Elasticity

Rigid body

A body is said to be a rigid body, if it suffers absolutely no change in its form (length, volume or shape) under the action of forces applied on it.
Such conditions are only ideal and in nature no body is perfectly rigid.

Elasticity

When a force is applied on a body, the body moves if it is free to do so. If the body does not move, then the force, changes the configuration (length, volume or shape) of the body. Such a force is called a deforming force. When this deforming force is withdrawn, the deformed body tends to regain its original form.
The property of a deformed body, by virtue of which it tends to regain its original configuration, when the deforming forces have been withdrawn, is called elasticity.
Bodies having properties of elasticity are called elastic bodies.
Bodies made of different substances have this property in different amount. Bodies which regain their original form completely, are called perfectly elastic. Quartz and phosphor bronze are the nearest approach to a perfectly elastic body.
There are bodies which, when deformed, have no tendency to regain their original form. They are called plastic bodies, and this property is known as plasticity. Mud, putty, wax etc., is the common example. Thus the property of plasticity is oppose to that of elasticity.
In fact, the property of elasticity also depends upon the magnitude of a deforming force. A body may regain its form completely when subjected to a small deforming force. But when deformed too much, it fails to regain its original form.

Stress

When a body is deformed by a deforming force, the molecules of the body are displaced from their original positions. These molecules oppose this displacement and exert a force, equal and opposite of the deforming force (within the limit of elasticity) to restore their original positions.
In deformed condition of the body, the deforming and the restoring forces are in equilibrium (Fig). When the deforming force is withdrawn, it is the restoring force, still persisting, which takes the body back into the original form.

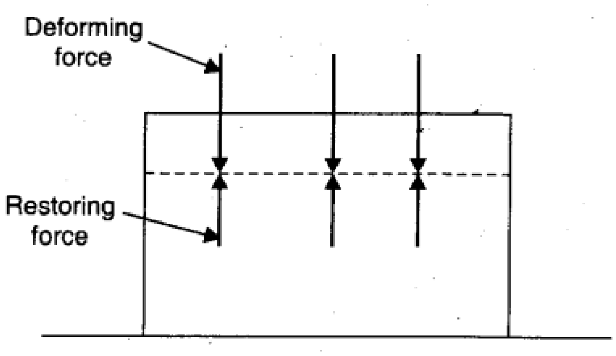
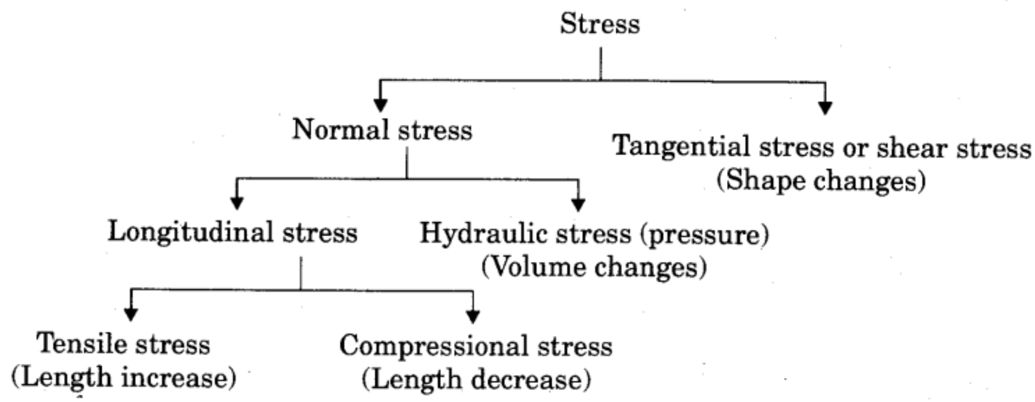


Fig. Stress.

The restoring force per unit area exerted by the molecules of a deformed body, from within the body, is called the stress. It is equal to and is measured by the external deforming force applied per unit area, within the elastic limit.
If a force F is applied on a surface having area a, then
Stress = F/a

The units of stress are dyne per square cm (dyne dyne cm^{-2}) in C.G.S. system and newton per square metre (N m^{-2}) pascal (Pa) in S.I. Stress is a scalar quantity.



If the force is applied normal to the surface, the stress is a normal stress. It causes a change of length or of volume of a body. The body is extended if the stress acts normally outward and is compressed when the stress acts normally inwards.
If the force is applied tangential to the surface, the stress is called tangential or shearing stress. It produces a change of shape in the body.

Strain

When a body is deformed by a deforming force, the molecules are displaced from their initial positions. In this condition the molecules, and for that reason the body, is said to be under strain. Thus, strain is a condition of the body under deforming force. As a quantity, the strain produced is measured by the proportional change in the form of the body.

The fractional change in the configuration (dimensions) of a body is called strain.

$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

Strain does not has unit and dimensions. It is scalar quantity.
The change in form may be due to either change in length or change in volume or change in shape. Accordingly the strain is of three types as follows :
(a) Longitudinal Strain. It is produced when deformation causes a change in length of the body. It is measured by the ratio of the change in length to the original length of the

body. Thus, if L be the original length of a body and l be the change of length produced (Fig.), then longitudinal strain = l/L
The corresponding stress is called longitudinal (normal) stress.

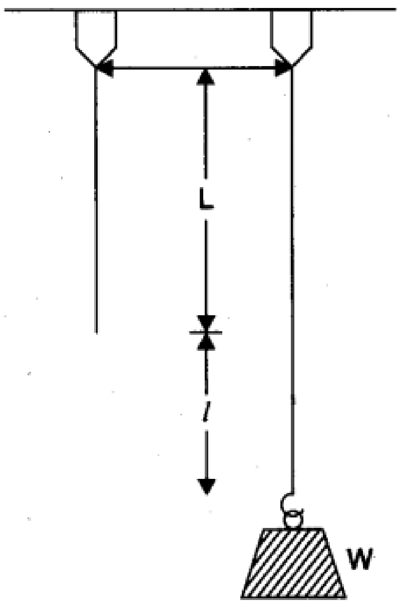


Fig. Longitudinal strain.

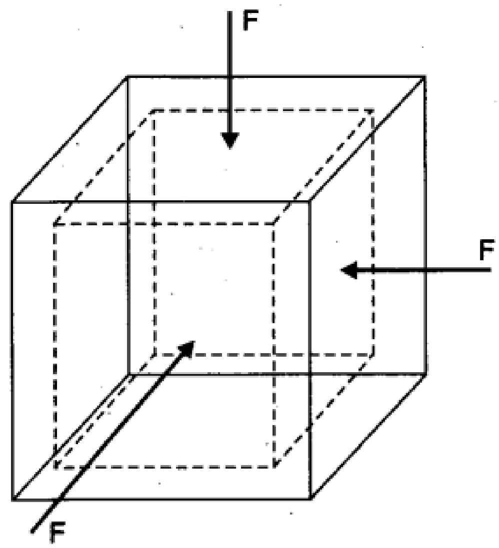


Fig. Volumetric strain.

(b) Volumetric Strain. It is produced when deformation causes a change in volume of the body. It is measured by the ratio of the change in volume to the original volume of the body.
Thus, if a block of original volume, V be compressed by the same normal stress (F) from all directions (Fig), its volume is decreased. All sides decrease in the same ratio. If υ be the decrease in volume, then Volumetric strain = υ/V.

(c) Shear Strain. It is produced when deformation Causes a change in shape of the body. It is measured by the ratio of the displacement of one plane of the body to its distance from the fixed plane of the body.
It is also measured by the angle turned by a line, originally perpendicular to the fixed plane. This angle is known as the angle of shear.
Suppose ABCDEFGH is the original shape of a body. A tangential forced is applied on top plane EFGH from left to right (Fig). The shape of the body changes to $ABCDE_1F_1G_1H_1$. There is no change in volume of the body.

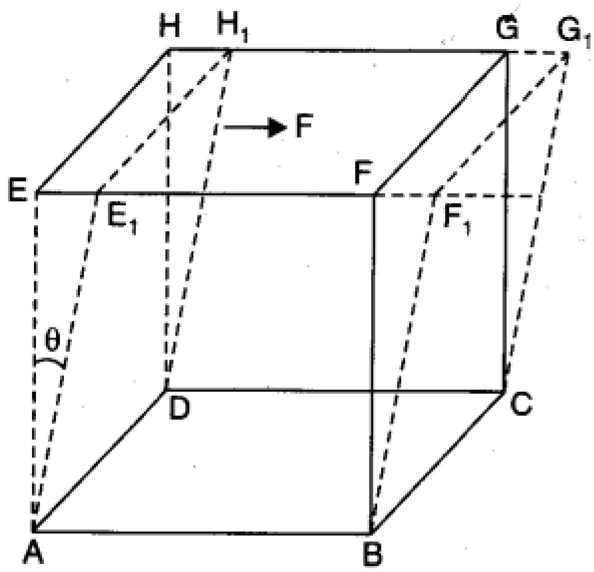


Fig. Shear strain.

The base plane ABCD of the body remains fixed. The plane EFGH is displaced by a distance EE_1. The distance of this plane from fixed plane is AE.
Then, shear strain $=EE_1/AE = \tan\theta = \theta$ (as θ is small) = Angle of shear

Hooke's law

Hooke's law is fundamental law of elasticity. Robert Hooke studied the relation between the tension of a wire and the longitudinal strain produced due to it. Results of his investigation were formulated by him in the form of a law in 1678-79. In its original form, the law may be stated as follows : "Within elastic limits, the extension of an elastic body is directly proportional to the tension (stretching force)".

Mathematically,

Extension (change of length) $l \propto$ *tension* (stretching force) F.

If the force be applied on area a and L be the initial length, then as a and L remain constant,

$$\frac{l}{L} \propto \frac{F}{a}$$

i.e., Strain ∝ Force applied per unit area.

Further as $\frac{F}{a}$ = Stress and comes into existence due to strain, the above relation can be written as *Stress* ∝ *Strain* (within elastic limit).

The above relation is true for all types of strains and is the modified form of Hooke's law. The law may now be stated as follows : "Within elastic limits, the stress is proportional to the strain." This modified statement of Hooke's law is due to Thomas

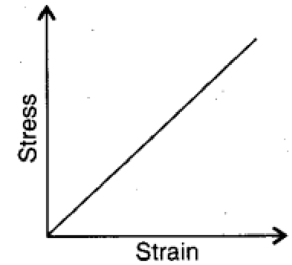
Young and is now the accepted form of Hooke's law.

Hence,

$$\text{Stress} \propto \text{strain}$$

$$\text{Stress} = E \times \text{strain}$$

Here, E = modulus of elasticity or coefficient of elasticity.



Longitudinal or young's modulus of elasticity

The ratio of the normal stress to the longitudinal strain is called the Young's modulus of elasticity. It is represented by the symbol Y.
If a wire of length L and area of cross-section a be stretched by a force F and if a change (increase) of length l is produced, then

$$\text{Normal stress} = \frac{\text{Force applied}}{\text{Area of crosssection}} = \frac{F}{a}$$

$$\text{Longitudinal strain} = \frac{\text{Change of length}}{\text{Original length}} = \frac{l}{L}$$

and Young's modulus of elasticity,

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

i.e.,

$$Y = \frac{\frac{F}{a}}{\frac{l}{L}} = \frac{FL}{al}.$$

The units of Young's modulus are dyne per square cm (dyne cm^{-2}) in C.G.S. system, and newton per square metre ($N\ m^{-2}$) in S.I.

If the extension be produced by suspending a load of mass M from a wire of radius r cm, then,

$$F = Mg \text{ dyne}$$

and

$$a = \pi r^2 \text{ cm}^2$$

Hence,

$$Y = \frac{FL}{al} = \frac{MgL}{\pi r^2 l} \text{ dyne cm}^{-2}.$$

The Young's modulus is different for different materials.

Volume or bulk modulus of elasticity

The ratio of the normal stress (hydro static stress) to the volume metric strain, is called the bulk modulus of elasticity. It is represented by the symbol K.
If a body of volume V be compressed normally by the same pressure P from all sides

and if a change (decrease) of volume v is produced, then

$$\text{Normal stress} = \frac{Force}{Area} = \text{Pressure} = P$$

$$\text{Volumetric strain} = \frac{\text{Change of volume}}{\text{Original volume}} = \frac{v}{V}$$

and Bulk modulus of elasticity,

$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

i.e.,

$$K = \frac{P}{v/V} = \frac{PV}{v}.$$

The reciprocal of bulk modulus *i.e.*, $\frac{1}{K}$ is called the **Compressibility** of the substance of the body.

The unit of bulk modulus is also dyne per square cm (dyne cm^{-2}) in C.G.S. system and newton square metre (N m^{-2}) in S.I.

Shear modulus or modulus of rigidity

The ratio of the tangential stress to the shearing strain in called the shear modulus or modulus of rigidity. It is represented by the symbol η (eta—a Greek letter).
If a tangential force F be applied on the top of a block, on an area a (Fig) and if the top-plane be displaced by an angle θ, then

$$\text{Tangential stress,} \qquad T = \frac{F}{a}$$

$$\text{Shear strain} = \theta$$

and Shear modulus or modulus of rigidity,

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{T}{\theta}.$$

The unit of modulus of rigidity, like other modulii of elasticity, is dyne per square cm (dyne cm^{-2}) in C.G/S. system, and newton per square metre (N m^{-2}) in S.I.

Elastic limit

The maximum stress from which a deformed body regains its original form, when deforming force is withdrawn, is called elastic limit of the substance (material) of the body.
Bodies deformed beyond elastic limit, stay permanently deformed.
Elastic limit of steel is more than that of rubber. Hence, steel is more elastic than rubber. (Though rubber is more stretchable than steel).