Introduction

Combination (selection)

Case-I abc

$n=3 \quad r=3$

(abc) bca/acb/bac/cab/cba → these selections are same.

orders of alphabet donot mattery.

= 1 way

$${}^nC_r = \frac{n!}{(n-r)!\,r!} \qquad n=3 \quad r=3$$

$${}^3C_3 = \frac{3!}{(3-3)!\,3!} = \frac{3!}{0!\,3!} = \frac{1}{1} = 1$$

Case-II abc taken two at a time

$n=3 \quad r=2$

ba/ab, bc/cb, ca/ac → 3 selections

$${}^nC_r = \frac{n!}{(n-r)!\,r!} \Rightarrow \frac{n!}{(n-r)!} = r! \times {}^nC_r$$

$${}^nP_r = r! \times {}^nC_r$$

$n=3 \quad r=2$

$${}^3C_2 = \frac{3!}{(3-2)!\,2!} = \frac{3!}{1! \times 2!} = \frac{3 \times 2!}{1 \times 2!} = 3$$

1. $n_{C_0} = 1$ (e.g. $5_{C_0} = 1$, $10_{C_0} = 1$)

2. $n_{C_n} = 1$ (e.g. $5_{C_5} = 1$ $3_{C_3} = 1$)

3. $n_{C_{n-1}} = n$ (e.g. $5_{C_4} = 5$, $11_{C_{10}} = 11$)

4. $n_{C_r} + n_{C_{r-1}} = {}^{n+1}C_r$

e.g. ${}^{n-5}C_r^{3} + {}^{5}C_{r-1}^{2} = {}^{5+1}C_3 = {}^{6}C_3$

5. $n_{C_a} = n_{C_b}$ either $a = b$ or $n = a+b$

1. If ${}^nC_8 = {}^nC_2$, find ${}^nC_2$.
2. Determine n if
   (i) ${}^{2n}C_3 : {}^nC_3 = 12 : 1$ (ii) ${}^{2n}C_3 : {}^nC_3 = 11 : 1$

HW

① $n_{C_8} = n_{C_2}$ find n_{C_2}

$n_{C_a} = n_{C_b}$

$a = b$ $\therefore$ $n = a+b = 8+2 = 10$

$n = 10$

$n_{C_2} = 10_{C_2} = \frac{10!}{(10-2)!\,2!} = \frac{10!}{8!\,2!} = \frac{10 \times 9 \times 8!}{8! \times 2} = 45$

② (ii) $2n_{C_3} : n_{C_3} = 11:1$ ——— (1)

$2n_{C_3} = \frac{(2n)!}{(2n-3)!\,3!} = \frac{2n\,(2n-1)\,(2n-2)\,(2n-3)!}{(2n-3)!\;6}$

$= \frac{n\,(2n-1)\,(2n-2)}{3}$

$n_{C_3} = \frac{n!}{(n-3)!\,3!} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!\;6}$

$= \frac{n(n-1)(n-2)}{6}$ Put in (1)

$$\frac{\frac{n(2n-1)(2n-2)}{3}}{\frac{n(n-1)(n-2)}{6}} = \frac{11}{1}$$

$$\frac{2(2n-1)\,2(n-1)}{} = \frac{11}{1}$$

$(n-1)(n-2)$

$4(2n-1) = 11$

$n-2$

$8n - 4 = 11n - 22$

$22 - 4 = 11n - 8n$

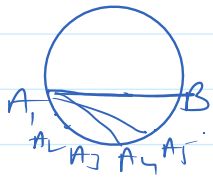
$3n = 18$

$n = \frac{18}{3} = 6$

नोट (इस)

3. How many chords can be drawn through 21 points on a circle?

Sol $n=21$ $r=2$



$$nC_2 = {}^{21}C_2 = \frac{21!}{(21-2)!\,2!} = \frac{21!}{19!\,2!}$$

$$= \frac{21 \times 20 \times 19!}{19! \times 2} = 210$$

4. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

$^5C_3 \times {}^4C_3$

$n=5$ $r=3$ | $n=4$ $r=3$

Sol no. of boys $= 5$
of girls $= 4$

we are to select team of 3 boys & 3 girls

Required no. of ways $= {}^5C_3 \times {}^4C_3$

$$= \frac{5!}{(5-3)!\,3!} \times 4$$

$$= \frac{5!}{2!\,3!} \times 4 = \frac{5 \times 4 \times 3!}{2 \times 3!} \times 4$$

$$= 40$$

5. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

$= {}^{6}C_3 \times {}^{5}C_3 \times {}^{5}C_3$ (Simplify it)

6. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Sol

Total Cards = 52

no. of ace = 4

Remaining Card = 52 − 4 = 48

We are to select 5 Card Combination

5 − 1 = 4

$= {}^{4}C_1 \times {}^{48}C_4$

$= 4 \times \frac{48!}{44!\,4!} = 4 \times \frac{48 \times 47 \times 46 \times 45 \times 44!}{44! \times 24}$

$= 4 \times 2 \times 47 \times 46 \times 45$

$= 8 \times 47 \times 2070$

$= 376 \times 2070$

$= 778320$

7. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Sol

Total players = 17 (out of 17, 5 can bowl)

select = 11

Total bowlers = 5

$= {}^{5}C_{4} \times {}^{17-5}C_{11-4} = {}^{5}C_{4} \times {}^{12}C_{7}$

$= 5 \times \frac{12!}{5!\,7!} = \frac{5 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7!}{10 \times 20 \times 7!} = 5 \times 11 \times 9 \times 8$

$= 5 \times 11 \times 72$

$= 5 \times 792$

$= 3960$

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

HW

9. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Sol.

Total Courses available = 9
To select = 5
Compulsory Course = 2.

$= {}^{9-2}C_{5-2}$

$= {}^{7}C_{3}$

$= \frac{7!}{4!\,3!} = \frac{7 \times 6 \times 5 \times 4!}{4! \times 6} = 35$

$n = 9 - 1 = 8, \quad r = 5$

$= {}^{8}C_{5}$