Chapter : 12. CIRCLES

Exercise : 12A

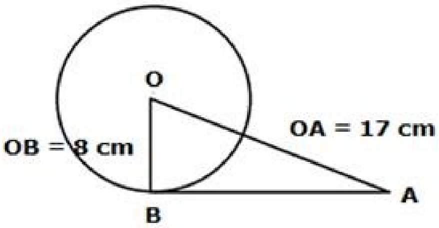
Question: 1

Find the length o

Solution:

Let us consider a circle with center O and radius 8 cm.

The diagram is given as:



Consider a point A 17 cm away from the center such that OA = 17 cm

A tangent is drawn at point A on the circle from point B such that OB = radius = 8 cm

To Find: Length of tangent AB = ?

As seen OB ⊥ AB

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled ΔAOB, By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OA)^2 = (OB)^2 + (AB)^2$

$(17)^2 = (8)^2 + (AB)^2$

$289 = 64 + (AB)^2$

$(AB)^2 = 225$

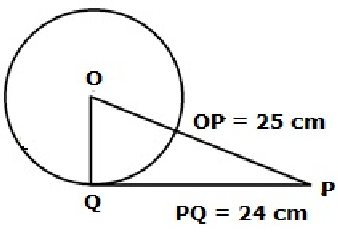
AB = 15 cm

∴ The length of the tangent is 15 cm.

Question: 2

A point P is 25 c

Solution:



Let us consider a circle with center O.

Consider a point P 25 cm away from the center such that OP = 25 cm

A tangent PQ is drawn at point Q on the circle from point P such that PQ = 24 cm

To Find : Length of radius OQ = ?

Now, OQ ⊥ PQ

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled △POQ,

By Pythagoras Theorem,

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OQ)^2 + (PQ)^2$

$(25)^2 = (OQ)^2 + (24)^2$

$625 = (OQ)^2 + 576$

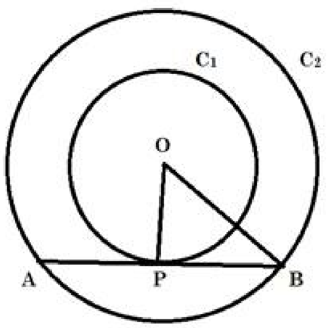
$(OQ)^2 = 49$

OQ = 7 cm

Question: 3

Two concentric ci

Solution:



Given: Two concentric circles (say C_1 and C_2) with common center as O and radius r_1 = 6.5 cm and r_2 = 2.5 cm respectively.

To Find: Length of the chord of the larger circle which touches the circle C_2. i.e. Length of AB.

As AB is tangent to circle C_2 and we know that "Tangent at any point on the circle is perpendicular to the radius through point of contact"

So, we have,

$OP \perp AB$

∴ OPB is a right - angled triangle at P,

By Pythagoras Theorem in △OPB

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

We have,

$(OP)^2 + (PB)^2 = (OB)^2$

${r_2}^2 + (PB)^2 = {r_1}^2$

$(2.5)^2 + (PB)^2 = (6.5)^2$

$6.25 + (PB)^2 = 42.25$

$(PB)^2 = 36$

PB = 6 cm

Now, AP = PB ,

[as perpendicular from center to chord bisects the chord and $OP \perp AB$]

So,

AB = AP + PB = PB + PB

= 2PB = 2(6)

= 12 cm

Question: 4

In the given figu

Solution:

Let AD = x cm, BE = y cm and CF = z cm

As we know that,

Tangents from an external point to a circle are equal,

In given Figure we have

AD = AF = x [Tangents from point A]

BD = BE = y [Tangents from point B]

CF = CE = z [Tangents from point C]

Now, Given: AB = 12 cm

AD + BD = 12

x + y = 12

y = 12 – x.... [1]

and BC = 8 cm

BE + EC = 8

y + z = 8

12 - x + z = 8 [From 1]

z = x – 4.... [2]

and

AC = 10 cm

AF + CF = 10

x + z = 10 [From 2]

x + x - 4 = 10

2x = 14

x = 7 cm

Putting value of x in [1] and [2]

y = 12 - 7 = 5 cm

z = 7 - 4 = 3 cm

So, we have AD = 7 cm, BE = 5 cm and CF = 3 cm

Question: 5

In the given figu

Solution:

Given: PA and PB are tangents to a circle with center O

To show : A, O, B and P are concyclic i.e. they lie on a circle i.e. AOBP is a cyclic quadrilateral.

Proof:

$OB \perp PB$ and $OA \perp AP$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle OBP = \angle OAP = 90°$

$\angle OBP + \angle OAP = 90 + 90 = 180°$

AOBP is a cyclic quadrilateral i.e. A, O, B and P are concyclic.

[As we know if the sum of opposite angles in a quadrilateral is 180° then quadrilateral is cyclic]

Hence Proved.

Question: 6

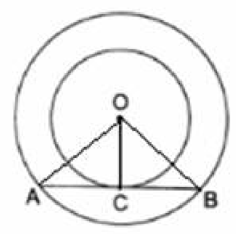
In the given figu

Solution:

Given: Two concentric circles with common center as O

To Prove: AC = CB

Construction: Join OC, OA and OB



Proof :

$OC \perp AB$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

In $\triangle OAC$ and $\triangle OCB$, we have

OA = OB

[∵ radii of same circle]

OC = OC

[∵ common]

$\angle OCA = \angle OCB$

[∵ Both 90° as $OC \perp AB$]

$\triangle OAC \cong \triangle OCB$

[By Right Angle - Hypotenuse - Side]

AC = CB

[Corresponding parts of congruent triangles are congruent]

Hence Proved.

Question: 7

From an external

Solution:

Given : From an external point P, two tangents, PA and PB are drawn to a circle with center O. At a point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. And PA = 14 cm

To Find : Perimeter of $\triangle PCD$

As we know that, Tangents drawn from an external point to a circle are equal.

So we have

AC = CE ...[1] [Tangents from point C]

ED = DB ...[2] [Tangents from point D]

Now Perimeter of Triangle PCD

= PC + CD + DP

= PC + CE + ED + DP

= PC + AC + DB + DP [From 1 and 2]

= PA + PB

Now,

PA = PB = 14 cm as tangents drawn from an external point to a circle are equal

So we have

Perimeter = PA + PB = 14 + 14 = 28 cm

Question: 8

A circle is inscr

Solution:

As we know that tangents drawn from an external point to a circle are equal ,

In the Given image we have,

AP = AR = 7 cm ....[1]

[tangents from point A]

CR = QC = 5 cm ....[2]

[tangents from point C]

BQ = PB ...[3]

[tangents from point B]

Now,

AB = 10 cm [Given]

AP + PB = 10 cm

7 + PB = 10 [From 1]

PB = 3 cm

BQ = 3 cm .....[4]

[From 3]

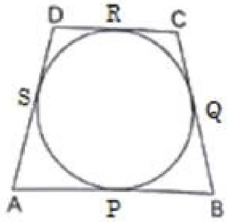
BC = BQ + QC = 5 + 3 = 8 cm [From 2 and 4]

Question: 9

In the given figu

Solution:

Let sides AB, BC, CD, and AD touches circle at P, Q, R and S respectively.



As we know that tangents drawn from an external point to a circle are equal,

In the given image we have,

AP = AS = w (say) [Tangents from point A]

BP = BQ = x (say) [Tangents from point B]

CP = CR = y (say) [Tangents from point C]

DR = DS = z (say) [Tangents from point D]

Now,

Given,

AB = 6 cm

AP + BP = 6

w + x = 6 ...[1]

BC = 7 cm

BP + CP = 7

x + y = 7 ....[2]

CD = 4 cm

CR + DR = 4

y + z = 4 ....[3]

Also,

AD = AS + DS = w + z ....[4]

Add [1] and [3] and substracting [2] from the sum we get,

w + x + y + z - (x + y) = 6 + 4 - 7

w + z = 3 cm ; From [4]

AD = 3 cm

Question: 10

In the given figu

Solution:

As we know that tangents drawn from an external point to a circle are equal,

BR = BP [Tangents from point B] [1]

QC = CP [Tangents from point C] [2]

AR = AQ [Tangents from point A] [3]

As ABC is an isosceles triangle,

AB = BC [Given] [4]

Now substract [3] from [4]

AB - AR = BC - AQ

BR = QC

BP = CP [From 1 and 2]

∴ P bisects BC

Hence Proved.

Question: 11

In the given figu

Solution:

In given Figure,

OA ⊥ AP

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled △OAP,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OA)^2 + (PA)^2$

Given, PA = 10 cm and OA = radius of outer circle = 6 cm

$(OP)^2 = (6)^2 + (100)^2$

$(OP)^2 = 36 + 100 = 136$ [1]

Also,

OB ⊥ BP

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled △OBP,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OB)^2 + (PB)^2$

Now, OB = radius of inner circle = 4 cm

And from [2]

$(OP)^2 = 136$

$136 = (4)^2 + (PB)^2$

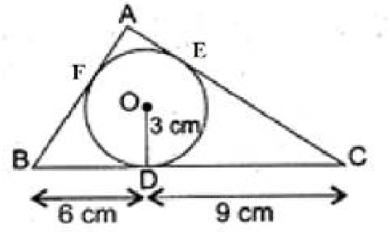
$(PB)^2 = 136 - 16 = 120$

PB = 10.9 cm

Question: 12

In the given figu

Solution:



Given : △ABC that is drawn to circumscribe a circle with radius r = 3 cm and BD = 6 cm DC = 9cm

Also, area(△ABC) = 54 cm^2

To Find : AB and AC

Now,

As we know tangents drawn from an external point to a circle are equal.

Then,

FB = BD = 6 cm [Tangents from same external point B]

DC = EC = 9 cm [Tangents from same external point C]

AF = EA = x (let) [Tangents from same external point A]

Using the above data, we get

AB = AF + FB = x + 6 cm

AC = AE + EC = x + 9 cm

BC = BD + DC = 6 + 9 = 15 cm

Now we have heron's formula for area of triangles if its three sides a, b and c are given

$$ar = \sqrt{s(s-a)(s-b)(s-c)}$$

Where,

$$\Rightarrow s = \frac{a+b+c}{2}$$

So for △ABC

a = AB = x + 6

b = AC = x + 9

c = BC = 15 cm

$$\Rightarrow s = \frac{x+6+x+9+15}{2} = x + 15$$

And

$$ar(\triangle ABC) = \sqrt{(x+15)\big(x+15-(x+6)\big)\big(x+15-(x+9)\big)(x+15-15)}$$

$$\Rightarrow 54 = \sqrt{(x+15)(9)(6)(x)}$$

Squaring both sides, we get,

54(54) = 54x(x + 15)

x^2 + 15x - 54 = 0

x^2 + 18x - 3x - 54 = 0

x(x + 18) - 3(x + 18) = 0

(x - 3)(x + 18) = 0

x = 3 or - 18

but x = - 18 is not possible as length can't be negative.

So

AB = x + 6 = 3 + 6 = 9 cm

AC = x + 9 = 3 + 9 = 12 cm

Question: 13

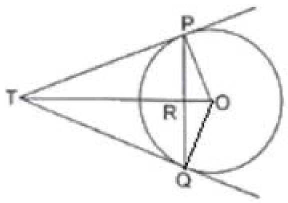
PQ is a chord of

Solution:

Given : A circle with center O and radius 3 cm and PQ is a chord of length 4.8 cm. The tangents at P and Q intersect at point T

To Find : Length of TP

Construction : Join OQ



Now in △OPT and △OQT

OP = OQ [radii of same circle]

PT = PQ

[tangents drawn from an external point to a circle are equal]

OT = OT [Common]

△OPT ≅ △OQT [By Side - Side - Side Criterion]

∠POT = ∠OQT

[Corresponding parts of congruent triangles are congruent]

or ∠POR = ∠OQR

Now in △OPR and △OQR

OP = OQ [radii of same circle]

OR = OR [Common]

∠POR = ∠OQR [Proved Above]

△OPR ≅ △OQT [By Side - Angle - Side Criterion]

∠ORP = ∠ORQ

[Corresponding parts of congruent triangles are congruent]

Now,

∠ORP + ∠ORQ = 180° [Linear Pair]

∠ORP + ∠ORP = 180°

∠ORP = 90°

⇒ OR ⊥ PQ

⇒ RT ⊥ PQ

As OR ⊥ PQ and Perpendicular from center to a chord bisects the chord we have

$$PR = QR = \frac{PQ}{2} = \frac{4.8}{2} = 2.4 \text{ cm}$$

∴ In right - angled △OPR,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OR)^2 + (PR)^2$

$(3)^2 = (OR)^2 + (2.4)^2$

$9 = (OR)^2 + 5.76$

$(OR)^2 = 3.24$

OR = 1.8 cm

Now,

In right angled $\triangle$TPR,

By Pythagoras Theorem

$(PT)^2 = (PR)^2 + (TR)^2$ ...[1]

Also, OP $\perp$ OT

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

In right angled $\triangle$OPT, By Pythagoras Theorem

$(PT)^2 + (OP)^2 = (OT)^2$

$(PR)^2 + (TR)^2 + (OP)^2 = (TR + OR)^2$ ...[From 1]

$(2.4)^2 + (TR)^2 + (3)^2 = (TR + 1.8)^2$

$4.76 + (TR)^2 + 9 = (TR)^2 + 2(1.8)TR + (1.8)^2$

$13.76 = 3.6TR + 3.24$

$3.6TR = 10.52$

TR = 2.9 cm [Appx]

Using this in [1]

$PT^2 = (2.4)^2 + (2.9)^2$

$PT^2 = 4.76 + 8.41$

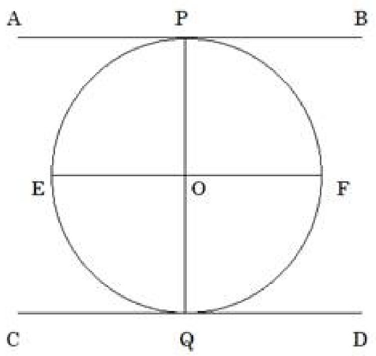
$PT^2 = 13.17$

PT = 3.63 cm [Appx]

Question: 14

Prove that the li

Solution:



Given: A circle with center O and AB and CD are two parallel tangents at points P and Q on the circle.

To Prove: PQ passes through O

Construction: Draw a line EF parallel to AB and CD and passing through O

Proof :

$\angle OPB = 90°$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

Now, AB || EF

$\angle OPB + \angle POF = 180°$

$90° + \angle POF = 180°$

$\angle POF = 90°$ ...[1]

Also,

$\angle OQD = 90°$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

Now, CD || EF

$\angle OQD + \angle QOF = 180°$

$90° + \angle QOF = 180°$

$\angle QOF = 90°$ [2]

Now From [1] and [2]

$\angle POF + \angle QOF = 90 + 90 = 180°$

So, By converse of linear pair POQ is a straight Line

i.e. O lies on PQ

Hence Proved.

Question: 15

In the given figu

Solution:

In quadrilateral POQB

$\angle OPB = 90°$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle OQB = 90°$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle PQB = 90°$ [Given]

By angle sum property of quadrilateral PQOB

$\angle OPB + \angle OQB + \angle PBQ + \angle POQ = 360°$

$90° + 90° + 90° + \angle POQ = 360°$

$\angle POQ = 90°$

As all angles of this quadrilaterals are 90° The quadrilateral is a rectangle

Also, OP = OQ = r

i.e. adjacent sides are equal, and we know that a rectangle with adjacent sides equal is a square

∴ POQB is a square

And OP = PB = BQ = OQ = r [1]

Now,

As we know that tangents drawn from an external point to a circle are equal

In given figure, We have

DS = DR = 5 cm

[Tangents from point D and DS = 5 cm is given]

AD = 23 cm [Given]

AR + DR = 23

AR + 5 = 23

AR = 18 cm

Now,

AR = AQ = 18 cm

[Tangents from point A]

AB = 29 cm [Given]

AQ + QB = 29

18 + QB = 29

QB = 11 cm

From [1]

QB = r = 11 cm

Hence Radius of circle is 11 cm.

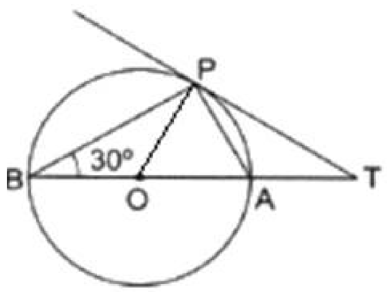
Question: 16

In the given figu

Solution:

In Given Figure, we have a circle with center O let the radius of circle be r.

Construction : Join OP



Now, In $\triangle APB$

$\angle ABP = 30°$

$\angle APB = 90°$

[Angle in a semicircle is a right angle]

By angle sum Property of triangle,

$\angle ABP + \angle APB + \angle PAB = 180$

$30° + 90° + \angle PAB = 180$

$\angle PAB = 60°$

OP = OA = r [radii]

$\angle PAB = \angle OPA = 60°$

[Angles opposite to equal sides are equal]

By angle sum Property of triangle

$\angle OPA + \angle OAP + \angle AOP = 180°$

$60° + \angle PAB + \angle AOP = 180$

$60 + 60 + \angle AOP = 180$

$\angle AOP = 60°$

As all angles of $\triangle OPA$ equals to 60°, $\triangle OPA$ is an equilateral triangle

So, we have, OP = OA = PA = r [1]

$\angle OPT = 90°$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle OPA + \angle APT = 90$

$60 + \angle APT = 90$

$\angle APT = 30°$

Also,

$\angle PAB + \angle PAT = 180°$ [Linear pair]

$60° + \angle PAT = 180°$

$\angle PAT = 120°$

In $\triangle APT$

$\angle APT + \angle PAT + \angle PTA = 180°$

$30° + 120° + \angle PTA = 180°$

$\angle PTA = 30°$

So,

We have

$\angle APT = \angle PTA = 30°$

AT = PA

[Sides opposite to equal angles are equal]

AT = r [From 1] [2]

Now,

AB = OA + OB = r + r = 2r [3]

From [2] and [3]

AB : AT = 2r : r = 2 : 1

Hence Proved !

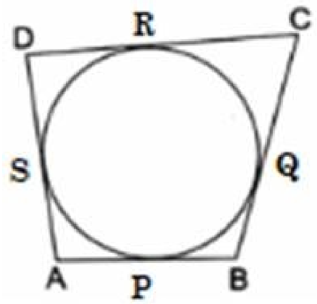
Exercise : 12B

Question: 1

In the adjoining

Solution:

Let sides AB, BC, CD, and AD touches circle at P, Q, R and S respectively.



As we know that tangents drawn from an external point to a circle are equal ,

In the given image we have,

AP = AS = w (say) [Tangents from point A]

BP = BQ = x (say) [Tangents from point B]

CP = CR = y (say) [Tangents from point C]

DR = DS = z (say) [Tangents from point D]

Now,

Given,

AB = 6 cm

AP + BP = 6

w + x = 6 [1]

BC = 9 cm

BP + CP = 9

x + y = 9 [2]

CD = 8 cm

CR + DR = 8

y + z = 8 [3]

Also,

AD = AS + DS = w + z [4]

Add [1] and [3] and substracting [2] from the sum we get,

w + x + y + z - (x + y) = 6 + 8 - 9

w + z = 5 cm

From [4]

AD = 5 cm

Question: 2

In the given figu

Solution:

In the given figure, PA and PB are two tangents from common point P

∴ PA = PB

[Tangents drawn from an external point are equal]

∠PBA = ∠PAB

[Angles opposite to equal angles are equal] [1]

By angle sum property of triangle in △APB

∠APB + ∠PBA + ∠PAB = 180°

50° + ∠PAB + ∠PAB = 180° [From 1]

2∠PAB = 130°

∠PAB = 65° [2]

Now,

∠OAP = 90°

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠OAB + ∠PAB = 90°

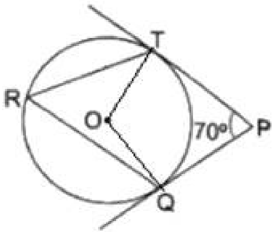
∠OAB + 65° = 90° [From 2]

∠OAB = 25°

Question: 3

In the given figu

Solution:



Given: In the figure, PT and PQ are two tangents and ∠TPQ = 70°

To Find: ∠TRQ

Construction: Join OT and OQ

In quadrilateral OTPQ

∠OTP = 90°

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠OQP = 90°

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠TPQ = 70° [Common]

By Angle sum of Quadrilaterals,

In quadrilateral OTPQ we have

∠OTP + ∠OQP + ∠TPQ + ∠TOQ = 360°

90° + 90° + 70° + ∠TOQ = 360°

250° + ∠TOQ = 360

∠TQO = 110°

Now,

As we Know the angle subtended by an arc at the center is double the angle subtended by it at any

point on the remaining part of the circle.

∴ we have

∠TOQ = 2∠TRQ

110° = 2 ∠TRQ

$\angle TRQ = 55°$

Question: 4

In the given figu

Solution:

Given: AB and CD are two tangents to two circles which intersects at E .

To Prove: AB = CD

Proof:

As

AE = CE ...[1]

[Tangents drawn from an external point to a circle are equal]

And

EB = ED ...[2]

[Tangents drawn from an external point to a circle are equal]

Adding [1] and [2]

AE + EB = CE + ED

AB = CD

Hence Proved.

Question: 5

If PT is a tangen

Solution:

Given: PT is a tangent to a circle with center O and PQ is a chord of the circle such that $\angle QPT = 70°$

To Find: $\angle POQ$ = ?

Now,

$\angle OPT = 90°$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle OPQ + \angle QPT = 90°$

$\angle OPQ + 70° = 90°$

$\angle OPQ = 20°$

Also,

OP = OQ [Radii of same circle]

$\angle OQP = \angle OPQ = 20°$

[Angles opposite to equal sides are equal]

In $\triangle OPQ$ By Angle sum property of triangles,

$\angle OPQ + \angle OQP + \angle POQ = 180°$

$20° + 20° + \angle POQ = 180°$

$\angle POQ = 140°$

Question: 6

In the given figu

Solution:

Given: △ABC that is drawn to circumscribe a circle with radius r = 2 cm and BD = 4 cm DC = 3cm

Also, area(△ABC) = 21 cm^2

To Find: AB and AC

Now,

As we know tangents drawn from an external point to a circle are equal.

Then,

FB = BD = 4 cm [Tangents from same external point B]

DC = EC = 3 cm [Tangents from same external point C]

AF = EA = x (let) [Tangents from same external point A]

Using the above data, we get

AB = AF + FB = x + 4 cm

AC = AE + EC = x + 3 cm

BC = BD + DC = 4 + 3 = 7 cm

Now we have heron's formula for area of triangles if its three sides a, b and c are given

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where, $s = \frac{a+b+c}{2}$

So, for △ABC

a = AB = x + 4

b = AC = x + 3

c = BC = 7 cm

$$\Rightarrow s = \frac{x+4+x+3+7}{2} = x + 7$$

And

$$ar(\triangle ABC) = \sqrt{(x+7)\big(x+7-(x+4)\big)\big(x+7-(x+3)\big)(x+7-7)}$$

$$\Rightarrow 21 = \sqrt{(x+7)(3)(4)(x)}$$

Squaring both sides,

21(21) = 12x(x + 7)

$12x^2 + 84x - 441 = 0$

$4x^2 + 28x - 147 = 0$

As we know roots of a quadratic equation in the form $ax^2 + bx + c = 0$ are,

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

So roots of this equation are,

$$x = \frac{-28 \pm \sqrt{(28)^2-4(4)(-147)}}{2(4)}$$

$$\Rightarrow x = \frac{-28 \pm \sqrt{3136}}{8}$$

$\Rightarrow\ x = \frac{-28 \pm 56}{8} = 3.5 \text{ or } -10.5$

but x = - 10.5 is not possible as length can't be negative.

So

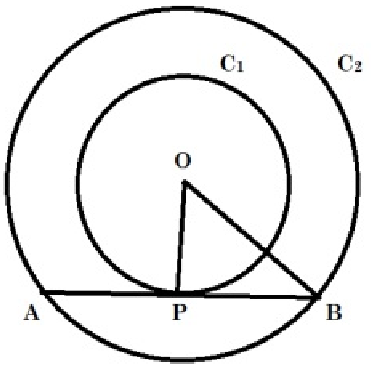
AB = x + 4 = 3.5 + 4 = 7.5 cm

AC = x + 3 = 3.5 + 3 = 6.5 cm

Question: 7

Two concentric ci

Solution:



Given : Two concentric circles (say C_1 and C_2) with common center as O and radius r_1 = 5 cm and r_2 = 3 cm respectively.

To Find : Length of the chord of the larger circle which touches the circle C_2. i.e. Length of AB.

As AB is tangent to circle C_2 and,

We know that "Tangent at any point on the circle is perpendicular to the radius through point of contact"

So, we have,

$OP \perp AB$

$\therefore$ OPB is a right - angled triangle at P,

By Pythagoras Theorem in $\triangle OPB$

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

We have,

$(OP)^2 + (PB)^2 = (OB)^2$

$r_2^2 + (PB)^2 = r_1^2$

$(3)^2 + (PB)^2 = (5)^2$

$9 + (PB)^2 = 25$

$(PB)^2 = 16$

PB = 4 cm

Now, AP = PB,

[as perpendicular from center to chord bisects the chord and $OP \perp AB$]

So,

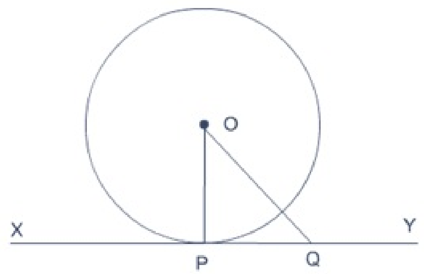
AB = AP + PB = PB + PB

= 2PB = 2(4) = 8 cm

Question: 8

Prove that the pe

Solution:



Let us consider a circle with center O and XY be a tangent

To prove : Perpendicular at the point of contact of the tangent to a circle passes through the center i.e. the radius OP ⊥ XY

Proof :

Take a point Q on XY other than P and join OQ .

The point Q must lie outside the circle. (because if Q lies inside the circle, XY

will become a secant and not a tangent to the circle).

∴ OQ is longer than the radius OP of the circle. That is,

OQ > OP.

Since this happens for every point on the line XY except the point P, OP is the

shortest of all the distances of the point O to the points of XY.

So OP is perpendicular to XY.

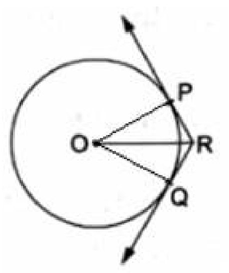
[As Out of all the line segments, drawn from a point to points of a line not passing through the point, the smallest is the perpendicular to the line.]

Question: 9

In the given figu

Solution:

Given : In the figure ,



Two tangents RQ and RP are drawn from an external point R to the circle with center O and ∠PRQ = 120°

To Prove: OR = PR + RQ

Construction: Join OP and OQ

Proof :

In △△OPR and △OQR

OP = OQ [radii of same circle]

OR = OR [Common]

PR = PQ ...[1]

[Tangents drawn from an external point are equal]

$\triangle OPR \cong \triangle OQR$

[By Side - Side - Side Criterion]

$\angle ORP = \angle ORQ$

[Corresponding parts of congruent triangles are congruent]

Also,

$\angle PRQ = 120°$

$\angle ORP + \angle ORQ = 120°$

$\angle ORP + \angle ORP = 120°$

$2\angle ORP = 120°$

$\angle ORP = 60°$

Also, $OP \perp PR$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, In right angled triangle OPR,

$$\cos\angle ORP = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{PR}{OR}$$

$$\cos 60° = \frac{PR}{OR} = \frac{1}{2}$$

$\therefore$ OR = 2PR

OR = PR + PR

OR = PR + RQ [From 1]

Hence Proved.

Question: 10

In the given figu

Solution:

Let AD = x cm, BE = y cm and CF = z cm

As we know that,

Tangents from an external point to a circle are equal,

In given Figure we have

AD = AF = x

[Tangents from point A]

BD = BE = y

[Tangents from point B]6CF = CE = z [Tangents from point C]

Now, Given: AB = 14 cm

AD + BD = 14

x + y = 14

y = 14 - x ... [1]

and BC = 8 cm

BE + EC = 8

y + z = 8

14 - x + z = 8 ... [From 1]

z = x - 6 [2]

and

CA = 12 cm

AF + CF = 12

x + z = 12 [From 2]

x + x - 6 = 12

2x = 18

x = 9 cm

Putting value of x in [1] and [2]

y = 14 - 9 = 5 cm

z = 9 - 6 = 3 cm

So, we have AD = 9 cm, BE = 5 cm and CF = 3 cm

Question: 11

In the given figu

Solution:

Given : PA and PB are tangents to a circle with center O

To show : AOBP is a cyclic quadrilateral.

Proof :

OB ⊥ PB and OA ⊥ AP

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle OBP = \angle OAP = 90°$

$\angle OBP + \angle OAP = 90 + 90 = 180°$

AOBP is a cyclic quadrilateral

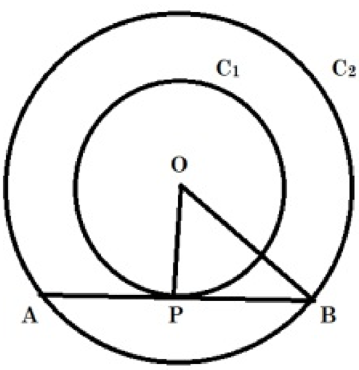
[As we know if the sum of opposite angles in a quadrilateral is 180° then quadrilateral is cyclic]

Hence Proved.

Question: 12

In two concentric

Solution:



Let us consider circles C_1 and C_2 with common center as O. Let AB be a tangent to circle C_1 at point P and chord in circle C_2. Join OB

In △OPB

$OP \perp AB$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∴ OPB is a right - angled triangle at P,

By Pythagoras Theorem,

[i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$]

$(OB)^2 = (OP)^2 + (PB)^2$

Now, 2PB = AB

[As we have proved above that $OP \perp AB$ and Perpendicular drawn from center to a chord bisects the chord]

2PB = 8 cm

PB = 4 cm

$(OB)^2 = (5)^2 + (4)^2$

[As OP = 5 cm, radius of inner circle]

$(OB)^2 = 41$

$\Rightarrow OB = \sqrt{41}$ cm

Question: 13

In the given figu

Solution:

Given : , PQ is a chord of a circle with center 0 and PT is a tangent and ∠QPT = 60°.

To Find : ∠PRQ

∠OPT = 90°

∠OPQ + ∠QPT = 90°

∠OPQ + 60° = 90°

∠OPQ = 30° ... [1]

Also.

OP = OQ [radii of same circle]

∠OQP = ∠OPQ [Angles opposite to equal sides are equal]

From [1], ∠OQP = ∠OQP = 30°

In △OPQ , By angle sum property

∠OQP + ∠OPQ + ∠POQ = 180°

30° + 30° + ∠POQ = 180°

∠POQ = 120°

As we know, the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

So, we have

2∠PRQ = reflex ∠POQ

2∠PRQ = 360° - ∠POQ

2∠PRQ = 360° - 120° = 240°

∠PRQ = 120°

Question: 14

In the given figu

Solution:

In the given figure, PA and PB are two tangents from common point P

∴ PA = PB

[∵ Tangents drawn from an external point are equal]

$\angle PBA = \angle PAB$

[∵ Angles opposite to equal angles are equal] ...[1]

By angle sum property of triangle in $\triangle APB$

$\angle APB + \angle PBA + \angle PAB = 180°$

$60° + \angle PAB + \angle PAB = 180°$ [From 1]

$2\angle PAB = 120°$

$\angle PAB = 60°$ ...[2]

Now,

$\angle OAP = 90°$ [Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle OAB + \angle PAB = 90°$

$\angle OAB + 60° = 90°$ [From 2]

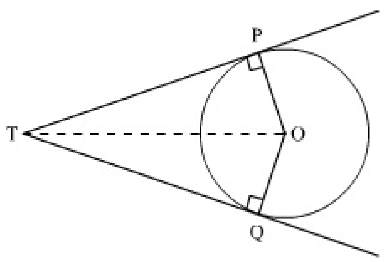
$\angle OAB = 30°$

Exercise : MULTIPLE CHOICE QUESTIONS (MCQ)

Question: 1

The number of tan

Solution:



The maximum number of tangents that can be drawn from an external point to a circle is two and they are equal in length.

Question: 2

In the given figu

Solution:

As SQ is diameter and OQ is radius in the given circle,

∴ 2OQ = SQ [As 2 × radius) = diameter]

2OQ = 6 cm

OQ = 3 cm

Now, QR is tangent

∴ $OQ \perp QR$

In right - angled $\triangle OQR$,

By Pythagoras Theorem,

[i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$]

$(QR)^2 + (OQ)^2 = (OR)^2$

$(4)^2 + (3)^2 = (OR)^2$

$16 + 9 = (OR)^2$

$(OR)^2 = 25$

OR = 5 cm

Question: 3

In a circle of ra

Solution:

We have given, PT is a tangent drawn at point T on the circle.

$\therefore OT \perp TP$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, In $\triangle OTP$ we have,

By Pythagoras Theorem,

[i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$]

$(OP)^2 = (OT)^2 + (PT)^2$

$(OP)^2 = (7)^2 + (24)^2$

$(OP)^2 = 49 + 576$

$(OP)^2 = 625$

$\Rightarrow OP = 25$ cm

Question: 4

Which of the foll

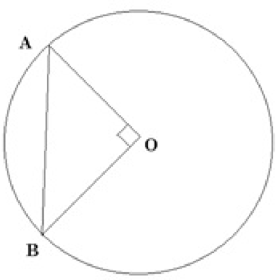
Solution:

As all diameters of a circle passes through center O it is not possible to have two parallel diameters in a circle.

Question: 5

The chord of a ci

Solution:



Let us consider a circle with center O and AB be any chord that subtends 90° angle at its center.

Now, In $\triangle OAB$

OA = OB = 10 cm

And as $\angle AOB = 90°$,

By Pythagoras Theorem,

[i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$]

$(OA)^2 + (OB)^2 = (AB)^2$

$(10)^2 + (10)^2 = (AB)^2$

$100 + 100 = (AB)^2$

$\Rightarrow AB = \sqrt{200} = 10\sqrt{2}$

So, Correct option is C .

Question: 6

In the given figu

Solution:

We have given, PT is a tangent drawn at point T on the circle.

$\therefore OT \perp TP$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, In $\triangle OTP$ we have,

By Pythagoras Theorem,

[i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$]

$(OP)^2 = (OT)^2 + (PT)^2$

$(10)^2 = (6)^2 + (PT)^2$

$(PT)^2 = 100 - 36$

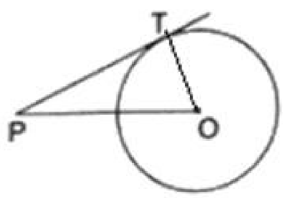
$(PT)^2 = 64$

$\Rightarrow PT = 8$ cm

Question: 7

In the given figu

Solution:



We have given, PT is a tangent drawn at point T on the circle and OP = 26 cm and PT = 24 cm

Join OT

$\therefore OT \perp TP$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, In $\triangle OTP$ we have,

By Pythagoras Theorem,

[i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$]

$(OP)^2 = (OT)^2 + (PT)^2$

$(26)^2 = (OT)^2 + (24)^2$

$(OT)^2 = 676 - 576$

$(OT)^2 = 100$

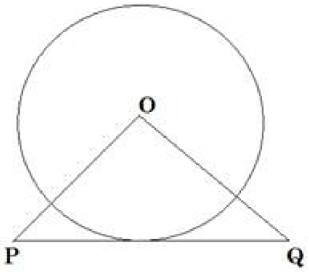
OT = 10 cm

Hence, radius of circle is 10 cm.

Question: 8

PQ is a tangent t

Solution:



Let us consider a circle with center O and PQ is a tangent

on the circle, Joined OP and OQ

But OPQ is an isosceles triangle, ∴ OP = PQ

$\angle OQP = \angle POQ$

[Angles opposite to equal sides are equal]

In $\triangle OQP$

$\angle OQP + \angle OPQ + \angle POQ = 180°$

[Angle sum property of triangle]

$\angle OQP + 90° + \angle OPQ = 180°$

$2\ \angle OPQ = 90°$

$\angle OPQ = 45°$

Question: 9

In the given figu

Solution:

As AB and AC are tangents to given circle,

We have,

$OB \perp AB$ and $OC \perp AC$

[∵ Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, $\angle OBA = \angle OCA = 90°$

In quadrilateral AOBC, By angle sum property of quadrilateral, we have,

$\angle OBA + \angle OCA + \angle BOC + \angle BAC = 360°$

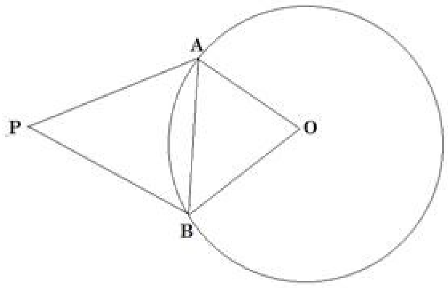
$90° + 90° + \angle BOC + 40° = 360°$

$\angle BOC = 140°$

Question: 10

If a chord AB sub

Solution:



Let us consider a circle with center O and AB be a chord such that ∠AOB = 60°

AP and BP are two intersecting tangents at point P at point A and B respectively on the circle.

To find : Angle between tangents, i.e. ∠APB

As AP and BP are tangents to given circle,

We have,

OA ⊥ AP and OB ⊥ BP [Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, ∠OAP = ∠OBP = 90°

In quadrilateral AOBP, By angle sum property of quadrilateral, we have

∠OAP + ∠OBP + ∠APB + ∠AOB = 360°

90° + 90° + ∠APB + 60° = 360°

∠APB = 120°

Question: 11

In the given figu

Solution:

Given: Two concentric circles (say C_1 and C_2) with common center as O and radius r_1 = 6 cm(inner circle) and r_2 = 10 cm (outer circle) respectively.

To Find : Length of the chord AB.

As AB is tangent to circle C_1 and we know that "Tangent at any point on the circle is perpendicular to the radius through point of contact"

So, we have,

OP ⊥ AB

∴ OPB is a right - angled triangle at P,

By Pythagoras Theorem in △OPB

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

We have,

$(OP)^2 + (PA)^2 = (OA)^2$

$r_1^2 + (PA)^2 = r_2^2$

$(6)^2 + (PA)^2 = (10)^2$

$36 + (PA)^2 = 100$

$(PA)^2 = 64$

PA = 8 cm

Now, PA = PB ,

[as perpendicular from center to chord bisects the chord and OP ⊥ AB]

So,

AB = PA + PB = PA + PA = 2PA = 2(8) = 16 cm

Question: 12

In the given figu

Solution:

As AB is tangent to the circle at point B

OB ⊥ AB

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

In right angled triangle AOB,

By Pythagoras Theorem,

[i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$]

$(OA)^2 = (OB)^2 + (AB)^2$

$(17)^2 = (8)^2 + (AB)^2$

[As OA = 17 cm is given and OB is radius]

$289 = 64 + (AB)^2$

$(AB)^2 = 225$

AB = 15 cm

Now, AB = AC [Tangents drawn from an external point are equal]

∴ AC = 15 cm

Question: 13

In the given figu

Solution:

In △ABC

∠ABC = 90°

[Angle in a semicircle is a right angle]

∠ACB = 50° [Given]

By angle sum Property of triangle,

∠ACB + ∠ABC + ∠CAB = 180°

90° + 50° + ∠CAB = 180°

∠CAB = 40°

Now,

∠CAT = 90°

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠CAB + ∠BAT = 90°

40° + ∠BAT = 90°

∠BAT = 50°

Question: 14

In the given figu

Solution:

In $\triangle OPQ$

$\angle POQ = 70°$ [Given]

$OP = OQ$ [radii of same circle]

$\angle OQP = \angle OPQ$ [Angles opposite to equal sides are equal]

By angle sum Property of triangle,

$\angle POQ + \angle OQP + \angle OPQ = 180°$

$70° + \angle OPQ + \angle OPQ = 180°$

$2\ \angle OPQ = 110°$

$\angle OPQ = 55°$

Now,

$\angle OPT = 90°$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle OPQ + \angle TPQ = 90°$

$55° + \angle TPQ = 90°$

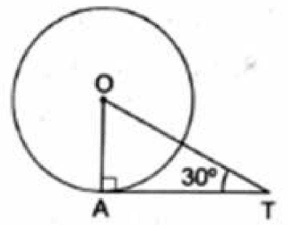
$\angle TPQ = 35°$

Question: 15

In the given figu

Solution:

Given: AT is a tangent to the circle with center O such that OT = 4 cm and $\angle OTA = 30°$.**To find:** The value of AT.**Solution:**



In $\triangle$ OAT ,

$OA \perp AT$ [Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\therefore$OAT is a right - angled triangle at A and

$$\cos\angle OTA = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AT}{OT}$$

$$\cos 30° = \frac{AT}{4}$$

$$\frac{\sqrt{3}}{2} = \frac{AT}{4}$$

$AT = 2\sqrt{3}$ cm

Question: 16

If PA and PB are

Solution:

As AP and BP are tangents to given circle,

We have,

$OA \perp AP$ and $OB \perp BP$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, $\angle OAP = \angle OBP = 90°$

In quadrilateral AOBP,

By angle sum property of quadrilateral, we have

$\angle OAP + \angle OBP + \angle AOB + \angle APB = 360°$

$90° + 90° + 110° + \angle APB = 360°$

$\angle APB = 70°$

Question: 17

In the given figu

Solution:

As we know,

Tangents drawn from an external point are equal, We have

AF = AE = 4 cm

[Tangents from common point A]

BF = BD = 3 cm

[Tangents from common point B]

CE = CD = x (say)

[Tangents from common point C]

Now,

AC = AE + CE

11 = 4 + x

x = 7 cm [1]

and, BC = BD + BC

BC = 3 + x = 3 + 7 = 10 cm

Question: 18

In the given figu

Solution: We know that the sum of angles subtended by opposite sides of a quadrilateral having a circumscribed circle is 180ºTherefore,$\angle AOD + \angle BOC = 180º135º + \angle BOC = 180º\angle BOC = 45º$

Question: 19

In the given figu

Solution:

In the given figure PT is a tangent to circle ∴ we have

$\angle OPT = 90°$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle OPQ + \angle QPT = 90°$

$\angle OPQ + 50° = 90°$

$\angle OPQ = 40°$

Now, In $\triangle POQ$

OP = OQ

$\angle PQO = \angle QPO = 40°$

[Angles opposite to equal sides are equal]

Now,

$\angle PQO + \angle QPO + \angle POQ = 180°$

[By angle sum property of triangle]

$40° + 40° + \angle POQ = 180°$

$\angle POQ = 100°$

Question: 20

In the given figu

Solution:

In the given figure, PA and PB are two tangents from common point P

$\therefore PA = PB$

[Tangents drawn from an external point are equal]

$\angle PBA = \angle PAB$...[1]

[Angles opposite to equal angles are equal]

By angle sum property of triangle in $\triangle APB$

$\angle APB + \angle PBA + \angle PAB = 180°$

$60° + \angle PAB + \angle PAB = 180°$ [From 1]

$2\angle PAB = 120°$

$\angle PAB = 60°$...[2]

Now,

$\angle OAP = 90°$ [Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle OAB + \angle PAB = 90°$

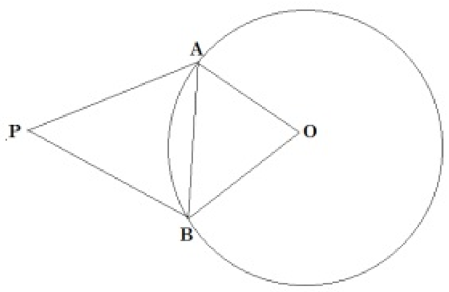
$\angle OAB + 60° = 90°$ [From 2]

$\angle OAB = 30°$

Question: 21

If two tangents i

Solution:



Let us consider a circle with center O and AP and BP are two tangents such that angle of inclination i.e. $\angle APB = 60°$

Joined OA, OB and OP.

To Find : Length of tangents

Now,

PA = PB [Tangents drawn from an external point are equal] [1]

In $\triangle AOP$ and $\triangle BOP$

PA = PB [By 1]

OP = OP [Common]

OA = OB [radii of same circle]

$\triangle AOP \cong \triangle BOP$

[By Side - Side - Side Criterion]

$\angle OPA = \angle OPB$

[Corresponding parts of congruent triangles are congruent]

Now,

$\angle APB = 60°$ [Given]

$\angle OPA + \angle OPB = 60°$

$\angle OPA + \angle OPA = 60°$

$2\ \angle OPA = 60°$

$\angle OPA = 30°$

In $\triangle AOP$

$OA \perp PA$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact

$\therefore$ AOP is a right - angled triangle.

So, we have

$$\tan \angle OPA = \frac{\text{Perependicular}}{\text{Base}} = \frac{OA}{PA}$$

$$\tan 30° = \frac{3}{PA}$$

$$\frac{1}{\sqrt{3}} = \frac{3}{PA}$$

$\Longrightarrow PA = 3\sqrt{3}$ cm

From [1]

PA = PB = 4 cm

i.e. length of each tangent is $3\sqrt{3}$ cm

Question: 22

In the given figu

Solution:

In Given Figure,

PQ = PR...[1]

[Tangents drawn from an external point are equal]

In $\triangle AOP$ and $\triangle BOP$

PQ = PR [By 1]

AP = AP [Common]

AQ = AR [radii of same circle]

$\triangle AQP \cong \triangle ARP$ [By Side - Side - Side Criterion]

$\angle QPA = \angle RPA$

[Corresponding parts of congruent triangles are congruent]

Now,

∠QPA + ∠RPA = ∠QPR

∠QPA + ∠QPA = ∠QPR

2 ∠QPA = ∠QPR

∠QPR = 2(27) = 54°

As PQ and PQ are tangents to given circle,

We have,

AQ ⊥ PQ and AR ⊥ PR

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, ∠AQP = ∠ARP = 90°

In quadrilateral AQRP, By angle sum property of quadrilateral, we have

∠AQP + ∠ARP + ∠QAR + ∠QPR = 360°

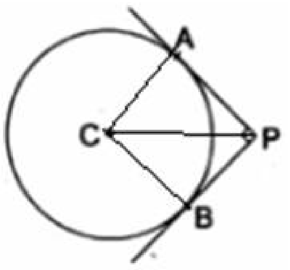
90° + 90° + ∠QAR + 54° = 360°

∠QAR = 126°

Question: 23

In the given figu

Solution:



Join AC, BC and CP

To Find: Length of tangents

Now,

PA = PB...[1]

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

In △ACP and △BCP

PA = PB [By 1]

CP = CP [Common]

CA = CB [radii of same circle]

△ACP ≅△BCP [By Side - Side - Side Criterion]

∠CPA = ∠CPB

[Corresponding parts of congruent triangles are congruent]

Now,

∠APB = 90°

[Given that PA ⊥ PB]

∠CPA + ∠CPB = 90°

∠CPA + ∠CPA = 90°

2 ∠CPA = 90°

$\angle CPA = 45°$

In $\triangle ACP$

$CA \perp PA$ [Tangents drawn at a point on circle is perpendicular to the radius through point of contact

$\therefore$ ACP is a right - angled triangle.

So, we have

$$\tan \angle CPA = \frac{\text{Perependicular}}{\text{Base}} = \frac{CA}{PA}$$

$$\tan 45° = \frac{4}{PA}$$

$$1 = \frac{4}{PA}$$

$\Longrightarrow PA = 4$ cm

From [1]

PA = PB = 4 cm

i.e. length of each tangent is 4 cm

Question: 24

If PA and PB are

Solution:

In Given Figure,

PA = PB...[1]

[Tangents drawn from an external point are equal]

In $\triangle AOP$ and $\triangle BOP$

PA = PB [By 1]

OP = OP [Common]

OA = OB

[radii of same circle]

$\triangle AOP \cong \triangle BOP$

[By Side - Side - Side Criterion]

$\angle OPA = \angle OPB$

[Corresponding parts of congruent triangles are congruent]

Now,

$\angle APB = 80°$ [Given]

$\angle OPA + \angle OPB = 80°$

$\angle OPA + \angle OPA = 80°$

$2 \angle OPA = 80°$

$\angle OPA = 40°$

In $\triangle AOP$,

$\angle OAP = 90°$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

And

$\angle OAP + \angle OPA + \angle AOP = 180°$

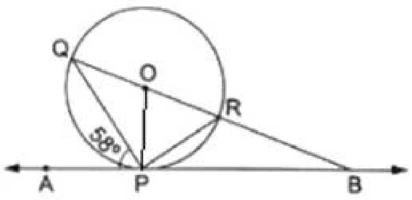
$90° + 40° + \angle AOP = 180°$

$\angle AOP = 50°$

Question: 25

In the given figu

Solution:



In the given Figure, Join OP

Now, $OP \perp AB$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\therefore \angle OPA = 90°$

$\angle OPQ + \angle APQ = 90°$

$\angle OPQ + 58° = 90°$

[Given, $\angle APQ = 58°$]

$\angle OPQ = 32°$

In $\triangle OPQ$

$OP = OQ$

[Radii of same circle]

$\angle OQP = \angle OPQ$

[Angles opposite to equal sides are equal]

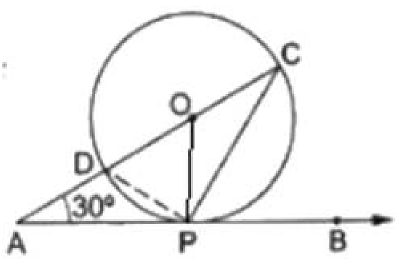
$\angle PQB = 32°$

[As $\angle OQP = \angle PQB$]

Question: 26

In the given figu

Solution:



In given Figure, Join OP

In $\triangle OPC$,

$OP = OC$ [Radii of same circle]

$\angle OCP = \angle OPC$

[Angles opposite to equal sides are equal]

$\angle ACP = \angle OPC$

[As ∠OCP = ∠ACP] ...[1]

Now,

∠OPB = 90°

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠OPC + ∠CPB = 90°

∠ACP + ∠CPB = 90° [By 1]

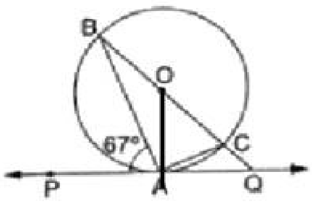
So,

∠CPB + ∠ACP = 90°

Question: 27

In the given figu

Solution:



In the given Figure, Join OA

Now,

OA ⊥ PQ

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠OAP = ∠OAQ = 90° [1]

∠OAB + ∠PAB = 90°

∠OAB + 67° = 90°

∠OAB = 23°

Now,

∠BAC = 90°

[Angle in a semicircle is a right angle]

∠OAB + ∠OAC = 90°

23° + ∠OAC = 90°

∠OAC = 67°

∠OAQ = 90° [From 1]

∠OAC + ∠CAQ = 90°

67° + ∠CAQ = 90°

∠CAQ = 23° [2]

Now,

OA = OC

[radii of same circle]

∠OCA = ∠OAC

[Angles opposite to equal sides are equal]

∠OCA = 67°

$\angle OCA + \angle ACQ = 180°$ [Linear Pair]

$67° + \angle ACQ = 180°$

$\angle ACQ = 113°$ [3]

Now, In $\triangle ACQ$ By Angle Sum Property of triangle

$\angle ACQ + \angle CAQ + \angle AQC = 180°$

$113° + 23° + \angle AQC = 180°$ [By 2 and 3]

$\angle AQC = 44°$

Question: 28

In the given figu

Solution:

Question: 29

O is the center o

Solution:

In Given Figure,

PQ = PR...[1]

[Tangents drawn from an external point are equal]

In $\triangle QOP$ and $\triangle ROP$

PQ = PR [By 1]

OP = OP [Common]

OQ = OR [radii of same circle]

$\triangle QOP \cong \triangle ROP$

[By Side - Side - Side Criterion]

area(Δ QOP) = area(Δ ROP)

[Congruent triangles have equal areas]

area(PQOR) = area(Δ QOP) + area(Δ ROP)

area(PQOR) = area(Δ QOP) + area(Δ QOP) = 2[area(Δ QOP)]

Now,

$OQ \perp PQ$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, QOP is a right - angled triangle at Q with OQ as base and PQ as height.

In $\triangle QOP$,

By Pythagoras Theorem in $\triangle OPB$

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OQ)^2 + (PQ)^2 = (OP)^2$

$(5)^2 + (PQ)^2 = (13)^2$

$25 + (PQ)^2 = 169$

$(PQ)^2 = 144$

PQ = 12 cm

Area(ΔQOP) = 1/2 × Base × Height

= 1/2 × OQ × PQ

= 1/2 × 5 × 12

= 30 cm^2

So,

Area(PQOR) = 2(30) = 60 cm^2

Question: 30

In the given figu

Solution:

In given figure, as PR is a tangent

OQ ⊥ PR

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

⟹LQ ⊥ PR

⟹LQ ⊥ AB

[As, AB || PR]

AL = LB

[Perpendicular from center to the chord bisects the chord]

Now,

∠LQR = 90°

∠LQB + ∠BQR = 90°

∠LQB + 70° = 90°

∠LQB = 20°...[1]

In △AQL and △BQL

∠ALQ = ∠BLQ [Both 90° as LQ ⊥ AB]

AL = LB [Proved above]

QL = QL [Common]

△AQL ≅ △BQL

[Side - Angle - Side Criterion]

∠LQA = ∠LQB

[Corresponding parts of congruent triangles are congruent]

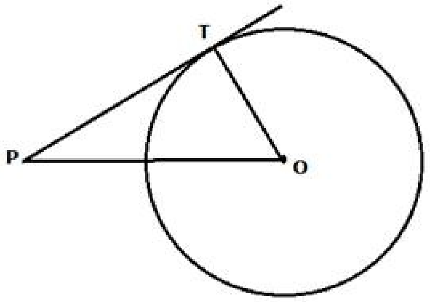
∠AQB = ∠LQA + ∠LQB = ∠LQB + ∠LQB

= 2∠LQB = 2(20) = 40° [By 1]

Question: 31

The length of the

Solution:



Let us consider a circle with center O and TP be a tangent at point A on the circle, Joined OT and OP

Given Length of tangent, TP = 10 cm, and OT = 5 cm [radius]

To Find : Distance of center O from P i.e. OP

Now,

OP ⊥ TP

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So OPT is a right - angled triangle,

By Pythagoras Theorem in ΔOPB

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OT)^2 + (TP)^2 = (OP)^2$

$(OP)^2 = (5)^2 + (10)^2$

$(OP)^2 = 25 + 100 = 125$

OP = $\sqrt{125}$ cm

Question: 32

In the given figu

Solution:

In △BOP

OB = OP [radii of same circle]

∠OPB = ∠PBO

[Angles opposite to equal sides are equal]

As, ∠PBO = 30°

∠OPB = 30°

Now,

∠OPT = 90°

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠BPT = ∠OPB + ∠OPT = 30° + 90° = 120°

Now, In ΔBPT

∠BPT + ∠PBO + ∠PTB = 180°

120° + 30° + ∠PTB = 180°

∠PTB = 30°

∠PTA = ∠PTB = 30°

Question: 33

In the given figu

Solution:

Given : In the given figure, a circle touches the side DF of ΔEDF at H and touches ED and EF produced at K and M respectively and EK = 9 cm

To Find : Perimeter of △EDF

As we know that, Tangents drawn from an external point to a circle are equal.

So, we have

KD = DH ...[1]

[Tangents from point D]

HF = FM ...[2]

[Tangents from point F]

Now Perimeter of Triangle PCD

= ED + DF + EF

= ED + DH + HF + EF

= ED + KD + FM + EF [From 1 and 2]

= EK + EM

Now,

EK = EM = 9 cm as tangents drawn from an external point to a circle are equal

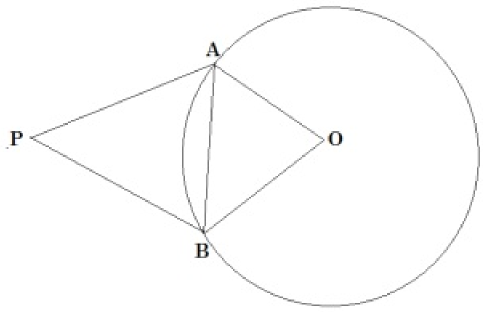
So, we have

Perimeter = EK + EM = 9 + 9 = 18 cm

Question: 34

To draw a pair of

Solution:



Let us consider a circle with center O and PA and PB are two tangents from point P, given that angle of inclination i.e. ∠APB = 45°

As PA and PB are tangents to given circle,

We have,

OA ⊥ PA and OB ⊥ PB [Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, ∠OAP = ∠OBP = 90°

In quadrilateral AQRP,

By angle sum property of quadrilateral, we have

∠OAP + ∠OBP + ∠ABP + ∠AOB = 360°

90° + 90° + 45° + ∠AOB = 360°

$\angle AOB = 135°$

Question: 35

In the given figu

Solution:

As PL and PM are tangents to given circle,

We have,

$OR \perp PM$ and $OQ \perp PL$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, $\angle ORM = \angle OQL = 90°$

$\angle ORM = \angle ORS + \angle SRM$

$90° = \angle ORS + 60°$

$\angle ORS = 30°$

And $\angle OQL = \angle OQS + \angle SQL$

$90° = \angle OQS + 50°$

$\angle OQS = 40°$

Now, In $\triangle SOR$

OS = OQ [radii of same circle]

$\angle ORS = \angle OSR$

[Angles opposite to equal sides are equal]

$\angle OSR = 30°$

[as $\angle ORS = 30°$]

Now, In $\triangle SOR$

OS = SQ [radii of same circle]

$\angle OQS = \angle OSQ$

[Angles opposite to equal sides are equal]

$\angle OSQ = 40°$ [as $\angle OQS = 40°$]

As,

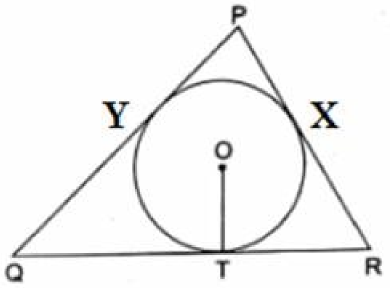
$\angle QSR = \angle OSR + \angle OSQ$

$\angle QSR = 30° + 40° = 70°$

Question: 36

In the given figu

Solution:



Given : $\triangle PQR$ that is drawn to circumscribe a circle with radius r = 6 cm and QT = 12 cm QR = 9cm

Also, area(△PQR) = 189 cm^2

Let tangents PR and PQ touch the circle at X and Y respectively.

To Find : PQ and QR

Now,

As we know tangents drawn from an external point to a circle are equal.

Then,

QT = QY = 12 cm

[Tangents from same external point B]

TR = RX = 9 cm

[Tangents from same external point C]

PX = PY = x (let)

[Tangents from same external point A]

Using the above data we get

PQ = PY + QT = x + 12 cm

PR = PC + RX = x + 9 cm

QR = QT + TR = 12 + 9 = 21 cm

Now we have heron's formula for area of triangles if its three sides a, b and c are given

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where,

$$s = \frac{a+b+c}{2}$$

So for △PQR

a = PQ = x + 12

b = PR = x + 9

c = QR = 21 cm

$$s = \frac{x+12+x+9+21}{2} = x + 21$$

And

$$ar(PQR) = \sqrt{(x+21)\big(x+21-(x+12)\big)\big(x+21-(x+9)\big)(x+21-21)}$$

$$189 = \sqrt{(x+21)(9)(12)(x)}$$

Squaring both side

189(189) = 108(x + 21)

7(189) = 4(x + 21)

$4x^2 + 84x - 1323 = 0$

As we know roots of a quadratic equation in the form $ax^2 + bx + c = 0$ are,

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

So, roots of this equation are,

$$x = \frac{-84 \pm \sqrt{(84)^2 - 4(4)(-1323)}}{2(4)}$$

$$x = \frac{-84 \pm \sqrt{28224}}{8}$$

$$x = \frac{-84 \pm 168}{8}$$

$$x = 10.5 \text{ or } -31.5$$

but x = - 31.5 is not possible as length can't be negative.

So

PQ = x + 12 = 10.5 + 12 = 22.5 cm

Question: 37

In the given figu

Solution:

Let the bigger circle be C1 and Smaller be C2,

Now,

PQ and PT are two tangents to circle C1,

∴ PT = QP

[Tangents drawn from an external point are equal]

QP = 3.8 cm

[As PT = 3.8 cm is given]

Also,

PR and PT are two tangents to circle C2,

∴ PT = PR

[Tangents drawn from an external point are equal]

PR = 3.8 cm

[As PT = 3.8 cm is given]

QR = QP + PR = 3.8 + 3.8 = 7.6 cm

Question: 38

In the given figu

Solution:

As we know Tangents drawn from an external point are equal]

In the given Figure, we have

AP = AQ = 5 cm

[Tangents from point A] [AP = 5 cm is given]

BQ = BR = x(say)

[Tangents from point B]

CR = CS = 3 cm [∵ CS = 3 cm is given]

[Tangents from point C]

Given,

BC = 7 cm

CR + BR = 7

3 + x = 7 cm

x = 4 cm

Now,

AB = AQ + BQ = 5 + x = 5 + 4 = 9 cm

Question: 39

In the given figu

Solution:

As we know Tangents drawn from an external point are equal]

In the given Figure, we have

AP = AS = 6 cm [AP = 6 cm is given]

[∵ Tangents from point A]

BP = BQ = 5 cm [BP = 5 cm is given]

[∵ Tangents from point B]

CR = CQ = 3 cm [CQ = 3 cm is given]

[∵ Tangents from point C]

DR = DS = 4 cm][DR = 4 cm is given]

[∵ Tangents from point D]

Now,

Perimeter of ABCD = AB + BC + CA + DA

= AP + BP + BQ + CQ + CR + DR + DS + AS

= 6 + 5 + 5 + 3 + 3 + 4 + 4 + 6 = 36 cm

Question: 40

In the given figu

Solution:

In $\triangle AOB$

OA = OB [radii of same circle]

$\angle OBA = \angle OAB$ [Angles opposite to equal sides are equal]

Also, By Triangle sum Property

$\angle AOB + \angle OBA + \angle OAB = 180°$

$100 + \angle OAB + \angle OAB = 180°$

$2\ \angle OAB = 90°$

$\angle OAB = 40°$

As AT is tangent to given circle,

We have,

$OA \perp AT$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, $\angle OAT = 90°$

$\angle OAB + \angle BAT = 90°$

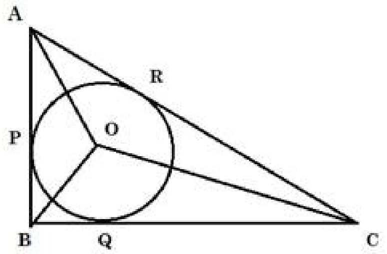
$40° + \angle BAT = 90°$

$\angle BAT = 50°$

Question: 41

In a right triang

Solution:



Let AB, BC and AC touch the circle at points P, Q and R respectively.

As ABC is a right triangle,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(AC)^2 = (BC)^2 + (AB)^2$

$(AC)^2 = (12)^2 + (5)^2$

$(AC)^2 = 144 + 25 = 169$

AC = 13 cm

Let O be the center of circle, Join OP, OQ and PR

Let the radius of circle be r,

We have

r = OP = OQ = OR

[radii of same circle] [1]

Now,

$ar(\triangle ABC) = ar(\triangle AOB) + ar(\triangle BOC) + ar(\triangle AOC)$

As we know,

Area of triangle is 1/2 × Base × Height (Altitude)

Now,

$OP \perp AB$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\therefore$ OP is the altitude in $\triangle AOB$

$OQ \perp BC$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\therefore$ OQ is the altitude in $\triangle BOC$

$OR \perp AC$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\therefore$ OR is the altitude in $\triangle AOC$

So, we have

$1/2 \times BC \times AB = (1/2 \times AB \times OP) + (1/2 \times BC \times OQ) + (1/2 \times AC \times OR)$

12(5) = 5(r) + 12(r) + 13(r) [Using 1]

60 = 30r

r = 2 cm

Question: 42

In the given figu

Solution:

In quadrilateral ORDS

∠ORD = 90°

[∵ Tangent at any point on the circle is perpendicular to the radius through point of contact]

∠OSD = 90°

[∵ Tangent at any point on the circle is perpendicular to the radius through point of contact]

∠SDR = 90° [AD ⊥ CD]

By angle sum property of quadrilateral PQOB

∠ORD + ∠OSD + ∠SDR + ∠SOR = 360°

90° + 90° + 90° + ∠SOR = 360°

∠SOR = 90°

As all angles of this quadrilaterals are 90° The quadrilateral is a rectangle

Also, OS = OR = r

i.e. adjacent sides are equal, and we know that a rectangle with adjacent sides equal is a square

∴ POQB is a square

And OS = OR = DR = DS = r = 10 cm [1]

Now,

As we know that tangents drawn from an external point to a circle are equal

In given figure, We have

CQ = CR ...[2]

[∵ tangents from point C]

PB = BQ = 27 cm

[∵Tangents from point B and PB = 27 cm is given]

BC = 38 cm [Given]

BQ + CQ = 38

27 + CQ = 38

CQ = 11 cm

From [2]

CQ = CR = 11 cm

Now,

CD = CR + DR

CD = 11 + 10 = 21 cm [from 1, DR = 10 cm]

Question: 43

In the given figu

Solution:

As ABC is a right triangle,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(AC)^2 = (BC)^2 + (AB)^2$

$(AC)^2 = (6)^2 + (8)^2$

$(AC)^2 = 36 + 64 = 100$

AC = 10 cm

Now,

$ar(\triangle ABC) = ar(\triangle AOB) + ar(\triangle BOC) + ar(\triangle AOC)$

As we know,

Area of triangle is $1/2 \times \text{Base} \times \text{Height(Altitude)}$

Now,

$OP \perp AB$ [Given]

$\therefore$ OP is the altitude in $\triangle AOB$

$OQ \perp BC$ [Given]

$\therefore$ OQ is the altitude in $\triangle BOC$

$OR \perp AC$ [Given]

$\therefore$ OR is the altitude in $\triangle AOC$

So, we have

$1/2 \times BC \times AB = (1/2 \times AB \times OP) + (1/2 \times BC \times OQ) + (1/2 \times AC \times OR)$

$6(8) = 8(x) + 6(x) + 10(x)$

[$\because$ OP = OQ = OR = x, Given]

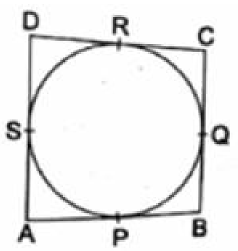
$48 = 24x$

x = 2 cm

Question: 44

Quadrilateral ABC

Solution:



Let sides AB, BC, CD, and AD touches circle at P, Q, R and S respectively.

As we know that tangents drawn from an external point to a circle are equal,

So, we have,

AP = AS = w (say)

[$\because$ Tangents from point A]

BP = BQ = x (say)

[∵Tangents from point B]

CP = CR = y (say)

[∵Tangents from point C]

DR = DS = z (say)

[∵Tangents from point D]

Now,

Given,

AB = 6 cm

AP + BP = 6

w + x = 6 ...[1]

BC = 7 cm

BP + CP = 7

x + y = 7 ...[2]

CD = 4 cm

CR + DR = 4

y + z = 4 ...[3]

Also,

AD = AS + DS = w + z ...[4]

Add [1] and [3] and substracting [2] from the sum we get,

w + x + y + z - (x + y) = 6 + 4 - 7

w + z = 3 cm

From [4]

AD = 3 cm

Question: 45

In the given figu

Solution:

In the given Figure,

As PA and PB are tangents from common external point P, we have

PA = PB

[∵ tangents drawn from an external point are equal]

$\angle PBA = \angle PAB$

[∵ Angles opposite to equal sides are equal]

Now,

In $\triangle APB$, By Angle sum Property of triangle

$\angle APB + \angle PBA + \angle PAB = 180°$

$60° + \angle PAB + \angle PAB = 180°$

$2 \angle PAB = 120°$

$\angle PAB = 60°$

So, We have

$\angle PBA = \angle PAB = \angle APB = 60°$

i.e. APB is an equilateral triangle

so, we have

PA = PB = AB = 5 cm [As PA = 5 cm]

Question: 46

In the given figu

Solution:

.

Question: 47

In the given figu

Solution:

In the given Figure

AR = AP = x(let) [Radii of same circle]

BP = BQ = y(let) [Radii of same circle]

CR = CQ = z(let) [Radii of same circle]

Now,

AB = 5 cm [Given]

AP + BP = 5

$x + y = 5$

$y = 5 - x$ ...[1]

BC = 7 cm [Given]

BQ + CQ = 7

$y + z = 7$

$5 - x + z = 7$ [using 1]

$z = 2 + x$ ...[2]

and

AC = 6 cm [Given]

$x + z = 6$

$x + 2 + x = 6$ [Using 2]

$2x = 4$

$x = 2$ cm

Question: 48

In the given figu

Solution:

Let tangent BC touch the circle at point R

As we know tangents drawn from an external point to a circle are equal.

We have

AP = AQ

[tangents from point A]

BP = BR ...[1]

[tangents from point B]

CQ = CR ...[2]

[tangents from point C]

Now,

AP = AQ

⇒ AB + BP = AC + CQ

⇒ 5 + BR = 6 + CR [From 1 and 2]

⇒CR = BR - 1 ...[3]

Now,

BC = 4 cm

BR + CR = 4

BR + BR - 1 = 4 [Using 3]

2BR = 5 cm

BR = 2.5 cm

BP = BR = 2.5 cm [Using 2]

AP = AB + BP = 5 + 2.5 = 7.5 cm

Question: 49

In the given figu

Solution:

In given Figure,

OA ⊥ AP [Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled △OAP,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OA)^2 + (PA)^2$

Given, PA = 12 cm and OA = radius of outer circle = 5 cm

$(OP)^2 = (5)^2 + (12)^2$

$(OP)^2 = 25 + 144 = 136$

OP = 13 cm ...[1]

Also,

OB ⊥ BP [Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled △OBP,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OB)^2 + (PB)^2$

Now, OB = radius of inner circle = 3 cm

And, from [2] (OP) = 13 cm

$(13)^2 = (3)^2 + (PB)^2$

$(PB)^2 = 169 - 9 = 160$

PB = $4\sqrt{10}$ cm

Question: 50

Which of the foll

Solution:

A circle cannot have more than two tangents parallel, because tangents to be parallel they should be at diametrically ends and a diameter has two ends only.

Question: 51

Which of the foll

Solution:

A straight line can meet a circle at two points in case if it is a chord or diameter or a line intersecting the circle at two points.

Question: 52

Which of the foll

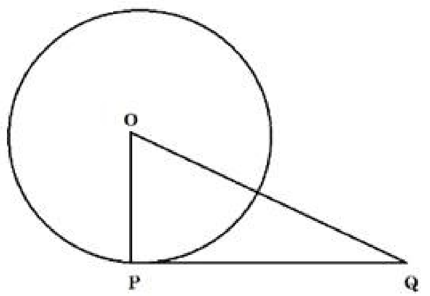
Solution:

If a tangent is drawn from a point inside a circle, it will intersect the circle at two points, so no tangent can be drawn from a point inside the circle.

Question: 53

Assertion - and -

Solution:



Let us consider a circle with center O and radius 12 cm

A tangent PQ is drawn at point P such that PQ = 16 cm

To Find : Length of OQ

Now, OP ⊥ PQ [Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled △POQ,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OQ)^2 = (OP)^2 + (PQ)^2$

$(OQ^2 = (12)^2 + (16)^2$

625 = 144 + 256

$(OQ)^2 = 400$

OQ = 20 cm

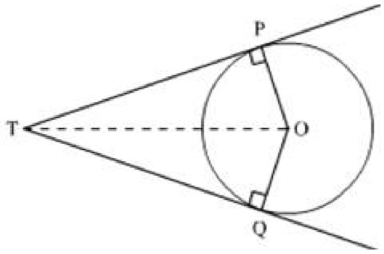
So,

Assertion is correct, and Reason is also correct.

Question: 54

Assertion - and -

Solution:



Let PT and PQ are two tangents from external point P to a circle with center O

In △OPT and △OQT

OP = OQ

[radii of same circle]

OT = OT

[common]

PT = PQ

[Tangents drawn from an external point are equal]

△OPT ≅ △OQT

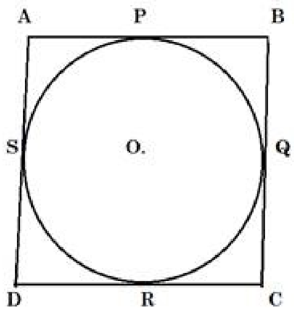
[By Side - Side - Side Criterion]

∠POT = ∠QOT

[Corresponding parts of congruent triangles are congruent]

i.e. Assertion is true

Now,



Consider a circle circumscribed by a parallelogram ABCD, Let side AB, BC, CD and AD touch circles at P, Q, R and S respectively.

As ABCD is a parallelogram

AB = CD and BC = AD

[opposite sides of a parallelogram are equal] [1]

Now, As tangents drawn from an external point are equal.

We have

AP = AS

[tangents from point A]

BP = BQ

[tangents from point B]

CR = CQ

[tangents from point C]

DR = DS

[tangents from point D]

Add the above equations

AP + BP + CR + DR = AS + BQ + CQ + DS

AB + CD = AS + DS + BQ + CQ

AB + CD = AD + BC

AB + AB = BC + BC [From 1]

AB = BC ...[2]

From [1] and [2]

AB = BC = CD = AD

And we know,

A parallelogram with all sides equal is a rhombus

So, reason is also true, but not a correct reason for assertion.

Hence, B is correct option .

Question: 55

Assertion - and -

Solution:

For Assertion :

In the given Figure,

As tangents drawn from an external point are equal.

We have

AP = AS

[tangents from point A]

BP = BQ

[tangents from point B]

CR = CQ

[tangents from point C]

DR = DS

[tangents from point D]

Add the above equations

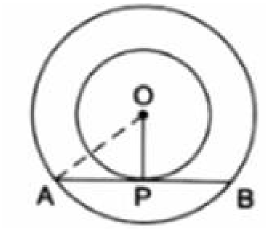
AP + BP + CR + DR = AS + BQ + CQ + DS

AB + CD = AS + DS + BQ + CQ

AB + CD = AD + BC

So, assertion is not true

For Reason,



Consider two concentric circles with common center O and AB is a chord to outer circle and is tangent to inner circle P.

Now,

OP ⊥ AB

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

We know, that perpendicular from center to chord bisects the chord.

So, P bisects AB.

Reason is true

Hence, Assertion is false, But Reason is true.

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

In the given figu

Solution:

In the given figure PT is a tangent to circle ∴ we have

∠OPT = 90°

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠OPQ + ∠QPT = 90°

∠OPQ + 50° = 90°

∠OPQ = 40°

Now, In △POQ

OP = OQ

∠PQO = ∠QPO = 40°

[Angles opposite to equal sides are equal]

Now,

∠ PQO + ∠QPO + ∠ POQ = 180° [

By angle sum property of triangle]

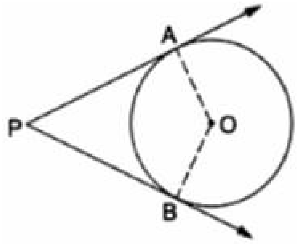
40° + 40° + ∠POQ = 180°

∠POQ = 100°

Question: 2

If the angle betw

Solution:



Let us consider a circle with center O and OA and OB are two radii such that ∠AOB = 60° .

AP and BP are two intersecting tangents at point P at point A and B respectively on the circle .

To find : Angle between tangents, i.e. ∠APB

As AP and BP are tangents to given circle,

We have,

OA ⊥ AP and OB ⊥ BP

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, ∠OAP = ∠OBP = 90°

In quadrilateral AOBP,

By angle sum property of quadrilateral, we have

$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$

$90° + 90° + \angle APB + 130° = 360°$

$\angle APB = 50°$

Question: 3

If tangents PA an

Solution:

In △AOP and △BOP

AP = BP

[Tangents drawn from an external point are equal]

OP = OP [Common]

OA = OB

[Radii of same circle]

$\triangle AOP \cong \triangle BOP$

[By Side - Side - Side criterion]

$\angle APO = \angle BPO$

[Corresponding parts of congruent triangles are congruent]

$\angle APB = \angle APO + \angle BPO$

$80 = \angle APO + \angle APO$

$2\angle APO = 80$

$\angle APO = 40°$

In △AOP

$\angle APO + \angle AOP + \angle OAP = 180°$

[By angle sum property]

$40° + \angle AOP + 90° = 180°$

[$\angle OAP = 90°$ as $OA \perp AP$ because Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle AOP = 50°$

Question: 4

In the given figu

Solution:

Given : From an external point A, two tangents, AD and AE are drawn to a circle with center O. At a point F on the circle tangent is drawn which intersects AE and AD at B and C, respectively. And AE = 5 cm

To Find : Perimeter of $\triangle ABC$

As we know that, Tangents drawn from an external point to a circle are equal.

So we have

BE = BF ...[1]

[Tangents from point B]

CF = CD ...[2]

[Tangents from point C]

Now Perimeter of Triangle abc

= AB + BC + AC

= AB + BF + CF + AC

= AB + BE + CD + AC ...[From 1 and 2]

= AE + AD

Now,

AE = AD = 5 cm as tangents drawn from an external point to a circle are equal

So we have

Perimeter = AE + AD = 5 + 5 = 10 cm

Question: 5

In the given figu

Solution:

As we know, Tangents drawn from an external point are equal.

CR = CQ [tangents from point C]

CQ = 3 cm [as CR = 3 cm]

Also,

BC = BQ + CQ

7 = BQ + 3 [BC = 7 cm]

BQ = 4 cm

Now,

BP = BQ [tangents from point B]

BP = 4 cm ...[1]

AP = AS [tangents from point A]

AP = 5 cm [As AC = 5 cm] ....[2]

AB = AP + BP = 5 + 4 = 9 cm [From 1 and 2]

AB = x = 9 cm

Question: 6

In the given figu

Solution:

Given : PA and PB are tangents to a circle with center O

To show : A, O, B and P are concyclic i.e. they lie on a circle i.e. AOBP is a cyclic quadrilateral.

Proof :

OB ⊥ PB and OA ⊥ AP

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle OBP = \angle OAP = 90°$

$\angle OBP + \angle OAP = 90 + 90 = 180°$

AOBP is a cyclic quadrilateral i.e. A, O, B and P are concyclic.

[As we know if the sum of opposite angles in a quadrilateral is 180° then quadrilateral is cyclic]

Hence Proved.

Question: 7

In the given figu

Solution:

In the given Figure,

PA = PB

[Tangents drawn from an external points are equal]

$\angle PBA = \angle PAB$

[Angles opposite to equal sides are equal]

$\angle PBA = \angle PAB = 65°$

In $\triangle APB$

$\angle PAB + \angle PBA + \angle APB = 180°$

$65° + 65° + \angle APB = 180°$

$\angle APB = 50°$

Also,

OB ⊥ AP

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle OAP = 90°$

$\angle OAB + \angle PAB = 90°$

$\angle OAB + 65° = 90°$

$\angle OAB = 25°$

Question: 8

Two tangent segme

Solution:

Given : A circle with center O , BC and BD are two tangents such that $\angle CBD = 120°$

To Proof : OB = 2BC

Proof :

In $\triangle BOC$ and $\triangle BOD$

BC = BD

[Tangents drawn from an external point are equal]

OB = OB

[Common]

OC = OD

[Radii of same circle]

$\triangle BOC \cong \triangle BOD$ [By Side - Side - Side criterion]

$\angle OBC = \angle OBD$

[Corresponding parts of congruent triangles are congruent]

$\angle OBC + \angle OBD = \angle CBD$

$\angle OBC + \angle OBC = 120°$

$2\ \angle OBC = 120°$

$\angle OBC = 60°$

In $\triangle OBC$

$$\cos\angle OBC = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{OB}$$

$$\cos 60° = \frac{BC}{OB} = \frac{1}{2}$$

$\Rightarrow OB = 2BC$

Hence Proved !

Question: 9

Fill in the blank

Solution:

(i) secant

(ii) two

(iii) point of contact

(iv) infinitely many

Question: 10

Prove that the le

Solution:

Let us consider a circle with center O.

TP and TQ are two tangents from point T to the circle.

To Proof : PT = QT

Proof :

$OP \perp PT$ and $OQ \perp QT$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle OPT = \angle OQT = 90°$

In △TOP and △QOT

∠OPT = ∠OQT

[Both 90°]

OP = OQ

[Common]

OT = OT

[Radii of same circle]

△TOP ≅ △QOT

[By Right Angle - Hypotenuse - Side criterion]

PT = QT

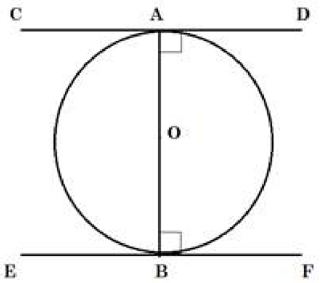
[Corresponding parts of congruent triangles are congruent]

Hence Proved.

Question: 11

Prove that the ta

Solution:



Let AB be the diameter of a circle with center O.

CD and EF are two tangents at ends A and B respectively.

To Prove : CD || EF

Proof :

OA ⊥ CD and OB ⊥ EF

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠OAD = ∠OBE = 90°

∠OAD + ∠OBE = 90° + 90° = 180°

Considering AB as a transversal

⇒ CD || EF

[Two sides are parallel, if any pair of the interior angles on the same sides of transversal is supplementary]

Question: 12

In the given figu

Solution:

We know, that tangents drawn from an external point are equal.

AD = AF

[tangents from point A] [1]

BD = BE

[tangents from point B] [2]

CF = CE

[tangents from point C] [3]

Now,

AB = AC [Given] ...[4]

Substracting [1] From [4]

AB - AD = AC - AF

BD = CF

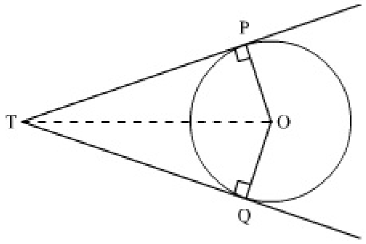
BE = CE [From 2 and 3]

Hence Proved.

Question: 13

If two tangents a

Solution:



Let PT and PQ are two tangents from external point P to a circle with center O

To Prove : PT and PQ subtends equal angles at center i.e. ∠POT = ∠QOT

In △OPT and △OQT

OP = OQ [radii of same circle]

OT = OT [common]

PT = PQ [Tangents drawn from an external point are equal]

△OPT ≅ △OQT [By Side - Side - Side Criterion]

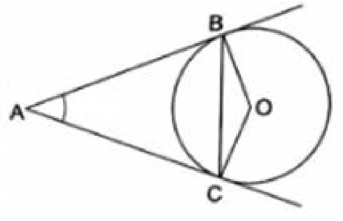
∠POT = ∠QOT [Corresponding parts of congruent triangles are congruent]

Hence, Proved.

Question: 14

Prove that the ta

Solution:



Let us consider a circle with center O and BC be a chord, and AB and AC are tangents drawn at end of a chord

To Prove : AB and AC make equal angles with chord, i.e. ∠ABC = ∠ACB

Proof :

In △ABC

AB = PC

[Tangents drawn from an external point to a circle are equal]

∠ACB = ∠ABC

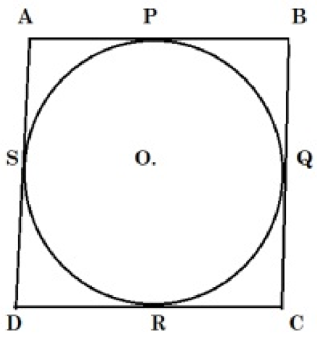
[Angles opposite to equal sides are equal]

Hence Proved.

Question: 15

Prove that the pa

Solution:



Consider a circle circumscribed by a parallelogram ABCD, Let side AB, BC, CD and AD touch circles at P, Q, R and S respectively.

To Proof : ABCD is a rhombus.

As ABCD is a parallelogram

AB = CD and BC = AD ...[1]

[opposite sides of a parallelogram are equal]

Now, As tangents drawn from an external point are equal.

We have

AP = AS

[tangents from point A]

BP = BQ

[tangents from point B]

CR = CQ

[tangents from point C]

DR = DS

[tangents from point D]

Add the above equations

AP + BP + CR + DR = AS + BQ + CQ + DS

AB + CD = AS + DS + BQ + CQ

AB + CD = AD + BC

AB + AB = BC + BC [From 1]

AB = BC ...[2]

From [1] and [2]

AB = BC = CD = AD

And we know,

A parallelogram with all sides equal is a rhombus

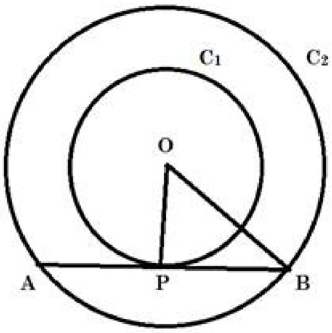
So, ABCD is a rhombus.

Hence Proved.

Question: 16

Two concentric ci

Solution:



Given : Two concentric circles (say C_1 and C_2) with common center as O and radius r_1 = 5 cm and r_2 = 3 cm respectively.

To Find : Length of the chord of the larger circle which touches the circle C_2. i.e. Length of AB.

As AB is tangent to circle C_2 and we know that "Tangent at any point on the circle is perpendicular to the radius through point of contact"

So, we have,

OP ⊥ AB

∴ OPB is a right - angled triangle at P,

By Pythagoras Theorem in △OPB

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

We have,

$(OP)^2 + (PB)^2 = (OB)^2$

$r_2^2 + (PB)^2 = r_1^2$

$(3)^2 + (PB)^2 = (5)^2$

$9 + (PB)^2 = 25$

$(PB)^2 = 16$

PB = 4 cm

Now, AP = PB ,

[as perpendicular from center to chord bisects the chord and OP ⊥ AB]

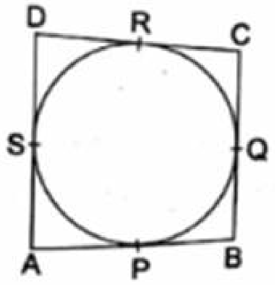
So,

AB = AP + PB = PB + PB = 2PB = 2(4) = 8 cm

Question: 17

A quadrilateral i

Solution:



Let us consider a quadrilateral ABCD, And a circle is circumscribed by ABCD

Also, Sides AB, BC, CD and DA touch circle at P, Q, R and S respectively.

To Proof : Sum of opposite sides are equal, i.e. AB + CD = AD + BC

Proof :

In the Figure,

As tangents drawn from an external point are equal.

We have

AP = AS

[tangents from point A]

BP = BQ

[tangents from point B]

CR = CQ

[tangents from point C]

DR = DS

[tangents from point D]

Add the above equations

AP + BP + CR + DR = AS + BQ + CQ + DS

AB + CD = AS + DS + BQ + CQ

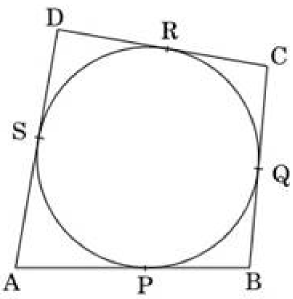
AB + CD = AD + BC

Hence Proved.

Question: 18

Prove that the op

Solution:



Consider a quadrilateral, ABCD circumscribing a circle with center O and AB, BC, CD and AD touch the circles at point P, Q, R and S respectively.

Joined OP, OQ, OR and OS and renamed the angles (as in diagram)

To Prove : Opposite sides subtends supplementary angles at center i.e.

$\angle AOB + \angle COD = 180°$ and $\angle BOC + \angle AOD = 180°$

Proof :

In $\triangle AOP$ and $\triangle AOS$

AP = AS

[Tangents drawn from an external point are equal]

AO = AO

[Common]

OP = OS

[Radii of same circle]

$\triangle AOP \cong \triangle AOS$

[By Side - Side - Side Criterion]

$\angle AOP = \angle AOS$

[Corresponding parts of congruent triangles are congruent]

$\angle 1 = \angle 2$ ...[1]

Similarly, We can Prove

$\angle 3 = \angle 4$ ....[2]

$\angle 5 = \angle 6$ ....[3]

$\angle 7 = \angle 8$ ....[4]

Now,

As the angle around a point is 360°

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 6 + \angle 6 + \angle 7 + \angle 7 = 360°$ [From 1, 2, 3 and 4]

$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

$\angle AOB + \angle COD = 180°$

[As, $\angle 2 + \angle 3 = \angle AOB$ and $\angle 5 + \angle 6 = \angle COD$] [5]

Also,

$\angle AOB + \angle BOC + \angle COD + \angle AOD = 360°$

[Angle around a point is 360°]

$\angle AOB + \angle COD + \angle BOC + \angle AOD = 360°$

$180° + \angle BOC + \angle AOD = 360°$ [From 5]

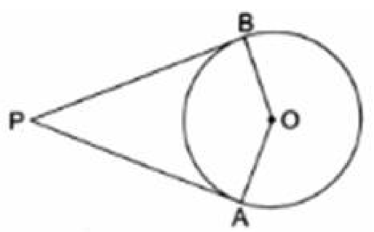
$\angle BOC + \angle AOD = 180°$

Hence Proved

Question: 19

Prove that the an

Solution:



Let us consider a circle with center O and PA and PB are two tangents to the circle from an external point P

To Prove : Angle between two tangents is supplementary to the angle subtended by the line segments joining the points of contact at center, i.e. $\angle APB + \angle AOB = 180°$

Proof :

As AP and BP are tangents to given circle,

We have,

$OA \perp AP$ and $OB \perp BP$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, $\angle OAP = \angle OBP = 90°$

In quadrilateral AOBP, By angle sum property of quadrilateral, we have

$\angle OAP + \angle OBP + \angle AOB + \angle APB = 360°$

$90° + 90° + \angle AOB + \angle APB = 360°$

$\angle AOB + \angle APB = 180°$

Hence Proved

Question: 20

PQ is a chord of

Solution:

Given : A circle with center O and radius 3 cm and PQ is a chord of length 4.8 cm. The tangents at P and Q intersect at point T

To Find : Length of PT

Construction : Join OQ

Now in $\triangle OPT$ and $\triangle OQT$

$OP = OQ$

[radii of same circle]

$PT = PQ$

[tangents drawn from an external point to a circle are equal]

$OT = OT$

[Common]

$\triangle OPT \cong \triangle OQT$

[By Side - Side - Side Criterion]

$\angle POT = \angle OQT$

[Corresponding parts of congruent triangles are congruent]

or $\angle POR = \angle OQR$

Now in $\triangle OPR$ and $\triangle OQR$

$OP = OQ$

[radii of same circle]

$OR = OR$ [Common]

$\angle POR = \angle OQR$ [Proved Above]

$\triangle OPR \cong \triangle OQT$

[By Side - Angle - Side Criterion]

$\angle ORP = \angle ORQ$

[Corresponding parts of congruent triangles are congruent]

Now,

$\angle ORP + \angle ORQ = 180°$

[Linear Pair]

$\angle ORP + \angle ORP = 180°$

$\angle ORP = 90°$

$\Rightarrow OR \perp PQ$

$\Rightarrow RT \perp PQ$

As $OR \perp PQ$ and Perpendicular from center to a chord bisects the chord we have

$PR = QR = PQ/2 = 16/2 = 8$ cm

$\therefore$ In right - angled $\triangle OPR$,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OR)^2 + (PR)^2$

$(10)^2 = (OR)^2 + (8)^2$

$100 = (OR)^2 + 64$

$(OR)^2 = 36$

$OR = 6$ cm

Now,

In right angled $\triangle TPR$, By Pythagoras Theorem

$(PT)^2 = (PR)^2 + (TR)^2$ [1]

Also, $OP \perp OT$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

In right angled $\triangle OPT$, By Pythagoras Theorem

$(PT)^2 + (OP)^2 = (OT)^2$

$(PR)^2 + (TR)^2 + (OP)^2 = (TR + OR)^2$ [From 1]

$(8)^2 + (TR)^2 + (10)^2 = (TR + 6)^2$

$64 + (TR)^2 + 100 = (TR)^2 + 2(6)TR + (6)^2$

$164 = 12TR + 36$

$12TR = 128$

$TR = 10.7$ cm [Appx]

Using this in [1]

$PT^2 = (8)^2 + (10.7)^2$

$PT^2 = 64 + 114.49$

$PT^2 = 178.49$

$PT = 13.67$ cm [Appx]