3. SKEWNESS

EXERCISE 3.1

1) For a distribution, mean = 100, mode = 127 and SD = 60. Find the Pearson coefficient of skewness Skp.

Sol: Given: $\bar{x} = 100$

$z = 127$

$\sigma = 60$

Pearsonian coefficient of skewness,

$$Skp = \frac{\bar{x} - z}{\sigma}$$

$$= \frac{100 - 127}{60}$$

$$= \frac{-27}{60}$$

$\therefore$ Skp = - 0.45

2) The mean and variance of the distribution is 60 and 100 respectively. Find the mode and the median of the distribution if Skp = - 0.3

Sol: Given: $\bar{x} = 60$

Variance = 100

$\because \sigma = \sqrt{Variance} = \sqrt{100} = 10$

Skp = - 0.3

Pearsonian coefficient of skewness,

$$Skp = \frac{\bar{x} - z}{\sigma}$$

$$\therefore -0.3 = \frac{60 - z}{10}$$

$\therefore -0.3 \times 10 = 60 - z$

$\therefore -0.3 - 60 = -z$

$\therefore -63 = -z$

$\therefore z = 63$

Skp $= \frac{\overline{3(x-M)}}{\sigma}$

$\therefore -0.3 = \frac{3(60-M)}{10}$

$\therefore -0.3 \times 10 = 3(60 - M)$

$\therefore \frac{-3}{3} = 60 - M$

$\therefore -1 - 60 = -M$

$\therefore -61 = -M$

$\therefore M = 61$

Hence, Model, $z = 63$ and Median, $M = 61$

3) For a data set, sum of upper and lower quartiles is 100, difference between upper and lower quartiles is 40 and median is 30. Find the coefficient of skewness.

Sol: Given: $Q_3 + Q_1 = 100$

$Q_3 + Q_1 = 40$

$M = 30$

Bowley's co-efficient of skewness,

Skb $= \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$

$= \frac{100-2(30)}{40}$

$= \frac{100-60}{40}$

$= \frac{40}{40}$

∴ Skb = 1

4) For a data set with upper quartile equal to 55 and median equal to 42. If the distribution is symmetric, find the value of lower quartile.

Sol: Given: $Q3 = 55$
M = 42
Distribution is symmetric

∴ Skb = 0

$\therefore \text{Skb} = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$

Bowley's co-efficient of skewness,

$\therefore \quad 0 = \frac{55 + Q_1 - 2(42)}{55 - Q_1}$

$\therefore \quad 0 = 55 + Q1 - 84$

$\therefore \quad 0 = -29 + Q1$

$\therefore \quad Q1 = 29$

5) Obtain the coefficient of skewness by formula and comment on mature of the distribution.

Height in inches	No. of females
Less than 60	10
60-64	20
64-68	40
68-72	10
72-76	2

Sol:

Height in inches	No. of females(f)	C.F. (Less than)
Less than 60	10	10
60-64	20	30

64-68	40	70
68-72	10	80
72-76	2	82
	N = 2	

Here, N = 82

Q 1 Calculation:

For Q_1, $\frac{N}{4} = \frac{82}{4} = 20.5$

$\therefore$ Q_1 lies in the class 60 - 64

L = 60, f = 20; c.f. = 10; h = 4

$\therefore$ $Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f\right)$

$= 60 + \frac{4}{20}(20.5\text{-}10)$

$= 60 + \frac{1}{5} \times 10.5$

$= 60 + 2.1$

$\therefore Q_1 = 62.1$

Q 3 Calculation:

For Q_3, $\frac{3N}{4} = 3 \times 20.5 = 61.5$

$\therefore$ Q_3 lies in the class 64 - 68

L = 64, f = 40; c.f. = 30; h = 4

$\therefore$ $Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f\right)$

$= 64 + \frac{4}{40}(61.5\text{-}30)$

$= 64 + \frac{1}{10} \times 31.5$

$= 64 + 3.15$

$\therefore Q_3 = 67.15$

Median calculation:

For M, $\frac{N}{2} = \frac{82}{2} = 41$

∴ Median lies in the class 64 - 68

L = 64, f = 40; c.f. = 30; h = 4

$\therefore$ M= $L + \frac{h}{f}\left(\frac{N}{2} - c.f\right)$

$= 64 + \frac{4}{40}$ (41-30)

$= 64 + \frac{1}{10} \times 11$

$= 64 + 1.1$

$\therefore M = 67.15$

Bowley's co-efficient of skewness,

$\therefore$ Skb = $\frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$

$= \frac{67.15 + 62.1 - 2(65.1)}{67.15 + 62.1}$

$= \frac{129.25 - 130.2}{5.05}$

$= \frac{-0.95}{5.05}$

$\therefore$ Skb = $^{-}0.1881$

$\because$ **Skb < 0, the distribution is negatively skewed.**

6) Find Skb for the following set of observations.

17,17,21,14,15,20,19,16,13,17,18

Sol:

x_i	x_i^2
17	289
17	289
21	441
14	196
15	225
20	400
19	361
16	256
13	169
17	289
18	324
$\sum x_i = 187$	$\sum x_i^2 = 3239$

Here, $n = 11$

Mean, $\bar{x} = \frac{\sum x_i}{n}$

$= \frac{187}{11}$

$\therefore \quad \bar{x} = 17$

Standard deviation,

$\sigma = \sqrt{\frac{\sum x_i^2}{n}} - (\bar{x})\ 2$

$= \sqrt{\frac{3239}{11}} - (17)2$

$= \sqrt{294.45 - 289}$

$= \sqrt{5.45}$

$\therefore \quad \sigma \quad = 2.33$

Mode,

$\because$ 17 is repeated maximum times i.e. 3 times

$\therefore$ Mode, $z = 17$

$\therefore$ Skp $= \frac{\bar{x}-z}{\sigma}$

$= \frac{17-17}{2.33}$

$\therefore$ **Skp $= 0$**

7) Calculate Skb for the following set of observations of yield of heat in kg from 13 plots.
4.6,3.5,4.8,5.1,4.7,5.5,4.7,3.6,4.2,3.5,3.6,5.2
Sol: Arranging the data in ascending order,
3.5,3.5,3.5,3.6,3.6,4.2,4.6,4.7,4.7,4.8,5.1.5.2.5.5
Here, $n = 13$

First Quartile:

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ observation

$= \left(\frac{13+1}{4}\right)^{th}$ observation

$= \left(\frac{14}{4}\right)^{th}$ observation

= 3.5th observation

= 3rd observation + 0.5 (4th observation-3rd observation)

= 3.5 + 0.5 (3.6-3.5)

= 3.5 + 0.5 × 0.1

= 3.5 + 0.05

$\therefore Q_1 = 3.55$

Third quartile:
$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ observation

= 3(3.5)th observation

= 10.5th observation

= 10th observation + 0.5 (11th observation-10th observation)

$= 4.8 + 0.5\ (5.1\text{-}4.8)$

$= 4.8 + 0.5 \times 0.3$

$= 4.8 + 0.15$

$\therefore\ Q_3 = 4.95$

Median:

$M = \left(\frac{n+1}{2}\right)^{th}$ observation

$= \left(\frac{13+1}{2}\right)^{th}$ observation

$= \left(\frac{14}{2}\right)^{th}$ observation

= 7th observation

$\therefore\ M = 4.6$

Bowley's co-efficient of skewness,

$\therefore Skb = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$

$= \frac{4.95 + 3.55 - 2(4.6)}{4.5+3.55}$

$= \frac{8.5 - 9.2}{1.4}$

$= \frac{-0.7}{1.4}$

$\therefore$ Skb = $^-$0.5

8) For a frequency distribution $Q_3 - Q_2 = 90$ and $Q_2 - Q_1 = 120$, find Skb.

Sol: Given: $Q_2 - Q_1 = 120$

$Q_3 - Q_2 = 90$

Bowley's co-efficient of skewness,

$$\therefore \text{Skb} = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{90 - 120}{90 + 120}$$

$$= \frac{-30}{210}$$

$$\therefore \text{Skb} = {}^{-}0.143$$