CBSE Class 11 Mathematics

Sample Papers 03 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part-B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. If A = [x : x $\in$ N, x is a factor of 12} and B = {x : x $\in$ N, x is a factor of 18}, find A$\cap$B.

OR

If A = {3, 6, 9, 12, 15, 18, 21}, B = {4, 8, 12,16, 20}, C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20}, find: D - A.

2. If a point lies in xy-plane then what is its z-coordinate?
3. Find the value of $\cot\left(\frac{29\pi}{4}\right)$

OR

Express the sum or difference of sines and cosines: 2 sin 4x sin 3x.

4. Express the complex numbers $(5i)\left(-\frac{3}{5}i\right)$ in the form a + ib.
5. How many 3-digit numbers are there when a digit may be repeated any numbers of time?

OR

Evaluate: $^{52}C_{52}$.

6. The third term of GP is 4. Find the product of its first 5 terms.
7. Reduce the equation y + 5 = 0 to slope-intercept form, and hence find the slope and the y-intercept of the line.

OR

Reduce the equation $\sqrt{3}x + y - 8 = 0$ into normal form. Find the values of p and ω.

8. Find the equation of the hyperbola with vertices at (0, ± 6) and $e = \frac{5}{3}$ Find its foci.
9. Is G = {0} null set?

OR

Express the set as an interval: B = {x : x ∈ R, 0 ≤ x < 3}

10. If A, B and C are three arbitrary events. Find the expression for the event, in the context of A, B and C. Two and no more occur.
11. Name the octants in which the following points lie

    (1, 2, 3), (4, -2, 3), (4, -2, -5), (-4, 2, -5), (-4, 2, 5), (-4, 2, 5), (-3, -1, 6), (2, -4, -7)
12. Evaluate: $^{8}P_{8}$

13. Prove that $\frac{\cos\theta}{\sin(90^\circ+\theta)}+\frac{\sin(-\theta)}{\sin(180^\circ+\theta)}-\frac{\tan(90^\circ+\theta)}{\cot\theta}=3$

14. If tan (A + B) = p and tan (A - B) = q, then write the value of tan 28.

15. Check that the plane $5x + 2y \leq 5$ contains origin or not.

16. Describe {x : x is positive integer and a divisor of 9} in Roster form.

Section - II

17. **Read the Case study given below and attempt any 4 sub parts:**

Father of Ashok is a builder, He planned a 12 story building in Gurgaon sector 5. For this, he bought a plot of 500 square yards at the rate of Rs 1000 /yard². The builder planned ground floor of 5 m height, first floor of 4.75 m and so on each floor is 0.25 m less than its previous floor.

Now Answer the following questions:

i. What is the height of the last floor?
   a. 2.5 m
   b. 2.75 m
   c. 2.25 m
   d. 3 m

ii. Which floor no is of 3 m height?
   a. 5
   b. 7
   c. 10
   d. 9

iii. What is the total height of the building?

a. 40 m
b. 43.5
c. 40.5 m
d. 44 m

iv. Up to which floor the height is 33 m?

a. 8
b. 7
c. 10
d. 9

v. Which floor no. is half in height of ground floor?

a. 10
b. 9
c. 12
d. 11

18. **Read the Case study given below and attempt any 4 subparts:**

A locker in a bank has 3 digit lock. Each place's digits may vary fro 0 to 9. Mahesh has a locker in the bank where he can put all his property papers and important documents. Once he needs one of the document but he forgot his password and was trying all possible combinations.

Read the above information and answer the following questions:

i. He took 6 seconds for each try. How much time will be needed by Mahesh to try all the combinations:

a. 90 minutes
b. 120 minutes
c. 60 minutes
d. 100 minutes

ii. How many permutations of 3 different digits are there, chosen from the ten digits 0 to 9 inclusive:

   a. 84
   b. 120
   c. 504
   d. 720

iii. The total number of 9-digit numbers that have all different digits is:

   a. 10!
   b. 9!
   c. $9 \times 9!$
   d. $10 \times 10!$

iv. A bank has 6 digit account number with no repetition of digits within an account number. The first and last digit of the account numbers is fixed to be 4 and 7. How many such account numbers are possible:

   a. 10080
   b. 5040
   c. 890
   d. 1680

v. If ${}^{n}C_{12} = {}^{n}C_{8}$, then n is equal to:

   a. 20
   b. 12
   c. 6
   d. 30

Part - B Section - III

19. Let A and B be two sets and U be the universal set such that n(A) = 25, n(B) = 28 and n(U) =50. Find

   i. the greatest value of $n(A \cup B)$
   ii. the least value of $n(A \cap B)$

20. Draw the graph of the function $f : R \to R$ Such that $f(x) = |x - 2|$.

OR

Let f and g be real function, defined by f(x) = $\sqrt{x+2}$ and g(x) = $\sqrt{4+x^2}$. Find: (ff)(x)

21. If α and β are different complex numbers such that $|\beta| = 1$, show that $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$.

22. Simplify 2(3 + 4i) + i(5 - 6i) and express it in the form a + ib.

23. Evaluate: $\left(i^{41} + \frac{1}{i^{257}}\right)^9$.

OR

Express $\frac{(2+3i)^2}{(2-i)}$ in the form (a + ib).

24. Two dice are thrown simultaneously. Find the probability of getting a doublet.

25. Evaluate: $\lim\limits_{x \to 2} \frac{x-2}{\sqrt[3]{x}-\sqrt[3]{2}}$.

26. Find the probability that the birth days of six different persons will fall in exactly two calender months.

27. Calculate the mean and standard deviation of first n natural numbers.

28. Prove that $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi+\theta)\ \left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right] = 1$

OR

If $\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos B} = q$, find tan A and tan B.

Section - IV

29. The mean and standard deviation of 20 observation are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in cases of it is replaced by 12 .

30. Find the domain and range of the real function f defined by $f(x) = \sqrt{x-1}$.

31. Insert three arithmetic means between 23 and 7.

OR

Insert six arithmetic means between 11 and -10

32. Find the equation of ellipse having Foci (± 3, 0), a = 4

33. Determine n if

(i) ${}^{2n}C_3 : {}^{n}C_2 = 12 : 1$

(ii) ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$

34. The points A (2, 3), B(4, -1) and C (-1, 2) are the vertices of $\triangle$ABC. Find the length of

perpendicular from C on AB and hence find the area of $\triangle$ABC.

OR

Find the points on the x-axis, where distances from the line $\frac{x}{3}+\frac{y}{4}=1$ are 4 units.

35. From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student have passed in at least one of the subject, 37 passed in Mathematics, 24 in Physics and 43 in Chemistry. At most 19 passed in Mathematics and Physics, at most 29 in Mathematics and Chemistry and at most 20 in Physics and Chemistry. Find the largest possible number that could have passed all three examinations.

Section - V

36. Evaluate: $\lim_{x\to 2}\frac{x^3+3x^2-9x-2}{x^3-x-6}$.

OR

Evaluate $\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}$

37. What is the fundamental difference between a relation and a function? Is every relation a function?

OR

Let $f=\left[\left(x,\frac{x^2}{1+x^2}\right):x\in R\right]$ be a function from R into R. Determine the range of f.

38. Solve for x, $|x+1|$ + $|x|$ > 3

OR

Solve the system of inequality graphically: x – 2y $\leq$ 3, 3x + 4y $\geq$ 12, x $\geq$ 0 , y $\geq$ 1

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Solution

Part - A Section - I

1. Here,it is given
   A = {x : x $\in$ N, x is a factor of 12} = {1, 2, 3, 4, 6, 12},
   B = {x : x $\in$ N, x is a factor of 18} = {1, 2, 3, 6, 9, 18}.
   $\therefore$ A $\cap$ B = {1, 2, 3, 4, 6, 12} $\cap$ {1, 2, 3, 6, 9, 18} = {1, 2, 3, 6}.

   OR

   Here A = {3, 6,9, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20}, C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20}
   D - A = {5,10,15,20} - {3,6,9,12,15,18,21}
   = {5,10,20}

2. If a point lies in XY-plane then its z-coordinate is 0.
3. Let y = $\cot\left(\frac{29\pi}{4}\right)$, then
   y = $\cot\left(\frac{29\pi}{4}\right) = \cot\left(7\pi + \frac{\pi}{4}\right)$
   $= \cot\frac{\pi}{4}\ [\because \cot(n\pi + \theta) = \cot\theta]$
   = 1

   OR

   Given: 2 sin 4x sin 3x
   $\because$ 2sin A sin B = cos (A - B) - cos (A + B)
   $\Rightarrow$ 2 sin 4x sin 3x = cos (4x - 3x) - cos (4x+ 3x)
   = cos x - cos 7x.

4. $(5i)\left(-\frac{3}{5}i\right) = -3i^2 = -3 \times -1(\because i^2 = -1)$
   = 3 = 3+0i
5. To make a 3 digit number,
   Let 1st box can be filled with nine numbers(1, 2, 3, 4, 5, 6, 7, 8, 9) if we include 0 in 1st box

then it become 2 digit number(i.e 010 is 2 digit number not 3 digit)

2nd box can be filled with ten numbers(1, 2, 3, 4, 5, 6, 7, 8, 9, 0) as repetition is allowed.

Similarly 3rd box can be filled with ten numbers(1, 2, 3, 4, 5, 6, 7, 8, 9, 0)

Total number of ways is $9 \times 10 \times 10 = 900$

OR

$^{52}C_{52} = 1$ [since, $^{n}C_{n} = 1$]

6. Let a be the first term and r be the common ratio.

   Given, third term = 4 i.e., $T_3 = 4 \Rightarrow ar^2 = 4$ ...(i)

   Now, product of first 5 terms

   $= a_1 \times a_2 \times a_3 \times a_4 \times a_5$

   $= a\,(ar)\,(ar^2)\,(ar^3)\,(ar^4) = a^5\,r^{10}$

   $= (ar^2)^5 = (4)^5$ [from Eq. (i)]

7. given that y + 5 = 0

   We can rewrite it as y = -5

   This equation is in the slope-intercept form, i.e. it is the form of $y = m \times x + c$, where m is the slope of the line and c is y-intercept of the line.

   Therefore, m = 0 and c = -5

   Conclusion: Slope is 0 and y-intercept is -5.

OR

We have,

$\sqrt{3}x + y - 8 = 0$ ... (1)

Dividing (1) by $\sqrt{(\sqrt{3})^2 + (1)^2} = 2$, we obtain

$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$ or $\cos 30^\circ x + \sin 30^\circ y = 4$ ....(2)

Comparing (2) with x cos ω + y sin ω = p, then we get p = 4 and $\omega = 30^\circ$.

8. Since the vertices are on the y-axes (with origin at the mid-point), the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

   As vertices are (0, ± 6), a = 6, $b^2 = a^2\,(e^2 - 1) = 36\,(\frac{25}{9} - 1) = 64$

   $\Rightarrow b = 8$

So the required equation of the hyperbola is $\frac{y^2}{36} - \frac{x^2}{64} = 1$ and the foci are (0, $\pm$ ae) = (0, $\pm$ 10).

9. We hace, 0 $\in$ G

   the set is not empty.

   $\therefore$ It is not a null set.

OR

The answer is B = [0, 3)

B = {x : x $\in$ R, 0 $\leq$ x < 3} is an open interval from 0 to 3, including 0 but excluding 3.

10. For two and no more occurs we have.$(A \cap B \cap \bar{C}) \cup (\bar{A} \cap B \cap C) \cup (A \cap \bar{B} \cap C)$.

11. Point (1, 2, 3) lies in Ist Octant.

    Point (4, - 2, 3) lies in IVth Octant. Point (4, -2, -5) lies in VIIIth Octant.

    Point (4, 2, -5) lies in Vth Octant. Point (-4, 2, -5) lies in VIth octant.

    Point (-4, 2, 5) lies in IInd Octant. Point (-3, -1, 6) lies in VIIIth octant.

    Point (2, -4, -7) lies in VIIIth Octant.

12. We have,

    ${}^{8}P_{8}$ = 8!

    = $(8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)$

    = 40320

13. Take L.H.S we have

    L.H.S = $\frac{\cos\theta}{\cos(90^\circ+\theta)} + \frac{\sin(-\theta)}{\sin(180^\circ+\theta)} - \frac{\tan(90^\circ+\theta)}{\cot\theta}$

    $= \frac{\cos\theta}{\cos\theta} + \frac{-\sin\theta}{-\sin\theta} - \frac{-\cot\theta}{\cot\theta}$

    = 1 + (1) + (1) $\Rightarrow$ 1 + 1 + 1 = 3

14. It is given that

    tan(A + B) = p and tan(A - B) = q

    Let 2B = tan(B + B)

    = tan(A + B -(A - B))

    $= \frac{tan(A+B)-tan(A-B)}{1+tan(A+B)tan(A-B)}$

    $= \frac{p-q}{1+pq}$ [$\because$ tan(A + B) = p and tan(A - B) = q]

15. We have, 5x + 2y $\leq$ 5

On putting x = y = 0, we get 5(0) + 2(0) $\leq$ 5

$\Rightarrow 0 \leq 5$, which is true.

$\therefore$ The Given plane contains the origin.

16. Since x is a positive integer and a divisor of 9.

So, x can take possible values as 1, 3, 9.Therefore,we have,

{x: x is a positive integer and a divisor of 9} = {1, 3, 9},which is the required roster form

Section - II

17. i. (c) 2.25 m
    ii. (d) 9
    iii. (b) 43.5
    iv. (a) 8 m
    v. (d) 11
18. i. (d) 100 minutes
    ii. (d) 720
    iii. (c) $9 \times 9!$
    iv. (d) 1680
    v. (a) 20

Part - B Section - III

19. i. We know that

$$A \cup B \subset U$$
$$\Rightarrow n(A \cup B) \leq n(U)$$
$$\Rightarrow n(A \cup B) \leq 50$$

Hence, the greatest value of n(A $\cup$ B) is 50.

ii. From equation (i), we have

$$n(A \cup B) \leq 50$$

We know that, n(A$\cup$B) = n(A) + n(B) − n(A$\cap$B)

$$\Rightarrow n(A) + n(B) - n(A \cap B) \leq 50$$
$$\Rightarrow 25 + 28 - n(A \cap B) \leq 50$$
$$\Rightarrow n(A \cap B) \geq 3$$

Therefore, the least value of n(A$\cup$B) is 3.

20. Clearly $y = |x-2| = \begin{cases} x-2, & \text{if } x-2 \geq 0 \\ -(x-2), & \text{if } x-2 < 0 \end{cases}$

$$= \begin{cases} x-2, & \text{if } x \geq 2 \\ 2-x, & \text{if } x < 2 \end{cases}$$

We know that a linear equation in x and y represents a line. For drawing a line, we need only two points.

For $y = x - 2$

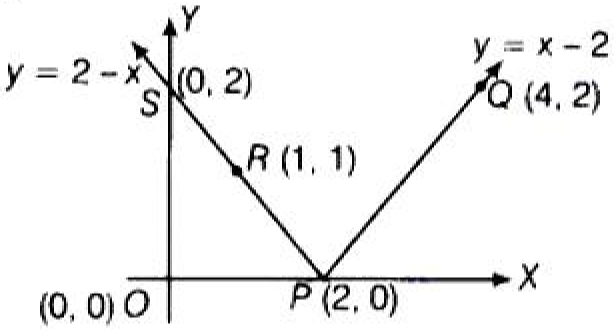
x	2	4
y	0	2

So, plot the points $P(2,0), Q(4,2)$ and join PQ to get the graph of $y = x - 2$.

For $y = 2 - x$

x	1	0	2
y	1	2	0

Plot the points $R(1,1), S(0,2)$ and $P > (2,0)$ and join RS to get the graph of $y = 2 - x$.



OR

Here we have, f(x) = $\sqrt{x+2}$ and g(x) = $\sqrt{4+x^2}$

Clearly, f(x) = $\sqrt{x+2}$ is defined for all x $\in$ R such that x + 2 $\geq$ 0, i.e., x $\geq$ - 2

$\therefore$ dom (f) = [-2, ∞).

Again, g(x) = $\sqrt{4+x^2}$ is defined for all x $\in$ R such that 4 - $x^2 \geq 0$

But, 4 - $x^2 \geq 0 \Rightarrow x^2 - 4 \leq 0 \Rightarrow$ (x + 2)(x - 2) $\leq 0 \Rightarrow$ x $\in$ [-2, 2]

$\therefore$ dom (g) = [-2, 2).

$\therefore$ dom (f) $\cap$ dom (g) = [-2, ∞) $\cap$ [-2, 2] = [-2, 2]

(ff) : [-2, 2] $\rightarrow$ R is given by

(ff)(x) = f(x) $\times$ g(x) = $\left(\sqrt{x+2}\right)\left(\sqrt{x+2}\right)$ = (x + 2)

21. We have

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2=\left\{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\}\left\{\overline{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}}\right\}\left[\because |z|^2=z\bar{z}\right]$$

$$=\frac{(\beta-\alpha)}{(1-\bar{\alpha}\beta)}\cdot\frac{(\bar{\beta}-\bar{\alpha})}{(1-\alpha\bar{\beta})}=\frac{(\beta-\alpha)(\bar{\beta}-\bar{\alpha})}{(1-\bar{\alpha}\beta)(1-\alpha\bar{\beta})}$$

$$=\frac{\beta\bar{\beta}+\alpha\bar{\alpha}-\beta\bar{\alpha}-\alpha\bar{\beta}}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+(\alpha\bar{\alpha})(\beta\bar{\beta})}$$

$$=\frac{|\beta|^2+|\alpha|^2-(\alpha\bar{\beta}+\beta\bar{\alpha})}{1-(\alpha\bar{\beta}+\beta\bar{\alpha})+|\alpha|^2|\beta|^2}\left[\because \alpha\bar{\alpha}=|\alpha|^2,\beta\bar{\beta}=|\beta|^2\right]$$

$$=\frac{1+|\alpha|^2-(\alpha\bar{\beta}+\beta\bar{\alpha})}{1+|\alpha|^2-(\alpha\bar{\beta}+\beta\bar{\alpha})}=1\ [\because |\beta|=1]$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=1$$

Hence Proved

22. Given that, 2(3 + 4i) + i(5 – 6i)

Firstly, we open the brackets

= (2 × 3) + (2 × 4i) + (i × 5) – (i × 6i)

= 6 + 8i + 5i – $6i^2$

= 6 + 13i – 6(-1) [$\because i^2 = -1$]

= 6 + 13i + 6

= 12 + 13i

23. $\left(i^{41}+\frac{1}{i^{257}}\right)^9=\left(i^{4\times10+1}+\frac{1}{i^{4\times64+1}}\right)^9$

= $\left[\left(i^4\right)^{10}\times i+\frac{1}{\left(i^4\right)^{64}\times i}\right]^9$

= $\left(i+\frac{1}{i}\right)^9\ (\because i^4=1)$

= $\left(i+\frac{i}{i^2}\right)^9$

= $(i - i)^9$ ($\because i^2 = -1$)

= 0

OR

Let z = $\frac{(2+3i)^2}{(2-i)}$

Now, we rationalize the above equation by multiply and divide by the conjugate of (2 - i)

$$=\frac{(2+3i)^2}{(2-i)}\times\frac{(2+i)}{(2+i)}$$

$$=\frac{(2+3i)^2(2+i)}{(2-i)(2+i)}$$

$= \frac{(4+9i^2+12i)(2+i)}{(2)^2-(i)^2}$ [$\because$ (a + b)(a - b) = (a^2 - b^2)]

$= \frac{[4+9(-1)+12i](2+i)}{4-i^2}\left[\because i^2 = -1\right]$

$= \frac{[4-9+12i](2+i)}{4-(-1)}$

$= \frac{(-5+12i)(2+i)}{5}$

$= \frac{-10-5i+24i+12i^2}{5}$

$= \frac{-10+19i+12(-1)}{5}$

$= \frac{-10-12+19i}{5}$

$= \frac{-10-12+19i}{5}$

$= \frac{-22+19i}{5}$

$z = -\frac{22}{5} + \frac{19}{5}i$

24. We know that in a single throw of two dice, the total number of possible outcomes is (6 × 6) = 36.

Let S be the sample space of the event is given by

n(S) = 36.

Let E_1 = event of getting a doublet. Then,

E_1 = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}.

$\therefore$ n(E_1) = 6

$\therefore$ P(E_1) = $\frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

25. We have to find $\lim\limits_{x\to 2} \frac{x-2}{\sqrt[3]{x}-\sqrt[3]{2}}$

We have,

$$\lim_{x\to 2} \frac{x-2}{x^{\frac{1}{3}}-2^{\frac{1}{3}}} = \frac{1}{\lim\limits_{x\to 2} \frac{x^{\frac{1}{3}}-2^{\frac{1}{3}}}{x-2}} = \frac{1}{\frac{1}{3}\left(2^{\frac{1}{3-1}}\right)} = \frac{1}{\frac{1}{3}\times\left(2^{\frac{-2}{3}}\right)} = 3\left(2^{\frac{2}{3}}\right)$$

26. We have to find the probability that the birth days of six different persons will fall in exactly two calender months

Since each person can have his birthday in any one of the 12 calendar months.

So, there are 12 options for each person.

$\therefore$ Total number of ways in which 6 persons can have their birthdays = 12 × 12 × 12 × 12 × 12 × 12 = 12^6

Out of 12 months, 2 months can be chosen in ${}^{12}C_2$ ways.

Now, birthdays of six persons can fall in these two months in 2^6 ways.

Out of these 2^6 ways, there are two ways when all six birthdays fall in one month.

So, there are (2^6 - 2) ways in which six birthdays fall in the chosen 2 months.

$\therefore$ Number of ways in which six birthdays fall in exactly two calendar month = ${}^{12}C_2 \times (2^6$ - 2)

Hence, required probability = $\frac{12C_2 \times (2^6 - 2)}{12^6} = \frac{341}{12^5}$

27. Here x_i = i where i = 1, 2, ..., n.

Let $\overline{X}$ be the mean and σ be the S.D. Then,

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}\sum_{i=1}^{n} i = \frac{1}{n}(1+2+3+\ldots+n) = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

Now, $\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n} x_i^2\right) - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = \frac{1}{n}\left(\sum_{i=1}^{n} x_i^2\right) - (\bar{X})^2$

$$\Rightarrow \quad \sigma^2 = \frac{1}{n}\left(1^2+2^2+\ldots+n^2\right) - \left(\frac{n+1}{2}\right)^2$$

$$\Rightarrow \quad \sigma^2 = \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2 = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = \frac{n^2-1}{12}$$

$$\therefore \quad \text{Mean } = \frac{n+1}{2} \text{ and } S.D. = \sqrt{\frac{n^2-1}{12}}.$$

28. To prove: $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right] = 1$

We have

L.H.S = $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right]$

$= \cos\left[\pi+\left(\frac{\pi}{2}+\theta\right)\right]\cos(2\pi+\theta)\left[\cot\left\{\pi+\left(\frac{\pi}{2}-\theta\right)\right\}+\cot(2\pi+\theta)\right]$

$= -\cos\left(\frac{\pi}{2}+\theta\right)\cos\theta\left[\cot\left(\frac{\pi}{2}-\theta\right)+\cot\theta\right]$

$[\because \cos(\pi+x) = -\cos x, \cot(\pi+x)$ = cot x and $\cot(2\pi+x) = \cot x]$

$= \sin\theta\cos\theta[\tan\theta+\cot\theta]\left[\because \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta\right]$

$= (\sin\theta\cos\theta)\left[\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right] = (\sin\theta\cos\theta)\cdot\frac{(\sin^2\theta+\cos^2\theta)}{(\sin\theta\cos\theta)} = 1$

OR

We have,

$\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos B} = q$

$\Rightarrow \frac{\sin A}{\sin B}\cdot\frac{\cos B}{\cos A} = \frac{p}{q}$

$\Rightarrow \frac{\tan A}{\tan B} = \frac{p}{q} \Rightarrow \frac{\tan A}{p} = \frac{\tan B}{q} = \lambda(\text{say})$

$\Rightarrow \tan A = p\lambda$ and $\tan B = q\lambda$ ... (i)

Now sin A = p sin B

$\Rightarrow \frac{\tan A}{\sqrt{1+\tan^2 A}} = p\frac{\tan B}{\sqrt{1+\tan^2 B}}$

$\Rightarrow \frac{p\lambda}{\sqrt{1+p^2\lambda^2}} = p\frac{q\lambda}{\sqrt{1+q^2\lambda^2}}$

$\Rightarrow p^2\left(1+q^2\lambda^2\right) = p^2q^2\left(1+p^2\lambda^2\right)$

$\Rightarrow \lambda^2\left(q^2-p^2q^2\right) = q^2-1 \Rightarrow \lambda^2 = \frac{q^2-1}{q^2(1-p^2)} \Rightarrow \lambda = \pm\frac{1}{q}\sqrt{\frac{q^2-1}{1-p^2}}$

$\therefore \tan \text{A} = \pm\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$ and, $\tan \text{B} = \pm\sqrt{\frac{q^2-1}{1-p^2}}$ [Using (i)].

Section - IV

29. Here we are given that, n = 20, $\bar{x} = 10$ and $\sigma = 2$

$\therefore\ \bar{x} = \frac{1}{n}\Sigma x_i \Rightarrow n \times \bar{x} = \Sigma x_i$

$\Rightarrow \Sigma x_i = 20 \times 10 = 200$

Therefore Incorrect $\Sigma x_i = 200$

Now $\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2 = \sigma^2$

$\Rightarrow \frac{1}{20}\Sigma x_i^2 - (10)^2 = 4 \Rightarrow \Sigma x_i^2 = 2080$

If it is replaced by 12,

When wrong item 8 is replaced by 12

Therefore, Correct Σx_i = Incorrect Σx_i - 8 + 12

= 200 - 8 + 12 = 204

$\therefore$ Correct mean = $\frac{204}{20}$ = 10.2

Also correct Σx_i^2 = Incorrect Σx_i^2 - $(8)^2$ + $(12)^2$

= 2080 - 64 + 144 = 2160

$\therefore$ Correct variance $= \frac{1}{20}(correct\ \Sigma x_1^2)$ - (correct mean)^2

$= \frac{2160}{20} - \left(\frac{204}{20}\right)^2$

$= \frac{2160}{20} - \frac{41616}{400} = \frac{43200-41616}{400} = \frac{1584}{400}$

Correct S.D. $= \sqrt{\frac{1584}{400}} = \sqrt{3.96}$ = 1.989

30. Here f (x) = $\sqrt{x-1}$, f (x) assumes real values if $x-1 \geqslant 0 \Rightarrow x \geqslant 1$

$\Rightarrow x \in [1,\infty) \Rightarrow x \in [1,\infty)$

$\therefore$ Domain of f(x) = $[1,\infty)$

For $x \geqslant 1, f(x) \geqslant 0$

Range of f (x) = all real numbers $\geqslant 0$

$= [0, \infty)$

31. We have to find: Three arithmetic means between 23 and 7

Formula used: (i) d = $\frac{b-a}{n+1}$, where, d is the common difference

n is the number of arithmetic means

(ii) A_n = a + nd

We have 23 and 7

We know that, d = $\frac{b-a}{n+1}$

d = $\frac{7-23}{3+1}$

d = $\frac{-16}{4}$

d = -4

We know that, A_n = a + nd

First arithmetic mean, A_1 = a + d

= 23 + (-4) = 19

Second arithmetic mean, A_2 = a + 2d

= 23 + 2(-4)

= 23 + (-8) = 15

Third arithmetic mean, A_3 = a + 3d

= 23 + 3(-4)

= 23 + (-12) = 11

Therefore, the three arithmetic means between 23 and 7 are 19, 15 and 11

OR

We have to find: Three arithmetic means between 23 and 7

Formula used:

i. d = $\frac{b-a}{n+1}$, where, d is the common difference
   n is the number of arithmetic means
ii. A_n = a + nd

We have 11 and -10

We know that, d = $\frac{b-a}{n+1}$

d = $\frac{-10-(11)}{6+1}$

d = $\frac{-21}{7}$

d = -3

We know that, A_n = a + nd

First arithmetic mean, A_1 = a + d

= 11 + (-3) = 8

Second arithmetic mean, A_2 = a + 2d

= 11 + 2(-3)

= 11 + (-6) = 5

Third arithmetic mean, A_3 = a + 3d

= 11 + 3(-3)

= 11 + (-9) = 2

Third arithmetic mean, A_4 = a + 4d

= 11 + 4(-3)

= 11 + (-12) = -1

Third arithmetic mean, A_5 = a + 5d

= 11 + 5(-3)

= 11 + (-15) = -4

Third arithmetic mean, A_6 = a + 6d

= 11 + 6(-3)

= 11 + (-18) = -7

Therefore,the three arithmetic means between 11 and -10 are 8, 5, 2, -1, -4 and -7

32. The foci (±3, 0) lie on x-axis

So the equation of ellipse in standard form is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Now foci (± c, 0) is (±3, 0) ⇒ c = 3

We know that $c^2 = a^2 - b^2$

$\therefore (3)^2 = (4)^2 - b^2 \Rightarrow b^2 = 16 - 9 = 7$

Thus equation of required ellipse is

$\frac{x^2}{16} + \frac{y^2}{7} = 1$

33. Here ${}^{2n}C_3 : {}^{n}C_2 = 12 : 1$

$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{2!(n-2)!}{n!} = \frac{12}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3\times 2!(2n-3)!} \times \frac{2!(n-2)!}{n(n-1)(n-2)!} = \frac{12}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = \frac{12}{1}.$

$\Rightarrow \frac{4(2n-1)}{3} = \frac{12}{1} \Rightarrow 8n-4 = 36 \Rightarrow n = 5$

(ii) Here ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$

$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{11}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$

$\Rightarrow \frac{4(2n-1)}{n-2} = \frac{11}{1} \Rightarrow 8n-4 = 11n-22$

$\Rightarrow$ 3n = 18 $\Rightarrow$ n = 6

34. Given: points A(2, 3), B(4, -1) and C(-1, 2) are the vertices of $\triangle$ABC

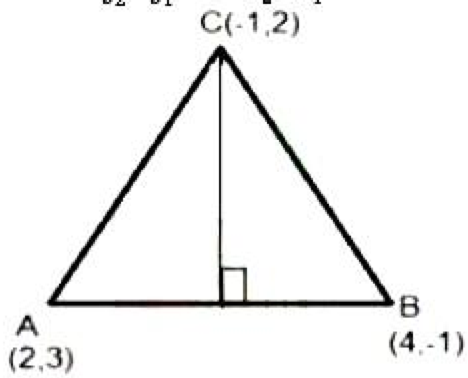
We have to find: length of the perpendicular from C on AB and the area of $\triangle$ABC

We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$

The equation of the line joining the points (x_1, y_1) and (x_2, y_2) is given by

$$\Longrightarrow \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$



The equation of the line joining the points A(2,3) and B(4,-1) is

Here,we have $x_1 = 2 y_1 = 3$ and $x_2 = 4 y_2 = -1$

$\Longrightarrow \frac{y-3}{x-2} = \frac{-1-3}{4-2} = \frac{-4}{2} = -2$

$\Longrightarrow$ y - 3 = -2x + 4

$\Longrightarrow$ 2x + y - 7 = 0

The equation of the line 2x + y - 7 = 0

The length of perpendicular from c(-1, 2) to the line AB.

The given equation of the line is 2x + y – 7 =0

Here ,we have m= -1 and n = 2, a = 2, b = 1, c = -7

$D = \frac{|2(-1)+1(2)-7|}{\sqrt{2^2+1^2}}$

$D = \frac{-2+2-7}{\sqrt{4+1}} = \frac{|-7|}{\sqrt{5}} = \frac{|-7|}{\sqrt{5}} = \frac{7}{\sqrt{5}}$

$D = \frac{7}{\sqrt{5}}$

The length of the perpendicular from C on AB is $\frac{7}{\sqrt{5}}$ units.

Height of the triangle is $\frac{7}{\sqrt{5}}$ units.

The distance between points A(x_1, y_1) and B(x_2, y_2) is given by

$\text{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here,we have x_1 = 2 and y_1 = 3, x_2 = 4 and y_2 = -1

$\text{AB} = \sqrt{(4-2)^2 + (-1-3)^2} = \sqrt{2^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$\text{Base AB} = 2\sqrt{5}$ units

Area of the triangle $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 2\sqrt{5} \times \frac{7}{\sqrt{5}} = 7$

Area of the triangle ABC = 7 square units.

OR

Let the coordinates of the point on x-axis be $(\alpha, 0)$.

∴ Perpendicular distance of the point $(\alpha, 0)$ from the line 4x + 3y - 12 = 0 is

$\left|\frac{4\alpha+3(0)-12}{\sqrt{(4)^2+(3)^2}}\right| = \left|\frac{4\alpha-12}{5}\right|$

It is given that $\left|\frac{4\alpha-12}{5}\right| = 4$

$\Rightarrow \frac{4\alpha-12}{5} = \pm 4$

Now $\frac{4\alpha-12}{5} = 4$ or $\frac{4\alpha-12}{5} = -4$

$\Rightarrow 4\alpha - 12 = 20$ or $4\alpha - 12 = -20$

$\Rightarrow 4\alpha = 32$ or $4\alpha = -8$

$\Rightarrow \alpha = 8$ or $\alpha = -2$

Thus the points on x-axis are (8, 0) and (-2, 0)

35. Suppose M be the set of students passing in Mathematics, P be the set of students passing in Physics and C be the set of students passing in Chemistry.

Given, $n(M \cup P \cup C) = 50$

$n(M) = 37$

$n(P) = 24$

$n(C) = 43$

$n(M \cap P) \leq 19$

$n(M \cap C) \leq 29$

$n(P \cap C) \leq 20$

Now, $n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C)$

$-n(P \cap C) + n(M \cap P \cap C) \leq 50$

$\Rightarrow 37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C) \leq 50$

$\Rightarrow \quad n(M \cap P \cap C) \leq 50 - 36$

$\Rightarrow \quad n(M \cap P \cap C) \leq 14$

Thus, the largest possible number that could have passed all the three examinations is 14.

Section - V

36. We have to find the value of $\lim\limits_{x \to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6}$

We have

$\lim\limits_{x \to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6}$

Divide $x^3 + 3x^2 - 9x - 2$ by $x^3 - x - 6$

```
                    1
x^3 - x - 6 ) x^3 + 3x^2 - 9x - 2
              +x^3       -x   -6
               -          +    +
              ---------------------
                   3x^2 - 8x + 4
```

$\Rightarrow \quad \lim\limits_{x \to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6} = \lim\limits_{x \to 2} 1 + \lim\limits_{x \to 2} \frac{3x^2-8x+4}{x^3-x-6}$

$= 1 + \lim\limits_{x \to 2} \frac{3x^2-2x-6x+4}{x^3-x-6}$

$= 1 + \lim\limits_{x \to 2} \frac{3x^2-2x-6x+4}{x^3-x-6}$

$\Rightarrow \lim\limits_{x \to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6} = 1 + \lim\limits_{x \to 2} \frac{(3x-2)(x-2)}{x^3-x-6}$

Divide $x^3 - x - 6$ by $x - 2$

$$
\begin{array}{r}
x^2 + 2x + 3 \\
x - 2 \,\overline{)\, x^3 - x - 6} \\
\underline{\pm x^3 - 2x^2} \\
\overset{+}{} \\
2x^2 - x - 6 \\
\underline{2x^2 \mp 4x} \\
3x - 6 \\
\underline{3x - 6} \\
\times
\end{array}
$$

$\Rightarrow \quad \lim\limits_{x \to 2} \frac{x^3+3x^2-9x-2}{x^3-x-6} = 1 + \lim\limits_{x \to 2} \frac{(3x-2)(x-2)}{(x-2)(x^2+2x+3)}$

$= 1 + \lim\limits_{x \to 2} \frac{(3x-2)}{(x^2+2x+3)}$

$= 1 + \frac{3\times 2-2}{2^2+2\times 2+3}$

$=1 + \frac{4}{11}$

$= \frac{15}{11}$

OR

To evaluate: $\lim\limits_{h \to 0} \frac{1}{h}\left\{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right\}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} g(x) = 0$ or $\pm\infty$ then

$\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$

As x $\to$ 0, we have

$\lim\limits_{h \to 0} \frac{1}{h}\left\{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right\} = \frac{0}{0}$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$\lim\limits_{h \to 0} \frac{1}{h}\left\{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right\} = \lim\limits_{h \to 0} \frac{\frac{d}{dh}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)}{\frac{d}{dh}(h)}$

$\lim\limits_{h \to 0} \frac{1}{h}\left\{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right\} = \lim\limits_{h \to 0} \frac{\frac{-1}{2\sqrt{x+h}} + \frac{1}{2\sqrt{x}}}{1} = \frac{-\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}}{1}$

$\lim_{h \to 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\} = 0$

37. Fundamental difference between Relation and Function:

Every function is a relation, but every relation need not be a function.

A relation f from A to B is called a function if

i. Dom(f) = A
ii. no two different ordered pairs in f have the same first component.

For. e.g. Let A = {a, b, c, d} and B = {1, 2, 3, 4, 5}

Some relations f, g and h are defined as follows: f = {(a, 1),(b, 2),(c, 3),(d, 4)}

g = {(a, 1),(b, 3),(c, 5)}

h = {(a, 1),(b, 2),(b, 3),(c, 4),(d, 5)}

In the relation f

f = {(a, 1),(b, 2),(c, 3),(d, 4)}

i. Dom (f) = A
ii. All first components are different.

So, f is a function.

In the relation g

1. Dom (g) $\neq$ A

So, the condition is not satisfied. Thus, g is not a function.

In the relation h

h = {(a, 1),(b, 2),(b, 3),(c, 4),(d, 5)}

i. Dom (h) = A
ii. Two first components are the same, i.e. b has two different images.

So, h is not a function.

No, every relation is not a function

OR

Here $f(x) = \frac{x^2}{1+x^2}$

Put $y = \frac{x^2}{1+x^2} \Rightarrow y + yx^2 = x^2 \Rightarrow x^2(1-y) = y$

$\Rightarrow x^2 = \frac{y}{1-y} \Rightarrow x = \pm\sqrt{\frac{y}{1-y}}$

$\frac{y}{1-y} \geqslant 0$

$\Rightarrow \frac{y}{y-1} \leqslant 0$

$\Rightarrow 0 \leqslant y < 1$

$\Rightarrow y \in [0, 1)$

$\therefore$ Range of f(x) = [0, 1)

38. We have, $|x+1| + |x| > 3$

Put x + 1 = 0 and x = 0 $\Rightarrow$ x = - 1 and x = 0

$\therefore$ x = - 1, 0 are critical point.

So, we will consider three intervals $(-\infty, -1)$, $[-1, 0)$, $[0, \infty)$

Case I: When $-\infty < x < -1$, then $|x+1| = -(x+1)$ and $|x| = -x$

$\therefore |x+1| + |x| > 3$

$\Rightarrow -x - 1 - x > 3$

$\Rightarrow -2x - 1 > 3$

$\Rightarrow -2x > 4$

$\Rightarrow x < -2$

Case II: When $-1 \leq x < 0$, then $|x+1| = x + 1$ and $|x| = -x$

$\therefore |x+1| + |x| > 3$

$\Rightarrow x + 1 - x > 3 \Rightarrow 1 > 3$ [not possible]

Case III: When $0 \leq x < \infty$, then $|x+1| = x + 1$ and $|x| = x$

$\therefore |x+1| + |x| > 3$

$\Rightarrow x + 1 + x > 3$

$\Rightarrow 2x + 1 > 3 \Rightarrow 2x > 2$

$\therefore x > 1$

On combining the results of cases I, II and III, we get

$x < -2$ and $x > 1$

$\therefore x \in (-\infty, -2) \cup (1, \infty)$

OR

The given inequality is $x - 2y \leqslant 3$

Draw the graph of the line x - 2y = 3

Table of values satisfying the equation x - 2y = 3

X	1	5
Y	-1	1

Putting (0, 0) in the given inequation, we have

$0 - 2 \times 0 \leqslant 3 \Rightarrow 0 \leqslant 3$, which is true.

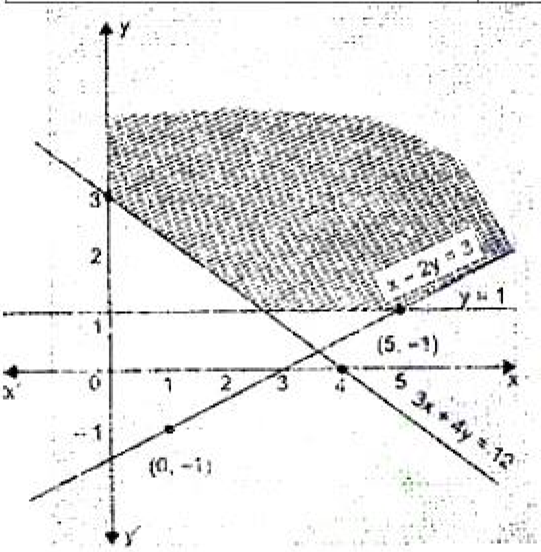
$\therefore$ Half plane of $x - 2y \leqslant 3$ is towards origin.

Also the given inequality is $3x + 4y \geqslant 12$

Draw the graph of the line 3x + 4y = 12

Table of values satisfying the equation 3x + 4y = 12

X	4	0
Y	0	3



Putting (0, 0) in the given inequation, we have

$3 \times 0 + 4 \times 0 \geqslant 12 \Rightarrow 0 \geqslant 12$, which is false.

$\therefore$ Half plane of $3x + 4y \geqslant 2$ is away from origin.

The given inequality is $y \geqslant 1$.

Draw the graph of the line y = 1.

Putting (0, 0) in the given inequation, we have

$0 \geqslant 1$, which is false.

$\therefore$ Half plane of $y \geqslant 1$ is away from origin.