ગુજરાત રાજ્યના શિક્ષણવિભાગના પત્ર-ક્રમાંક
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STATISTICS

(Part 1)

Standard 12

PLEDGE

India is my country.
All Indians are my brothers and sisters.
I love my country and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall respect my parents, teachers and all my elders and treat everyone with courtesy.
I pledge my devotion to my country and its people.
My happiness lies in their well-being and prosperity.

રાજ્ય સરકારની વિનામૂલ્યે યોજના હેઠળનું પુસ્તક

Gujarat State Board of School Textbooks
'Vidyayan', Sector 10-A, Gandhinagar-382010

Subject Adviser
Dr. R. T. Ratani

Authors
Dr. M. N. Patel (Convener)
Prof. Shubha A. Lagvankar
Dr. Chirag J. Trivedi
Dr. Kunjal H. Shah
Dr. Parag B. Shah
Shri Maheshbhai A. Patel
Shri Yatin A. Parikh

Translation
Dr. M. N. Patel
Prof. Shubha A. Lagvankar
Dr. Chirag J. Trivedi
Dr. Kunjal H. Shah
Dr. Parag B. Shah

Reviewers
Shri Lalita K. Patel
Shri Chirag S. Shah
Shri Haresh N. Doshi
Smt. Deepa K. Lalwani
Shri Girish C. Naik
Shri C. P. Rajpurohit
Smt. Aparna S. Pathak
Shri Manoj G. Naidu

Language Reviewer
Dr. Chetan Mewada

Artist
Media Graphics

Co-ordinator
Dr. Chirag N. Shah
(Subject Co-ordinator : Commerce)

Preparation and Planning
Dr. Kamlesh N. Parmar
(Dy. Director : Academic)

Lay-out and Planning
Shri Haresh S. Limbachiya
(Dy. Director : Production)

PREFACE

Gujarat State Board of School Textbooks has prepared new textbooks as per the new curricula developed by the Gujarat State Secondary and Higher Secondary Education Board and which has been sanctioned by the Education Department of the Government of Gujarat. A panel of experts from Universities/Colleges, Teachers Training Colleges and Schools have put lot of efforts in preparing the manuscript of the subject. It is then reviewed by another panel of experts to suggest changes and filter out the mistakes, if any. The suggestions of the reviewers are considered thoroughly and necessary changes are made in the manuscript. Thus, the Textbook Board takes sufficient care in preparing an error-free manuscript. The Board is vigilant even while printing the textbooks.

The Board expresses the pleasure to publish the Textbook of **Statistics (Part 1)** for **Std. 12** which is a translated version of Gujarati. The Textbook Board is thankful to all those who have helped in preparing this textbook. However, we welcome suggestions to enhance the quality of the textbook.

H. N. Chavda
Director
Date : 15-02-2017

Dr. Nitin Pethani
Executive President
Gandhinagar

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FUNDAMENTAL DUTIES

It shall be the duty of every citizen of India : *

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures;

(h) to develop scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

(k) to provide opportunities for education by the parent, the guardian, to his child, or a ward between the age of 6-14 years as the case may be.

*Constitution of India : Section 51-A

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1

Index Number

Contents :

1.1 Definition and Meaning of Index Number

Price of an item, national income, supply, production, employment, unemployment, investment, import-export, cost of living, population of a country, birth rate and death rate vary continuously with time. Generally, the proportion and direction of these variations also keep changing. It is important to study the variations in the price and quantity of an item with respect to change in time. The planning for the future can be suitably done from the knowledge of these changes. The changes taking place in the values of the variable at two different time periods can be measured by the following methods :

(1) Method of absolute measure (difference) and (2) Method of relative measure (ratio)

We will understand this concept by the following illustration :

Suppose the data regarding the average price per kilogram of two items, wheat and rice, for a month in the year 2015 and year 2016 are as follows :

Item	Price per kilogram ₹	
	Year 2015	Year 2016
Wheat	24	30
Rice	40	46

Let us understand the comparision of variations in the prices of wheat and rice using the two methods stated above.

(1) Method of Absolute Measure (difference) : The price of wheat in the year 2015 was ₹ 24 which increased to ₹ 30 in the year 2016. Thus, the price per kilogram increased in the year 2016 by ₹ 6 with respect to the year 2015. Similarly, the price of rice has also increased by ₹ 6. This is obtained by the absolute difference. Thus, it can be said that there is same rise in price in both the items. But this is not true in reality because the prices per unit of these items are not same in the year 2015. Thus, the base for comparative study of prices in 2016 is different. Hence, this method is not appropriate to compare the variations in a variable. We shall now study the method of relative measure which is used in such situations.

(2) Method of Relative Measure (ratio) : A ratio of price of the commodity in the year 2016 is obtained with the price in the year 2015 in this method to find the relative changes of the price of the item in the year 2016.

Thus, ratio of prices of wheat $= \frac{30}{24} = 1.25$

ratio of prices of rice $= \frac{46}{40} = 1.15$

It can be known from these ratios that the relative increase in the prices of wheat and rice in the year 2016 is not same. The price of wheat in the year 2016 is 1.25 times the price in the year 2015, whereas the price of rice is 1.15 times its price in the year 2015. Thus, it can be said that change in the price of wheat is more than the change in the price of rice.

The ratios of prices of wheat and rice given here indicate the changes in prices at two different time periods. It is also called as relative change or price relative. Generally, the ratios are expressed as percentages to facilitate comparison. Hence,

Percentage change in the price of wheat $= 1.25 \times 100 = 125$ and

Percentage change in the price of rice $= 1.15 \times 100 = 115$

This is the relative percentage measure for the changes. Such a relative measure is called index number.

Thus, the percentage change in the value of a variable associated with any item for the given (current) period compared to its value in a fixed (base) period is called an index number.

Now, we shall obtain a relative measure for the collective change in the prices of these two mutually related items. The absolute method is not useful to find a measure for the overall change because many times the units expressing the prices of these two items may be different and it is not possible to combine the changes in these prices. The method of relative measure is used in such a situation. Since the relative measure is free from the unit of measurement, it is possible to combine the changes in the prices of the two items and it is convenient to find a mathematical measure for these changes. Now, we shall take a relative measure for the overall change from the changes in prices of grains, wheat and rice. We shall denote the price for the year 2016 as p_1 and the price of the year 2015 by p_0. p_0 is called base year price and p_1 is called the current year price. The ratio $\frac{p_1}{p_0}$ is called the price relative of that item.

We shall present this in a tabular form :

Item	**Price of base year 2015 (₹)** p_0	**Price of current year 2016 (₹)** p_1	**Price relative or Relative change** $= \frac{p_1}{p_0}$	**Percentage of Price relative** $= \frac{p_1}{p_0} \times 100$
Wheat	24	30	$\frac{30}{24} = 1.25$	125
Rice	40	46	$\frac{46}{40} = 1.15$	115
Total			**2.40**	**240**

The index obtained by multiplying the average of price relatives of the current year for these two items by 100 is called the price index number of the items for the current year. It is denoted by *I*. Thus,

Price index number of wheat and rice for the current year

$$= \frac{\text{Price relative of wheat} + \text{Price relative of rice}}{\text{No. of items}} \times 100$$

$$= \frac{1.25 + 1.15}{2} \times 100$$

$$= 120$$

Hence, price index number of wheat and rice for the current year $I = 120$. The price index number of wheat and rice $I = 120$ indicates that there is an overall rise of 20 percent in the prices of the two items in the year 2016 as compared to the year 2015. Index number is a relative measure based on ratio. Similarly, measure for the overall change can be obtained using the relative method by combining changes in the values of more than two variables. We can define the general index number for a group as follows.

"The average of the percentage change in the value of a variable associated with one or more items for the given (current) period compared to its value in the fixed (base) period is called a general index number for the group."

General index number for the group $I = \frac{\Sigma\left[\frac{p_{1i}}{p_{0i}}\right]}{n} \times 100$

Where, general index number I = Index number of current period with respect to comparison period

p_{1i} = Value of variable i for current period $\left(i = 1, 2, 3, ..., n\right)$

p_{0i} = Value of variable i for comparison period $\left(i = 1, 2, 3, ..., n\right)$

n = Number of values of the variable

The simple mean is used in the definition of general index number for the group of n items. But the weighted mean or geometric mean can also be used in the definition of general index number, which will be discussed later in this chapter.

In practice, several mutually related items are to be included and the data regarding their prices should be obtained to find a price index number. For example, wheat, rice, pulses, oil, ghee, jaggery, spices, vegetables are included in the category of food items. Thus, price index number for food is an index associated with the relative change or price relative for the prices of several related items.

Now, if we take a group of n such mutually related items then an index is found using relative change in price of each item in that group. An average measure obtained from them is called the price index numbers for the group. It can be written as the following formula :

General price index number $I = \frac{\Sigma\left[\frac{p_{1i}}{p_{0i}}\right]}{n} \times 100$

Where, p_{1i} = price of item i in current period $\left(i = 1, 2, 3, ..., n\right)$

p_{0i} = price of item i in base period $\left(i = 1, 2, 3, ..., n\right)$

n = number of items

Further, if we take a group of n such mutually related items then an index is found using relative change in the quantity of each item in that group. An average measure obtained from them is called the quantity index number for the group.

Note : The index number for production, import, export, unemployment, industrial output, etc. can be obtained by the above formula.

1.2 Characteristics of Index Number

Some of the characteristics of index number deduced from its definition are as follows :

(1) Index number is free from the unit as it is a relative measure.

(2) The changes in the values of the variable having different units can be compared using index number. Hence, index number is a comparative measure.

(3) Index number is a relative measure showing percentage change.

(4) Index number is a special average. It has all the characteristics of an average.

(5) The situation at two different periods can be compared by ratio with the standard (base) period using an index number.

1.3 Uses of Index Number

A general notion about the index number is, an index number is only used to find a measure for changes in the value of a variable or the price level. But now its use is not limited to the study of change in the price level. The index number is used in various fields in the current revolutionary age. Index number is a useful statistical tool to study the challenges in the given economic, political, social and industrial activities. Index number provides important guidance for planning the economic development of a country as it gives a comparative study of economic and industrial scenario of the country. Some of the uses of index number are as follows :

(1) Index Number for Trade : This index number provides useful guidance to study the general situation of economic activities of business and trade in the country.

(2) Wholesale Price Index Number : This index number measures the changes in the general price level in the country. This index number is useful to the government, producers and businessmen to take policy decisions such as knowing the demand and supply of items in the economy, estimating the future values and planning the future. The Reserve Bank of India uses this index number to take necessary steps to control inflation by studying the changes in price levels. Using the wholesale price index number, the rate of inflation is found as follows.

$$\text{Rate of inflation} = \frac{\left(\begin{array}{c}\text{Wholesale index}\\ \text{number of current year}\end{array}\right) - \left(\begin{array}{c}\text{Wholesale index number}\\ \text{of previous year}\end{array}\right)}{\text{Wholesale index number of previous year}} \times 100$$

(3) Cost of Living Index Number : This index number is useful to study the changes in the cost of living of people of different sections. This index number helps to determine purchasing power of money, salary to employees, dearness allowance, bonus, to calculate real wage and to devise tax policies by the government.

(4) Index Number of Human Development : This index number is useful to determine the state of human resource, standard of living, life expectancy and level of education and it gives information about human resource development.

(5) Index Number of National Income : This index number is useful to evaluate economic condition of the country and to determine targets for the five year plans by the government. The suggestions for increasing the national income, production and per capita income of the country can also be given using this index number by studying the changes in the national income of the country.

(6) Index Number of Industrial Production : This index number is very useful to study the changes in the production in industrial and craft fields. This index number is helpful to increase the rate of development of the country, planning industrial and trading activities.

(7) Index Number of Agricultural Production : This index number is useful to study the changes in the prices of agricultural production. The government plans agricultural policies using this index number. Moreover, this index number is helpful in forming policy to give proper support price to the farmers for their production.

(8) Index Number of Import-Export : This index number is useful to determine import-export policy, exchange rate, foreign exchange requirement and the rate of excise on goods and to provide necessary suggestions.

(9) Index Number for Employment : This index number provides the picture of employment, unemployment prevailing in the country. This shows the problems of unemployment which facilitates human resource planning.

(10) Index Number of Capital Investment : The changes in prices of shares and stocks, debentures, government securities and flow of capital investment can be studied by this index number. It also helps to estimate the trend of prices of shares and stocks.

(11) Index Number of Raw Material : This index number provides necessary guidance to traders, businessmen, economists, etc. for the policies of production-sales.

As barometer is used to predict weather, air pressure, cyclone and rain, the index number is a necessary tool for the measurement and comparative study of changes in the economic, business and social activities of the country. Hence, an index number is called the barometer of the economy of a country.

1.4 Base Year

In the construction of index numbers, the value of a variable for the current period is compared with the value of the variable with a fixed period (usually from the past). This fixed period or year is called the base year. The fixed year from the past can be the preceding year or any year before that. The period or year for which the value is to be compared with the base period or year is called the current period or year. For example, if the price of an item in the year 2016 is to be compared with the price of the same item in the year 2015, the year 2015 is called base year and the year 2016 is called the current year.

The year selected as base year should be standard or normal. It should be free from natural calamities like floods, draught, earthquake, abnormal man-made events like war, revolt, riot, strike, agitation, political events, economic disturbance or any unusual events. It is also necessary that the base year should not be from a distant past. If the base year selected is an unusual year and the values of the variable are unusually high or low then the value of index number could be misleading and it will not reveal the realistic picture of the current situation. Thus, the base year should be carefully selected while constructing index number.

The base year can be selected in two ways : (1) Fixed Base Method (2) Chain Base Method

1.4.1 Fixed Base Method

In this method, a stable period or year with usual events or situation is selected as a normal year or base year. But sometimes it becomes difficult to select a normal or base year. In this case, an average value of certain years is taken as the value of the variable for the base year. Index number is obtained by comparing value of the variable in the current year with the value of variable for the base year. The base year should be changed periodically so that it does not become a year of the distant past. The index number by fixed base method is obtained from the following formula :

$$\text{Index number } I = \frac{\text{Value of the variable in current year (period)}}{\text{Value of the variable in base year (period)}} \times 100$$

$$= \frac{p_1}{p_0} \times 100$$

Where, p_1 = Value of the variable in current year (period)

p_0 = Value of the variable in base year (period)

Illustration 1 : The data about wholesale prices of wheat in a region are as follows. Taking the year 2005 as the base year, prepare the index numbers for the price of the item for the remaining years. State the percentage increase in the price of wheat in the year 2013 from these index numbers.

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013
Price per Quintal (₹)	1650	1690	1730	1750	1810	1850	1870	1900	1950

We will find the fixed base index number as the year 2005 is to be taken as the base year. The index number for the price of wheat in the year 2005 will be taken as 100.

Year	**Price of wheat per Quintal (₹)**	**Index number = $\frac{p_1}{p_0} \times 100$**
2005	1650	$\frac{1650}{1650} \times 100 = 100$
2006	1690	$\frac{1690}{1650} \times 100 = 102.42$
2007	1730	$\frac{1730}{1650} \times 100 = 104.85$
2008	1750	$\frac{1750}{1650} \times 100 = 106.06$
2009	1810	$\frac{1810}{1650} \times 100 = 109.70$
2010	1850	$\frac{1850}{1650} \times 100 = 112.12$
2011	1870	$\frac{1870}{1650} \times 100 = 113.33$
2012	1900	$\frac{1900}{1650} \times 100 = 115.15$
2013	1950	$\frac{1950}{1650} \times 100 = 118.18$

It can be said that the increase in the price of wheat in the year 2013 is (118.18 – 100) = 18.18 % with respect to the year 2005.

Illustration 2 : The prices per unit (₹) of six food items in the year 2014 and 2015 are given in the following table. Taking 2014 as the base year, compute the general index number for the price of food items and state the overall rise in prices of these food items.

Item	Unit	Price per unit (₹) of the item	
		Year 2014	Year 2015
Bread	Packet	25	28
Eggs	Dozen	30	35
Ghee	Tin	375	380
Milk	Litre	36	40
Cheese	Kilogram	440	500
Butter	Kilogram	265	300

A general index number for the price of these items for the current year 2015 is to be obtained with the base year 2014. We will find price relatives $\frac{p_1}{p_0}$ by taking base year price as p_0 and current year price as p_1. The calculation is shown in the following table :

Item	Price of item (₹)		Price relative = $\frac{p_1}{p_0}$	
	p_0	p_1		
Bread	25	28	$\frac{28}{25}$	= 1.1200
Eggs	30	35	$\frac{35}{30}$	= 1.1666
Ghee	375	380	$\frac{380}{375}$	= 1.0133
Milk	36	40	$\frac{40}{36}$	= 1.1111
Cheese	440	500	$\frac{500}{440}$	= 1.1364
Butter	265	300	$\frac{300}{265}$	= 1.1321
Total				**= 6.6795**

General index number of six food items $I = \dfrac{\Sigma\left[\frac{p_1}{p_0}\right]}{n} \times 100$

$= \dfrac{6.6795}{6} \times 100$

$= 111.33$

$\therefore$ General price index number of six food items is $I = 111.33$.

It can be seen from the value of the index number I that there is an overall rise in prices of food items by (111.33 – 100) = 11.33 % in the year 2015 as compared to the year 2014.

Illustration 3 : The data about sugar production of a sugar manufacturing company from the year 2008 to 2015 are as follows. Prepare index number by fixed base method from these data by taking average production of the years 2009, 2010 and 2011 as the production of the base year.

Year	2008	2009	2010	2011	2012	2013	2014	2015
Production (thousand tons)	186	196	202	214	229	216	226	230

The average production of the years 2009, 2010 and 2011 = $\dfrac{196 + 202 + 214}{3} = \dfrac{612}{3} = 204$

Year	**Production (thousand tons)**	**Index number by fixed base method** $= \dfrac{p_1}{p_0} \times 100$
2008	186	$\frac{186}{204} \times 100 = 91.18$
2009	196	$\frac{196}{204} \times 100 = 96.08$
2010	202	$\frac{202}{204} \times 100 = 99.02$
2011	214	$\frac{214}{204} \times 100 = 104.90$
2012	220	$\frac{220}{204} \times 100 = 107.84$
2013	216	$\frac{216}{204} \times 100 = 105.88$
2014	226	$\frac{226}{204} \times 100 = 110.78$
2015	230	$\frac{230}{204} \times 100 = 112.75$

Merits and limitations of fixed base method

Merits : (1) Uniformity is maintained in calculation and comparison of the relative changes in the values of the variable as the base year is constant in this method.

(2) This method is useful to compare the long term changes in the values of the variable.

(3) This method is easy to understand and compute.

Limitations : (1) The taste, habits and fashion of consumers change with time and hence there is a change in the items used by the consumer. The items with reduced usage which were used in the past can not be removed in this method.

(2) It is not always possible to have a standard year with normal conditions as the base year. Therefore, selection of the base year is difficult.

The reliability of the index number reduces if the base year is not selected appropriately.

(3) This method is not suitable to compare the short term changes in the value of the variable.

(4) The quality of selected items keeps changing. It is not possible to make necessary change in their weights in this method.

(5) If the base year is a year of very remote past, the comparison can not be considered to be appropriate.

1.4.2 Chain Base Method

A fixed year or period is not taken as a base year or period in this method. For every current year, its preceding year is taken as a base year. For example, the year 2015 is taken as a base year for the index number of the year 2016. The base year keeps changing in this method. Since the base year is repeatedly changed, this method is called chain base method. The current situation is compared with the recent past situation in this method. The index number by this method is found using the following formula :

$$\text{Index number} = \frac{\text{Value of the variable for current year (period)}}{\text{Value of the variable for preceding year (period)}} \times 100$$

$$\therefore \quad I = \frac{p_1}{p_0} \times 100$$

Illustration 4 : The data about bi-monthly closing prices of shares of a company in the year 2014 are given. Compute the chain base index numbers from these data.

Month	January	March	May	July	September	November
Price (₹)	22	21.20	22	23	24.70	26.00

The price for the month before January is not given here. Hence, we will take the index number for January, 2014 as 100. The calculation of index numbers for remaining months using the chain base method are shown in the following table.

Month	Price of share (₹)	Chain base index number $= \frac{\text{Value of the variable in current month}}{\text{Value of the variable in preceding month}} \times 100$
January	22.00	= 100
March	21.20	$\frac{21.20}{22.00} \times 100$ = 96.36
May	22.00	$\frac{22.00}{21.20} \times 100$ = 103.77
July	23.00	$\frac{23.00}{22.00} \times 100$ = 104.55
September	24.70	$\frac{24.70}{23.00} \times 100$ = 107.39
November	26.00	$\frac{26.00}{24.70} \times 100$ = 105.26

Merits and limitations of chain base method

Merits : (1) The problem of selecting the base year does not arise in this method because at any given time the preceding year (period) is taken as the base year (period).

(2) As the comparison is with the preceding year, new items can be included according to the taste and choice of the consumers. It is possible to remove the items not in use.

(3) This method is useful in the fields of economics, trade and commerce as the value of the variable in the current period is compared with the period in the recent past.

Limitations : (1) This method is suitable only for short term comparison of the value of the variable in the current period as the preceding year is taken as the base year. The method is not very convenient for long term comparison.

(2) If there is an error in the calculation of index number by this method then the effect of that error continues in the interpretation of the index number of the succeeding year.

(3) There is no uniformity in the computation of index numbers obtained by this method.

(4) If the information for a year is not available then the index number for the next year can not be obtained.

Illustration 5 : The data about the purchase of groundnut by an edible oil mill from the year 2008 to 2015 are as follows. Prepare the index numbers by fixed base method with the year 2008 as the base year, with chain base and by taking the average quantity purchased in the year 2010 and 2011 as the purchase for the base year.

Year	2008	2009	2010	2011	2012	2013	2014	2015
Purchase of groundnut (ton)	230	250	230	250	270	280	300	300

Year	Quantity Purchase of groundnut (ton)	Index number with base year 2008 $= \frac{\text{Value of variable in current year}}{\text{Value of variable in base year}} \times 100$	Chain base Index number $= \frac{\text{Value of variable in current year}}{\text{Value of variable in preceding year}} \times 100$	Index number by taking average of quantity in year 2010 and 2011 $= \frac{230 + 250}{2} = 240$ as base year quantity
2008	230	$= 100$	$= 100$	$\frac{230}{240} \times 100 = 95.83$
2009	250	$\frac{250}{230} \times 100 = 108.70$	$\frac{250}{230} \times 100 = 108.70$	$\frac{250}{240} \times 100 = 104.17$
2010	230	$\frac{230}{230} \times 100 = 100$	$\frac{230}{250} \times 100 = 92.00$	$\frac{230}{240} \times 100 = 95.83$
2011	250	$\frac{250}{230} \times 100 = 108.70$	$\frac{250}{230} \times 100 = 108.70$	$\frac{250}{240} \times 100 = 104.17$
2012	270	$\frac{270}{230} \times 100 = 117.39$	$\frac{270}{250} \times 100 = 108$	$\frac{270}{240} \times 100 = 112.5$
2013	280	$\frac{280}{230} \times 100 = 121.74$	$\frac{280}{270} \times 100 = 103.70$	$\frac{280}{240} \times 100 = 116.67$
2014	300	$\frac{300}{230} \times 100 = 130.43$	$\frac{300}{280} \times 100 = 107.14$	$\frac{300}{240} \times 100 = 125$
2015	300	$\frac{300}{230} \times 100 = 130.43$	$\frac{300}{300} \times 100 = 100$	$\frac{300}{240} \times 100 = 125$

Illustration 6 : The data about sale of three grain flour wheat, bajri and chana at a flour mill from the year 2011 to 2015 are as follows. Compute the general index number using simple average with (i) fixed base method (taking base year 2011) and (ii) Chain base method.

Year → / Grain flour	Sale (lakh ₹)				
	2011	2012	2013	2014	2015
Wheat flour	40	46	50	56	64
Bajri flour	20	30	36	42	54
Chana flour	50	64	80	96	112

(i) Fixed base method

$$\text{Fixed base index number } I = \frac{\text{Value of variable in current year (period)}}{\text{Value of variable in base year (period)}} \times 100$$

Year / Grain flour	2011	2012	2013	2014	2015
Wheat flour	100	$\frac{46}{40} \times 100 = 115$	$\frac{50}{40} \times 100 = 125$	$\frac{56}{40} \times 100 = 140$	$\frac{64}{40} \times 100 = 160$
Bajri flour	100	$\frac{30}{20} \times 100 = 150$	$\frac{36}{20} \times 100 = 180$	$\frac{42}{20} \times 100 = 210$	$\frac{54}{20} \times 100 = 270$
Chana flour	100	$\frac{64}{50} \times 100 = 128$	$\frac{80}{50} \times 100 = 160$	$\frac{96}{50} \times 100 = 192$	$\frac{112}{50} \times 100 = 224$
Total	300	393	465	542	654
General index number of sale $= \frac{\text{Total}}{3}$	$\frac{300}{3}$ $= 100$	$\frac{393}{3}$ $= 131$	$\frac{465}{3}$ $= 155$	$\frac{542}{3}$ $= 180.67$	$\frac{654}{3}$ $= 218$

(ii) General index number by chain base method :

$$\text{Chain base index number } I = \frac{\text{Value of variable in current year (period)}}{\text{Value of variable in preceding year (period)}} \times 100$$

Year / Grain flour	2011	2012	2013	2014	2015
Wheat flour	100	$\frac{46}{40} \times 100 = 115$	$\frac{50}{46} \times 100 = 108.70$	$\frac{56}{50} \times 100 = 112$	$\frac{64}{56} \times 100 = 114.29$
Bajri flour	100	$\frac{30}{20} \times 100 = 150$	$\frac{36}{30} \times 100 = 120$	$\frac{42}{36} \times 100 = 116.67$	$\frac{54}{42} \times 100 = 128.57$
Chana flour	100	$\frac{64}{50} \times 100 = 128$	$\frac{80}{64} \times 100 = 125$	$\frac{96}{80} \times 100 = 120$	$\frac{112}{96} \times 100 = 116.67$
Total	300	393	353.7	348.67	359.53
Aggregate index number $= \frac{\text{Total}}{3}$	$\frac{300}{3}$ $= 100$	$\frac{393}{3}$ $= 131$	$\frac{353.7}{3}$ $= 117.90$	$\frac{348.67}{3}$ $= 116.22$	$\frac{359.53}{3}$ $= 119.84$

Illustration 7 : The following data are available about the crimes in a city. Find the general index number by fixed base method considering the year 2010 as base year.

Type of Crime \ Year	2007	2008	2009	2010
Murder	110	128	134	129
Violence and rape	30	45	40	48
Robbery	610	720	770	830
Theft of property	2450	2630	2910	2890

$$\text{Fixed base index number } I = \frac{\text{Value of variable in current year (period)}}{\text{Value of variable in base year (period)}} \times 100$$

Type of Crime \ Year	2007	2008	2009	2010
Murder	$\frac{110}{129} \times 100 = 85.27$	$\frac{128}{129} \times 100 = 99.22$	$\frac{134}{129} \times 100 = 103.88$	$\frac{129}{129} \times 100 = 100$
Violence and rape	$\frac{30}{48} \times 100 = 62.5$	$\frac{45}{48} \times 100 = 93.75$	$\frac{40}{48} \times 100 = 83.33$	$\frac{48}{48} \times 100 = 100$
Robbery	$\frac{610}{830} \times 100 = 73.49$	$\frac{720}{830} \times 100 = 86.75$	$\frac{770}{830} \times 100 = 92.77$	$\frac{830}{830} \times 100 = 100$
Theft of property	$\frac{2450}{2890} \times 100 = 84.78$	$\frac{2630}{2890} \times 100 = 91.00$	$\frac{2910}{2890} \times 100 = 100.69$	$\frac{2890}{2890} \times 100 = 100$
Total	306.04	370.72	380.67	400
General Index number of crime $= \frac{\text{Total}}{4}$	$\frac{306.04}{4} = 76.51$	$\frac{370.72}{4} = 92.68$	$\frac{380.67}{4} = 95.17$	$\frac{400}{4} = 100$

EXERCISE 1.1

1. The data about average daily wage of a group of workers employed in a factory in a city during the year 2008 to 2015 are as follows. Find the index number by (1) Fixed base method (taking base year 2008) (2) Chain base method (3) Fixed base method by taking average of average daily wages of the years 2011 to 2013 as the wage for the base year.

Year	2008	2009	2010	2011	2012	2013	2014	2015
Average daily wage (₹)	275	284	289	293	297	313	328	345

2. From the following data about the retail prices of sugar in a city, find the index numbers of price of sugar by (1) Fixed base method with year 2008 as base year (2) Chain base method (3) taking the average price of sugar for the year 2009 and 2010 as the base year price.

Year	2008	2009	2010	2011	2012	2013	2014	2015
Price of Sugar per kilogram (₹)	28	28.50	29.50	30	31	32	34	36

3. The following data are obtained about the annual average prices of wheat, rice and sugar in the wholesale market of a city. Find the general index number for three items by fixed base method with base year 2011 and by chain base method.

Item \ **Year**	2011	2012	2013	2014	2015
Wheat	18	18.50	18.90	19	19.50
Rice	30	36	38	38	39
Sugar	30	31	32	34	36

4. The prices of five fuel related items in the years 2012 and 2014 are as follows. Calculate the general index number for five fuel items by taking the year 2012 as the base year and state the overall increase in the prices of fuel items.

Item	Electricity	Gas	Match Box	Kerosene	Wood
Unit	Unit	Cylinder	Box	Litre	Kilogram
Price in 2012 (₹)	3	345	1.00	15	12
Price in 2014 (₹)	3.5	370	1.50	20	15

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1.5 Conversion from Fixed Base to Chain Base and from Chain Base to Fixed Base

Generally, whenever the fixed base or chain base index numbers only are available instead of the original information about the values of the variable, the conversion of base is necessary for the following reasons. If the need arises to find the short term changes in the values of the variable then it becomes difficult to find it from the fixed base index numbers. It is easier to find the short term variations after converting the given fixed base index numbers into chain base index numbers.

Sometimes, it is necessary to compare the value of the variable at a given period to the value of another period in a series of values of the variable. This is not possible if only chain base index numbers are available. The above comparison is possible in this situation if the chain base index numbers are converted to the fixed base index numbers. Thus, it is necessary to convert the chain base index numbers into the fixed base index numbers. Hence, the base conversion is carried out as follows :

Conversion of the fixed base index number to the chain base index numbers : The formula for the conversion of the fixed base index numbers into the chain base index numbers is as follows.

$$\text{Chain base index number} = \frac{\text{Fixed base index number of current year}}{\text{Fixed base index number of preceding year}} \times 100$$

Note : If the base year is not mentioned then we will take the chain base index number for the first year as 100. If the base year is mentioned then the fixed base index number of the first year will be taken as its chain base index number.

Illustration 8 : Convert the following index numbers obtained by fixed base method about the production of craft industry of a state into the chain base index numbers.

Year	2009	2010	2011	2012	2013	2014
Fixed base index numbers	120	132	96	144	138	108

Since the base year is not mentioned here, we will take 100 as the chain base index number for the first year.

$$\text{Chain base index number} = \frac{\text{Fixed base index number of current year}}{\text{Fixed base index number of preceding year}} \times 100$$

Year	**Index number**	**Chain base index number**
2009	120	= 100
2010	132	$\frac{132}{120} \times 100$ = 110
2011	96	$\frac{96}{132} \times 100$ = 72.73
2012	144	$\frac{144}{96} \times 100$ = 150
2013	138	$\frac{138}{144} \times 100$ = 95.83
2014	108	$\frac{108}{138} \times 100$ = 78.26

Illustration 9 : The wholesale price index numbers for commodities with the base year 2007-08 are as follows. Compute the chain base index numbers.

Year	2008-09	2009-10	2010-11	2011-12	2012-13	2013-14	2014-15	2015-16
Wholesale price index number	126	130.8	143.3	156.1	167.6	177.6	181.2	177.2

The base year 2007-08 is mentioned here. Hence, we will take the given fixed base index number for the year 2008-09 as chain base index number. Thus, the chain base index number for the first year is 126.

$$\text{Chain base index number} = \frac{\text{Fixed base index number of the current year}}{\text{Fixed base index number of the preceding year}} \times 100$$

Year	Wholesale price index number of commodities	Chain base index number
2008-09	126	= 126
2009-10	130.8	$\frac{130.8}{126} \times 100$ = 103.81
2010-11	143.3	$\frac{143.3}{130.8} \times 100$ = 109.56
2011-12	156.1	$\frac{156.1}{143.3} \times 100$ = 108.93
2012-13	167.6	$\frac{167.6}{156.1} \times 100$ = 107.37
2013-14	177.6	$\frac{177.6}{167.6} \times 100$ = 105.97
2014-15	181.2	$\frac{181.2}{177.6} \times 100$ = 102.03
2015-16	177.2	$\frac{177.6}{181.2} \times 100$ = 97.79

Conversion of chain base index numbers to fixed base index number : If the year-wise chain base index numbers are given, the fixed base index number can be found accordingly. To obtain the fixed base index numbers, the chain base index number of that year is multiplied by the fixed base index number of the previous year and the product is divided by 100.

$$\text{Thus, Fixed base index number of current year} = \frac{\left(\begin{array}{c}\text{Chain base index number}\\ \text{of the current year}\end{array}\right) \times \left(\begin{array}{c}\text{Fixed base index number of the}\\ \text{preceding year to current year}\end{array}\right)}{100}$$

Let us understand this method with an illustration.

Illustration 10 : The chain base index numbers obtained for food items from the year 2008-09 to 2015-16 are as follows. Compute the fixed base index numbers. (Take 2007-08 as base year)

Year	2008 – 09	2009 – 10	2010 – 11	2011 – 12	2012 – 13	2013 – 14	2014 – 15	2015 – 16
Index number of food items	134.8	115.28	115.57	107.29	109.91	112.80	106.24	102.48

The year 2007-08 is to be taken as the base year here. Hence, the fixed base index number for the year 2008-09 will not change.

$$\text{Current year fixed base index number} = \frac{\left(\begin{matrix}\text{Current year chain base}\\ \text{index number}\end{matrix}\right) \times \left(\begin{matrix}\text{Fixed base index number of}\\ \text{preceding year to current year}\end{matrix}\right)}{100}$$

Year	**Index number of food items**	**Fixed base index number**
2008-09	134.8	= 134.8
2009-10	115.28	$\frac{115.28 \times 134.8}{100}$ = 155.40
2010-11	115.57	$\frac{115.57 \times 155.40}{100}$ = 179.60
2011-12	107.29	$\frac{107.29 \times 179.60}{100}$ = 192.69
2012-13	109.91	$\frac{109.91 \times 192.69}{100}$ = 211.79
2013-14	112.80	$\frac{112.80 \times 211.79}{100}$ = 238.9
2014-15	106.24	$\frac{106.24 \times 238.9}{100}$ = 253.81
2015-16	102.48	$\frac{102.48 \times 253.81}{100}$ = 260.10

EXERCISE 1.2

1. The chain base index numbers of agricultural production of a state from the year 2008 to 2014 are as follows. Compute the fixed base index numbers. (Take 2007 as base year.)

Year	2008	2009	2010	2011	2012	2013	2014
Index number of agricultural production	100	110	95	108	120	106	110

2. Obtain the chain base index number from the fixed base index numbers given below with the year 2007-08 as the base year for the wholesale prices of machines and equipments.

Year	2008 – 09	2009 – 10	2010 – 11	2011 – 12	2012 – 13	2013 – 14	2014 – 15
Index number of machines and equipments	117.4	118	121.3	125.1	128.4	131.6	134.6

3. The fixed base index numbers of food from the month of January to October in the year 2015 for the industrial workers of Ahmedabad are as given below. Compute the chain base index numbers.

Month	January	February	March	April	May	June	July	August	September	October
Index number of food	271	270	268	268	278	283	283	293	293	299

4. The chain base index numbers for sales of a certain type of scooter from the year 2010 to 2015 are as follows. Find fixed base index numbers.

Year	2010	2011	2012	2013	2014	2015
Index number of sale	110	112	109	108	105	111

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1.6 Specific Formulae for Computing Index Number

We have seen that the index number is useful to study the changes in the values of variable for an item or the values of variables for items in a group. Simple average is used in the construction of an index number and every item is given equal weightage. But the importance of every item is generally not same in practice. For example, the importance given to grains is not same as the importance given to vegetables, pulses or edible oil. Thus, the index number of food will be more realistic and meaningful if each item is assigned weight according to its importance.

The weights of items included are decided in the construction of different types of index numbers. Generally, the weights given to the items for constructing the index number are determined on the basis of their quantity consumed. We shall study some specific formulae for computing index numbers by taking this fact into consideration where different methods of selecting weights are taken for the construction of index number.

Method of weighted average : Suppose I_i is the index number of the ith group among the groups of items (or items) wheat, rice and pulses with the corresponding weight W_i, then the general index number of these groups is obtained using the following formula.

General index number $I = \frac{\Sigma I_i W_i}{\Sigma W_i} = \frac{\Sigma IW}{\Sigma W}$

Note : We shall ignore the suffix 'i' for the simplicity of calculation.

For example, if the index numbers of these groups are 120, 150 and 300 respectively and their corresponding weights are 3, 2 and 1 then the general index number for the group of items is

$$I = \frac{\Sigma IW}{\Sigma W}$$

$$= \frac{120 \times 3 + 150 \times 2 + 300 \times 1}{3 + 2 + 1}$$

$$= \frac{360 + 300 + 300}{6}$$

$$= \frac{960}{6}$$

$$= 160$$

Laspeyre's Formula

This method of finding the index number is given by Laspeyere. It is one of the important methods of finding index number. In this method, base year price is denoted by p_0 and the quantity is denoted by q_0 whereas the prices of items in the current year are denoted by p_1. The expenditure $p_0 q_0$ is assigned as weight to the price relative $\frac{p_1}{p_0}$. The formula of weighted index number thus obtained is called the formula of **Laspeyre's index**, which is denoted by I_L. The Laspeyre's formula is as follows :

$$\textbf{Laspeyre's index number } I_L = \frac{\Sigma\left[\frac{p_1}{p_0}\right] \times p_0 q_0}{\Sigma p_0 q_0} \times 100$$

$$= \frac{\Sigma \frac{p_1}{p_0} \times p_0 q_0}{\Sigma p_0 q_0} \times 100$$

$$\therefore \quad I_L = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

Paasche's Formula

This method is given by an economist named Paasche. If we denote p_0 as base year price, p_1 as current year price and q_1 as current year quantity then the expenditure $p_0 q_1$ is assigned as weight for the price relative $\frac{p_1}{p_0}$. The formula of weighted index number thus obtained is called the formula of **Paasche's index number**, which is denoted by I_P. The formula of Paasche's index number is as follows :

$$\textbf{Paasche's index number } I_P = \frac{\Sigma\left[\frac{p_1}{p_0}\right] \times p_0 q_1}{\Sigma p_0 q_1} \times 100$$

$$= \frac{\Sigma \frac{p_1}{p_0} \times p_0 q_1}{\Sigma p_0 q_1} \times 100$$

$$\therefore \quad I_P = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

Fisher's Formula

The base year and current year quantities are taken into condsideration for computing the weight in Laspeyre's and Paasche's method respectively. Prof. Irving Fisher has constructed an index number by considering quantities of both the years. The geometric mean of Laspeyre's and Paasche's index numbers is called **Fisher's index number**, which is denoted by I_F. The formula of Fisher's index number is as follows :

Fisher's index number $I_F = \sqrt{I_L \times I_P}$ or

Fisher's index number $I_F = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$

The Fisher's index number is called ideal index number due to the following reasons :

(1) The quantities of both the years, base year and current year, are taken in the computation for constructing this index number.

(2) This index number satisfies both the important fundamental tests, time reversal and factor reversal tests, of index numbers.

(3) The geometric mean is used to calculate this index number which is the best average for the construction of index number.

(4) This index number is free from bias as it balances the demerits of Laspeyre's and Paasche's index number.

Thus, Fisher's index number is an ideal index number.

Illustration 11 : Find the index number for the year 2016 with base year 2011 by weighted average method from the following data of price and weights of five different items.

Item	Weight	Price (₹)	
		Year 2011	Year 2016
A	40	160	200
B	25	400	600
C	5	50	70
D	20	10	18
E	10	2	3

The weights of different items are given here. We shall compute the general index number from the price relatives of the year 2016 based on the prices of the year 2011.