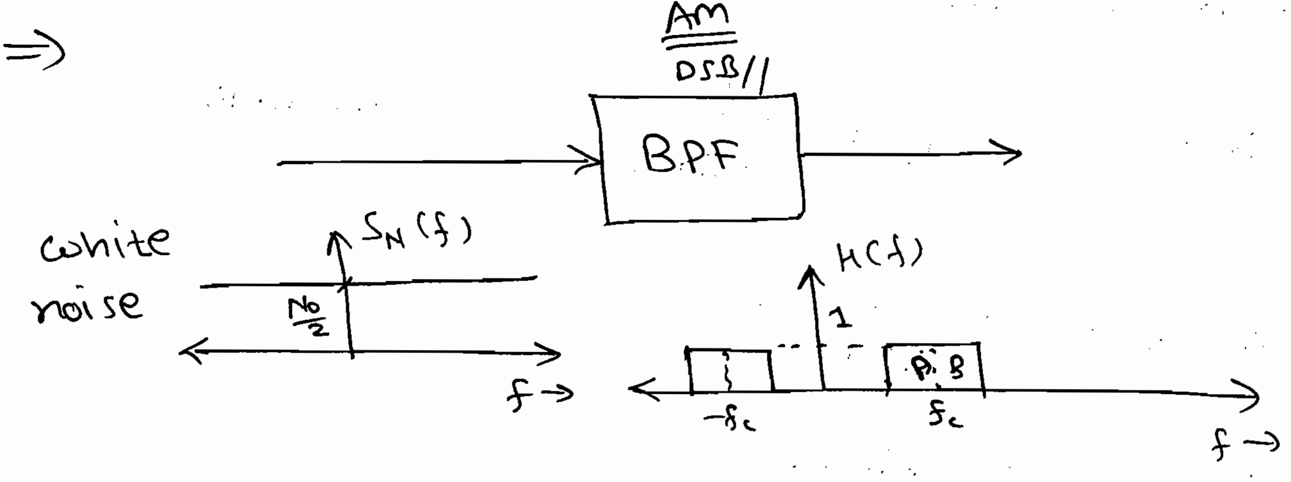
Now, Noise Power $= \frac{4 N_o}{4RC}$ watts

$$= \frac{8 \times 10^{-3}}{4} \times 4\pi \times 1000$$

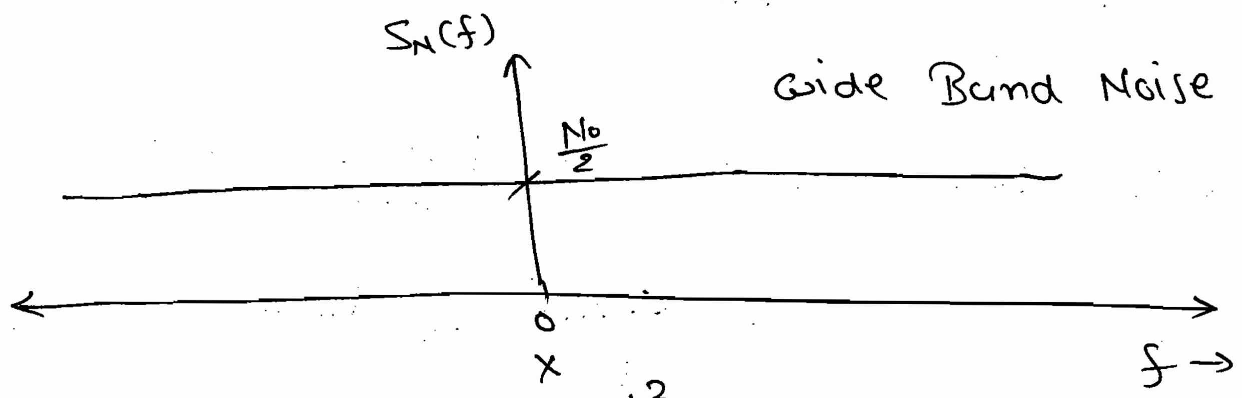
$$= 25.12 \text{ watts.}$$

② When White noise Passed through BPF :

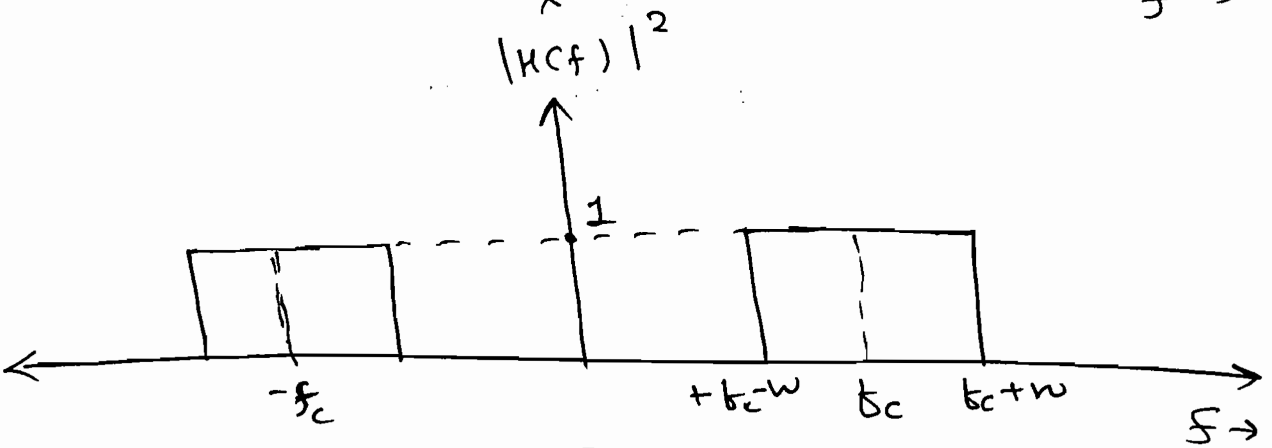
⇒



⇒



X



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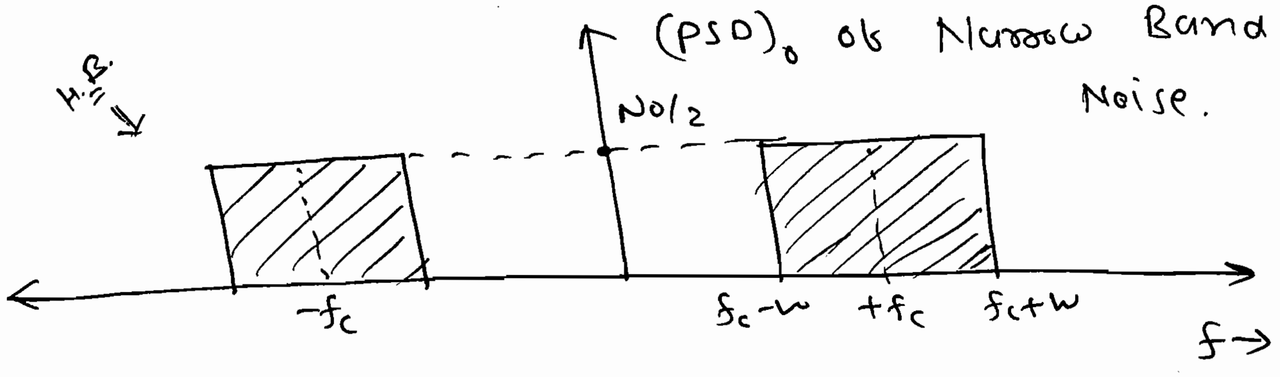


⇒ Noise Power = Area under $(PSD)_0$

$$= 2 \times \frac{N_0}{2} \times 2W$$

Noise Power $= 2 N_0 W$

=> $\boxed{\text{Noise Power} = 2N_oW.}$ ← H.B

→ $\boxed{R(\tau) = 2N_ow \text{ sinc}(2w\tau)\cdot \cos 2\pi f_c t}$

=> $\cos 2\pi f_c t$ terms is come because $(PSD)_o$ shifted left as well as right.

=> If the white noise is passed through a BPF, then the resultant Noise at the o/p of the BPF is called as the <u>Narrow Band Noise.</u>

=> Narrow Band Noise represent mathematically as, ← H.B

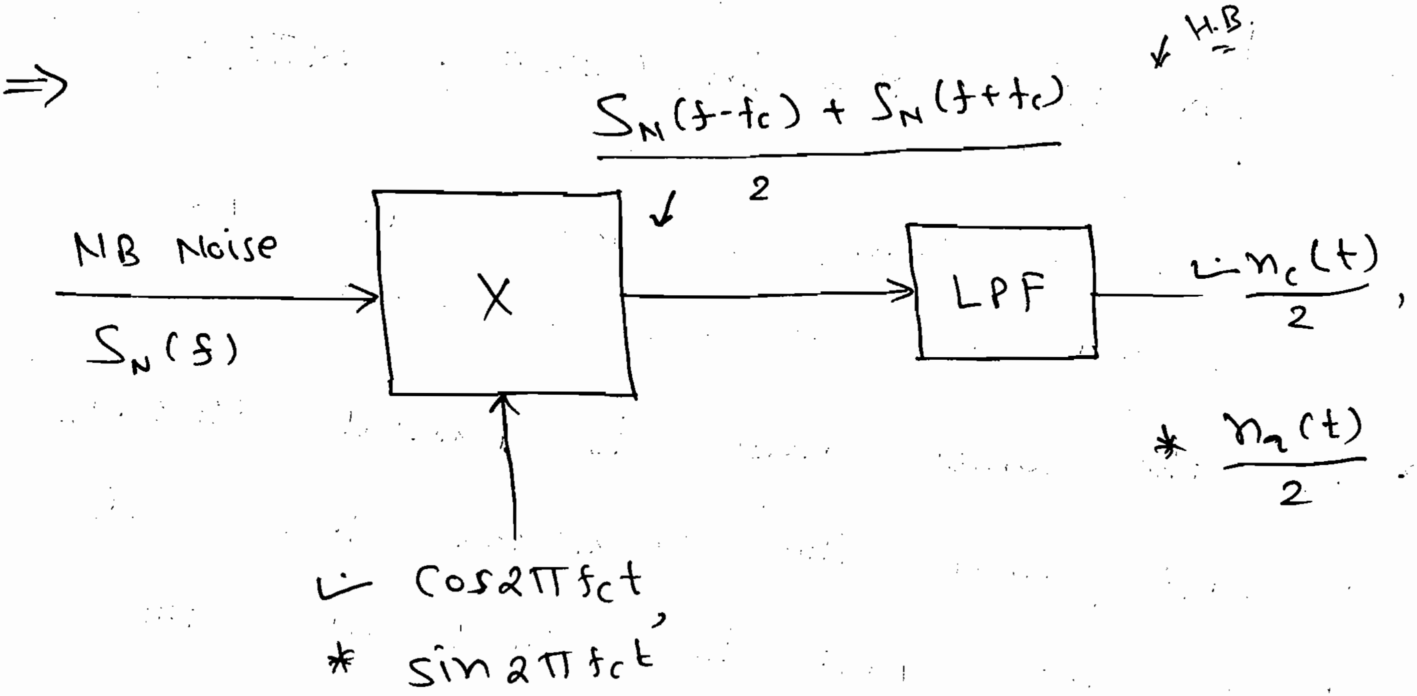
~~Sols~~

$$\boxed{\underbrace{n_c(t)}_{\text{Inphase Component}}\cdot\cos 2\pi f_c t + \underbrace{n_q(t)}_{\text{Quadrature component}}\cdot\sin 2\pi f_c t.}$$

=> To determine the inphase (or) Quadrature Componenet, the following method is used.

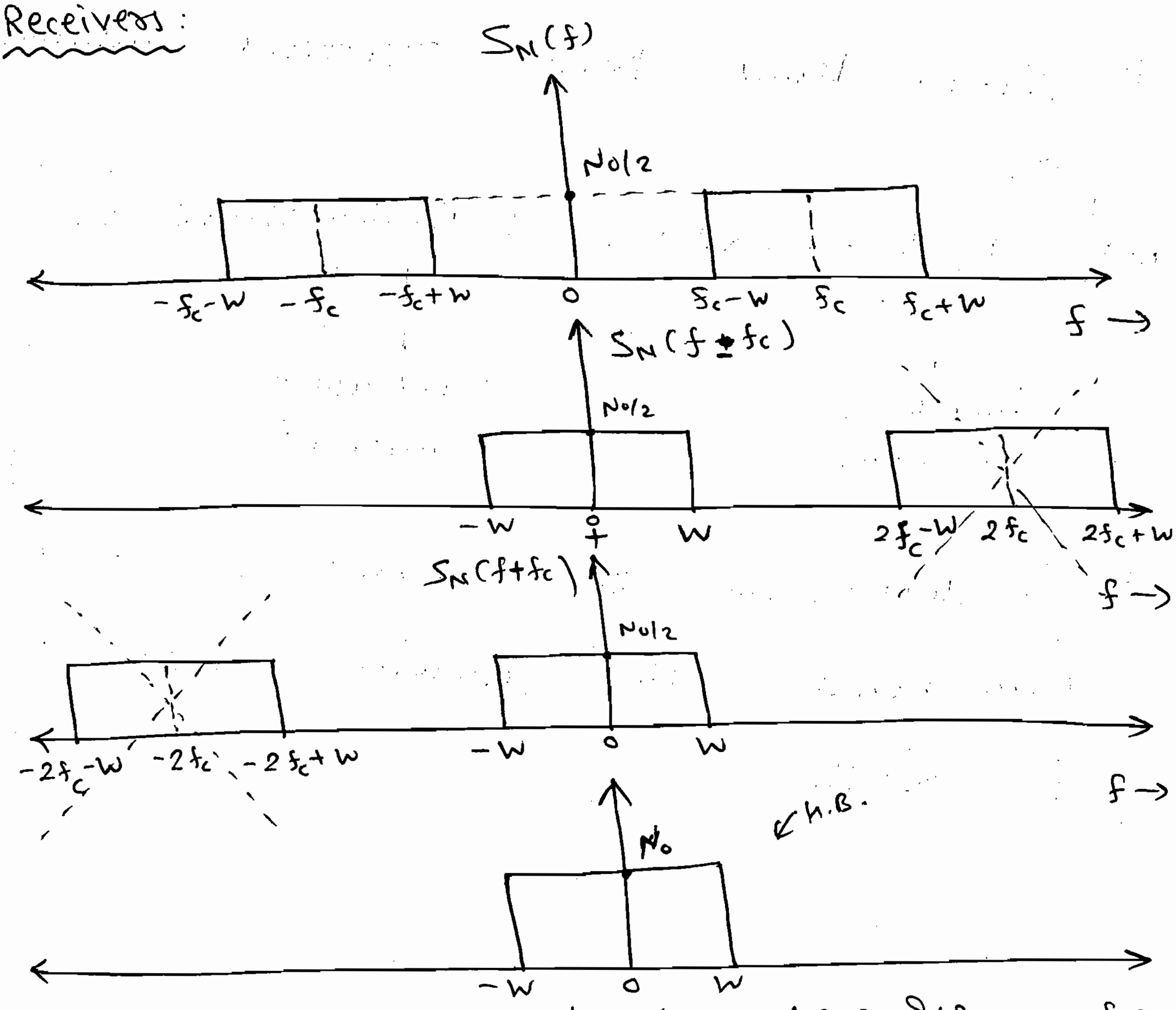
=> Method to determine $n_c(t)$ & $n_q(t)$:

=>



* PSD of Inphase Component in AM & DSB Receivers:

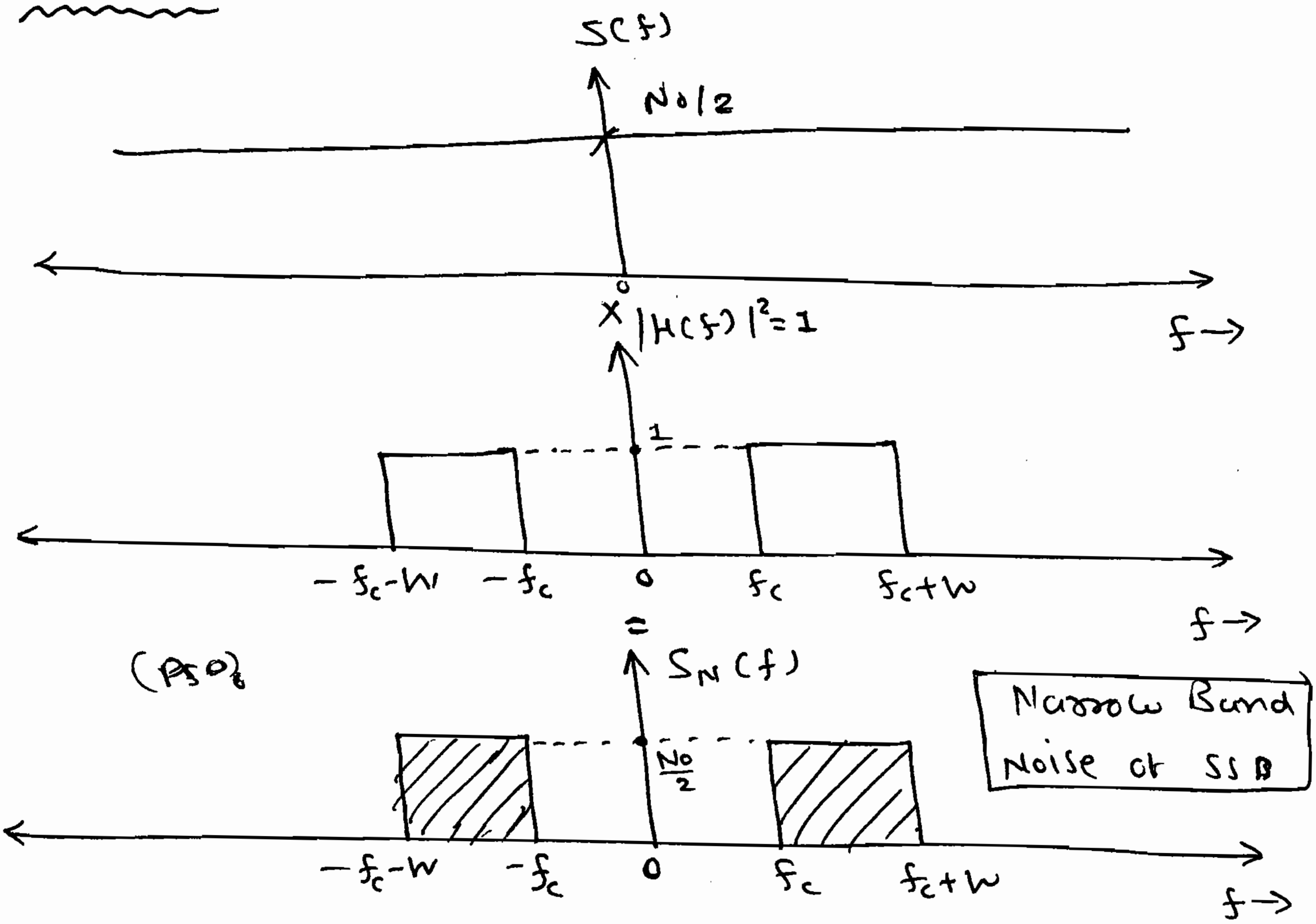
=>

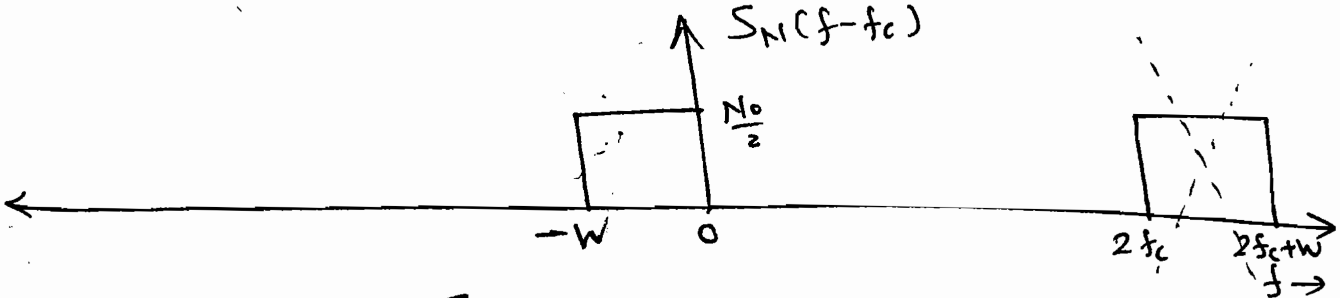


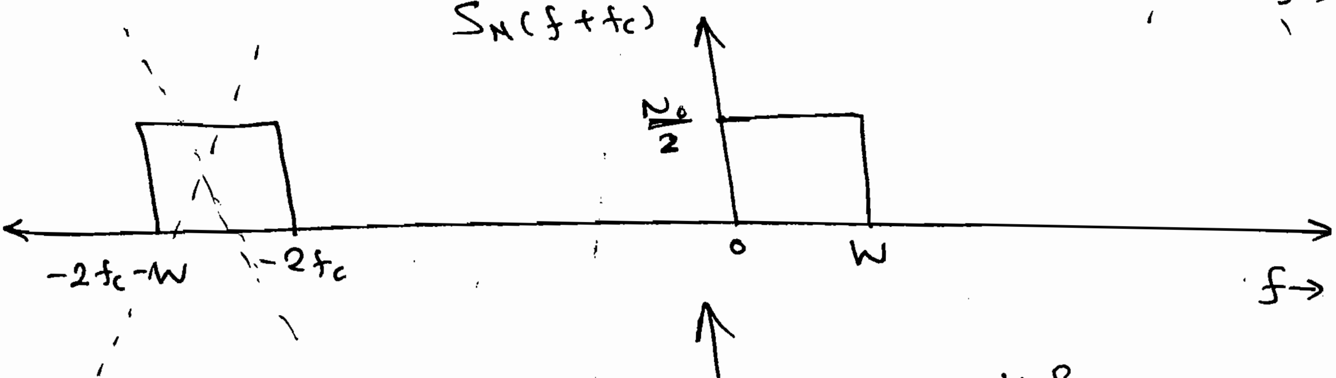
PSD of $n_c(t)$ for AM & DSB

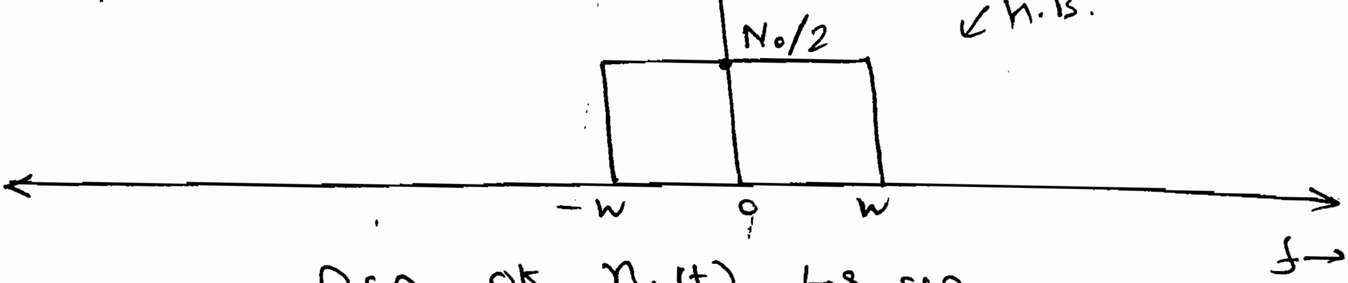
* PSD of Inphase Component in SSB Receivers:

=>



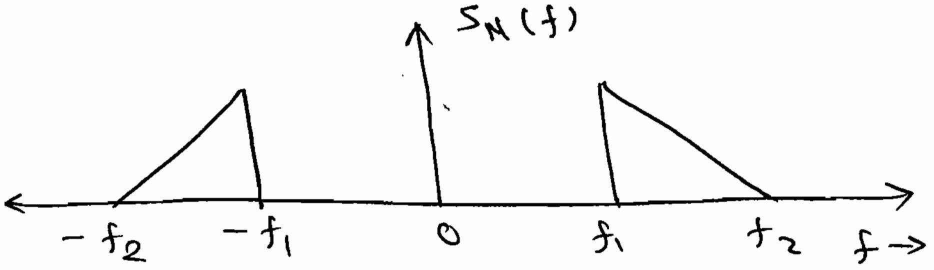






PSD of $n_c(t)$ for SSB

$\boxed{Q}$ The Power spectral density of NB Noise is shown in the fig.

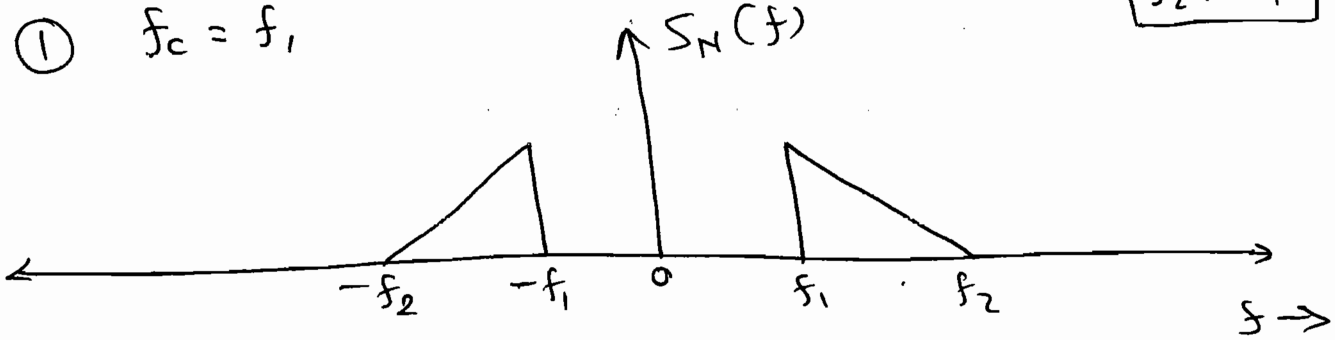


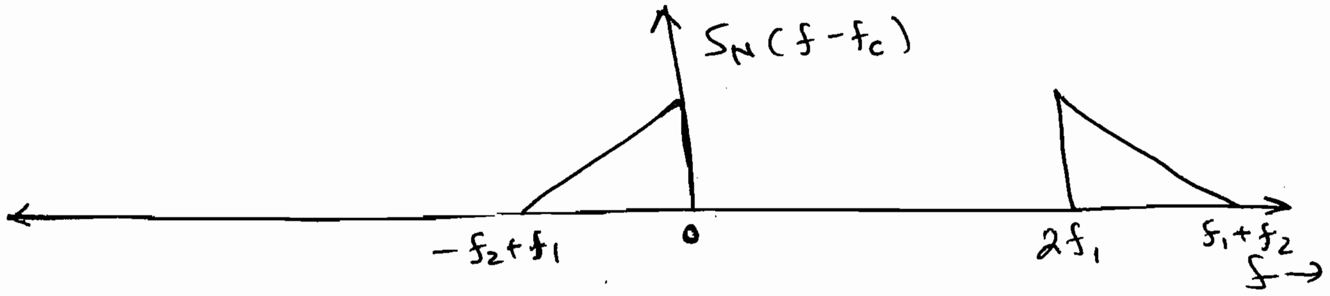
Sketch the PSD of inphase component when the carrier freq is

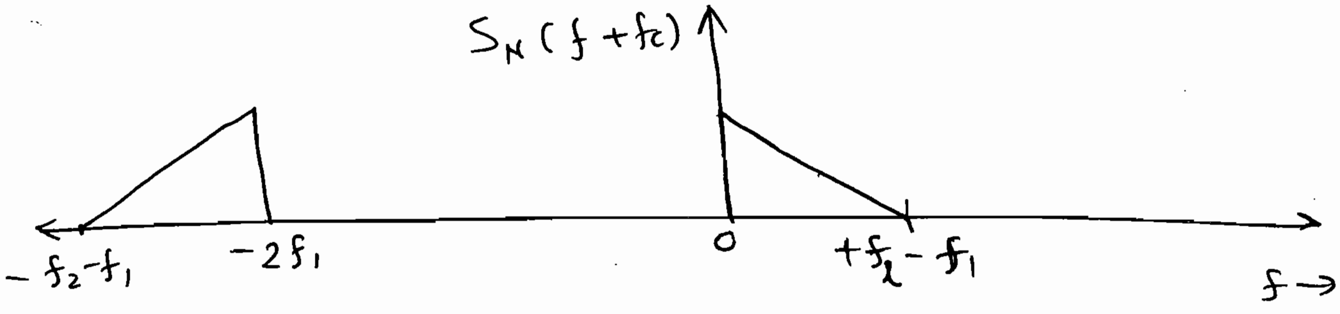
① $f_c = f_1$ ② $f_c = f_2$ ③ $f_c = \frac{f_1+f_2}{2}$.

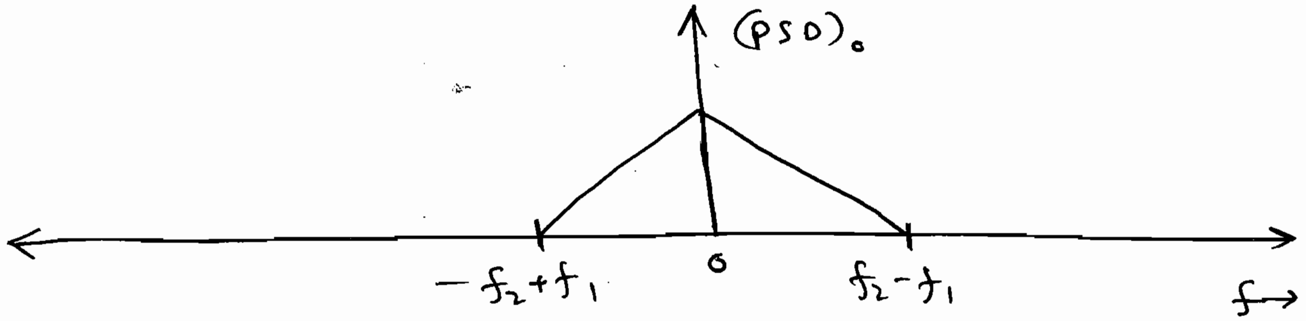
$\boxed{f_2 > f_1}$

Sol^n: ① $f_c = f_1$

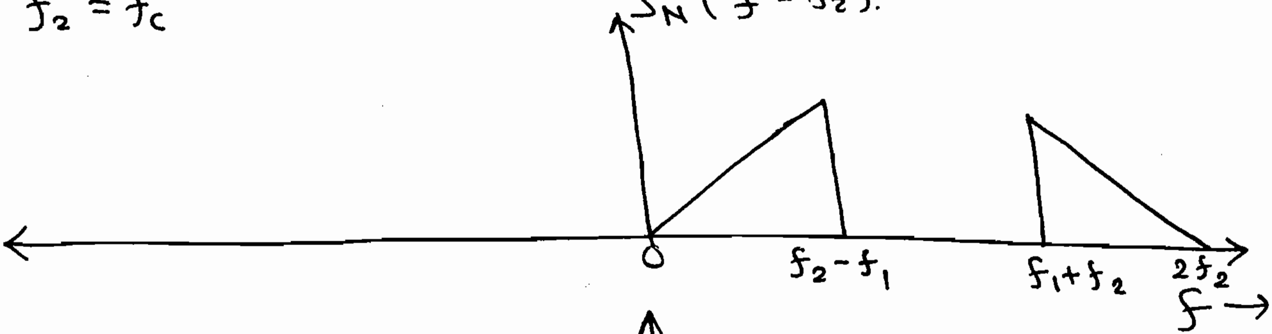


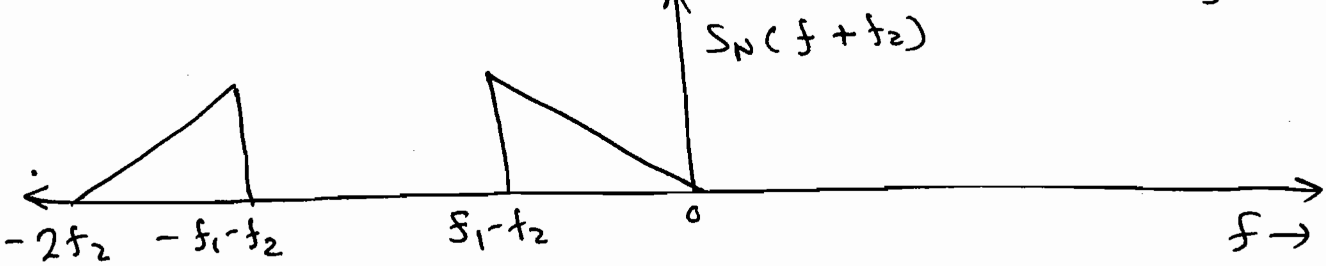


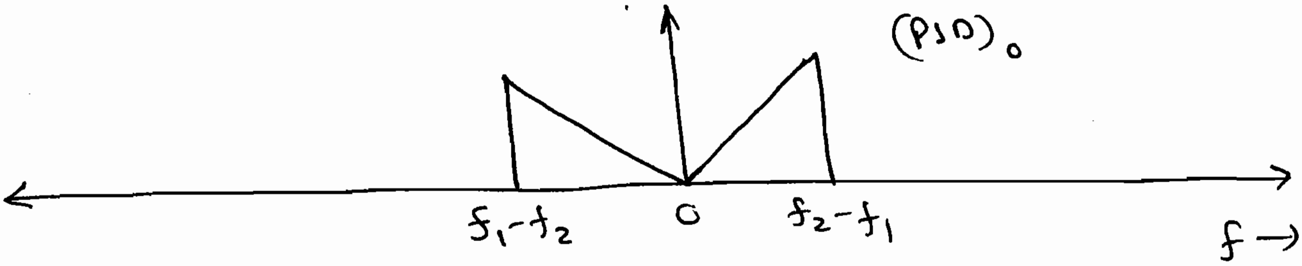




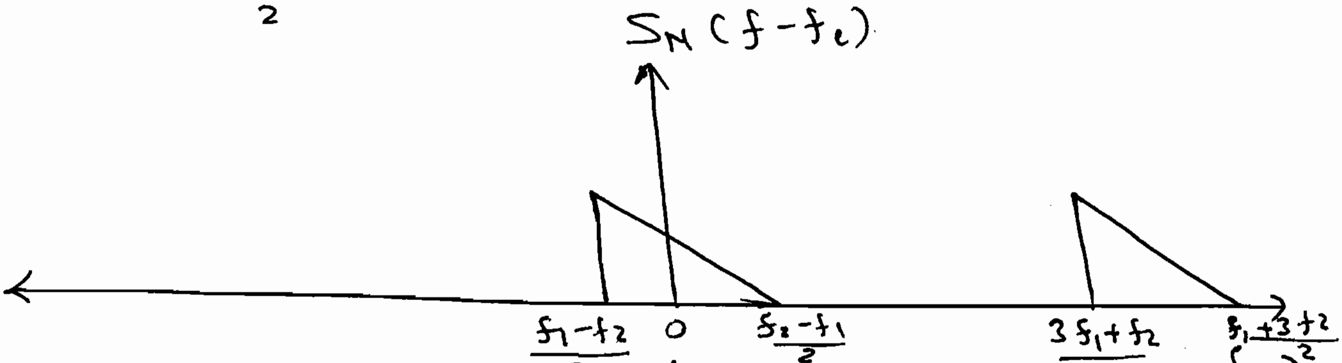
② $f_2 = f_c$

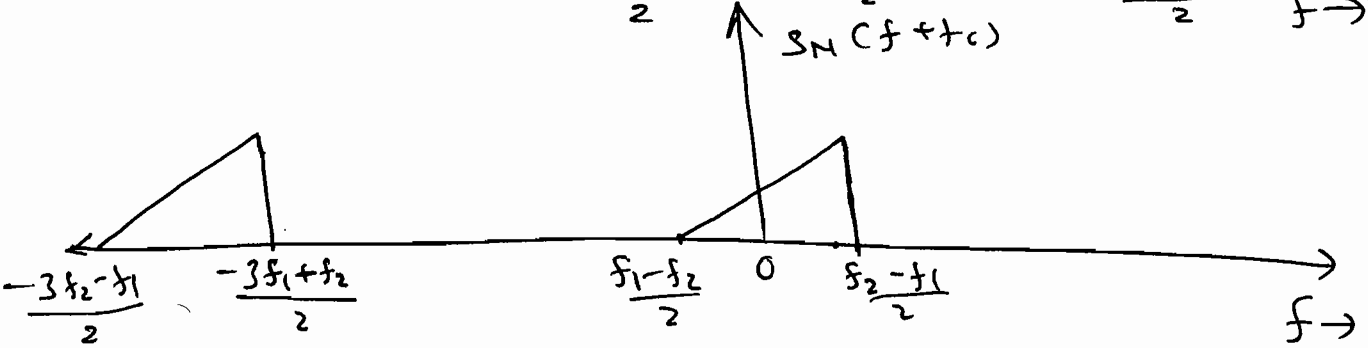


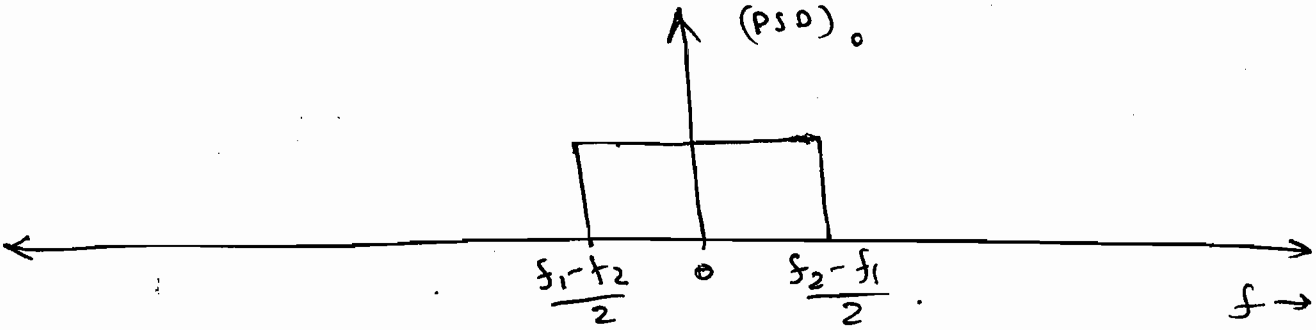




② $f_c = \frac{f_1 + f_2}{2}$







$\boxed{Q}$ PSD of noise is $10^{-8}\left[1-\frac{|f|}{10^{8}}\right]$ in freq. range $-10^{8} \le f \le 10^{8}$ & '0' otherwise. The noise is passed through an ideal BPF having a centre freq. of 50 MHz. & a BW of 2 MHZ. ① Sketch the PSD of at i/p & o/p at B.P.F.

② Find the noise power at o/p.

③ Sketch PSD of inphase component when $f_c = 50$ MHz.

Solⁿ: Let, PSD of Noise $S(f) = Y = 10^{-8}\left[1-\frac{|f|}{10^{8}}\right]$

Let, $S(f) = Y$
$f = x.$

$$\therefore Y = 10^{-8}\left[1-\frac{|X|}{10^{8}}\right].$$

$$\therefore Y = 10^{-8} - 10^{-16}|x|.$$

$$\therefore 10^{-16}|x| + Y = 10^{-8}$$

$$\therefore \frac{|x|}{10^{8}} + \frac{Y}{10^{-8}} = 1.$$

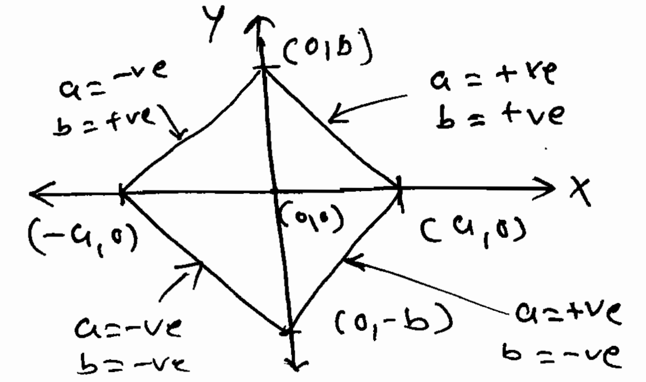
So, ① $X = +ve.$

$$\therefore \frac{X}{10^{8}} + \frac{Y}{10^{-8}} = 1 \quad -①$$

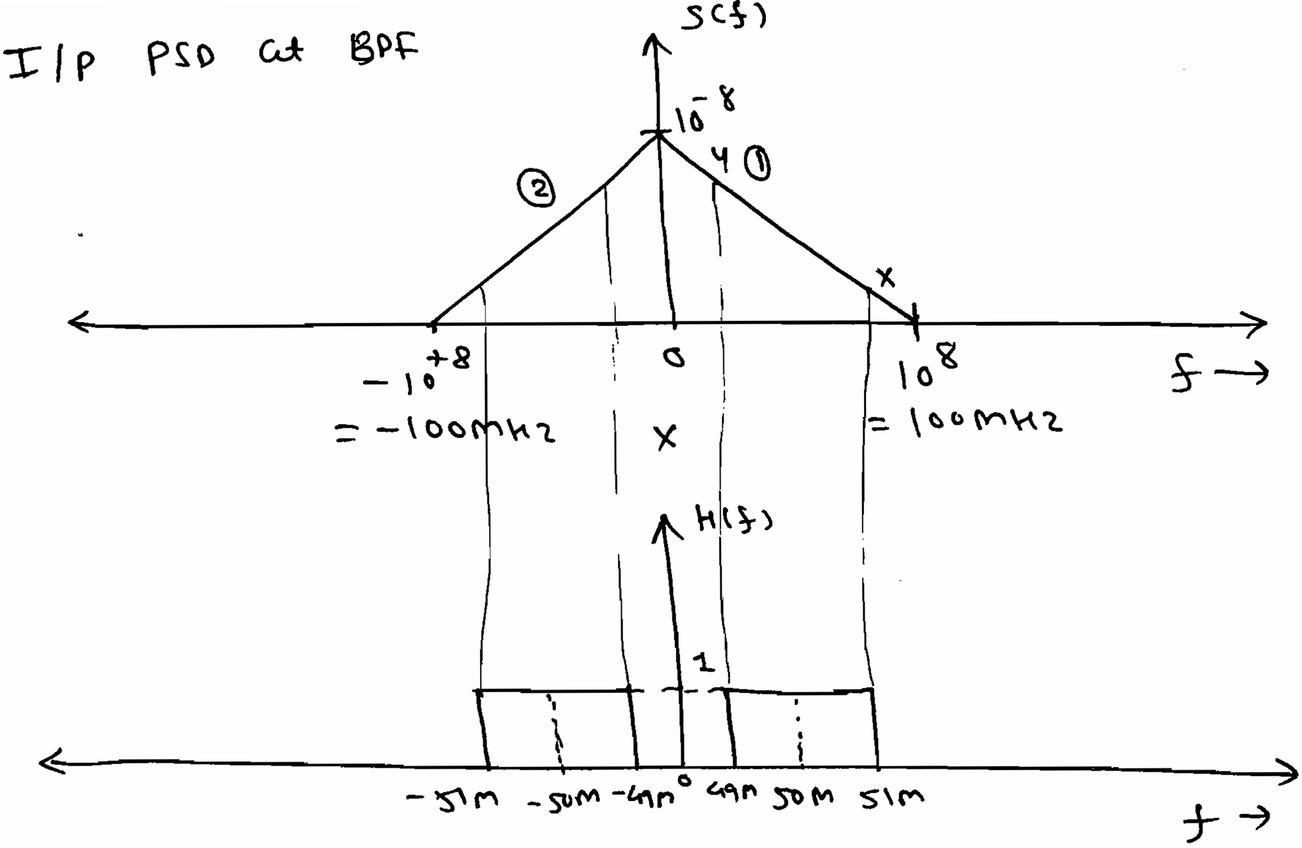
② $X = -ve$

$$\frac{-X}{10^{8}} + \frac{Y}{10^{-8}} = 1 \quad -②$$

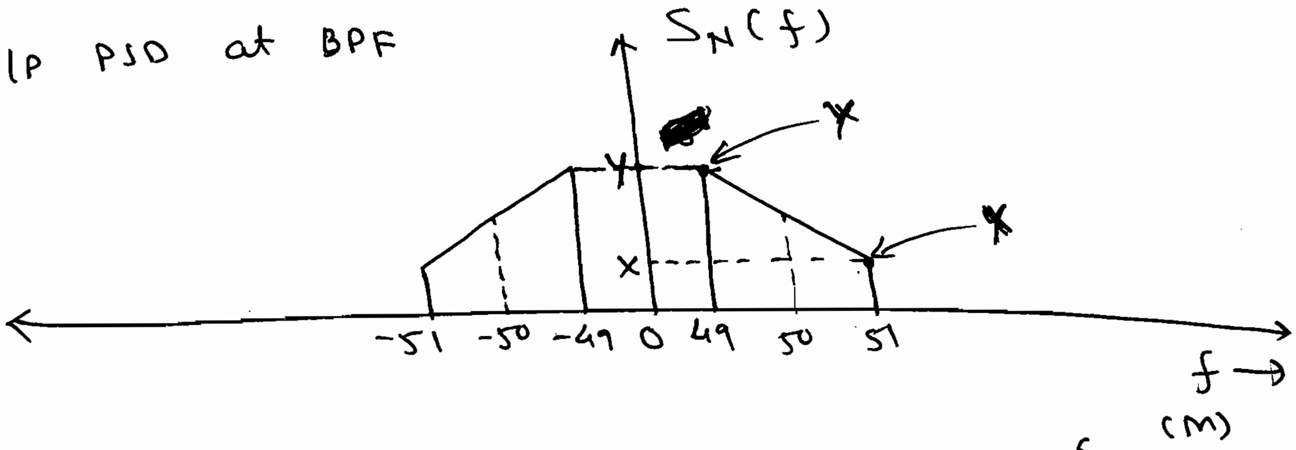
$$\pm\frac{X}{a} \pm \frac{Y}{b} = 1.$$



① I/p PSD at BPF



O/P PSD at BPF



⇒ at $f = 51M$ $\quad S_N(f) = 10^{-8}\left[1 - \frac{51\times10^{6}}{10^{8}}\right]$

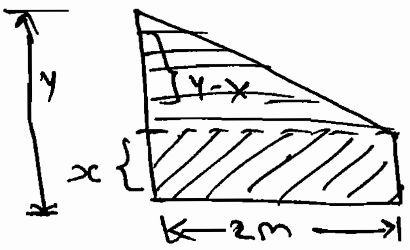
$x = S_N(f) = 0.49\times10^{-8}$ W/Hz.

⇒ at $f = 49M$ $\quad S_N(f) = 10^{-8}\left[1 - \frac{49}{100}\right]$

$y = S_N(f) = 0.51\times10^{-8}$ W/Hz.

⇒ Noise Power = Area under $(PSD)_o$

Noise Power $= 2\left[(x\times 2M) + \frac{1}{2}(2M\times(y-x))\right]$



Noise power

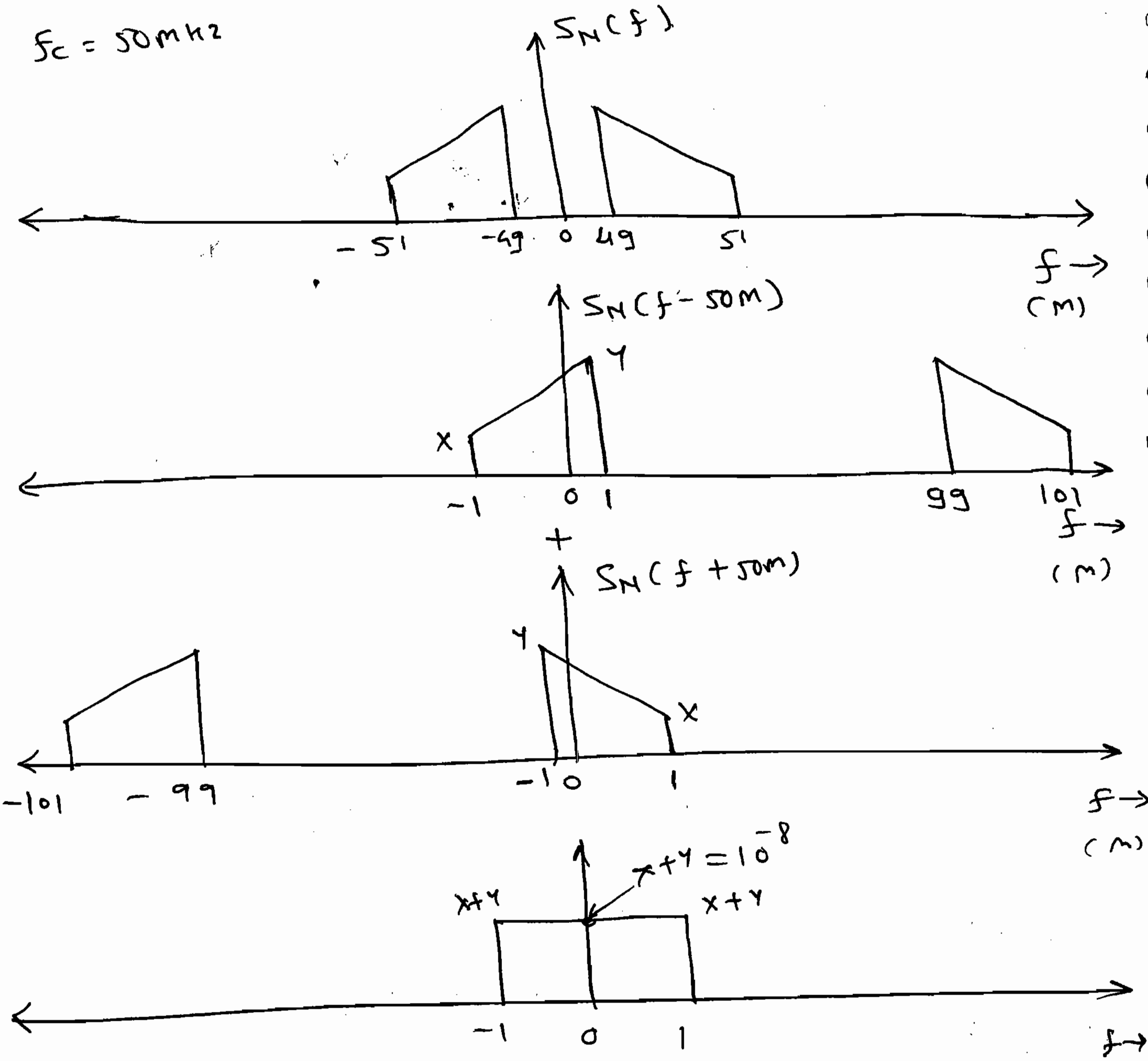
$$= 2\left[\left(2\times10^{+6}\times0.49\times10^{-8}\right)+\left(\frac{1}{\cancel{2}}\times\cancel{2}\times10^{6}\times2\times10^{-10}\right)\right]$$

$$= 2\left[9.8\times10^{-3} + 0.2\times10^{-3}\right]$$

$$= 2\times10\times10^{-3}$$

$$\therefore \boxed{\text{Noise Power} = 20\,\text{mW}}$$

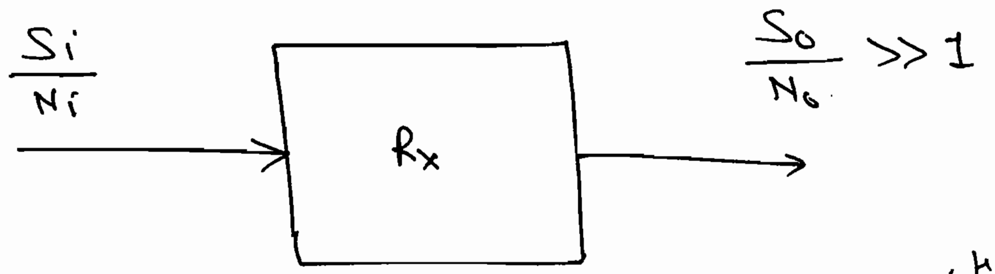
③ $f_c = 50\,\text{MHz}$



☆ Noise Analysis of Analog Communication:

⇒ To determine the Performance of the modulation technique, Figure of Merit (FoM) is used.

⇒



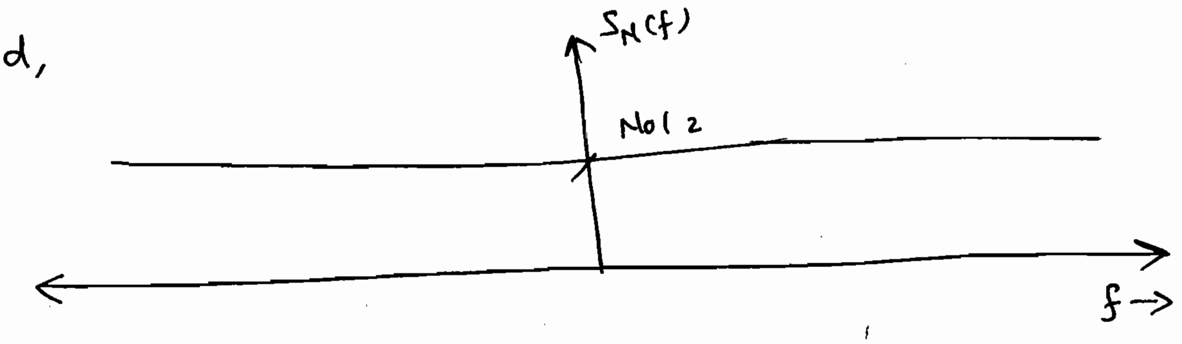
↙ H.B.

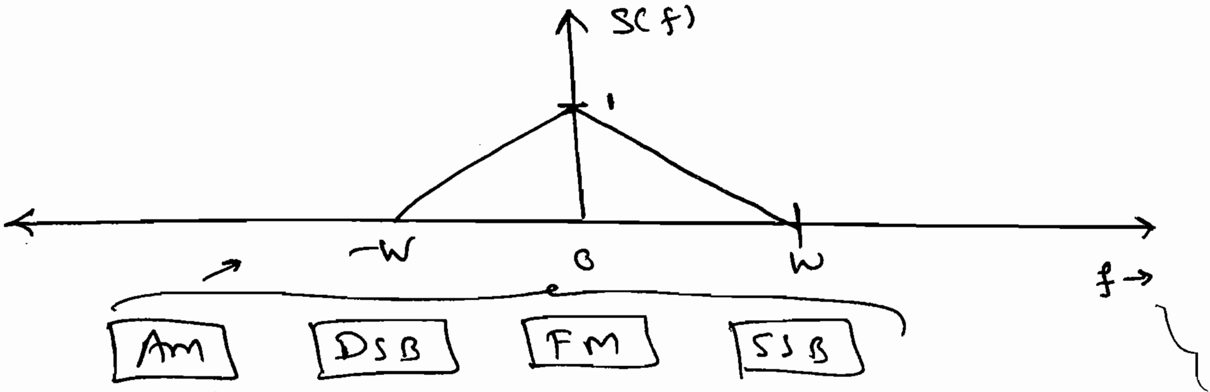
⇒ $$\text{Figure of Merit} = \frac{S_o/N_o}{S_i/N_i} >> 1$$

↙ H.B.

⇒ $$\text{Noise Figure} = \frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}} << 1.$$

⇒ To determine n_i following Procedure is used,





$n_i =$ noise Power in message BW

= area

$\therefore n_i = \left(\frac{N_0}{2} \times 2W\right)$

$\therefore \boxed{n_i = N_0 W \quad \text{watts.}}$ ↙ H.B.

* Figure of Merit of a DSB Receivers:

⟹ Power

dc ⟶ $(dc)^2$

ac ⟶ $(rms)^2$

$10\cos 2\pi f_c t \longrightarrow \left(\frac{10}{\sqrt{2}}\right)^2 = 50W$

$\Rightarrow \boxed{K\,(10\cos 2\pi f_c t) \longrightarrow K^2\,(50W).}$ ✓ H.B.

→ Time domain eqn of DSB is,

$s(t) = A_c\, m(t).\ \cos 2\pi f_c t.$

$\therefore S_i = \left(\frac{A_c m(t)}{\sqrt{2}}\right)^2$

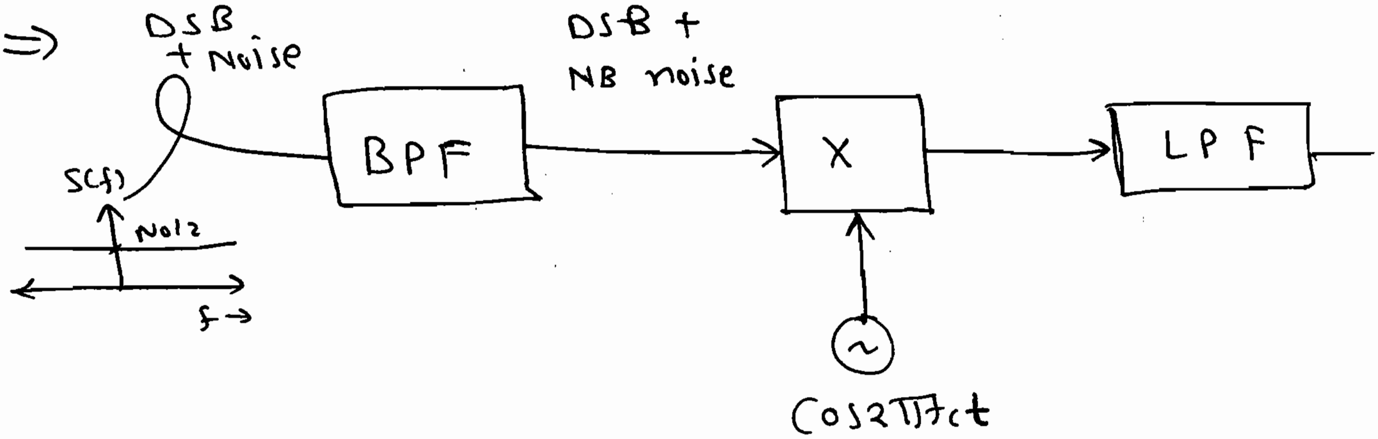
$S_i = \frac{A_c^2}{2}.\ m^2(t).$, $n_i = N_0 W$, S_i 2

let, P = Power of message signal

$S_i = \frac{A_c^2 . P}{2}$

$\therefore \boxed{(SNR)_i = \frac{S_i}{N_i} = \frac{A_c^2 . P}{2 N_0 W}.}$ ← H.B.

⇒



⇒ The o/p of multiplier is,

[DSB + NB noise] $\cos 2\pi f_c t$.

$$= \left[A_c \cdot m(t) \cdot \cos 2\pi f_c t + n_c(t) \cos 2\pi f_c t + n_q(t) \cdot \sin 2\pi f_c t\right] \cos 2\pi f_c t$$

$$= A_c \cdot m(t) \cdot \cos^2 2\pi f_c t + n_c(t) \cdot \cos^2 2\pi f_c t + n_q(t) \cdot \sin 2\pi f_c t \cdot \cos 2\pi f_c t$$

⇒ o/p of LPF is.

$m(t) \longrightarrow P$

$\frac{A_c}{2} \cdot m(t) \rightarrow \frac{A_c^2 \cdot P}{4}$

$$\underbrace{\boxed{\frac{A_c \cdot m(t)}{2}}}_{\text{signal}} + \boxed{\frac{n_c(t)}{2}} \rightarrow \text{noise.}$$

$$\Rightarrow S_o = \left(\frac{A_c \cdot m(t)}{2\sqrt{2}}\right)^2 = \frac{A_c^2 \cdot P}{2 \times 2} = \frac{A_c^2 \cdot P}{4}$$

$m(t) \longrightarrow P$

$\frac{A_c}{2} \cdot m(t) \rightarrow \frac{A_c^2}{4} \cdot P$

$n_o =$ ~~$n_c(t)$~~

Power

$\Rightarrow n_c(t) \longrightarrow (2N_0W)$.

$\therefore \frac{1}{2} n_c(t) \longrightarrow 2N_0W \times \frac{1}{4} = \frac{2N_0W}{4} = n_0$

So, $S_o = \frac{A_c^2 \cdot P}{4}$, $n_o = \frac{2 N_o W}{4}$.

$$\therefore \boxed{SNR_o = \frac{S_o}{N_o} = \frac{A_c^2 \cdot P}{2 N_o W}.} \leftarrow \underline{H.B.}$$

⇒ Figure of Merit $= \frac{(SNR)_o}{(SNR)_i}$

$$\underline{H.B.} \nearrow \boxed{FoM = 1}$$ for DSB Receiver.

* Figure of Merit of a SSB Receiver.

⇒ $(S_i)_{SSB} = 50\% \cdot (S_i)_{DSB}$.

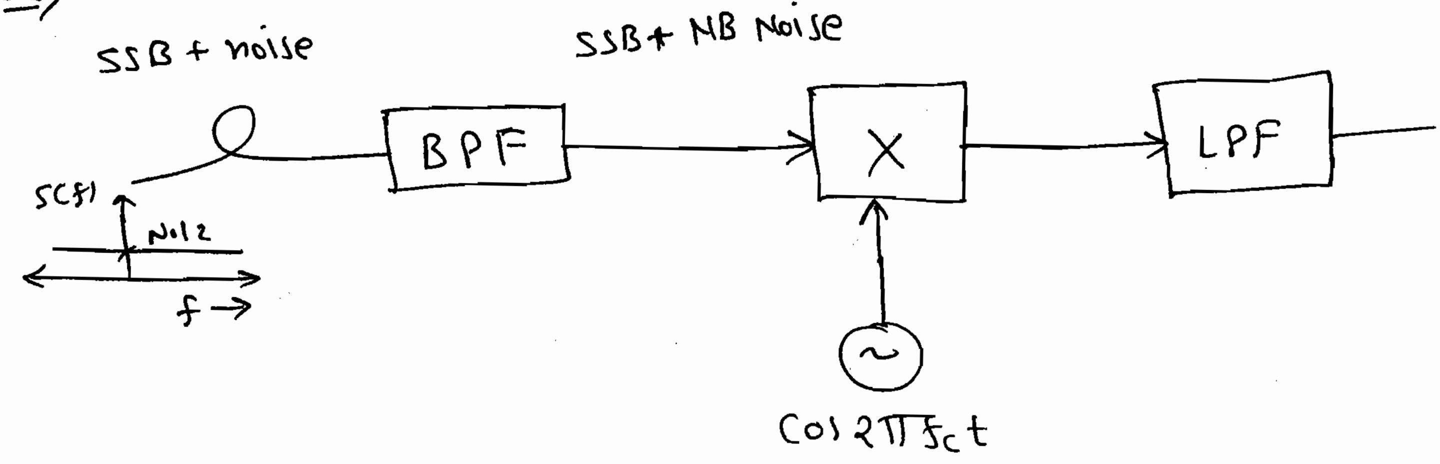
$\therefore (S_i)_{SSB} = \frac{1}{2} \cdot (S_i)_{DSB}$.

$\therefore (S_i)_{SSB} = \frac{A_c^2 \cdot P}{4}$.

$n_i = N_o \cdot W$.

$$\therefore \boxed{(SNR)_i = \frac{S_i}{n_i} = \frac{A_c^2 \cdot P}{4 N_o W}.} \leftarrow H.B.$$

⇒



$\Rightarrow$ The o/p of multiplier is,

$(\text{SSB} + \text{NB noise}) \cos 2\pi f_c t.$

$$= \left[\frac{A_c}{2} m(t) \cos 2\pi f_c t \mp \frac{A_c}{2} \hat{m}(t).\ \sin 2\pi f_c t + n_c(t).\ \cos 2\pi f_c t + n_q(t).\ \sin 2\pi f_c t\right] \cos 2\pi f_c t$$

$$= \frac{A_c}{2} m(t).\ \cos^2 2\pi f_c t \mp A_c \hat{m}\ \cancel{(t)}.\ \sin 2\pi f_c t.\ \cos 2\pi f_c t + n_c(t).\ \cos^2 2\pi f_c t + n_q(t).\ \cancel{\sin} 2\pi f_c t.\ \cos 2\pi f_c t.$$

$\Rightarrow$ o/p of the LPF is,

$$\therefore = \underbrace{\boxed{\frac{A_c}{4} m(t)}}_{\text{signal}} + \boxed{\frac{n_c(t)}{2}}. \longrightarrow \text{Noise},$$

$$\Rightarrow \quad m(t) \longrightarrow P$$

$$\rightarrow \frac{A_c}{4}.m(t) \longrightarrow \frac{A_c^2}{16}.\ P.$$

$$\rightarrow \underline{n_c(t)} \longrightarrow N_o.W$$

$$\frac{1}{2} n_c(t) \longrightarrow \frac{1}{4}.\ N_o.W.$$

So, $S_o = \frac{A_c^2.P}{16}, \quad n_o = \frac{N_o.W}{4}.$

$$\therefore (SNR)_o = \frac{S_o}{N_o} = \frac{\frac{A_c^2.P}{16}}{\frac{N_o.W}{4}}.$$

$$\therefore \boxed{(SNR)_o = \frac{A_c^2 \cdot P}{4 N_o W}} \leftarrow \underline{\underline{H.B.}}$$

$$\Rightarrow \quad \text{Figure of Merit} = \frac{(SNR)_o}{(SNR)_o} = 1$$

$$\therefore \boxed{F.o.M = 1} \quad \text{for SSB Receiver.}$$

* Figure of Merit of AM:

$$\Rightarrow \quad s(t) = A_c \cos 2\pi f_c t + A_c K_a \cdot m(t) \cdot \cos 2\pi f_c t.$$

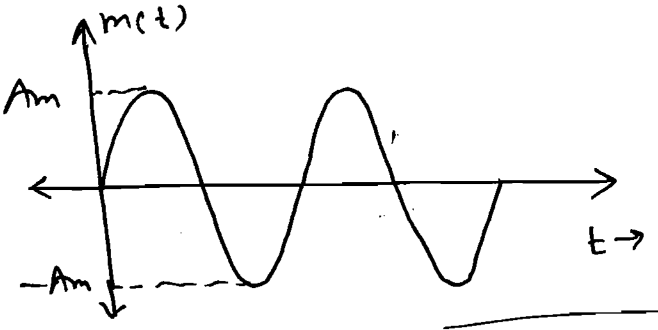
$$\therefore \quad S_i = \frac{A_c^2}{2} + \frac{A_c^2 \cdot K_a^2 \cdot m^2(t)}{2}$$

$$m(t) \xrightarrow{\text{Power}} P$$

$$\therefore \quad S_i = \frac{A_c^2}{2} + \frac{A_c^2 \cdot K_a^2 \cdot P}{2}.$$

$$\therefore \boxed{S_i = \frac{A_c^2}{2} \cdot \left[1 + \frac{K_a^2 \cdot P}{2}\right].} \swarrow \underline{\underline{H.B.}}$$

$$\Rightarrow \text{ if } \quad m(t) = A_m \cos 2\pi f_m t = \text{sinusoidal signal.}$$



$$rms = V_m / \sqrt{2}$$

$$= \frac{A_m}{\sqrt{2}}$$

$$\therefore \quad P = \frac{(rms)^2}{R} = \frac{A_m^2}{2R}$$

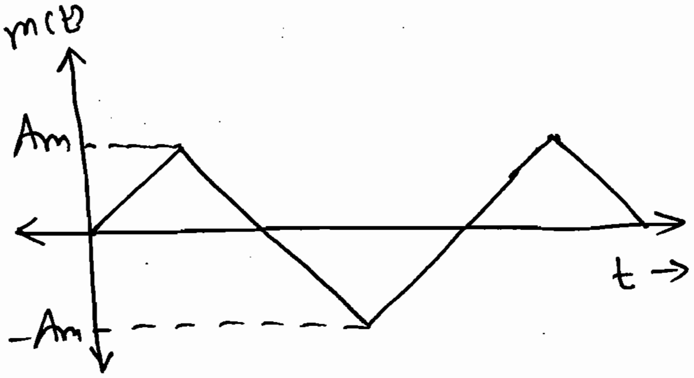
let, $R = 1\Omega$

$$S_i = \frac{A_c^2}{2}\left[1 + \frac{K_a^2 A_m^2}{2}\right] \boxed{= \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]} \swarrow H.B$$

$\Rightarrow$ if $m(t) =$ triangular wave.

$\Rightarrow$ $rms = \frac{Am}{\sqrt{3}}$.

$\therefore$ $P = \frac{Am^2}{3}$.



$\Rightarrow$ $\therefore$ $S_i = \frac{Ac^2}{2}\left[1 + \frac{Ka^2 \cdot Am^2}{3}\right]$

as $\mu = Ka \cdot Am$.

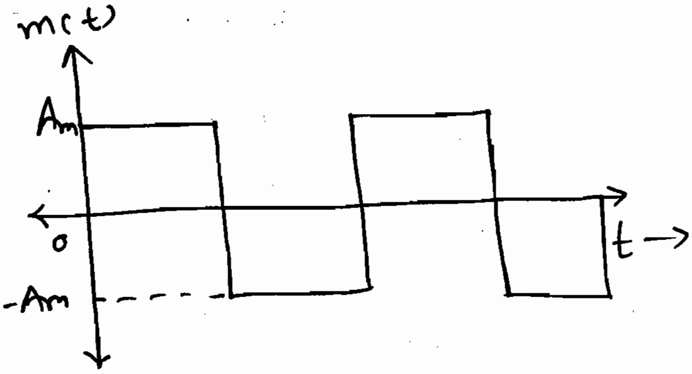
$\therefore$ $\boxed{S_i = \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{3}\right].}$ ← H.B.

$\Rightarrow$ if $m(t) =$ Square wave.

$\therefore$ $rms = Am$

$\therefore$ $P = Am^2$.

$\therefore$ $\boxed{S_i = \frac{A_c^2}{2}\left[1 + \mu^2\right].}$ ← H.B.



$\Rightarrow$ $S_i = \underset{\text{Carrier power}}{\frac{Ac^2}{2}} + \underset{\text{sideband power}}{\frac{Ac^2 \cdot Ka^2 P}{2}}$.

$\therefore$ $\eta = \frac{P_{SB}}{P_t}$.

$$\therefore \eta = \frac{P_{SB}}{P_t}$$

$$\therefore \eta = \frac{\frac{A_c^2 K_a^2 P}{2}}{\frac{A_c^2}{2} + \frac{A_c^2 K_a^2 P}{2}}$$

$$\therefore \boxed{\eta = \frac{K_a^2 P}{1 + K_a^2 P}} \leftarrow \underline{\underline{\text{h.B.}}}$$

$\therefore$ ① $\underline{\text{sine}}$ $\Rightarrow$ $P \rightarrow A_m^2/2$.

$$\therefore \eta = \frac{K_a^2 A_m^2/2}{1 + K_a^2 \cdot A_m^2/2}$$

when $\underline{\underline{\mu = 1}}$

$$\boxed{\eta = 33.33\ \%}$$

$$\therefore \eta = \frac{K_a^2 A_m^2}{2 + K_a^2 \cdot A_m^2}$$

$$\therefore \boxed{\eta = \frac{\mu^2}{\mu^2 + 2}} \leftarrow \underline{\underline{\text{h.B}}}$$

② $\underline{\text{Square}}$ $\underline{\text{wave}}$ $\Rightarrow$ $P = A_m^2$.

$$\therefore \eta = \frac{K_a^2 A_m^2}{1 + K_a^2 A_m^2}$$

when $\underline{\underline{\mu = 1}}$

$$\therefore \boxed{\eta = \frac{\mu^2}{\mu^2 + 1}} \leftarrow \underline{\underline{\text{h.B}}}$$

$$\boxed{\eta = 50\ \%}$$

③ $\underline{\text{Triangular}}$ $\underline{\text{wave}}$ $\Rightarrow$ $P = A_m^2/3$.

$$\therefore \eta = \frac{K_a^2 \cdot A_m^2/3}{1 + K_a^2 \cdot A_m^2/3}$$

when $\underline{\underline{\mu = 1}}$

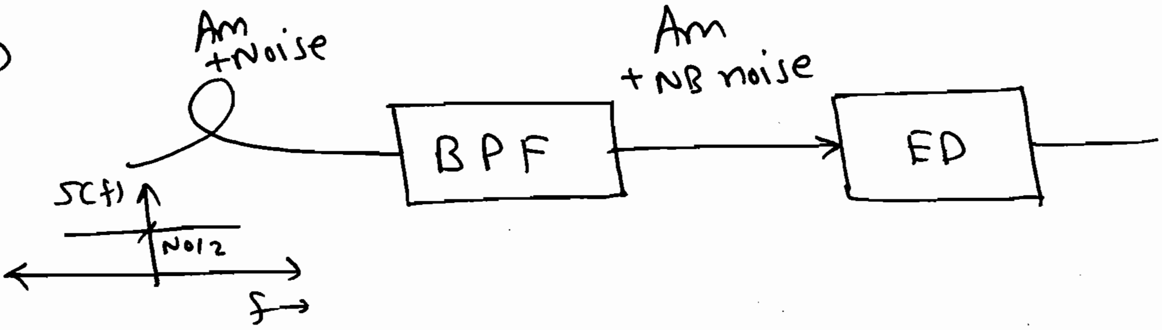
$$\therefore \boxed{\eta = \frac{\mu^2}{\mu^2 + 3}} \leftarrow \underline{\text{h.B.}}$$

$$\boxed{\eta = 25\ \%}$$

$$\Rightarrow \quad (SNR)_i = \frac{S_i}{n_i}$$

$$\therefore \boxed{(SNR)_i = \frac{A_c^2\,[1 + K_a^2 . m^2(t)]}{2 N_0 W}} \leftarrow \underline{\underline{H.B}}$$

⇒



⇒ O/P of BPF is.

AM + NB noise.

$$= A_c\,[1 + K_a m(t)] \cos 2\pi f_c t \; + n_c(t) . \cos 2\pi f_c t$$
$$+ n_q(t) . \sin 2\pi f_c t$$

$$= \left[A_c\,[1 + K_a m(t)] \; + n_c(t)\right] \cos 2\pi f_c t$$
$$+ n_q(t) \sin 2\pi f_c t$$

Now, O/P of ED is

$$\sqrt{[A_c\,[1 + K_a m(t)] + n_c(t)]^2 + \underbrace{(n_q(t))^2}_{\rightarrow \text{very small.}}}$$

So, O/P of ED is

$$= A_c\,[1 + K_a m(t)] + n_c(t)$$

$$= \underset{dc}{\cancel{A_c}} + \underbrace{A_c K_a m(t)}_{\text{signal}} + \underbrace{n_c(t)}_{\text{noise.}}$$

$$\Rightarrow \quad m(t) \longrightarrow P$$

$$A_c K_a\, m(t) \longrightarrow A_c^2 . K_a^2 . P$$

$$\therefore \quad S_o = A_c^2 . K_a^2 . P .$$

$$n_c(t) \longrightarrow 2 N_o W .$$

$$\therefore \quad N_o = 2 N_o W .$$

H.B.

$$\therefore \quad \boxed{(SNR)_o = \frac{S_o}{N_o} = \frac{A_c^2 . K_a^2 . P}{2 N_o W}}$$

$$\therefore \quad \text{Figure of Merit} = \frac{(SNR)_o}{(SNR)_i} .$$

$$= \frac{\dfrac{\cancel{A_c^2} . K_a^2 . P}{\cancel{2 N_o W}}}{\dfrac{\cancel{A_c^2}\left[1 + K_a^2 m^2(t)\right]}{\cancel{2 N_o W}}}$$

H.B

$$\therefore \quad \boxed{FoM = \frac{K_a^2 . P}{1 + K_a^2 . P} = \eta < 1 .}$$

for AM.

* FoM of FM:

$\Rightarrow$ $$\text{FoM} = \frac{3K_f^2 P}{W^2}.$$

$\therefore$ for single tone $P = \frac{A_m^2}{2}$.

$$\therefore \text{FoM} = \frac{3}{2} \cdot \frac{K_f^2 \cdot A_m^2}{f_m^2}.$$

$$\therefore \text{FoM} = \frac{3}{2} \cdot \left(\frac{K_f \cdot A_m}{f_m}\right)^2.$$

$$\beta = \frac{K_f \cdot A_m}{f_m}.$$

$$\therefore \boxed{\text{FoM} = 3/2 \cdot \beta^2.} \leftarrow \underline{\text{M.B}}$$

for $\beta = 10$ FoM $= 150$.

* FoM of pm:

$\Rightarrow$ $$\text{FoM} = K_p^2 \cdot P.$$

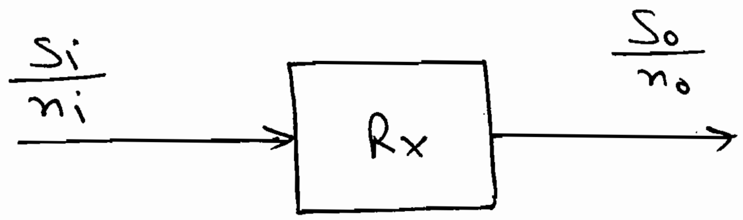
for single tone modulation $P = \frac{A_m^2}{2}$.

$$\therefore \text{FoM} = \frac{K_p^2 \cdot A_m^2}{2}.$$

$$\therefore \beta = \Delta\phi = K_p \cdot A_m.$$

$$\therefore \boxed{\text{FoM} = \frac{\beta^2}{2} = \frac{\Delta\phi^2}{2}.} \leftarrow \underline{\text{M.B}}$$

*



$\Rightarrow$ For $\left.\begin{array}{l} DSB \\ SSB \\ VSB \end{array}\right\} \longrightarrow FOM = 1 \Rightarrow (SNR)_o = (SNR)_i$

$\Rightarrow$ For AM $\longrightarrow$ $FOM = \frac{1}{3} \Rightarrow (SNR)_o = \frac{1}{3}(SNR)_i$

when $[\mu = 1]$

$\Rightarrow$ For FM $\longrightarrow$ $FOM = 150 \Rightarrow (SNR)_o = 150 (SNR)_i$

when $[\beta = 10]$

$\Rightarrow$ For PM $\longrightarrow$ $FOM = 50 \Rightarrow (SNR)_o = 50 (SNR)_i$

$[\beta = 10]$

$\boxed{Q}$ An Audio signal is Band limited to 15 KHz and transmitted through a channel after modulation. The signal is affected by white noise having a two sided PSD of 10^{-10} watts/Hz. The Power loss in the channel is 50 dB. The SNR required at the o/p of the receiver is 40 dB. Determine the transmitted power when the modulation used is,

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① DSB

② Am with $\mu=1$

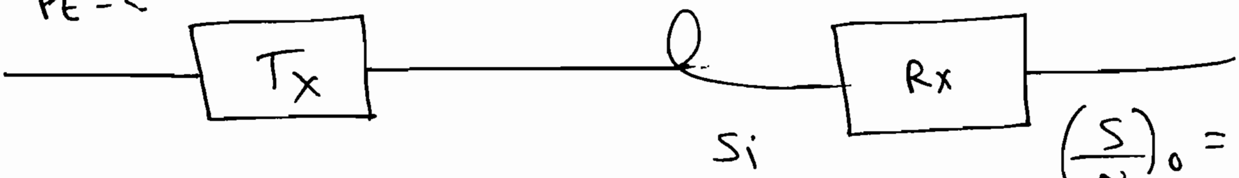
③ FM $(\beta=5]$.

Ans:

$P_L = 50\,dB$

$P_L = 10^5\ W$

$P_t = 2$



$\left(\frac{S}{N}\right)_o = 40\,dB$

$\left(\frac{S}{N}\right)_o = 10^4\ W.$

① DSB:

For DSB FOM = 1.

$\frac{N_o}{2} = 10^{-10}$ watts/Hz

$$\therefore \frac{S_o}{N_o} = \frac{S_i}{n_i}.$$

$$\therefore S_i = n_i \times 10^4.$$

$$n_i = N_o W = \begin{matrix} 2\times10^{-10} \\ \times 15\times10^3 \end{matrix} \text{ watts/Hz.}$$

$$\therefore S_i = 2\times10^{-10} \times 15\times10^3 \times 10^4.$$

$$\therefore \boxed{S_i = 30\text{ m watts}}$$

$$\therefore (S_i)_{dB} = (P_t)_{dB} - (P_L)_{dB}.$$

$$\therefore 20\log(s_i)_{dB} = 20\log(P_t)_{dB} - 20\log(P_L)_{dB}.$$

$$\therefore \boxed{S_i = \frac{P_t}{P_c}.}$$

$\Rightarrow \quad P_t = S_i \times P_L.$

$\therefore \quad P_t = 30 \times 10^{-3} \times 10^{5}$

$\boxed{P_t = 3 \text{ kW}}$

② AM with $\mu = 1.$

$\therefore \quad \text{FoM} = \frac{\mu^2}{\mu^2 + 2}$

$\text{FoM} = \frac{1}{3}.$

$\therefore \quad \frac{S_o}{n_o} = \frac{1}{3} \cdot \frac{S_i}{n_i}.$

$\therefore \quad S_i = 3 \times n_i \times \frac{n_o}{S_o}.$

$= 3 \times 2 N_o \omega \times 10^{4}.$

$= 3 \times 2 \times 10^{-10} \times 15 \times 10^{3} \times 10^{4}.$

$\therefore \quad \boxed{S_i = 90 \text{ mW}.}$

$\therefore \quad P_t = S_i \cdot P_L.$

$\therefore \quad P_t = 90 \times 10^{-3} \times 10^{5}$

$\boxed{P_t = 9 \text{ W}.}$

③ FM $[\beta = 5].$

$\therefore \quad \text{FoM} = 3/2 \cdot \beta^2 = \cancel{150} \cdot \frac{75}{2}.$

$\therefore \quad \left(\frac{S}{N}\right)_o = \frac{150}{4} \times \frac{S_i}{n_i}.$

∴ ~~$S_i = N_0 w \times \frac{150}{4} \times 10^4$.~~

∴ ~~$S_i = 10^{-10} \times 2 \times 15 \times 10^3 \times \frac{75}{2} \times 10^4$.~~

$$\therefore \quad S_i = \frac{2}{75} \times 10^4 \times N_0 . w.$$

$$= \frac{2}{\cancel{75}\,5} \times 10^4 \times 2 \times 10^{-10} \times \cancel{15} \times 10^3$$

$$S_i = \frac{4}{8} \times 10^{-3}$$

$$\therefore \quad S_i = 0.8\ mW.$$

$$\therefore \quad P_t = S_i \times P_L.$$

$$\therefore \quad P_t = 0.8 \times 10^{-3} \times 10^5.$$

$$\therefore \quad \boxed{P_t = 80\ W}$$

⇒ FM require very less Power compured to other modulcation technique. But BW is very large compured to AM.

④ PM $[\beta = 5]$.

$$\therefore \quad FOM = \beta^2/2 = \frac{25}{2}.$$

$$(FOM)_{pm} = \frac{1}{3}\,(FOM)_{FM}.$$

$$\therefore \quad P_t = 3 \times 80$$

$$\therefore \quad \boxed{P_t = 240\ W}$$

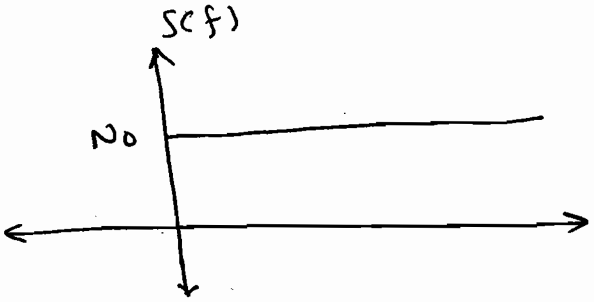
Q-2 1 mw Vedio signal Band limited to 100 MHz is transmitted through a channel the Power loss in the channel is 40 dB. The signal is affected by white noise having single sided power spectral density of 10^{-20} watts/Hz. Determine the $(SNR)_i$.

Ans:

$P_t = 1\ mw.$, $W = f_m = 100 \times 10^6$ Hz

$P_L = 40 dB$

$P_L = 10^4$ W.



$\therefore n_i = N_0 \cdot W$

$\therefore n_i = 10^8 \times 10^{-20}$

$\therefore n_i = 10^{-12}$ W.

$\therefore S_i = P_t / P_L.$

$\therefore S_i = \dfrac{10^{-3}}{10^4} = 10^{-7}.$

$\therefore (SNR)_i = \dfrac{S_i}{n_i} = \dfrac{10^{-7}}{10^{-12}}.$

$\therefore (SNR)_i = 10^5$

$$\therefore \boxed{(SNR)_{i\,dB} = 50\ dB.}$$