Therefore, $(-20x^4) \div 10x^2 = \dfrac{-2 \times 2 \times 5 \times x \times x \times x \times x}{2 \times 5 \times x \times x} = -2 \times x \times x = -2x^2$

(ii) $7x^2y^2z^2 \div 14xyz$

$$= \frac{7 \times x \times x \times y \times y \times z \times z}{2 \times 7 \times x \times y \times z}$$

$$= \frac{x \times y \times z}{2} = \frac{1}{2}xyz$$

TRY THESE

Divide.

(i) $24xy^2z^3$ by $6yz^2$ (ii) $63a^2b^4c^6$ by $7a^2b^2c^3$

6.3.2 Division of a polynomial by a monomial

Let us consider the division of the trinomial $4y^3 + 5y^2 + 6y$ by the monomial $2y$.

$$4y^3 + 5y^2 + 6y = (2 \times 2 \times y \times y \times y) + (5 \times y \times y) + (2 \times 3 \times y)$$

(Here, we expressed each term of the polynomial in factor form) we find that $2 \times y$ is common in each term. Therefore, separating $2 \times y$ from each term. We get

$$4y^3 + 5y^2 + 6y = 2 \times y \times (2 \times y \times y) + 2 \times y \times \left(\frac{5}{2} \times y\right) + 2 \times y \times 3$$

$$= 2y\,(2y^2) + 2y\left(\frac{5}{2}y\right) + 2y\,(3)$$

$$= 2y\left(2y^2 + \frac{5}{2}y + 3\right)$$ (The common factor $2y$ is shown separately.

Therefore, $(4y^3 + 5y^2 + 6y) \div 2y$

$$= \frac{4y^3 + 5y^2 + 6y}{2y} = \frac{2y(2y^2 + \frac{5}{2}y + 3)}{2y} = 2y^2 + \frac{5}{2}y + 3$$

Here, we divide each term of the polynomial in the numerator by the monomial in the denominator.

Alternatively, we could divide each term of the trinomial by the monomial using the cancellation method.

$$(4y^3 + 5y^2 + 6y) \div 2y = \frac{4y^3 + 5y^2 + 6y}{2y}$$

$$= \frac{4y^3}{2y} + \frac{5y^2}{2y} + \frac{6y}{2y} = 2y^2 + \frac{5}{2}y + 3$$

Example 14: Divide $24(x^2yz + xy^2z + xyz^2)$ by $8xyz$ using both the methods.

Solution: $24\,(x^2yz + xy^2z + xyz^2)$

$= 2 \times 2 \times 2 \times 3 \times [(x \times x \times y \times z) + (x \times y \times y \times z) + (x \times y \times z \times z)]$

$= 2 \times 2 \times 2 \times 3 \times x \times y \times z \times (x + y + z) = 8 \times 3 \times xyz \times (x + y + z)$ (By taking out the common factor)

Therefore, $24\,(x^2yz + xy^2z + xyz^2) \div 8xyz$

$$= \frac{8 \times 3 \times xyz \times (x + y + z)}{8 \times xyz} = 3 \times (x + y + z) = 3\,(x + y + z)$$

Alternately, $24(x^2yz + xy^2z + xyz^2) \div 8xyz = \dfrac{24x^2yz}{8xyz} + \dfrac{24xy^2z}{8xyz} + \dfrac{24xyz^2}{8xyz}$

$= 3x + 3y + 3z = 3(x + y + z)$

6.4 Division of Algebraic Expressions Continued (Polynomial ÷ Polynomial)

- Consider $(7x^2 + 14x) \div (x + 2)$

We shall factorise $(7x^2 + 14x)$ first to check and match factors with the denominator:

$$7x^2 + 14x = (7 \times x \times x) + (2 \times 7 \times x)$$
$$= 7 \times x \times (x + 2) = 7x(x + 2)$$

Will it help here to divide each term of the numerator by the binomial in the denominator?

Now $\quad (7x^2 + 14x) \div (x + 2) = \dfrac{7x^2 + 14x}{x + 2}$

$= \dfrac{7x(x + 2)}{x + 2} = 7x \quad$ (Cancelling the factor $(x + 2)$)

Example 15: Divide $44(x^4 - 5x^3 - 24x^2)$ by $11x(x - 8)$

Solution: Factorising $44(x^4 - 5x^3 - 24x^2)$, we get

$$44(x^4 - 5x^3 - 24x^2) = 2 \times 2 \times 11 \times x^2(x^2 - 5x - 24)$$

(taking the common factor x^2 out of the bracket)

$$= 2 \times 2 \times 11 \times x^2(x^2 - 8x + 3x - 24)$$
$$= 2 \times 2 \times 11 \times x^2\,[x(x - 8) + 3(x - 8)]$$
$$= 2 \times 2 \times 11 \times x^2\,(x + 3)\,(x - 8)$$

Therefore, $\;44(x^4 - 5x^3 - 24x^2) \div 11x(x - 8)$

$$= \frac{2 \times 2 \times 11 \times x \times x \times (x + 3) \times (x - 8)}{11 \times x \times (x - 8)}$$

$$= 2 \times 2 \times x\,(x + 3) = 4x(x + 3)$$

We cancel the factors 11, x and $(x - 8)$ common to both the numerator and denominator.

Example 16: Divide $z(5z^2 - 80)$ by $5z(z + 4)$

Solution: Dividend $= z(5z^2 - 80)$

$= z[(5 \times z^2) - (5 \times 16)]$

$= z \times 5 \times (z^2 - 16)$

$= 5z \times (z + 4)\,(z - 4) \qquad$ [using the identity $a^2 - b^2 = (a + b)\,(a - b)$]

Thus, $\qquad z(5z^2 - 80) \div 5z(z + 4) = \dfrac{5z(z - 4)\,(z + 4)}{5z(z + 4)} = (z - 4)$

EXERCISE 6.3

1. Carry out the following divisions.

(i) $28x^4 \div 56x$ (ii) $-36y^3 \div 9y^2$ (iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$ (v) $12a^8b^8 \div (-6a^6b^4)$

2. Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$ (ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$ (iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

3. Work out the following divisions.

(i) $(10x - 25) \div 5$ (ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$ (iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

4. Divide as directed.

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$ (ii) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

(iii) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

(iv) $20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$ (v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

5. Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$ (ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$ (iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$ (vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

6.5 Can you Find the Error?

Task 1 While solving an equation, Sarita does the following.

$$3x + x + 5x = 72$$

Therefore $8x = 72$

and so, $x = \frac{72}{8} = 9$

Where has she gone wrong? Find the correct answer.

Coefficient 1 of a term is usually not shown. But while adding like terms, we include it in the sum.

Task 2 Appu did the following:

For $x = -3$, $5x = 5 - 3 = 2$

Is his procedure correct? If not, correct it.

Remember to make use of brackets, while substituting a negative value.

Task 3 Namrata and Salma have done the multiplication of algebraic expressions in the following manner.

Remember, when you multiply the expression enclosed in a bracket by a constant (or a variable) outside, each term of the expression has to be multiplied by the constant (or the variable).

	Namrata	**Salma**
(a)	$3(x - 4) = 3x - 4$	$3(x - 4) = 3x - 12$

(b) $(2x)^2 = 2x^2$ $\qquad$ $(2x)^2 = 4x^2$

Remember, when you square a monomial, the numerical coefficient and each factor has to be squared.

Make sure, before applying any formula, whether the formula is really applicable.

(c) $(2a - 3)(a + 2)$ $\qquad$ $(2a - 3)(a + 2)$
$= 2a^2 - 6$ $\qquad$ $= 2a^2 + a - 6$

(d) $(x + 8)^2 = x^2 + 64$ $\qquad$ $(x + 8)^2$
$= x^2 + 16x + 64$

(e) $(x - 5)^2 = x^2 - 25$ $\qquad$ $(x - 5)^2 = x^2 - 10x + 25$

Is the multiplication done by both Namrata and Salma correct? Give reasons for your answer.

Task 4 Joseph does a division as : $\dfrac{a+5}{5} = a + 1$

While dividing a polynomial by a monomial, we divide each term of the polynomial in the numerator by the monomial in the denominator.

His friend Sirish has done the same division as: $\dfrac{a+5}{5} = a$

And his other friend Suman does it this way: $\dfrac{a+5}{5} = \dfrac{a}{5} + 1$

Who has done the division correctly? Who has done incorrectly? Why?

Some fun!

Atul always thinks differently. He asks Sumathi teacher, "If what you say is true, then why do I get the right answer for $\dfrac{64}{16} = \dfrac{4}{1} = 4$?" The teacher explains, " This is so because 64 happens to be 16×4; $\dfrac{64}{16} = \dfrac{16\times 4}{16\times 1} = \dfrac{4}{1}$. In reality, we cancel a factor of 16 and not 6, as you can see. In fact, 6 is not a factor of either 64 or of 16." The teacher adds further, "Also, $\dfrac{664}{166} = \dfrac{4}{1}$, $\dfrac{6664}{1666} = \dfrac{4}{1}$, and so on". Isn't that interesting? Can you help Atul to find some other examples like $\dfrac{64}{16}$?

EXERCISE 6.4

Find and correct the errors in the following mathematical statements.

1. $4(x - 5) = 4x - 5$
2. $x(3x + 2) = 3x^2 + 2$
3. $2x + 3y = 5xy$
4. $x + 2x + 3x = 5x$
5. $5y + 2y + y - 7y = 0$
6. $3x + 2x = 5x^2$
7. $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$
8. $(2x)^2 + 5x = 4x + 5x = 9x$
9. $(3x + 2)^2 = 3x^2 + 6x + 4$

10. Substituting $x = -3$ in

(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$

11. $(y - 3)^2 = y^2 - 9$ **12.** $(z + 5)^2 = z^2 + 25$

13. $(2a + 3b)(a - b) = 2a^2 - 3b^2$ **14.** $(a + 4)(a + 2) = a^2 + 8$

15. $(a - 4)(a - 2) = a^2 - 8$ **16.** $\frac{3x^2}{3x^2} = 0$

17. $\frac{3x^2 + 1}{3x^2} = 1 + 1 = 2$ **18.** $\frac{3x}{3x + 2} = \frac{1}{2}$ **19.** $\frac{3}{4x + 3} = \frac{1}{4x}$

20. $\frac{4x + 5}{4x} = 5$ **21.** $\frac{7x + 5}{5} = 7x$

WHAT HAVE WE DISCUSSED?

1. When we factorise an expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions.
2. An irreducible factor is a factor which cannot be expressed further as a product of factors.
3. A systematic way of factorising an expression is the common factor method. It consists of three steps: (i) Write each term of the expression as a product of irreducible factors (ii) Look for and separate the common factors and (iii) Combine the remaining factors in each term in accordance with the distributive law.
4. Sometimes, all the terms in a given expression do not have a common factor; but the terms can be grouped in such a way that all the terms in each group have a common factor. When we do this, there emerges a common factor across all the groups leading to the required factorisation of the expression. This is the method of regrouping.
5. In factorisation by regrouping, we should remember that any regrouping (i.e., rearrangement) of the terms in the given expression may not lead to factorisation. We must observe the expression and come out with the desired regrouping by trial and error.
6. A number of expressions to be factorised are of the form or can be put into the form : $a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$, $a^2 - b^2$ and $x^2 + (a + b) + ab$. These expressions can be easily factorised using Identities I, II, III and IV, given in Chapter 9,

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^2 - b^2 = (a + b)(a - b)$$
$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

7. In expressions which have factors of the type $(x + a)(x + b)$, remember the numerical term gives ab. Its factors, a and b, should be so chosen that their sum, with signs taken care of, is the coefficient of x.
8. We know that in the case of numbers, division is the inverse of multiplication. This idea is applicable also to the division of algebraic expressions.

9. In the case of division of a polynomial by a monomial, we may carry out the division either by dividing each term of the polynomial by the monomial or by the common factor method.
10. In the case of division of a polynomial by a polynomial, we cannot proceed by dividing each term in the dividend polynomial by the divisor polynomial. Instead, we factorise both the polynomials and cancel their common factors.
11. In the case of divisions of algebraic expressions that we studied in this chapter, we have Dividend = Divisor × Quotient.
    In general, however, the relation is
    Dividend = Divisor × Quotient + Remainder
    Thus, we have considered in the present chapter only those divisions in which the remainder is zero.
12. There are many errors students commonly make when solving algebra exercises. You should avoid making such errors.

CHAPTER 7 Visualising Solid Shapes

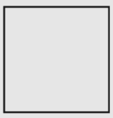
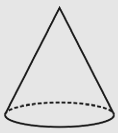
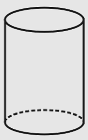
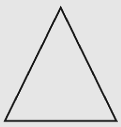
B9I3M1

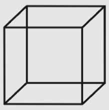
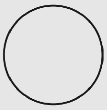
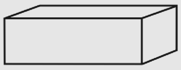
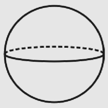
7.1 Introduction

In Class VII, you have learnt about plane shapes and solid shapes. Plane shapes have two measurements like length and breadth and therefore they are called two-dimensional shapes whereas a solid object has three measurements like length, breadth, height or depth. Hence, they are called three-dimensional shapes. Also, a solid object occupies some space. Two-dimensional and three-dimensional figures can also be briefly named as 2-D and 3-D figures. You may recall that triangle, rectangle, circle etc., are 2-D figures while cubes, cylinders, cones, spheres etc. are three-dimensional figures.

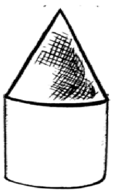
DO THIS

Match the following: (First one is done for you)

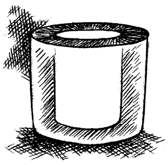
Shape	Type of Shape	Name of the shape
	3-dimensional	Sphere
	2-Dimensional	Cylinder
	3-dimensional	Square
	2-dimensional	Circle

	3-dimensional	Cuboid
	3- dimensional	Cube
	2-dimensional	Cone
	3-dimensional	Triangle

Note that all the above shapes are single. However, in our practical life, many a times, we come across combinations of different shapes. For example, look at the following objects.

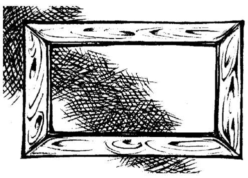


A tent
A cone surmounted on a cylinder



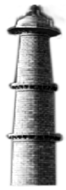
A tin
A cylinderical shell

Softy (ice-cream)
A cone surmounted by a hemisphere



A photoframe
A rectangular path

A bowl
A hemispherical shell

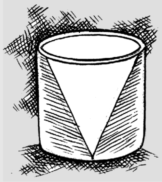
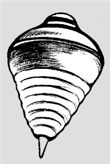
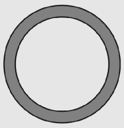
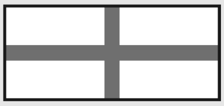


Tomb on a pillar
Cylinder surmounted by a hemisphere

DO THIS

Match the following pictures (objects) with their shapes:

Picture (object)		**Shape**
(i) An agricultural field		Two rectangular cross paths inside a rectangular park.

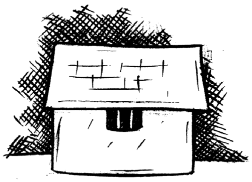
(ii) A groove		A circular path around a circular ground.
(iii) A toy		A triangular field adjoining a square field.
(iv) A circular park		A cone taken out of a cylinder.
(v) A cross path		A hemisphere surmounted on a cone.

7.2 Views of 3D-Shapes

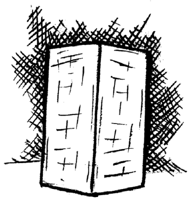
You have learnt that a 3-dimensional object can look differently from different positions so they can be drawn from different perspectives. For example, a given hut can have the following views.

A hut

Front view



Side view

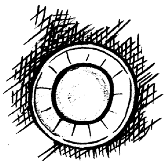


Top view

similarly, a glass can have the following views.

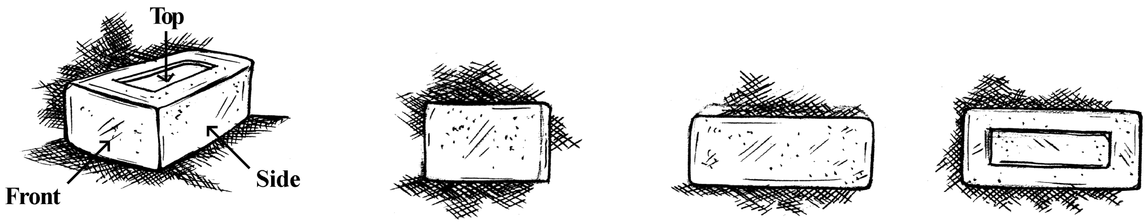
A glass

Side view



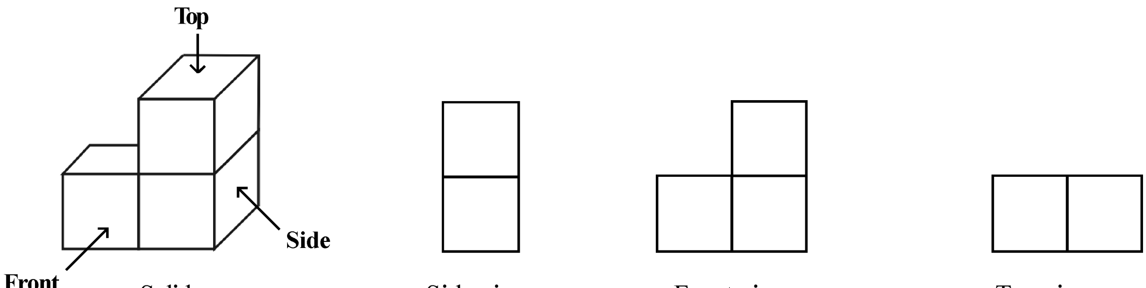
Top view

Why is the top view of the glass a pair of concentric circles? Will the side view appear different if taken from some other direction? Think about this! Now look at the different views of a brick.

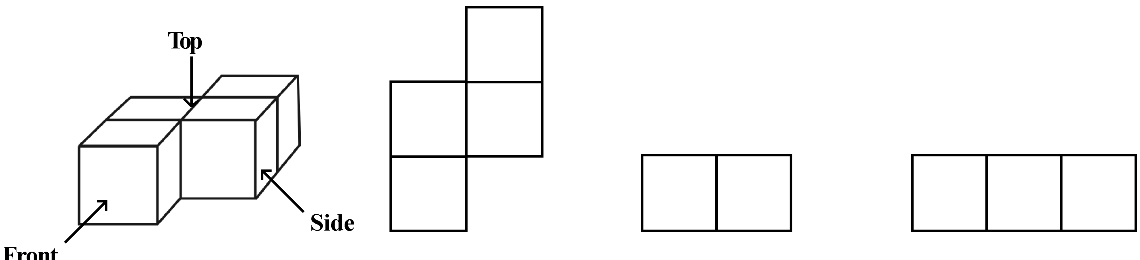


A brick   Front view   Side view   Top view

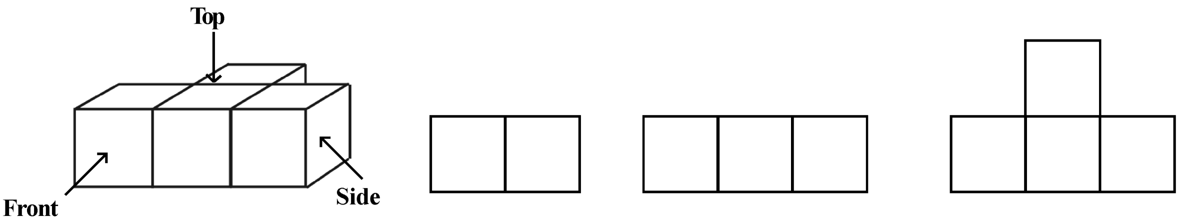
We can also get different views of figures made by joining cubes. For example.



Solid made of three cubes   Side view   Front view   Top view



Solid made of four cubes   Top view   Front view   Side view



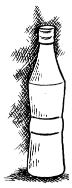
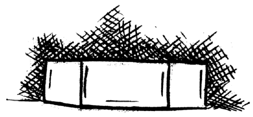
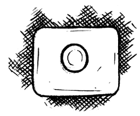
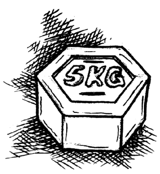
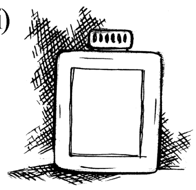
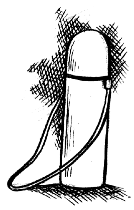
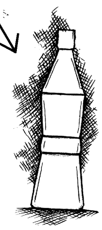
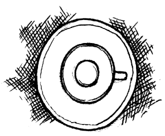
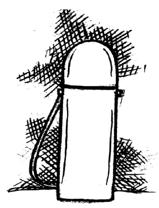
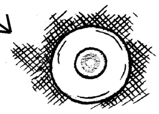
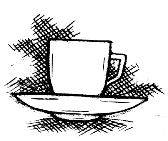
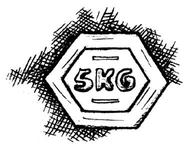
Solid made of four cubes   Side view   Front view   Top view

DO THIS

Observe different things around you from different positions. Discuss with your friends their various views.

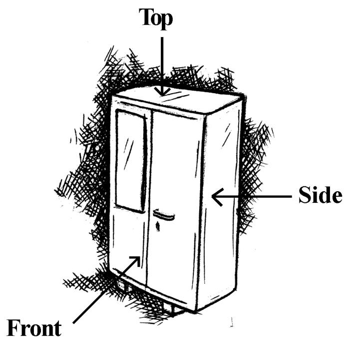
EXERCISE 7.1

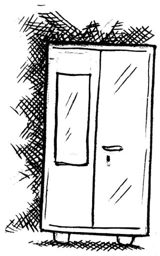
1. For each of the given solid, the two views are given. Match for each solid the corresponding top and front views. The first one is done for you.

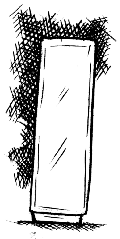
Object	**Side view**	**Top view**
(a)  A bottle	(i) 	(i) 
(b)  A weight	(ii) 	(ii) 
(c)  A flask	(iii) 	(iii) 
(d)  Cup and Saucer	(iv) 	(iv) 
(e)  Container	(v) 	(v) 

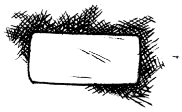
2. For each of the given solid, the three views are given. Identify for each solid the corresponding top, front and side views.

(a) **Object** (i) (ii) (iii)



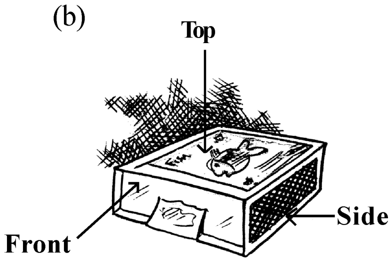


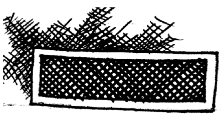


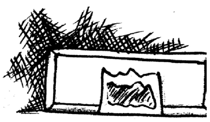


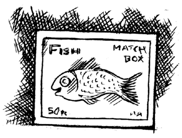
An almirah

(b)



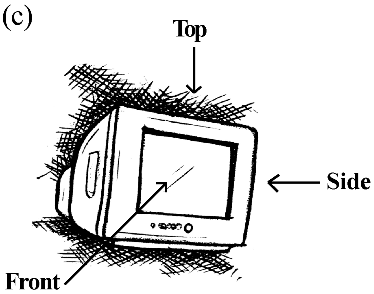


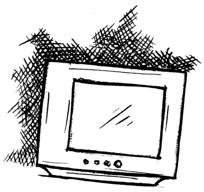


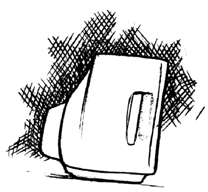


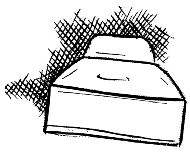
A Match box

(c)



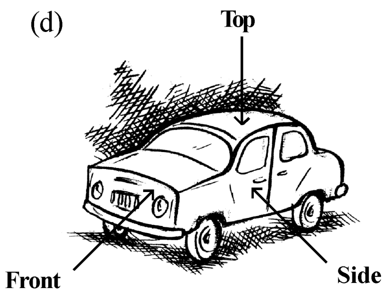


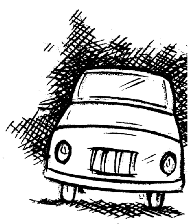


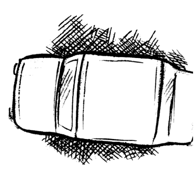


A Television

(d)



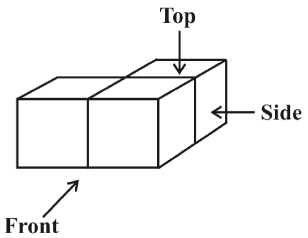


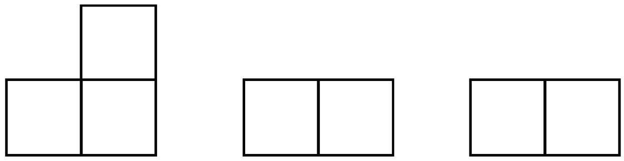


A car

3. For each given solid, identify the top view, front view and side view.

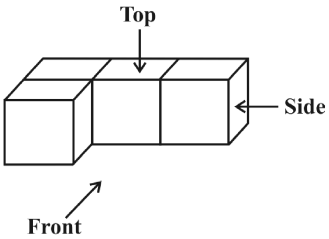
(a)

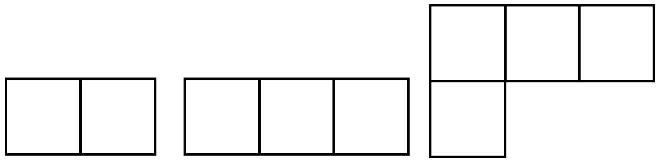




(i) (ii) (iii)

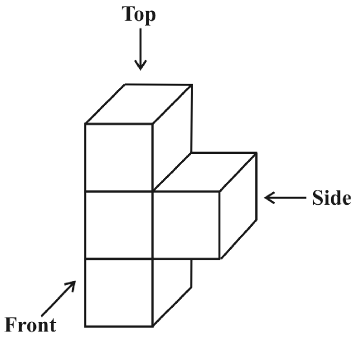
(b)

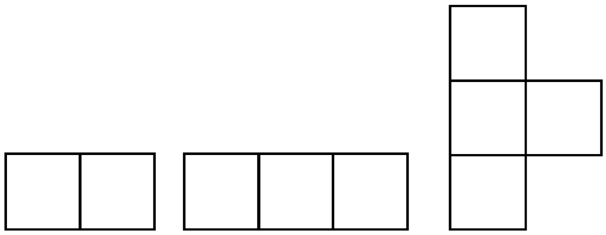




(i) (ii) (iii)

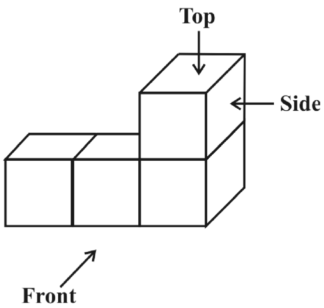
(c)

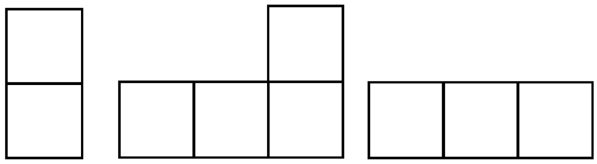




(i) (ii) (iii)

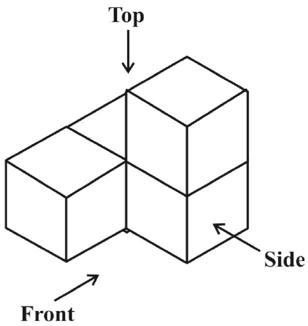
(d)

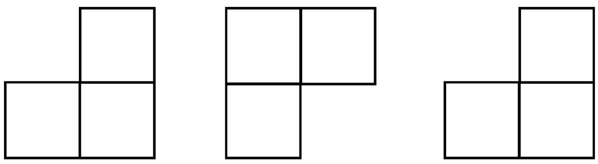




(i) (ii) (iii)

(e)

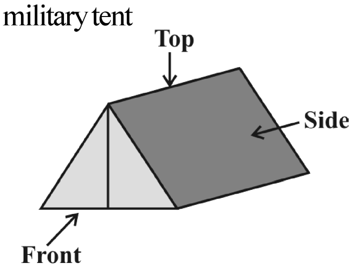




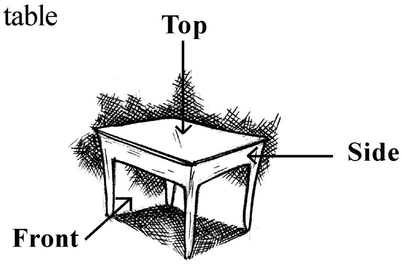
(i) (ii) (iii)

4. Draw the front view, side view and top view of the given objects.

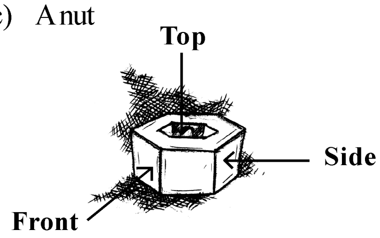
(a) A military tent



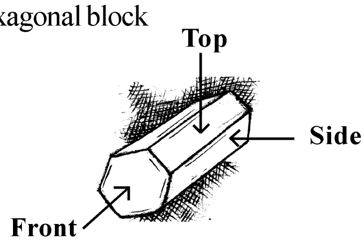
(b) A table



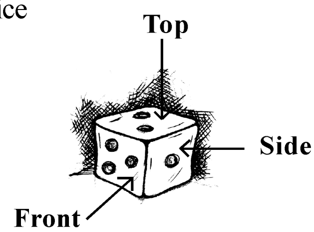
(c) A nut



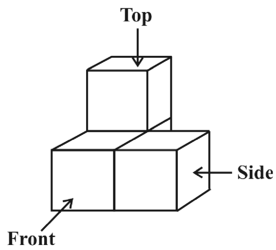
(d) A hexagonal block



(e) A dice



(f) A solid

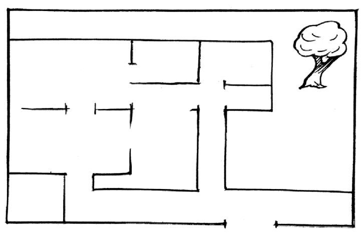


7.3 Mapping Space Around Us

You have been dealing with maps since you were in primary, classes. In Geography, you have been asked to locate a particular State, a particular river, a mountain etc., on a map. In History, you might have been asked to locate a particular place where some event had occured long back. You have traced routes of rivers, roads, railwaylines, traders and many others.

How do we read maps? What can we conclude and understand while reading a map? What information does a map have and what it does not have? Is it any different from a picture? In this section, we will try to find answers to some of these questions. Look at the map of a house whose picture is given alongside (Fig 7.1).

Fig 7.1



What can we conclude from the above illustration? When we draw a picture, we attempt to represent reality as it is seen with all its details, whereas, a map depicts only the location of an object, in relation to other objects. Secondly, different persons can give descriptions of pictures completely different from one another, depending upon the position from which they are looking at the house. But, this is not true in the case of a map. The map of the house remains the same irrespective of the position of the observer. In other words, **perspective is very important for drawing a picture but it is not relevant for a map.**

Now, look at the map (Fig 7.2), which has been drawn by seven year old Raghav, as the route from his house to his school:

From this map, can you tell –

(i) how far is Raghav's school from his house?

(ii) would every circle in the map depict a round about?

(iii) whose school is nearer to the house, Raghav's or his sister's?

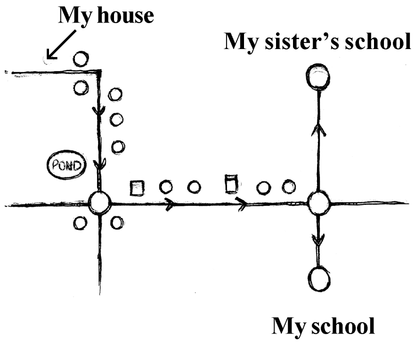


Fig 7.2

It is very difficult to answer the above questions on the basis of the given map. Can you tell why?

The reason is that we do not know if the distances have been drawn properly or whether the circles drawn are roundabouts or represent something else.

Now look at another map drawn by his sister, ten year old Meena, to show the route from her house to her school (Fig 7.3).

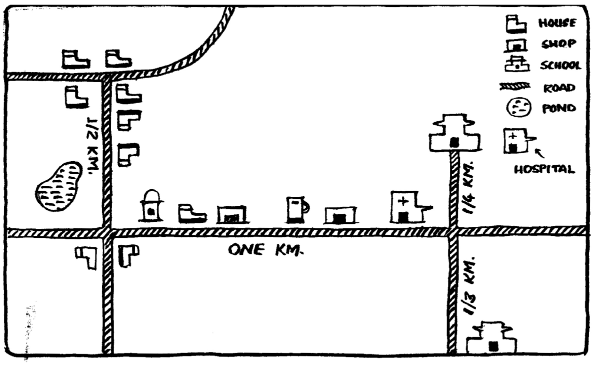


Fig 7.3

This map is different from the earlier maps. Here, Meena has used different symbols for different landmarks. Secondly, longer line segments have been drawn for longer distances and shorter line segments have been drawn for shorter distances, i.e., she has drawn the map to a scale.

Now, you can answer the following questions:

- How far is Raghav's school from his residence?
- Whose school is nearer to the house, Raghav's or Meena's?
- Which are the important landmarks on the route?

Thus we realise that, use of certain symbols and mentioning of distances has helped us read the map easily. Observe that the distances shown on the map are proportional to the actual distances on the ground. This is done by considering a proper scale. While drawing (or reading) a map, one must know, to what scale it has to be drawn (or has been drawn), i.e., how much of actual distance is denoted by 1mm or 1cm in the map. This means, that if one draws a map, he/she has to decide that 1cm of space in that map shows a certain fixed distance of say 1 km or 10 km. This scale can vary from map to map but not within a map. For instance, look at the map of India alongside the map of Delhi.

You will find that when the maps are drawn of same size, scales and the distances in the two maps will vary. That is 1 cm of space in the map of Delhi will represent smaller distances as compared to the distances in the map of India.

The larger the place and smaller the size of the map drawn, the greater is the distance represented by 1 cm.

Thus, we can summarise that:

1. A map depicts the location of a particular object/place in relation to other objects/places.
2. Symbols are used to depict the different objects/places.
3. There is no reference or perspective in map, i.e., objects that are closer to the observer are shown to be of the same size as those that are farther away. For example, look at the following illustration (Fig 7.4).

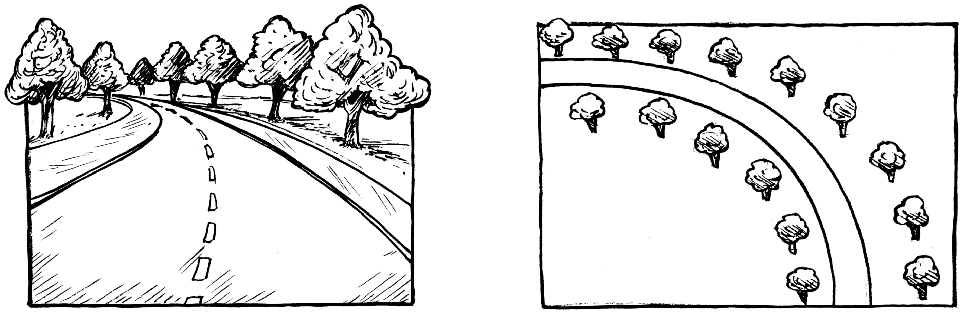


Fig 7.4

4. Maps use a scale which is fixed for a particular map. It reduces the real distances proportionately to distances on the paper.

DO THIS

1. Look at the following map of a city (Fig 7.5).

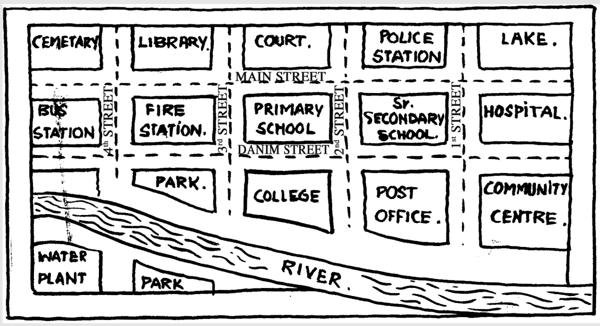


Fig 7.5

(a) Colour the map as follows: Blue-water, Red-fire station, Orange-Library, Yellow-schools, Green-Parks, Pink-Community Centre, Purple-Hospital, Brown-Cemetry.

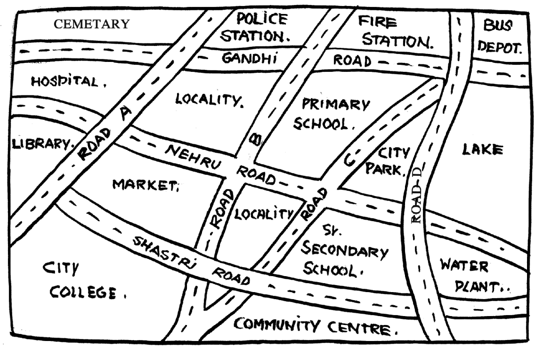
(b) Mark a Green 'X' at the intersection of 2nd street and Danim street. A Black 'Y' where the river meets the third street. A red 'Z' at the intersection of main street and 1st street.

(c) In magenta colour, draw a short street route from the college to the lake.

2. Draw a map of the route from your house to your school showing important landmarks.

EXERCISE 7.2

1. Look at the given map of a city.



Answer the following.

(a) Colour the map as follows: Blue-water, red-fire station, orange-library, yellow - schools, Green - park, Pink - College, Purple - Hospital, Brown - Cemetery.

(b) Mark a green 'X' at the intersection of Road 'C' and Nehru Road, Green 'Y' at the intersection of Gandhi Road and Road A.

(c) In red, draw a short street route from Library to the bus depot.

(d) Which is further east, the city park or the market?

(e) Which is further south, the primary school or the Sr. Secondary School?

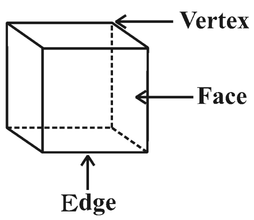
2. Draw a map of your class room using proper scale and symbols for different objects.

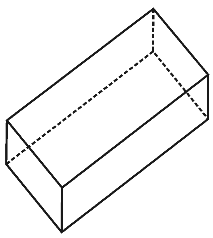
3. Draw a map of your school compound using proper scale and symbols for various features like play ground main building, garden etc.

4. Draw a map giving instructions to your friend so that she reaches your house without any difficulty.

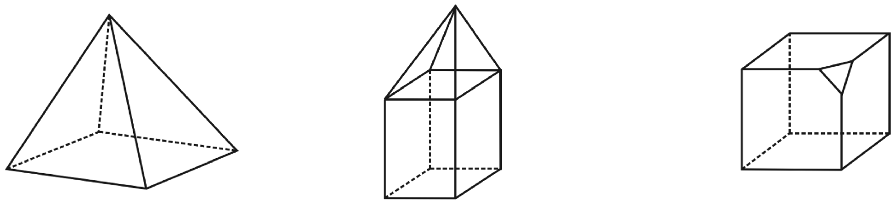
7.4 Faces, Edges and Vertices

Look at the following solids!

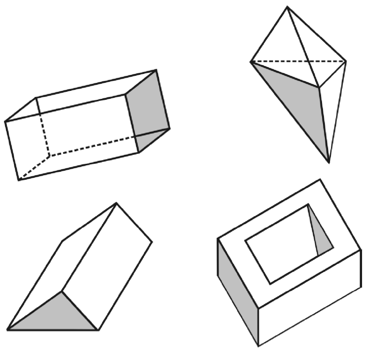


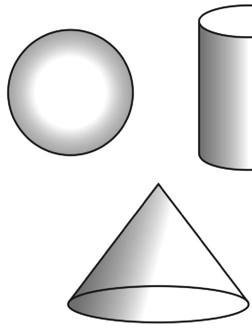


Riddle
I have no vertices. I have no flat faces. Who am I?



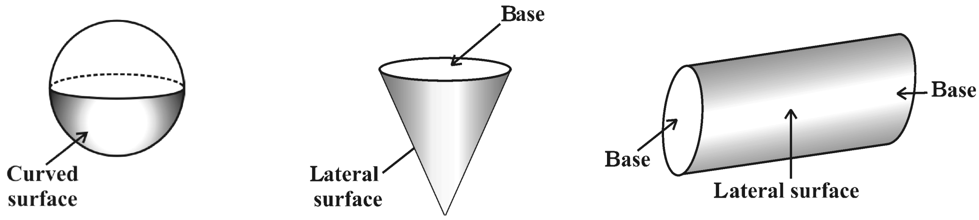
Each of these solids is made up of polygonal regions which are called its **faces**; these faces meet at **edges** which are line segments; and the edges meet at vertices which are **points**. Such solids are called **polyhedrons**.





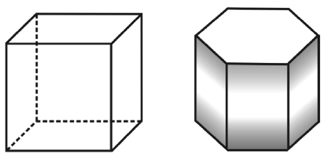
These are polyhedrons **These are not polyhedrons**

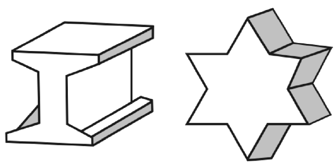
How are the polyhedrons different from the non-polyhedrons? Study the figures carefully. You know three other types of common solids.



Sphere **Cone** **Cylinder**

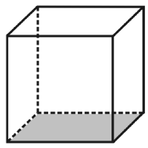
Convex polyhedrons: You will recall the concept of convex polygons. The idea of convex polyhedron is similar.



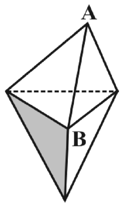


These are convex polyhedrons **These are not convex polyhedrons**

Regular polyhedrons: A polyhedron is said to be **regular** if its faces are made up of regular polygons and the same number of faces meet at **each** vertex.

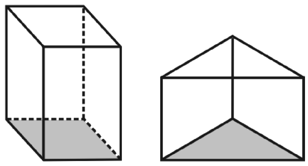


This polyhedron is regular.
Its faces are congruent, regular polygons. Vertices are formed by the same number of faces

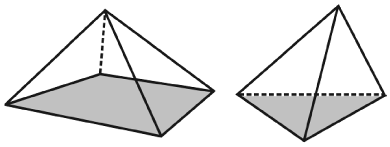


This polyhedon is not regular. All the sides are congruent; but the vertices are not formed by the same number of faces.
3 faces meet at A but
4 faces meet at B.

Two important members of polyhedron family around are prisms and pyramids.



These are prisms



These are pyramids

We say that **a prism** is a polyhedron whose base and top are congruent polygons and whose other faces, i.e., lateral faces are parallelograms in shape.

On the other hand, **a pyramid** is a polyhedron whose base is a polygon (of any number of sides) and whose lateral faces are triangles with a common vertex. (If you join all the corners of a polygon to a point not in its plane, you get a model for pyramid).

A prism or a pyramid is named after its base. Thus a hexagonal prism has a hexagon as its base; and a triangular pyramid has a triangle as its base. What, then, is a rectangular prism? What is a square pyramid? Clearly their bases are rectangle and square respectively.

DO THIS

Tabulate the number of faces, edges and vertices for the following polyhedrons: (Here 'V' stands for number of vertices, 'F' stands for number of faces and 'E' stands for number of edges).

Solid	F	V	E	F+V	E+2
Cuboid					
Triangular pyramid					
Triangular prism					
Pyramid with square base					
Prism with square base					

What do you infer from the last two columns? In each case, do you find F + V = E + 2, i.e., F + V – E = 2? This relationship is called **Euler's formula.** In fact this formula is true for any polyhedron.

THINK, DISCUSS AND WRITE

What happens to F, V and E if some parts are sliced off from a solid? (To start with, you may take a plasticine cube, cut a corner off and investigate).

EXERCISE 7.3

1. Can a polyhedron have for its faces
   (i) 3 triangles? (ii) 4 triangles?
   (iii) a square and four triangles?

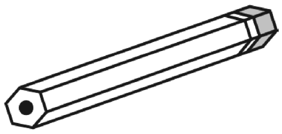
2. Is it possible to have a polyhedron with any given number of faces? (**Hint:** Think of a pyramid).

3. Which are prisms among the following?

(i)

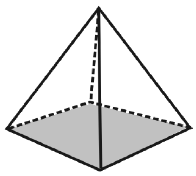
A nail

(ii)



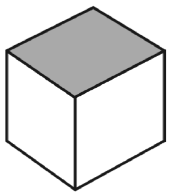
Unsharpened pencil

(iii)



A table weight

(iv)

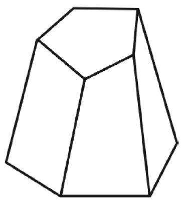


A box

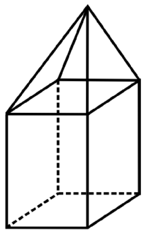
4. (i) How are prisms and cylinders alike?
   (ii) How are pyramids and cones alike?

5. Is a square prism same as a cube? Explain.

6. Verify Euler's formula for these solids.



(i)



(ii)

7. Using Euler's formula find the unknown.

Faces	?	5	20
Vertices	6	?	12
Edges	12	9	?

8. Can a polyhedron have 10 faces, 20 edges and 15 vertices?

WHAT HAVE WE DISCUSSED?

1. Recognising 2D and 3D objects.
2. Recognising different shapes in nested objects.
3. 3D objects have different views from different positions.
4. A map is different from a picture.
5. A map depicts the location of a particular object/place in relation to other objects/places.
6. Symbols are used to depict the different objects/places.
7. There is no reference or perspective in a map.
8. Maps involve a scale which is fixed for a particular map.
9. For any polyhedron,

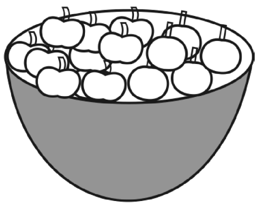
$$F + V - E = 2$$

where 'F' stands for number of faces, V stands for number of vertices and E stands for number of edges. This relationship is called **Euler's formula**.

CHAPTER

8 Comparing Quantities

8.1 Recalling Ratios and Percentages



We know, ratio means comparing two quantities.
A basket has two types of fruits, say, 20 apples and 5 oranges.
Then, the ratio of the number of oranges to the number of apples = 5 : 20.

The comparison can be done by using fractions as, $\frac{5}{20} = \frac{1}{4}$

The number of oranges is $\frac{1}{4}$th the number of apples. In terms of ratio, this is 1 : 4, read as, "1 is to 4"

OR

Number of apples to number of oranges = $\frac{20}{5} = \frac{4}{1}$ which means, the number of apples is 4 times the number of oranges. This comparison can also be done using percentages.

> There are 5 oranges out of 25 fruits.
> So percentage of oranges is
> $$\frac{5}{25} \times \frac{4}{4} = \frac{20}{100} = 20\%$$
> [Denominator made 100].

OR

> By unitary method:
> Out of 25 fruits, number of oranges are 5.
> So out of 100 fruits, number of oranges
> $$= \frac{5}{25} \times 100 = 20.$$

Since  contains only apples and oranges,

So, percentage of apples + percentage of oranges = 100

or percentage of apples + 20 = 100

or percentage of apples = 100 – 20 = 80

Thus the basket has 20% oranges and 80% apples.

Example 1: A picnic is being planned in a school for Class VII. Girls are 60% of the total number of students and are 18 in number.

The picnic site is 55 km from the school and the transport company is charging at the rate of ₹ 12 per km. The total cost of refreshments will be ₹ 4280.

Can you tell.

1. The ratio of the number of girls to the number of boys in the class?
2. The cost per head if two teachers are also going with the class?
3. If their first stop is at a place 22 km from the school, what per cent of the total distance of 55 km is this? What per cent of the distance is left to be covered?

Solution:

1. To find the ratio of girls to boys.

   Ashima and John came up with the following answers.

   They needed to know the number of boys and also the total number of students.

Ashima did this		John used the unitary method
Let the total number of students be x. 60% of x is girls. Therefore, 60% of x = 18 $\frac{60}{100} \times x = 18$ or, $x = \frac{18 \times 100}{60} = 30$ Number of students = 30.	OR	There are 60 girls out of 100 students. There is one girl out of $\frac{100}{60}$ students. So, 18 girls are out of how many students? Number of students = $\frac{100}{60} \times 18$ $= 30$

   So, the number of boys = 30 – 18 = 12.

   Hence, ratio of the number of girls to the number of boys is 18 : 12 or $\frac{18}{12} = \frac{3}{2}$.

   $\frac{3}{2}$ is written as 3 : 2 and read as 3 is to 2.

2. To find the cost per person.

   Transportation charge = Distance both ways × Rate

   = ₹ (55 × 2) × 12

   = ₹ 110 × 12 = ₹ 1320

   Total expenses = Refreshment charge + Transportation charge

   = ₹ 4280 + ₹ 1320

   = ₹ 5600

   Total number of persons = 18 girls + 12 boys + 2 teachers

   = 32 persons

   Ashima and John then used unitary method to find the cost per head.

   For 32 persons, amount spent would be ₹ 5600.

   The amount spent for 1 person = ₹ $\frac{5600}{32}$ = ₹ 175.

3. The distance of the place where first stop was made = 22 km.

To find the percentage of distance:

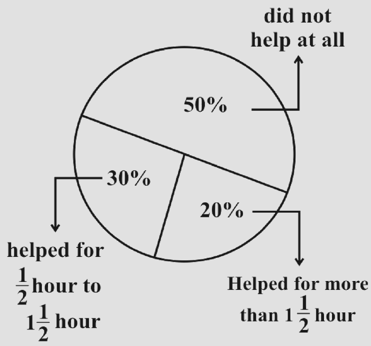
Ashima used this method:		John used the unitary method:
$\frac{22}{55} = \frac{22}{55} \times \frac{100}{100} = 40\%$ $\left[\text{She is multiplying the ratio by } \frac{100}{100} = 1 \text{ and converting to percentage.}\right]$	OR	Out of 55 km, 22 km are travelled. Out of 1 km, $\frac{22}{55}$ km are travelled. Out of 100 km, $\frac{22}{55} \times 100$ km are travelled. That is 40% of the total distance is travelled.

Both came out with the same answer that the distance from their school of the place where they stopped at was 40% of the total distance they had to travel.
Therefore, the percent distance left to be travelled = 100% – 40% = 60%.

TRY THESE

In a primary school, the parents were asked about the number of hours they spend per day in helping their children to do homework. There were 90 parents who helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$ hours. The distribution of parents according to the time for which, they said they helped is given in the adjoining figure ; 20% helped for more than $1\frac{1}{2}$ hours per day;
30% helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$ hours; 50% did not help at all.
Using this, answer the following:

(i) How many parents were surveyed?
(ii) How many said that they did not help?
(iii) How many said that they helped for more than $1\frac{1}{2}$ hours?



EXERCISE 8.1

1. Find the ratio of the following.
   (a) Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.
   (b) 5 m to 10 km
   (c) 50 paise to ₹ 5
2. Convert the following ratios to percentages.
   (a) 3 : 4
   (b) 2 : 3
3. 72% of 25 students are interested in mathematics. How many are not interested in mathematics?
4. A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all?
5. If Chameli had ₹ 600 left after spending 75% of her money, how much did she have in the beginning?

6. If 60% people in a city like cricket, 30% like football and the remaining like other games, then what per cent of the people like other games? If the total number of people is 50 lakh, find the exact number who like each type of game.

8.2 Finding the Increase or Decrease Per cent

We often come across such information in our daily life as.

(i) 25% off on marked prices (ii) 10% hike in the price of petrol

Let us consider a few such examples.

Example 2: The price of a scooter was ₹ 34,000 last year. It has increased by 20% this year. What is the price now?

Solution:

Amita said that she would first find the increase in the price, which is 20% of ₹ 34,000, and then find the new price.

20% of ₹ 34000 = ₹ $\frac{20}{100} \times 34000$

= ₹ 6800

New price = Old price + Increase

= ₹ 34,000 + ₹ 6,800

= ₹ 40,800

OR

Sunita used the unitary method.
20% increase means,
₹ 100 increased to ₹ 120.
So, ₹ 34,000 will increase to?

Increased price = ₹ $\frac{120}{100} \times 34000$

= ₹ 40,800

Similarly, a percentage decrease in price would imply finding the actual decrease followed by its subtraction the from original price.

Suppose in order to increase its sale, the price of scooter was decreased by 5%. Then let us find the price of scooter.

Price of scooter = ₹ 34000

Reduction = 5% of ₹ 34000

= ₹ $\frac{5}{100} \times 34000$ = ₹ 1700

New price = Old price – Reduction

= ₹ 34000 – ₹ 1700 = ₹ 32300

We will also use this in the next section of the chapter.

8.3 Finding Discounts

Discount is a reduction given on the Marked Price (MP) of the article.

This is generally given to attract customers to buy goods or to promote sales of the goods. You can find the discount by subtracting its sale price from its marked price.

So, Discount = Marked price – Sale price

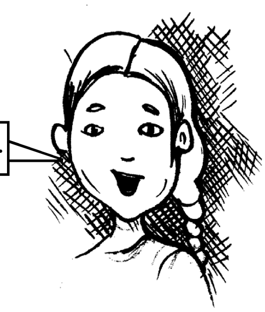
Example 3: An item marked at ₹ 840 is sold for ₹ 714. What is the discount and discount %?

Solution: Discount = Marked Price – Sale Price

= ₹ 840 – ₹ 714

= ₹ 126

Since discount is on marked price, we will have to use marked price as the base.

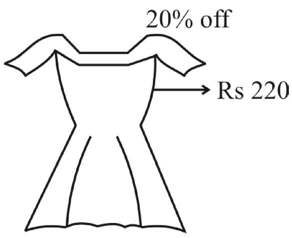


On marked price of ₹ 840, the discount is ₹ 126.
On MP of ₹ 100, how much will the discount be?

$$\text{Discount} = \frac{126}{840} \times 100\% = 15\%$$

You can also find discount when discount % is given.

Example 4: The list price of a frock is ₹ 220. A discount of 20% is announced on sales. What is the amount of discount on it and its sale price.



Solution: Marked price is same as the list price.
20% discount means that on ₹ 100 (MP), the discount is ₹ 20.

By unitary method, on ₹1 the discount will be ₹ $\frac{20}{100}$.

On ₹ 220, discount = ₹ $\frac{20}{100} \times 220$ = ₹ 44

The sale price = (₹ 220 – ₹ 44) or ₹ 176

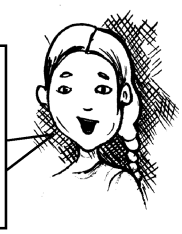
Rehana found the sale price like this —

A discount of 20% means for a MP of ₹ 100, discount is ₹ 20. Hence the sale price is ₹ 80. Using unitary method, when MP is ₹ 100, sale price is ₹ 80;

When MP is ₹ 1, sale price is ₹ $\frac{80}{100}$.

Hence when MP is ₹ 220, sale price = ₹ $\frac{80}{100} \times 220$ = ₹ 176.

Even though the discount was not found, I could find the sale price directly.



TRY THESE

1. A shop gives 20% discount. What would the sale price of each of these be?
   (a) A dress marked at ₹ 120
   (b) A pair of shoes marked at ₹ 750
   (c) A bag marked at ₹ 250
2. A table marked at ₹ 15,000 is available for ₹ 14,400. Find the discount given and the discount per cent.
3. An almirah is sold at ₹ 5,225 after allowing a discount of 5%. Find its marked price.

8.3.1 Estimation in percentages

Your bill in a shop is ₹ 577.80 and the shopkeeper gives a discount of 15%. How would you estimate the amount to be paid?

(i) Round off the bill to the nearest tens of ₹ 577.80, i.e., to ₹ 580.

(ii) Find 10% of this, i.e., ₹ $\frac{10}{100} \times 580 =$ ₹ 58.

(iii) Take half of this, i.e., $\frac{1}{2} \times 58 =$ ₹ 29.

(iv) Add the amounts in (ii) and (iii) to get ₹ 87.

You could therefore reduce your bill amount by ₹ 87 or by about ₹ 85, which will be ₹ 495 approximately.

1. Try estimating 20% of the same bill amount. 2. Try finding 15% of ₹ 375.

8.4 Prices Related to Buying and Selling (Profit and Loss)

For the school fair (mela) I am going to put a stall of lucky dips. I will charge ₹ 10 for one lucky dip but I will buy items which are worth ₹ 5.

So you are making a profit of 100%.

No, I will spend ₹ 3 on paper to wrap the gift and tape. So my expenditure is ₹ 8.

This gives me a profit of ₹ 2, which is, $\frac{2}{8} \times 100\% = 25\%$ only.

Sometimes when an article is bought, some additional expenses are made while buying or before selling it. These expenses have to be included in the cost price.

These expenses are sometimes referred to as **overhead charges**. These may include expenses like amount spent on repairs, labour charges, transportation etc.

8.4.1 Finding cost price/selling price, profit %/loss%

Example 5: Sohan bought a second hand refrigerator for ₹ 2,500, then spent ₹ 500 on its repairs and sold it for ₹ 3,300. Find his loss or gain per cent.

Solution: Cost Price (CP) = ₹ 2500 + ₹ 500 (overhead expenses are added to give CP)

= ₹ 3000

Sale Price (SP) = ₹ 3300

As SP > CP, he made a profit = ₹ 3300 – ₹ 3000 = ₹ 300

His profit on ₹ 3,000, is ₹ 300. How much would be his profit on ₹ 100?

Profit $= \frac{300}{3000} \times 100\% = \frac{30}{3}\% = 10\%$

$$\boxed{P\% = \frac{P}{CP} \times 100}$$

TRY THESE

1. Find selling price (SP) if a profit of 5% is made on
   (a) a cycle of ₹ 700 with ₹ 50 as overhead charges.
   (b) a lawn mower bought at ₹ 1150 with ₹ 50 as transportation charges.
   (c) a fan bought for ₹ 560 and expenses of ₹ 40 made on its repairs.

Example 6: A shopkeeper purchased 200 bulbs for ₹ 10 each. However 5 bulbs were fused and had to be thrown away. The remaining were sold at ₹ 12 each. Find the gain or loss %.

Solution: Cost price of 200 bulbs = ₹ 200 × 10 = ₹ 2000

5 bulbs were fused. Hence, number of bulbs left = 200 – 5 = 195

These were sold at ₹ 12 each.

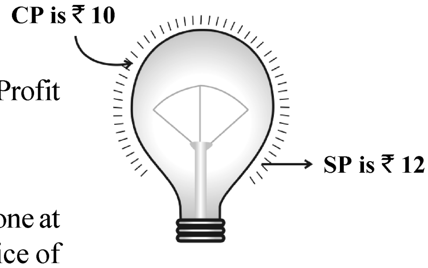
The SP of 195 bulbs = ₹ 195 × 12 = ₹ 2340

He obviously made a profit (as SP > CP).

Profit = ₹ 2340 – ₹ 2000 = ₹ 340

On ₹ 2000, the profit is ₹ 340. How much profit is made on ₹ 100? Profit

$= \frac{340}{2000} \times 100\% = 17\%.$

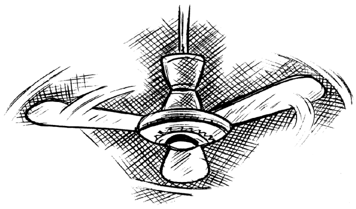


Example 7: Meenu bought two fans for ₹ 1200 each. She sold one at a loss of 5% and the other at a profit of 10%. Find the selling price of each. Also find out the total profit or loss.

Solution: Overall CP of each fan = ₹ 1200. One is sold at a loss of 5%.

This means if CP is ₹ 100, SP is ₹ 95.

Therefore, when CP is ₹ 1200, then SP = ₹ $\frac{95}{100} \times 1200$ = ₹ 1140



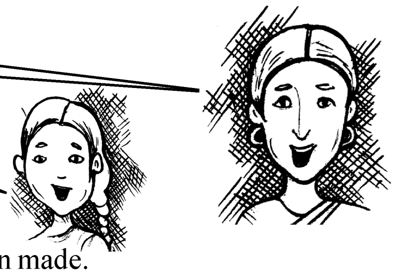
Also second fan is sold at a profit of 10%.

It means, if CP is ₹ 100, SP is ₹ 110.

Therefore, when CP is ₹ 1200, then SP = ₹ $\frac{110}{100} \times 1200$ = ₹ 1320

Was there an overall loss or gain?

We need to find the combined CP and SP to say whether there was an overall profit or loss.



Total CP = ₹ 1200 + ₹ 1200 = ₹ 2400

Total SP = ₹ 1140 + ₹ 1320 = ₹ 2460

Since total SP > total CP, a profit of ₹ (2460 – 2400) or ₹ 60 has been made.

TRY THESE

1. A shopkeeper bought two TV sets at ₹ 10,000 each. He sold one at a profit 10% and the other at a loss of 10%. Find whether he made an overall profit or loss.

8.5 Sales Tax/Value Added Tax/Goods and Services Tax

The teacher showed the class a bill in which the following heads were written.

Bill No.			**Date**	
Menu				
S.No.	Item	Quantity	Rate	Amount
		Bill amount + ST (5%)		
	Total			

Sales tax (ST) is charged by the government on the sale of an item. It is collected by the shopkeeper from the customer and given to the government. This is, therefore, always on the selling price of an item and is added to the value of the bill. There is another type of tax which is included in the prices known as **Value Added Tax (VAT)**.

From July 1, 2017, Government of India introduced GST which stands for Goods and Services Tax which is levied on supply of goods or services or both.

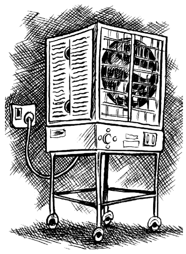
Example 8: (**Finding Sales Tax**) The cost of a pair of roller skates at a shop was ₹ 450. The sales tax charged was 5%. Find the bill amount.

Solution: On ₹ 100, the tax paid was ₹ 5.

On ₹ 450, the tax paid would be = ₹ $\frac{5}{100} \times 450$

= ₹ 22.50

Bill amount = Cost of item + Sales tax = ₹ 450 + ₹ 22.50 = ₹ 472.50.



Example 9: (**Value Added Tax (VAT)**) Waheeda bought an air cooler for ₹ 3300 including a tax of 10%. Find the price of the air cooler before VAT was added.

Solution: The price includes the VAT, i.e., the value added tax. Thus, a 10% VAT means if the price without VAT is ₹ 100 then price including VAT is ₹ 110.

Now, when price including VAT is ₹ 110, original price is ₹ 100.

Hence when price including tax is ₹ 3300, the original price = ₹ $\frac{100}{110} \times 3300$ = ₹ 3000.

Example 10: Salim bought an article for ₹ 784 which included GST of 12% . What is the price of the article before GST was added?

Solution: Let original price of the article be ₹ 100. GST = 12%.

Price after GST is included = ₹ (100+12) = ₹ 112

When the selling price is ₹ 112 then original price = ₹ 100.

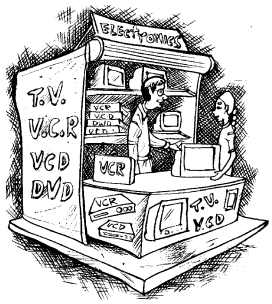
When the selling price is ₹ 784, then original price = ₹ $\frac{100}{12} \times 784$ = ₹ 700

THINK, DISCUSS AND WRITE

1. Two times a number is a 100% increase in the number. If we take half the number what would be the decrease in per cent?
2. By what per cent is ₹ 2,000 less than ₹ 2,400? Is it the same as the per cent by which ₹ 2,400 is more than ₹ 2,000?

EXERCISE 8.2

1. A man got a 10% increase in his salary. If his new salary is ₹ 1,54,000, find his original salary.
2. On Sunday 845 people went to the Zoo. On Monday only 169 people went. What is the per cent decrease in the people visiting the Zoo on Monday?



3. A shopkeeper buys 80 articles for ₹ 2,400 and sells them for a profit of 16%. Find the selling price of one article.
4. The cost of an article was ₹ 15,500. ₹ 450 were spent on its repairs. If it is sold for a profit of 15%, find the selling price of the article.
5. A VCR and TV were bought for ₹ 8,000 each. The shopkeeper made a loss of 4% on the VCR and a profit of 8% on the TV. Find the gain or loss percent on the whole transaction.

6. During a sale, a shop offered a discount of 10% on the marked prices of all the items. What would a customer have to pay for a pair of jeans marked at ₹ 1450 and two shirts marked at ₹ 850 each?
7. A milkman sold two of his buffaloes for ₹ 20,000 each. On one he made a gain of 5% and on the other a loss of 10%. Find his overall gain or loss. (**Hint:** Find CP of each)
8. The price of a TV is ₹ 13,000. The sales tax charged on it is at the rate of 12%. Find the amount that Vinod will have to pay if he buys it.
9. Arun bought a pair of skates at a sale where the discount given was 20%. If the amount he pays is ₹ 1,600, find the marked price.
10. I purchased a hair-dryer for ₹ 5,400 including 8% VAT. Find the price before VAT was added.
11. An article was purchased for ₹ 1239 including GST of 18%. Find the price of the article before GST was added?

8.6 Compound Interest

You might have come across statements like "one year interest for FD (fixed deposit) in the bank @ 9% per annum" or 'Savings account with interest @ 5% per annum'.

Interest is the extra money paid by institutions like banks or post offices on money deposited (kept) with them. Interest is also paid by people when they borrow money. We already know how to calculate **Simple Interest**.

Example 11: A sum of ₹ 10,000 is borrowed at a rate of interest 15% per annum for 2 years. Find the simple interest on this sum and the amount to be paid at the end of 2 years.

Solution: On ₹ 100, interest charged for 1 year is ₹ 15.

So, on ₹ 10,000, interest charged $= \frac{15}{100} \times 10000 =$ ₹ 1500

Interest for 2 years = ₹ 1500 × 2 = ₹ 3000

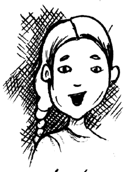
Amount to be paid at the end of 2 years = Principal + Interest

= ₹ 10000 + ₹ 3000 = ₹ 13000

TRY THESE

Find interest and amount to be paid on ₹ 15000 at 5% per annum after 2 years.

My father has kept some money in the post office for 3 years. Every year the money increases as more than the previous year.



We have some money in the bank. Every year some interest is added to it, which is shown in the passbook. This interest is not the same, each year it increases.

Normally, the interest paid or charged is never simple. The interest is calculated on the amount of the previous year. This is known as interest compounded or **Compound Interest** (C.I.).

Let us take an example and find the interest year by year. Each year our sum or principal changes.

Calculating Compound Interest

A sum of ₹ 20,000 is borrowed by Heena for 2 years at an interest of 8% compounded annually. Find the Compound Interest (C.I.) and the amount she has to pay at the end of 2 years.

Aslam asked the teacher whether this means that they should find the interest year by year. The teacher said 'yes', and asked him to use the following steps :

1. Find the Simple Interest (S.I.) for one year.
   Let the principal for the first year be P_1. Here, P_1 = ₹ 20,000

   SI_1 = SI at 8% p.a. for 1st year = ₹ $\frac{20000 \times 8}{100}$ = ₹ 1600

2. Then find the amount which will be paid or received. This becomes principal for the next year.

   Amount at the end of 1st year = $P_1 + SI_1$ = ₹ 20000 + ₹ 1600

   = ₹ 21600 = P_2 (Principal for 2nd year)

3. Again find the interest on this sum for another year.

$$SI_2 = \text{SI at 8\% p.a.for 2nd year} = ₹\ \frac{21600 \times 8}{100}$$

$$= ₹\ 1728$$

4. Find the amount which has to be paid or received at the end of second year.

$$\text{Amount at the end of 2nd year} = P_2 + SI_2$$

$$= ₹\ 21600 + ₹\ 1728$$

$$= ₹\ 23328$$

$$\text{Total interest given} = ₹\ 1600 + ₹\ 1728$$

$$= ₹\ 3328$$

Reeta asked whether the amount would be different for simple interest. The teacher told her to find the interest for two years and see for herself.

$$\text{SI for 2 years} = ₹\ \frac{20000 \times 8 \times 2}{100} = ₹\ 3200$$

Reeta said that when compound interest was used Heena would pay ₹ 128 more.

Let us look at the difference between simple interest and compound interest. We start with ₹ 100. Try completing the chart.

		Under Simple Interest	Under Compound Interest
First year	Principal	₹ 100.00	₹ 100.00
	Interest at 10%	₹ 10.00	₹ 10.00
	Year-end amount	₹ 110.00	₹ 110.00
Second year	Principal	₹ 100.00	₹ 110.00
	Interest at 10%	₹ 10.00	₹ 11.00
	Year-end amount	₹(110 + 10) = ₹ 120	₹ 121.00
Third year	Principal	₹ 100.00	₹ 121.00
	Interest at 10%	₹ 10.00	₹ 12.10
	Year-end amount	₹(120 + 10) = ₹ 130	₹ 133.10

Which means you pay interest on the interest accumulated till then!

Note that in 3 years,

Interest earned by Simple Interest = ₹ (130 – 100) = ₹ 30, whereas,

Interest earned by Compound Interest = ₹ (133.10 – 100) = ₹ 33.10

Note also that the Principal remains the same under Simple Interest, while it changes year after year under compound interest.

8.7 Deducing a Formula for Compound Interest

Zubeda asked her teacher, 'Is there an easier way to find compound interest?' The teacher said 'There is a shorter way of finding compound interest. Let us try to find it.'

Suppose P_1 is the sum on which interest is compounded annually at a rate of R% per annum.

Let P_1 = ₹ 5000 and R = 5. Then by the steps mentioned above

1. $SI_1 = ₹\ \frac{5000 \times 5 \times 1}{100}$ or $SI_1 = ₹\ \frac{P_1 \times R \times 1}{100}$

so, $A_1 = ₹\ 5000 + \frac{5000 \times 5 \times 1}{100}$ or $A_1 = P_1 + SI_1 = P_1 + \frac{P_1 R}{100}$

$= ₹\ 5000\left(1 + \frac{5}{100}\right) = P_2$ $\qquad = P_1\left(1 + \frac{R}{100}\right) = P_2$

2. $SI_2 = ₹\ 5000\left(1 + \frac{5}{100}\right) \times \frac{5 \times 1}{100}$ or $SI_2 = \frac{P_2 \times R \times 1}{100}$

$= ₹\ \frac{5000 \times 5}{100}\left(1 + \frac{5}{100}\right)$ $\qquad = P_1\left(1 + \frac{R}{100}\right) \times \frac{R}{100}$

$$= \frac{P_1 R}{100}\left(1 + \frac{R}{100}\right)$$

$A_2 = ₹\ 5000\left(1 + \frac{5}{100}\right) + ₹\ \frac{5000 \times 5}{100}\left(1 + \frac{5}{100}\right)$ $\qquad A_2 = P_2 + SI_2$

$= ₹\ 5000\left(1 + \frac{5}{100}\right)\left(1 + \frac{5}{100}\right)$ $\qquad = P_1\left(1 + \frac{R}{100}\right) + P_1\frac{R}{100}\left(1 + \frac{R}{100}\right)$

$= ₹\ 5000\left(1 + \frac{5}{100}\right)^2 = P_3$ $\qquad = P_1\left(1 + \frac{R}{100}\right)\left(1 + \frac{R}{100}\right)$

$$= P_1\left(1 + \frac{R}{100}\right)^2 = P_3$$

Proceeding in this way the amount at the end of n years will be

$$A_n = P_1\left(1 + \frac{R}{100}\right)^n$$

Or, we can say $\qquad A = P\left(1 + \frac{R}{100}\right)^n$

So, Zubeda said, but using this we get only the formula for the amount to be paid at the end of n years, and not the formula for compound interest.

Aruna at once said that we know CI = A – P, so we can easily find the compound interest too.

Example 12: Find CI on ₹ 12600 for 2 years at 10% per annum compounded annually.

Solution: We have, $A = P\left(1+\frac{R}{100}\right)^n$, where Principal (P) = ₹ 12600, Rate (R) = 10, Number of years (n) = 2

$$= ₹\ 12600\left(1+\frac{10}{100}\right)^2 = ₹\ 12600\left(\frac{11}{10}\right)^2$$

$$= ₹\ 12600 \times \frac{11}{10} \times \frac{11}{10} = ₹\ 15246$$

CI = A – P = ₹ 15246 – ₹ 12600 = ₹ 2646

TRY THESE

1. Find CI on a sum of ₹ 8000 for 2 years at 5% per annum compounded annually.

8.8 Rate Compounded Annually or Half Yearly (Semi Annually)

You may want to know why 'compounded annually' was mentioned after 'rate'. Does it mean anything?

It does, because we can also have interest rates compounded half yearly or quarterly. Let us see what happens to ₹ 100 over a period of one year if an interest is compounded annually or half yearly.

Time period and rate when interest not compounded annually

The time period after which the interest is added each time to form a new principal is called the **conversion period**. When the interest is compounded half yearly, there are two conversion periods in a year each after 6 months. In such situations, the half yearly rate will be half of the annual rate. What will happen if interest is compounded quarterly? In this case, there are 4 conversion periods in a year and the quarterly rate will be one-fourth of the annual rate.

P = ₹ 100 at 10% per annum compounded annually	P = ₹ 100 at 10% per annum compounded half yearly
The time period taken is 1 year	The time period is 6 months or $\frac{1}{2}$ year
$I = ₹\ \frac{100 \times 10 \times 1}{100} = \text{Rs } 10$	$I = ₹\ \frac{100 \times \boxed{10 \times \frac{1}{2}}}{100} = ₹\ 5$
A = ₹ 100 + ₹ 10 = ₹ 110	A = ₹ 100 + ₹ 5 = ₹ 105 Now for next 6 months the P = ₹ 105
	So, $I = ₹\ \frac{105 \times 10 \times \frac{1}{2}}{100} = ₹\ 5.25$ and A = ₹ 105 + ₹ 5.25 = ₹ 110.25

Rate becomes half

Do you see that, if interest is compounded half yearly, we compute the interest two times. So time period becomes twice and rate is taken half.

TRY THESE

Find the time period and rate for each .

1. A sum taken for $1\frac{1}{2}$ years at 8% per annum is compounded half yearly.
2. A sum taken for 2 years at 4% per annum compounded half yearly.

THINK, DISCUSS AND WRITE

A sum is taken for one year at 16% p.a. If interest is compounded after every three months, how many times will interest be charged in one year?

Example 13: What amount is to be repaid on a loan of ₹ 12000 for $1\frac{1}{2}$ years at 10% per annum compounded half yearly.

Solution:

Principal for first 6 months = ₹ 12,000	**Principal for first 6 months = ₹ 12,000**
There are 3 half years in $1\frac{1}{2}$ years.	Time = 6 months = $\frac{6}{12}$ year = $\frac{1}{2}$ year
Therefore, compounding has to be done 3 times.	Rate = 10%
Rate of interest = half of 10%	I = ₹ $\dfrac{12000 \times 10 \times \frac{1}{2}}{100}$ = ₹ 600
= 5% half yearly	A = P + I = ₹ 12000 + ₹ 600
$A = P\left(1+\frac{R}{100}\right)^n$	= ₹12600. It is principal for next 6 months.
= ₹ 12000 $\left(1+\frac{5}{100}\right)^3$	I = ₹ $\dfrac{12600 \times 10 \times \frac{1}{2}}{100}$ = ₹ 630
= ₹ 12000 $\times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$	Principal for third period = ₹ 12600 + ₹ 630
= ₹ 13,891.50	= ₹ 13,230.
	I = ₹ $\dfrac{13230 \times 10 \times \frac{1}{2}}{100}$ = ₹ 661.50
	A = P + I = ₹ 13230 + ₹ 661.50
	= ₹ 13,891.50

TRY THESE

Find the amount to be paid

1. At the end of 2 years on ₹ 2,400 at 5% per annum compounded annually.
2. At the end of 1 year on ₹ 1,800 at 8% per annum compounded quarterly.

Example 14: Find CI paid when a sum of ₹ 10,000 is invested for 1 year and 3 months at $8\frac{1}{2}$% per annum compounded annually.

Solution: Mayuri first converted the time in years.

$$1 \text{ year } 3 \text{ months} = 1\frac{3}{12} \text{ year} = 1\frac{1}{4} \text{ years}$$

Mayuri tried putting the values in the known formula and came up with:

$$A = ₹\ 10000 \left(1+\frac{17}{200}\right)^{1\frac{1}{4}}$$

Now she was stuck. She asked her teacher how would she find a power which is fractional? The teacher then gave her a hint:

Find the amount for the whole part, i.e., 1 year in this case. Then use this as principal to get simple interest for $\frac{1}{4}$ year more. Thus,

$$A = ₹\ 10000 \left(1+\frac{17}{200}\right)$$

$$= ₹\ 10000 \times \frac{217}{200} = ₹\ 10,850$$

Now this would act as principal for the next $\frac{1}{4}$ year. We find the SI on ₹ 10,850 for $\frac{1}{4}$ year.

$$SI = ₹\ \frac{10850 \times \frac{1}{4} \times 17}{100 \times 2}$$

$$= ₹\ \frac{10850 \times 1 \times 17}{800} = ₹\ 230.56$$

Interest for first year = ₹ 10850 – ₹ 10000 = ₹ 850

And, interest for the next $\frac{1}{4}$ year = ₹ 230.56

Therefore, total compound Interest = 850 + 230.56 = ₹ 1080.56.

8.9 Applications of Compound Interest Formula

There are some situations where we could use the formula for calculation of amount in CI. Here are a few.

(i) Increase (or decrease) in population.

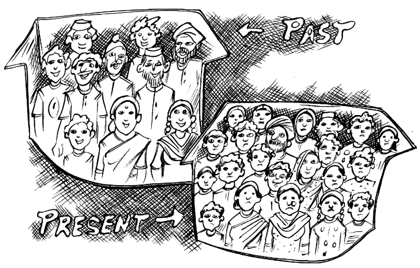
(ii) The growth of a bacteria if the rate of growth is known.

(iii) The value of an item, if its price increases or decreases in the intermediate years.

Example 15: The population of a city was 20,000 in the year 1997. It increased at the rate of 5% p.a. Find the population at the end of the year 2000.

Solution: There is 5% increase in population every year, so every new year has new population. Thus, we can say it is increasing in compounded form.

Population in the beginning of 1998 = 20000 (we treat this as the principal for the 1st year)



Increase at 5% = $\frac{5}{100} \times 20000 = 1000$

Population in 1999 = 20000 + 1000 = 21000

Treat as the Principal for the 2nd year.

Increase at 5% = $\frac{5}{100} \times 21000 = 1050$

Population in 2000 = 21000 + 1050

= 22050

Treat as the Principal for the 3rd year.

Increase at 5% = $\frac{5}{100} \times 22050$

= 1102.5

At the end of 2000 the population = 22050 + 1102.5 = 23152.5

or, Population at the end of 2000 = $20000\left(1+\frac{5}{100}\right)^3$

$= 20000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$

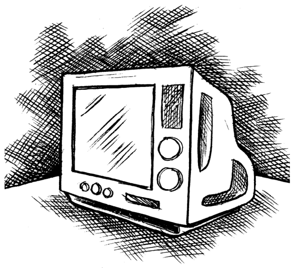
= 23152.5

So, the estimated population = 23153.

Aruna asked what is to be done if there is a decrease. The teacher then considered the following example.

Example 16: A TV was bought at a price of ₹ 21,000. After one year the value of the TV was depreciated by 5% (Depreciation means reduction of value due to use and age of the item). Find the value of the TV after one year.

Solution:



$$\text{Principal} = ₹\ 21{,}000$$
$$\text{Reduction} = 5\% \text{ of } ₹\ 21000 \text{ per year}$$
$$= ₹\ \frac{21000 \times 5 \times 1}{100} = ₹\ 1050$$
$$\text{value at the end of 1 year} = ₹\ 21000 - ₹\ 1050 = ₹\ 19{,}950$$

Alternately, We may directly get this as follows:

$$\text{value at the end of 1 year} = ₹\ 21000 \left(1 - \frac{5}{100}\right)$$
$$= ₹\ 21000 \times \frac{19}{20} = ₹\ 19{,}950$$

TRY THESE

1. A machinery worth ₹ 10,500 depreciated by 5%. Find its value after one year.
2. Find the population of a city after 2 years, which is at present 12 lakh, if the rate of increase is 4%.

EXERCISE 8.3

1. Calculate the amount and compound interest on

(a) ₹ 10,800 for 3 years at $12\frac{1}{2}$% per annum compounded annually.

(b) ₹ 18,000 for $2\frac{1}{2}$ years at 10% per annum compounded annually.

(c) ₹ 62,500 for $1\frac{1}{2}$ years at 8% per annum compounded half yearly.

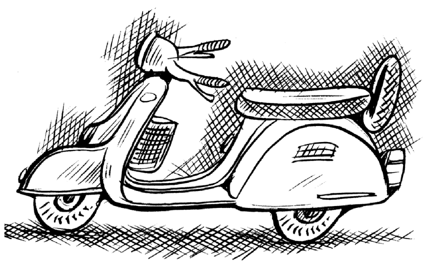
(d) ₹ 8,000 for 1 year at 9% per annum compounded half yearly.
(You could use the year by year calculation using SI formula to verify).

(e) ₹ 10,000 for 1 year at 8% per annum compounded half yearly.

2. Kamala borrowed ₹ 26,400 from a Bank to buy a scooter at a rate of 15% p.a. compounded yearly. What amount will she pay at the end of 2 years and 4 months to clear the loan?

(**Hint:** Find A for 2 years with interest is compounded yearly and then find SI on the 2nd year amount for $\frac{4}{12}$ years).

3. Fabina borrows ₹ 12,500 at 12% per annum for 3 years at simple interest and Radha borrows the same amount for the same time period at 10% per annum, compounded annually. Who pays more interest and by how much?
4. I borrowed ₹ 12,000 from Jamshed at 6% per annum simple interest for 2 years. Had I borrowed this sum at 6% per annum compound interest, what extra amount would I have to pay?
5. Vasudevan invested ₹ 60,000 at an interest rate of 12% per annum compounded half yearly. What amount would he get
   (i) after 6 months?
   (ii) after 1 year?
6. Arif took a loan of ₹ 80,000 from a bank. If the rate of interest is 10% per annum, find the difference in amounts he would be paying after $1\frac{1}{2}$ years if the interest is
   (i) compounded annually.
   (ii) compounded half yearly.
7. Maria invested ₹ 8,000 in a business. She would be paid interest at 5% per annum compounded annually. Find
   (i) The amount credited against her name at the end of the second year.
   (ii) The interest for the 3rd year.
8. Find the amount and the compound interest on ₹ 10,000 for $1\frac{1}{2}$ years at 10% per annum, compounded half yearly. Would this interest be more than the interest he would get if it was compounded annually?
9. Find the amount which Ram will get on ₹ 4096, if he gave it for 18 months at $12\frac{1}{2}\%$ per annum, interest being compounded half yearly.
10. The population of a place increased to 54,000 in 2003 at a rate of 5% per annum
   (i) find the population in 2001.
   (ii) what would be its population in 2005?
11. In a Laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5, 06,000.
12. A scooter was bought at ₹ 42,000. Its value depreciated at the rate of 8% per annum. Find its value after one year.



WHAT HAVE WE DISCUSSED?

1. **Discount** is a reduction given on marked price.
   Discount = Marked Price – Sale Price.
2. Discount can be calculated when discount percentage is given.
   Discount = Discount % of Marked Price
3. Additional expenses made after buying an article are included in the cost price and are known as **overhead expenses**.
   CP = Buying price + Overhead expenses
4. Sales tax is charged on the sale of an item by the government and is added to the Bill Amount.
   Sales tax = Tax% of Bill Amount
5. GST stands for Goods and Services Tax and is levied on supply of goods or services or both.
6. Compound interest is the interest calculated on the previous year's amount (A = P + I)
7. (i) Amount when interest is compounded annually

   $$= P\left(1+\frac{R}{100}\right)^n ; \quad \text{P is principal, R is rate of interest, } n \text{ is time period}$$

   (ii) Amount when interest is compounded half yearly

   $$= P\left(1+\frac{R}{200}\right)^{2n} \quad \begin{cases} \frac{R}{2} \text{ is half yearly rate and} \\ 2n = \text{number of 'half-years'} \end{cases}$$