EXERCISE 15(A)

Question 1:

State, true or false:

(i) Two similar polygons are necessarily congruent.

(ii) Two congruent polygons are necessarily similar.

(iii) all equiangular triangles are similar

(iv) all isosceles triangles are similar.

(v) Two isosceles – right triangles are similar

(vi) Two isosceles triangles are similar, if an angle of one is congruent to the corresponding angle of the other.

(vii) The diagonals of a trapezium, divide each other into proportional segments.

Solution 1:

(i) False

(ii) True

(iii) True

(iv) False

(v) True

(vi) True

(vii) True

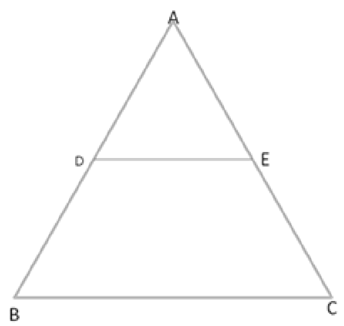
Question 2:

In triangle ABC, DE is parallel to BC; where D and E are the points on AB and AC respectively.

Prove: $\Delta ADE \sim \Delta ABC$.

Also, find the length of DE, if AD = 12 cm, BD = 24 cm BC = 8 cm.

Solution 2:



In Δ ADE and Δ ABC, DE is parallel to BC, so corresponding angles are equal.

$\angle ADE = \angle ABC$

$\angle AED = \angle ACB$

Hence, $\Delta ADE \sim \Delta ABC$ (By AA similarity criterion)

$\therefore \frac{AD}{AB} = \frac{DE}{BC}$

$\frac{12}{12+24} = \frac{DE}{8}$

$DE = \frac{12}{36} \times 8 = \frac{8}{3} = 2\frac{2}{3}$

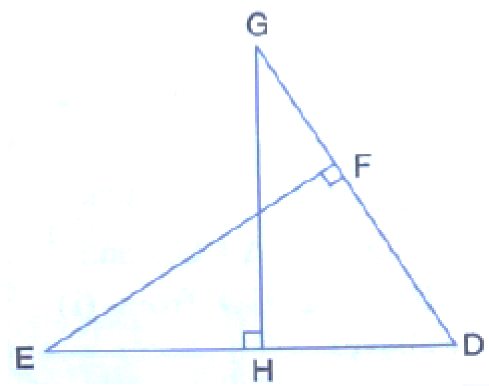
Hence, $DE = 2\frac{2}{3}$ cm

Question 3:

Given: $\angle GHE = \angle DFE = 90^\circ$,

DH = 8, DF = 12,

DG = 3x – 1 and DE = 4x + 2.



Find: the lengths of segments DG and DE

Solution 3:

In ΔDHG and ΔDFE,

$\angle GHD = \angle DFE = 90^\circ$

$\angle D = \angle D$ (Common)

$\therefore \Delta DHG \sim \Delta DFE$

$\Rightarrow \frac{DH}{DF} = \frac{DG}{DE}$

$$\Rightarrow \frac{8}{12} = \frac{3x-1}{4x+2}$$
$$\Rightarrow 32x + 16 = 36x - 12$$
$$\Rightarrow 28 = 4x$$
$$\Rightarrow x = 7$$
$$\therefore\ DG = 3\times 7 - 1 = 20$$
$$DE = 4\times 7 + 2 = 30$$

Question 4:

D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that: $CA^2 = CB \times CD$

Solution 4:

In ΔADC and ΔBAC,

$\angle ADC = \angle BAC$ (Given)

$\angle ACD = \angle ACB$ (Common)

$\therefore \Delta ADC \sim \Delta BAC$

$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

Hence, $CA^2 = CB \times CD$

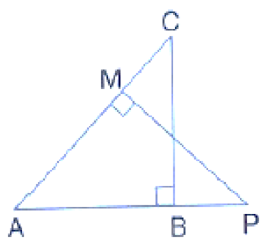
Question 5:

In the given figure, Δ ABC and Δ AMP are right angled at B and M respectively.

Given AC = 10 cm, AP = 15 cm and PM = 12 cm.

(i) Prove $\Delta ABC \sim \Delta AMP$

(ii) Find AB and BC



Solution 5:

(i) In Δ ABC and Δ AMP,

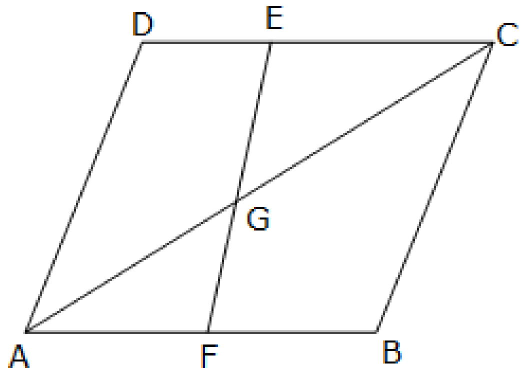
$\angle BAC = \angle PAM$ [Common]

∠ ABC = ∠ PMA [Each = 90°]
Δ ABC ~ Δ AMP [AA Similarity]
(ii)
AM = $\sqrt{AP^2 - PM^2} = \sqrt{15^2 - 12^2} = 11$
Since Δ ABC − Δ AMP,
$\frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$
$\Rightarrow \frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$
$\Rightarrow \frac{AB}{11} = \frac{BC}{12} = \frac{10}{15}$
From this we can write,
$\frac{AB}{11} = \frac{10}{15}$
$\Rightarrow AB = \frac{10 \times 11}{15} = 7.33$
$\frac{BC}{12} = \frac{10}{15}$
$\Rightarrow BC = 8cm$

Question 6:

E and F are the points in sides DC and AB respectively of parallelogram ABCD. If diagonal AC and segment EF intersect at G; prove that:
AG × EG = FG × CG

Solution 6:



In ΔEGC and ΔFGA

$\angle ECG = \angle FAG$ (Alternate angles as AB ∥ CD)
$\angle EGC = \angle FGA$ (Vertically opposite angles)
$\Delta EGC \sim \Delta FGA$ (By AA – similarity)

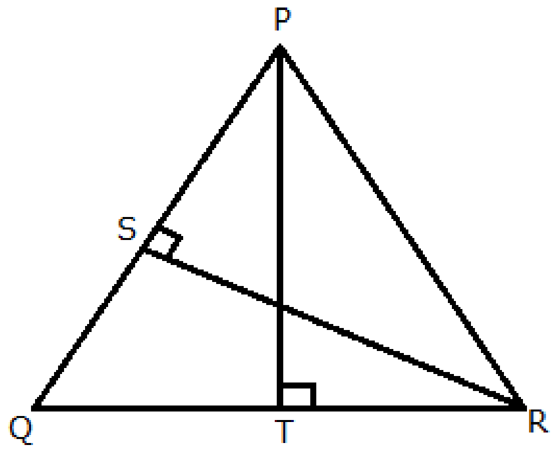
$$\therefore \frac{EG}{FG} = \frac{CG}{AG}$$

$AG \times EG = FG \times CG$

Question 7:

Given: RS and PT are altitudes of ΔPQR. Prove that:
(i) $\Delta PQT \sim \Delta QRS$
(ii) $PQ \times QS = RQ \times QT$

Solution 7:



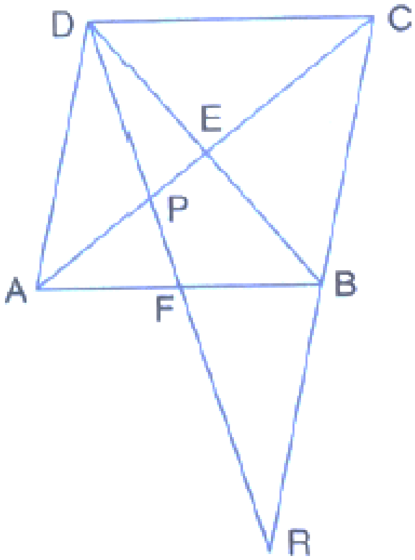
(i)
In ΔPQT and ΔQRS,
$\angle PTQ = \angle RSQ = 90^\circ$ (Given)
$\angle PQT = \angle RQS$ (Common)
$\Delta PQT \sim \Delta RQS$ (By AA similarity)
(ii)
Since, triangle PQT and RQS are similar

$$\therefore \frac{PQ}{RQ} = \frac{QT}{QS}$$

$\Rightarrow PQ \times QS = RQ \times QT$

Question 8:

Given: ABCD is a rhombus, DRP and CBR are straight lines.



Prove that:

DP × CR = DC × PR

Solution 8:

In ΔDPA and ΔRPC,

∠DPA = ∠RPC (Vertically opposite angles)

∠PAD = ∠PCR (Alternate angles)

ΔDPA ~ ΔRPC

$$\therefore \frac{DP}{PR} = \frac{AD}{CR}$$

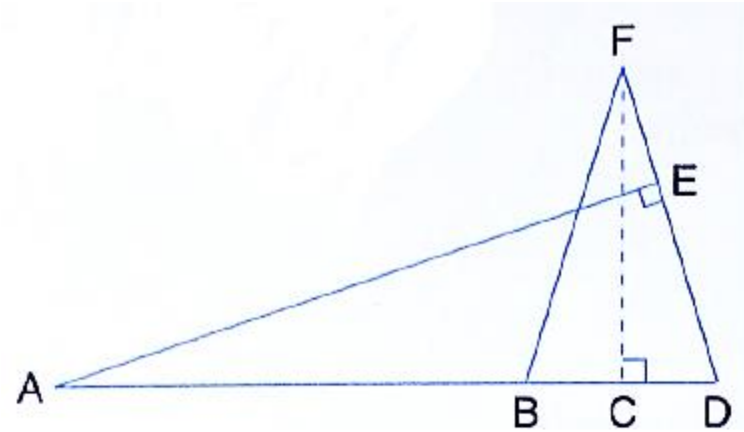
$$\frac{DP}{PR} = \frac{DC}{CR} \quad \text{(AD = DC, as ABCD is rhombus)}$$

Hence, DP × CR = DC × PR

Question 9:

Given: FB = FD, AE ⊥ FD and FC ⊥ AD

Prove: $\frac{FB}{AD} = \frac{BC}{ED}$



Solution 9:

Given, FB = FD

$\therefore \angle FDB = \angle FBD$ ……….(1)

In ΔAED = ΔFCB,

$\angle AED = \angle FCB = 90^\circ$

$\angle ADE = \angle FBC$ [using (1)]

ΔAED ~ ΔFCB [By AA similarity]

$$\therefore \frac{AD}{FB} = \frac{ED}{BC}$$

$$\frac{FB}{AD} = \frac{BC}{ED}$$

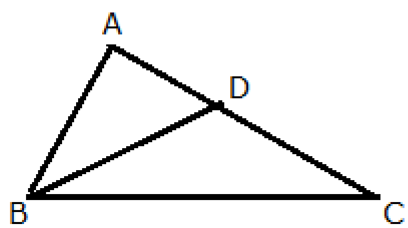
Question 10:

In Δ ABC, ∠B = 2 ∠C and the bisector of angle B meets CA at point D. Prove that:

(i) Δ ABC and Δ ABD are similar,

(ii) DC: AD = BC: AB

Solution 10:



(i) Since, BD is the bisector of angle B,

$\angle ABD = \angle DBC$

Also, given $\angle B = 2\angle C$

$\therefore \angle ABD = \angle DBC = \angle ACB$ .... (1)

In Δ ABC and Δ ABD,

$\angle BAC = \angle DAB$ (Common)

$\angle ACB = \angle ABD$ (Using (1))

$\therefore \Delta ABC \sim \Delta ADB$ (By AA similarity)

(ii) Since, triangles ABC and ADB are similar,

$$\therefore \frac{BC}{BD} = \frac{AB}{AD}$$

$$\frac{BC}{AB} = \frac{BD}{AD}$$

$$\frac{BC}{AB} = \frac{DC}{AD} \quad (\angle DBC = \angle DCB \Rightarrow DC = BD)$$

BC : AB = DC : AD

Question 11:

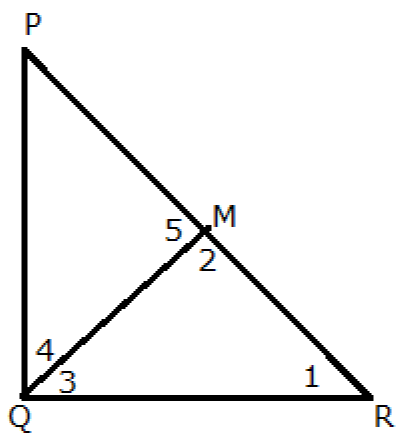
In ΔPQR, $\angle Q = 90°$ and QM is perpendicular to PR. Prove that:

(i) $PQ^2 = PM \times PR$

(ii) $QR^2 = PR \times MR$

(iii) $PQ^2 + QR^2 = PR^2$

Solution 11:



(i) In ΔPQM and ΔPQR,

$\angle PMQ = \angle PQR = 90^\circ$

$\angle QPM = \angle RPQ$ (Common)

$\therefore \Delta PQM \sim \Delta PRQ$ (By AA Similarity)

$\therefore \dfrac{PQ}{PR} = \dfrac{PM}{PQ}$

$\Rightarrow PQ^2 = PM \times PR$

(ii) In ΔQMR and ΔPQR,

$\angle QMR = \angle PQR = 90^\circ$

$\angle QRM = \angle QRP$ (Common)

$\therefore \Delta\ QRM \sim \Delta PQR$ (By AA similarity)

$\therefore \dfrac{QR}{PR} = \dfrac{MR}{QR}$

$\Rightarrow QR^2 = PR \times MR$

(iii) Adding the relations obtained in (i) and (ii), we get,

$PQ^2 + QR^2 = PM \times PR + PR \times MR$

$= PR(PM + MR)$

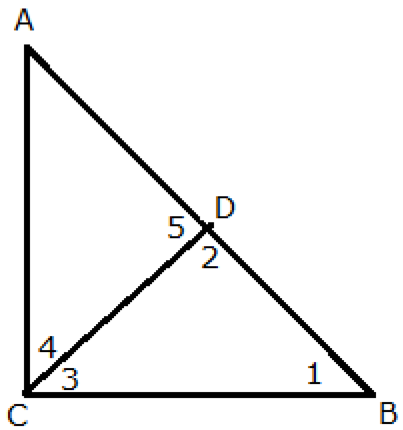
$= PR \times PR$

$= PR^2$

Question 12:

In ΔABC, right – angled at C, $CD \perp AB$. Prove: $CD^2 = AD \times DB$

Solution 12:



In Δ CDB,

$\angle 1 + \angle 2 + \angle 3 = 180^o$

$\angle 1 + \angle 3 = 90^o$ ...... (1)(Since, $\angle 2 = 90^o$)

$\angle 3 + \angle 4 = 90^o$ ...... (2) (Since, $\angle ACB = 90^o$)

From (1) and (2),

$\angle 1 + \angle 3 = \angle 3 + \angle 4$

$\angle 1 = \angle 4$
Also, $\angle 2 = \angle 5 = 90^o$
$\therefore \Delta BDC \sim \Delta CDA$ (By AA similarity)

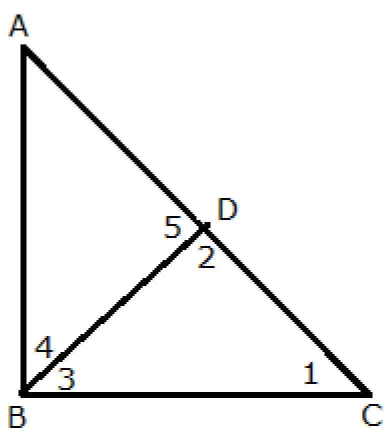
$$\Rightarrow \frac{DB}{CD} = \frac{CD}{AD}$$

$$\Rightarrow CD^2 = AD \times DB$$

Question 13:

In ΔABC, $\angle B = 90°$ and $BD \perp AC$.
(i) If CD = 10 cm and BD = 8 cm; find AD.
(ii) IF AC = 18 cm and AD = 6cm; find BD.
(iii) If AC = 9 cm and AB = 7cm; find AD.

Solution 13:



(i) In Δ CDB,
$\angle 1 + \angle 2 + \angle 3 = 180^o$
$\angle 1 + \angle 3 = 90^o$ ..... (1)(Since, $\angle 2 = 90^o$)
$\angle 3 + \angle 4 = 90^o$ .....(2) (Since, $\angle ABC = 90^o$)
From (1) and (2),
$\angle 1 + \angle 3 = \angle 3 + \angle 4$
$\angle 1 = \angle 4$
Also, $\angle 2 = \angle 5 = 90^o$
$\therefore \Delta CDB \sim \Delta BDA$ (By AA similarity)

$$\Rightarrow \frac{CD}{BD} = \frac{BD}{AD}$$

$$\Rightarrow BD^2 = AD \times CD$$

$\Rightarrow (8)^2 = AD \times 10$

$\Rightarrow AD = 6.4$

Hence, AD = 6.4 cm

(ii) Also, by similarity, we have:

$\frac{BD}{DA} = \frac{CD}{BD}$

$BD^2 = 6 \times (18-6)$

$BD^2 = 72$

Hence, $BD = 8.5$ cm

(iii)

Clearly, $\Delta ADB \sim \Delta ABC$

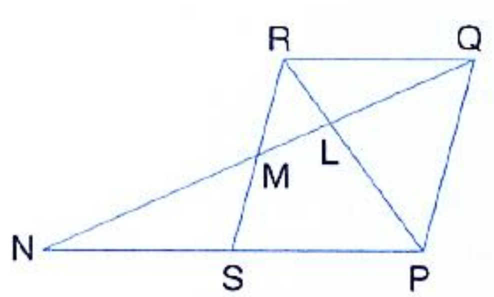
$\therefore \frac{AD}{AB} = \frac{AB}{AC}$

$AD = \frac{7 \times 7}{9} = \frac{49}{9} = 5\frac{4}{9}$

Hence, $AD = 5\frac{4}{9}$ cm

Question 14:

In the figure, PQRS is a parallelogram with PQ = 16 cm and QR = 10cm, L is a point on PR such that RL: LP = 2: 3. QL produced meets RS at M and PS produced at N.



Find the lengths of PN and RM.

Solution 14:

In ΔRLQ and ΔPLN,

$\angle RLQ = \angle PLN$ (Vertically opposite angles)

$\angle LRQ = \angle LPN$ (Alternate angles)

$\Delta RLQ \sim \Delta PLN$ (AA Similarity)

$\therefore \frac{RL}{LP} = \frac{RQ}{PN}$

$\frac{2}{3} = \frac{10}{PN}$

PN = 15 cm

In ΔRLM and ΔPLQ

$\angle RLM = \angle PLQ$ (Vertically opposite angles)

$\angle LRM = \angle LPQ$ (Alternate angles)

$\Delta RLM \sim \Delta PLQ$ (AA Similarity)

$\therefore \frac{RM}{PQ} = \frac{RL}{LP}$

$\frac{RM}{16} = \frac{2}{3}$

$RM = \frac{32}{3} = 10\frac{2}{3}$ cm

Question 15:

In quadrilateral ABCD, diagonals AC and BD intersect at point E such that

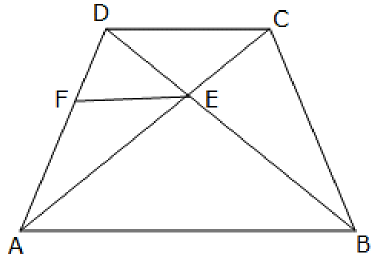
AE: EC = BE: ED

Show that ABCD is a parallelogram.

Solution 15:

Given, AE : EC = BE : ED

Draw EF || AB



In ΔABD, EF || AB

Using Basic Proportionality theorem,

$\frac{DF}{FA} = \frac{DE}{EB}$

But, given $\frac{DE}{EB} = \frac{CE}{EA}$

$\therefore \dfrac{DF}{FA} = \dfrac{CE}{EA}$

Thus, in ΔDCA, E and F are points on CA and DA respectively such that $\dfrac{DF}{FA} = \dfrac{CE}{EA}$

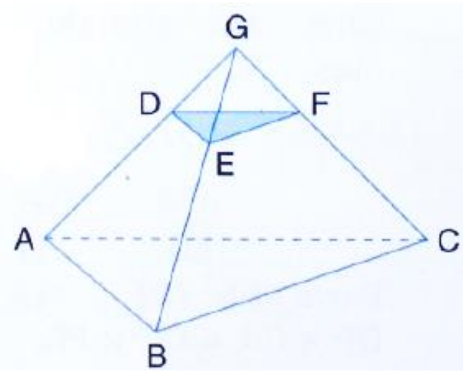
Thus, by converse of Basic proportionality theorem, FE || DC.
But, FE || AB.
Hence, AB || DC.
Thus, ABCD is a trapezium.

Question 16:

Given: AB || DE and BC || EF. Prove that:

(i) $\dfrac{AD}{DG} = \dfrac{CF}{FG}$

(ii) $\Delta DFG \sim \Delta ACG$



Solution 16:

(i) In ΔAGB, DE || AB , by Basic proportionality theorem,

$\dfrac{GD}{DA} = \dfrac{GE}{EB}$ …………(1)

In ΔGBC, EF || BC, by Basic proportionality theorem,

$\dfrac{GE}{EB} = \dfrac{GF}{FC}$ ……………(2)

From (1) and (2), we get,

$\dfrac{GD}{DA} = \dfrac{GF}{FC}$

$\dfrac{AD}{DG} = \dfrac{CF}{FG}$

(ii)

From (i), we have:

$$\frac{AD}{DG} = \frac{CF}{FG}$$

$\angle DGF = \angle AGC$ (Common)

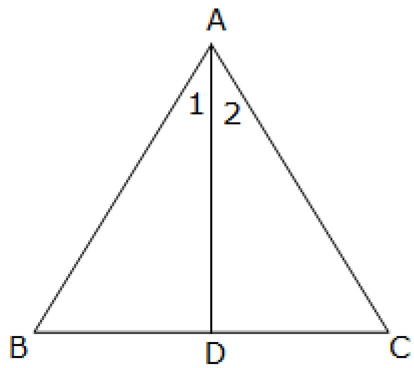
$\therefore \Delta DFG \sim \Delta ACG$ (SAS similarity)

Question 17:

In triangle ABC, AD is perpendicular to side BC and $AD^2 = BD \times DC$.

Show that angle BAC = 90°

Solution 17:



Given $AD^2 = BD \times DC$

$$\frac{AD}{DC} = \frac{BD}{AD}$$

$\angle ADB = \angle ADC = 90^\circ$

$\therefore \Delta DBA \sim \Delta DAC$ (SAS similarity)

So, these two triangles will be equiangular.

$\therefore \angle 1 = \angle C$ and $\angle 2 = \angle B$

$\angle 1 + \angle 2 = \angle B + \angle C$

$\angle A = \angle B + \angle C$

By angle sum property,

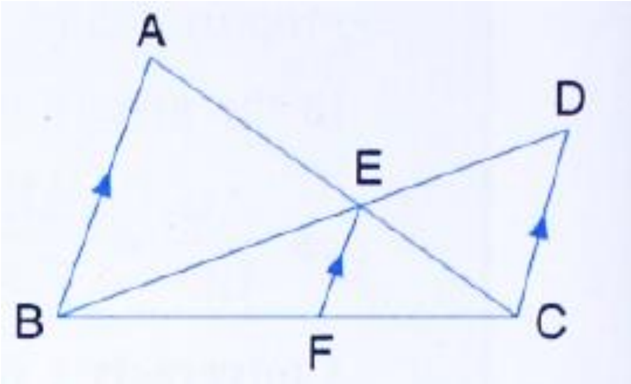
$\angle A + \angle B + \angle C = 180^\circ$

$2\angle A = 180^\circ$

$\angle A = \angle BAC = 90^\circ$

Question 18:

In the given figure, AB ∥ EF ∥ DC; AB = 67.5 cm, DC = 40.5 cm and AE = 52.5 cm.



(i) Name the three pairs of similar triangles.

(ii) Find the lengths of EC and EF.

Solution 18:

(i) The three pair of similar triangles are:

ΔBEF and ΔBDC

ΔCEF and ΔCAB

ΔABE and ΔCDE

(ii) Since, ΔABE and ΔCDE are similar,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{67.5}{40.5} = \frac{52.5}{CE}$$

CE = 31.5 cm

Since, ΔCEF and ΔCAB are similar,

$$\frac{CE}{CA} = \frac{EF}{AB}$$

$$\frac{31.5}{52.5+31.5} = \frac{EF}{67.5}$$

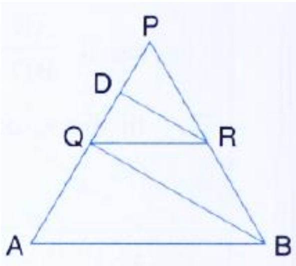
$$\frac{31.5}{84} = \frac{EF}{67.5}$$

$$EF = \frac{2126.25}{84}$$

$$EF = \frac{405}{16} = 25\frac{5}{16}\text{cm}$$

Question 19:

In the given figure, QR is parallel to AB and DR is parallel to AB and DR is parallel to QB. Prove that:



$PQ^2 = PD \times PA$

Solution 19:

Given, QR is parallel to AB. Using Basic proportionality theorem,

$$\Rightarrow \frac{PQ}{PA} = \frac{PR}{PB} \quad ....(1)$$

Also, DR is parallel to QB. Using Basic proportionality theorem,

$$\Rightarrow \frac{PD}{PQ} = \frac{PR}{PB} \quad ....(2)$$

From (1) and (2), we get,

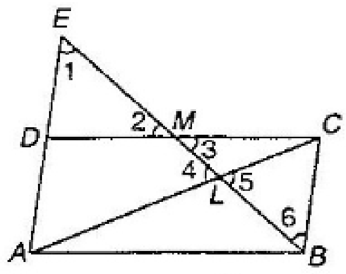
$$\frac{PQ}{PA} = \frac{PD}{PQ}$$

$PQ^2 = PD \times PA$

Question 20:

Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting diagonal AC in L and AD produced in E. Prove that: EL = 2 BL.

Solution 20:



$\angle 1 = \angle 6$ (Alternate interior angles)

$\angle 2 = \angle 3$ (Vertically opposite angles)

DM = MC (M is the mid-point of CD)

∴ ΔDEM ≅ ΔCBM (AAS congruence criterion)

So, DE = BC (Corresponding parts of congruent triangles)

Also, AD = BC (Opposite sides of a parallelogram)

⇒ AE = AD + DE = 2BC

Now, ∠1 = ∠6 and ∠4 = ∠5

∴ ΔELA ~ ΔBC (AA similarity)

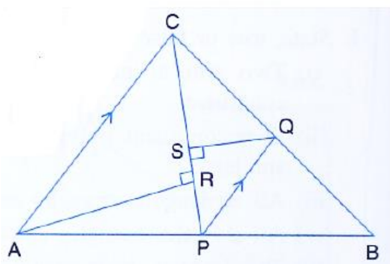
$$\Rightarrow \frac{EL}{BL} = \frac{EA}{BC}$$

$$\Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} = 2$$

$$\Rightarrow E = 2BL$$

Question 21:

In the figure, given below, P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC.



(i) Calculate the ratio PQ : AC, giving reason for your answer.

(ii) In triangle ARC, ∠ARC = 90°. Given QS = 6cm, calculate the length of AR.

Solution 21:

(i) Given, AP : PB = 4 : 3.

Since, PQ || AC. Using Basic Proportionality theorem,

$$\frac{AP}{PB} = \frac{CQ}{QB}$$

$$\Rightarrow \frac{CQ}{QB} = \frac{4}{3}$$

$$\Rightarrow \frac{BQ}{BC} = \frac{3}{7} \quad \ldots\ldots\ldots\ldots(1)$$

Now, ∠PQB = ∠ACB (Corresponding angles)

∠QPB = ∠CAB (Corresponding angles)

∴ΔPBQ ~ ΔABC (AA similarity)

$\Rightarrow \frac{PQ}{AC} = \frac{BQ}{BC}$

$\Rightarrow \frac{PQ}{AC} = \frac{3}{7}$ [using (1)]

(ii) $\angle ARC = \angle QSP = 90^o$

$\angle ACR = \angle SPQ$ (Alternate angles)

$\therefore \Delta ARC \sim \Delta QSP$ (AA similarity)

$\Rightarrow \frac{AR}{QS} = \frac{AC}{PQ}$

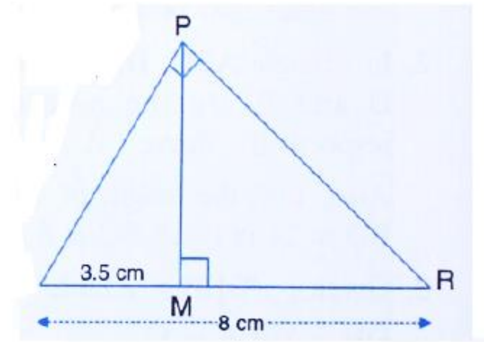
$\Rightarrow \frac{AR}{QS} = \frac{7}{3}$

$\Rightarrow AR = \frac{7 \times 6}{3} = 14cm$

Question 22:

In the right-angled triangle QPR, PM is an altitude.

Given that QR = 8cm and MQ = 3.5 cm, calculate the value of PR.



Solution 22:

We have

$\angle QPR = \angle PMR = 90°$

$\angle PRQ = \angle PRM$ (common)

$\Delta PQR \sim \Delta MPR$ (AA similarity)

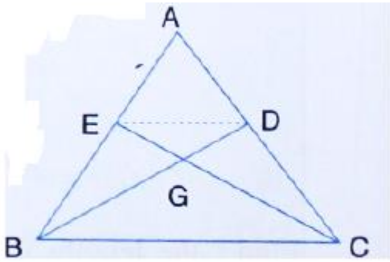
$\therefore \frac{QR}{PR} = \frac{PR}{MR}$

$PR^2 = 8 \times 4.5 = 36$

PR = 6 cm

Question 23:

In the figure, given below, the medians BD and CE of a triangle ABC meet at G. Prove that:
(i) $\Delta EGD \sim \Delta CGB$ and
(ii) BG = 2 GD from (i) above.



Solution 23:

(i) Since, BD and CE are medians.
AD = DC
AE = BE
Hence, by converse of Basic Proportionality theorem,
ED || BC
In ΔEGD and ΔCGB,
$\angle DEG = \angle GCB$ (alternate angles)
$\angle EGD = \angle BGC$ (Vertically opposite angles)
$\Delta EGD \sim \Delta CGB$ (AA similarity)
(ii) since, $\Delta EGD \sim \Delta CGB$

$$\frac{GD}{GB} = \frac{ED}{BC} \quad \ldots\ldots\ldots (1)$$

In ΔAED and ΔABC,
$\angle AED = \angle ABC$ (Corresponding angles)
$\angle EAD = \angle BAC$ (Common)
$\Delta EAD \sim \Delta BAC$ (AA similarity)

$$\therefore \frac{ED}{BC} = \frac{AE}{AB} = \frac{1}{2} \quad \text{(since, E is the mid – point of AB)}$$

$$\Rightarrow \frac{ED}{BC} = \frac{1}{2}$$

From (1),

$$\frac{GD}{GB} = \frac{1}{2}$$

$$GB = 2GD$$

EXERCISE. 15(B)

Question 1:

(i) The ratio between the corresponding sides of two similar triangles is 2 is to 5. Find the ratio between the areas of these triangles.

(ii) Area of two similar triangles are 98 sq.cm and 128 sq.cm. Find the ratio between the lengths of their corresponding sides.

Solution 1:

We know that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

(i) Required ratio $= \frac{2^2}{5^2} = \frac{4}{25}$

(ii) Required ratio $= \sqrt{\frac{98}{128}} = \sqrt{\frac{49}{64}} = \frac{7}{8}$

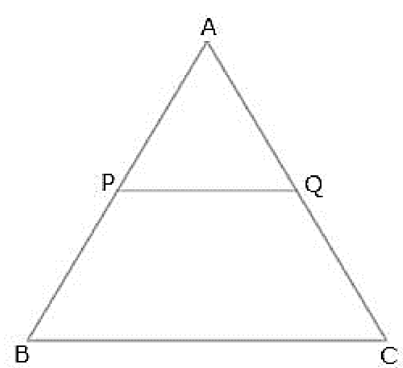
Question 2:

A line PQ is drawn parallel to the base BC of ΔABC which meets sides AB and AC at points P and Q respectively. If AP $= \frac{1}{3}$ PB; find the value of:

(i) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta APQ}$

(ii) $\frac{\text{Area of } \Delta APQ}{\text{Area of trapezium PBCQ}}$

Solution 2:



(i) AP $= \frac{1}{3}$ PB $\Rightarrow \frac{AP}{PB} = \frac{1}{3}$

In ΔAPQ and ΔABC,

As PQ ∥ BC, corresponding angles are equal

$\angle APQ = \angle ABC$

$\angle AQP = \angle ACB$

$\Delta APQ \sim \Delta ABC$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta APQ} = \frac{AB^2}{AP^2}$$

$$= \frac{4^2}{1^2} = 16 : 1$$

$$\left(\frac{AP}{PB} = \frac{1}{3} \Rightarrow \frac{AB}{AP} = \frac{4}{1}\right)$$

$$\frac{\text{Area of } \Delta APQ}{\text{Area of trapezium PBCQ}}$$

$$= \frac{\text{Area of } \Delta APQ}{\text{Area of } \Delta ABC - \text{Area of } \Delta APQ}$$

$$= \frac{1}{16-1} = 1 : 5$$

Question 3:

The perimeter of two similar triangles are 30 cm and 24 cm. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

Solution 3:

Let $\Delta ABC \sim \Delta DEF$

Then, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB+BC+AC}{DE+EF+DF}$

$$= \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$$

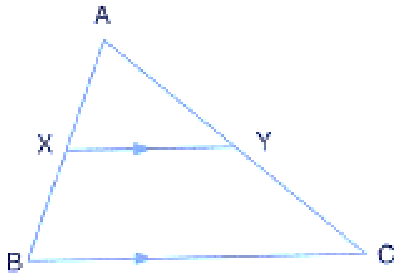
$$\Rightarrow \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB}{DE}$$

$$\Rightarrow \frac{30}{24} = \frac{12}{DE}$$

$\Rightarrow DE = 9.6$ cm

Question 4:

In the given figure, AX : XB = 3: 5



Find:

(i) the length of BC, if the length of XY is 18 cm.

(ii) the ratio between the areas of trapezium XBCY and triangle ABC.

Solution 4:

Given, $\frac{AX}{XB} = \frac{3}{5} \Rightarrow \frac{AX}{AB} = \frac{3}{8}$ ....(1)

(i)

In ΔAXY and ΔABC,

As XY || BC, Corresponding angles are equal

$\angle AXY = \angle ABC$

$\angle AYX = \angle ACB$

$\Delta AXY \sim \Delta ABC$

$\Rightarrow \frac{AX}{AB} = \frac{XY}{BC}$

$\Rightarrow \frac{3}{8} = \frac{18}{BC}$

$\Rightarrow$ BC = 48 cm

(ii)

$$\frac{\text{Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \frac{AX^2}{AB^2} = \frac{9}{64}$$

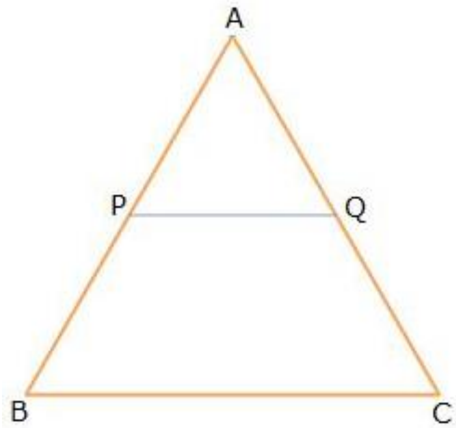
$$\frac{\text{Area of } \Delta ABC \text{ -Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \frac{64-9}{64} = \frac{55}{64}$$

$$\frac{\text{Area of trapezium XBCY}}{\text{Area of } \Delta ABC} = \frac{55}{64}$$

Question 5:

ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that PQ || BC and divides triangle ABC into two parts equal in area. Find the value of ratio BP : AB.

Solution 5:



From the given information, we have:

$$ar(\Delta APQ) = \frac{1}{2}ar(\Delta ABC)$$

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{2}$$

$$\Rightarrow \frac{AP^2}{AB^2} = \frac{1}{2}$$

$$\Rightarrow \frac{AP}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{AB - BP}{AB} = \frac{1}{\sqrt{2}}$$

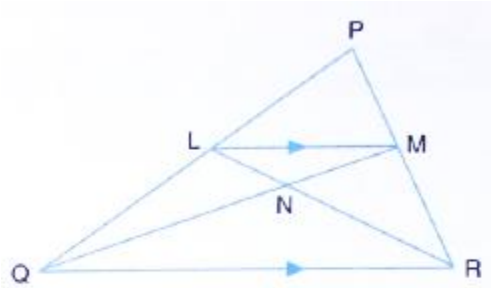
$$\Rightarrow 1 - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}} = \frac{2 - \sqrt{2}}{2}$$

Question 6:

In the given triangle PQR, LM is parallel to QR and PM : MR = 3: 4



Calculate the value of ratio:

(i) $\frac{PL}{PQ}$ and then $\frac{LM}{QR}$

(ii) $\frac{\text{Area of } \Delta LMN}{\text{Area of } \Delta MNR}$ (iii) $\frac{\text{Area of } \Delta LQM}{\text{Area of } \Delta LQN}$

Solution 6:

(i)

In ΔPLM and ΔPQR,

As LM || QR, Corresponding angles are equal

$\angle PLM = \angle PQR$

$\angle PML = \angle PRQ$

$\Delta PLM \sim \Delta PQR$

$$\Rightarrow \frac{3}{7} = \frac{LM}{QR}\left(\because \frac{PM}{MR} = \frac{3}{4} \Rightarrow \frac{PM}{PR} = \frac{3}{7}\right)$$

Also, by using basic proportionality theorem, we have:

$$\frac{PL}{LQ} = \frac{PM}{MR} = \frac{3}{4}$$

$$\Rightarrow \frac{LQ}{PL} = \frac{4}{3}$$

$$\Rightarrow 1 + \frac{LQ}{PL} = 1 + \frac{4}{3}$$

$$\Rightarrow \frac{PL + LQ}{PL} = \frac{3+4}{3}$$

$$\Rightarrow \frac{PQ}{PL} = \frac{7}{3}$$

$$\Rightarrow \frac{PL}{PQ} = \frac{3}{7}$$

(ii) Since ΔLMN and ΔMNR have common vertex at M and their bases LN and NR are along the same straight line

$$\therefore \frac{\text{Area of } \Delta LMN}{\text{Area of } \Delta MNR} = \frac{LN}{NR}$$

Now, in ΔLNM and ΔRNQ

$\angle NLM = \angle NRQ$ (Alternate angles)

$\angle LMN = \angle NQR$ (Alternate angles)

$\Delta LMN \sim \Delta RNQ$ (AA Similarity)

$$\therefore \frac{MN}{QN} = \frac{LN}{NR} = \frac{LM}{QR} = \frac{3}{7}$$

$$\therefore \frac{\text{Area of } \Delta LMN}{\text{Area of } \Delta MNR} = \frac{LN}{NR} = \frac{3}{7}$$

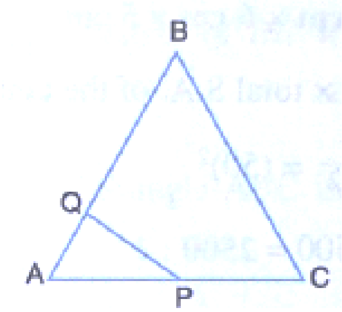
(iii) Since ΔLQM and ΔLQN have common vertex at L and their bases QM and QN are along the same straight line

$$\frac{\text{Area of } \Delta LQM}{\text{Area of } \Delta MNR} = \frac{QM}{QN} = \frac{10}{7}$$

$$\left(\because \frac{MN}{QN} = \frac{3}{7} \Rightarrow \frac{QM}{QN} = \frac{10}{7}\right)$$

Question 7:

The given diagram shows two isosceles triangles which are similar also. In the given diagram, PQ and BC are not parallel; PC = 4, AQ = 3, QB = 12, BC = 15 and AP = PQ



Calculate:

(i) the length of AP,

(ii) the ratio of the areas of triangle APQ and triangle ABC.

Solution 7:

(i)

Given, $\Delta AQP \sim \Delta ACB$

$$\Rightarrow \frac{AQ}{AC} = \frac{AP}{AB}$$

$$\Rightarrow \frac{3}{4+AP} = \frac{AP}{3+12}$$

$\Rightarrow AP^2 + 4AP - 45 = 0$

$\Rightarrow (AP + 9)(AP - 5) = 0$

$\Rightarrow AP = 5$ units (as length cannot be negative)

(ii)

Since, $\Delta AQP \sim \Delta ACB$

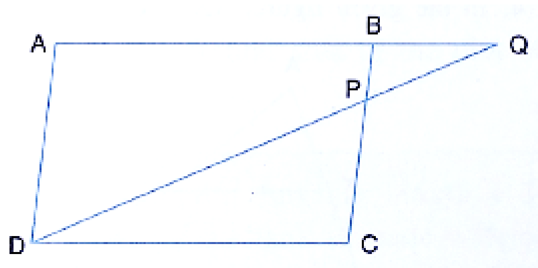
$$\therefore \frac{ar(\Delta APQ)}{ar(\Delta ACB)} = \frac{PQ^2}{BC^2}$$

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{BC^2} \quad (PQ = AP)$$

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \left(\frac{5}{15}\right)^2 = \frac{1}{9}$$

Question 8:

In the figure, given below, ABCD is a parallelogram. P is a point on BC such that BP : PC = 1: 2. DP produced meets AB produces at Q. Given the area of triangle CPQ = 20 cm^2.



Calculate:

(i) area of triangle CDP,

(ii) area of parallelogram ABCD.

Solution 8:

(i) In ΔBPQ and ΔCPD

$\angle BPQ = \angle CPD$ (Vertically opposite angles)

$\angle BQP = \angle PDC$ (Alternate angles)
$\Delta BPQ \sim \Delta CPD$ (AA similarity)

$$\therefore \frac{BP}{PC} = \frac{PQ}{PD} = \frac{BQ}{CD} = \frac{1}{2} \left(\because \frac{BP}{PC} = \frac{1}{2}\right)$$

$$\text{Also, } \frac{ar(\Delta BPQ)}{ar(\Delta CPD)} = \left(\frac{BP}{PC}\right)^2$$

$$\Rightarrow \frac{10}{ar(\Delta CPD)} = \frac{1}{4} \quad [ar(\Delta BPQ) = \frac{1}{2} \times ar(\Delta CPQ), ar(CPQ)=20]$$

$\Rightarrow ar(\Delta CPD) = 40\ cm^2$
(ii) In ΔBAP and ΔAQD
As BP || AD, corresponding angles are equal
$\angle QBP = \angle QAD$
$\angle BQP = \angle AQD$ (Common)
$\Delta BQP \sim \Delta AQD$ (AA similarity)

$$\therefore \frac{AQ}{BQ} = \frac{QD}{QP} = \frac{AD}{BP} = 3 \left(\because \frac{BP}{PC} = \frac{PQ}{PD} = \frac{1}{2} \Rightarrow \frac{PQ}{QD} = \frac{1}{3}\right)$$

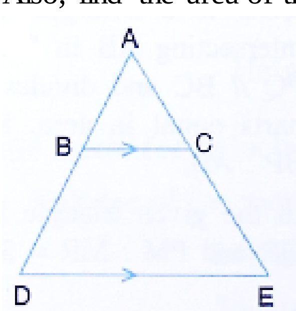
$$\text{Also, } \frac{ar(\Delta AQD)}{ar(\Delta BQP)} = \left(\frac{AQ}{BQ}\right)^2$$

$$\Rightarrow \frac{ar(\Delta AQD)}{10} = 9$$

$\Rightarrow ar(\Delta AQD) = 90\ cm^2$
$\therefore ar(ADPB) = ar(\Delta AQD) - ar(\Delta BQP) = 90\ cm^2 - 10\ cm^2 = 80\ cm^2$
$ar(ABCD) = ar(\Delta CDP) + ar(ADPB) = 40\ cm^2 + 80\ cm^2 = 120\ cm^2$

Question 9:

In the given figure, BC is parallel to DE. Area of triangle ABC = 25 cm^2 , Area of trapezium BCED = 24 cm^2 and DE = 14 cm. Calculate the length of BC.
Also, find the area of triangle BCD.



Solution 9:

In ΔABC and ΔADE,

As BC || DE, corresponding angles are equal

$\angle ABC = \angle ADE$

$\angle ACB = \angle AED$

$\Delta ABC \sim \Delta ADE$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{BC^2}{DE^2}$$

$$\frac{25}{49} = \frac{BC^2}{14^2} \text{ (ar } (\Delta ADE) = ar(\Delta ABC) + ar(\text{trapezium } BCED))$$

$BC^2 = 100$

BC = 10 cm

In trapezium BCED,

$$\text{Area} = \frac{1}{2}(\text{Sum of parallel sides}) \times h$$

Given : Area of trapezium BCED = 24 cm^2, BC = 10 cm, DE = 14cm

$$\therefore 24 = \frac{1}{2}(10+14) \times h$$

$$\Rightarrow h = \frac{48}{(10+14)}$$

$$\Rightarrow h = \frac{48}{24}$$

$\Rightarrow h = 2$

$$\text{Area of } \Delta BCD = \frac{1}{2} \times \text{base} \times \text{height}$$

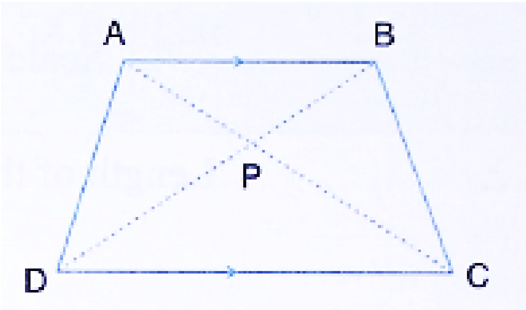
$$= \frac{1}{2} \times BC \times h$$

$$= \frac{1}{2} \times 10 \times 2$$

$\therefore$ Area of ΔBCD = 10 cm^2

Question 10:

The given figure shows a trapezium in which AB is parallel to DC and diagonals AC and BD intersect at point P. If AP : CP = 3: 5,



Find:

(i) ΔAPB : ΔCPB (ii) ΔDPC : ΔAPB

(iii) ΔADP : ΔAPB (iv) ΔAPB : ΔADB

Solution 10:

(i) Since ΔAPB and ΔCPB have common vertex at B and their bases AP and PC are along the same straight line

$$\therefore \frac{ar(\Delta APB)}{ar(\Delta CPB)} = \frac{AP}{PC} = \frac{3}{5}$$

(ii) Since ΔDPC and ΔBPA are similar

$$\therefore \frac{ar(\Delta DPC)}{ar(\Delta BPA)} = \left(\frac{PC}{AP}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

(iii) Since ΔADP and ΔAPB have common vertex at A and their bases DP and PB are along the same straight line

$$\therefore \frac{ar(\Delta ADP)}{ar(\Delta APB)} = \frac{DP}{PB} = \frac{5}{3}$$

$$\left(\Delta APB \sim \Delta CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5}\right)$$

(iv) Since ΔAPB and ΔADB have common vertex at A and their bases BP and BD are along the same straight line.

$$\therefore \frac{ar(\Delta APB)}{ar(\Delta ADB)} = \frac{PB}{BD} = \frac{3}{8}$$

$$\left(\Delta APB \sim \Delta CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5} \Rightarrow \frac{BP}{BD} = \frac{3}{8}\right)$$

Question 11:

On a map, drawn to a scale of 1 : 250000, a triangular plot PQR of land has the following measurements : PQ = 3cm, QR = 4 cm and angles PQR = 90°

Calculate:

(i) the actual lengths of QR and PR in kilometer.

(ii) the actual area of the plot in sq . km.

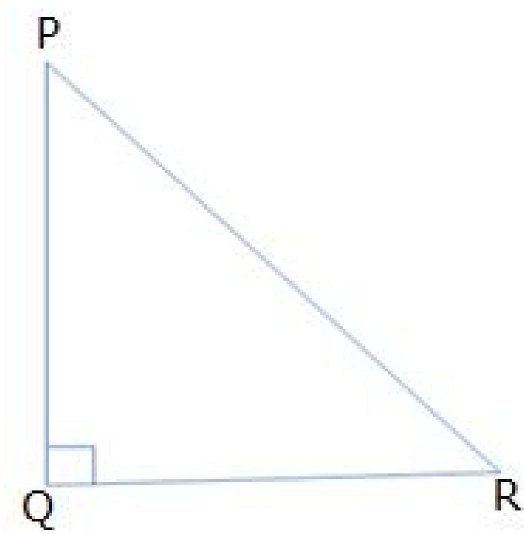
Solution 11:

Scale:-1 : 250000

∴ 1 cm represents 250000cm

$= \frac{250000}{1000 \times 100} = 2.5$ km

∴ 1 cm represents 2.5 km



(i)

Actual length of PQ = 3 × 2.5 = 7.5 km

Actual length of QR = 4 × 2.5 = 10 km

Actual length of PR = $\sqrt{(7.5)^2 + (10)^2}$ km $= 12.5$km

(ii)

Area of Î”PQR = $\frac{1}{2} \times PQ \times QR = \frac{1}{2}(3)(4)\text{cm}^2 = 6\text{cm}^2$

1cm represents 2.5 km

1 cm^2 represents 2.5 × 2.5 km^2

The area of plot = 2.5 × 2.5 × 6 km^2 = 37.5 km^2

Question 12:

A model of a ship if made to a scale of 1 : 200.

(i) The length of the model is 4 m; calculate the length of the ship.

(ii) The area of the deck of the ship is 160000 m^2; find the area of the deck of the model.

(iii) The volume of the model is 200 litres; calculate the volume of the ship in m^3.

Solution 12:

Scale factor = k = $\frac{1}{200}$

(i) Length of model = k × actual length of the ship

⟹ Actual length of the ship = 4 × 200 = 800 m

(ii) Area of the deck of the model = k^2 × area of the deck of the ship

$$=\left(\frac{1}{200}\right)^2 \times 160000\ m^2 = 4\ m^2$$

(iii) Volume of the model = k^3 × volume of the ship

Volume of the ship

$$=\frac{1}{k^3}\times 200 \text{ liters}$$

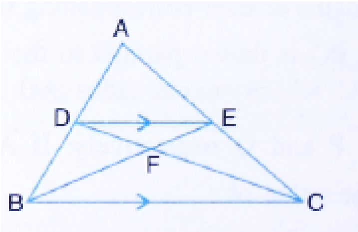
$$=(200)^3 \times 200 \text{ liters}$$

$$=1600000000 \text{ liters}$$

$$=1600000\ m^3$$

Question 13:

In the given figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB}=\frac{3}{2}$

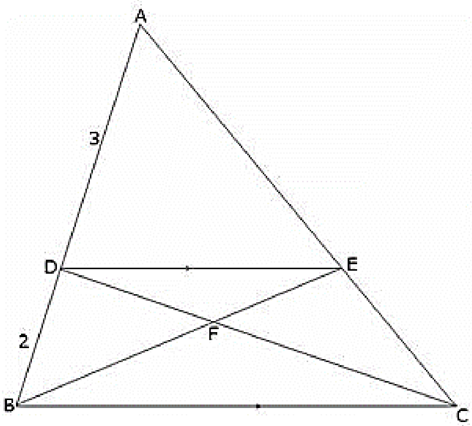


(i) Determine the ratios $\frac{AD}{AB}$ and $\frac{DE}{BC}$

(ii) Prove that ΔDEF is similar to ΔCBF Hence, find $\frac{EF}{FB}$.

(iii) What is the ratio of the areas of ΔDEF and ΔBFC.

Solution 13:



(i) Given, DE || BC and $\frac{AD}{DB} = \frac{3}{2}$

In ΔADE and ΔABC,

$\angle A = \angle A$ (Corresponding Angles)

$\angle ADE = \angle ABC$ (Corresponding Angles)

$\therefore \Delta ADE \sim \Delta ABC$ (By AA- similarity)

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now $\frac{AD}{AB} = \frac{AD}{AD+DB} = \frac{3}{3+2} = \frac{3}{5}$

Using (1), we get $\frac{AD}{AE} = \frac{3}{5} = \frac{DE}{BC} \quad \ldots\ldots\ldots\ldots\ldots (2)$

(ii) ΔIn DEF and ΔCBF,

$\angle FDE = \angle FCB$ (Alternate Angle)

$\angle DFE = \angle BFC$ (Vertically Opposite Angle)

$\therefore \Delta DEF \sim \Delta CBF$ (By AA- similarity)

$\frac{EF}{FB} = \frac{DE}{BC} = \frac{3}{5}$ Using (2)

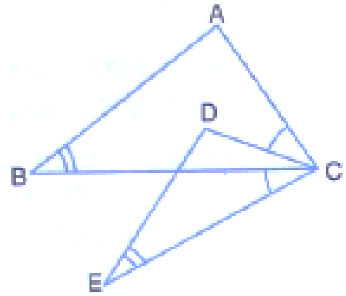
$\frac{EF}{FB} = \frac{3}{5}$

(iii) Since the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides, therefore.

$$\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CBF} = \frac{EF^2}{FB^2} = \frac{3^2}{5^2} = \frac{9}{25}$$

Question 14:

In the given figure, $\angle B = \angle E$, $\angle ACD = \angle BCE$, AB = 10.4cm and DE = 7.8 cm. Find the ratio between areas of the ΔABC and Δ DEC



Solution 14:

Given, $\angle ACD = \angle BCE$

$\angle ACD + \angle BCD = \angle BCE + \angle BCD$

$\angle ACB = \angle DCE$

Also, given $\angle B = \angle E$

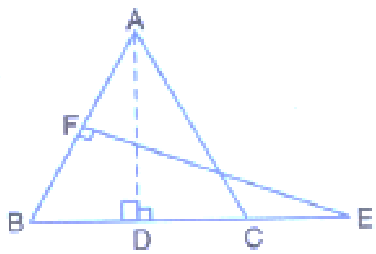
$\therefore \Delta ABC \sim \Delta DEC$

$$\frac{ar(\Delta ABC)}{ar(\Delta DEC)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{10.4}{7.8}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Question 15:

Triangle ABC is an isosceles triangle in which AB = AC = 13 cm and BC = 10 cm. AD is perpendicular to BC. If CE = 8 cm and EF $\perp$ AB, find:

(i) $\frac{\text{area of } \Delta ADC}{\text{area of } \Delta FEB}$ (ii) $\frac{\text{area of } \Delta AFEB}{\text{area of } \Delta ABC}$



Solution 15:

(i) AB = AC(Given)

$\therefore \angle FBE = \angle ACD$

$\angle BFE = \angle ADC$

$\Delta EFB \sim \Delta ADC$ (AA similarity)

$$\therefore \frac{ar(\Delta ADC)}{ar(\Delta EFB)} = \left(\frac{AC}{BE}\right)^2$$

$$= \left(\frac{AC}{BC + CE}\right)^2$$

$$= \left(\frac{13}{18}\right)^2 = \frac{169}{324} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

(ii) Similarly, it can be proved that $\Delta ADB \sim \Delta EFB$

$$\therefore \frac{ar(\Delta ADB)}{ar(\Delta EFB)} = \left(\frac{AB}{BE}\right)^2$$

$$= \left(\frac{13}{18}\right)^2$$

$$= \frac{169}{324} \quad \ldots\ldots\ldots\ldots\ldots..(2)$$

From (1) and (2),

$$\frac{ar(\Delta ABC)}{ar(\Delta EFB)} = \frac{169 + 169}{324} = \frac{338}{324} = \frac{169}{162}$$

$\therefore$ ar(ΔEFB) : ar(ΔABC) = 162 : 169

Question 16:

An aeroplane is 30 m long and its model is 15 cm long. If the total outer surface area of the model is 150 cm^2, find the cost of painting the outer surface of the aeroplane at the rate of Rs. 120 per sq. m. Given that 50 sq. m of the surface of the aeroplane is left for windows.

Solution 16:

15cm represents = 30 m

1cm represents $\frac{30}{15} = 2\text{m}$

1 cm^2 represents 2m × 2m = 4 m^2

Surface area of the model = 150 cm^2

Actual surface area of aeroplane = 150 × 2 × 2 m^2= 600 m^2

50 m^2 is left out for windows

Area to be painted = 600 − 50 =50 m^2

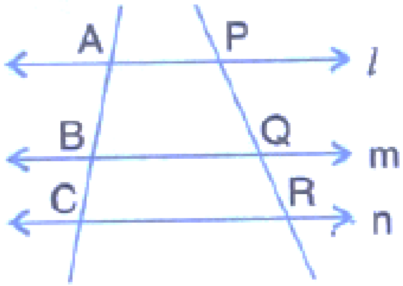
Cost of painting per m^2 = Rs. 120

Cost of painting 550 m^2= 120 × 550 = Rs. 66000

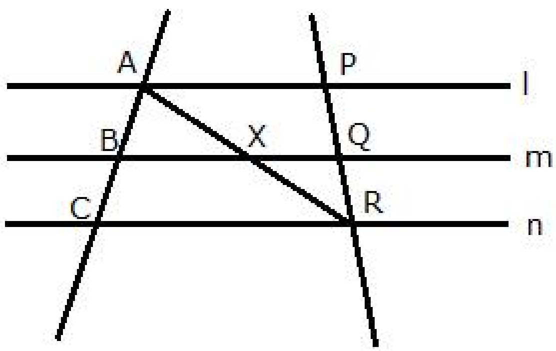
EXERCISE. 15 (C)

Question 1:
In the given figure, lines l, m and n are such that ls || m || n.
Prove that:

$$\frac{AB}{BC}=\frac{PQ}{QR}$$



Solution 1:
Join AR.



In ΔACR, BX || CR. By Basic Proportionality theorem,

$$\frac{AB}{BC}=\frac{AX}{XR} \quad \ldots\ldots\ldots\ldots..(1)$$

In ΔAPR, XQ || AP. By Basic Proportionality theorem,

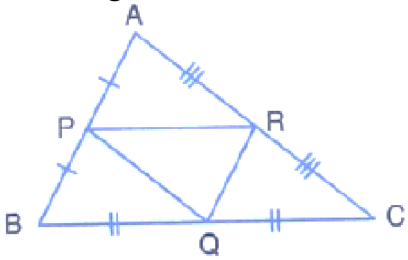
$$\frac{PQ}{QR}=\frac{AX}{XR} \quad \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

From (1) and (2), we get,

$$\frac{AB}{BC}=\frac{PQ}{QR}$$

Question 2:

In the given triangle P, Q and R are the mid points of sides AB, BC and AC respectively. Prove that triangle PQR is similar to triangle ABC.



Solution 2:

In ΔABC, PR || BC. By Basic proportionality theorem,

$$\frac{AP}{PB} = \frac{AR}{RC}$$

Also, in ΔPAR and ΔABC,

$\angle PAR = \angle BAC$ (common)

$\angle APR = \angle ABC$ (Corresponding angles)

$\Delta PAR \sim \Delta BAC$ (AA similarity)

$$\frac{PR}{BC} = \frac{AP}{AB}$$

$$\frac{PR}{BC} = \frac{1}{2} \quad \text{(As P is the mid-point of AB)}$$

$$\frac{PR}{BC} = \frac{1}{2}BC$$

Similarity, $PQ = \frac{1}{2}AC$

$RQ = \frac{1}{2}AB$

Thus, $\frac{PR}{BC} = \frac{PQ}{AC} = \frac{RQ}{AB}$

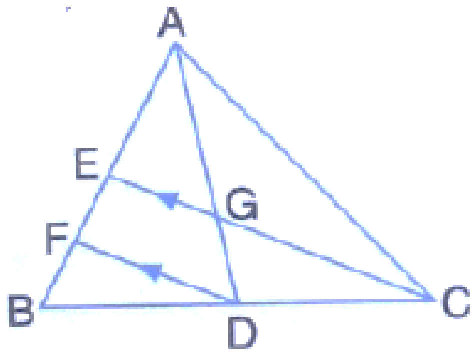
$\Rightarrow \Delta QRP \sim \Delta ABC$ (SSS similarity)

Question 3:

In the following figure, AD and CE are medians of ΔABC. DF is drawn parallel to CE. Prove that :

(i) EF = FB,

(ii) AG : GD = 2 : 1



Solution 3:

(i)

In ΔBFD and ΔBEC,

$\angle BFD = \angle BEC$ (Corresponding angles)

$\angle FBD = \angle EBC$ (Common)

$\Delta BFD \sim \Delta BEC$ (AA Similarity)

$$\therefore \frac{BF}{BE} = \frac{BD}{BC}$$

$$\frac{BF}{BE} = \frac{1}{2} \quad \text{(As D is the mid – point of BC)}$$

$BE = 2BF$

$BF = FE = 2BF$

Hence, $EF = FB$

(ii) In ΔAFD, $EG \parallel FD$. Using Basic Proportionality theorem,

$$\frac{AE}{EF} = \frac{AG}{GD} \quad \ldots\ldots\ldots (1)$$

Now, $AE = EB$ (as E is the mid-point of AB)

$AE = 2EF$ (Since, $EF = FB$, by (i))

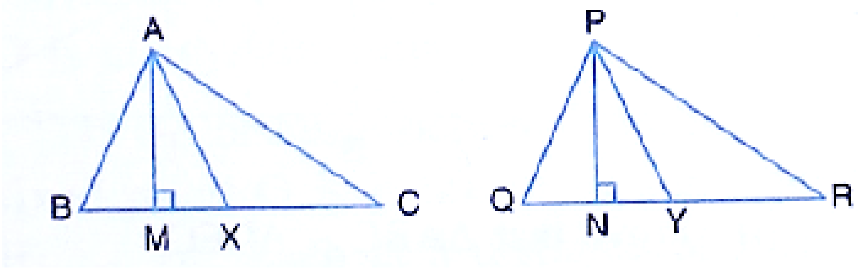
From (1),

$$\frac{AG}{GD} = \frac{2}{1}$$

Hence, $AG : GD = 2 : 1$

Question 4:

In the given figure, triangle ABC is similar to triangle PQR. AM and PN are altitudes whereas AX and PY are medians.



Prove that : $\frac{AM}{PN} = \frac{AX}{PY}$

Solution 4:

Since $\Delta ABC \sim \Delta PQR$

So, their respective sides will be in proportion

Or, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

Also, $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$

In ΔABM and ΔPQN,

$\angle ABM = \angle PQN$ (Since, ABC and PQR are similar)

$\angle AMB = \angle PNQ = 90°$

$\Delta ABM \sim \Delta PQN$ (AA similarity)

$\therefore \frac{AM}{PN} = \frac{AB}{PQ}$ ……………….. (1)

Since, AX and PY are medians so they will divide their opposite sides.

Or, $BX = \frac{BC}{2}$ and $QY = \frac{QR}{2}$

Therefore, we have:

$\frac{AB}{PQ} = \frac{BX}{QY}$

$\angle B = \angle Q$

So, we had observed that two respective sides are in same proportion in both triangles and also angle included between them is respectively equal.

Hence, $\Delta ABX \sim \Delta PQY$ (by SAS similarity rule)

So, $\frac{AB}{PQ} = \frac{AX}{PY}$ ………………..(2)

From (1) and (2),

$\frac{AM}{PN} = \frac{AX}{PY}$

Question 5:

The two similar triangles are equal in area. Prove that the triangles are congruent.

Solution 5:

Let us assume two similar triangles as $\Delta ABC \sim \Delta PQR$

Now $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Since area (ΔABC) = area (ΔPQR)

Therefore, AB = PQ

BC = QR

AC = PR

So, respective sides of two similar triangles

Are also of same length

So, $\Delta ABC \cong \Delta PQR$ (by SSS rule)

Question 6:

The ratio between the altitudes of two similar triangles is 3 : 5; write the ratio between their:
(i) medians (ii) perimeters (iii) areas

Solution 6:

The ratio between the altitudes of two similar triangles is same as the ratio between their sides.
(i) The ratio between the medians of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 3 : 5
(ii) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 3 : 5
(iii) The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides.
∴ Required ratio = (3)2 : (5)2 = 9 : 25

Question 7:

The ratio between the areas of two similar triangles is 16 : 25, Find the ratio between their:
(i) perimeters (ii) altitudes (iii) medians

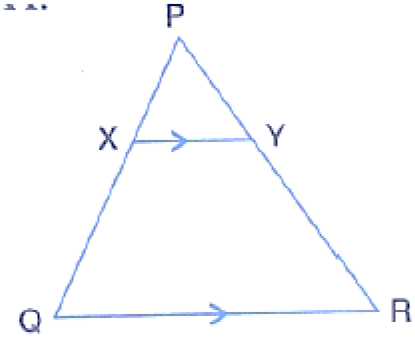
Solution 7:

The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides.
So, the ratio between the sides of the two triangles = 4 : 5

(i) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 4 : 5
(ii) The ratio between the altitudes of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 4 : 5
(iii) The ratio between the medians of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 4 : 5

Question 8:

The following figure shows a triangle PQR in which XY is parallel to QR. If PX : XQ = 1 : 3 and QR = 9 cm. find the length of XY.



Further, if the area of Δ PXY = x cm^2; find, in terms of x the area of:
(i) triangle PQR (ii) trapezium XQRY

Solution 8:

In ΔPXY and ΔPQR, XY is parallel to QR, so corresponding angles are equal.
∠PXY = ∠PQR
∠PYX = ∠PRQ
Hence, ΔPXY ~ ΔPQR (By AA similarity criterion)

$$\frac{PX}{PQ} = \frac{XY}{QR}$$

$$\Rightarrow \frac{1}{4} = \frac{XY}{QR} \quad (PX : XQ = 1 : 3 \Rightarrow PX : PQ = 1 : 4)$$

$$\Rightarrow \frac{1}{4} = \frac{XY}{9}$$

⇒ XY = 2.25 cm

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\frac{Ar(\Delta PXY)}{Ar(\Delta PQR)} = \left(\frac{PX}{PQ}\right)^2$$

$$\frac{x}{Ar(\Delta PQR)} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

$$Ar(\Delta PQR) = 16x \text{ cm}^2$$

(ii) Ar (trapezium XQRY) = Ar (ΔPQR) - Ar (ΔPXY)

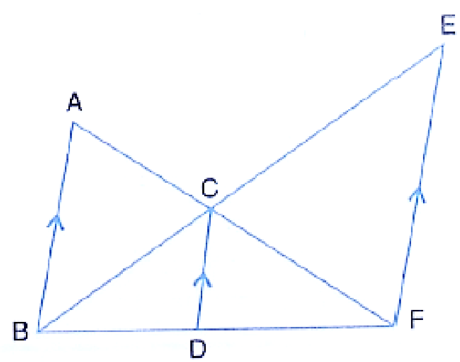
= (16x – x) cm^2

= 15x cm^2

Question 9:

In the following figure, AB, CD and EF are parallel lines. AB = 6cm, CD = y cm, EF = 10 cm, AC = 4 cm and CF = x cm.

Calculate x and y



Solution 9:

In ΔFDC and ΔFBA,

∠FDC = ∠FDA (Corresponding angles)

∠DFC = ∠BFA (Common)

ΔFDC ~ ΔFBA (AA similarity)

$$\frac{CD}{AB} = \frac{FC}{FA}$$

$$\frac{Y}{6} = \frac{x}{x+4} \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

In ΔFCE and ΔACB,

$\angle FCE = \angle ACB$ (vertically opposite angles)
$\angle CFE = \angle CAB$ (Alternate angles)
$\Delta FCE \sim \Delta ACB$ (AA similarity)

$$\frac{FC}{AC} = \frac{EF}{AB}$$

$$\frac{x}{4} = \frac{10}{6} \Rightarrow x = \frac{20}{3} = 6\frac{2}{3}\text{cm}$$

From (1):

$$y = \frac{6 \times \frac{20}{3}}{\frac{20}{3} + 4} = 3.75$$

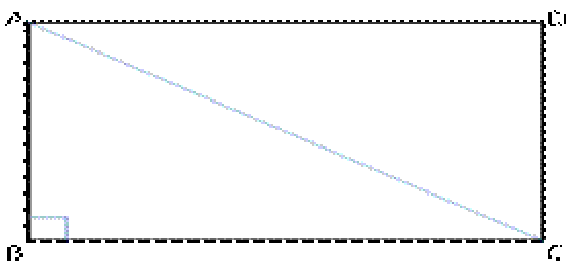
Question 10:

On a map, drawn to a scale of 1 : 20000, a rectangular plot of land ABCD has AB = 24cm and BC = 32 cm. Calculate:
(i) the diagonal distance of the plot in kilometer
(ii) the area of the plot in sq.km

Solution 10:

Scale :- 1 : 20000

$$1 \text{ cm represents } 20000 \text{ cm} = \frac{20000}{1000 \times 100} = 0.2 \text{ km}$$



(i)
$AC^2 = AB^2 + BC^2$
$= 24^2 + 32^2$
$= 576 + 1024 = 1600$
AC = 40 cm
Actual length of diagonal $= 40 \times 0.2$ km $= 8$ km
(ii)

1 cm represents 0.2 km
1 cm^2 represents $0.2 \times 0.2\ km^2$
The area of the rectangle ABCD = AB × BC
= 24 × 32 = 768 cm^2
Actual area of the plot = $0.2 \times 0.2 \times 768\ km^2 = 30.72\ km^2$

Question 11:

The dimensions of the model of a multistoreyed building are 1 m by 60 cm by 1.20 m. if the scale factor is 1 : 50, find the actual dimensions of the building.
Also find:
(i) the floor area of a room of the building, if the floor area of the corresponding room in the model is 50 sq. cm
(ii) the space (volume) inside a room of the model, if the space inside the corresponding room of the building is 90 m^3.

Solution 11:

The dimensions of the building are calculated as below.
Length = 1 × 50 m = 50 m
Breadth = 0.60 × 50 m = 30 m
Height = 1.20 × 50 m = 60 m
Thus, the actual dimensions of the building are 50 m × 30 m × 60 m.
(i)
Floor area of the room of the building =

$$50\times\left(\frac{50}{1}\right)^2 = 125000\ cm^2 = \frac{125000}{100\times100} = 12.5\ m^2$$

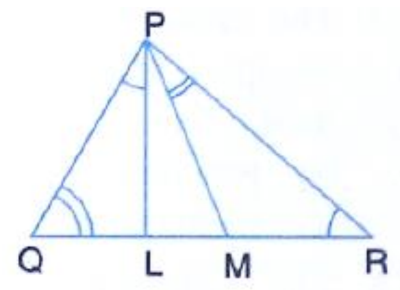
(ii)
Volume of the model of the building

$$= 90\left(\frac{1}{50}\right)^3 = 90\times\left(\frac{1}{50}\right)\times\left(\frac{1}{50}\right)\times\left(\frac{1}{50}\right) = 90\times\left(\frac{100\times100\times100}{50\times50\times50}\right)cm^3$$

$$= 720\ cm^3$$

Question 12:

In a triangle PQR, L and M are two points on the base QR, such that ∠LPQ = ∠QRP and ∠RPM = ∠RQP. Prove that:
(i) ΔPQL and ΔRMP
(ii) $QL \times RM = PL \times PM$
(iii) $PQ^2 = QR \times QL$



Solution 12:

In ΔPQL and ΔRMP

$\angle LPQ = \angle QRP$ (Given)

$\angle RQP = \angle RPM$ (Given)

$\Delta PQL \sim \Delta RMP$ (AA similarity)

(ii)

As $\Delta PQL \sim \Delta RMP$ (proved above)

$$\frac{PQ}{RP} = \frac{QL}{PM} = \frac{PL}{RM}$$

$$\Rightarrow QL \times RM = PL \times PM$$

(iii)

$\angle LPQ = \angle QRP$ (Given)

$\angle Q = \angle Q$ (Common)

$\Delta PQL \sim \Delta RQP$ (AA similarity)

$$= \frac{PQ}{RQ} = \frac{QL}{QP} = \frac{PL}{PR}$$

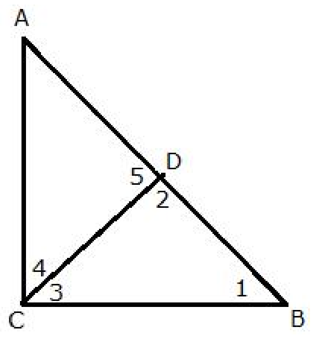
$$\Rightarrow PQ^2 = QR \times QL$$

Question 13:

In ΔABC, $\angle ACB = 90°$ and $CD \perp AB$.

Prove that : $\dfrac{BC^2}{AC^2} = \dfrac{BD}{AD}$

Solution 13:



In ΔCDB,

$\angle 1 + \angle 2 + \angle 3 = 180^o$

$\angle 1 + \angle 3 = 90^o$ … (1) (Since, $\angle 2 = 90^0$)

$\angle 3 + \angle 4 = 90^o$ … (2) (Since, $\angle ACB = 90^0$)

From (1) and (2),

$\angle 1 + \angle 3 = \angle 3 + \angle 4$

$\angle 1 = \angle 4$

Also, $\angle ADC = \angle ACB = 90^o$

$\therefore \Delta ACD \sim \Delta ABC$ (AA similarity)

$$\therefore \frac{AC}{AB} = \frac{AD}{AC}$$

$AC^2 = AB \times AD$ ………….. (1)

Now $\angle BDC = \angle ACB = 90^0$

$\angle CBD = \angle ABC$ (common)

$\Delta BCD \sim \Delta BAC$ (AA similarity)

$$\therefore \frac{BC}{BA} = \frac{BD}{BC} \quad \text{…………. (2)}$$

$BC^2 = BA \times BD$

From (1)and (2), we get,

$$\frac{BC^2}{AC^2} = \frac{BA \times BD}{AB \times AD} = \frac{BD}{AD}$$

Question 14:

A triangle ABC with AB = 3 cm, BC = 6 cm and AC = 4 cm is enlarged to ΔDEF such that the longest side of ΔDEF = 9 cm. Find the scale factor and hence, the lengths of the other sides of ΔDEF.

Solution 14:

Triangle ABC is enlarged to DEF. So, the two triangles will be similar.

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Longest side in ΔABC = BC = 6 cm

Corresponding longest side in ΔDEF = EF = 9 cm

Scale factor = $\frac{EF}{BC} = \frac{9}{6} = \frac{3}{2} = 1.5$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{2}{3}$$

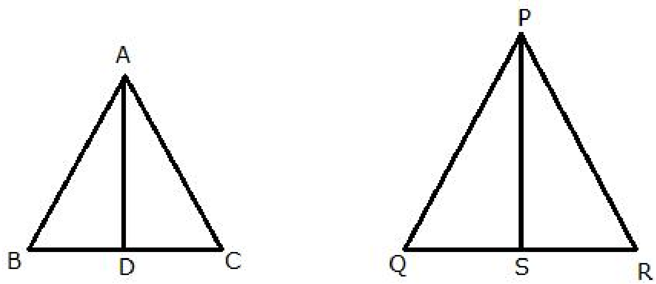
$$DE = \frac{3}{2}AB = \frac{9}{2} = 4.5 \text{ cm}$$

$$DF = \frac{3}{2}AC = \frac{12}{2} = 6 \text{ cm}$$

Question 15:

Two isosceles triangles have equal vertical angles. Show that the triangles are similar.
If the ratio between the areas of these two triangles is 16 : 25, find the ratio between their corresponding altitudes.

Solution 15:



Let ABC and PQR be two isosceles triangles.

Then, $\frac{AB}{AC} = \frac{1}{1}$ and $\frac{PQ}{PR} = \frac{1}{1}$

Also, $\angle A = \angle P$ (Given)

$\therefore \Delta ABC \sim \Delta PQR$ (SAS similarity)

Let AD and PS be the altitude in the respective triangles.

We know that the ratio of areas of two similar triangles is equal to the square of their corresponding altitudes.

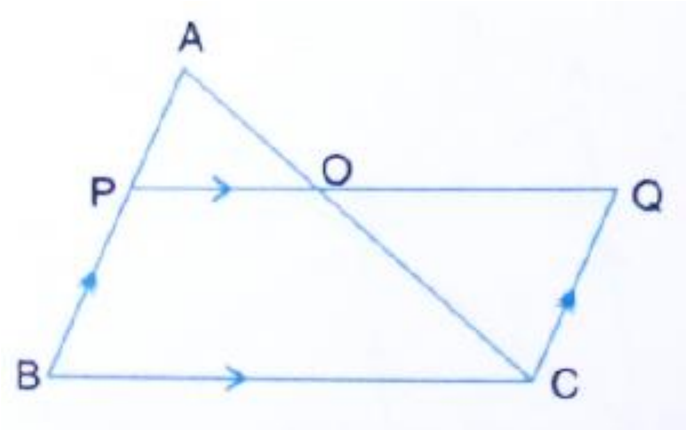
$$\frac{Ar(\triangle ABC)}{Ar(\triangle PQR)} = \left(\frac{AD}{PS}\right)^2$$

$$\frac{16}{25} = \left(\frac{AD}{PS}\right)^2$$

$$\frac{AD}{PS} = \frac{4}{5}$$

Question 16:

In ΔABC, AP : PB = 2 : 3. PO is parallel to BC and is extended to Q so that CQ is parallel to BA.



Find:

(i) area ΔAPO : area Δ ABC.

(ii) area ΔAPO : area Δ CQO.

Solution 16:

In triangle ABC, PO || BC. Using Basic proportionality theorem,

$$\frac{AP}{PB} = \frac{AO}{OC}$$

$$\Rightarrow \frac{AO}{OC} = \frac{2}{3} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

(i) $\angle PAO = \angle BAC$ (common)

$\angle APO = \angle ABC$ (Corresponding angles)

$\Delta APO \sim \Delta ABC$ (AA similarity)

$$\therefore \frac{Ar(\Delta APO)}{Ar(\Delta ABC)} = \left(\frac{AO}{AC}\right)^2 = \left(\frac{2}{2+3}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

(ii)

$\angle POA = \angle COQ$ (vertically opposite angles)

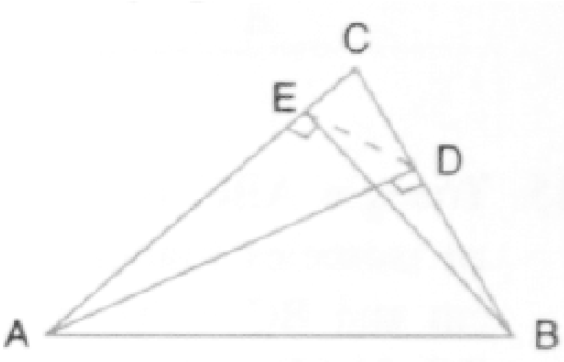
$\angle PAO = \angle QCO$ (alternate angles)

$\Delta AOP \sim \Delta COQ$ (AA similarity)

$$\therefore \frac{Ar(\Delta AOP)}{Ar(\Delta COQ)} = \left(\frac{AO}{CO}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Question 17:

The following figure shows a triangle ABC in which AD and BE are perpendiculars to BC and AC respectively.



Show that;

(i) $\Delta ADC \sim \Delta BEC$

(ii) $CA \times CE = CB \times CD$

(iii) $\Delta ABC \sim \Delta DEC$

(iv) $CD \times AB = CA \times DE$

Solution 17:

$\angle ADC = \angle BEC = 90°$

$\angle ACD = \angle BCE$ (Common)

$\Delta ADC \sim \Delta BEC$ (AA similarity)

(ii) From part (i),

$$\frac{AC}{BC} = \frac{CD}{EC} \quad \text{.........} \ (1)$$

$\Rightarrow CA \times CE = CB \times CD$

(iii) In ΔABC and ΔDEC,

From (1),

$$\frac{AC}{BC} = \frac{CD}{EC} \Rightarrow \frac{AC}{CD} = \frac{BC}{EC}$$

$\angle DCE = \angle BCA$ (Common)

$\Delta ABC \sim \Delta DEC$ (SAS similarity)

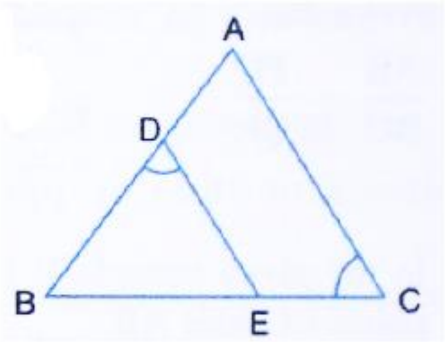
(iv) From part (iii),

$$\frac{AC}{DC} = \frac{AB}{DE}$$

$\Rightarrow CD \times AB = CA \times DE$

Question 18:

In the give figure, ABC is a triangle with ∠EDB = ∠ACB. Prove that ΔABC ~ ΔEBD.
If BE =6 cm, EC = 4cm, BD = 5cm and area of ΔBED = 9 cm^2. Calculate the:
(i) length of AB
(ii) area of Δ ABC



Solution 18:

In ΔABC and ΔEBD,
∠ACB = ∠EDB (given)
∠ABC = ∠EBD (common)
ΔABC ~ ΔEBD (by AA – similarity)

(i) we have, $\frac{AB}{BE} = \frac{BC}{BD} \Rightarrow AB = \frac{6 \times 10}{5} = 12\text{ cm}$

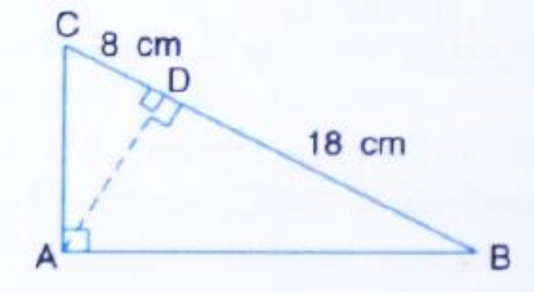
(ii) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BED} = \left(\frac{AB}{BE}\right)^2$

$\Rightarrow$ Area of ΔABC $= \left(\frac{12}{6}\right)^2 \times 9\text{ cm}^2$

$= 4 \times 9\text{ cm}^2 = 36\text{ cm}^2$

Question 19:

In the given figure, ABC is a right angled triangle with ∠BAC = 90°.



(i) Prove $\Delta ADB \sim \Delta CDA$.

(ii) If BD = 18 cm and CD = 8cm, find AD.

(iii) Find the ratio of the area of ΔADB is to area of ΔCDA.

Solution 19:

(i) let $\angle CAD = x$

$\Rightarrow m\angle dab = 90^{\circ} - x$

$\Rightarrow m\angle DBA = 180° - (90° + 90° - x) = x$

$\Rightarrow \angle CDA = \angle DBA$ ……… (1)

In ΔADB and ΔCDA,

$\angle ADB = \angle CDA$ …… [each 90°]

$\angle ABD = \angle CAD$ …… [From (1)]

$\therefore \Delta ADB \sim \Delta CDA$ …….. [By A.A]

(ii) Since the corresponding sides of similar triangles are proportional, we have.

$$\frac{BD}{AD} = \frac{AD}{CD}$$

$$\Rightarrow \frac{18}{AD} = \frac{AD}{8}$$

$\Rightarrow AD^2 = 18 \times 8 = 144$

$\Rightarrow AD = 12$ cm

(iii) The ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

$$\Rightarrow \frac{Ar(\Delta ADB)}{Ar(\Delta CDA)} = \frac{AD^2}{CD^2} = \frac{12^2}{8^2} = \frac{144}{64} = \frac{9}{4} = 9:4$$