8 Chapter Ratio, Proportion and Variation

INTRODUCTION

The concept of ratio, proportion and variation is an important one for the aptitude examinations. Questions based on this chapter have been regularly asked in the CAT exam (direct or application based). In fact, questions based on this concept regularly appear in all aptitude tests (XLRI, CMAT, NMIMS, SNAP, NIFT, IRMA, Bank PO, etc.).

Besides, this concept is very important in the area of Data Interpretation, where ratio change and ratio comparisons are very popular question types.

RATIO

When comparing any two numbers, sometimes, it is necessary to find out how many times one number is greater (or less) than the other. In other words, we often need to express one number as a fraction of the other.

In general, the ratio of a number x to a number y is defined as the quotient of the numbers x and y.

The numbers that form the ratio are called the terms of the ratio. The numerator of the ratio is called the *antecedent* and the denominator is called the *consequent* of the ratio.

The ratio may be taken for homogenous quantities or for heterogeneous quantities. In the first case, the ratio has no unit (or is unitless), while ih222n the second case, the unit of the ratio is based on the units of the numerator and that of the denominator.

Ratios can be expressed as percentages. To express the value of a ratio as a percentage, we multiply the ratio by 100.

Thus, 4/5 = 0.8 = 80%

The Calculation of a ratio:

Percentage and decimal values

The calculation of ratio is principally on the same lines as the calculation of a percentage value.

Hence, you should see it as:

The ratio 2/4 has a percentage value of 50% and it has a decimal value of 0.5.

It should be pretty obvious to you that in order to find out the decimal value of any ratio, calculate the percentage value using the percentage rule method illustrated in the chapter of percentage and then shift the decimal point 2 places to the left.

Thus a ratio which has a percentage value of 62.47% will have a decimal value of 0.6247.

Some Important Properties of Ratios

1. If we multiply the numerator and the denominator of a ratio by the same number, the ratio remains unchanged.

   That is, $\frac{a}{b} = \frac{ma}{mb}$

2. If we divide the numerator and the denominator of a ratio by the same number, the ratio remains unchanged. Thus

   $a/b = \frac{(a/d)}{(b/d)}$

3. Denominator equation method:

   The magnitudes of two ratios can be compared by equating the denominators of the two ratios and then checking for the value of the numerator.

   Thus, if we have to check for

   8/3 vs 11/4

   We can compare $\frac{(8 \times 1.33)}{(3 \times 1.33)}$ vs $\frac{11}{4}$

   That is, $\frac{10.66}{4} < \frac{11}{4}$

   In fact, the value of a ratio has a direct relationship with the value of the numerator of the ratio. At the same time, it has an inverse relationship with the denominator of the ratio. Since the denominator has an inverse relationship with the ratio's value, it involves an unnecessary inversion in the minds of the reader. Hence, in my opinion, we should look at maintaining constancy in the denominator and work all the requisite calculations on the numerator's basis.

   The reader should recall here the Product Constancy Table (or the denominator change to ratio change table) explained in the chapter of percentages to understand the mechanics of how a change in the denominator affects the value of the ratio. A clear understanding of these dynamics will help the student become much faster in solving the problems based on ratios.

4. The ratio of two fractions can be expressed as a ratio of two integers. Thus the ratio:

   $a/b : c/d = \frac{(a/b)}{(c/d)} = \frac{ad}{bc}$

5. If either or both the terms of a ratio are a surd quantity, then the ratio will never evolve into integral numbers unless the surd quantities are equal. Use this principle to spot options in questions having surds.

   Example: $\frac{\sqrt{3}}{\sqrt{2}}$ can never be represented by integers.

This principle can also be understood in other words as follows:

Suppose while solving a question, you come across a situation where $\sqrt{3}$ appears as a part of the process. In such a case, it would be safe to assume that $\sqrt{3}$ will also be part of the answer. Since the only way the $\sqrt{3}$ can be removed from the answer is by multiplying or dividing the expression by $\sqrt{3}$.

Thus for instance, the formula for the area of an equilateral triangle is ($\sqrt{3}$ /4)a^2.

Hence, you can safely assume that the area of any equilateral triangle will have $\sqrt{3}$ in its answer. The only case when this gets negated would be when the value of the side has a component which has the fourth root of three.

6. The multiplication of the ratios $\frac{a}{b}$ and $\frac{c}{d}$ yields:

   $a/b \times c/d = \frac{ac}{bd}$

7. When the ratio a/b is compounded with itself, the resulting ratio is a^2/b^2 and is called the duplicate ratio. Similarly, a^3/b^3 is the triplicate ratio and $a^{0.5}/b^{0.5}$ is the sub-duplicate ratio of a/b.

8. If $a/b = c/d = e/f = g/h = k$ then

   $k = \frac{(a+c+e+g)}{(b+d+f+h)}$

9. If a_1/b_1, a_2/b_2, a_3/b_3 ... a_n/b_n are unequal fractions

   Then the ratio:

   $\frac{(a_1 + a_2 + a_3 + \ldots + a_n)}{(b_1 + b_2 + b_3 + \ldots b_n)}$

   lies between the lowest and the highest of these fractions.

10. If we have two equations containing three unknowns as

    $a_1x + b_1y + c_1z = 0$ (1)

    and $a_2x + b_2y + c_2z = 0$ (2)

    Then, the value of x, y and z cannot be resolved without having a third equation.

    However, in the absence of a third equation, we can find the proportion $x : y : z$. This will be given by $b_1c_2 - b_2c_1 : c_1a_2 - c_2a_1 : a_1b_2 - a_2b_1$.

    This can be remembered by writing as follows:

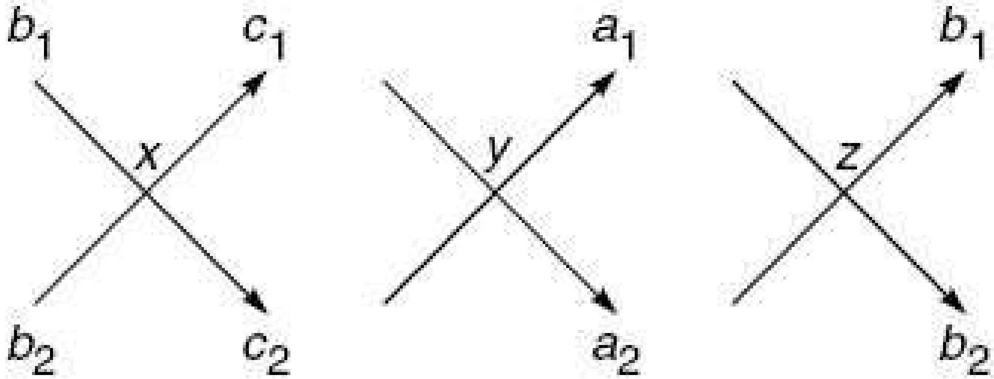


Fig. 8.1

Multiply the coefficients across the arrow indicated always taking a multiplication as positive if the arrow points downwards and taking it as negative if the arrow points upwards.

Thus x corresponds to $b_1c_2 - b_2c_1$ and so on.

11. If the ratio $a/b > 1$ (called a ratio of greater inequality) and if k is a positive number:

    $(a + k)/(b + k) < a/b$ and $(a - k)/(b - k) > a/b$

    Similarly if $a/b < 1$ then

    $(a + k)/(b + k) > a/b$ and $(a - k)/(b - k) < a/b$

    [The student should try assuming certain values and check the results]

12. Maintenance of equality when numbers are added in both the numerator and the denominators.

    This if best illustrated through an example:

    $20/30 = (20 + 2)/(30 + 3)$

    i.e. $a/b = (a + c)/(b + d)$ if and only if $c/d = a/b$. In other words, the ratio of the additions should be equal to the original ratio to maintain equality of ratios when two different numbers are added in the numerator and denominator.

    Consequently, if $c/d > a/b$ then $(a + c)/(b + d) > a/b$

    and if $c/d < a/b$ then $(a + c)/(b + d) < a/b$

    The practical applications of (11) and (12) is of immense importance for all aptitude exams.

MATHEMATICAL USES OF RATIOS

Use 1

As a bridge between 3 or more quantities:

Suppose you have a ratio relationship given between the salaries of two individuals *A* and *B*. Further, if there is another ratio relationship between *B* and *C*. Then, by combining the two ratios, you can come up with a single consolidated ratio between *A*, *B* and *C*. This ratio will give you the relationship between *A* and *C*.

Illustration

The Ratio of *A*'s salary to *B*'s salary is 2:3. The ratio of *B*'s salary to *C*'s salary is 4:5. What is the ratio of A's salary to C's salary?

Using the conventional process in this case:

Take the LCM of 3 and 4 (the two values representing B's amount). The LCM is 12.

Then, convert B's value in each ratio to 12.

Thus, Ratio 1 = 8/12 and Ratio 2 = 12/15

Thus, $A:B:C$ = 8:12:15

Hence, $A:C$ = 8:15

Further, if it were given that A's salary was 800, you could derive the values of C's salary (as 1500).

SHORTCUT for this process:

The LCM process gets very cumbersome especially if you are trying to create a bridge between more than 3 quantities.

Suppose, you have the ratio train as follows:

$A:B$ = 2:3

$B:C$ = 4:5

$C:D$ = 6:11

$D:E$ = 12:17

In order to create one consolidated ratio for this situation using the LCM process becomes too long.

The short cut goes as follows:

$A:B:C:D:E$ can be written directly as:

$2 \times 4 \times 6 \times 12 : 3 \times 4 \times 6 \times 12 : 3 \times 5 \times 6 \times 12 : 3 \times 5 \times 11 \times 17$

The thought algorithm for this case goes as:

To get the consolidated ratio $A:B:C:D:E$, A will correspond to the product of all numerators ($2 \times 4 \times 6 \times 12$) while B will take the first denominator and the last 3 numerators ($3 \times 4 \times 6 \times 12$). C on the other hand takes the first two denominators and the last 2 numerators ($3 \times 5 \times 6 \times 12$), D takes the first 3 denominators and the last numerator ($3 \times 5 \times 11 \times 12$) and E takes all the four denominators ($3 \times 5 \times 11 \times 17$).

In mathematical terms this can be written as:

If $a/b = N_1/D_1$, $b/c = N_2/D_2$, $c/d = N_3/D_3$ and $d/e = N_4/D_4$ then $a : b : c : d : e = N_1N_2N_3N_4 : D_1N_2N_3N_4 : D_1D_2N_3N_4 : D_1D_2D_3N_4 : D_1D_2D_3D_4$

Use 2

Ratio as a Multiplier

This is the most common use of Ratios:

If $A:B$ is 3:1, then the value of B has to be multiplied by 3 to get the value of A.

CALCULATION METHODS related to RATIOS

(A) Calculation methods for Ratio comparisons:

There could be four broad cases when you might be required to do ratio comparisons:

The table below clearly illustrates these:

	Numerator	*Denominator*	*Ratio*	*Calculations*
Case 1	Increases	Decreases	Increase	Not required
Case 2	Increases	Increases	May Increase or Decrease	Required
Case 3	Decreases	Increases	Decreases	Not required
Case 4	Decreases	Decreases	May Increase or Decrease	Required

In case 2 and 4 in the table, calculations will be necessitated. In such a situation, the following process can be used for ratio comparisons.

1. The Cross Multiplication Method

Two ratios can be compared using the cross multiplication method as follows. Suppose you have to compare

12/17 with 15/19

Then, to test which ratio is higher cross multiply and compare 12×19 and 15×17.

If 12×19 is bigger the Ratio 12/17 will be bigger. If 15×17 is higher, the ratio 15/19 will be higher.

In this case, 15×17 being higher, the Ratio 15/19 is higher.

Note: In real time usage (esp. in D.I.) this method is highly impractical and calculating the product might be more cumbersome than calculating the percentage values.

Thus, this method will not be able to tell you the answer if you have to compare $\frac{3743}{5624}$ with $\frac{3821}{5783}$

2. Percentage value comparison method:

Suppose you have to compare: $\frac{173}{212}$ with $\frac{181}{241}$

In such a case just by estimating the 10% ranges for each ratio you can clearly see that —

he first ratio is > 80% while the second ratio is < 80%

Hence, the first ratio is obviously greater.

This method is extremely convenient if the two ratios have their values in different 10% ranges.

However, this problem will become slightly more difficult, if the two ratios fall in the same 10% range. Thus, if you had to compare $^{173}/_{212}$ with $^{181}/_{225}$, both the values would give values between 80 – 90%. The next step would be to calculate the 1% range.

The first ratio here is 81 – 82% while the second ratio lies between 80 – 81%

Hence the first ratio is the larger of the two.

Note: For this method to be effective for you, you'll first need to master the percentage rule method for calculating the percentage value of a ratio. Hence if you cannot see that 169.6 is 80% of 212 or for that matter that 81% of 212 is 171.72 and 82% is 172.84 you will not be able to use this method effectively. (This is also true for the next method.) However, once you can calculate percentage values of 3 digit ratios to 1% range, there is not much that can stop you in comparing ratios. The CAT and all other aptitude exams normally do not challenge you to calculate further than the 1% range when you are looking at ratio comparisons.

3. Numerator denominator percentage change method:

There is another way in which you can compare close ratios like 173/212 and 181/225. For this method, you need to calculate the percentage changes in the numerator and the denominator.

Thus:

173 Æ 181 is a % increase of 4 – 5%

While 212 Æ 225 is a % increase of 6 – 7%.

In this case, since the denominator is increasing more than the numerator, the second ratio is smaller.

This method is the most powerful method for comparing close ratios—provided you are good with your percentage rule calculations.

(B) Method for calculating the value of a percentage change in the ratio:

PCG (Percentage Change Graphic) gives us a convenient method to calculate the value of the percentage change in a ratio.

Suppose, you have to calculate the percentage change between 2 ratios. This has to be done in two stages as:

$$\text{Original Ratio} \xrightarrow[\text{numerator}]{\text{Effect of}} \text{Intermediate Ratio}$$

$$\xrightarrow[\text{Denominator}]{\text{Effect of}} \text{Final Ratio}$$

Thus if 20/40 becomes 22/50

Effect of numerator = 20 Æ 22(10% increase)

Effect of denominator = 50 Æ 40(25% decrease) (reverse fashion)

Overall effect on the ratio:

$$100 \xrightarrow[\substack{\text{Numerator}\\\text{Effect}}]{10\%\uparrow} 110 \xrightarrow[\substack{\text{Denominator}\\\text{Effect}}]{25\%\downarrow} 82.5$$

Hence, overall effect = 17.5% decrease.

PROPORTION

When two ratios are equal, the four quantities composing them are said to be proportionals. Thus if $a/b = c/d$, then a, b, c, d are proportionals. This is expressed by saying that a is to b as c is to d, and the proportion is written as

$a : b : : c : d$

or

$a : b = c : d$

• The terms a and d are called the extremes while the terms b and c are called the means.

• If four quantities are in proportion, the product of the extremes is equal to the product of the means.

Let a, b, c, d be the proportionals.

Then by definition $a/b = c/d$

\ad = bc

Hence if any three terms of proportion are given, the fourth may be found. Thus if a, c, d are given, then b

$= ad/c$.

• **If three quantities a, b and c are in continued proportion, then $a : b = b : c$**

\ $ac = b^2$

In this case, b is said to be a *mean proportional* between a and c; and c is said to be a *third proportional* to a and b.

• **If three quantities are proportionals the first is to the third is the duplicate ratio of the first to the second.**

That is: for $a : b : : b : c$

$a : c = a^2 : b^2$

• If four quantities a, b, c and d form a proportion, many other proportions may be deduced by the properties of fractions. The results of these operations are very useful. These operations are

1. **Invertendo:** If $a/b = c/d$ then $b/a = d/c$
2. **Alternando:** If $a/b = c/d$, then $a/c = b/d$
3. **Componendo:** If $a/b = c/d$, then $\left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right)$
4. **Dividendo:** If $a/b = c/d$, then $\left(\frac{a-b}{b}\right) = \left(\frac{c-d}{d}\right)$
5. **Componendo and Dividendo:** If $a/b = c/d$, then $(a + b)/(a – b) = (c + d)/(c – d)$

VARIATION

Essentially there are two kinds of proportions that two variables can be related by:

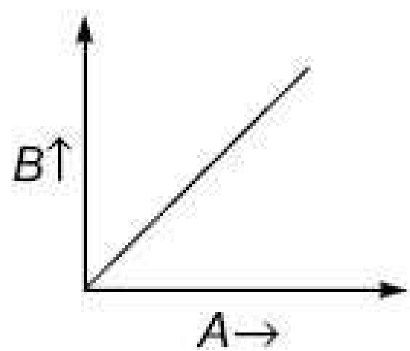
(1) Direct Proportion

When it is said that A varies directly as B, you should understand the following implications:

(a) **Logical implication:** When A increases B increases

(b) **Calculation implication:** If A increases by 10%, B will also increase by 10%

(c) **Graphical implications:** The following graph is representative of this situation.



(d) **Equation implication:** The ratio A/B is constant.

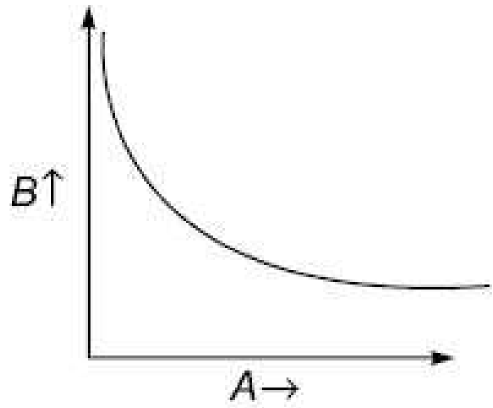
(2) Inverse Proportion:

When A varies inversely as B, the following implication arise.

(a) **Logical implication:** When A increases B decreases

(b) **Calculation implication:** If A decreases by 9.09%, B will increase by 10%.

(c) Graphical implications: The following graph is representative of this situation.



(d) Equation implication: The product $A \times B$ is constant.

A quantity '*A*' is said to vary directly as another '*B*' when the two quantities depend upon each other in such a manner that if *B* is changed, *A* is changed in the same ratio.

Note: The word directly is often omitted, and *A* is said to vary as *B*.

The symbol µ *is used to denote variation. Thus, A* µ *B is read "A varies as B".*

If *A* µ *B* then, $A = KB$ where *K* is any constant.

Thus to find $K = A/B$, we need one value of *A* and a corresponding value of *B*.

where $K = 3/12 = 1/4$ fi $A = B \times (1/4)$.

A quantity *A* is said to vary inversely as another *B* when *A* varies directly as the reciprocal of *B*. Thus if *A* varies inversely as *B*, $A = m/B$, where *m* is constant.

A quantity is said to vary jointly as a number of others when it varies directly as their product. Thus *A* varies jointly as *B* and *C*, when $A = mBC$.

If *A* varies as *B* when *C* is constant, and *A* varies as *C* when *B* is constant, then *A* will vary as *BC* when both *B* and *C* vary.

The variation of *A* depends partly on that of *B* and partly on that of *C*. Assume that each letter variation takes place separately, each in its turn producing its own effect on *A*.

WORKED-OUT PROBLEMS

Problem 8.1 ` 5783 is divided among Sherry, Berry, and Cherry in such a way that if t` 28, t` 37 and t` 18 be deducted from their respective shares, they have money in the ratio 4 : 6 : 9. Find Sherry's share.

(a) ` 1256 (b) ` 1228
(c) ` 1456 (d) ` 1084

Solution The problem clearly states that when we reduce 28, 37 and 18 rupees respectively from Sherry's, Berry's and Cherry's shares, the resultant ratio is: 4 : 6 : 9.

Thus, if we assume the reduced values as

$4x$, $6x$ and $9x$, we will have Æ

Sherry's share Æ $4x + 28$, Berry's share Æ $6x + 37$ and Cherry's share Æ $9x + 18$ and thus we have

$(4x + 28) + (6x + 37) + (9x + 18) = 5783$

Æ $19x = 5783 – 83 = 5700$

Hence, $x = 300$.

Hence, Sherry's share is t` 1228.

Note: For problems based on this chapter we are always confronted with ratios and proportions between different number of variables. For the above problem we had three variables which were in the ratio of 4 : 6 : 9. When we have such a situation we normally assume the values in the same proportion, using one unknown 'x' only (in this example we could take the three values as $4x$, $6x$ and $9x$ respectively).

Then, the total value is represented by the addition of the three giving rise to a linear equation, which on solution, will result in the answer to the value of the unknown 'x'.

However, the student should realise that most of the time this unknown 'x' is not needed to solve the problem. This is illustrated through the following alternate approach to solving the above problem:

Assume the three values as 4, 6 and 9

Then we have

(4 + 28) + (6 + 37) + (9 + 18) = 5783

Æ 19 = 5783–83 = 5700 Æ 1 = 300

Hence, 4 + 28 = 1228.

While adopting this approach the student should be careful in being able to distinguish the numbers in bold as pointing out the unknown variable.

Problem 8.2 Two numbers are in the ratio P : Q. When 1 is added to both the numerator and the denominator, the ratio gets changed to R/S. Again, when 1 is added to both the numerator and the denominator, it becomes 1/2. Find the sum of P and Q.

(a) 3 (b) 4
(c) 5 (d) 6

Solution The normal process of solving this problem would be through the writing of equations.

Approach 1: We have: Final ratio is $x/2x$.

Then, $\frac{x-2}{2x-2} = P/Q$

Then, $Qx - 2Q = 2Px - 2P$

$2(P - Q) = x(2P - Q)$ (At this stage we see that the solution is a complex one)

Approach 2: $\frac{R+1}{S+1} = \frac{1}{2}$

$2R + 2 = S + 1$ Æ $R = \frac{S-1}{2}$

Now, $\frac{P+1}{Q+1} = \frac{R}{S} = \frac{S-1}{2S}$ (At this time we realise that we are getting stuck)

Start from front:

$\frac{P+2}{Q+2} = 1/2$ Æ $2P + 4 = Q + 2$ (Again the solution is not visible and we are likely to get stuck)

Note: Such problems should never be attempted by writing the equations since this process takes more time than is necessary to solve the problem and is impractical in the exam situation due to the amount of time required in writing.

Besides, in complex problems where the final solution is not visible to the student while starting off, many a times the student has to finally abort the problem midway. This results in an unnecessary wastage of time if the student has attempted to write equations.

In fact, the student should realise that selecting the correct questions to solve in aptitude exams like the CAT is more important than being aware of how all the problems are solved.

The following process will illustrate the option based solution process.

Option A: It has $P + Q = 3$. The possible values of P/Q are 1/2 or 2/1.

Using 1/2, we see that on adding 2 to both the numerator and the denominator we get

3/4 (Not the required value.)

Similarly, we see that 2/1 will also not give the answer. We should also realise that the numerator has to be lower than the denominator to have the final value of 1/2.

Next we try **Option B,** where we have

1/3 as the only possible ratio.

Then we get the final value as 3/5 (Not equal to 1/2) Hence, we reject option B.

Next we try **Option C,** where we have

1/4 or 2/3

Checking for 1/4 we get 3/6 = 1/2. Hence, the option is correct.

Problem 8.3 If 10 persons can clean 10 floors by 10 mops in 10 days, in how many days can 8 persons clean 8 floors by 8 mops?

(a) 12 ½ days (b) 8 days

(c) 10 days (d) 8 ⅓ days

Solution Do not get confused by the distractions given in the problem. 10 men and 10 days means 100 man-days are required to clean 10 floors.

That is, 1 floor requires 10 man-days to get cleaned. Hence, 8 floors will require 80 man-days to clean.

Therefore, 10 days are required to clean 8 floors.

Problem 8.4 Three quantities *A*, *B*, *C* are such that $AB = KC$, where *K* is a constant. When *A* is kept constant, *B* varies directly as *C*; when *B* is kept constant, *A* varies directly *C* and when *C* is kept constant, *A* varies inversely as *B*.

Initially, *A* was at 5 and $A : B : C$ was 1 : 3 : 5. Find the value of *A* when *B* equals 9 at constant *C*.

(a) 8 (b) 8.33

(c) 9 (d) 9.5

Solution Initial values are 5, 15 and 25.

Thus we have $5 \times 15 = K \times 25$.

Hence, $K = 3$.

Thus, the equation is $AB = 3C$.

For the problem, keep *C* constant at 25. Then, $A \times 9 = 3 \times 25$.

i.e. $A = 75/9 = 8.33$

Problem 8.5 If $x/y = 3/4$, then find the value of the expression, $(5x - 3y)/(7x + 2y)$.

(a) 3/21 (b) 5/29

(c) 3/29 (d) 5/33

Solution Assume the values as $x = 3$ and $y = 4$.

Then we have

$$\frac{(15-12)}{(21+8)} = 3/29$$

Problem 8.6 ` 3650 is divided among 4 engineers, 3 MBAs and 5 CAs such that 3 CAs get as much as 2 MBAs and 3 Engineers as much as 2 CAs. Find the share of an MBA.

(a) 300 (b) 450

(c) 475 (d) None of these

Solution

$4E + 3M + 5C = 3650$

Also, $3C = 2M$, that is, $M = 1.5\ C$

and $3E = 2C$ that is, $E = 0.66C$

Thus, $4 \times 0.66C + 3 \times 1.5\ C + 5C = 3650$

$C = 3650/12.166$

That is, $C = 300$

Hence, $M = 1.5\ C = 450$

Problem 8.7 The ratio of water and milk in a 30 litre mixture is 7 : 3. Find the quantity of water to be added to the mixture in order to make this ratio 6 : 1.

(a) 30 (b) 32

(c) 33 (d) 35

Solution Solve while reading Æ As you read the first sentence, you should have 21 litres of water and 9 litres of milk in your mind.

In order to get the final result, we keep the milk constant at 9 litres.

Then, we have 9 litres, which corresponds to 1

Hence, '?' corresponds to 6.

Solving by using unitary method we have

54 litres of water to 9 litres of milk.

Hence, we need to add 33 litres of water to the original mixture.

Alternatively, we can solve this by using options. The student should try to do the same.

Problem 8.8 Three containers *A*, *B* and *C* are having mixtures of milk and water in the ratio of 1 : 5, 3 : 5 and 5 : 7 respectively. If the capacities of the containers are in the ratio 5 : 4 : 5, find the ratio of milk to water, if the mixtures of all the three containers are mixed together.

Solution Assume that there are 500, 400 and 500 litres respectively in the 3 containers.

Then we have, 83.33,150 and 208.33 litres of milk in each of the three containers.

Thus, the total milk is 441.66 litres. Hence, the amount of water in the mixture is

1400–441.66 = 958.33 litres.

Hence, the ratio of milk to water is

441.66 : 958.33 Æ 53 : 115 (Using division by 0.33333)

The calculation thought process should be:

(441 × 3 + 2) : (958 × 3 + 1) = 1325 : 2875.

Dividing by 25 Æ 53 : 115.

LEVEL OF DIFFICULTY (I)

1. Divide t` 1870 into three parts in such a way that half of the first part, one-third of the second part and one-sixth of the third part are equal.

(a) 241, 343, 245 (b) 400, 800, 670

(c) 470, 640, 1160 (d) None of these

2. Divide t` 500 among *A*, *B*, *C* and *D* so that *A* and *B* together get thrice as much as *C* and *D* together, *B* gets four times of what *C* gets and *C* gets 1.5 times as much as *D*. Now the value of what *B* gets is

(a) 300 (b) 75

(c) 125 (d) 150

3. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$, then each fraction is equal to

(a) $(a + b + c)^2$ (b) 1/2

(c) 1/4 (d) 0

4. If $6x^2 + 6y^2 = 13xy$, what is the ratio of x to y?

(a) 1 : 4 (b) 3 : 2

(c) 4 : 5 (d) 1 : 2

(Hint: Use options to solve fast)

5. If $a : b = c : d$ then the value of $\frac{a^2+b^2}{c^2+d^2}$ is

(a) 1/2 (b) $\frac{a+b}{c+d}$

(c) $\frac{a-b}{c-d}$ (d) $\frac{ab}{cd}$

6. A crew can row a certain course up the stream in 84 minutes; they can row the same course down stream in 9 minutes less than they can row it in still water. How long would they take to row down with the stream.

(a) 45 or 23 minutes (b) 63 or 12 minutes

(c) 60 minutes (d) 19 minutes

7. If a, b, c, d are in continued proportion then $\frac{a-d}{b-c} \geq x$. What is the value of x.

(a) 2 (b) 3

(c) 1 (d) 0

8. If 4 examiners can examine a certain number of answer books in 8 days by working 5 hours a day,

for how many hours a day would 2 examiners have to work in order to examine twice the number of answer books in 20 days.

(a) 6 (b) 7 ½

(c) 8 (d) 9

9. If a, b, c, d are proportional, then $(a - b)(a - c)/a =$

(a) $a + c + d$ (b) $a + d - b - c$

(c) $a + b + c + d$ (d) $a + c - b - d$

10. In a mixture of 40 litres, the ratio of milk and water is 4 : 1. How much water must be added to this mixture so that the ratio of milk and water becomes 2 : 3.

(a) 20 litres (b) 32 litres

(c) 40 litres (d) 30 litres

11. If A varies as C, and B varies as C, then which of the following is false:

(a) $(A + B)$ µ C (b) $(A - B)$ µ $1/C$

(c) $\sqrt{AB}$ µ C (d) AB µ C^2

12. If three numbers are in the ratio of 1 : 2 : 3 and half the sum is 18, then the ratio of squares of the numbers is:

(a) 6 : 12 : 13 (b) 1 : 2 : 4

(c) 36 : 144 : 324 (d) 3 : 5 : 7

13. The ratio between two numbers is 3 : 4 and their LCM is 180. The first number is:

(a)6. (b) 45

(c)1. (d) 20

14. A and B are two alloys of argentum and brass prepared by mixing metals in proportions 7 : 2 and 7 : 11 respectively. If equal quantities of the two alloys are melted to form a third alloy C, the proportion of argentum and brass in C will be:

(a) 5 : 9 (b) 5 : 7

(c) 7 : 5 (d) 9 : 5

15. If 30 men working 7 hours a day can do a piece of work in 18 days, in how many days will 21 men working 8 hours a day do the same work?

(a) 24 days (b) 22.5 days

(c) 30 days (d) 45 days

16. The incomes of A and B are in the ratio 3 : 2 and their expenditures are in the ratio 5 : 3. If each saves t` 1000, then, A's income can be:

(a) ` 3000 (b) ` 4000

(c) ` 6000 (d) ` 9000

17. If the ratio of sines of angles of a triangle is 1 : 1 : $\sqrt{2}$, then the ratio of square of the greatest

side to sum of the squares of other two sides is

(a) 3 : 4 (b) 2 : 1

(c) 1 : 1 (d) 1 : 2

18. Divide t` 680 among *A*, *B* and *C* such that *A* gets 2/3 of what *B* gets and *B* gets 1/4th of what *C* gets. Now the share of *C* is:

(a) ` 480 (b) ` 300

(c) ` 420 (d) ` 360

19. *A*, *B*, *C* enter into a partnership. A contributes one-third of the whole capital while *B* contributes as much as *A* and *C* together contribute. If the profit at the end of the year is t` 84,000, how much would each receive?

(a) 24,000, 20,000, 40,000

(b) 28,000, 42,000, 14,000

(c) 28,000, 42,000, 10,000

(d) 28,000, 14,000, 42,000

20. The students in three batches at Mindworkzz are in the ratio 2 : 3 : 5. If 20 students are increased in each batch, the ratio changes to 4 : 5 : 7. The total number of students in the three batches before the increases were

(a) 1. (b) 90

(c) 100 (d) 150

21. The speeds of three cars are in the ratio 2 : 3 : 4. The ratio between the times taken by these cars to travel the same distance is

(a) 2 : 3 : 4 (b) 4 : 3 : 2

(c) 4 : 3 : 6 (d) 6 : 4 : 3

22. If a, b, c and d are proportional then the mean proportion between $a^2 + c^2$ and $b^2 + d^2$ is

(a) ac/bd (b) $ab + cd$

(c) $a/b + d/c$ (d) $a^2/b^2 + c^2/d^2$

23. A number z lies between 0 and 1. Which of the following is true?

(a) $z > \sqrt{z}$ (b) $z > 1/z$

(c) $z^3 > z^2$ (d) $1/z > \sqrt{z}$

24. ` 2250 is divided among three friends Amar, Bijoy and Chandra in such a way that 1/6th of Amar's share, 1/4th of Bijoy's share and 2/5th of Chandra's share are equal. Find Amar's share.

(a) ` 720 (b) ` 1080

(c) ` 450 (d) ` 1240

25. After an increment of 7 in both the numerator and denominator, a fraction changes to 3/4. Find the original fraction.

(a) 5/12 (b) 7/9

(c) 2/5 (d) 3/8

26. The difference between two positive numbers is 10 and the ratio between them is 5 : 3. Find the product of the two numbers.

(a) 375 (b) 175

(c) 275 (d) 125

27. If 30 oxen can plough 1/7th of a field in 2 days, how many days 18 oxen will take to do the remaining work?

(a) 30 days (b) 20 days

(c) 15 days (d) 18 days

28. A cat takes 5 leaps for every 4 leaps of a dog, but 3 leaps of the dog are equal to 4 leaps of the cat. What is the ratio of the speed of the cat to that of the dog?

(a) 11 : 15 (b) 15 : 11

(c) 16 : 15 (d) 15 : 16

29. The present ratio of ages of *A* and *B* is 4 : 5. 18 years ago, this ratio was 11 : 16. Find the sum total of their present ages.

(a) 90 years (b) 105 years

(c) 110 years (d) 80 years

30. Four numbers in the ratio 1 : 3 : 4 : 7 add up to give a sum of 105. Find the value of the biggest number.

(a) 4. (b) 35

(c) 4. (d) 63

31. Three men rent a farm for t` 7000 per annum. *A* puts 110 cows in the farm for 3 months, *B* puts 110 cows for 6 months and *C* puts 440 cows for 3 months. What percentage of the total expenditure should *A* pay?

(a) 20% (b) 14.28%

(c) 16.66% (d) 11.01%

32. 10 students can do a job in 8 days, but on the starting day, two of them informed that they are not coming. By what fraction will the number of days required for doing the whole work get increased?

(a) 4/5 (b) 3/8

(c) 3/4 (d) 1/4

33. A dishonest milkman mixed 1 litre of water for every 3 litres of milk and thus made up 36 litres of milk . If he now adds 15 litres of milk to the mixture, find the ratio of milk and water in the new mixture.

(a) 12 : 5 (b) 14 : 3

(c) 7 : 2
(d) 9 : 4

34. ` 3000 is distributed among *A*, *B* and *C* such that *A* gets 2/3rd of what *B* and *C* together get and *C* gets 1/2 of what *A* and *B* together get. Find *C*'s share.

(a) ` 750
(b) ` 1000
(c) ` 800
(d) ` 1200

35. If the ratio of the ages of Maya and Chhaya is 6 : 5 at present, and fifteen years from now, the ratio will get changed to 9 : 8, then find Maya's present age.

(a) 24 years
(b) 30 years
(c) 18 years
(d) 33 years

36. At constant temperature, pressure of a definite mass of gas is inversely proportional to the volume. If the pressure is reduced by 20%, find the respective change in volume.

(a) –16.66%
(b) +25%
(c) –25%
(d) +16.66%

37. If t` 58 is divided among 150 children such that each girl and each boy gets 25 p and 50 p respectively. Then how many girls are there?

(a) 5.
(b) 54
(c) 6.
(d) 62

38. If 391 bananas were distributed among three monkeys in the ratio 1/2 : 2/3 : 3/4, how many bananas did the first monkey get?

(a) 102
(b) 108
(c) 112
(d) 104

39. A mixture contains milk and water in the ratio 5 : 1. On adding 5 litres of water, the ratio of milk to water becomes 5 : 2. The quantity of milk in the mixture is:

(a) 16 litres
(b) 25 litres
(c) 32.5 litres
(d) 22.75 litres

40. A beggar had ten paise, twenty paise and one rupee coins in the ratio 10 : 17 : 7 respectively at the end of day. If that day he earned a total of t` 57, how many twenty paise coins did he have?

(a) 114
(b) 171
(c) 95
(d) 85

41. Vijay has coins of the denomination of Re. 1, 50 p and 25 p in the ratio of 12 : 10 : 7. The total worth of the coins he has is t` 75. Find the number of 25 p coins that Vijay has

(a) 48
(b) 72
(c) 60
(d) None of these

42. If two numbers are in the ratio of 5 : 8 and if 9 be added to each, the ratio becomes 8 : 11. Now find the lower number.

(a) 5
(b) 10

(c) 15 (d) None of these

43. What number must be taken from each term of the fraction 27/35 that it may become 2 : 3?

(a) 9 (b) 10

(c) 11 (d) None of these

44. If x varies inversely as $y^2 - 1$ and is equal to 24 when $y = 10$, find x when $y = 5$.

(a) 99 (b) 101

(c) 91 (d) 93

45. If x varies as y, and $y = 7$ when $x = 18$, find x when $y = 21$

(a) 36 (b) 54

(c) 72 (d) 18

46. A varies jointly as B and C; and $A = 6$ when $B = 3$, $C = 2$; find A when $B = 5$, $C = 7$.

(a) 17.5 (b) 35

(c) 70 (d) 105

47. If x varies as y directly, and as z inversely, and $x = 14$ when $y = 10$; find z when $x = 49$, $y = 45$.

(a) 14/10 (b) 10

(c) 10/14 (d) Cannot be determined

48. A cask contains a mixture of 49 litres of wine and water in the proportion 5 : 2. How much water must be added to it so that the ratio of wine to water may be 7 : 4?

(a) 3.5 (b) 6

(c) 7 (d) None of these

49. A cask contains 12 gallons of mixture of wine and water in the ratio 3 : 1. How much of the mixture must be drawn off and water substituted, so that wine and water in the cask may become half and half.

(a) 3 litres (b) 5 litres

(c) 6 litres (d) None of these

50. The total number of pupils in three classes of a school is 333. The number of pupils in classes I and II are in the ratio 3 : 5 and those in classes II and III are in the ratio 7 : 11. Find the number of pupils in the class that had the highest number of pupils.

(a) 63 (b) 105

(c) 165 (d) 180

LEVEL OF DIFFICULTY (II)

1. If the work done by $(x - 1)$ men in $(x + 1)$ days is to the work done by $(x + 2)$ men in $(x - 1)$ days is in the ratio 9 : 10, then the value of x is

   (a) 10 (b) 12

   (c) 8 (d) 15

2. The duration of a railway journey varies as the distance and inversely as the velocity; the velocity varies directly as the square root of the quantity of coal used, and inversely as the number carriages in the train. In a journey of 50 km in half an hour with 18 carriages, 100 kg of coal is required. How much coal will be consumed in a journey of 42 km in 28 minutes with 16 carriages.

   (a) 64 kg (b) 49 kg

   (c) 25 kg (d) 36 kg

3. The weight of a circular disc varies as the square of the radius when the thickness remains the same; it also varies as the thickness when the radius remains the same. Two discs have their thicknesses in the ratio of 9:8; the ratio of the radii if the weight of the first is twice that of the second is

   (a) 4 : 3 (b) 5 : 2

   (c) 2 : 1 (d) 1 : 2

4. If a and b are positive integers then $\sqrt{2}$ always lies between:

   (a) $(a + b)/(a - b)$ and ab

   (b) a/b and $(a + 2b)/(a + b)$

   (c) a and b

   (d) $ab/(a + b)$ and $(a - b)/ab$

5. The cost of digging a pit was t` 1,347. How much will it cost (approximately) if the wages of workmen per day had been increased by 1/8 of the former wages and length of the working day increased by 1/20 of the former period?

   (a) ` 1443 (b) ` 1234

   (c) ` 1439 (d) ` 1000

6. A vessel contains a litres of wine, and another vessel contains b litres of water. c litres are taken out of each vessel and transferred to the other if $c \times (a + b) = ab$. If A and B are the respective values of the amount of wine contained in the respective containers after this operation, then what can be said about the relationship between A and B.

   (a) $A = B$ (b) $\frac{A - C}{B - C} > 2$

   (c) $A - B = 4c$ (d) None of these

7. If sum of the roots and the product of the roots of a quadratic equation S are in the ratio of 2 : 1,

then which of the following is true?

(a) $f(S) < 0$ (b) $(b^2 - 4ac) < 0$

(c) S is a perfect square (d) None of these

8. A factory employs skilled workers, unskilled workers and clerks in the proportion 8 : 5 : 1, and the wages of a skilled worker, an unskilled worker and a clerk are in the ratio 5 : 2 : 3. When 20 unskilled workers are employed, the total daily wages of all amount to ` 318. The wages paid to each category of workers are

(a) ` 240, t` 60, t` 18

(b) ` 200, t` 90, t` 28

(c) ` 150, t` 108, t` 60

(d) ` 250, t` 50, t` 18

9. If $a : b = c : d$, and $e : f = g : h$, then $(ae + bf) : (ae - bf) = ?$

(a) $\frac{(e+f)}{(e-f)}$ (b) $\frac{(cg+dh)}{(cg-dh)}$

(c) $\frac{(cg+dh)}{(cg-dh)}$ (d) $\frac{e-f}{e+f}$

10. Brass is an alloy of copper and zinc. Bronze is an alloy containing 80% of copper, 4 % of zinc and 16% of tin. A fused mass of brass and bronze is found to contain 74% of copper, 16% of zinc, and 10% of tin. The ratio of copper to zinc in Brass is:

(a) 64% and 36% (b) 33% and 67%

(c) 50% and 75% (d) 35% and 65%

11. The Lucknow Indore Express without its rake can go 24 km an hour, and the speed is diminished by a quantity that varies as the square root of the number of wagons attached. If it is known that with four wagons its speed is 20 km/h, the greatest number of wagons with which the engine can just move is

(a) 144 (b) 140

(c) 143 (d) 124

12. If x varies as y then $x^2 + y^2$ varies as

(a) $x + y$ (b) $x - y$

(c) $x^2 - y^2$ (d) None of these

13. If $f(x) = \frac{(x+1)}{(x-1)}$, then the ratio of x to $f(y)$ where $y = f(x)$ is

(a) $x : y$ (b) $x^2 : y^2$

(c) $1 : 1$ (d) $y : x$

14. A contractor employs 200 men to build a bund. They finish 5/6 of the work in 10 weeks. Then rain sets in and not only does the work remain suspended for 4 weeks but also half of the work already

done is washed away. After the rain, when the work is resumed, only 140 men turn up. The total time in which the contractor is able to complete the work assuming that there are no further disruptions in the schedule is

(a) 25 weeks (b) 26 weeks

(c) 24 weeks (d) 20 weeks

15. In a journey of 48 km performed by tonga, rickshaw and cycle in that order, the distance covered by the three ways in that order are in the ratio of 8 : 1 : 3 and charges per kilometre in that order are in the ratio of 8 : 1 : 4. If the tonga charges being 24 paise per kilometre, the total cost of the journey is

(a) ` 9.24 (b) ` 10

(c) ` 12 (d) None of these

16. A bag contains 25 paise, 50 paise and 1 Re. coins. There are 220 coins in all and the total amount in the bag is t` 160. If there are thrice as many 1 Re. coins as there are 25 paise coins, then what is the number of 50 paise coins?

(a) 60 (b) 40

(c) 120 (d) 80

Directions for Questions 17 to 19: Read the following and answer the questions that follow.

Tuliram runs in a triathlon consisting of three phases in the following manner. Running 12 km, cycling 24 km and swimming 5 km. His speeds in the three phases are in the ratio 2 : 6 : 1. He completes the race in *n* minutes. Later, he changes his strategy so that the distances he covers in each phase are constant but his speeds are now in the ratio 3 : 8 : 1. The end result is that he completes the race taking 20 minutes more than the earlier speed. It is also known that he has not changed his running speed when he changes his strategy.

17. What is his initial speed while swimming?

(a) 1/2 km/min (b) 0.05 km/min

(c) 0.15 km/min (d) None of these

18. If his speeds are in the ratio 1:3:1, with the running time remaining unchanged, what is his finishing time?

(a) 500/3 min (b) 250/3 min

(c) 200/3 min (d) 350/3 min

19. What is Tuliram's original speed of running?

(a) 9 kmph (b) 18 kmph

(c) 54 kmph (d) 12 kmph

20. Concentrations of three wines *A*, *B* and *C* are 10%, 20% and 30% respectively. They are mixed in the ratio 2 : 3 : *x* resulting in a 23% concentration solution. Find *x*.

(a) 7 (b) 6

(c) 5 (d) 4

21. The cost of an article (which is composed of raw materials and wages) was 3 times the value of the raw materials used. The cost of raw materials increased in the ratio 3 : 7 and wages increased in the ratio 4 : 9. Find the present cost of the article if its original cost was t` 18.

(a) ` 41
(b) ` 30
(c) ` 40
(d) ` 46

22. In a co-educational school there are 15 more girls than boys. If the number of girls is increased by 10% and the number of boys is also increased by 16%, there would be 9 more girls than boys. What is the number of students in the school?

(a) 140
(b) 125
(c) 265
(d) 255

23. At IIM Bangalore class of 1995, Sonali, a first year student has taken 10 courses, earning grades *A* (worth 4 points each), *B* (worth 3 points each) and *C* (worth 2 points each). Her grade point average is 3.2, and if the course in which she get *C*'s were deleted, her GPA in the remaining courses would be 3.333. How many *A*'s, *B*'s and *C*'s did she get?

(a) 3, 1 and 6
(b) 1, 3 and 6
(c) 3, 6 and 1
(d) 1, 6 and 3

24. Total expenses of running the hostel at Harvard Business School are partly fixed and partly varying linearly with the number of boarders. The average expense per boarder is \$70 when there are 25 boarders and \$60 when there are 50 boarders. What is the average expense per boarder when there are 100 boarders?

(a) 55
(b) 56
(c) 54
(d) 50

25. The speed of the engine of Gondwana Express is 42 km/h when no compartment is attached, and the reduction in speed is directly proportional to the square root of the number of compartments attached. If the speed of the train carried by this engine is 24 km/h when 9 compartments are attached, the maximum number of compartments that can be carried by the engine is

(a) 49
(b) 48
(c) 46
(d) 47

Three drunkards agree to pool their vodka and decided to share it with a fourth drunkard (who had no vodka) at a price equal to 5 roubles a litre. The first drunkard contributed 1 litre more than the second and the second contributed a litre more than the third. Then all four of them divided the vodka equally and drank it. The fourth drunkard paid money, which was divided in the ratio of each drunkard's contribution towards his portion. It was found that the first drunkard should get twice as much money as the second. Based on this information answer the questions 26–28. (Assume that all shares are integral).

26. How much money did the second drunkard get (in roubles)?

(a) 8
(b) 10
(c) 5
(d) Data insufficient

27. How many litres of vodka was consumed in all by the four of them?

(a) 12

(b) 16

(c) 10

(d) None of these

28. What proportion of the fourth drunkard's drink did the second drunkard contribute?

(a) 1/3

(b) 2/3

(c) 1/2

(d) None of these

29. In Ramnagar Colony, the ratio of school going children to non-school going children is 5 : 4. If in the next year, the number of non-school going children is increased by 20%, making it 35,400, what is the new ratio of school going children to non-school going children?

(a) 4 : 5

(b) 3 : 2

(c) 25 : 24

(d) None of these

30. A precious stone weighing 35 grams worth t` 12,250 is accidentally dropped and gets broken into two pieces having weights in the ratio of 2 : 5. If the price varies as the square of the weight then find the loss incurred.

(a) ` 5750

(b) ` 6000

(c) ` 5500

(d) ` 5000

31. On his deathbed, Mr. Kalu called upon his three sons and told them to distribute all his assets worth ` 525,000 in the ratio of 1/15 : 1/21 : 1/35 amongst themselves. Find the biggest share amongst the three portions.

(a) 17,500

(b) 245,000

(c) 10,500

(d) 13,250

32. Three jackals—Paar, Maar and Taar together have 675 loaves of bread. Paar has got three times as much as Maar but 25 loaves more than Taar. How many does Taar have?

(a) 175

(b) 275

(c) 375

(d) None of these

33. King Sheru had ordered the distribution of apples according to the following plan : for every 20 apples the elephant gets, the zebra should get 13 apples and the deer should get 8 apples. Now his servant Shambha jackal is in a fix. Can you help him by telling how much should he give to the elephant if there were 820 apples in total?

(a) 140

(b) 160

(c) 200

(d) 400

34. In the famous Bhojpur island, there are four men for every three women and five children for every three men. How many children are there in the island if it has 531 women?

(a) 454

(b) 1180

(c) 1070

(d) 389

35. Which of the following will have the maximum change in their values if 5 is added to both the numerator and denominator of all the fractions?

(a) 3/4

(b) 2/3

(c) 4/7 (d) 5/7

36. 40 men could have finished the whole project in 28 days but due to the inclusion of a few more men, work got done in 3/4 of the time. Find out how many more men were included (in whole numbers).

(a) 12 (b) 13

(c) 14 (d) None of these

37. Mr *AM*, the magnanimous cashier at *XYZ* Ltd., while distributing salary, adds whatever money is needed to make the sum a multiple of 50. He adds t` 10 and ` 40 to *A*'s and *B*'s salary respectively and then he realises that the salaries of *A*, *B* and *C* are now in the ratio 4 : 5 : 7. The salary of *C* could be

(a) ` 2300 (b) ` 2150

(c) ` 1800 (d) ` 2100

38. A mother divided an amount of t` 61,000 between her two daughters aged 18 years and 16 years respectively and deposited their shares in a bond. If the interest rate is 20% compounded annually and if each received the same amount as the other when she attained the age of 20 years, their shares are

(a) ` 35,600 and t` 25,400

(b) ` 30500 each

(c) ` 24,000 and t` 37000

(d) None of these

Directions for Questions 39 to 41: Read the passage below and answer the questions that follow:

Anshu gave Bobby and Chandana as many pens as each one of them already had. Then Chandana gave Anshu and Bobby as many pens as each already had. Now each had an equal number of pens.The total number of pens is 72.

39. How many pens did Bobby have initially?

(a) 24 (b) 18

(c) 12 (d) 6

40. How many pens did Chandana have initially?

(a) 24 (b) 18

(c) 12 (d) 6

41. How many pens did Anshu have initially?

(a) 30 (b) 36

(c) 42 (d) 48

42. The volume of a pyramid varies jointly as its height and the area of its base; and when the area of the base is 60 square dm and the height 14 dm, the volume is 280 cubic dm. What is the area of the base of a pyramid whose volume is 390 cubic dm and whose height is 26 dm?

(a) 40 (b) 45

(c) 50 (d) None of these

43. The expenses of an all boys' institute are partly constant and partly vary as the number of boys. The expenses were t` 10,000 for 150 boys and t` 8400 for 120 boys. What will the expenses be when there are 330 boys?

(a) 18,000 (b) 19,600

(c) 22,400 (d) None of these

44. The distance of the horizon at sea varies as the square root of the height of the eye above sea-level. When the distance is 14.4 km, the height of the eye is 18 metres. Find, in kilometres, the distance when the height of the eye is 8 metres.

(a) 4.8 km (b) 7.2 km

(c) 9.6 km (d) 12 km

45. A mixture of cement, sand and gravel in the ratio of 1 : 2 : 4 by volume is required. A person wishes to measure out quantities by weight. He finds that the weight of one cubic foot of cement is 94 kg, of sand 100 kg and gravel 110 kg. What should be the ratio of cement, sand and gravel by weight in order to give a proper mixture?

(a) 47 : 100 : 220 (b) 94 : 100 : 220

(c) 47 : 200 : 440 (d) None of these

LEVEL OF DIFFICULTY (III)

1. An alloy of gold and silver is taken in the ratio of 1 : 2, and another alloy of the same metals is taken in the ratio of 2 : 3. How many parts of the two alloys must be taken to obtain a new alloy consisting of gold and silver that are in the ratio 3 : 5?

   (a) 3 and 5 (b) 2 and 9

   (c) 2 and 5 (d) 1 and 5

2. There are two quantities of oil, with the masses differing by 2 kg. The same quantity of heat, equal to 96 kcal, was imparted to each mass, and the larger mass of oil was found to be 4 degrees cooler than the smaller mass. Find the mass of oil in each of the two quantities.

   (a) 6 and 8 (b) 4 and 6

   (c) 2 and 9 (d) 4 and 9

3. There are two alloys of gold and silver. In the first alloy, there is twice as much gold as silver, and in the second alloy there is 5 times less gold than silver. How many times more must we take of the second alloy than the first in order to obtain a new alloy in which there would be twice as much silver as gold?

   (a) Two times (b) Three times

   (c) Four times (d) Ten times

4. Calculate the weight and the percentage of zinc in the zinc-copper alloy being given that the latter's alloy with 3 kg of pure zinc contains 90 per cent of zinc and with 2 kg of 90% zinc alloy contains 84% of zinc.

   (a) 2.4 kg or 80% (b) 1.4 kg or 88%

   (c) 3.4 kg or 60% (d) 7.4 kg or 18%

5. Two solutions, the first of which contains 0.8 kg and the second 0.6 kg of salt, were poured together and 10 kg of a new salt solution were obtained. Find the weight of the first and of the second solution in the mixture if the first solution is known to contain 10 per cent more of salt than the second.

   (a) 4 kg, 6 kg (b) 3 kg, 7 kg

   (c) 4 kg, 9 kg (d) 5 kg, 9 kg

6. From a full barrel containing 729 litres of honey we pour off 'a' litre and add water to fill up the barrel. After stirring the solution thoroughly, we pour off 'a' litre of the solution and again add water to fill up he barrel. After the procedure is repeated 6 times, he solution in the barrel contains 64 litres of honey. Find a.

   (a) 243 litres (b) 81 litres

   (c) 2.7 litres (d) 3 litres

7. In two alloys, the ratios of nickel to tin are 5 : 2 and 3 : 4 (by weight). How many kilogram of the first alloy and of the second alloy should be alloyed together to obtain 28 kg of a new alloy with equal contents of nickel and tin?

(a) 9 kg of the first alloy and 22 kg of the second

(b) 17 kg of the first alloy and 11 kg of the second

(c) 7 kg of the first alloy and 21 kg of the second

(d) 8 kg and 20 kg respectively

8. In two alloys, aluminium and iron are in the ratios of 4 : 1 and 1 : 3. After alloying together 10 kg of the first alloy, 16 kg of the second and several kilograms of pure aluminium, an alloy was obtained in which the ratio of aluminium to iron was 3 : 2. Find the weight of the new alloy.

(a) 15 (b) 35

(c) 65 (d) 95

9. There are two alloys of gold, silver and platinum. The first alloy is known to contain 40 per cent of platinum and the second alloy 26 per cent of silver. The percentage of gold is the same in both alloys. Having alloyed 150 kg of the first alloy and 250 kg of the second, we get a new alloy that contains 30 per cent of gold. How many kilogram of platinum is there in the new alloy?

(a) 170 kg (b) 175 kg

(c) 160 kg (d) 165 kg

10. Two alloys of iron have different percentage of iron in them. The first one weighs 6 kg and second one weighs 12 kg. One piece each of equal weight was cut off from both the alloys and the first piece was alloyed with the second alloy and the second piece alloyed with the first one. As a result, the percentage of iron became the same in the resulting two new alloys. What was the weight of each cut-off piece?

(a) 4 kg (b) 2 kg

(c) 3 kg (d) 5 kg

11. Two litres of a mixture of wine and water contain 12% water. They are added to 3 litres of another mixture containing 7% water, and half a litre of water is then added to whole. What is the percentage of water in resulting concoction?

(a) 17(2/7)% (b) 15 (7/11)%

(c) 17(3/11)% (d) 16 (2/3)%

12. Three vessels having volumes in the ratio of 1 : 2 : 3 are full of a mixture of coke and soda. In the first vessel, ratio of coke and soda is 2 : 3, in second, 3 : 7 and in third, 1 : 4. If the liquid in all the three vessels were mixed in a bigger container, what is the resulting ratio of coke and soda?

(a) 4:11 (b) 5:7

(c) 7:11 (d) 7:5

13. Two types of tea are mixed in the ratio of 3 : 5 to produce the first quality and if they are mixed in the ratio of 2 : 3, the second quality is obtained. How many kilograms of the first quality has to be mixed with 10 kg of the second quality so that a third quality having the two varieties in the ratio of 7 : 11 may be produced?

(a) 5 kg (b) 10 kg

(c) 8 kg (d) 9 kg

14. A toy weighing 24 grams of an alloy of two metals is worth t` 174, but if the weights of metals in alloy be interchanged, the toy would be worth t` 162. If the price of one metal be t` 8 per gram, find the price of the other metal in the alloy used to make the toy.

(a) ` 10 per gram (b) ` 6 per gram

(c) ` 4 per gram (d) ` 5 per gram

15. The weight of three heaps of gold are in the ratio 5 : 6 : 7. By what fractions of themselves must the first two be increased so that the ratio of the weights may be changed to 7 : 6 : 5?

(a) $\frac{24}{25}, \frac{2}{5}$ (b) $\frac{48}{50}, \frac{4}{5}$

(c) $\frac{48}{50}, \frac{3}{5}$ (d) $\frac{24}{25}, \frac{3}{7}$

16. An alloy of gold, silver and bronze contains 90% bronze, 7% gold and 3% silver. A second alloy of bronze and silver only is melted with the first and the mixture contains 85% of bronze, 5% of gold and 10% of silver. Find the percentage of bronze in the second alloy.

(a) 75% (b) 72.5%

(c) 70% (d) 67.5%

17. Gunpowder can be prepared by saltpetre and nitrous oxide. Price of saltpetre is thrice the price of nitrous oxide. Notorious gangster Kallu Bhai sells the gunpowder at t` 2160 per 10 g, thereby making a profit of 20%. If the ratio of saltpetre and nitrous oxide in the mixture be 2 : 3, find the cost price of saltpetre.

(a) ` 210/gm (b) ` 300/gm

(c) ` 120/gm (d) None of these

18. Two boxes *A* and *B* were filled with a mixture of rice and dal—in *A* in the ratio of 5 : 3, and in *B* in the ratio of 7 : 3. What quantity must be taken from the first to form a mixture that shall contain 8 kg of rice and 3 kg of dal?

(a) 4 kg (b) 5 kg

(c) 6 kg (d) This cannot be achieved

19. A person buys 18 local tickets for ` 110. Each first class ticket costs ` 10 and each second class ticket costs ` 3. What will another lot of 18 tickets in which the number of first class and second class tickets are interchanged cost?

(a) 112 (b) 118

(c) 121 (d) 124

20. Two jars having a capacity of 3 and 5 litres respectively are filled with mixtures of milk and water. In the smaller jar 25% of the mixture is milk and in the larger 25% of the mixture is water. The jars are emptied into a 10 litre cask whose remaining capacity is filled up with water. Find the percentage of milk in the cask.

(a) 55%	(b) 50%

(c) 45%	(d) None of these

21. Two cubes of bronze have their total weight equivalent to 60 kg. The first piece contains 10 kg of pure zinc and the second piece contains 8 kg of pure zinc. What is the percentage of zinc in the first piece of bronze if the second piece contains 15 per cent more zinc than the first?

(a) 15%	(b) 25%

(c) 55%	(d) 24%

22. Sonu gets a jewellery made of an alloy of copper and silver. The alloy with a weight of 8 kg contains p per cent of copper. What piece of a copper-silver alloy containing 40 per cent of silver must be alloyed with the first piece in order to obtain a new alloy with the minimum percentage of copper if the weight of the second piece is 2 kg?

(a) 2 kg for $p > 60$, a kg, where a Œ [0, 2], for $p = 60$, 0 kg for $0 < p < 60$

(b) 0 kg for $p > 60$, a kg, where a Œ [0, 2], for $p = 60$, 2 kg for $0 < p < 60$

(c) 0 kg for $p > 60$, a kg, where a Œ [0, 3], for $p = 70$, 0 kg for $0 < p < 70$

(d) None of these

23. From a vessel filled up with pure spirit to the brim, two litres of spirit was removed and 2 litres of water were added. After the solution was mixed, 2 litres of the mixture was poured off and again 2 litres of water was added. The solution was stirred again and 2 litres of the mixture was removed and 2 litres of water was added. As a result of the above operations, the volume of water in the vessel increased by 3 litres than the volume of spirit remaining in it. How many litres of spirit and water were there in the vessel after the above procedure was carried out?

(a) 0.7 litre of spirit and 3.7 litres of water

(b) 1.5 litres of spirit and 4.5 litres of water

(c) 8.5 litre of spirit and 11.5 litres of water

(d) 0.5 litre of spirit and 3.5 litres of water

24. There are two qualities of milk—Amul and Sudha having different prices per litre, their volumes being 130 litres and 180 litres respectively. After equal amounts of milk was removed from both, the milk removed from Amul was added to Sudha and vice-versa. The resulting two types of milk now have the same price. Find the amount of milk drawn out from each type of milk.

(a) 58.66	(b) 75.48

(c) 81.23	(d) None of these

25. Assume that the rate of consumption of coal by a locomotive varies as the square of the speed and is 1000 kg per hour when the speed is 60 km per hour. If the coal costs the railway company t` 15 per 100 kg and if the other expenses of the train be t` 12 per hour, find a formula for the cost in paise per kilometre when the speed is S km per hour.

(a) $1200 + \frac{5S^2}{18}$	(b) $1200 + \frac{75S^2}{18}$

(d) None of these

(c) $\frac{1200}{S} + \frac{75S}{18}$

ANSWER KEY

Level of Difficulty (I)

1. (d)	2. (a)	3. (b)	4. (b)
5. (d)	6. (b)	7. (b)	8. (c)
9. (b)	10. (c)	11. (b)	12. (c)
13. (b)	14. (c)	15. (b)	16. (c)
17. (c)	18. (a)	19. (b)	20. (c)
21. (d)	22. (b)	23. (d)	24. (b)
25. (c)	26. (a)	27. (b)	28. (d)
29. (a)	30. (c)	31. (b)	32. (d)
33. (b)	34. (b)	35. (b)	36. (b)
37. (c)	38. (a)	39. (b)	40. (d)
41. (d)	42. (c)	43. (c)	44. (a)
45. (b)	46. (b)	47. (d)	48. (b)
49. (d)	50. (c)		

Level of Difficulty (II)

1. (c)	2. (a)	3. (a)	4. (b)
5. (a)	6. (d)	7. (d)	8. (a)
9. (b)	10. (a)	11. (c)	12. (d)
13. (c)	14. (c)	15. (a)	16. (a)
17. (d)	18. (b)	19. (b)	20. (c)
21. (a)	22. (c)	23. (c)	24. (a)
25. (b)	26. (c)	27. (a)	28. (a)
29. (c)	30. (d)	31. (b)	32. (b)
33. (d)	34. (b)	35. (b)	36. (c)
37. (d)	38. (d)	39. (d)	40. (a)
41. (c)	42. (b)	43. (b)	44. (c)
45. (a)			

Level of Difficulty (III)

1. (a)	2. (a)	3. (a)	4. (a)
5. (a)	6. (a)	7. (c)	8. (b)
9. (a)	10. (a)	11. (c)	12. (a)
13. (c)	14. (b)	15. (a)	16. (b)
17. (b)	18. (d)	19. (d)	20. (c)
21. (b)	22. (a)	23. (d)	24. (b)
25. (c)			

Hints

Level of Difficulty (II)

1. Check the equation: $\frac{(x-1)(x+1)}{(x+2)(x-1)} = \frac{9}{10}$

   Through options

2. $S = k\frac{\sqrt{Q}}{N}$ where Q is the quantity of coal used per km and N is the number of carriages.

   $100 = k\frac{\sqrt{2}}{18}$

   \ $k = \frac{1800}{\sqrt{2}}$ $900\sqrt{2}$ and

   $S = 900\sqrt{2} \times \frac{\sqrt{Q}}{N}$

   Then $150 = 900\sqrt{2} \times \frac{\sqrt{Q}}{16}$

3. $\frac{w_1}{w_2} = \frac{r_1^2 t_1}{r_2^2 t_2} = \frac{2w_2}{w_2}$

5. $1347 \times 1.125 \times 0.9523$

6. Assume values of a, b and c such that they satisfy $c(a + b) = ab$.

   Then $A = a - c$ and $B = c$

8. Number of workers—32 skilled, 20 unskilled and 4 clerks.

   Then, wages are divided in the ratio 160 : 40 : 12.

11. $S = 24 - \mathrm{k}\sqrt{N}$

13. $y = \frac{(x+1)}{(x-1)}$, $f(y) = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = x$

14. 2000 man weeks = $\frac{5}{6}$ of the total work.

    \ Total work = 2400 man weeks

    Work left after disruption = 1400 men weeks.

17–19. Let the running speed be $2x$, then $6x$ and x are the cycling and swimming speeds respectively

Then, $\frac{n}{60} = \frac{12}{2x} + \frac{24}{6x} + \frac{5}{x}$ (1)

Also, $\frac{n+20}{60} = \frac{12}{3y} + \frac{24}{8y} + \frac{5}{y}$ (2)

And $3y = 2x$ (3)

23. Solve using options

24. Variable cost per boarder = $\frac{60 \times 50 - 70 \times 25}{25} = 50$

26–28. Let the contributions be C, $C + 1$ and $C + 2$. Total Vodka = $3C + 3 = 3(C + 1)$

The total Vodka should be divisible into 4 parts.

Hence, C can take values like 3, 7, 11 etc.

Proceed with trial and error and check the value for which all conditions are met.

31. The ratio is 7 : 5 : 3

34. Ratios are 3 : 4 and 3 : 5 i.e. 9 : 12 : 20

37. The question asks for a possible answer and not for a definite answer.

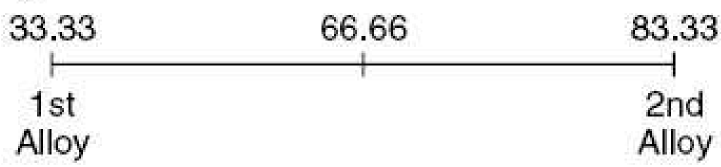
44. $D = k\sqrt{h}$

$k = \frac{14.4}{3\sqrt{2}}$

Level of Difficulty (III)

1. One alloy contains 33.33% gold, the other contains 40% gold. The mixture must contain 37.5% gold. Solve using alligation.

2. $\frac{96}{x} - \frac{96}{(x+2)} = 4$ (Required difference). Check using options.

3. 

gives 1 : 2

4. Check using options whether the given conditions of mixing are met.

Option (a) gives : 2.4 kg of zinc @ 80% concentration. i.e. 3 kg alloy of 80% zinc concentration is mixed with 3 kg of pure zinc. Satisfies the given condition.

5. Solve using options the following equation $\frac{0.8}{x} - \frac{0.6}{10 - x} = 0.1$

7. Check the options.

8. 80% aluminium (4: 1) and 25% aluminium (1 : 3) have to be mixed with pure aluminium to obtain an alloy with 60% aluminium.

\ $\frac{10 \times 0.8 + 16 \times 0.25 + x}{10 + 16 + x} = 0.6$

9. Since the percentage of gold in both alloys is the same, any mixture of the two will contain the same percentage concentration of gold.

Hence, we get

First alloy :	Gold :	Silver :	Platinum
	30 :	30 :	40
AND Second alloy:	Gold :	Silver :	Platinum
	30 :	26 :	44

10. Let 'w' be the weight of the cut off piece.

Then, $\frac{6-w}{w} = \frac{w}{12-w}$

13. First alloy has 37.5% of the first tea type. Similarly, the second alloy has 40% of the first tea type. The mixture should contain 42.85% of the first tea type. This is not possible.
16. When one alloy having 7% gold is mixed with another alloy having no gold, the result is a new alloy with 5% gold. Hence, ratio of mixing is 2 : 5.
19. $10x + (18 - x) \times 3 = 110$
20. Out of 8 liters milk and water mixture poured into the 10 liter cask, the milk is $0.25 \times 3 + 0.75 \times 5 = 4.5$.
21. $\frac{8}{x} - \frac{10}{60-x} = 0.15$.
22. Since the second alloy contains 60% copper, the requirement for the minimisation of copper will be fulfilled by option 2. Note, that the values of the number of kgs required of the second alloy will depend on the value of p.
24. Solve through options.
25. Total cost = Other expenses (paise/km) + Coal cost

(paise/km).

Coal Consumption = $k \times s^2$

\ $1000 = k \times 60^2$

$k = \frac{5}{18}$ and Coal consumption = $\frac{5}{18} \times s^2$

\ Required expression is

Total cost = $\frac{1200}{s} + \frac{5}{18} s \times 15$.

Solutions and Shortcuts

Level of Difficulty (I)

1. Solve this question using options. 1/2 of the first part should equal 1/3rd of the second part and $\frac{1}{6}$ of the third part. This means that the first part should be divisible by 2, the second one by 3, and the third one by 6. Looking at the options, none of the first 3 options has its third number divisible by 6. Thus, option (d) is correct.
2. $(A + B) = 3(C + D)$ Æ $A + B = 375$ and $C + D = 125$.

   Also, since C gets 1.5 times D we have

   $C = 75$ and $D = 50$, and $B = 4C = 300$.
3. The given condition has a, b and c symmetrically placed. Thus, if we use $a = b = c = 2$ (say) we get each fraction as 1/2.

4. Solve using options. Since $x > y > 0$ it is clear that a ratio of x:y as 3:2 fits the equation.

5. 1 : 2 = 3 : 6 So, $(a^2 + b^2)/(c^2 + d^2) = 5/45 = 1/9$

   From the given options, only ab/cd gives us this value.

6. If you try to solve this question through equations, the process becomes too long and almost inconclusive. The best way to approach this question is by trying to use options.

   The question asks us to find the time in which the boat can move downstream.

   The basic situation in this question is:

   Percentage increase over still water speed while going downstream = Percentage decrease over still water speed while going upstream.

   (Since: $S_{downstream} = S_{boat} + S_{stream}$ and $S_{upstream} = S_{boat} - S_{stream}$)

   Hence, the percentage increase in time while going upstream should match the percentage decrease in time while going downstream in such a way that the percentage change in the speed is same in both the cases).

   Testing for option (a):

   $Time_{upstream}$ = 84 minutes (given)

   $Time_{downstream}$ = 45 minutes (first value from the option)

   $Time_{Stillwater}$ = 54 minutes (45 + 9)

   % increase in time when going upstream = 30/54

   [***Note:*** The percentage increase should be written as 30*100/54. However, as I have repeatedly pointed out right from the chapter of percentages, you need to be able to look at % values of ratios directly by using the Percentage rule for calculations)

   % decrease in time when going downstream = 9/54 =16.66%

   Since, the % decrease is 16.66%, this should correspond to a % increase in speed by 20% (Since, product speed × time is constant).

   This means that the speed should drop by 20% while going upstream and hence the time should increase by 25% while going upstream. But, 30/54 does not give us a value of 25% increase. Hence this option is incorrect.

   Testing for option (b):

   $Time_{upstream}$ = 84 minutes (given)

   $Time_{downstream}$ = 63 minutes (first value from the option)

   $Time_{Stillwater}$ = 72 minutes (63 + 9)

   % increase in time when going upstream = 12/72 = 16.66%

   % decrease in time when going downstream = 9/72 =12.5%

   Since, the % decrease is 12.5%, this should correspond to *a* % increase in speed by 14.28% (Since, product speed × time is constant).

   This means that the speed should drop by 14.28% while going upstream and hence the time should increase by 16.66% while going upstream. This is actually occurring. Hence, this option is correct.

   Options (c) & (d) can be seen to be incorrect in this context.

7. Experimentally if you were to take the value of a, b, c, and d as 1 : 2 : 4 : 8, you get the value of the expression as 3.5. If you try other values for a, b, c and d experimentally you can see that while you can approach 3, you cannot get below that.

   For instance,

   1 : 1.1 : : 1.21 : 1.331

   Gives us: – 0.331/–0.11 which is slightly greater than 3.

8. 4 × 8 × 5 = 160 man-hours are required for ‘x’ no. of answer sheets. So, for ‘2x’ answer sheets we would require 320 man-hours = 2 × 20 × n Æ n = 8 Hours a day.

9. Assume a set of values for a, b, c, d such that they are proportional i.e. $a/b = c/d$. Suppose we take a:b as 1:4 and c:d as 3:12 we get the given expression:

   $(a – b)(a – c)/a$ = –3 × –2/1 = 6. This value is also given by $a + d – b – c$ and hence option (b) is correct.

10. In 40 litres, milk = 32 and water = 8. We want to create 2 : 3 milk to water mixture, for this we would need: 32 milk and 48 water. (Since milk is not increasing). Thus, we need to add 40 litres of water.

11. Option (b) is not true.

12. 1 : 2 : 3 Æ x, 2x and 3x add up to 36.

    So the numbers are: 6, 12 and 18.

    Ratio of squares = 36 : 144 : 324.

13. The numbers would be 3x and 4x and their LCM would be 12x. This gives us the values as 45 and 60. The first number is 45.

14. Since equal quantities are being mixed, assume that both alloys have 18 kgs (18 being a number which is the LCM of 9 and 18).

    The third alloy will get, 14 kg of argentum from the first alloy and 7 kg of argentum from the second alloy. Hence, the required ratio: 21:15 = 7:5

15. The total number of manhours required = 30 × 7 × 18 = 3780

    21 × 8 × no. of days Æ 3780/168 = 22.5 days

    Note, you could have done this directly by: (30 × 7 × 18)/(21 × 8).

16. Solve using options. Option (c) fits the situation as if you take A’s income as t` 6,000, $B$’s income will become t` 4,000 and if they each save t` 1,000, their expenditures would be t` 5,000 , t` 3,000 respectively. This gives the required 5: 3 ratio.

17. The given ratio for sines would only be true for a 45-45-90 triangle. The sides of such a triangle are in the ratio $1:1:\sqrt{2}$. The square of the longest side is 2 while the sum of the squares of the other two sides is also 2. Hence, the required ratio is 1:1.

18. Solve by options. Option (a) C = 480 fits perfectly because if C = 480, B = 120 and A = 80.

19. A’s contribution = 33.33%

    B’s contribution = 50%

    C’s contribution = 16.66%

    Ratio of profit sharing = Ratio of contribution

    = 2 : 3 : 1

Thus, profit would be shared as : 28000 : 42000 : 14000.

20. $2x + 20 : 3x + 20 : 5x + 20 = 4 : 5 : 7$ Æ $x = 10$ and initially the number of students would be 20, 30 and 50 Æ a total of 100.

21. The ratio of time would be such that speed × time would be constant for all three. Thus if you take the speeds as $2x$, $3x$ and $4x$ respectively, the times would be $6y$, $4y$ and $3y$ respectively.

22. Again in order to solve this question, try to assume values for a, b, c and d such that $a{:}b = c{:}d$ (i.e. a, b, c and d are proportional). Let us say we assume $a = 1$, $b = 4$, $c = 3$ and $d = 12$ we get:

    $a^2 + c^2 = 10$ and $b^2 + d^2 = 160$. The mean proportional between 10 and 160 is 40. $ab + cd$ gives us this value and can be checked by taking another set of values to see that it still works.

23. Option (d) is true since $1/z$ will be greater than 1 and $\sqrt{z}$ would be less then 1.

24. Amar's share should be divisible by 6. Option d gets rejected by this logic.

    Further: $A + B + C = 2250$. If Amar's share is 720 (acc. To option a) Bijoy's share should be 480 & Chandra's share should be 300. (Gives us a total of $720 + 480 + 300 = 1500$).

    But the required total is 2250 (50% more than 1500). Since all relationships are linear, 1500 will increase to 2250 if we increase all values by 50%. Hence, Amar's share should be 1080.

25. Trial and error would give us 2/5 as the original fraction.

26. Their ratio being 5:3, the difference according to the ratio is 2. But this difference is 10. To get the values, expand the ratio 5 times. This gives 25 and 15 as the required values. Hence, the product is 375.

27. 60 oxen days = 1/7 of the field Æ 420 oxen days are required to plough the field. Thus, the remaining work would be 360 oxen days. With 18 oxen, it would take 20 days.

28. Assume that 1 cat leap is equal to 3 metres and 1 dog leap is equal to 4 metres.

    Then the speed of the cat in one unit time = $3 \times 5 = 15$ meters.

    Also, the speed of the dog in one unit time = $4 \times 4 = 16$ meters.

    The required ratio is 15:16

29. $4x$, and $5x$ are their current ages. According to the problem, $4x - 18 : 5x - 18 = 11{:}16$ Æ $x = 10$ and hence the sum total of their present ages is 90 years (40 + 50).

30. $x + 3x + 4x + 7x = 105$ Æ $x = 7$

    Thus, $7x = 49$.

31. The share of the rent is on the basis of the ratio of the number of cow months. A uses 330 cow months (110 × 3), B uses 660(110 × 6) and C uses 1320 cow months (440 × 3)

    Hence, the required ratio is: 330:660:1320 = 1:2:4

32. 10 × 8 = 80 man days is required for the job. If only 8 students turn up, they would require 10 days to complete the task. The number of days is increasing by 1/4.

33. The initial amount of water is 9 liters and milk is 27 liters. By adding 15 liters of milk the mixture becomes 42 milk and 9 water Æ 14:3 the required ratio.

34. From the first statement $A = 1200$ and $B + C = 1800$. From the second statement $C = 1000$ and $A + B = 2000$.

35. $6x + 15 : 5x + 15 = 9 : 8$

    Æ $45x + 135 = 48x + 120$

$3x = 15$ Æ $x = 5$

Maya's present age = $6x = 30$

36. Since pressure and volume are inversely proportional, we get that if one is reduced by 20% the other would grow by 25%. Option (b) is correct.

37. Solve using options.

For option (c), 68 girls. Hence, 82 boys

Amount with Girls = $68 \times 0.25 = 17$

Amount with Boys = $82 \times 0.5 = 41$.

Total of t` 58.

Thus, option (c) fits the conditions.

38. The ratio : 1/2 : 2/3 : 3/4

Converts to 6 : 8 : 9 (on multiplying by 12)

Thus, the first monkey would get $(391/23) \times 6 = 102$ bananas.

39. Let the values of milk and water be $5x$ and x respectively. Then when we add 5 liters of water to this mixture, water would become $x + 5$.

Now: $5x/(x + 5) = 5:2$ Æ $x = 5$. Thus, $5x$ is 25.

40. The ratio of the values of the three coins are:

$10 \times 10 : 17 \times 20 : 7 \times 100 = 100:340:700$

= 5:17:35 is the ratio of division of value of coins.

Thus, 20 paise coins correspond to t` 17. Hence, there will be 85 coins.

41. Ratio of no. of coins = 12 : 10 : 7

Ratio of individual values of coins = 1 : 0.5 : 0.25

Ratio of gross value of coins = 12 : 5 : 1.75

= 48 : 20 : 7 Æ 75 t`

Thus, he has t` 7 in 25 paisa coins. Which means that he would have 28 such coins.

42. Solve using options. 15/24 becomes 24/33 Æ 8/11

43. $\dfrac{27-11}{35-11} = 16/24 = 2/3$.

Thus option (c) is correct.

44. $x = k/(y^2 – 1)$. This gives $k = 24 \times 99 = 2376$.

The equation becomes $x = 2376/(24) = 99$.

45. $x = ky$ Æ $18 = 7k$ Æ $k = 18/7$

Hence, $x = 18/7 \times y$

When $y = 21$, $x = 54$.

46. $A = K \times B \times C$ Æ It is known that when $A = 6$, $B = 3$ and $C = 2$. Thus we get $6 = 6K$ Æ $K = 1$.

Thus, our relationship between A, B and C becomes $A = B \times C$. Thus, when $B = 5$ and $C = 7$ we get $A = 35$.

47. $x = ky/z$

We cannot determine the value of k from the given information and hence cannot answer the question.

48. Initial wine = 35 litres

Initial water = 14 litres

Since, we want to create 7 : 4 mixture of wine and water by adding only water, it mean that the amount of wine is constant at 35 litres. Thus 7 : 4 = 35 : 20. So, we need 6 litres of water.

49. If we were to draw out 4 litres of wine and substitute it with plain water, the ratio of wine to water would become 1:1. Hence, option (d) is correct.

50. The overall ratio is: 21:35:55. Dividing 333 in 111 parts (21 + 35 + 55) each part will be 3 and Class III will have the highest number of pupils Æ 55 × 3 = 165

Level of Difficulty (II)

1. By taking the value of $x = 8$ from Option (c), the required ratio of 9:10 is achieved.

2. $T = K$D/V. $V = (K_1Q^{1/2})/N$ where K and K_1 are constants, T is the time duration of the journey, Q is the quantity of coal used and N is the number of carriages.

Thus, $T = (KDN)/(K_1Q^{1/2})$ or $T = (K_2DN)/(Q^{1/2})$ Æ if we take K/K_1 as K_2.

From the information provided in the question: $30 = (K_2 \times 50 \times 18)/10$ 24 $K_2= 1/3$

Thus, the equation becomes: $T = (DN)/(3Q^{1/2})$. Then, when $D = 42$, $T = 28$, and $N = 16$ we get:

$28 = 42 \times 16/(3Q^{1/2})$ Æ $Q = 64$

3. $\frac{2w_2}{w_2} = \frac{9r_1^2}{8r_2^2}$

Thus, $r_1/r_2 = 4:3$

4. Suppose you take $a = 3$ and $b = 2$. It can be clearly seen that the square root of 2 does not lie between 2 and 3. Hence, option c is incorrect.

Further with these values for a and b option a also can be ruled out since it means that the value should lie between 5 and 6 which it obviously does not.

Also, Option d gives 6/5 and 1/6. This means that the value should lie between 0.1666 and 1.2 (which it obviously does not). Hence, option (b) is correct.

5. (1347 × 1.125)/1.05 = 1443.

6. The constraint given to us for the values of a, b, and c is

$c(a + b) = a \times b$

So, if we take $a = 6$, $b = 3$ and $c = 2$, we have 18 = 18 and a feasible set of values for a, b and c respectively. With this set of values, we can complete the operation as defined and see what happens.

A.Wine left in the first vessel = 4 = (6 – 2)

B.Wine in the second vessel = 2

With these values none of the first 3 options matches. Thus, option (d) is correct.

7. The only information available here is that $–b/c$ should be equal to 2/1. This is not sufficient to make any of the first three options as conclusions. Hence, option (d) is correct.

8. The ratio of total salaries will be:

40:10:3. This gives 53 corresponds to 318. Hence, 1 corresponds to 6.

Thus the wages are: 240, 60 and 18 respectively.

9. Solve by taking values of a, b, c, d and e, f, g, and h independently of each other

$a = 1$, $b = 2$, $c = 3$, $d = 6$

and $e = 3$, $f = 9$, $g = 4$

and $h = 12$

gives $(ae + bf) : (ae - bf) = 21 : -15 = -7/5$

Option (b) $(cg + dh)/(cg - dh) = 84/-60 = -7/5$.

10. Since Brass has 0% tin and bronze has 16% tin, the ratio of mixing in the fused mass must be 3:5. Using alligation as follows:

10 6

0% tin 10% tin 16% tin

Brass Fused mass Bronze

Hence, 6:10 or 3:5

Then, percentage of copper in brass can be got as follows:

x 74 80

Brass Fused mass Bronze

3 5

x = 64%

11. Speed = $24 - k \div N$.

Putting value of $N = 4$ we get:

$20 = 24 - 2k$. Hence, $k = 2$

Thus the equation is: $S = 24 - 2 \div N$

This means that when $N = 144$, the speed will become zero. Hence, the train can just move when 143 wagons are attached.

12. x varies as y, means $x = ky$. This does not have any relation to the variance of $x^2 + y^2$.

13. Let $x = 5$

Then $f(x) = 6/4 = 1.5 = y$

And $f(y) = 2.5/0.5 = 5$.

Thus, the ratio of $x : f(y) = 1 : 1$

Note: Even if you take some other value of y, you would still get the same answer.

14. 2000 man weeks before the rain, $5/6^{th}$ of the work is completed. Hence, 2400 men weeks will be the total amount of work. However, due to the rain half the work gets washed off Æ This means

that 1000 man weeks worth of work must have got washed off. This leaves 1400 men weeks of work to be completed by the 140 men. They will take 10 more weeks and hence the total time required is 24 weeks.

15. Total distances covered under each mode = 32, 4 and 12 km respectively.

Total charges = 32 × 24 + 4 × 3 + 12 × 12 = 924 paise

= t` 9.24.

16. The no. of coins of 1 Re = 3x and 25p = x.

Conventionally, we can solve this using equations as follows.

$A + B + C = 220$ (1)

$A = 3C$ (2)

$A + 0.5B + 0.25C = 160$ (3)

We have a situation with 3 equations and 3 unknowns. and we can solve for

A (no. of 1 Re coins),

B (no. of 50 paise coins)

and C (no. of 25 paise coins)

However, a much smarter approach would be to go through the options. If we check option (a) – no. of 50 paise coins = 60 we would get the number of 1 Re coins as 120 and the number of 25 paise coins as 40.

120 × 1 + 60 × 0.5 + 40 × 0.25 = 160

This fits the conditions perfectly and is hence the correct answer.

17–19. In order to solve this question, if you try going through equation and expressions, it would lead you in to a very long drawn solution.

Thus: $12/2x + 24/6x + 5/x = n/60$

and $12/3y + 24/8y + 5/y = n + 20/60$

We also know that $3y = 2x$.

In order to handle this expression, you can try be substituting the values of the speeds. Also, we know that his running speed (initially) is twice his swimming speed.

Question 17 is asking us his swimming speed, white 19 is asking us his running speed. So the answer of the two questions should be in the ratio 1 : 2.

However, a scrutiny of the options shows us that none of the 4 options (values) in question 17 have a value which is half the values provided for in 19. (you would need to check for this after converting the values into kmph).

So, we can start by checking individual options from question 19.

Amongst the 4 values (9, 18, 54, and 12) 12 kmph is the easiest to check.

Checking for it we have:

Scenario 1:12/12 + 24/36 + 5/6 = 1 + 1 hr + 40 mins

+ 50 mins = 2 hr 30 minutes.

Scenario 2:12/12 + 24/32 + 5/4 = 1hr + 45 minutes

+ 1 hr 15 minutes = 3 hrs.

This doesn’t match the condition of 20 minutes extra.

If you check for 18 kmph, you will get all values fitting in perfectly.

Scenario 1 would give you 1 hr 40 minutes and Scenario 2 would give you 2 hours.

Answers are: 17. d

18. b

19. b

20. $(20 + 60 + 30x)/(2 + 3 + x) = 23$ Æ $80 + 30x = 115 + 23x$ Æ $x = 5$.

21. Assume raw materials cost as 150 and total cost as 450. (Thus, wages cost is 300)

Since, the cost of raw materials goes up in the ratio of 3:7 the new raw material cost would become 350 and the new wages cost would become in the ratio 4:9 as 675.

The new cost would become, 1025.

Since 450 become 1025 (change in total cost), unitary method calculation would give us that 18 would become t` 41.

22. Solve using options. For option (c), we will get that initially there are 125 boys and 140 girls. After the given increases, the number of boys would be 145 and the number of girls would become 154 which gives a difference of 9 as required.

23. From the question, it is evident that after leaving out the *C* courses, Sonali's GPA goes to 3.33. This means that the number of subjects she must have had after leaving out the *C*'s must be a multiple of 3. This only occurs in Option c. Hence, that is the answer.

24. When there are 25 boarders, the total expenses are \$1750. When there are 50 boarders, the total expenses are \$ 3000. The change in expense due to the coming in of 25 boarders is \$ 1250. Hence, expense per boarder is equal to \$50. This also means that when there are 25 boarders, the variable cost would be 25×50 = \$1250. Hence, \$500 must be the fixed expenses.

So for 100 boarders, the total cost would be: \$ 500 (fixed) + \$ 5000 = \$5500

25. $S = 42 - k \div n$

$24 = 42 - k \times 3$ Æ $k = 6$

So, $S = 42 - 6 \div n$

For 49 compartments the train would not move. Hence it would move for 48 compartments.

26-28:

Let the third drunkard get in x litres. Then the second will contribute $x + 1$ and the first will contribute $x + 2$ litres. Thus in all they have $3x + 3$ litres of the drink. Using option a in question 27, this value is 12, giving x as 3.

Also, each drunkard will drink 3 litres.

Thus, the first drunkard brings 5 litres and the second 4 litres. Their contribution to the fourth drunkard will be in the ratio 2:1 and hence their share of money would be also in the ratio 2:1. Hence, this option is correct for question 27.

Hence, for question 26, the second drunkard will get 5 roubles (for his contribution of 1 litre to the fourth) and for question 28, the answer would be 1:3

29. 5 : 4 Æ 5 : 4.8 Æ 25 : 24.

Option (c) is correct.

30. $P = K \times W^2$ Æ $12250 = K \times 35^2$ Æ $K = 10$.

Thus our price and weight relationship is: $P = 10W^2$.

When the two pieces are in the ratio 2:5 (weight wise) then we know that their weights must be 10 grams and 25 grams respectively. Their values would be:

10 gram piece: 10×10^2 = t` 1000;

25 gram piece: 10×25^2 = t`6250.

Total Price = 1000 + 62450 = 7250. From an initial value of 12250, this represents a loss of t` 5000.

31. The ratio of distribution should be:

    21 × 35 : 15 × 35 : 15 × 21 Æ 147 : 105:63 Æ 7:5:3

    The biggest share will be worth: 7 × 525000/15 = 245000.

32. $P + M + T = 675$ Æ $3M + M + 3M - 25 = 675$ Æ $7M = 700$. Hence, $M = 100$. $P = 300$ and $T = 275$.

33. Ratio of distribution = 20 : 13 : 8

    So the elephant should get (20/41) × 820 = 400.

34. Women : Men = 3 : 4

    Men : Children = 3 : 5

    Æ Women : Men : children = 9 : 12 : 20

    In the ratio, 9 Æ 531 Women

    Thus, 20 Æ 1180 children.

35. 2/3 becomes 7/8 a change from 0.666 to 0.875 while the other changes are smaller than this. For instance 4/7 becomes 9/12 a change from 0.5714 to 0.75 which is smaller than the change in 2/3. Similarly the other options can be checked and rejected.

36. Since, the work gets done in 25% less time there must have been an addition of 33.33% men.

    This would mean 13.33 men extra Æ which would mean 14 extra men (in whole nos.)

37. From the given options, we just need to look for a multiple of 7. 2100 is the only option which is a multiple of 7 and is hence the correct answer.

38. This is a simple question if you can catch hold of the logic of the question. i.e. the younger daughter's share must be such after adding a CI of 20% for two years, she should get the same value as her elder sister.

    None of the options meets this requirement. Hence, None of these is correct.

39-41. You should realise that when Anshu gives her pens to Bobby & Chandana, the number of pens for both Bobby & Chandana should double. Also, the number of pens for Anshu & Bobby should also double when Chandana gives off her pens. Further the final condition is that each of them has 24 pens. The following table will emerge on the basis of this logic.

	Anshu	*Bobby*	*Chandana*
Final	24	24	24
Second round	12	12	48
Initial	42	6	24

42. $V = k\,AH$ Æ $280 = k \times 60 \times 14$ Æ $280 = 840k$. Thus, $k = 1/3$ and the equation becomes:

$V = AH/3$ and $390 = 26A/3$ Æ $A = 45$.

43. Expenses for 120 boys = 8400

    Expenses for 150 boys = 10000.

    Thus, variable expenses are t` 1600 for 30 boys.

    If we add 180 more boys to make it 330 boys, we will get an additional expense of 1600 × 6 = ` 9600.

    Total expenses are t` 19600.

44. Let the distance be d. Then, $d/14.4 = \sqrt{8}/\sqrt{18}$ Æ $d = 9.6$

45. 47 : 100: 220 would give: 0.5 cubic feet of Cement, 1cubic feet of sand and 2 cubic feet of gravel. Required ratio 1 : 2 : 4 is satisfied.

Level of Difficulty (III)

1. You can use alligation between 33.33% and 40% to get 37.5%. Hence the ratio of mixing must be 2.5:4.16 Æ 3:5

6. Check each of the options as follows:

   Suppose you are checking option b which gives the value of *a* as 81 litres.

   Then, it is clear that when you are pouring out 81 litres, you are leaving 8/9 of the honey in the barrel.

   Thus the amount of honey contained after 6 such operations will be given by:

   $729 \times (8/9)^6$. If this answer has to be correct this value must be equal to 64 (which it clearly is not since the value will be in the form of a fraction.)

   Hence, this is not the correct option. You can similarly rule out the other options.

7. It is clear that if 7 kg of the first is mixed with 21 kg of the second you will get 5 + 9 = 14 kg of nickel and 14 kg of tin. You do not need to check the other options since they will go into fractions.

10. The piece that is cut off should be such that the fraction of the first to the second alloy in each of the two new alloys formed should be equal.

    If you cut off 4 kg, the respective ratios will be:

    First alloy: 2 kg of first alloy and 4 kg of second alloy

    Second alloy: 4 kg of first alloy and 8 kg of the second alloy. It can easily be seen that the ratios are equal to 1:2 in each case.

13. This is again the typical alligation situation.

    The required ratio will be given by (7/18 – 3/8) : (2/5 – 7/18)

    Alternately, you can also look at it through options. It can be easily seen that if you take 8 kg of the first with 10 kg of the second you will get the required 7:11 ratio.

17. The cost of making one gram of gun powder would be t` 180. This will contain 0.4 gm of saltpetre and 0.6 gm of nitrous oxide. Check through options.

    At the rate of saltpetre of 300/gm, the nitrous oxide will cost t`100/gm. The total cost of 0.4 grams of saltpetre will be 120 and 0.6 grams of nitrous oxide will be t` 60 giving the total cost as 180.

20. There will be a total of 4.5 litres of milk (25% of 3 + 75% of 5) giving a total of 4.5. Hence, 45%.

23. Go through the options as follows:

According to option d, if the initial quantity of spirit is 4 litres, half the spirit is taken out when 2 litres are drawn out. Thus the spirit after three times of the operation would be:

$4 \times (1/2)^2 = 0.5$ litres. This matches the option. You can check for yourself that the first three options will not work.