

CBSE Board
Class X Mathematics (Standard)
Sample Paper - 2
Term 2 – 2021 - 22

Time: 2 hours

Total Marks: 40

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
 2. All questions are compulsory.
 3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
 4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
 5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.
-

Section A

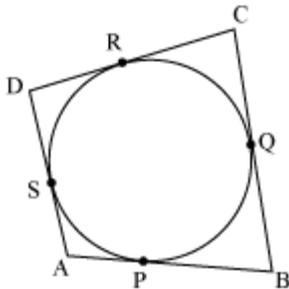
Q1 – Q6 are of 2 mark each.

- 1.** If 17th term of an AP exceeds its 10th term by 7. Find the common difference.

OR

Which term of the AP 3, 15, 27, 39, ... will be 132 more than its 54th term?

- 2.** Sum of the areas of two squares is 468 m^2 . If the difference of their perimeters is 24 m, find the sides of the two squares.
- 3.** A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$.



4. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.
5. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs.18. Find the missing frequency f .

Daily pocket allowance (in Rs)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Number of workers	7	6	9	13	f	5	4

6. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

OR

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Section B

Q7 – Q10 are of 3 mark each.

7. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode.

Number of students per teacher	Number of states/U.T
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	0
45 – 50	0
50 – 55	2

8. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

9. If the median of the distribution is given below is 28.5, find the values of x and y .

Class interval	Frequency
0 – 10	5
10 – 20	x
20 – 30	20
30 – 40	15
40 – 50	y
50 – 60	5
Total	60

10. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

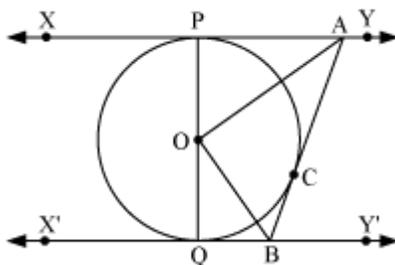
OR

A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.

Section C

Q11 – Q14 are of 4 mark each.

11. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.
12. In the given figure XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY and A and $X'Y'$ at B . Prove that $\angle AOB = 90^\circ$.



OR

Prove that the parallelogram circumscribing a circle is a rhombus.

- 13.** A bird flying at height can see the top of the two buildings of height 534 m and 300 m. The angles of depression from bird, to the top of first and second buildings are 30° and 60° respectively. If the distance between the two buildings is 142 m, and the bird is vertically above the midpoint of the distance between the two buildings, answer the following questions.



- i) Draw a labelled figure on the basis of the given information and find the approximate height of the bird from ground.
- ii) Find the distance between bird and top of building I.
- 14.** Rakesh is much worried about his upcoming assessment on A. P. He was vigorously practicing for the exam but unable to solve some questions. One of these questions is as shown.
- If the 3rd and the 9th terms of an A.P. are 4 and -8 respectively, then help Rakesh in solving the problem.
- i) Form an A.P using the given data
- ii) Find which term of the A.P. is -160 ?

Solution

Section A

1.

We know that,

For an AP, $a_n = a + (n - 1) d$

$$a_{17} = a + (17 - 1) d$$

$$a_{17} = a + 16d$$

Similarly, $a_{10} = a + 9d$

It is given that

$$a_{17} - a_{10} = 7$$

$$(a + 16d) - (a + 9d) = 7$$

$$7d = 7$$

$$d = 1$$

Therefore, the common difference is 1.

OR

Given AP is 3, 15, 27, 39, ...

$$a = 3$$

$$d = a_2 - a_1 = 15 - 3 = 12$$

$$a_{54} = a + (54 - 1) d$$

$$= 3 + (53) (12)$$

$$= 3 + 636 = 639$$

$$132 + 639 = 771$$

We have to find the term of this AP which is 771.

Let n^{th} term be 771.

$$a_n = a + (n - 1) d$$

$$771 = 3 + (n - 1) 12$$

$$768 = (n - 1) 12$$

$$(n - 1) = 64$$

$$n = 65$$

Therefore, 65th term was 132 more than 54th term.

2.

Let the sides of the two squares be x m and y m. Therefore, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively.

It is given that, $4x - 4y = 24$

$$x - y = 6$$

$$x = y + 6$$

$$\text{Also, } x^2 + y^2 = 468$$

$$\Rightarrow (6 + y)^2 + y^2 = 468$$

$$\Rightarrow 36 + y^2 + 12y + y^2 = 468$$

$$\Rightarrow 2y^2 + 12y - 432 = 0$$

$$\Rightarrow y^2 + 6y - 216 = 0$$

$$\Rightarrow y^2 + 18y - 12y - 216 = 0$$

$$\Rightarrow y(y + 18) - 12(y + 18) = 0$$

$$\Rightarrow (y + 18)(y - 12) = 0$$

$$\Rightarrow y = -18 \text{ or } 12.$$

However, side of a square cannot be negative.

Hence, the sides of the squares are 12 m and $(12 + 6) \text{ m} = 18 \text{ m}$

3.

It can be observed that:

$$DR = DS \quad (\text{tangents from point D})$$

$$CR = CQ \quad (\text{tangents from point C})$$

$$BP = BQ \quad (\text{tangents from point B})$$

$$AP = AS \quad (\text{tangents from point A})$$

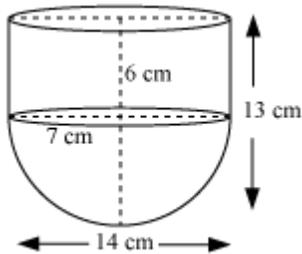
Adding the above four equations,

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$$CD + AB = AD + BC$$

4.



Radius (r) of cylindrical part and hemispherical part = 7 cm

Height of hemispherical part = radius = 7 cm.

Height of cylindrical part (h) = $13 - 7 = 6 \text{ cm}$

Inner surface area of the vessel = CSA of cylindrical part + CSA of hemispherical part

$$= 2\pi rh + 2\pi r^2$$

$$\begin{aligned} \text{Inner surface area of vessel} &= 2 \times \frac{22}{7} \times 7 \times 6 + 2 \times \frac{22}{7} \times 7 \times 7 \\ &= 44(6 + 7) = 44 \times 13 \\ &= 572 \text{ cm}^2 \end{aligned}$$

5.

We may find class mark (x_i) for each interval by using the relation.

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Given that mean pocket allowance $\bar{x} = \text{Rs.}18$

Now taking 18 as assured mean (a) we may calculate d_i and $f_i d_i$ as following.

Daily pocket allowance (in Rs.)	Number of children f_i	Class mark x_i	$d_i = x_i - 18$	$f_i d_i$
11 - 13	7	12	- 6	- 42
13 - 15	6	14	- 4	- 24
15 - 17	9	16	- 2	- 18
17 - 19	13	18	0	0
19 - 21	f	20	2	$2f$
21 - 23	5	22	4	20
23 - 25	4	24	6	24
Total	$\sum f_i = 44 + f$			$2f - 40$

From the table we may obtain

$$\sum f_i = 44 + f$$

$$\sum f_i d_i = 2f - 40$$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$18 = 18 + \left(\frac{2f - 40}{44 + f} \right)$$

$$0 = \left(\frac{2f - 40}{44 + f} \right)$$

$$2f - 40 = 0$$

$$2f = 40$$

$$f = 20$$

Hence the missing frequency f is 20.

6.

Let the speed of the train be x km/hr.

Time taken to cover 360 km = $\frac{360}{x}$ hr

According to the given question,

$$(x + 5) \left(\frac{360}{x} - 1 \right) = 360$$

$$\Rightarrow (x + 5) \left(\frac{360}{x} - 1 \right) = 360$$

$$\Rightarrow 360 - x + \frac{1800}{x} - 5 = 360$$

$$\Rightarrow x^2 + 5x - 1800 = 0$$

$$\Rightarrow x^2 + 45x - 40x - 1800 = 0$$

$$\Rightarrow x(x + 45) - 40(x + 45) = 0$$

$$\Rightarrow (x + 45)(x - 40) = 0$$

$$\Rightarrow x = 40, -45$$

However, speed cannot be negative.

Hence speed of train is 40 km/hr

OR

Let the shorter side of the rectangle be x m.

Then, larger side of the rectangle = $(x + 30)$ m

$$\text{Diagonal of the rectangle} = \sqrt{x^2 + (x + 30)^2}$$

It is given that the diagonal of the rectangle is 60 m more than the shorter side.

$$\therefore \sqrt{x^2 + (x + 30)^2} = x + 60$$

$$\Rightarrow x^2 + (x + 30)^2 = (x + 60)^2$$

$$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$$

$$\Rightarrow x^2 - 60x - 2700 = 0$$

$$\Rightarrow x^2 - 90x + 30x - 2700 = 0$$

$$\Rightarrow x(x - 90) + 30(x - 90)$$

$$\Rightarrow (x - 90)(x + 30) = 0$$

$$\Rightarrow x = 90, -30$$

However, side cannot be negative. Therefore, the length of the shorter side will be 90 m. Hence, length of the larger side will be $(90 + 30) \text{ m} = 120 \text{ m}$

Section B

7.

We may observe from the given data that maximum class frequency is 10 belonging to class interval 30 - 35.

So, modal class = 30 - 35

Class size (h) = 5

Lower limit (l) of modal class = 30

Frequency (f_1) of modal class = 10

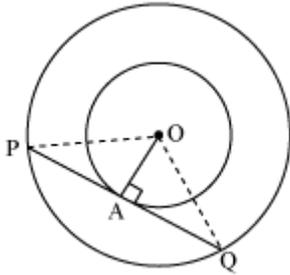
Frequency (f_0) of class preceding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 30 + \left(\frac{10 - 9}{2(10) - 9 - 3} \right) \times (5) \\ &= 30 + \left(\frac{1}{20 - 12} \right) 5 \\ &= 30 + \frac{5}{8} = 30.625 \end{aligned}$$

Mode = 30.6

8.



Let two concentric circles be centered at point O. Let PQ be the chord of the larger circle which touches the smaller circle at point A. So, PQ is tangent to smaller circle.

Since, OA is radius of circle, $OA \perp PQ$

Applying Pythagoras theorem in $\triangle OAP$,

$$OA^2 + AP^2 = OP^2$$

$$3^2 + AP^2 = 5^2$$

$$AP^2 = 16$$

$$AP = 4 \text{ cm}$$

In $\triangle OPQ$, as $OA \perp PQ$, $AP = AQ$

(Perpendicular from center of circle bisects the chord)

$$\therefore PQ = 2AP = 2 \times 4 \text{ cm} = 8 \text{ cm}$$

So, length of chord of larger circle is 8cm.

9. We may find cumulative frequency for the given data as following

Class interval	Frequency	Cumulative frequency
0 - 10	5	5
10 - 20	x	$5 + x$
20 - 30	20	$25 + x$
30 - 40	15	$40 + x$
40 - 50	y	$40 + x + y$
50 - 60	5	$45 + x + y$
Total (n)	60	

It is clear that $n = 60$

$$45 + x + y = 60$$

$$x + y = 15 \quad \dots(1)$$

Median of data is given as 28.5 which lies in interval 20 - 30.

So, median class = 20 - 30

Lower limit (l) of median class = 20

Cumulative frequency (cf) of class preceding the median class = $5 + x$

Frequency (f) of median class = 20

Class size (h) = 10

$$\text{Now, median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$28.5 = 20 + \left[\frac{\frac{60}{2} - (5 + x)}{20} \right] \times 10$$

$$8.5 = \left(\frac{25 - x}{2} \right)$$

$$17 = 25 - x$$

$$x = 8$$

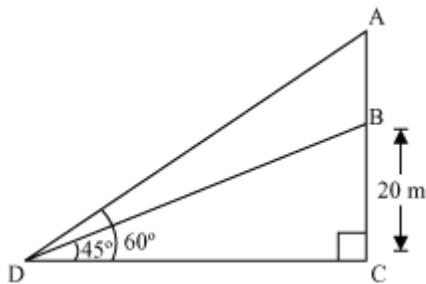
From equation (1)

$$8 + y = 15$$

$$y = 7$$

Hence values of x and y are 8 and 7 respectively.

10.



Let BC be the building, AB be the transmission tower, and D be the point on ground from where elevation angles are to be measured.

In $\triangle BCD$.

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{20}{CD} = 1$$

$$CD = 20 \text{ m} \quad \dots (i)$$

In $\triangle ACD$,

$$\frac{AC}{CD} = \tan 60^\circ$$

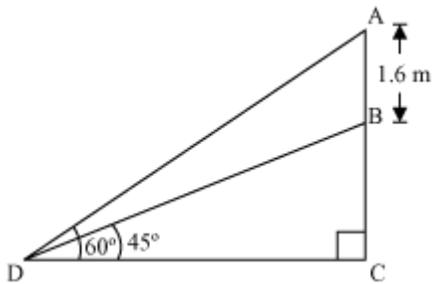
$$\frac{AB + BC}{CD} = \sqrt{3}$$

$$\frac{AB + 20}{20} = \sqrt{3} \quad [\text{From (i)}]$$

$$\begin{aligned} AB &= 20\sqrt{3} - 20 \\ &= 20(\sqrt{3} - 1) \end{aligned}$$

Thus, the height of the tower is $20(\sqrt{3} - 1)$ m.

OR



Let AB be the statue, BC be the pedestal and D be the point on ground from where elevation angles are to be measured.

In $\triangle BCD$,

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{BC}{CD} = 1$$

$$BC = CD \quad \dots (i)$$

In $\triangle ACD$,

$$\frac{AB + BC}{CD} = \tan 60^\circ$$

$$\frac{AB + BC}{BC} = \sqrt{3} \quad [\text{From (i)}]$$

$$1.6 + BC = BC\sqrt{3}$$

$$BC(\sqrt{3} - 1) = 1.6$$

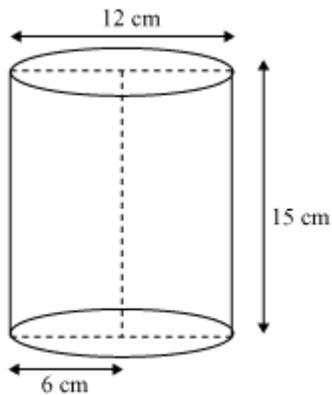
$$BC = \frac{(1.6)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{1.6(\sqrt{3} + 1)}{2} = 0.8(\sqrt{3} + 1)$$

Thus, the height of pedestal is $0.8(\sqrt{3} + 1)$ m.

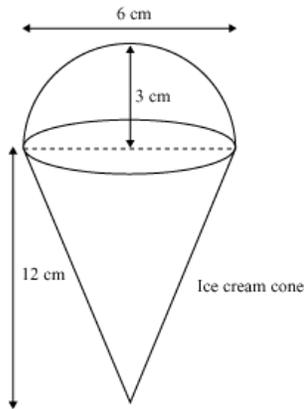
Section C

11.



Height (h_1) of cylindrical container = 15 cm

Radius (r_1) of circular end of container = $\frac{12}{2} = 6$ cm



Radius (r_2) of circular end of ice cream cone = $\frac{6}{2} = 3$ cm

Height (h_2) of conical part of ice cream cone = 12 cm

Let n ice cream cones be filled with the ice cream of container

Volume of ice cream in cylinder = $n \times$ volume of 1 ice-cream cone

$$\pi \times r_1^2 \times h_1 = n \times \left[\frac{1}{3} \pi \times r_2^2 \times h_2 + \frac{2}{3} \pi \times r_2^3 \right]$$

$$\pi \times 6^2 \times 15 = n \times \left[\frac{1}{3} \pi \times 3^2 \times 12 + \frac{2}{3} \pi \times 3^3 \right]$$

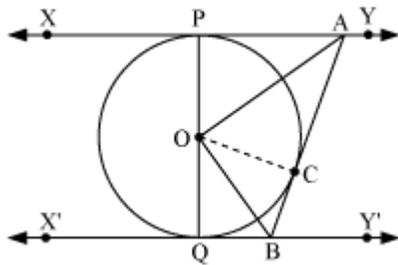
$$36 \times 15 = n [36 + 18]$$

$$n = \frac{36 \times 15}{54}$$

$$n = 10$$

So, number of ice-cream cones, filled with the ice cream of container is 10.

12.



Join OC.

In $\triangle OPA$ and $\triangle OCA$,

$OP = OC$ (Radius of the same circle)

$AP = AC$ (tangents from point A)

$AO = AO$ (common)

$\triangle OPA \cong \triangle OCA$ (SSS congruence rule)

$$\therefore \angle POA = \angle COA \quad \dots (1)$$

Similarly $\triangle OQB \cong \triangle OCB$

$$\therefore \angle QOB = \angle COB \quad \dots (2)$$

Since POQ is a diameter of circle we can say it is a straight line.

$$\text{So, } \angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$$

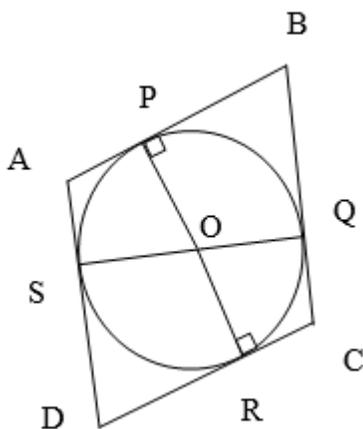
Now from equations (1) and (2),

$$2\angle COA + 2\angle COB = 180^\circ$$

$$(\angle COA + \angle COB) = 90^\circ$$

$$\angle AOB = 90^\circ$$

OR



Since, ABCD is a parallelogram,

$$AB = CD \quad (i)$$

$$BC = AD \quad (ii)$$

Now, it can be observed that:

$$DR = DS \quad \dots(\text{tangents on circle from point D})$$

$$CR = CQ \quad \dots(\text{tangents on circle from point C})$$

$$BP = BQ \quad \dots(\text{tangents on circle from point B})$$

$$AP = AS \quad \dots(\text{tangents on circle from point A})$$

Adding all the above four equations,

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$$CD + AB = AD + BC \quad \dots(iii)$$

From equation (i) (ii) and (ii):

$$2AB = 2BC$$

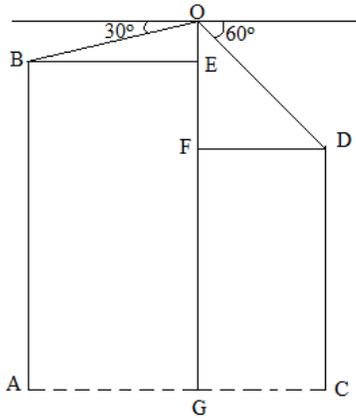
$$AB = BC$$

$$\therefore AB = BC = CD = DA$$

Hence, ABCD is a rhombus.

13.

- i) Let AB and CD represents buildings I and II respectively and O is the position of bird.

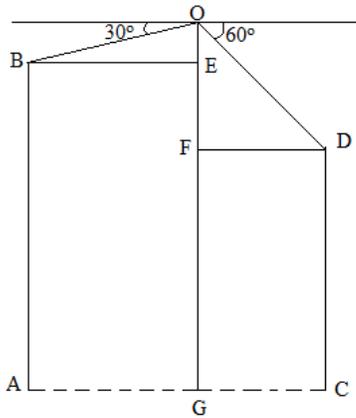


Distance between bird and ground = $OG = OE + EG$

$$OE = BE \times \tan(\angle OBE) = 71 \times \frac{1}{\sqrt{3}} = \frac{71}{\sqrt{3}} = 40.99 \text{ m}$$

$$\text{Therefore, } OG = 534 + 40.99 = 574.99 \text{ m}$$

ii)



$$BE = AC/2 = 71 \text{ m}$$

$$\cos(\angle OBE) = \frac{BE}{BO}$$

$$\Rightarrow \cos 30^\circ = \frac{71}{BO}$$

$$\Rightarrow BO = \frac{142}{\sqrt{3}} \text{ m}$$

14.

i) $a_3 = a + 2d = 4$ (i) and

$a_9 = a + 8d = -8$ (ii)

Subtracting (i) from (ii), we get

$$6d = -12 \Rightarrow d = -2$$

$$d = -2 \text{ and } a_3 = a + 2d = 4$$

$$\Rightarrow a = 4 - 2(-2) = 4 + 4 = 8$$

So, the AP is 8, 6, 4, 2, 0, -2,....

ii) Here, $a = 8$ and $d = -2$ and $a_n = -160$

$$\text{Also, } a_n = a + (n-1)d$$

$$\Rightarrow -160 = 8 + (n-1)(-2)$$

$$\Rightarrow -160 - 8 = -2n + 2$$

$$\Rightarrow n = 85$$