12. Proportion

Questions Pg-210

1. Question

A person in vests 10000 rupees and 15000 rupees in two different schemes. After one year, he got 900 rupees as interest for the first amount and 1200 rupees as interest for the second amount.

i) Are the interests proportional to the investments?

ii) What is the ratio of the interest to the amount invested in the first scheme? What about the second?

iii) What is the annual rate of interest in the first scheme? And in the second?

Answer

Given:

Amount invested: ₹10000 and ₹15000

Interest after 1 year: ₹ 900 for first amount and ₹1200 for second amount.

i). To check whether interest proportional to the investments.

Interest ratio $= \frac{900}{1200} = \frac{9}{12} = \frac{3}{4}$

Investment ratio $= \frac{10000}{15000} = \frac{10}{15} = \frac{2}{3}$

If Interest ratio is equal to investment ratio, then only both are in proportion.

As we can observe from above solution, interest ratio is not equal to investment ratio.

i.e. $\frac{3}{4} \neq \frac{2}{3}$

ii). Ratio of interest to investment

In first case, amount invested is ₹10000 and interest earned is ₹900

∴ Ratio $= \frac{900}{10000} = \frac{9}{100}$

Now,

In second case, amount invested is ₹15000 and interest earned is ₹1200

∴ Ratio $= \frac{1200}{15000} = \frac{12}{150} = \frac{4}{50} = \frac{2}{25}$

iii). Annual rate of the scheme

Rate $= \frac{\text{Interest earned}}{\text{Amount invested}} \times 100$

In first scheme, rate $= \frac{900}{10000} \times 100$

$\Rightarrow \frac{9}{100} \times 100 = 9\%$

∴ Annual rate in this scheme is 9% per annum.

In second scheme, rate $= \frac{1200}{15000} \times 100$

$\Rightarrow \frac{12}{150} \times 100$

$\Rightarrow \frac{12}{6} \times 4$

$\Rightarrow 2 \times 4 = 8\%$

∴ Annual rate in this scheme is 8% per annum.

2. Question

The area of A0 paper is one square metre. Calculate the lengths of the sides of A4 paper correct to a millimetre, using a calculator.

Answer

Given:

Area of A0 paper = $1m^2$

Area of A1 paper = half of $1 = \frac{1}{2}\mathrm{m}^2$

Area of A2 paper = half of $\frac{1}{2} = \frac{1}{4}\mathrm{m}^2$

Area of A3 paper = half of $\frac{1}{4} = \frac{1}{8}\mathrm{m}^2$

Area of A2 paper = half of $\frac{1}{8} = \frac{1}{16}\mathrm{m}^2$

Now we calculate the sides of A4 sizes:

Let the length be l and width be w

Area = Length × width

$\Rightarrow \frac{1}{16} = \mathrm{l} \times \mathrm{w}$

We know that, l = √2 × w

$\Rightarrow \frac{1}{16} = (\sqrt{2}\,\mathrm{w} \times \mathrm{w}$

$\Rightarrow \frac{1}{16} = \sqrt{2}\,\mathrm{w}^2$

$\Rightarrow \mathrm{w}^2 = \frac{1}{16\times\sqrt{2}}$

⇒ w^2 = 0.04419

⇒ w = √0.04419

⇒ w = 0.2102 m

⇒ w = 210.2 mm

So, length = √2 × 210.2 mm

= 1.414 × 210.2

= 297.3 mm

3. Question

In calcium carbonate, the masses of calcium, carbon and oxygen are in the ratio 10: 3: 12. When 150 grams of a compound was analysed, it was found to contain 60 grams of calcium, 20 grams of carbon and 70 grams of oxygen. Is it calcium carbonate?

Answer

Given:

Ratio of masses of calcium, carbon and oxygen in calcium carbonate is 10: 3: 12

Now,

In 150 grams, 60 gram of calcium, 20gram of carbon and 70gram of oxygen

Ratio of masses in 150gram of a compound

⇒ 60: 20: 70

⇒ 6: 2: 7

As we observe that ratio given for calcium carbonate is 10: 3: 12 but in 150 grams of compound the ratio is 6: 2: 7.

∴ 150gram compound is not the calcium carbonate.

Questions Pg-213

1 A. Question

For each pair of quantities given below, check whether the first is proportional to the second. For proportional quantities, calculate the constant of proportionality.

Perimeter and radius of circles.

Answer

Perimeter and radius of circle.

Let the perimeter be y and radius of circle be r.

Perimeter = $2\pi r$

⇒ $y = 2 \times \pi \times r$

⇒ $y = 2\pi r$

We can whatever value we like for x on right side of the above equation. So, x is a variable.

Whatever value we give for r, that the value will be multiplied by a constant 2π. So, y will always be proportional to nr.

1 B. Question

For each pair of quantities given below, check whether the first is proportional to the second. For proportional quantities, calculate the constant of proportionality.

Area and radius of circles.

Answer

Area and radius of circle

Let the area be y and radius be r

Area = πr^2

⇒ $y = \pi \times r^2$

⇒ $y = \pi x^2$

We can whatever value we like for r on right side of the above equation. So, r is a variable.

Whatever value we give for r, that the value will be multiplied by a constant π and the variable r. That means r is not multiplied by the same constant every time.

So, y is not proportional to r.

1 C. Question

For each pair of quantities given below, check whether the first is proportional to the second. For proportional quantities, calculate the constant of proportionality.

The distance travelled and the number of rotations of a circular ring moving along a line.

Answer

The distance travelled and the number of rotations of a circular ring moving along a line.

Let the distance travelled be d and the number of rotation be n.

We know,

The distance travelled in one rotation = $2\pi r$

where r is the radius of the circle

so, distance travelled in n rotation =d = $2n\pi r$

We can whatever value we like for n on right side of the above equation. So, n is a variable. But r is a constant, we are considering the movement of a single ring.

Whatever value we give for n, that the value will be multiplied by a constant $2\pi r$. So, d will always be proportional to n.

1 D. Question

For each pair of quantities given below, check whether the first is proportional to the second. For proportional quantities, calculate the constant of proportionality.

The interest got in any year and the amount deposited in a scheme in which interest in compounded annually.

Answer

The interest got in any year and the amount deposited in a scheme in which interest in compounded annually.

Let the interest got in year be ‘i' and the amount deposited be p.

We know that,

$i = p \times r$ (where r is the rate of interest)

We can whatever value we like for p on right side of the above equation. So, p is a variable. But r is a constant for a scheme.

Whatever value we give for p, that the value will be multiplied by a constant r. So, i will always be proportional to p.

1 E. Question

For each pair of quantities given below, check whether the first is proportional to the second. For proportional quantities, calculate the constant of proportionality.

The volume of water poured into a hollow prism and the height of the water level.

Answer

The volume of water poured into a hollow prism and the height of the water level.

Let the volume of hollow prism be v and height of water level be h.

Let the base area of the prism be ‘a’

The poured volume = volume of the water column formed in the prism

But volume for water column in the prism = base × height

$\Rightarrow v = a \times h$

$\Rightarrow v = ah$

$\Rightarrow h = \frac{v}{a}$

We can whatever value we like for v on right side of the above equation. So, v is a variable. But a is a constant for a prism.

Whatever value we give for n, that the value will be multiplied by a constant $\frac{1}{a}$. So, d will always be proportional to n.

2 A. Question

During rainfall, the volume of water falling in each square metre may be considered equal.

Prove that the volume of water falling in a region is proportional to the area of the region.

Answer

Let us consider a surface with three squares of height h and side 1m on a region.

On this surface, we can mark a random number of squares, each side of side 1m. So, each of such squares will have an area of $1m^2$.

It is stated in the question that, “the volume of water falling in each square metre may be considered equal”.

So, each of the three squares will receive equal volume of water.

If this water is not allowed to sun off and if there is no sewage into the ground, the water received will become a water column. There will be three water columns.

These water columns are prism with square base. And they have the same volume (V), because, according to volume of water in each square should be considered equal.

We know that,

Volume = base area × height

But base area of all prism = $1m^2$

$\Rightarrow V = 1 \times h$

$\Rightarrow V = h$

But V is same for all three prisms.

2 B. Question

During rainfall, the volume of water falling in each square metre may be considered equal.

Explain why the heights of rainwater collected in different sized hollow prisms kept near one another are equal.

Answer

Let us consider two hollow prisms kept near to one another with different base.

We can think of random number of water columns inside these prisms. Let the base of these columns be squares of area $1cm^2$.

Then each water column will be a square of prism of base area $1cm^2$.

We have seen in above part, that the heights of all water prisms be equal. Regardless of whether they are in the first hollow prism or second hollow prism.

Let the height be h.

Let the base area of first prism = a_1

Let the base area of second prism = a_2

Then total number of water prisms (each with base area $1cm^2$) in first prism $= \frac{a_1}{1} = a_1$

Then total number of water prisms (each with base area $1cm^2$) in second prism $= \frac{a_2}{1} = a_2$

All the a_1 and a_2 in the first prism and second prism will have the same heights h.

That means, the water level in both prism will be h.

So, whatever number of hollow prism (of different base sizes) we place near one another, after the rainfall, the height of water in all will be the same.

Consider two paddy fields in a locality. Let their areas be a_1 and a_2. If the water is allowed to run off and if there is no sewage into the ground, there will be two water prisms. Each will cover the entire area of the respective field.

The volume of first field = $v_1 = a_1h$

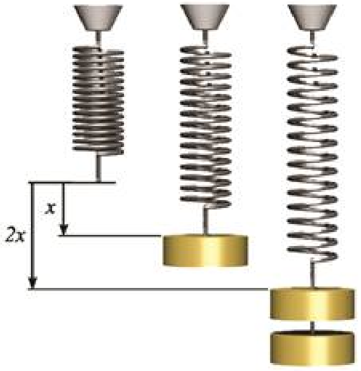
The volume of second field = $v_2 = a_2h$

h is a constant. So, if area increases, volume increases and if area decreases, volume decreases.

That means volume is proportional to the area.

3. Question

When a weight is suspended by a spring, the extension is proportional to the weight. Explain how this can be used to mark weights on a spring balance.



Answer

Let the different position of spring 0, 1 and 2.

Position 0 shows the situation when no load is applied on the spring

Position 1 shows the situation when a load of w_1 is applied on the spring.

We can see that there is an extension of x cm from the initial position.

Position 2 shows the situation when a load of w_2 is applied on the spring.

We can see that there is extension of x_2 cm from the initial position.

It is given in the question that; the extension is proportional to the applied load.

So, any extension x will be proportional to the weight w that produces that extension.

We can write: x = a constant k × w

$\Rightarrow x = kw$

$\Rightarrow k = \frac{x}{w}$

The constant ‘k’ is called the spring constant. Every spring will have a particular value of spring constant. We want to find this constant for our spring.

For that, we adopt the following procedure:

Put a known weight w_1. Measure the extension x_1.

Then $k = \frac{x_1}{w_1}$

Put another known weight w_2. Measure the extension x_2.

Then $k = \frac{x_2}{w_2}$

Since k is a constant, we will get the same value for k in both the weight.

Repeat the trial with several known weights. In all cases we will get the same k.

Once k is obtained, we can make the markings on the spring balance.

The spring is fixed inside an outer casting. Markings are made on this casing.

When the spring is at Zero load position, mark that position on the casing as 0kg.

Now we want to mark the 1kg position

Let the extension for a load of 1kg be x_1 kg

$x_1 = k \times 1$ kg = k

we have already calculated k. so we get x_1 kg.

Measure this x_1 kg from the 0kg mark on the casing and mark it as 1kg.

Now we want to mark the 2kg position

Let the extension for a load of 2kg be x_2 kg

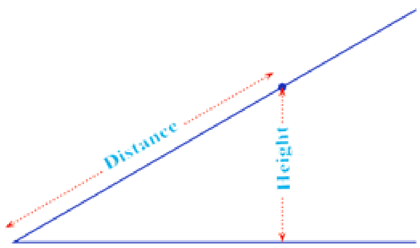
$X_2 = k \times 2$ kg = 2k

we have already calculated k. so we get x_2 kg.

Measure this x_2 kg from the 0kg mark on the casing and mark it as 2kg.

4. Question

In the angle shown below, for different points on the slanted line, as the distance from the vertex of the angle changes, the height from the horizontal line also changes.



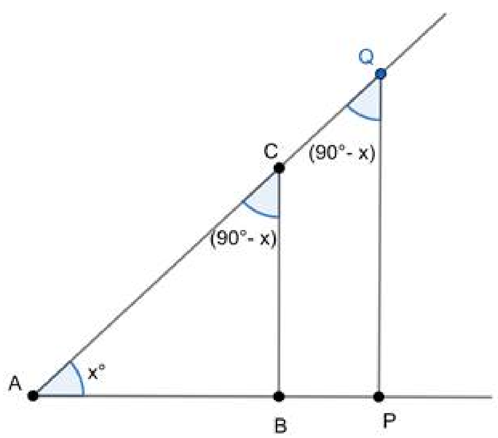
i) Prove that height is proportional to the distance.

ii) Calculate the constant of proportionality for 30°, 35° and 60° angles.

Answer

i). The given angle in question figure is reproduced in below figure.

Let the blue object which moves along the slanting line is marked as Q and perpendicular is dropped from Q to the horizontal leg of the given angle. The foot of perpendicular is marked as P.



So, we get a triangle Δ APQ

There is another right-angle ΔABC. This is our base triangle. That is, we are going to calculations based on Δ ABC:

We assume that BC is fixed at its position and also that all sides of Δ ABC are known.

But PQ is not fixed. Because, Q can be at any point along the slanting line.

Now, we find relation between Δ ABC and Δ APQ:

∠ A is denoted as x° (It is same for both the Δ)

∠ P and ∠ B are 90°

∠ c and ∠ Q will both be equal to (90 – x) °

So, both the triangles have the same angles. Therefore, they are similar.

i.e. Δ ABC ≅ Δ APQ

$$\frac{\text{Side opposite to } \angle x \text{ in } \Delta ABC}{\text{Side opposite to } \angle x \text{ in } \Delta APQ}$$

$$= \frac{\text{Side opposite to } \angle 90 - x° \text{ in } \Delta ABC}{\text{Side opposite to } \angle 90 - x° \text{ in } \Delta APQ}$$

$$= \frac{\text{Side opposite to } \angle 90 \text{ in } \Delta ABC}{\text{Side opposite to } \angle 90 \text{ in } \Delta APQ}$$

$$\Rightarrow \frac{BC}{PQ} = \frac{AB}{AP} = \frac{AC}{AQ}$$

From the above,

$$\frac{BC}{PQ} = \frac{AC}{AQ}$$

∵ BC and PQ are heights and AC and AQ are distance.

$$\Rightarrow \frac{BC}{PQ} \times AQ = AC$$

$$\Rightarrow \frac{AQ}{PQ} = \frac{AC}{BC}$$

$$\Rightarrow AQ = \frac{AC}{BC} \times PQ$$

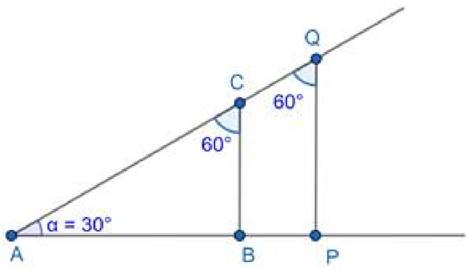
$\frac{AC}{BC}$ is constant bcause Δ ABC is fixed.

⇒ AQ = k × PQ

That means, the distance of Q from the vertex A is proportional to the height from Q from the horizontal line.

ii). In this part we explore the cases when the angle x at vertex A is 30°, 60° and 45°.

First, we will take 30°.



A base triangle Δ ABC is drawn such that AC = 2cm, BC = 1cm and AB = √3

As we see in figure, here also Δ ABC and Δ APQ are similar

So, we get:

$$\frac{\mathrm{BC}}{\mathrm{PQ}} = \frac{\mathrm{AB}}{\mathrm{AP}} = \frac{\mathrm{AC}}{\mathrm{AQ}}$$

∵ BC and PQ are heights and AC and AQ are distance.

$\Rightarrow \frac{\mathrm{BC}}{\mathrm{PQ}} \times \mathrm{AQ} = \mathrm{AC}$

$\Rightarrow \mathrm{AQ} = \frac{\mathrm{AC}}{\mathrm{BC}} \times \mathrm{PQ}$

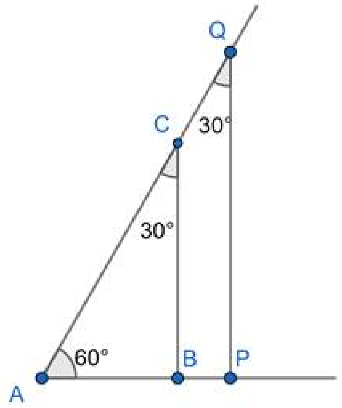
$\Rightarrow \mathrm{AQ} = \frac{2}{1} \times \mathrm{PQ}$

⇒ AQ = 2 PQ

In the above result, ‘2’ is constant. So AQ is proportional to PQ.

That means the distance of Q from the vertex A is proportional to the height Q from the horizontal line.

Now take x°= 60°,



A base triangle Δ ABC is drawn such that AC = 2cm, AB = 1cm and BC = √3

As we see in figure, here also Δ ABC and Δ APQ are similar

So, we get:

$$\frac{\mathrm{BC}}{\mathrm{PQ}} = \frac{\mathrm{AB}}{\mathrm{AP}} = \frac{\mathrm{AC}}{\mathrm{AQ}}$$

∵ BC and PQ are heights and AC and AQ are distance.

$\Rightarrow \frac{\mathrm{BC}}{\mathrm{PQ}} \times \mathrm{AQ} = \mathrm{AC}$

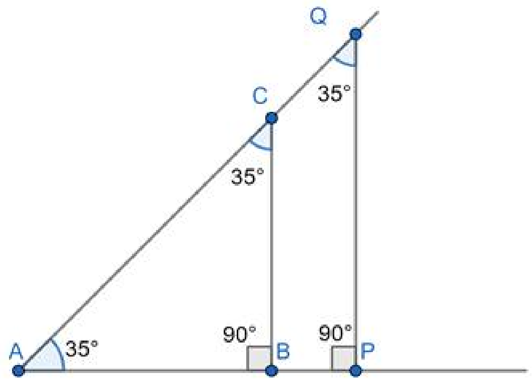
$\Rightarrow \mathrm{AQ} = \frac{\mathrm{AC}}{\mathrm{BC}} \times \mathrm{PQ}$

$\Rightarrow \mathrm{AQ} = \frac{2}{\sqrt{3}} \times \mathrm{PQ}$

In the above result, $\frac{2}{\sqrt{3}}$ is constant. So AQ is proportional to PQ.

That means the distance of Q from the vertex A is proportional to the height Q from the horizontal line.

Now take x°= 35°,



A base triangle Δ ABC is drawn such that AC = √2cm, AB = 1cm and BC = 1cm

As we see in figure, here also Δ ABC and Δ APQ are similar

So, we get:

$\frac{BC}{PQ} = \frac{AB}{AP} = \frac{AC}{AQ}$

∵ BC and PQ are heights and AC and AQ are distance.

$\Rightarrow \frac{BC}{PQ} \times AQ = AC$

$\Rightarrow AQ = \frac{AC}{BC} \times PQ$

$\Rightarrow AQ = \frac{\sqrt{2}}{1} \times PQ$

⇒ AQ = √2 PQ

In the above result, $\sqrt{2}$ is constant. So AQ is proportional to PQ.

That means the distance of Q from the vertex A is proportional to the height Q from the horizontal line.

Questions Pg-217

1 A. Question

Prove that for equilateral triangles, area is proportional to the square of the length of a side. What is the constant of proportionality?

Answer

We know that area of equilateral triangle is given by a $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

Where s is the length of side.

Put $s^2 = q$

Then $a = \frac{\sqrt{3}}{4} q$

Here, $\frac{\sqrt{3}}{4}$ is constant

When ‘q’ increases ‘a’ increases

When ‘q’ decreases ‘a’ decreases

So, ‘a’ is proportional to q. That means, ‘a’ is proportional to the square of the side

The constant of proportionality is $\frac{\sqrt{3}}{4}$.

1 B. Question

For squares, is area proportional to square of the length of a side? If so, what is the constant of proportionality?

Answer

Let us consider a square of side ‘s’ and its area ‘a’

Let s = 1cm

Area = $(\text{side})^2$

∴ a = 1 × 1 = $1cm^2$

Now, let us change the side and see how it affects the area:

Let s = 2 cm, now a = $(2)^2$ = 4 cm^2

Let s = 2.25 cm, now a = $(2.25)^2$ = 5.0625 cm^2

Let s = 3 cm, now a = $(3)^2$ = 9 cm^2

Let s = 0.4 cm, now a = $(0.4)^2$ = 0.16 cm^2

Now we will write the above result in a tabular form, and calculate a/s ratio in each case:

Side (s)	1	2	2.25	3	0.4
Area (a)	1	4	5.0625	9	0.16
a/s ratio	1	2	2.25	3	0.4

From the table, we can see that a/s ratio is not a constant. We will not get ‘a’ by multiplying ‘s’ by a fixed number. So, a is not proportional to s.

2. Question

In rectangles of area one square metre, as the length of one side changes, so does the length of the other side. Write the relation between the lengths as an algebraic equation. How do we say this in the language of proportions?

Answer

Let the area of rectangle be a, length of the rectangle be x and breadth be y.

Then the area of rectangle = Length × breadth

⇒ a = x × y

⇒ a = xy

Area is given as $1m^2$, which is constant.

⇒ xy = 1

This is the algebraic equation which gives the relation between length and breadth of a rectangle whose area is $1m^2$.

Now we see if there is any proportionality between length and breadth

We have xy = 1. So, length or breadth increases, the other decreases.

Also, if length or breadth decreases, the other increases

Their product will remain constant only if this simultaneous increase and decrease take place.

We can write: $x = \frac{1}{y}$

⇒ $x = 1 \times \frac{1}{y}$

So, length is proportional to the reciprocal of the breadth.

That means, length and width are inversely proportional.

The constant of proportionality is 1.

3. Question

In triangles of the same area, how do we say the relation between the length of the longest side and the length of the perpendicular from the opposite vertex? What if we take the length of the shortest side instead?

Answer

Let us consider triangles of same area.

That means area is constant. Let it be ‘a’.

Let the length of the longest side be x and length of the perpendicular from the opposite vertex to this longest side be y.

Then we have, $a = \frac{1}{2} \times x \times y$

$\Rightarrow a = \frac{1}{2}xy$

⇒ xy = 2a

Here 2a is constant. So, if x or y increases, the other decreases.

Also, if x or y decreases, the other increases.

Their product will remain constant only if this simultaneous increase and decrease take place.

We can write: $x = \frac{2a}{y}$

$\Rightarrow x = 2a \times \frac{1}{y}$

So, x is inversely proportional to y

If we change the shape of the triangle while keeping the area the same, the longest side that we considered may become the shortest side. Then y should increase proportionately so that area will remain the same.

4. Question

In regular polygons, what is the relation between the number of sides and the degree measure of an internal angle? Can it be stated in terms of proportion?

Answer

Let us consider a regular polygon.

We know how to calculate the sum of all its interior angles.

s = 180 (n – 2)

Where, s = sum

N is the number of sides of the regular polygon

Let us use this formula for a triangle:

For a triangle, n = 3

So, s = 180 × (3 – 2)

= 180 × 1

= 180

Let us use this formula for a square:

For a square, n = 4

So, s = 180 × (4 – 2)

= 180 × 2

= 360

Let us use this formula for a pentagon:

For a square, n = 5

So, s = 180 × (5 – 2)

= 180 × 3

= 540

Let us tabulate the results:

No. of Side (n)	3	4	5	6	7
Angle (s)	180	360	540	720	900
s/n ratio	60	90	108	120	128.57

From the above table we can see that s/n is not a constant. So, s is not proportional to n.

Let us modify the formula

Let s = 180 × m

Where s = sum

M = (n-2)

N is the number of side of the regular polygon

No. of Side	n	3	4	5	6	7
n-2	m	1	2	3	4	5
Angle	s	180	360	540	720	900
s/m ratio		180	180	180	180	180

We can see that s is proportional to m. The constant of proportionality is 180

In ordinary language, we can say this:

The sum of interior angle of a regular polygon is proportional to ‘2 less than the number of sides’.

5. Question

A fixed volume of water is to flow into a rectangular water tank. The rate of flow can be changed by using different pipes. Write the relations between the following quantities as an algebraic equation and in terms of proportions.

i) The rate of water flow and the height of the water level.

ii) The rate of water flow and the time taken to fill the tank.

Answer

i). Let the rate of flow be ‘r’ m^3/s.

That means, in 1 second, ‘r’ m^3 of water will enter the tank.

Let the base area of the tank be ‘a’m^2.

Then in the 1^{st} second, that is, when t = 1, the height of water level will be:

$\frac{\text{volume}}{\text{base area}} = \frac{r}{a}$

[∵ after 1 second, the volume in the tank will be r m^3]

In the 2^{nd} second, that is, when t = 2, the height of water level will be:

$\frac{\text{volume}}{\text{base area}} = \frac{2r}{a}$

[∵ after 2 seconds, the volume in the tank will be 2r m^3]

In the 3^{rd} second, that is, when t = 3, the height of water level will be:

$\frac{\text{volume}}{\text{base area}} = \frac{3r}{a}$

So, we can write:

The height of water after the n^{th} second = h $= \frac{nr}{a}$

$\Rightarrow \frac{n}{a} \times r$

n is a constant because we will put a particular value of n. We want the height of water at that n

a is also constant

So, $\frac{n}{a}$ is a constant.

Thus, we have a relation between two quantities:

Height 'h' at the n^{th} second

Rate of flow 'r'.

We can write: h = kr.

This is the algebraic equation.

Where k $= \frac{n}{a}$

From the equation, we can see that h is directly proportional to r.

The constant of proportionality is $\frac{k}{a}$.

ii). We can use the same equation used above, i.e.

The height of water after the n^{th} second = h $= \frac{nr}{a}$

$\Rightarrow h = \frac{n}{a} \times r$

In this case, h is a constant because of the following two reasons:

1. A fixed volume of water is flowing into the tank.

2. When the tank is filled, it will have a particular value of 'h'.

n is the number of seconds required to fill the tank. It will change if the rate 'r' is increased or decreased.

So, n and r are the variables. Let us bring them to opposite side of the '=' sign:

$$h = \frac{n}{a} \times r$$

$\Rightarrow r = \frac{ah}{n}$

$\Rightarrow r = ah \times \frac{1}{n}$

This is the algebraic equation.

‘ah’ is a constant. Let it be ‘k’.

We can write:

$$r = k \times \frac{1}{n}$$

So, r is proportional to the reciprocal of n. That means n is inversely proportional to r.

If r increases and ‘n’ decreases, indicating a lesser time sufficient to fill up the tank.

If r decreases and ‘n’ increases, indicating a greater time required to fill up the tank.