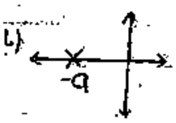
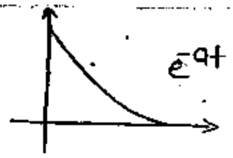
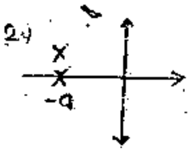
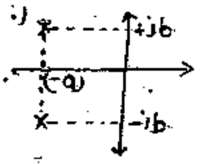
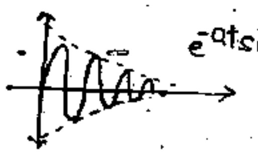
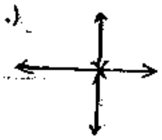
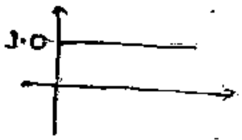
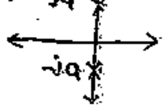
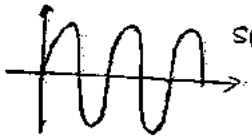
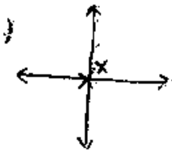
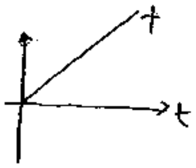
Chapter-03
Stability

* The stability of LTI system may be defined as when the sys. is subjected to bounded i/p the o/p should be bounded.
* BIBO implies the IR of the sys. should tend to zero as time t approaches ∞.
* The stability of a sys. depands on roots of the c/s eq^n

$$1+G(s)H(s)=0$$

i.e. closed loop poles.

* IR & stability →

Closed loop pole loc^n	Stability Criteria	Impulse Responce
1)  $F(s)=\frac{1}{s+a}$	absolutely stable	 e^{-at}
2)  $F(s)=\frac{1}{(s+a)^2}$	absolutely stable	 te^{-at}
3)  $F(s)=\frac{1}{(s+a)^2+b^2}$	absolutely stable	 $e^{-at}\sin bt$
4)  $F(s)=\frac{1}{s}$	marginaly stable/ criticaly stable	
5)  $F(s)=\frac{1}{s^2+a^2}$	marginaly/criticaly	 $\sin at$
6)  $F(s)=\frac{1}{s^2}$	Unstable	

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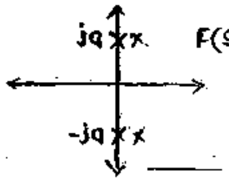
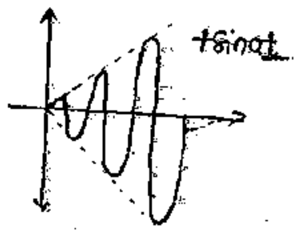


$F(s) = \frac{1}{(s^2+a^2)^2}$ Unstable



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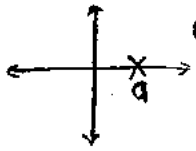
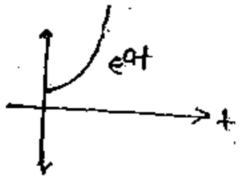


$F(s) = \frac{1}{(s-a)}$ Unstabe



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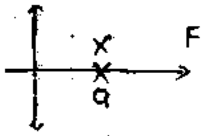
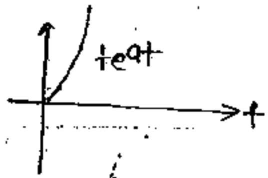


$F(s) = \frac{1}{(s-a)^2}$ Unstable



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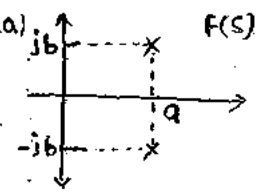
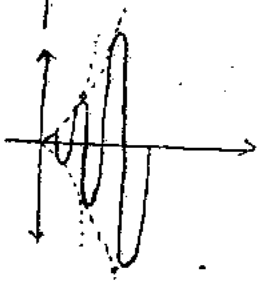


$F(s) = \frac{1}{(s-a)^2+b^2}$ Unstable



Routh-Hurwitz Criteria →

(2/5) $P(s) = s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$

ROUTH ARRAY =

s^4	1	18	5
s^3	8	16	0
s^2	$b_1 = 16$	$b_2 = 5$	0
s	$c_1 = 13.5$	0	0
s^0	$d_1 = 5$	0	0

$b_1 = \frac{8\times18 - 1\times16}{8} = 16$

$b_2 = \frac{8\times5 - 1\times0}{8} = 5$

$c_1 = \frac{16\times16 - 8\times5}{16} = 13.5$

$d_1 = \frac{13.5\times5 - 16\times0}{13.5} = 5$

(3/5) $P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$

s^5	1	2	3
s^4	1	2	15
s^3	$\cancel{0}\ \epsilon$	-12	0
s^2	$\frac{2\epsilon+12}{\epsilon}$	15	0
s^1	$\frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12}$	0	
s^0	15		0

To check for sign changes

(1.) $\lim_{\epsilon\to 0} \frac{2\epsilon+12}{\epsilon} = \frac{2(0)+12}{0} = +\infty$

(2.) $\lim_{\epsilon\to 0} \frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12} = \frac{-144}{12} = -12$

2 sign changes: $+\infty \to -12$.
$-12 \to +15$

2 poles are in RHS

Difficulty-1 → When the 1st element of any row is 0 while the rest of row has atleast one non-zero term then is such cases substitute $+\epsilon$ (small +ve no) in place of zero & evaluate the rest of RA (Routh array) in terms of ϵ.

∴ Check for sign changes by taking $\lim_{\epsilon\to 0}$ for the 1st column elements to comment on stability.

(4/65) $P(s) = s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

s^6	1	8	20	16
s^5	2	12	16	20
s^4	2	12	16	0
s^3	$\cancel{0}$ 8	$\cancel{0}$ 24	0	0
s^2	6	16	0	0
s^1	2.6	0	0	0
s^0	16	0	0	0

(i) Construct an auxiliary eq^n $A(s)$

$A(s) = 2s^4 + 12s^2 + 16$

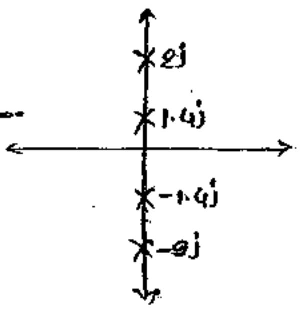
(ii) $\frac{d}{ds}A(s) = \cancel{8s^3+36s^2+16}\ 8s^3 + 24s$

$\boxed{\text{The roots of } A(s) = \text{poles symmetric about origin}}$

$\frac{-12 \pm \sqrt{144-8\times 16}}{4} = -2, -4$

$(s^2+2)(s^2+4) = 0$

$s = \pm j1.4, \pm j2$



$\left(\frac{5}{65}\right)$

s^5	2	4	2
s^4	1	2	1
s^3	$\cancel{0}\,4$	$\cancel{0}\,4$	0
s^2	1	1	0
s^1	$\cancel{0}\,2$	0	0
s^0	1	0	0

$A_1(s) = s^4 + 2s^2 + 1$

$\frac{d}{ds}A_1(s) = 4s^3 + 4s$

$A_2(s) = s^2 + 1$

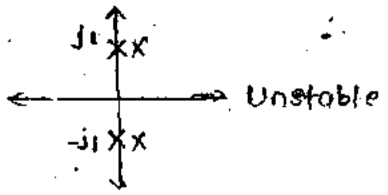
$\frac{d}{ds}A_2(s) = 2s$

The roots of $A_1(s)$

$$\frac{-2 \pm \sqrt{4-4}}{2} = -1, -1$$

$(s^2+1)(s^2+1) = 0$

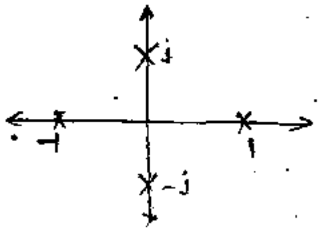
$s = \pm j, \pm j$



Poles symmetric about origin

$(s^2+1)(s^2-1) = 0$

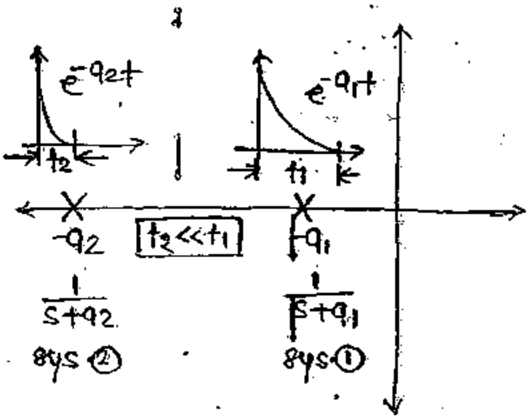
$s = \pm j, \; s = \pm 1$



Difficulty (02) → when one compete row of Routh array is 0, then in such cases construct an AE, A(s) differentiated to get new cofficient & evaluate rest of the RA.

* check the roots of AE which are poles symmetric about origin to comment on stability.

Relative stability Analysis →

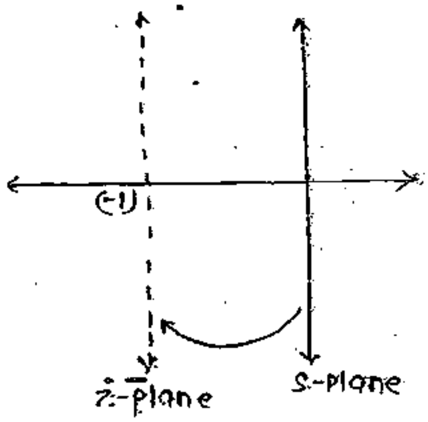


Both sys ① & sys ② are said to be absolutely stable

sys (2) is said to be relatively more stable than sys ① bcoz $t_2 \ll t_1$

$P(s) = s^3 + 7s^2 + 25s + 39 = 0$

check weather the roots are lying more −vely wrt = 1?



$s+1 = z$

$s = z-1$

$P(z) = (z-1)^3 + 7(z-1)^2 + 25(z-1) + 39 = 0$

$P(z) = z^3 + 4z^2 + 14z + 20 = 0$

z^3	1	14
z^2	4	20
z	9	0
z^0	20	0

Shortcut → * Put $s = -1$ in given $P(s)$ & if +ve value is coming means all the roots are lying in LHS

If $P(s) = 0$, then only 4 values are lying on LHS.

* Conditionaly stable → A sys is said to be conditionaly stable if its stability depends on one (or) more parameters.

$\frac{7}{6}$) $P(s) = s^4 + 2s^3 + 3s^2 + 2s + k = 0$

s^4	1	3	k
s^3	2	2	0
s^2	2	k	0
s^1	$\frac{4-2k}{2}$	0	0
s^0	k	0	0

(i) $\frac{4-2k}{2} > 0$; $k < 2$

(ii) $k > 0$

$\boxed{0 < k < 2}$

at $k = k_{mar} = 2$

s^1 row $= 0$

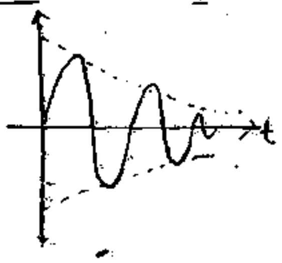
$A(s) = 2s^2 + k = 0$

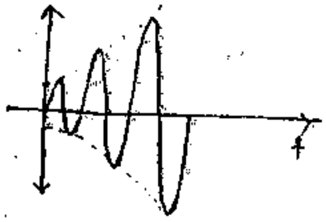
$2s^2 + 2 = 0$

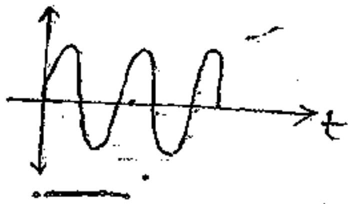
$s = \pm j \rightarrow (j\omega)$

$\boxed{\omega = \omega_{mar} = 1\ r/s}$

case(1) → $s = -\alpha \pm j\omega$ case(2) → $s = \alpha \pm j\omega$ case(3) → $s = \pm j\omega$







Q/06

$$1 + \frac{K(s-2)^2}{(s+2)^2} = 0$$

$$(s+2)^2 + K(s-2)^2 = 0$$

$$s^2(1+K) + s(4-4K) + (4+4K) = 0$$

s^2	$1+K$	$4+4K$
s^1	$4-4K$	0
s^0	$4+4K$	0

(i) $1+K>0$, $K>-1$ (2) $4-4K>0$, $K<1$

$\boxed{-1<K<1}$ $\boxed{0\le K<1}$

I/6

$$1 + \frac{10(K_p s + K_I)}{s(s^2+s+20)} = 0$$

$$s^3 + s^2 + s(20+10K_p) + 10K_I = 0$$

s^3	1	$20+10K_p$
s^2	1	$10K_I$
s	$20+10K_p-10K_I$	0
s^0	$10K_I$	0

(i) $10K_I>0$, $\boxed{K_I>0}$

(ii) $20+10K_p-10K_I>0$

$\boxed{K_p>K_I-2}$

Q/66

$$1 + \frac{K(s+2)^2}{s(s^2+1)(s+4)} = 0$$

$$s^4 + 4s^3 + s^2(1+K) + s(4+4K) + 4K = 0$$

s^4	1	$1+K$	$4K$
s^3	4	$(4+4K)$	0
s^2	$\cancel{0}\,\epsilon$	$4K$	0
s^1	$\frac{(4+4K)\epsilon-16K}{\epsilon}$	0	0
s^0	$4K'$	0	0

$\boxed{\text{the sys is unstable for all } K>0}$

8/66 $1+\frac{K}{(s^2+2s+2)(s+2)}=0$

Sol^n →

$(s^2+2s+2)(s+2)+K=0$

$s^3+4s^2+6s+(4+K)=0$

s^3	1	6	0
s^2	4	$(4+K)$	0
s^1	$\frac{24-(4+K)}{4}$	0	0
s^0	$4+K$	0	0

(i) $\frac{24-(4+K)}{4}>0$

$K<20$

(ii) $4+K>0$

$K>-4$

$\boxed{-4<K<20}$

at $K=K_{mar}=20$

$A(s)=4s^2+(4+K)=0$

$4s^2+(4+20)=0$

$s^2=-6$

$s=\pm j\sqrt{6}=j\omega$

$\boxed{\omega=\sqrt{6}\ r/s}$

Shortcut → (3rd order system) (only when all coefficients of eq^n are +ve)

$$s^3+4s^2+6s+(4+K)=0$$

$A(s)$

(i) Product of external coefficients < product of internal = stable

(ii) product of external coefficients > product of internal = unstable

(iii) " = " = marginaly stable

$$4+k=24$$

$$k_{mar}=20$$

$$\cancel{4+20=24}$$

$$A(s)=4s^2+(4+k)=0$$

$$4s^2+(4+20)=0$$

$$s^2=-6$$

$$s=\pm j\sqrt{6} \simeq j\omega$$

$$\boxed{\omega=\sqrt{6}\ r/s}$$

10 / 66

$$1+\frac{K(s+1)}{s^3+as^2+2s+1}=0$$

Sol^n →

$$s^3+as^2+s(2+k)+(k+1)=0$$

$k+1=a(k+2)$ (Given)

$$a=\frac{k+1}{k+2}$$

$$A(s)=as^2+(k+1)=0$$

$$s^2=\frac{-(k+1)}{a}$$

$$s^2=\frac{-\cancel{(k+1)}\times(k+2)}{\cancel{(k+1)}}$$

$$s=\pm j\sqrt{k+2} \simeq j\omega$$

$$\omega=\sqrt{k+2}$$

$$2=\sqrt{k+2}$$

$$k=2$$

$$a=\frac{2+1}{2+2}=\frac{3}{4}$$

$$\boxed{k=2,\ a=\frac{3}{4}}$$

Control (3)
66

Sol 4 (a) $1 + \dfrac{K(s+\alpha)}{s(s+2)(s+4)^2} = 0$

$$s^4 + 10s^3 + 32s^2 + (K+32)s + K\alpha = 0$$

s^4	1	32	$K\alpha$
s^3	10	$32+K$	0
s^2	$\dfrac{288-K}{10}$	$K\alpha$	0
s^1	A	0	0
s^0	$K\alpha$	0	0

where $A = \dfrac{\dfrac{(288-K)(K+32)}{10} - 10K\alpha}{\left(\dfrac{288-K}{10}\right)}$

* (i) $\dfrac{288-K}{10} > 0$

$\boxed{K < 288}$

* (ii) $\dfrac{\dfrac{(288-K)(K+32)}{10} - 10K\alpha}{\left(\dfrac{288-K}{10}\right)} > 0$

$(288-K)(K+32) - 100K\alpha > 0$

$K\alpha < \dfrac{(288-K)(K+32)}{100}$

$\boxed{0 < K\alpha < \dfrac{(288-K)(K+32)}{100}}$

K (b.) $e_{ss} = \lim_{s \to 0} \dfrac{s \cdot \frac{1}{s^2}}{1 + \dfrac{K(s+\alpha)}{s(s+2)(s+4)^2}}$

$e_{ss} = \dfrac{32}{K\alpha}$ $\quad$ Let $e_{ss} = 0.16$ (16%)

$0.16 = \dfrac{32}{200 \times \alpha}$ $\quad \boxed{\because \alpha = 1} \quad \boxed{K = 200}$

$K\alpha = 200 \times 1 = 200$

$200 < \dfrac{(288-200)(200+32)}{100}$

$\boxed{200 < 204}$

THE ROOT LOCUS Technique

- The Root locus is defined as the Locus of closed loop poles obtained when sys gain k is varied from 0 to ∞.
- The RL determines relative stability of the sys.

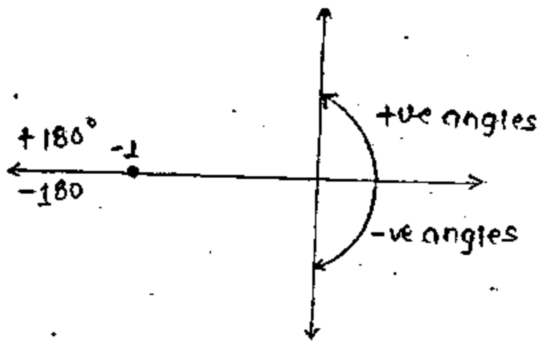
Angle & magnitude condⁿ →

- The angle condⁿ is used for checking weather certain points lie on RL (or) not & also the validity of RL for closed loop poles.

$$1 + G(S)\cdot H(S) = 0$$

$$G(S)\cdot H(S) = -1 + j0$$

$$\angle G(S)\cdot H(S) = +180^\circ = \tan^{-1}\left(\frac{0}{1}\right)\left(\frac{Img}{Real}\right) = 180^\circ \simeq \pm 180^\circ \simeq \pm(2q+1)\,180^\circ$$



- The angle condⁿ may be stated as for a point to lie on RL the angle evaluated at that point must be odd multiple of ±180°.
- The magnitude condⁿ is used for finding the sys gain k at any point on RL.

$$1 + G(S)\cdot H(S) = 0$$

$$G(S)\cdot H(S) = -1 + j0$$

$$|G(S)\cdot H(S)| = \sqrt{(-1)^2 + 0^2} = 1$$

$$\boxed{|G(S)\cdot H(S)| = 1}$$

(4/68) $s_1 = -3+4j$; $s_2 = -3-2j$

$$G(s)H(s) = \frac{k}{(s+1)^4}$$

$$G(s)\cdot H(s)\Big|_{(s=s_1=-3+4j)} = \frac{k+j0}{(-3+4j+1)^4} = \frac{0^\circ}{(-2+4j)^4} = \frac{0^\circ}{116.5^\circ\times 4} = -466^\circ$$

$$G(s)\cdot H(s)\Big|_{(s=s_2=-3-2j)} = \frac{k+j0}{(-3-2j+1)^4} = \frac{0^\circ}{-135\times 4} = +540^\circ$$

Because of the odd multiple is present in the s_2 then $180\times 3 = 540^\circ$ it is lying on RL.

(5/68)

$$G(s) = \frac{k}{s(s^2+7s+12)}$$

$$s = -1+j1$$

$$G(s)\Big|_{s=-1+j1} = \frac{k}{(-1+j)[(-1+j)^2+7(-1+j)+12]} = \frac{k+j0}{(-1+j)(5+5j)}$$

$$\angle G(s)\Big|_{s=-1+j} = \frac{0^\circ}{(135^\circ)(45^\circ)} = -180^\circ \text{ (lying on the R-L)}$$

$$|G(s)|_{s=-1+j} = 1 \Rightarrow \frac{\sqrt{k^2+0^2}}{\sqrt{(-1)^2+(1)^2}\sqrt{5^2+5^2}} \Rightarrow \frac{k}{\sqrt{2}\times\sqrt{50}} = 1 \quad \boxed{\therefore k=10}$$

Que → The OLTF of U.FB sys. is $G(s) = \dfrac{k}{s(s+1)(s+3)}$

A zero is added to the sys. so that the locus passes through $-1+j$.

The locn of zero would be?

(a) -1.33 (b) -2.33 (c) -1.66 (d) -2.66

Soln → Locus passes through $-1+j$ means it is lying on R-L.

$$G(s) = \frac{k}{s(s+1)(s+3)}$$

$$G(s)\Big|_{s=-1+j} = \frac{k(-1+j+z)}{(-1+j)(-1+j+1)(-1+j+3)}$$

$$= \frac{(k+j0)(z-1+j)}{(-1+j)(0+j)(2+j)}$$

$$\angle G(j\omega) = \frac{(0^\circ)\tan^{-1}\left(\frac{1}{z-1}\right)}{(135^\circ)(90^\circ)(26.5^\circ)}$$

$$\tan^{-1}\left(\frac{1}{z-1}\right) - 251.5^\circ = 180^\circ$$

$$\tan^{-1}\left(\frac{1}{z-1}\right) = 180 + 251.5^\circ$$

$$\left(\frac{1}{z-1}\right) = \tan(431.5^\circ)$$

$$\left(\frac{1}{z-1}\right) = 3$$

$$z = 1.33$$

$$(s+z) = 0$$

$$(s+1.33) = 0$$

$$\boxed{s = -1.33}$$

* Construction Rules of R-L →

(1.) The R-L is symmetrical about real axis.

(2.) Let p = no. of open Loop poles ; ~~z=~~

z = no. of open Loop zeros

& p > z ; then the no. of branches of R-L = p

(3) The no. of branches terminating at zeros = z

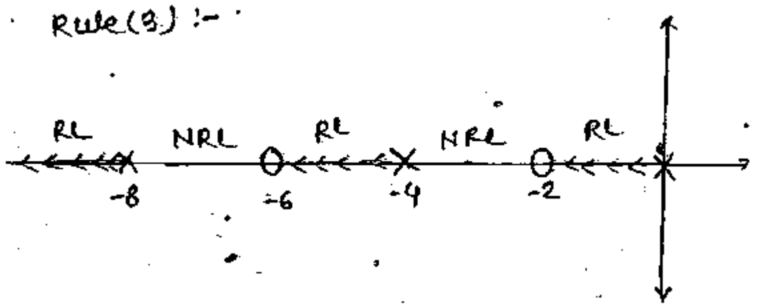
The no. of branches terminating at ∞ = p − z

~~(3) A~~ point on Real axis is said to be on RL if to the right side of this point the sum of open loop poles & zeros is odd.

eg:- $$G(s) = \frac{k(s+2)(s+6)}{s(s+4)(s+8)}$$

Rule (2) :- p = 3, z = 2, p − z = 1

Rule (3) :-



$$\frac{G(s)}{1+G(s)} = \frac{\frac{K(s+2)(s+6)}{s(s+4)(s+8)}}{1+\frac{K(s+2)(s+6)}{s(s+4)(s+8)}}$$

$$= \frac{K(s+2)(s+6)}{s(s+4)(s+8)+K(s+2)(s+6)}$$

Closed loop poles = $s(s+4)(s+8)+K(s+2)(s+6)$

when $K=0$

closed loop poles = $0, -4, -8$

(4) Angle of Asymptodes → The p-z branches terminate at ∞ along certain straight line known as asymptodes of RL.

Therefore no. of asymptodes = p-z.

$$\theta = \frac{(2q+1)\,180^\circ}{(p-z)} \qquad q = 0, 1, 2, 3 \ldots\ldots$$

eg:- $p-z=2$

$$\theta_1 = \frac{[2(0)+1]\times 180^\circ}{2} = 90^\circ \quad ; \quad \theta_2 = \frac{[2(1)+1]\times 180^\circ}{2} = 270^\circ$$

6/68

$$s(s+4)(s^2+2s+5)+K(s+1)=0$$

$$1 + \frac{K(s+1)}{s(s+4)(s^2+3s)} = 0$$

$$1 + G(s)\,H(s) = 0$$

$$G(s)\cdot H(s) = \frac{K(s+1)}{s(s+4)(s^2+3s)} = \frac{K(s+1)}{s^2(s+4)(s+3)}$$

(2) $p=4, \; z=1, \; p-z=3$

(4)
$$\theta_1 = \frac{[2(0)+1]\times 180^\circ}{3} = 60^\circ \qquad \theta_2 = \frac{[2(2)+1]\times 180^\circ}{3} = 300^\circ$$

$$\theta_2 = \frac{[2(1)+1]\times 180^\circ}{3} = 180^\circ$$

**
$$\boxed{\text{Angle b/n asymptodes} = \frac{2\pi}{p-z}}$$

5.) Centoriod → It is the intersection point of asymptodes on the real axis. It may (or) may not be a part of RL.

$$\text{Centoriod} = \frac{\sum \text{Real part of open Loop poles} - \sum \text{Real part of open Loop zeros}}{P-Z}$$

7/58)

$$s^3 + 5s^2 + (K+6)s + K = 0$$

$$s^3 + 5s^2 + 6s + Ks + K = 0$$

$$s^3 + 5s^2 + 6s + K(s+1) = 0$$

$$1 + \frac{K(s+1)}{s^3+5s^2+6s} = 0$$

$$1 + G(s)H(s) = 0$$

$$G(s) \cdot H(s) = \frac{K(s+1)}{s(s^2+5s+6)} = \frac{K(s+1)}{s(s+3)(s+2)}$$

(2.) $P = 3$; $Z = 1$, $P - Z = 2$

(5.) Centoriod →

Zero at $s = -1 + j0 = -1$

Poles at $s = 0 + j0$
$-2 + j0$
$-3 + j0$
-5

$$\text{Centoriod} = \frac{-5-(-1)}{2} = -2$$

Centoriod = $-2, 0$

K(6.) Break away points → They are those points where multiple roots of the c/s eqn occur.

Procedure → (1.) Construct $1 + G(s)H(s) = 0$
(2.) Write 'K' in terms of "s"
(3.) Find $\frac{dK}{ds} = 0$.

4.) The roots of $\frac{dk}{ds} = 0$ will give break away points.

(5.) To test valid BA points substitute in step (2.)

If k is +ve ⇒ valid BA points.

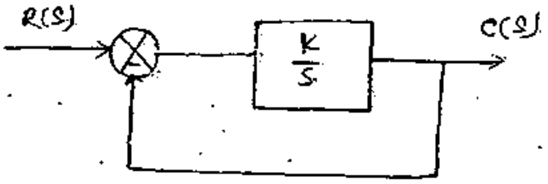
General predictions about BA points →

(1.) The branches of RL either approach (or) ~~live~~ leave the BA points at an angle of $\frac{\pm 180}{n}$, where n = no of branches approaching (or) ~~leaving~~ the BA point.

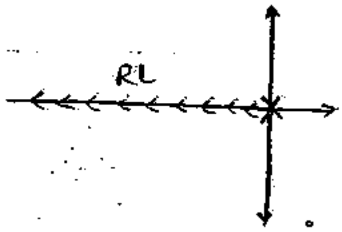
(2.) The complex congugate path for the branches of RL approaching (or) leaving the BA point is a circle.

(3.) Whenever there are 2 adjecently placed poles on the real axis with the section of real axis b/n them as a part of RL then there exist some BA point b/n the adjecently placed poles.

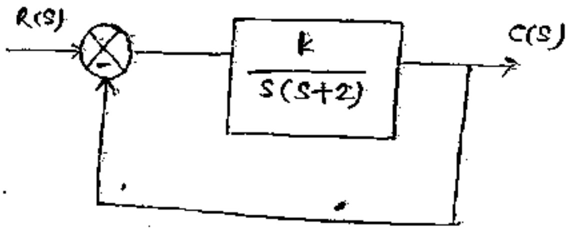
Con(64) 69 First order system →



$$\frac{C(S)}{R(S)} = \frac{K}{S+K} \qquad G(S) = \frac{K}{S}$$



* 2nd order system →

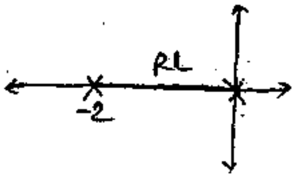


$$\frac{C(s)}{R(s)} = \frac{k}{s^2+2s+k}$$

$$G(s) = \frac{k}{s(s+2)}$$

(2.) $P = 2$; $z = 0$; $P - z = 2$

(3.)



(4.) $\theta_1 = 90^\circ$, $\theta_2 = 270^\circ$

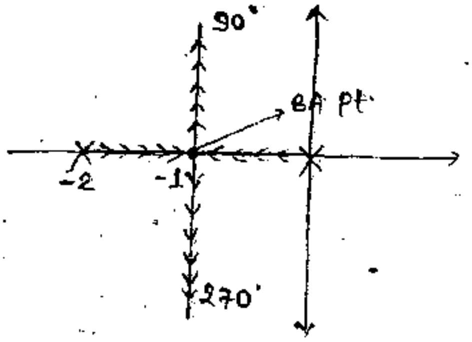
(5.) Centoriod $= \frac{0+(-2)}{2} = -1$

(6) BA Point →

$s^2+2s+k=0$

$k = -s^2-2s$

$\frac{dk}{ds} = 0$, $-2s-2=0$; $\boxed{s=-1}$

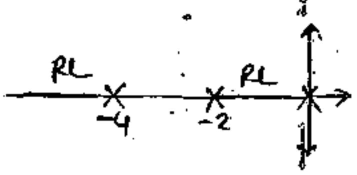


* 3rd order system →

$$G(s) = \frac{k}{s(s+2)(s+4)}$$

Effect of adding poles to a TF :—

(2) $P=3$, $z=0$, $P-z=3$ (3)



(4.) $\theta_1 = 60^\circ$, $\theta_2 = 180^\circ$, $\theta_3 = 300^\circ$

(5.) $\frac{0+(-2)+(-4)-0}{3} = -2$

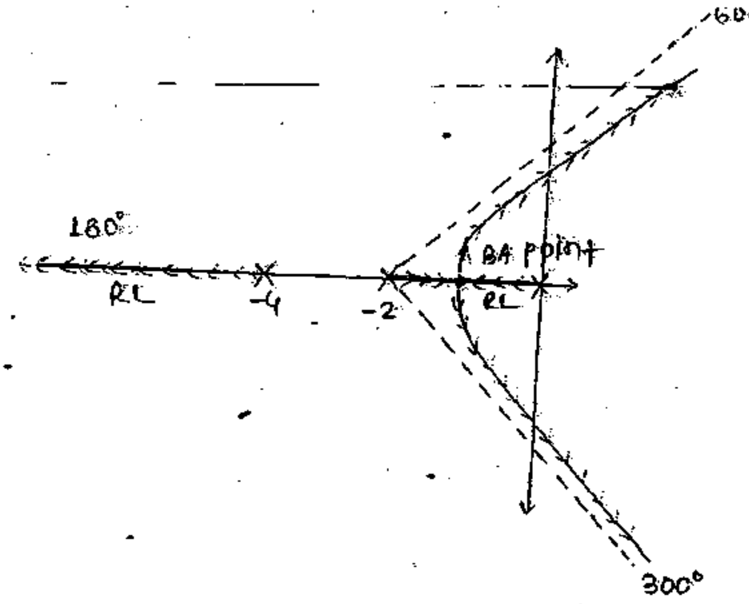
(6.) BA point

$s^3+6s^2+8s+k=0$

$k = -s^3-6s^2-8s$

$\frac{dk}{ds} = 0$; $3s^2+12s+8=0$

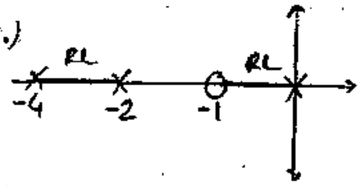
$s = -0.8, -3.15$ ✗



$*$ $G(s) = \dfrac{k(s+1)}{s(s+2)(s+4)}$ Effect of adding zero to a TF.

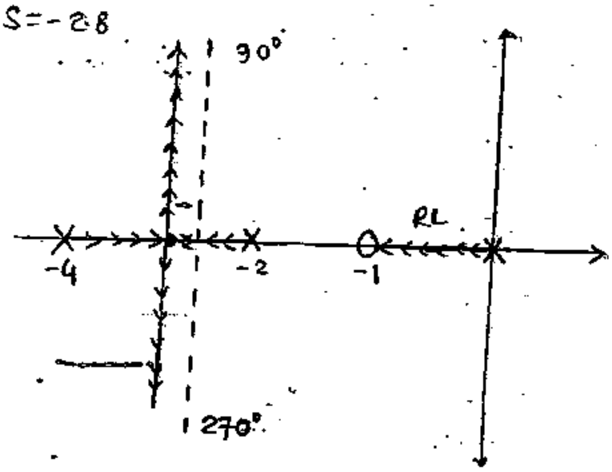
(2.) $P = 3$, $Z = 1$, $P - Z = 2$

(3.)



(4.) $\theta_1 = 90^\circ$, $\theta_2 = 270^\circ$ (5.) $\dfrac{0+(-2)+(-4)-(-1)}{2} = -2.5$

(6.) BA point $S = -2.8$



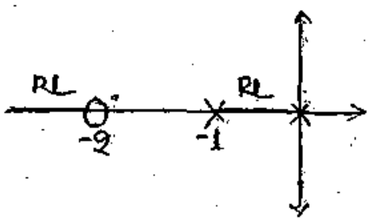
$\star\star\star$ Stability ↑
RL shifts to
LHS

Prediction (4) whenever there is a zero on real axis & to the left side of that zero there are no poles (or) zeros on the real axis with the entire section of real axis to the left side of zero as a part of RL then there exists a BA points to the left side of that zero.

Eg:- $G(s) = \dfrac{k(s+2)}{s(s+1)}$

(2.) $P = 2$, $Z = 1$, $P-Z = 1$

(3.)



(6.) BA points:-

$$s(s+1)+k(s+2)=0$$

$$k = \frac{-s^2-s}{s+2}$$

$$\frac{dk}{ds} = 0$$

$$\frac{(s+2)(-2s-1)-[(-s^2-s)\cdot 1]}{(s+2)^2} = 0$$

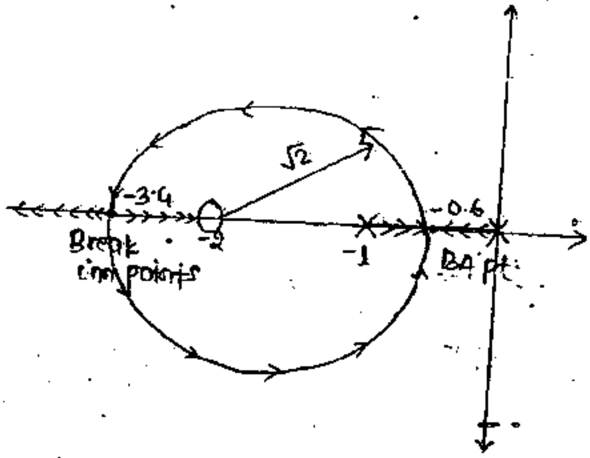
$$-2s^2+s-4s-2+s^2+s = 0$$

$$-s^2-4s-2=0$$

$$s^2+4s+2=0$$

$$s = \frac{-4 \pm \sqrt{16-8}}{2}$$

$$s = -2 \pm \sqrt{2} = -0.6, -3.4$$



To evaluate the center & radius →

$$G(s) = \frac{k(s+b)}{s(s+a)}$$

let $s = x+jy$

$$G(s) = \frac{k[x+jy+b]}{(x+jy)(x+jy+a)}$$

$$G(s) = \frac{k[(x+b)+jy]}{x^2+xjy+jyx-y^2+ax+ajy}$$

$$G(s) = \frac{K[(x+b)+jy]}{x^2+ax-y^2+j(2xy+ay)}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x+b}\right) - \tan^{-1}\left(\frac{2xy+ay}{x^2+ax-y^2}\right) = 180^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{A-B}{1+AB}\right) = 180^\circ$$

$$\Rightarrow \frac{A-B}{1+AB} = \tan(180^\circ)$$

$$\Rightarrow \frac{A-B}{1+AB} = 0$$

$$\Rightarrow A-B = 0$$

$$\Rightarrow \left(\frac{y}{x+b}\right) - \left(\frac{2xy+ay}{x^2+ax-y^2}\right) = 0$$

$$\Rightarrow x^2+ax-y^2-(2x^2+2xb+ax+ab) = 0$$

$$\Rightarrow (x+b)^2+y^2 = b(b-a)$$

$$\Rightarrow \text{center} = -b, 0 \;, = -2, 0$$

$$\Rightarrow \text{Radius} = \sqrt{b(b-a)} = \sqrt{2(2-1)} = \sqrt{2}$$

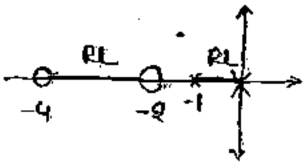
$$\boxed{\text{BA points shortcuts} = \text{Center} \pm \text{Radius}}$$

Prediction (5) Whenever there are 2 adjecently placed zeros on the real axis with the section of real axis b/n them as a part of RL then there exists BA point b/n the adjecently placed zeros.

eg:- $G(s) = \frac{K(s+2)(s+4)}{s(s+1)}$

(2) $P=2$, $Z=2$; $P-Z=0$

(3)



(6.) BA. points →

$$s(s+1) + K(s^2+6s+8) = 0$$

$$K = \frac{-s^2-s}{s^2+6s+8}$$

$$\frac{dK}{ds} = 0$$

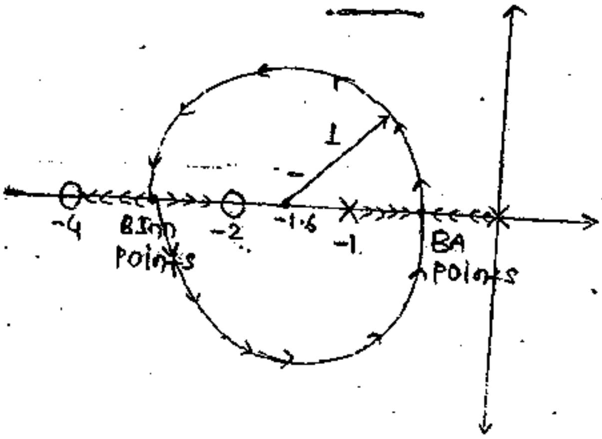
$$\frac{(s^2+6s+8)(-2s-1) - [(-s^2-s)(2s+6)]}{(s^2+6s+8)^2} = 0$$

$$\Rightarrow 5s^2 + 16s + 8 = 0$$

$$s = \frac{-16 \pm \sqrt{256-160}}{10}$$

$$s = -1.6 \pm 1 = -2.6, -0.6$$

(Center Radius)



Rule(7) → Intersection of RL with imaginary axis :- The roots of the AE A(s) at $K = K_{max}$ from Routh Array gives the intersection of RL with imaginary axis.

Eg:-

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

$$s^3 + 6s^2 + 8s + K = 0$$

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	0

(i) $\frac{48-K}{6} > 0$ (ii) $K > 0$

$K < 48$

$\boxed{0 < K < 48}$

$K = K_{max} = 48$

$A(s) = 6s^2 + K = 0$

$6s^2 + 48 = 0$

$s = \pm j\sqrt{8}$

$\boxed{s = \pm j2.8}$

~~Intersection of asymptodes~~ with $j\omega$ axis.

$\tan\theta = \frac{y}{x}$

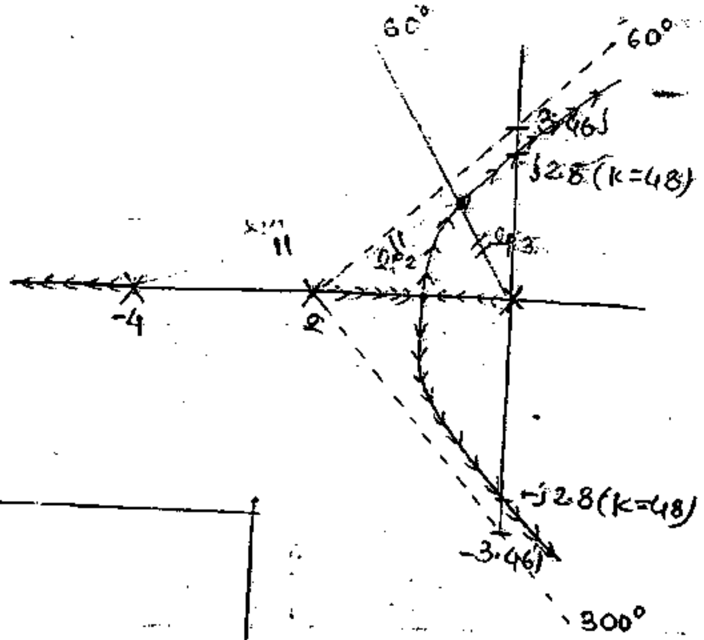
$\tan 60^\circ = \frac{y}{2}$

$y = \tan 60^\circ \times 2$

$y = \sqrt{3} \times 2$

$= 3.46$

$= \pm 3.48j$



Short cut method →
$G(s) = \frac{k}{s(s+a)(s+b)}$
Intersection of RL with $j\omega$ axis $= \pm j\sqrt{ab}$

Q → Find k when $\xi = 0.5$ from RL?

Soln →

$\theta = \cos^{-1}(0.5)$

$\theta = 60^\circ$

$k = \frac{\text{Product of vector lengths of poles}}{\text{Product of vector lengths of zeros}}$

$= \frac{l_{p_1} \times l_{p_2} \times l_{p_3}}{1}$

Linearity	$a_1x_1(t)+a_2x_2(t) \rightleftharpoons a_1c_1(n)+a_2c_2(n)$	$a_1x_1(t)+a_2x_2(t) \rightleftharpoons a_1X_1(\omega)+a_2X_2(\omega)$	$a_1f_1(t)+a_2f_2(t) \rightleftharpoons a_1F_1(s)+a_2F_2(s)$	$a_1x_1(n)+a_2x_2(n) \rightleftharpoons a_1X_1(z)+a_2X_2(z)$
Time-Reversal	$x(-t) \rightleftharpoons c_{(-n)}$	$x(-t) \rightleftharpoons X(-\omega)$	$f(-t) \rightleftharpoons F(-s)$	$x(-n) \rightleftharpoons X(z^{-1})$
Congugation	$x^*(t) \rightleftharpoons c^*_{-n}$	$x^*(t) \rightleftharpoons X^*(-\omega)$	$f^*(t) \rightleftharpoons f^*(s^*)$	$x^*(n) \rightleftharpoons X^*(z^*)$
Time-shifting	$x(t-t_0) \rightleftharpoons c_n e^{-jn\omega_0 t_0}$	$x(t-t_0) \rightleftharpoons X(\omega)e^{-j\omega t_0}$	$f(t-t_0) \rightleftharpoons F(s)e^{-st_0}$	$x(n-n_0) \rightleftharpoons X(z)z^{-n_0}$ $n_0=1 \Rightarrow x(n-1) \rightleftharpoons X(z)z^{-1}$
Freq. shifting	$x(t)e^{jm\omega_0 t} \rightleftharpoons c_{n-m}$ $x(t)e^{-jm\omega_0 t} \rightleftharpoons c_{n+m}$	$x(t)\cdot e^{j\omega_0 t} \rightleftharpoons X(\omega-\omega_0)$	$e^{-at}f(t) \rightleftharpoons F(s+a)$	
Convolution in time	$x_1(t) * x_2(t) \rightleftharpoons T_0(c_{1n}.c_{2n})$ where $T_0 = \text{Lcm}(T_1, T_2)$	$x_1(t) * x_2(t) = X_1(\omega)X_2(\omega)$	$F_1(t) * F_2(t) \rightleftharpoons F_1(s)\cdot F_2(s)$	$x_1(n) * x_2(n) \rightleftharpoons X_1(z)X_2(z)$
Multiplication in time	$x_1(t)\cdot x_2(t) \rightleftharpoons (c_{1n} * c_{2n})$	$x_1(t)\cdot x_2(t) = \frac{1}{2\pi}[X_1(\omega) * X_2(\omega)]$	$f_1(t)\cdot f_2(t) \rightleftharpoons \frac{1}{2\pi j}[F_1 s * F_2(s)]$	$x_1(n)\cdot x_2(n) \rightleftharpoons \frac{1}{2\pi j}[X_1(z) * X_2(z)]$
Integration in time.	$\int_{-\infty}^{t} x(k)dk = \frac{c_n}{jn\omega_0}$	$\int_{-\infty}^{t} x(k)dk = \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$ $X(0) = X(\omega)\vert_{\omega=0}$	$\int_{-\infty}^{t} x(t)\cdot dt \rightleftharpoons \begin{cases} \frac{F(s)}{s}, & \text{Bilateral LT} \\ \frac{F(s)}{s} + \frac{\int_{-\infty}^{0} f(t)dt}{s} & \text{Unilateral LT} \end{cases}$	$\sum_{k=-\infty}^{n} x(n) \rightleftharpoons \frac{X(z)}{1-z^{-1}}$
Diff in time	$\frac{d^m x(t)}{dt^m} = (jn\omega_0)^m c_n$	$\frac{d^n x(t)}{dt^n} = (j\omega)^n X(\omega)$	$\frac{d^n f(t)}{dt^n} \rightleftharpoons \begin{cases} s^n F(s) & \text{Bilateral LT} \\ s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) & \text{Uni LT} \end{cases}$	$\frac{dx(n)}{dn} = \frac{x(n)-x(n-1)}{n-(n-1)}$ $x(n)-x(n-1) \rightleftharpoons (1-z^{-1})X(z)$

Passeval's Power theorem	$P = \frac{1}{T_0}\int_{T_0} \lvert x(t)\rvert^2 dt$ $P = \sum_{n=-\infty}^{\infty} \lvert c_n\rvert^2$	$E = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$ $= \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$.	Initial value theorem:- $f(0) = \lim_{s\to\infty} s\cdot F(s)$ applicable only causal sys. $f(t)=0,\ t<0$	Initial value theorem $x(0) = \lim_{z\to\infty} X(z)$ $x(0) = \lim_{s\to\infty} sX(s)$
Time Scaling		$x(at)_{a\neq 0} = \frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$	$f(at)_{a\neq 0} \rightleftharpoons \frac{1}{\lvert a\rvert} F\left(\frac{s}{a}\right)$	Condn:- applicable only for causal type of sys. i.e. $x(n)=0,\ n<0$
modulation		$x(t)\cdot\cos\omega_0 t = \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$ $x(t)\cdot\sin\omega_0 t = \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right]$		Scaling of z:- $a^n x(n) \rightleftharpoons X(a^{-1}z)$
Differentiation in Freq.		$t^n x(t) = j^n \frac{d^n X(\omega)}{d\omega^n}$	$t^n f(t) \rightleftharpoons (-1)^n \frac{d^n F(s)}{ds^n}$	$n\cdot x(n) \rightleftharpoons -z\frac{dX(z)}{dz}$
Area under time domain		$X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t} dt$ $\downarrow \omega=0$ $X(0) = \int_{-\infty}^{\infty} x(t)dt$ $x(t) = X(\omega)\rvert_{\omega=0}$	Final value theorem→ $f(\infty) = \lim_{s\to 0} s\cdot F(s)$ (i) causal type $f(t)=0, t<0$ (ii) $sF(s)$ should have only LHS poles in s-plane.	Final value theorem→ $x(\infty) = \lim_{z\to 1} (1-z^{-1})X(z)$ $x(\infty) = \lim_{s\to 0} sX(s)$ Condn:- (i) applicable for causal signals i.e. $x(n)=0$, $n<0$
Area under Freq. domain		$X(\omega) = 2\pi x(t)\rvert_{t=0}$		
Integration in Freq.			$\frac{f(t)}{t} \rightleftharpoons \int_s^{\infty} F(s)ds$	(ii) $(1-z^{-1})X(z)$ should have poles inside unit circle in z-plane.

*** Rule no.(8) → Angle of departure & arrival →** The angle of departure is obtained when complex poles terminate at ∞

* The angle of arrival is obtained at complex zeros.

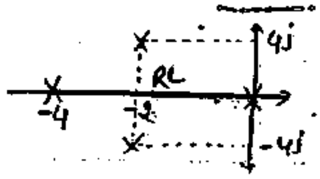
$$\phi_D = 180^\circ + \phi \quad ; \phi_A = 180^\circ - \phi$$

where; $\phi = \Sigma\phi_Z - \Sigma\phi_P$

(8/68) $$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

(2.) $P=4$; $Z=0$; $P-Z=4$

(3.)



(4.) $\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$, $\theta_3 = 225^\circ$, $\theta_4 = 315^\circ$

(5.) $$\frac{0+(-2)+(-2)+(-4)-0}{4} = -2$$

(6.) BA points →

(a.) Shortcut methode →

Avg. value of real poles $= \frac{0+(-4)}{2} = -2$

* If the avg. value of real poles = Real part of complex pole

There will be 3 BA points.

* The avg. value of real poles $\neq$ Real part of complex poles.

There will be 1 BA points

(b.) nature of BA points →

Absolute value of avg. value = 2
of real poles.

$2 \times x = 20$

$\boxed{x = 10}$

$x \geq 5 \Rightarrow$ There will be 1 real & 2 complex BA points

$x < 5 \Rightarrow$ There will be 3 real BA points.

*** Original method →**

$$s^4 + 8s^3 + 36s^2 + 80s + K = 0$$

$$K = -s^4 - 8s^3 - 36s^2 - 80s$$

$$\frac{dK}{ds} = -4s^3 - 24s^2 - 72s - 80 = 0$$

$$4s^3 + 24s^2 + 72s + 80 = 0$$

$$s = -2 \,;\, -2 \pm 2.45j$$

<u>Note:-</u> To check the validity of complex BA points we angle cond^n:

$$G(s)H(s)\Big|_{s=-2\pm 2.45j} = \frac{K}{(-2+2.45j)(2+j2.45)(0+6.45j)(0-1.55j)}$$

$$\underline{|G(s)\cdot H(s)}\Big|_{s=-2+j2.45} = \frac{0^\circ}{(130^\circ)(50^\circ)(90^\circ)(-90^\circ)} = -180^\circ$$

(7)

s^4	1	36	K
s^3	8	80	0
s^2	26	K	0
s^1	$\frac{2080-8K}{26}$	0	0
s^0	K	0	0

$$\frac{2080-8K}{26} > 0 \quad , \quad K > 0$$

$$K < 260$$

$$\boxed{0 < K < 260}$$

$$K_{mar} = 260$$

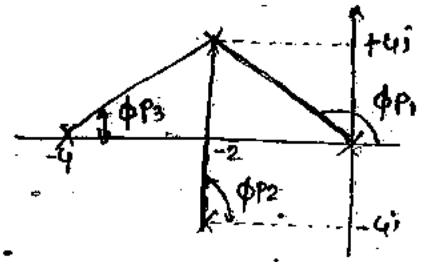
$$A(s) = 26s^2 + K = 0$$

$$26s^2 + 260 = 0$$

$$\boxed{s = \pm 3.16j}$$

$$\boxed{\begin{aligned} y &= \tan 45 \times 2 = 2 \\ &= \pm j2 \end{aligned}}$$

(8) <u>Angle of departure →</u>



$$\phi_{P_1} = 180^\circ - \tan^{-1}\left(\frac{4-0}{0-(-2)}\right)$$

$$= 116.6^\circ$$

$$\phi_{P_2} = 90^\circ$$

$$\phi_{P_3} = \tan^{-1}\left[\frac{4-0}{-2-(-4)}\right] = 63.4^\circ$$

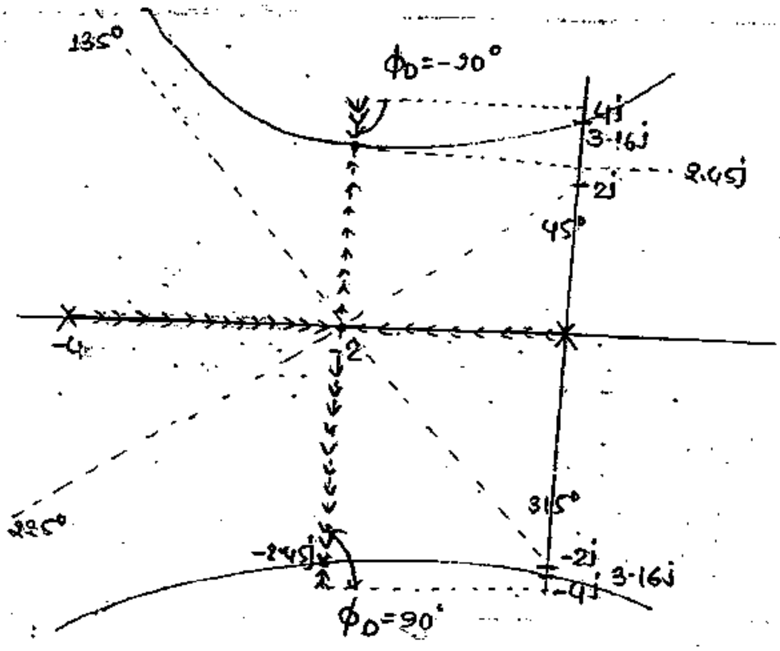
$$\phi = \Sigma\phi_z - \Sigma\phi_p$$

$$= 0 - [116.6 + 90 + 63.4]$$

$$\phi = -270^\circ$$

$$\phi_D = 180^\circ + \phi = 180^\circ - 270^\circ$$

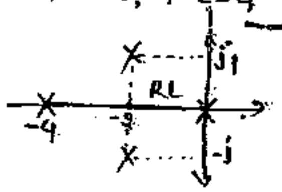
$$\boxed{\phi_D = -90^\circ}$$



(9/68) $$G(s) \cdot H(s) = \frac{K}{s(s+4)(s^2+4s+5)}$$

(i) $P = 4$; $Z = 0$, $P - Z = 4$

(3)



(4) $\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$, $\theta_3 = 225^\circ$, $\theta_4 = 315^\circ$

(5.) $\frac{0+(-2)+(-2)+(-4)-0}{4} = -2$

(6.) BA points →

$$s^4+8s^3+21s^2+20s+K=0$$

$$K=-s^4-8s^3-21s^2-20s$$

$$\frac{dK}{ds}=0.$$

$$-4s^3-24s^2-42s-20=0$$

$$4s^3+24s^2+42s+20=0$$

$$\boxed{s=-0.78,\ -2,\ -3.22}$$

(7.)

s^4	1	21	K
s^3	8	20	0
s^2	18.5	K	0
s^1	$\frac{370-8K}{18.5}$	0	0
s^0	K	0	0

$\frac{370-8K}{18.5}>0$, $K<46.25$

$\boxed{0<K<46.25}$

$K=K_{mar}=46.25$

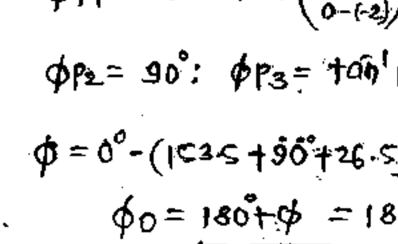
$$A(s)=18.5s^2+K=0$$

$$18.5s^2+46.25=0$$

$$\boxed{s=\pm j1.58}$$

$$y=\tan^{-1}45^\circ\times 2=2=\pm 2j$$

(8.) Angle of departure →



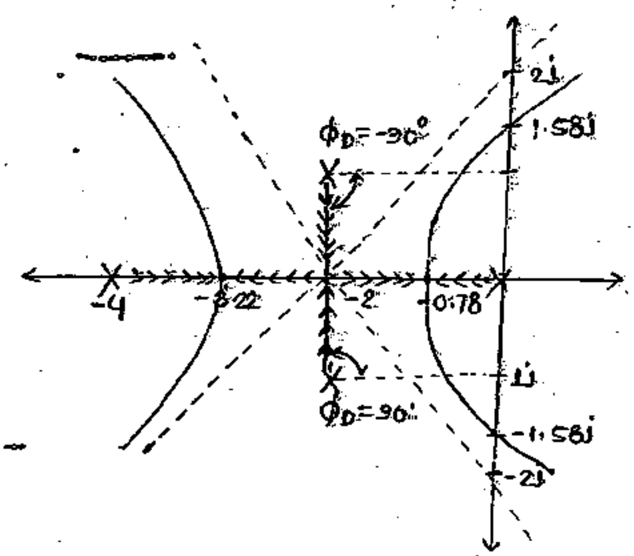
$$\phi P_1=180^\circ-\tan^{-1}\left(\frac{1-0}{0-(-2)}\right)=153.5^\circ$$

$$\phi P_2=90^\circ;\ \ \phi P_3=\tan^{-1}\left(\frac{1-0}{-2-(-4)}\right)=26.5^\circ$$

$$\phi=0^\circ-(153.5+90^\circ+26.5)=-270^\circ$$

$$\phi_0=180^\circ+\phi=180-270^\circ=-90$$

$$\boxed{\phi_0=-90^\circ}$$

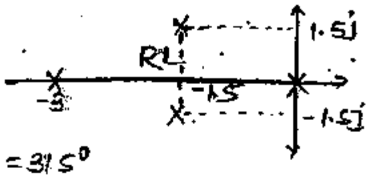


Conv-(1) 69

$$G(s) = \frac{K}{s(s+3)(s^2+3s+4.5)}$$

(2) $P=4$, $Z=0$, $P-Z=4$

(3)



(4) $\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$, $\theta_3 = 225^\circ$, $\theta_4 = 315^\circ$

(5) $$\frac{0+(-1.5)+(-1.5)+(-3)-0}{4} = -1.5$$

(6) BA points :-

$$s^4 + 6s^3 + 13.5s^2 + 13.5s + K = 0$$

$$K = -s^4 - 6s^3 - 13.5s^2 - 13.5s$$

$$\frac{dK}{ds} = -4s^3 - 18s^2 - 27s - 13.5 = 0$$

$$4s^3 + 18s^2 + 27s + 13.5 = 0$$

$$\boxed{s = -1.5, -1.5, -1.5}$$

(7)

s^4	1	13.5	K
s^3	6	13.5	0
s^2	11.25	K	0
s^1	$\frac{151.87-6K}{11.25}$	0	0
s^0	K	0	0

$$\frac{151.87-6K}{11.25} > 0$$

$$K < 25.3$$

$$\boxed{0 < K < 25.3}$$

at $K = K_{max} = 25.3$

$$A(s) = 11.25s^2 + K = 0$$

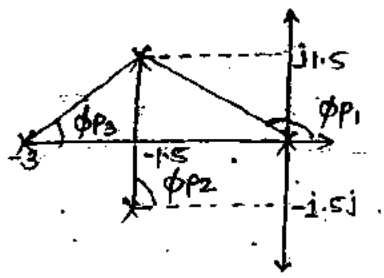
$$11.25s^2 + 25.3 = 0$$

$$\boxed{s = \pm j1.5}$$

$$y = \tan^{-1} 45^\circ \times 1.5 = 1.5$$

$$= \pm 1.5j$$

8.) Angle of departure →



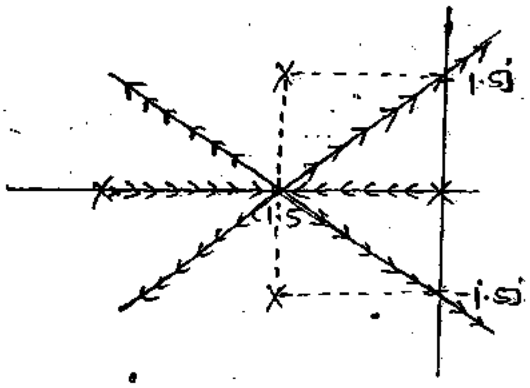
$$\phi p_1 = 180^\circ - \tan^{-1}\left(\frac{1.50}{0-(-1.5)}\right) = 135^\circ$$

$$\phi p_2 = 90^\circ, \quad \phi p_3 = \tan^{-1}\left(\frac{1.5-0}{-1.5-(-3)}\right] = 45^\circ$$

$$\phi = 0^\circ - (135^\circ + 90 + 45) = -270^\circ$$

$$\phi_D = 180^\circ - 270^\circ = -90^\circ$$

$$\boxed{\phi_D = -90^\circ}$$



$\boxed{\frac{\text{contd.(2)}}{69}}$

$$s(s+4)(s^2+2s+2)+K(s+1)=0$$

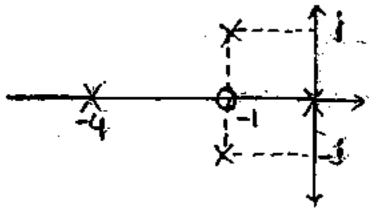
$$1+\frac{K(s+1)}{s(s+4)(s^2+2s+2)}=0$$

$$1+G(s)H(s)=0$$

$$G(s)H(s)=\frac{K(s+1)}{s(s+4)(s^2+2s+2)}$$

(2.) $P=4$; $Z=1$, $P-Z=3$

(3.)



(4.) $\theta_1=60°$, $\theta_2=180°$, $\theta_3=300°$

(5.) $$\frac{0+(-1)+(-1)+(-4)-(-1)}{3}=-1.6$$

(6.) BA points:- Nill.

(7.) $s^4+6s^3+10s^2+(K+8)s+K=0$

s^4	1	10	K
s^3	6	(K+8)	0
s^2	$\frac{52-K}{6}$	K	0
s^1	$\frac{\frac{(52-K)(K+8)}{6}-6K}{\left(\frac{52-K}{6}\right)}$	0	0
s^0	K	0	0

$$(52-K)(8+K)-36K=0$$

$$K^2-8K-416=0$$

$$K=24.78,\ -16.75$$

$$\boxed{K_{mar}=24.78}$$

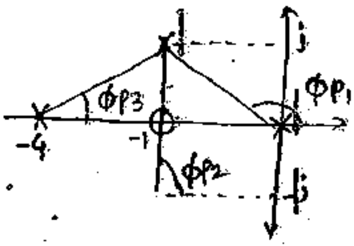
$$A(s)=\left(\frac{52-24.78}{6}\right)s^2+24.78=0$$

$$\boxed{s=\pm j2.34}$$

$$y=\tan 60°\times 1.6=\sqrt{3}\times 1.6=2.77$$

$$=\pm 2.77j$$

(8.)



$$\phi_{P1}=180°+\tan^{-1}\left(\frac{1-0}{0-(-1)}\right)=135°$$

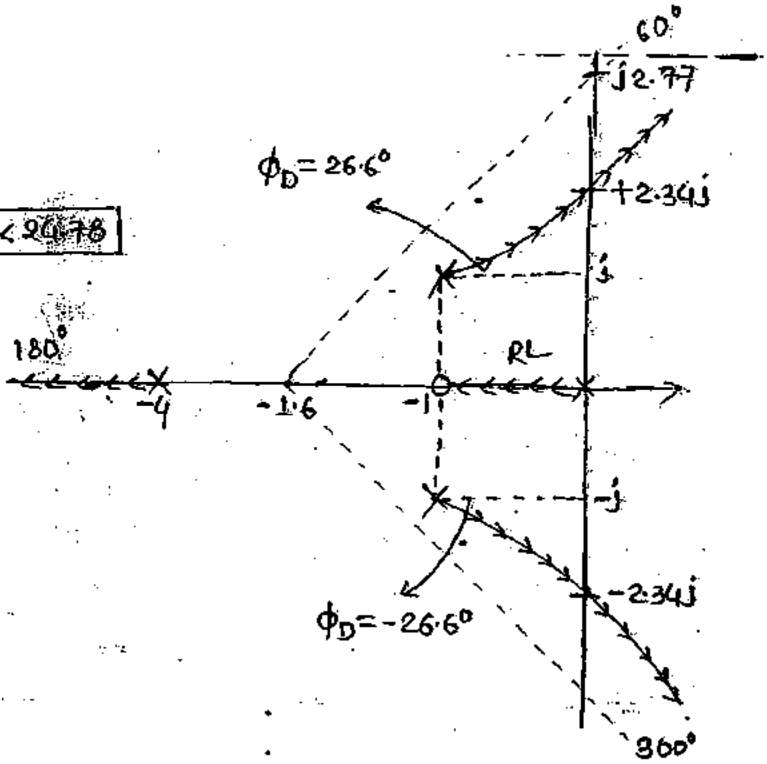
$$\phi_{P2}=90°;\ \phi_{Z}=90°$$

$$\phi_{P3}=\tan^{-1}\left(\frac{1-0}{-1-(-4)}\right)=18.4°$$

$$\phi=90°-(135°+90°+18.4)=-153.4°$$

$$\phi_D=180-153.4°=26.6°$$

$\boxed{0<K<2478}$

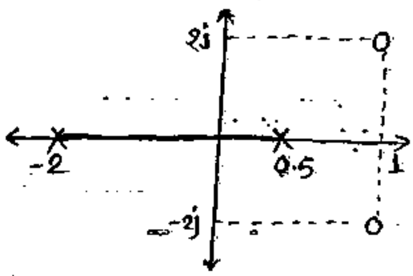


Q(3) / 69

$$G(s) = \frac{K(s^2-2s+5)}{(s+2)(s-0.5)}$$

(2.) $P = 2,\ Z = 2,\ P - Z = 0$

(3.)



(6.) <u>BA. point →</u>

$$(s+2)(s-0.5) + K(s^2-2s+5) = 0$$

$$s^2 + 1.5s - 1 + K(s^2-2s+5) = 0$$

$$K = \frac{-s^2-1.5s+1}{s^2-2s+5}$$

$$\frac{dK}{ds} = 0$$

$$\Rightarrow\ 3.5s^2 - 12s - 5.5 = 0$$

$$s = -0.4,\ 3.8$$

(7.) $s^2(1+K) + s(1.5-2K) + (5K-1) = 0$

s^2	$(1+K)$	$5K-1$
s^1	$(1.5-2K)$	0
s^0	$5K-1$	0

$1.5-2K=0$

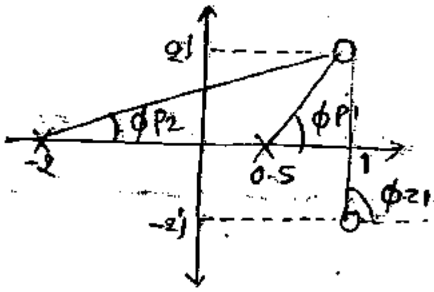
$K_{mar}=0.75$

$A(s)=(1+K)s^2+(5K-1)=0$

$(1+0.75)s^2+(5\times0.75-1)=0$

$\boxed{s=\pm j1.25}$

(8.) Angle of arrival →



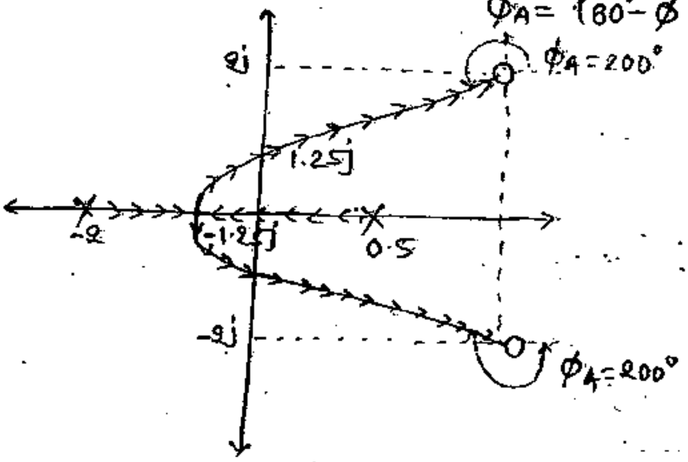
$\phi z_1 = 90^\circ$

$\phi P_1 = \tan^{-1}\left(\frac{2-0}{1-0.5}\right) = 76^\circ$

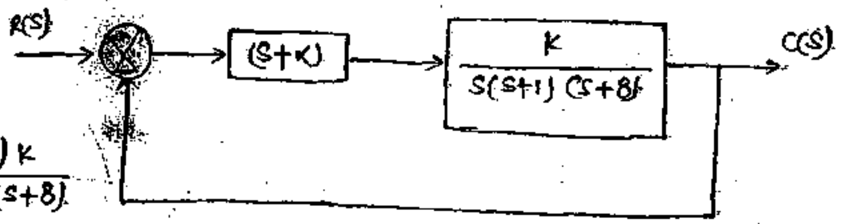
$\phi P_2 = \tan^{-1}\left(\frac{2-0}{1-(-2)}\right) = 34^\circ$

$\phi = 90^\circ-(76+34) = -20^\circ$

$\phi_A = 180^\circ-\phi = 180-(-20) = 200^\circ$



* ROOT Contour →

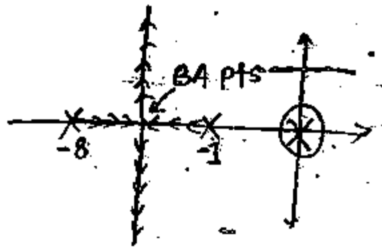


$$G(s) = \frac{K(s+\alpha)K}{s(s+1)(s+8)}$$

* These are multiple RL diagrams obtained by varying multiple parameter in a TF· drawn on same s-plane.

Case(i) → $\alpha = 0$

$$G(s) = \frac{K\cdot \cancel{s}}{\cancel{s}(s+1)(s+8)}$$



Case(2) →

$$1 + \frac{K(s+\alpha)}{s(s+1)(s+8)} = 0$$

$$s(s+1)(s+8) + K(s+\alpha) = 0$$

$$s(s+1)(s+8) + Ks + K\alpha = 0$$

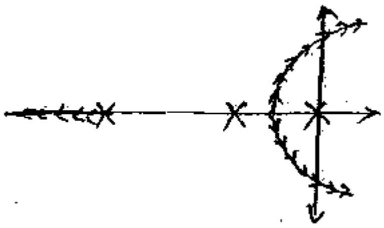
$$1 + \frac{K\alpha}{s(s+1)(s+8)+Ks} = 0$$

$$1 + G(s)\cdot H(s) = 0$$

$$G(s)\cdot H(s) = \frac{K\alpha}{s(s+1)(s+8)+Ks}$$

if value of K is not given then $K=1$;

$$G(s)\cdot H(s) = \frac{\alpha}{s(s+1)(s+8)+s} = \frac{\alpha}{s(s^2+9s+9)}$$



Q→ Find BA points for $k=10$?

Soln→ $G(s)\cdot H(s) = \dfrac{10\alpha}{s(s+1)(s+8)+10s}$

let $10\alpha = k'$

$$= \frac{k'}{s(s+1)(s+8)+10s}$$

$$= \frac{k'}{s(s^2+9s+18)}$$

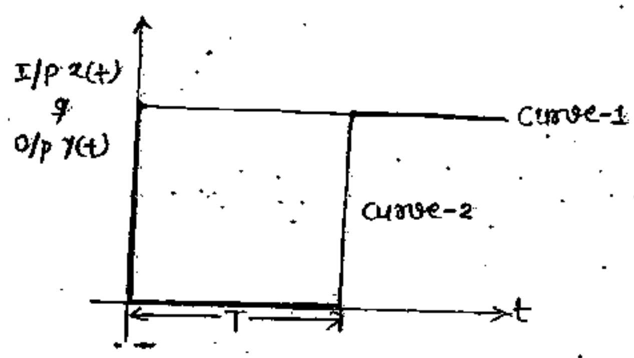
$$1 + \frac{k'}{s(s^2+9s+18)} = 0$$

$$s^3+9s^2+18s+k' = 0$$

$$k' = -s^3-9s^2-18s$$

$$\frac{dk'}{ds} = -3s^2-18s-18 = 0$$

* Analysis of sys having DEAD TIME (or) TRANSPORTATION LAG →



For curve-(1)

O/p $y(t) =$ I/p $x(t)$

For curve-(2)

O/p $y(t) = x(t-T)$

$$Y(s) = e^{-Ts}X(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = e^{-Ts}}$$

Time domain approximation → (Time domain analysis, RH, RL plots)

$$y(t) = x(t-T) = x(t) - T\dot{x}(t) + \frac{T^2}{2!}\ddot{x}(t) - \frac{T^3}{3!}\dddot{x}(t) + \cdots\cdots$$

$$y(t) = x(t) - T\dot{x}(t)$$

$$Y(s) = X(s)(1-Ts) \quad \& \quad Y(s) = X(s)\cdot e^{-Ts}$$

$$\boxed{e^{-Ts} \simeq (1-Ts)}$$

Ex:- $G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)}$

* Dead time is one of the forms of non-linearity & is approximated as zero in RHS of s-plane.
* TF having poles (or) zeros in RHS of s-plane are known as non-minm phase fn.

Non minm phase fn →

$$\boxed{\angle F(s)\Big|_{\omega\to\infty} \neq -(P-Z)\cdot 90^\circ}$$

eg:- $G(s) = \frac{Ke^{-s}}{s(s+3)} = \cancel{\frac{K(s-3)}{}}\ \frac{K(1-s)}{s(s+3)} = \frac{K(1-s)}{(s)\,3\left(1+\frac{s}{3}\right)}$

$$G(j\omega) = \frac{\left(\frac{K}{3}+j0\right)(1-j\omega)}{(0+j\omega)\left(1+\frac{j\omega}{3}\right)}$$

$$\angle G(j\omega) = \frac{(0^\circ)(-\tan^{-1}\omega)}{(90^\circ)\left(\tan^{-1}\frac{\omega}{3}\right)} = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}$$

$$\angle G(j\omega)\Big|_{\omega=\infty} = -90^\circ - 90 - 90 = -270^\circ \neq -(P-Z)90^\circ$$

* TF having poles & zeros in the LHS of s-plane are known as minm phase fn. LTI TF should be minm phase fn.

minm phase fn →

$$\boxed{\angle F(s)\Big|_{\omega\to\infty} = -(P-Z)\,90^\circ}$$

Eg:- $G(s) = \frac{k(1+s)}{s(s+3)} = \frac{(k/3)(1+s)}{s(1+\frac{s}{3})}$

$$G(j\omega) = \frac{(\frac{k}{3}+j0)(1+j\omega)}{(0+j\omega)(1+\frac{j\omega}{3})}$$

$$\underline{|G(j\omega)} = \frac{(0^\circ)[\tan^{-1}(\omega)]}{(90^\circ)(\tan^{-1}\frac{\omega}{3})} = -90^\circ + \tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}$$

$$\underline{|G(j\omega)}\Big|_{\omega=\infty} = -90 + 90^\circ - 90^\circ = -90$$

$$\boxed{-90^\circ = -(P-Z)90^\circ}$$

* Since s indicates time & can't be -ve (1-s) factor should be expressed as -(s-1) in time domain methods.

$$G(s) = \frac{ke^{-s}}{s(s+3)} = \frac{k(1-s)}{s(s+3)} = \frac{-k(s-1)}{s(s+3)}$$

* <u>Complemetry R-L (CRL) Or Inverse RL (IRL) →</u>

$$G(s)H(s) = 1+j0$$

(1.) Angle cond^n

$$\underline{|G(s)\cdot H(s)} = 0^\circ = \pm(2q)180^\circ$$

(2.) Mag. Cond^n

$$|G(s)\cdot H(s)| = 1$$

* <u>construction Rule of CRL →</u>

<u>Rule(1)</u>. The CRL is symmetrical about real axis

<u>Rule(2)</u> Same as RL.

<u>Rule(3)</u> A point on real axis is said to be on CRL is to the right side of this point the sum of open loop poles & zeros is even.

<u>Rule(4)</u> Angle of asymptodes →

$$\boxed{\theta = \frac{(2q)180}{P-Z}}$$

where, $q = 0, 1, 2, \cdots$

Rule(5) Centroid → same as RL

Rule(6) BA points → same as RL

Rule(7) Intersection of CRL with $j\omega$ axis → same as RL

Rule(8) Angle of departure & arrival →

$$\phi_D = 0^\circ + \phi$$

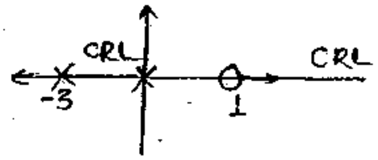
$$\phi_A = 0^\circ - \phi$$

where; $\phi = \Sigma\phi_z - \Sigma\phi_p$

Que.→ $G(s) = \dfrac{Ke^{-s}}{s(s+3)} = \dfrac{K(1-s)}{s(s+3)}$

Soln→ (2.) $P = 2$, $Z = 1$, $P - Z = 1$

(3.)



(6.) BA points :-

$$1 + \frac{K(1-s)}{s(s+3)} = 0$$

$$s(s+3) + K(1-s) = 0$$

$$K = \frac{-s^2 - 3s}{1-s}$$

$$\frac{dK}{ds} = 0$$

$$\frac{(1-s)(-2s-3) - [(-s^2-3s)(-1)]}{(1-s)^2} = 0$$

$$s^2 - 2s - 3 = 0$$

$$s = \frac{2 \pm \sqrt{4+12}}{2}$$

$$s = 1 \pm 2 = -1, 3$$

(7.) $s^2 + s(3-K) + K = 0$

s^2	1	K
s	3-K	0
s^0	K	0

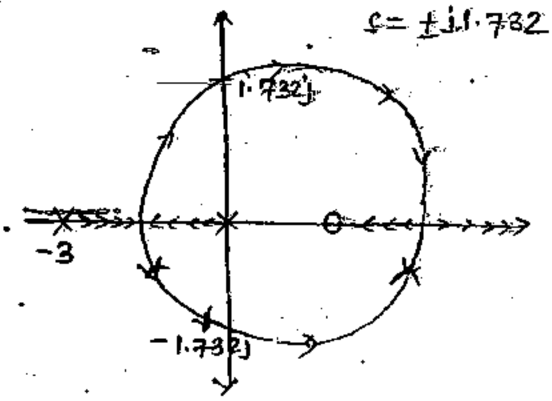
$3-K=0$

$K_{mar}=3$

$A(s)=s^2+K=0$

$s^2+3=0$

$s=\pm j1.732$



(1/67)

$$G(s)=\frac{K(s+a)}{s^2(s+b)}$$

ans (c)

$1+G(s)=0$

$$1+\frac{K(s+a)}{s^2(s+b)}=0$$

$s^3+bs^2+Ks+aK=0$

s^3	1	K
s^2	b	aK
s^1	$\frac{bK-aK}{b}$	0
s^0	aK	0

(i) $aK>0$, $K>0$

(ii) $\frac{bK-aK}{b}>0$

$K(b-a)>0$

$\boxed{K>0}$

$$\frac{bK-aK}{b}=0$$

$K(b-a)=0$

$K=K_{mar}=0$

$A(s)=bs^2+aK=0$

$bs^2+0=0$

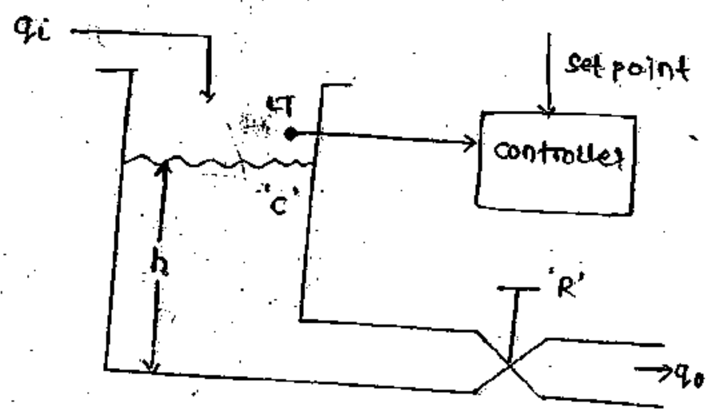
$\boxed{s=0}$

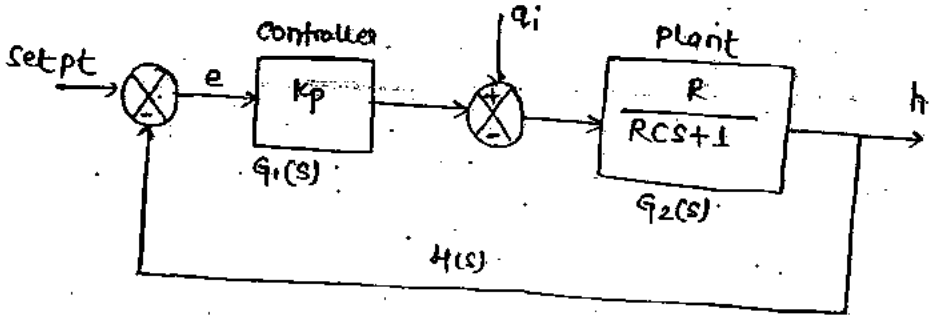
(10/69)

$0\le K<1\rightarrow$ overdamped (1, 3) ans.(c)

$K>5$ overdamped

INDUSTRIAL CONTROLLER





$$|e_{ss}| = \lim_{s \to 0} \frac{S \cdot Q_i(s)\, G_2(s)}{1 + G_1(s) G_2(s)}$$

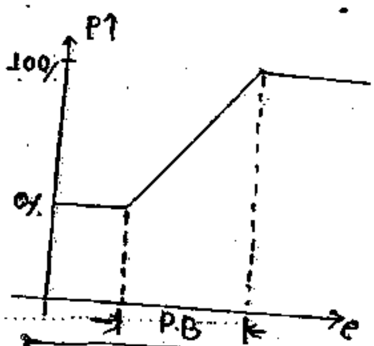
i) Proportional mode →

$P \propto e$

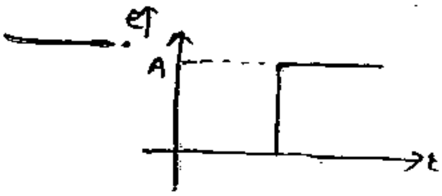
$P = K_p e$; K_p = proportional gain

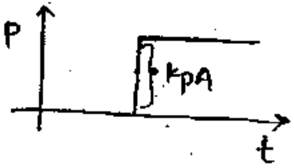
$P(s) = K_p E(s)$

$E(S) \rightarrow \boxed{K_p} \rightarrow P(S)$



$$\boxed{\text{Proportional Band (PB)} = \frac{100}{K_p}}$$





$P = K_p e \quad (e = A)$

$P = K_p A$

$$|e_{ss}| = \lim_{s \to 0} \frac{S \cdot \frac{A}{S} \cdot \frac{R}{RCs+1}}{1 + \frac{RK_p}{RCs+1}}$$

$$\boxed{|e_{ss}| = \frac{AR}{1 + RK_p}}$$
offset

$$\boxed{\text{Offset} \propto \frac{1}{K_p}}$$

* It is a natural extension of on/off controller.
* The band of error where every value of error has unique value of controller o/p is known as proportional band.
* The disadvantage of this controller is it exibits a permanant residual error known as offset.

(20/63)

$$P = K_p e$$
$$100 = K_p \times 1$$
$$K_p = 100$$
$$PB = \frac{100}{K_p} = \frac{100}{100} = 1$$

$$\% PB = 1 \times 100\% = 100\%$$
$$100\% \ PB = 100 V$$
$$20\% \ PB = ?$$
$$\frac{20}{100} \times 100 = 20 V$$

$\boxed{10 V, 20 V}$

(2) Integral mode →

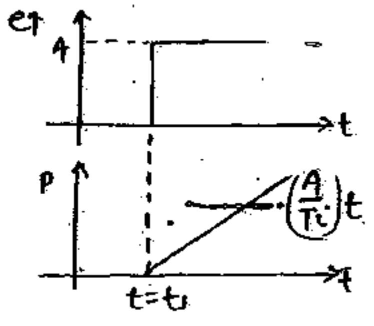
$$\frac{dp}{dt} \propto e$$
$$\frac{\partial p}{dt} = K_I e \quad (K_I = \text{Integral scaling})$$
$$p = K_I \int e \, dt$$
$$P(s) = \frac{K_I}{s} E(s)$$

$$E(s) \longrightarrow \boxed{\frac{K_I}{s}} \longrightarrow P(s)$$

Defining "RESET TIME"

$$T_i = \frac{1}{K_I}$$
$$P = \frac{1}{T_i} \int e \, dt$$
$$P(s) = \frac{1}{T_i s} E(s)$$

$$E(s) \longrightarrow \boxed{\frac{1}{T_i s}} \longrightarrow P(s)$$



$$P = \frac{1}{T_i} \int A \, dt \quad (e = A)$$
$$P = \left(\frac{A}{T_i}\right) t$$

Let $e = \sin\omega t$

$$P = \frac{1}{T_i} \int \sin\omega t \, dt$$
$$P = \frac{1}{\omega T_i} (-\cos\omega t)$$
$$P = \frac{1}{\omega T_i} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$|e_{ss}| = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{A}{\cancel{s}} \times \frac{R}{RCs+1}}{1 + \frac{R}{T_i s (T_1 s)}}$$

$$\boxed{|e_{ss}| = \frac{AR}{1+\infty} = 0}$$

* The <u>disadvantage</u> of integral controller is its responce to errors is <u>slow</u>. However it is capable of eliminating the error completely in the sys.

<u>3) Derivative (OR) Rate mode →</u>

$$P \propto \frac{de}{dt}$$

$$P = k_D \frac{de}{dt}$$

k_D = Rate constant

$$P(s) = k_D s \, E(s)$$

$$E(s) \longrightarrow \boxed{k_D s} \longrightarrow P(s)$$

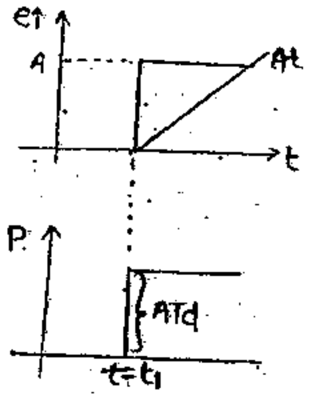
Defining "RATE TIME"

$$T_d = k_D$$

$$P = T_d \frac{de}{dt}$$

$$P(s) = T_d s \cdot E(s)$$

$$E(s) \longrightarrow \boxed{T_d s} \longrightarrow P(s)$$



$$P = T_d \frac{d}{dt}(A) \quad (e = A)$$

$$P = 0$$

$$P = T_d \frac{d}{dt}(At) \quad (e = At)$$

$$P = T_d A$$

$$|e_{ss}| = \lim_{s \to 0} \frac{s \cdot \frac{A}{s^2} \cdot \frac{R}{RCs+1}}{1 + T_d s \cdot \frac{R}{RCs+1}}$$

$$|e_{ss}| = \lim_{s \to 0} \frac{\frac{AR}{RCs+1}}{\frac{s + s^2 T_d R}{RCs+1}}$$

$$|e_{ss}| = \frac{AR}{0} = \infty$$

$$\boxed{|e_{ss}| = \infty}$$

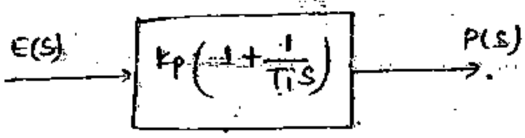
* The disadvantage of this controller is it can't respond to sudden error.
* It is also called as anticipatory controller because it sends a control signal in anticipation of error.
* This anticipatory nature may result in large instability in the system.

* Composite controller mode →

(1) P+I mode →

$$P = k_p e + \frac{k_p}{T_i}\int e\,dt$$

$$P(s) = \left[k_p\left(1+\frac{1}{T_i s}\right)\right]E(s)$$



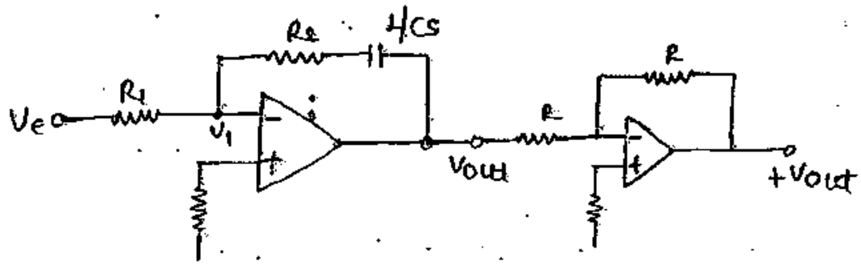
Effect on transient state

let $e = \sin\omega t$

$$P = k_p \sin\omega t + \frac{k_p}{T_i}\int \sin\omega t\,dt$$

$$P = k_p \sin\omega t + \left(\frac{-k_p}{\omega T_i}\right)\cos\omega t$$

$$P = \sqrt{k_p^2 + \left(\frac{k_p}{\omega T_i}\right)^2}\,\sin\left(\omega t - \tan^{-1}\frac{1}{\omega T_i}\right)$$



$$\frac{V_e - V_1}{R_1} = \frac{V_1 - V_{out}}{\frac{R_2 Cs + 1}{Cs}}$$

$\because V_1 = 0$

$$\frac{V_e(R_2 Cs + 1)}{R_1 Cs} = -V_{out}$$

$$-V_{out} = \frac{V_e R_2 \cancel{Cs}}{R_1 \cancel{Cs}} + \frac{V_e}{R_1 Cs}$$

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1}\frac{1}{R_2 C}\int V_e\,dt$$

$$\boxed{k_p = \frac{R_2}{R_1}\ ;\ T_i = R_2 C}$$

* It is capable of improving steady state responce c/s of the sys. i.e. elimination of steady state error b/n i/p & o/p.
* The integral controller eliminates offset of proportional controller.
* It is also known as proportional + Reset controller because the rate of change of controller o/p can be reset by changing the value of reset time Ti.
* For sinusoidal i/p the phase of the controller o/p lags by $\tan^{-1}\left(\frac{1}{\omega T_i}\right)$. Hence it is similar to phase lag compensator.
* It terms of filtering property it acts as low pass filter.
* The P+I controller increases the type & order of system by one.

Effect on performance specification →

1.) It increases rise time.

2.) It reduces BW.

3.) It reduces the stability of the system.

4.) It increases the damping ratio & hence reduces the peak overshoot.

5.) It eliminate steady state error.

2.) P+D mode →

$$P = K_p e + K_p T_d \frac{de}{dt}$$

$$P(s) = \left[K_p(1+T_d s)\right] E(s)$$

$$E(s) \longrightarrow \boxed{K_p(1+T_d s)} \longrightarrow P(s)$$

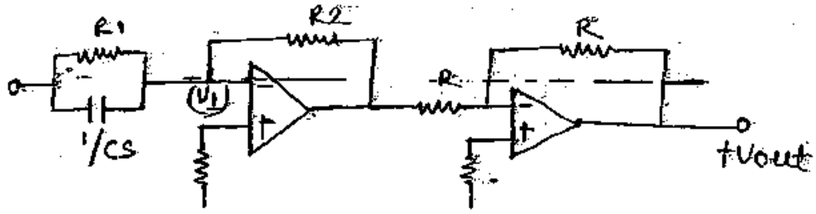
effect on transient state

let $e = \sin\omega t$

$$P = K_p \sin\omega t + K_p T_d \frac{d}{dt}\sin\omega t$$

$$P = K_p \sin\omega t + \omega K_p T_d \cos\omega t$$

$$P = \sqrt{K_p^2 + (\omega K_p T_d)^2}\, \sin\left(\omega t + \tan^{-1}\omega T_d\right)$$



$$\frac{\frac{Ve - V_1}{\frac{R_1}{R_1 cs + 1}}}{} = \frac{V_1 - Vout}{R_2}$$

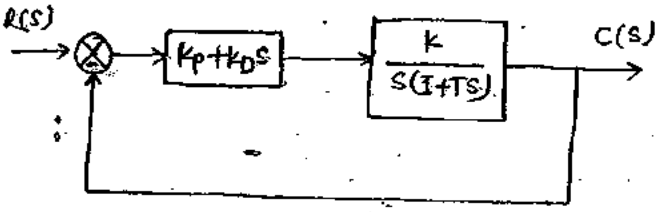
$$-Vout = \frac{Ve(R_1 cs + 1) R_2}{R_1}$$

$$\boxed{k_p = \frac{R_2}{R_1} ;\ T_d = R_1 C}$$

$$-Vout = \frac{R_2}{R_1} Ve + \frac{R_2}{R_1} R_1 cs Ve$$

$$+Vout = \frac{R_2}{V_1} ve + \frac{R_2}{R_1} R_1 C \frac{dve}{dt}$$

chapter(3)
Conv.(4.)



(1) without P+D controller →

$$G(s) = \frac{k}{S(1+TS)}$$

Type-1/order-2

with P+D controller

$$G(s) = \frac{k(k_p + k_D s)}{s(1+TS)}$$

Type-1/order-2

(2) with P-controller →

$$G(s) = \frac{kk_p}{S(1+TS)}$$

$$1 + \frac{kk_p}{S(1+TS)} = 0$$

$$S(1+TS) + kk_p = 0$$

$$TS^2 + s + kk_p = 0$$

$$S^2 + \frac{S}{T} + \frac{k k_p}{T} = 0$$

$$\boxed{\omega_n = \sqrt{\frac{kk_p}{T}}\ r/s}$$

$$2\xi\sqrt{\frac{kk_p}{T}} = \frac{1}{T}$$

$$\boxed{\xi = \frac{1}{2\sqrt{kk_pT}}}$$

(III) With P+D controller →

$$G(s) = \frac{K(K_P + K_D s)}{s(1+Ts)}$$

$$1 + \frac{K(K_P + K_D s)}{s(1+Ts)} = 0$$

$$Ts^2 + s + K_D s + KK_P = 0$$

$$s^2 + \frac{s(1+K_D)}{T} + \frac{KK_P}{T} = 0$$

$$\boxed{\omega_n = \sqrt{\frac{KK_P}{T}}\ r/s}$$

$2\xi\omega_n$

$$2\xi\sqrt{\frac{KK_P}{T}} = \frac{1+KK_D}{T}$$

$$\boxed{\xi = \frac{1+KK_D}{2\sqrt{KK_P T}}}$$

(IV) ess/ unit Ramp i/p →

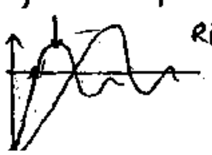
$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{K(K_P + K_D s)}{s(1+Ts)}}$$

$$\boxed{e_{ss} = \frac{1}{KK_P}}$$

* It is capable of improving the transient state c/s of the system only. i.e. it improves the speed of responce of sys.
* for sinusoidal i/p the phase of controller o/p leads by $\tan^{-1}\omega T_d$. Hence it is similar to phase lead compensator.
* In terms of filtering property it acts as HPF.
* The P+D controller does not affect the type & order of the system.

* Effect on performance specification →

1.) Reduces the Rise time.
2.) Increases the BW.
3.) It amplifies noise & hence reduces $\frac{S}{N}$ ratio.
4.) It increases the stability of the sys.
5.) It increases the damping ratio ξ & Hence reduces peak overshoot.



Rise time & peak overshoot ↓.

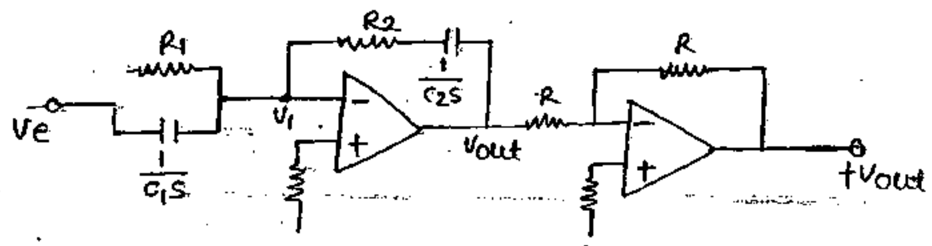
$\frac{14}{62}$ (d) $\frac{15}{63}$ (c) $\frac{16}{63}$ (c) $\frac{18}{63}$ (c) $\frac{8}{77}$ (b) $\frac{9}{77}$ (a)

(3) PID mode →

$$p = k_p e + \frac{k_p}{T_i}\int e\,dt + k_p T_d \frac{de}{dt}$$

$$P(s) = \left[k_p\left(1 + \frac{1}{T_i s} + T_d s\right)\right] E(s)$$

$$E(s) \longrightarrow \boxed{k_p\left(1+\frac{1}{T_i s}+T_d s\right)} \longrightarrow P(s)$$



$$\Rightarrow \frac{V_e - V_1}{\frac{R_1}{R_1C_1s+1}} = \frac{V_1 - V_{out}}{\frac{R_2C_2s+1}{C_2 s}}$$

$$\Rightarrow -V_{out} = \frac{V_e(R_1C_1s+1)(R_2C_2s+1)}{R_1C_2 s}$$

$$\Rightarrow -V_{out} = \frac{V_e(\cancel{R_1}C_1R_2\cancel{C_2}s\cancel{^2})}{\cancel{R_1}\cancel{C_2}\cancel{s}} + \frac{V_e\cancel{s}(R_1C_1+R_2C_2)}{R_1C_2\cancel{s}} + \frac{V_e}{R_1C_2 s}$$

$$\Rightarrow -V_{out} = V_e\left[\frac{R_1C_1}{R_1C_2} + \frac{R_2C_2}{R_1C_2}\right] + \frac{V_e}{R_1C_2 s} + R_2C_1 s V_e$$

$$\Rightarrow +V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}\frac{1}{R_2C_2}\int V_e\,dt + \frac{R_2}{R_1}R_1C_1\frac{dV_e}{dt}$$

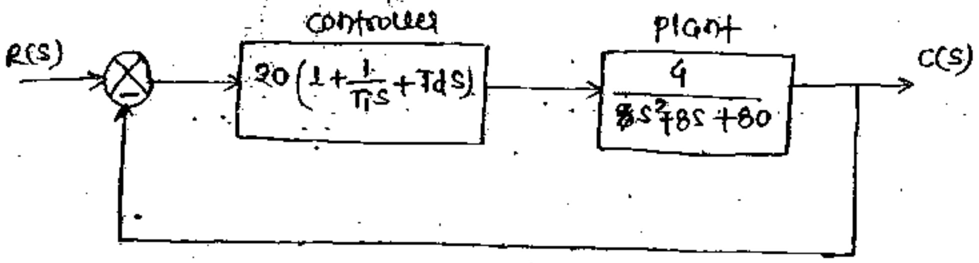
$$\boxed{k_p = \frac{R_2}{R_1},\ T_i = R_2C_2 \\ T_d = R_1C_1}$$

* It improves both transient state & steady state responce c/s.
* It is similar to lag lead compensator.
* In terms of filtering property it acts as band reject filter.

*** Effect on performance specification →**

(1) It reduces rise time.
2) It increases BW.
3) It amplifies noise & Hence reduces S/N ratio.
4) It increases stabilty of the sys.
5) It increases damping ratio ξ & Hence reduces peak overshoot.
6) It eliminates steady state error b/n i/p & o/p.
7) The PID controller increases type & order of sys by 1.

(1/78) $\theta_c(s) = \left\{20\left[1 + \frac{1}{T_i s} + T_d s\right]\right\} E(s)$



(a) $G(s) = \frac{4\times20(1+T_d s)}{s^2+8s+80}$

($T_i s \approx 0$ because given)

$1 + G(s) = 0$

$1 + \frac{80(1+T_d s)}{s^2+8s+80} = 0$

$s^2+8s+80+80(1+T_d s) = 0$

$s^2+s(8+80T_d)+160 = 0$

$\omega_n = \sqrt{160} = 12.64\ r/s$

$2\xi\times12.64 = 8+80T_d$

$\xi = 1$ (given)

$2\times1\times12.64 = 8+80T_d$

$\boxed{T_d = 0.2s}$

(b) $G(s) = \frac{20(T_i s+20+4T_i s^2)4}{T_i s(s^2+8s+40)}$

$1+G(s)=0$

$T_i s^3+8T_i s^2+80T_i s+80T_i s+80+16T_i s^2=0$

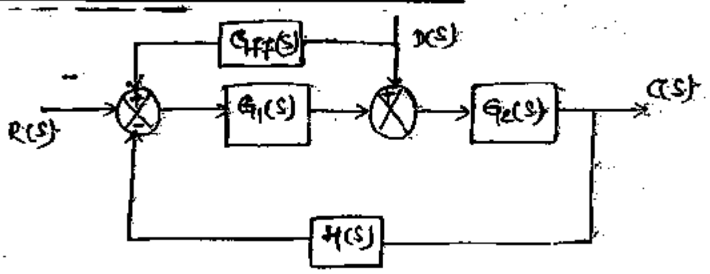
$T_i s^3+24T_i s^2+160T_i s+80=0$

$s^3+24s^2+160s+\frac{80}{T_i}=0$

$\frac{80}{T_i} = 24\times160$

$\boxed{T_i = 0.02s}$

* Feed forward Compensation →



$$\left.\frac{C(s)}{R(s)}\right|_{D(s)=0} = \frac{G_1(s)\cdot G_2(s)}{1+G_1(s)\cdot G_2(s)\cdot H(s)}$$

$$\left.\frac{C(s)}{D(s)}\right|_{R(s)=0} \quad \frac{G_2(s)+G_{ff}(s)\cdot G_1(s)\cdot G_2(s)}{1+G_1(s)\cdot G_2(s)\cdot H(s)}$$

$$C(s) = \frac{R(s)\left[G_1(s)\,G_2(s)\right] + D(s)\left[G_2(s)+G_{ff}(s)\,G_1(s)\,G_2(s)\right]}{1+G_1(s)\,G_2(s)\,H(s)}$$

To eliminate the effect of the disturbances in the sys., the condn for feed forward controller is

$$\boxed{G_{ff}(s) = \frac{-1}{G_1(s)}}$$

$\frac{Q(8)}{cha(8)}$

$$G_1(s) = \frac{k(s+a)(s+c)}{(s+b)(s+d)} \; ; \; G_2(s) = 1$$

$$G_c(s) = \frac{1}{G_1(s)} = \frac{(s+b)(s+d)}{k(s+a)(s+c)}$$

$$\boxed{G_c(s) = \frac{(s+b)(s+d)}{k(s+a)(s+c)}}$$