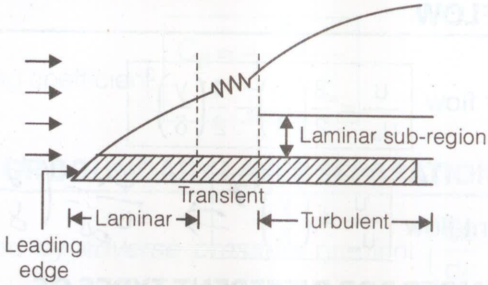
10. BOUNDARY LAYER THEORY

Boundary layer is a region in the immediate vicinity of the boundary surface in which the velocity of flowing ***fluid increases gradually*** from zero at the boundary surface to the velocity of the main stream.

DEVELOPMENT OF BOUNDARY LAYER REGION



VARIOUS TERMS ASSOCIATED WITH BOUNDARY LAYER THEORY

- **Boundary layer thickness (δ):** It is defined as the distance from the boundary surface in which the velocity reaches the 99% of the velocity of the main stream.

  $y = \delta$

  for $v = 0.99\ V_0$

- **Displacement Thickness (δ*):** It is the distance measured normal to the boundary, by which the solid boundary should be displaced in order to compensate for the reduction in mass flow rate due to boundary layer growth.

$$\delta^* = \int_o^\delta \left(1 - \frac{v}{V_0}\right) dy$$

The quantity $(V_0 - v)$ known as the velocity defect.

- **Momentum Thickness (θ)**

$$\theta = \int_o^\delta \frac{v}{V_0}\left(1 - \frac{v}{V_0}\right) dy$$

- **Energy Thickness (δ_e)**

$$\delta_e = \int_o^\delta \frac{v}{V_0}\left(1 - \frac{v^2}{V_0^2}\right) dy$$

Remember

- For $\frac{v}{V_0} = \frac{y}{\delta}$,

  $\delta^* = \frac{\delta}{2},\ \theta = \frac{\delta}{6},\ \delta_e = \frac{\delta}{4}$ thus $\delta^* > \delta_e > \theta$

- The ratio of displacement thickness to momentum thickness is called the ***shape factor*** (H). Its value should be always greater than 1.

GENERAL VELOCITY PROFILE FOR LAMINAR AND TURBULENT FLOW

- For laminar flow $\frac{u}{u_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

- For turbulent flow $\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$ ⇒ $\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$

REYNOLD NUMBER FOR DIFFERENT TYPES OF FLOW OVER FLAT PLATE

$R_e = \frac{\rho V x}{\mu}$

Here, x = Distance from where solid surface starts

$R_e < 5 \times 10^5$ – laminar flow

$R_e > 5 \times 10^5$ – turbulent flow

BLASSIUS EXPERIMENT RESULTS/WHEN VELOCITY PROFILE IS NOT GIVEN

Laminar	Turbulent
1. $\frac{\delta}{x} = \frac{5}{\sqrt{R_{ex}}}$	1. $\frac{\delta}{x} = \frac{0.576}{(R_{ex})^{1/5}}$
2. $C_{fx} = \frac{0.664}{\sqrt{R_{ex}}}$	2. $C_{fx} = \frac{0.059}{(R_{ex})^{1/5}}$
3. $C_d = \frac{1.328}{\sqrt{R_{el}}}$	3. $C_d = \frac{0.074}{(R_{el})^{1/5}}$

$\frac{\delta}{x} = \frac{0.3757}{(R_{ex})^{1/5}}$

For laminar flow $\delta \propto \sqrt{x}$ $\quad \tau \propto \frac{1}{\sqrt{x}}$

Here, δ = Boundary layer thickness

τ_o = Shear stress at solid surface

x = Distance from where solid surface starts

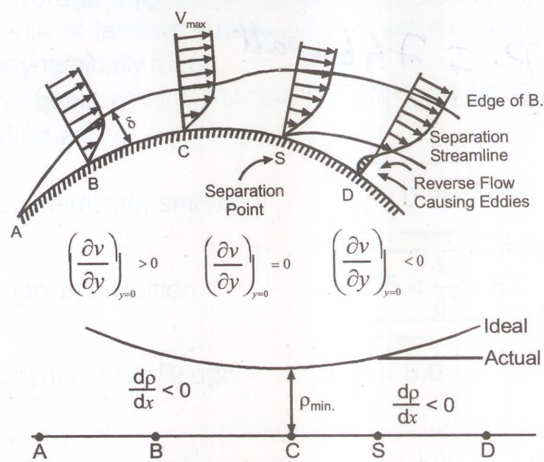
HARDNESS

Local skin friction coefficient $C_{fx} = \frac{\tau_0}{\frac{1}{2}\rho u_\infty^2}$

Average drag coefficient $C_d = \frac{F_d}{\frac{1}{2}\rho A U_\infty^2}$

CONDITION FOR BOUNDARY LAYER SEPARATION

It is caused by ***adverse pressure*** gradient $\left(\frac{dp}{dx} > 0\right)$



- **Location of Separation point**

The separation point S is determined from the condition $\left(\frac{\partial v}{\partial y}\right)_{y=0} = 0.$

For a given velocity it can be determined whether the B.L. has separated or on the verge of separation or will not separate from the following conditions:

- If $\left(\frac{\partial v}{\partial y}\right)_{y=0} < 0$: Flow has separated.

- If $\left(\frac{\partial v}{\partial y}\right)_{y=0} = 0$: Flow is on verge of separation.
- If $\left(\frac{\partial v}{\partial y}\right)_{y=0} > 0$: Flow is attached with the surface

METHODS OF PREVENTING SEPARATION

- Rotating the boundary in the direction of flow.
- Suction of the slow moving fluid by a suction slot.
- Supplying additional energy from a blower.
- Providing a bypass in the slotted wing.
- Providing guide blades in a bend.
- Injecting fluid into boundary layer.
- Streamlining of body shapes.

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