Chapter 4 Matrices and Determinants

Ex 4.2

Answer 1e.

The graph of a quadratic function of the form $y = a(x-h)^2 + k$ has its vertex at (h, k). We can easily identify the vertex of a graph if its equation is in this form.

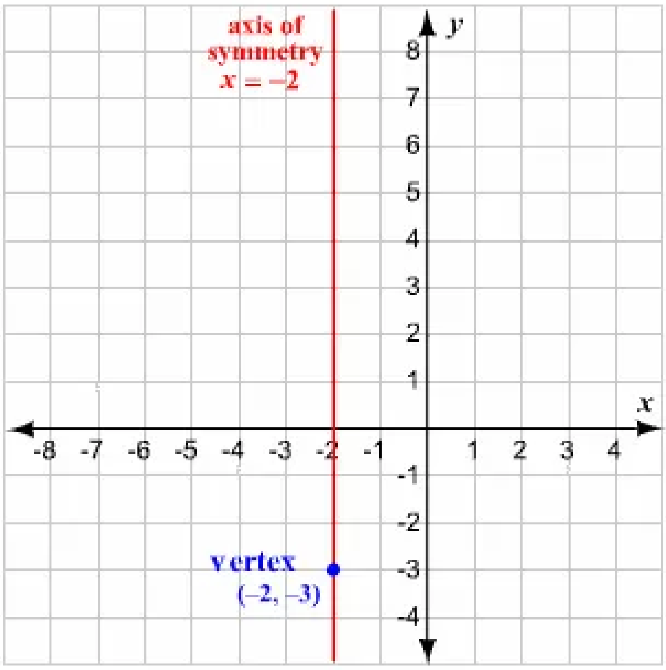
Thus, a quadratic function in the form $y = a(x-h)^2 + k$ is in vertex form.

Answer 1gp.

STEP 1 The graph of a quadratic function in vertex form $y = a(x-h)^2 + k$ has its vertex at (h, k) and $x = h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.
On comparing the given equation with the vertex form, we find that $a = 1$, $h = -2$, and $k = -3$. Thus, the vertex is $(h, k) = (-2, -3)$ and the axis of symmetry is $x = -2$.
Since $a > 0$, the parabola opens up.

STEP 2 Plot the vertex $(-2, -3)$ on a coordinate plane and draw the axis of symmetry $x = -2$.



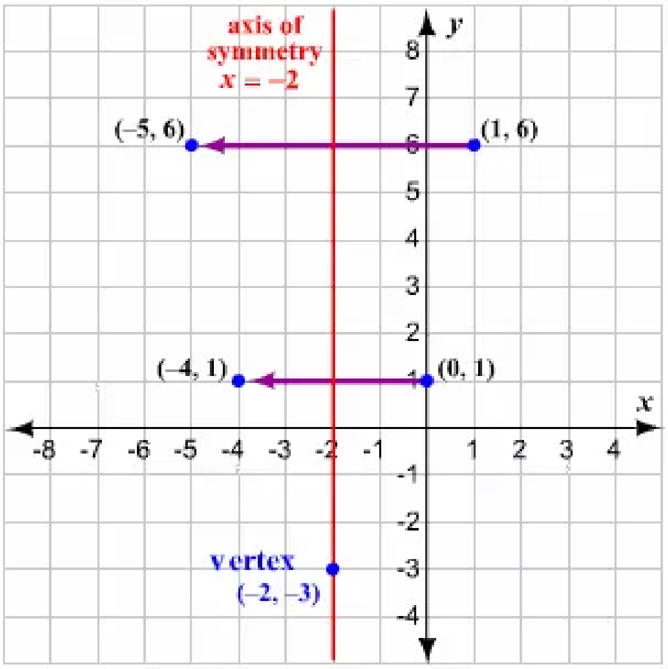
STEP 3 Evaluate the function for two values of x.

$$x = 0: \quad y = (0+2)^2 - 3 = 1$$

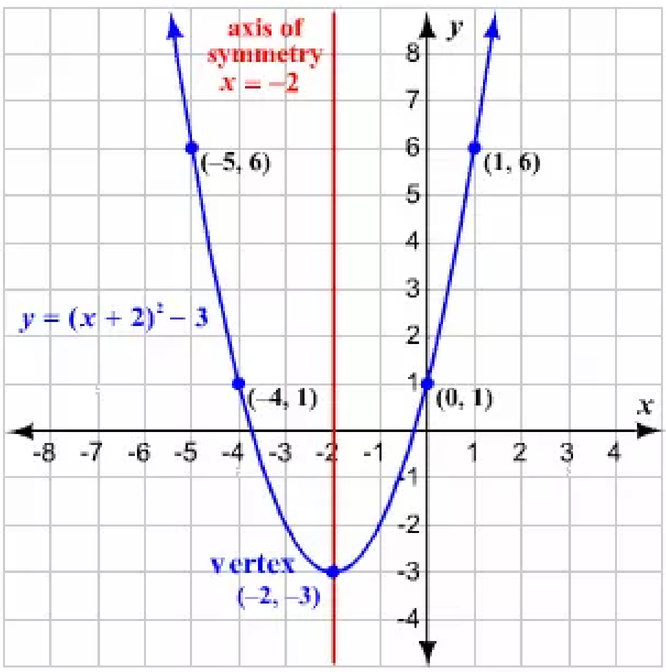
$$x = 1: \quad y = (1+2)^2 - 3 = 6$$

Thus, (0, 1) and (1, 6) are two points on the graph.

Now, plot the points (0, 1) and (1, 6) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.



Answer 2e.

If a quadratic function is given in intercept form.

that is $y = a(x-p)(x-q)$

We need to identify a as positive or negative, accordingly the given function will have minimum value or maximum value respectively.

In order to find the minimum value or maximum value as the case may be, we need to find the coordinates of the vertex.

Because, abscissa of the vertex is the point of minimum value or maximum value and the ordinate of the vertex is the minimum value or maximum value respectively.

Answer 2gp.

Consider the function

$y = -(x-1)^2 + 5$

It is need to graph the function and label the vertex and axes of symmetry.

Recall the concept that, the graph of $y = a(x-h)^2 + k$ has vertex at (h,k) and axis of symmetry is $x = h$ and the graph of opens up if $a > 0$ and down if $a < 0$.

By comparing the equation $y = -(x-1)^2 + 5$ with $y = a(x-h)^2 + k$, we get

$a = -1, h = 1$ and $k = 5$

Step-1:

The constants are $a = -1, h = 1$ and $k = 5$.

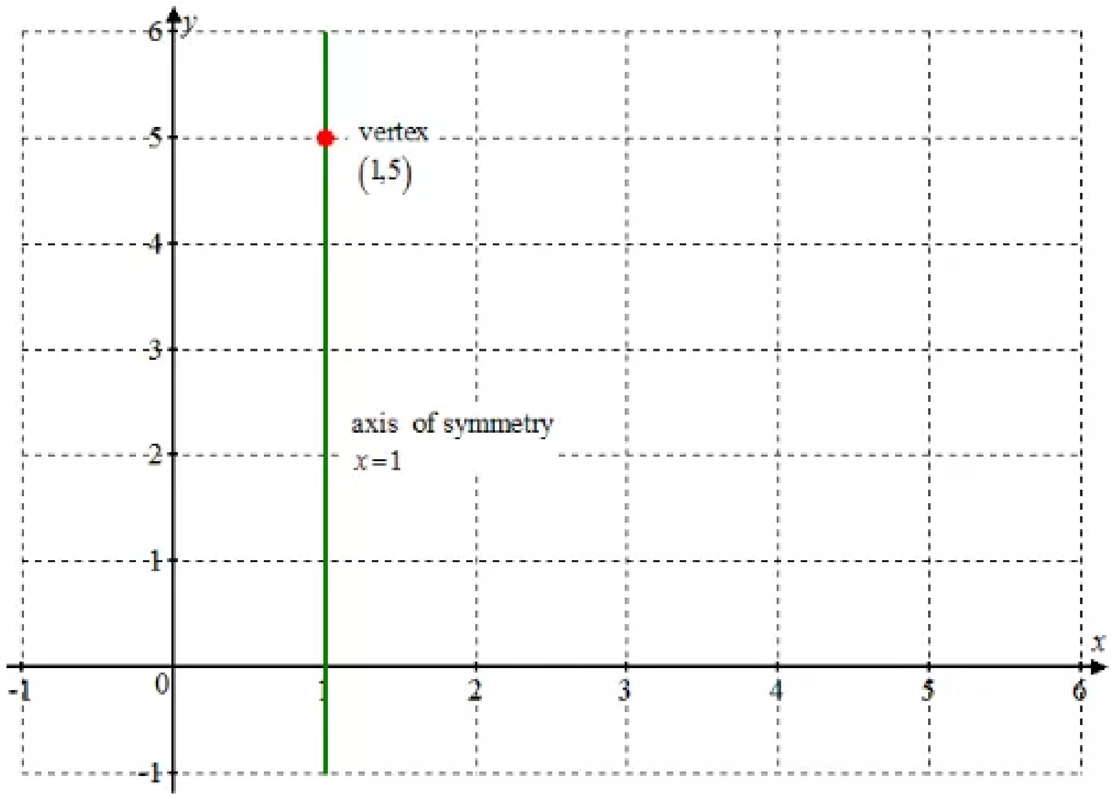
Because $a = -1 < 0$, the parabola opens down

Step-2:

Plot the vertex $(h,k) = (1,5)$ and draw the axis of symmetry

$x = h$

$= 1$

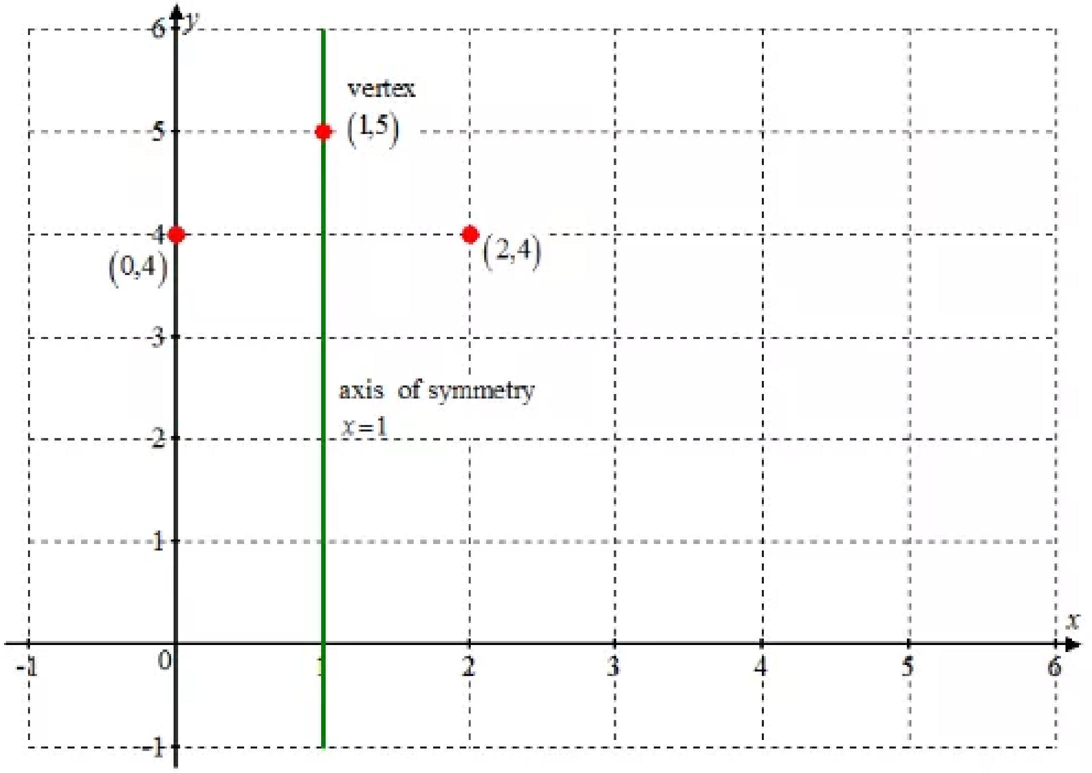


Step-3:
Evaluate the function for two values of x.

$$x=0: \quad y=-(0-1)^2+5$$
$$=-1+5$$
$$=4$$

$$x=2: \quad y=-(2-1)^2+5$$
$$=-1+5$$
$$=4$$

Plot the points $(0,4)$ and $(2,4)$ on the graph.

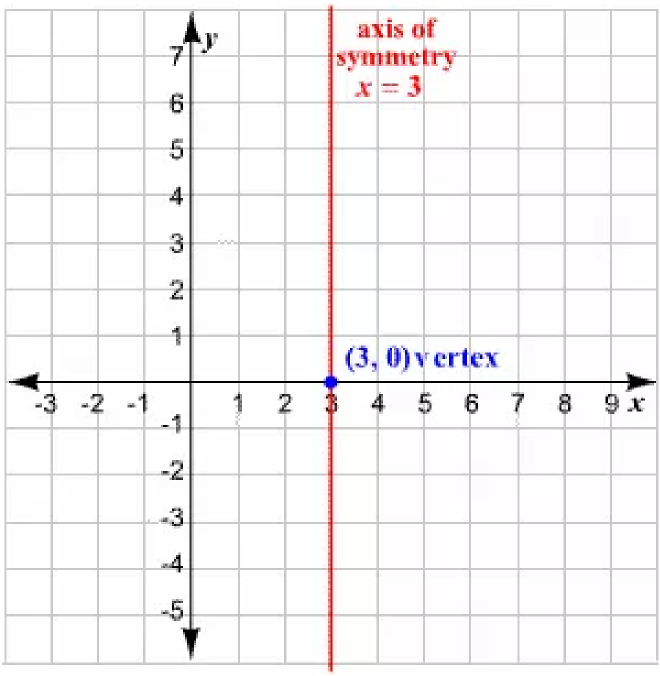


Answer 3e.

STEP 1 The graph of a quadratic function in vertex form $y=a(x-h)^2+k$ has its vertex at (h, k) and $x=h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.
On comparing the given equation with the vertex form, we find that $a=1$, $h=3$, and $k=0$. Thus, the vertex is $(h, k)=(3, 0)$ and the axis of symmetry is $x=3$
Since $a>0$, the parabola opens up.

STEP 2 Plot the vertex (3, 0) on a coordinate plane and draw the axis of symmetry $x = 3$.



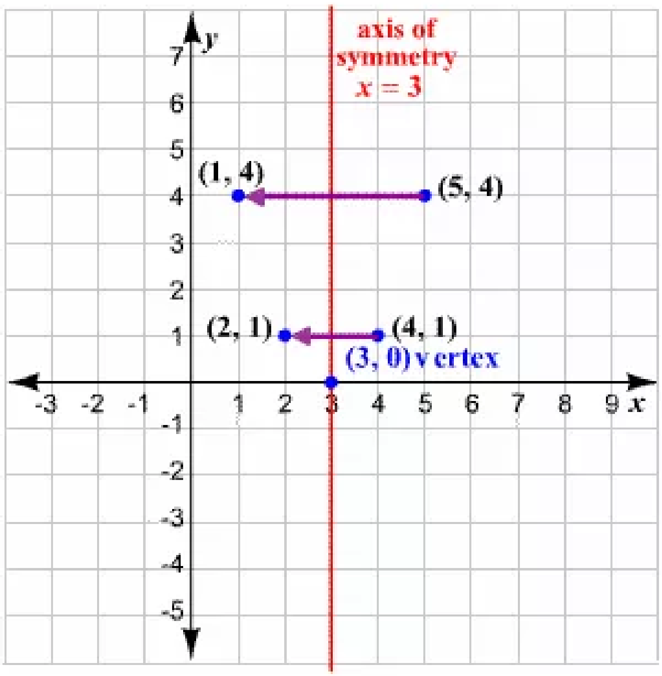
STEP 3 Evaluate the function for two values of x.

$x = 4: \quad y = (4-3)^2 = 1$

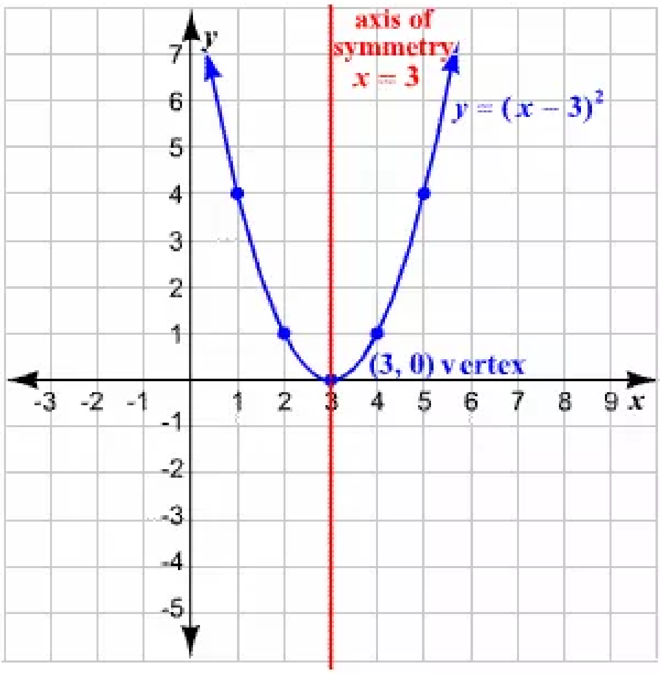
$x = 5: \quad y = (5-3)^2 = 4$

Thus, (4, 1) and (5, 4) are two points on the graph.

Now, plot the points (4, 1) and (5, 4) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.

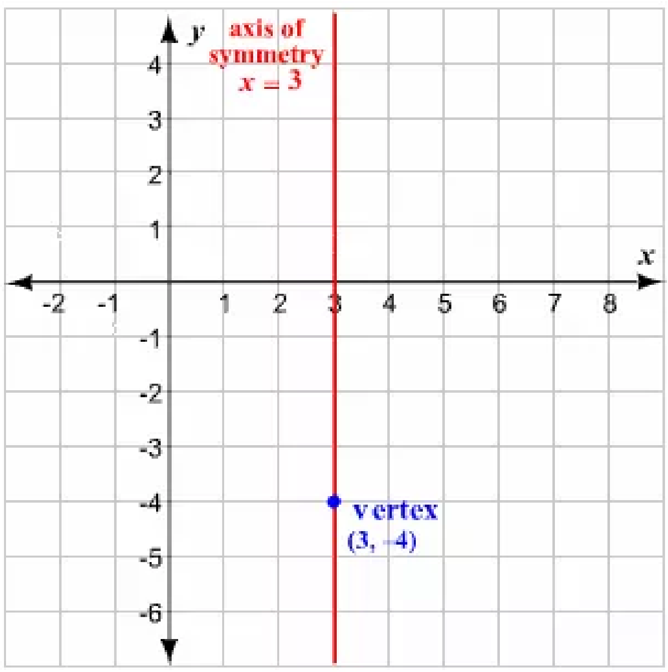


Answer 3gp.

STEP 1 The graph of a quadratic function in vertex form $y = a(x - h)^2 + k$ has its vertex at (h, k) and $x = h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.
On comparing the given equation with the vertex form, we find that $a = \frac{1}{2}$, $h = 3$, and $k = -4$. Thus, the vertex is $(h, k) = (3, -4)$ and the axis of symmetry is $x = 3$.
Since $a > 0$, the parabola opens up.

STEP 2 Plot the vertex $(3, -4)$ on a coordinate plane and draw the axis of symmetry $x = 3$.



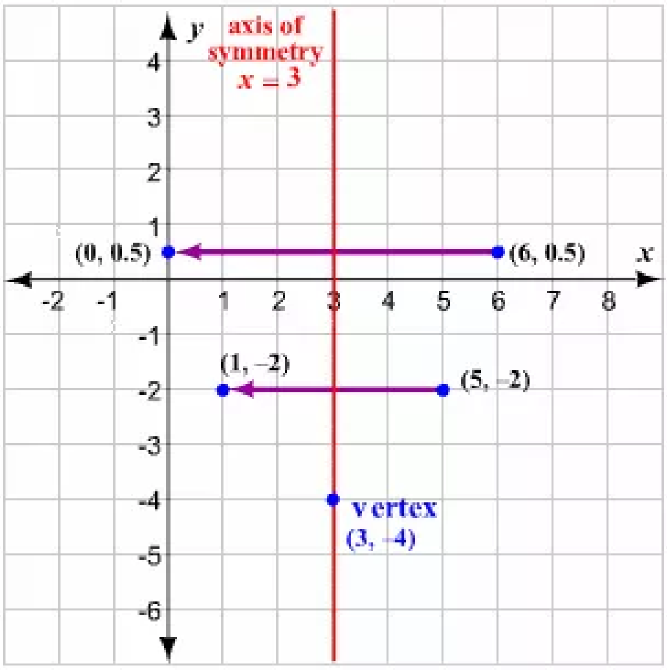
STEP 3 Evaluate the function for two values of x.

$$x = 5: \quad y = \frac{1}{2}(5-3)^2 - 4 = -2$$

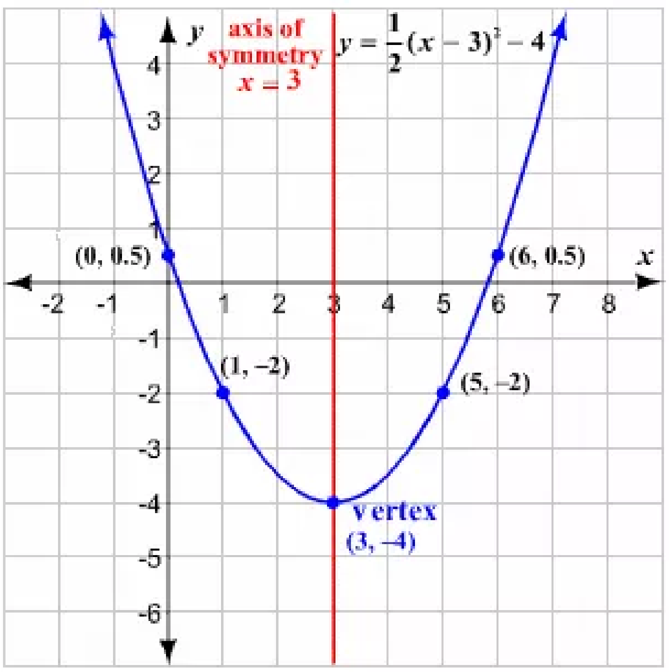
$$x = 6: \quad y = \frac{1}{2}(6-3)^2 - 4 = 0.5$$

Thus, (5, −2) and (6, 0.5) are two points on the graph.

Now, plot the points (5, −2) and (6, 0.5) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.



Answer 4e.

Consider the function

$y=(x+4)^2$

It is need to graph the function and label the vertex and axes of symmetry.

Recall the concept that, the graph of $y=a(x-h)^2+k$ has vertex at (h,k) and axis of symmetry is $x=h$ and the graph of opens up if $a>0$ and down if $a<0$.

By comparing the equation $y=(x-(-4))^2$ with $y=a(x-h)^2+k$, we get

$a=1, h=-4$ and $k=0$

Step-1:

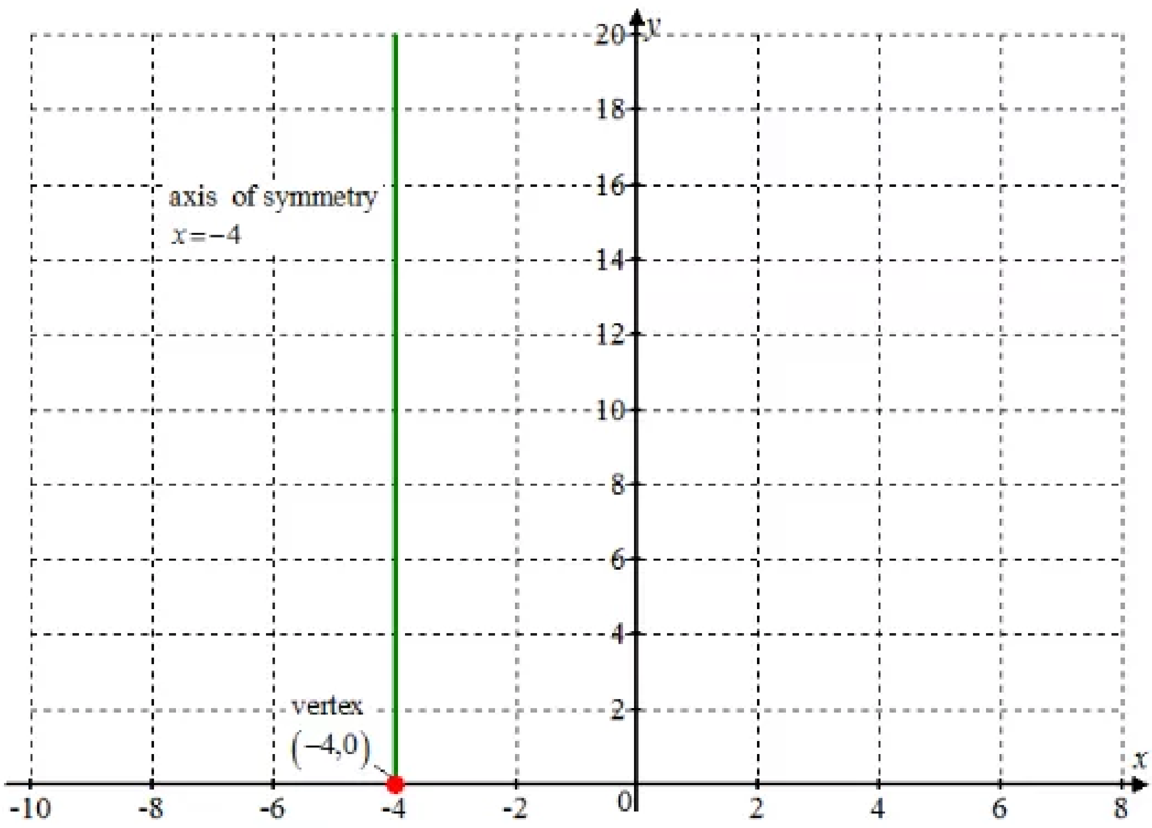
The constants are $a=1, h=-4$ and $k=0$

Because $a=1>0$, the parabola opens upward.

Step-2:

Plot the vertex $(h,k)=(-4,0)$ and draw the axis of symmetry

$$x=h$$
$$=-4$$

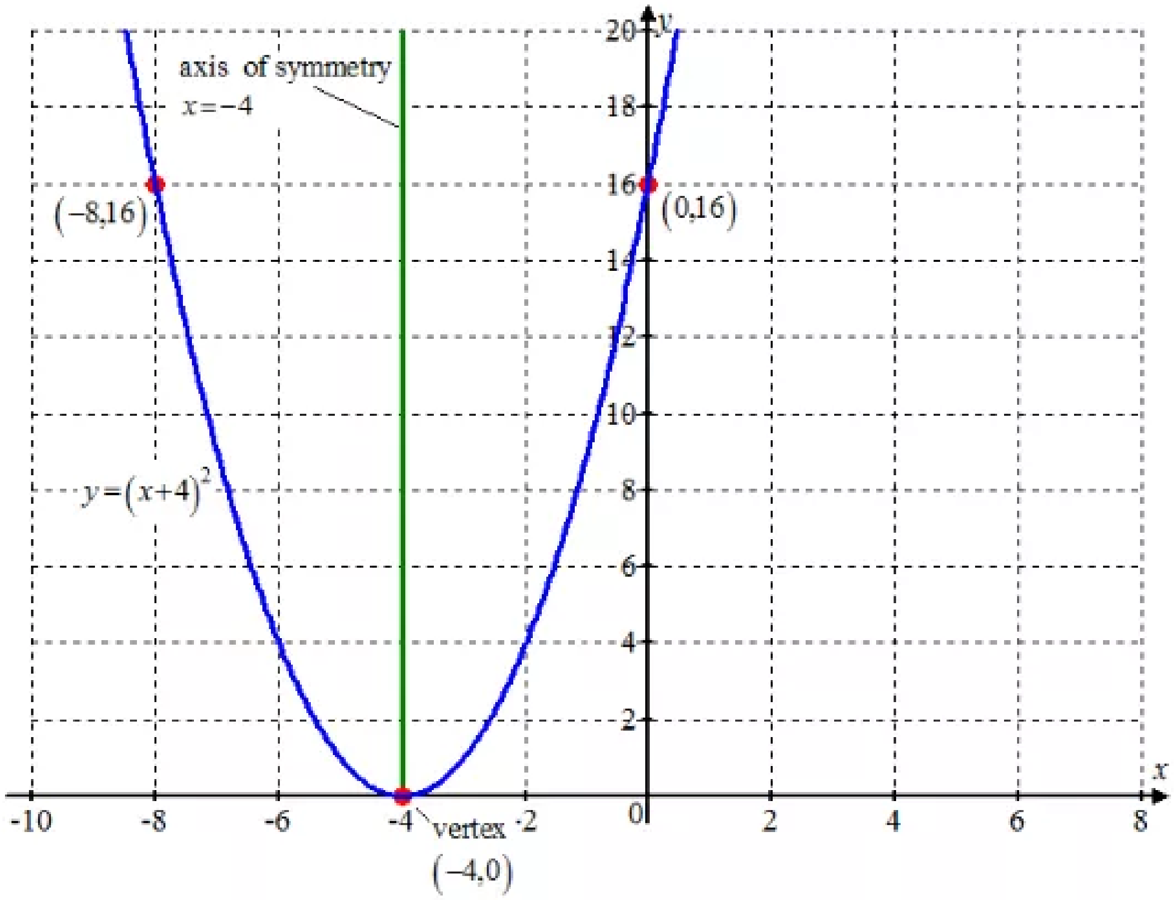


Step-3:
Evaluate the function for two values of x.

$x=0$: $y=(0+4)^2$
$=16$

$x=-8$: $y=(-8+4)^2$
$=16$

Plot the points $(0,16)$ and $(-8,16)$ on the graph.

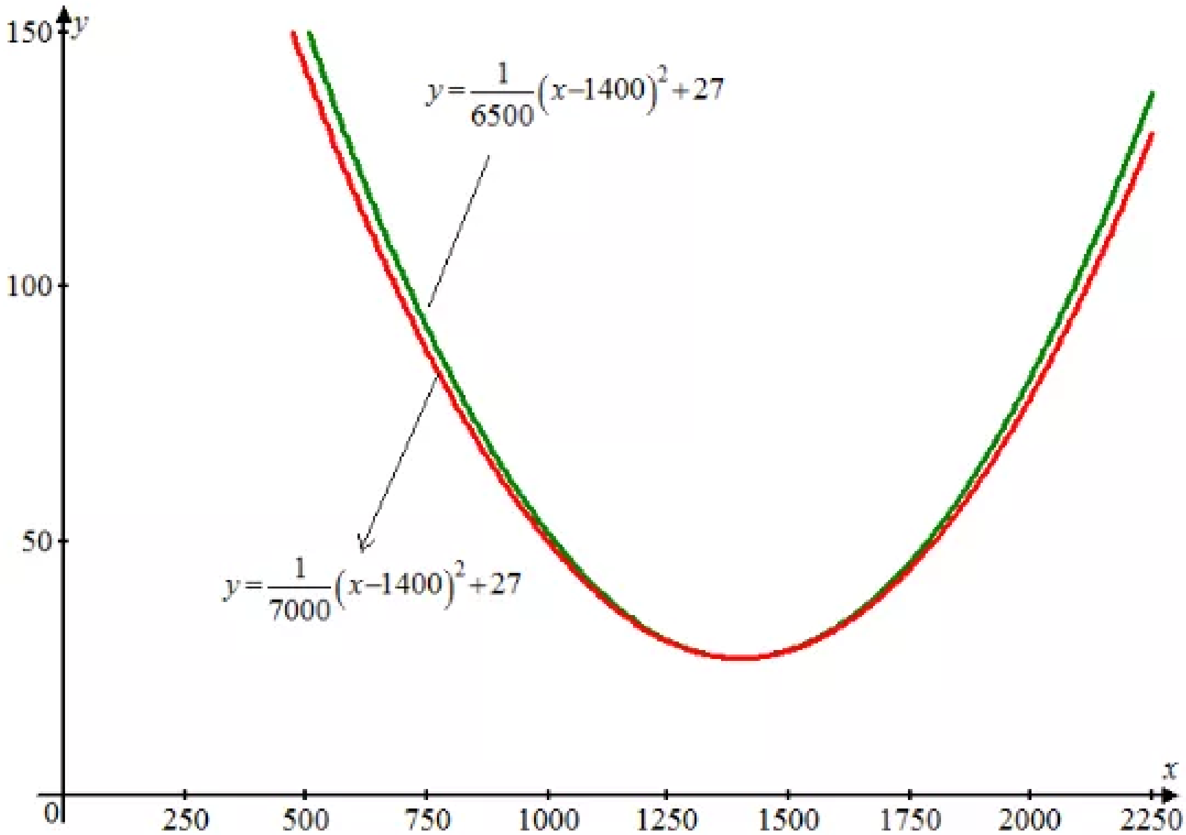


Answer 4gp.

Suppose that an architect designs a bridge with cables that can be modeled by $y=\frac{1}{6500}(x-1400)^2+27$ where x and y are measured in feet.

We need to compare the graph of the above function with the graph of $y=\frac{1}{7000}(x-1400)^2+27$

The following diagram contains the graphs of the functions $y = \frac{1}{6500}(x-1400)^2 + 27$ and $y = \frac{1}{7000}(x-1400)^2 + 27$.



Answer 5e.

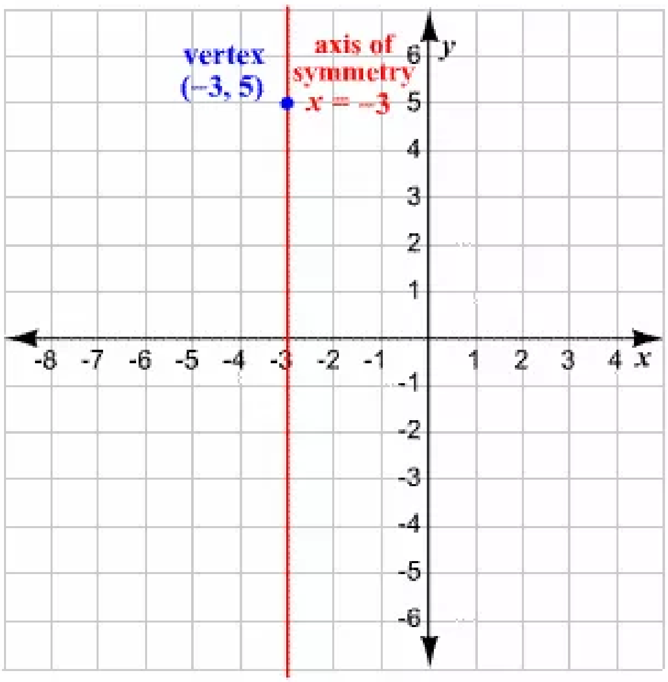
STEP 1 The graph of a quadratic function in vertex form $y = a(x-h)^2 + k$ has its vertex at (h, k) and $x = h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.

On comparing the given equation with the vertex form, we find that $a = -1$, $h = -3$, and $k = 5$. Thus, the vertex is $(h, k) = (-3, 5)$ and the axis of symmetry is $x = -3$.

Since $a < 0$, the parabola opens down.

STEP 2 Plot the vertex (–3, 5) on a coordinate plane and draw the axis of symmetry $x = -3$.



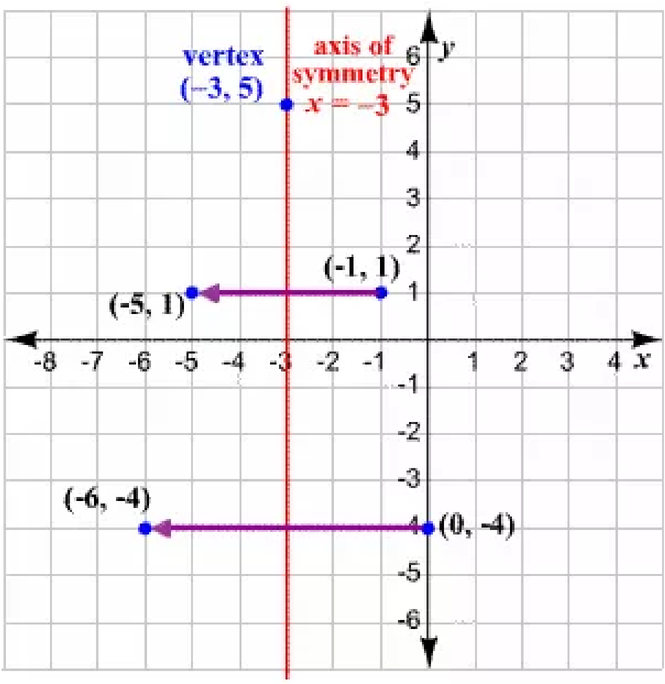
STEP 3 Evaluate the function for two values of x.

$$x = 0: \quad y = -(0+3)^2 + 5 = -4$$

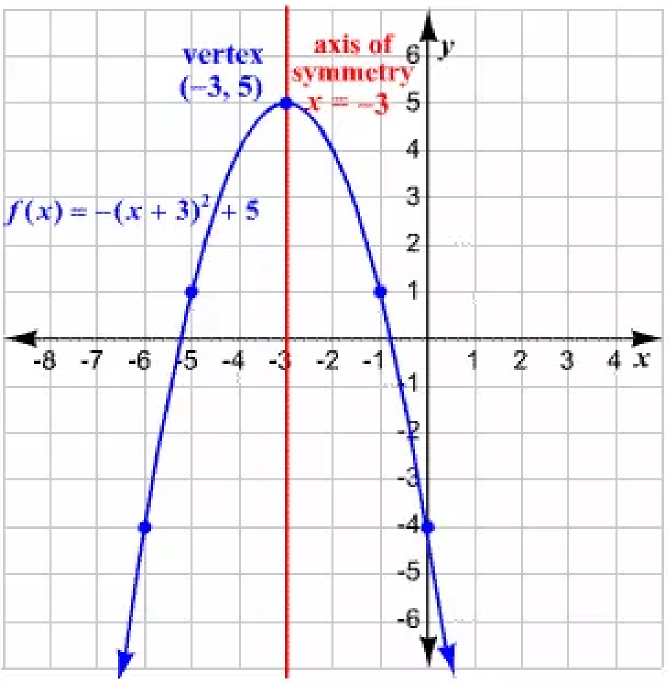
$$x = -1: \quad y = -(-1+3)^2 + 5 = 1$$

Thus, (0, –4) and (–1, 1) are two points on the graph.

Now, plot the points (0, –4) and (–1, 1) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.



Answer 5gp.

STEP 1 The intercept form of a quadratic function is $y = a(x-p)(x-q)$, where p and q are the x-intercepts and $x = \frac{p+q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.
On comparing the given equation with the intercept form, we find that $a = 1, p = 3$, and $q = 7$. Thus, the x-intercepts occur at (3, 0) and (7, 0). Since $a > 0$, the parabola opens up.

STEP 2 Now, find the coordinates of the vertex. Substitute for p and q in $x = \frac{p+q}{2}$ and evaluate.

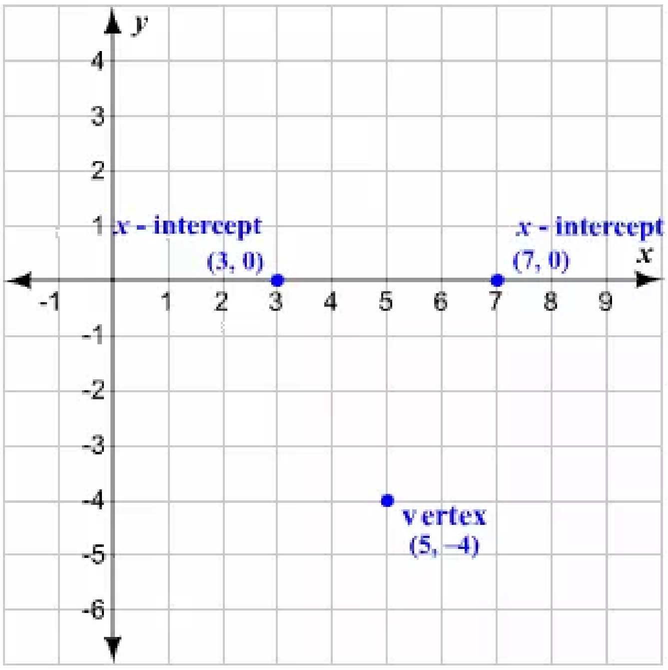
$$x = \frac{3+7}{2} = 5$$

Substitute 5 for x in the given function and evaluate y.

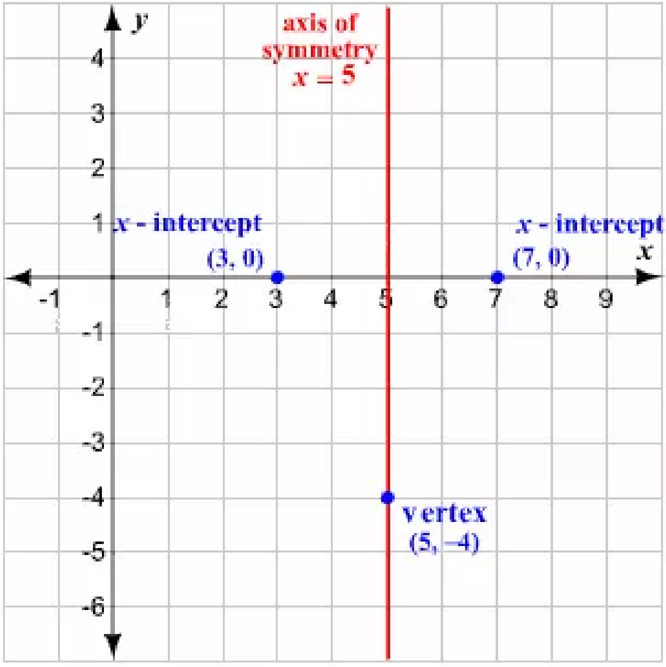
$$\begin{aligned} y &= (5-3)(5-7) \\ &= (2)(-2) \\ &= -4 \end{aligned}$$

Thus, the vertex is (5, −4).

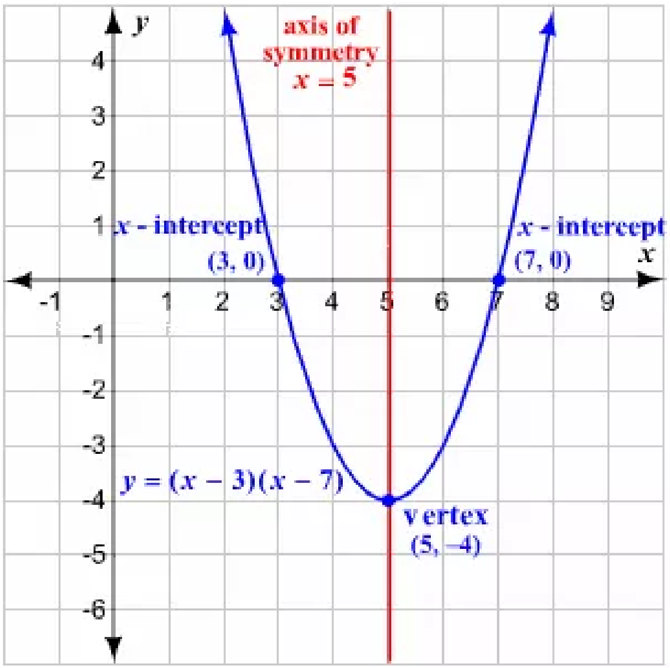
STEP 3 Now, plot the vertex and the points where the x-intercepts occur on a coordinate plane.



Draw the axis of symmetry $x = 5$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 6e.

Consider the function

$$y = 3(x-7)^2 - 1$$

By comparing with the quadratic function in the vertex form,

$$y = a(x-h)^2 + k.$$

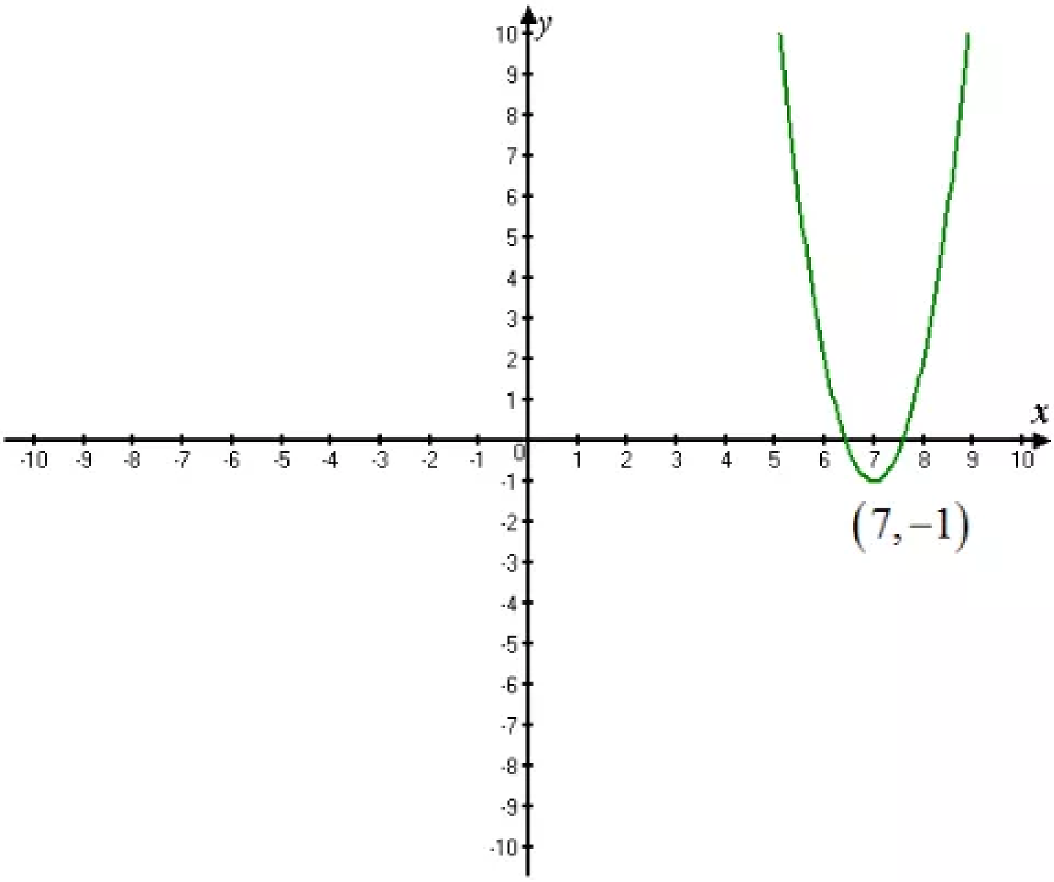
We get $a = 3, h = 7, k = -1$

The vertex is $(h,k) = (7,-1)$

The axis of symmetry is $x = h \Rightarrow x = 7$

If $a = 3 > 0$ the graph opens up

The following diagram contains the graph of the function $y=3(x-7)^2-1$



Answer 6gp.

Consider the function

$f(x)=2(x-4)(x+1)$

It is need to graph the function and label the vertex and axes of symmetry and $x-$intercepts.

Recall the concept that, the graph of $y=a(x-p)(x-q)$has $x-$intercepts are p and q and the axis of symmetry is halfway between $(p,0)$ and $(q,0)$. it has equation $x=\frac{p+q}{2}$. The graph of opens up if $a>0$ and down if $a<0$.

By comparing the equation $f(x)=2(x-4)(x+1)$ with $y=a(x-p)(x-q)$, we get $a=2, p=4$ and $q=-1$

Step-1:

Identify the $x-$intercepts. Because $p=4$ and $q=-1$, the $x-$intercepts are occur at the points $(4,0)$ and $(-1,0)$.

Step-2:

Find the coordinates of the vertex.

$$\begin{aligned} x &= \frac{p+q}{2} \\ &= \frac{4-1}{2} \\ &= \frac{3}{2} \end{aligned}$$

And

$$f\left(\frac{3}{2}\right)=2\left(\frac{3}{2}-4\right)\left(\frac{3}{2}+1\right) \qquad \text{By substuting } x=\frac{3}{2} \text{ in } f(x)$$

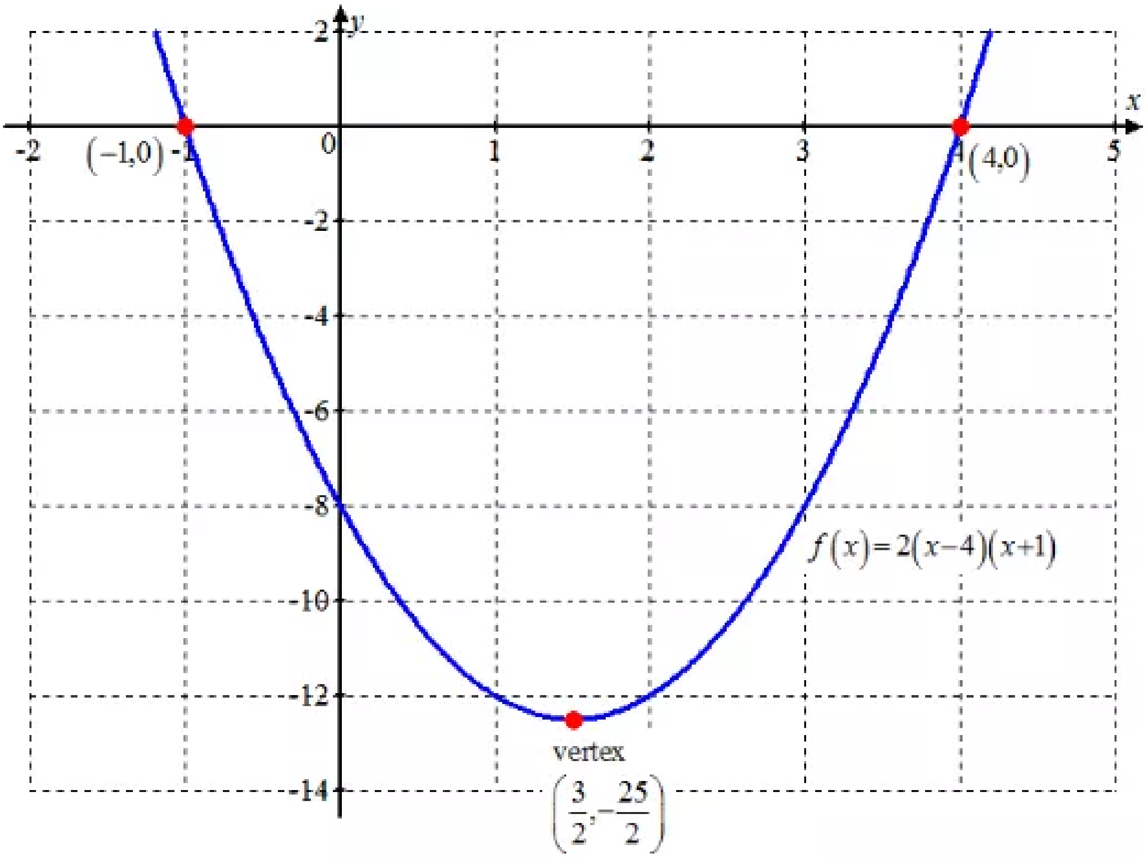
$$=2\left(\frac{3-8}{2}\right)\left(\frac{3+2}{2}\right)$$

$$=-\frac{25}{2}$$

So the vertex is $(x, f(x))=\left(\frac{3}{2}, -\frac{25}{2}\right)$

Step-3:

Draw a parabola through the vertex and the points where the $x-$ intercepts occur.



Answer 7e.

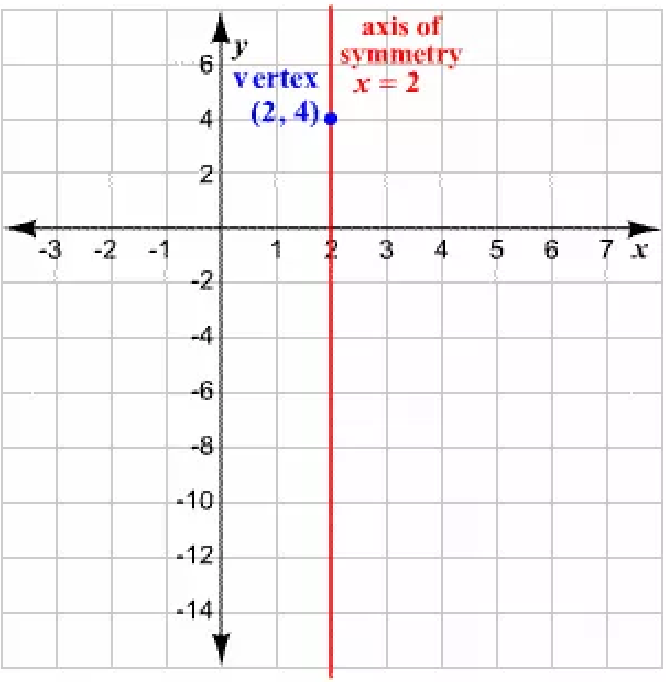
STEP 1 The graph of a quadratic function in vertex form $y=a(x-h)^2+k$ has its vertex at (h, k) and $x=h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.

On comparing the given equation with the vertex form, we find that $a=-4$, $h=2$, and $k=4$. Thus, the vertex is $(h, k)=(2, 4)$ and the axis of symmetry is $x=2$.

Since $a<0$, the parabola opens down.

STEP 2 Plot the vertex (2, 4) on a coordinate plane and draw the axis of symmetry $x = 2$.



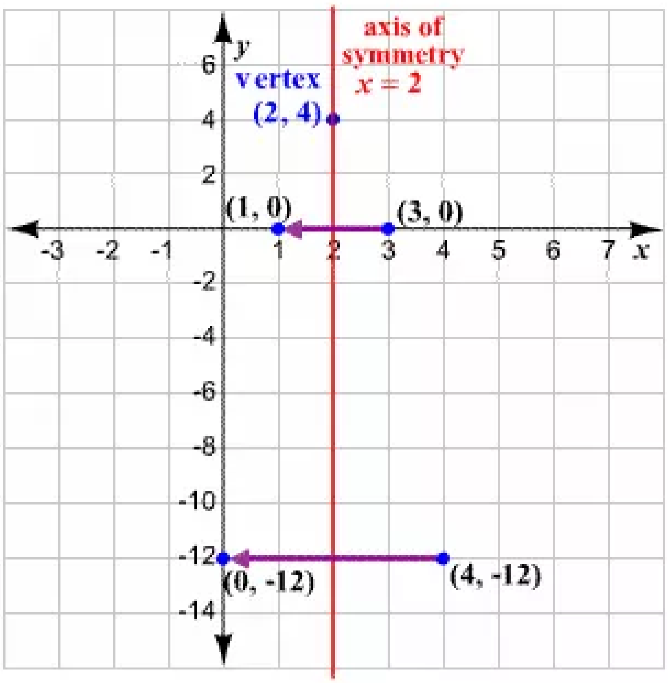
STEP 3 Evaluate the function for two values of x.

$x = 3: \quad y = -4(3-2)^2 + 4 = 0$

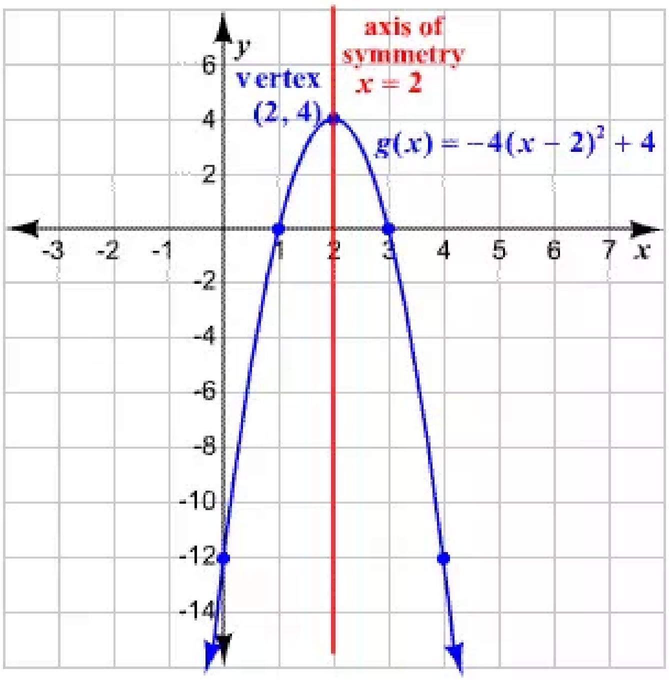
$x = 4: \quad y = -4(4-2)^2 + 4 = -12$

Thus, (3, 0) and (4, −12) are two points on the graph.

Now, plot the points (3, 0) and (4, −12) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.



Answer 7gp.

STEP 1 The intercept form of a quadratic function is $y = a(x-p)(x-q)$, where p and q are the x-intercepts and $x = \frac{p+q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.
On comparing the given equation with the intercept form, we find that $a = -1, p = -1$, and $q = 5$. Thus, the x-intercepts occur at $(-1, 0)$ and $(5, 0)$.
Since $a < 0$, the parabola opens down.

STEP 2 Now, find the coordinates of the vertex. Substitute for p and q in $x = \frac{p+q}{2}$ and evaluate.

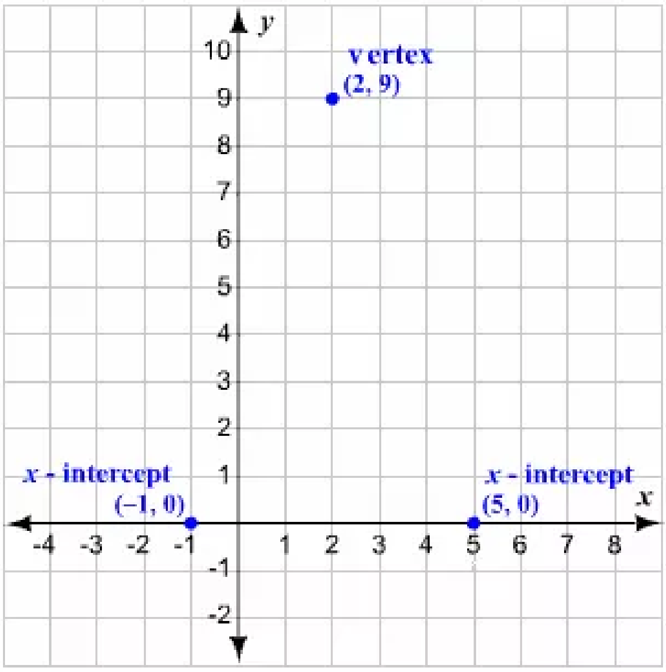
$$x = \frac{-1+5}{2} = 2$$

Substitute 2 for x in the given function and evaluate y.

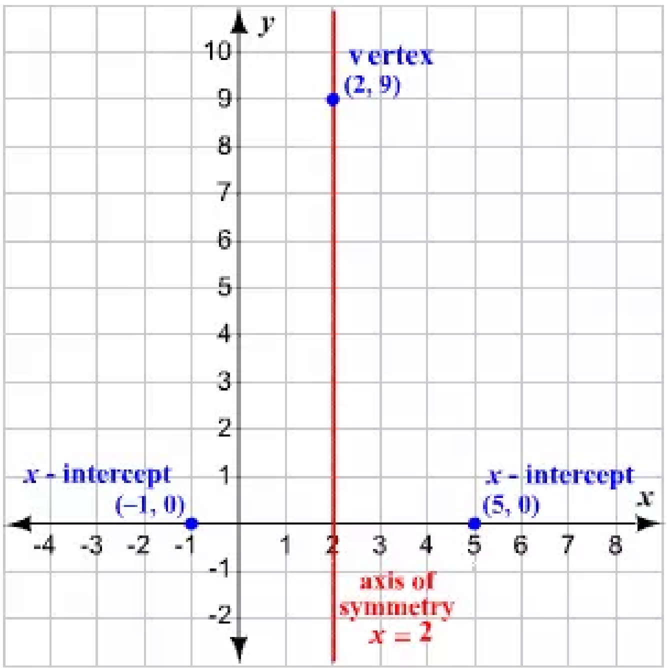
$$y = -(2+1)(2-5)$$
$$= -(3)(-3)$$
$$= 9$$

Thus, the vertex is $(2, 9)$.

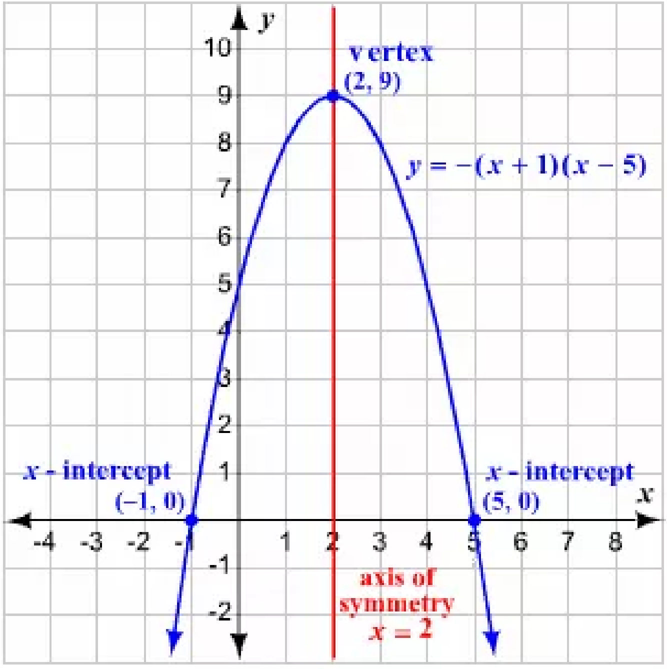
STEP 3 Now, plot the vertex and the points where the x-intercepts occur on a coordinate plane.



Draw the axis of symmetry $x = 2$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 8e.

Consider the function

$$y = 2(x+1)^2 - 3$$

By comparing with the quadratic function in the vertex form,

$$y = a(x-h)^2 + k.$$

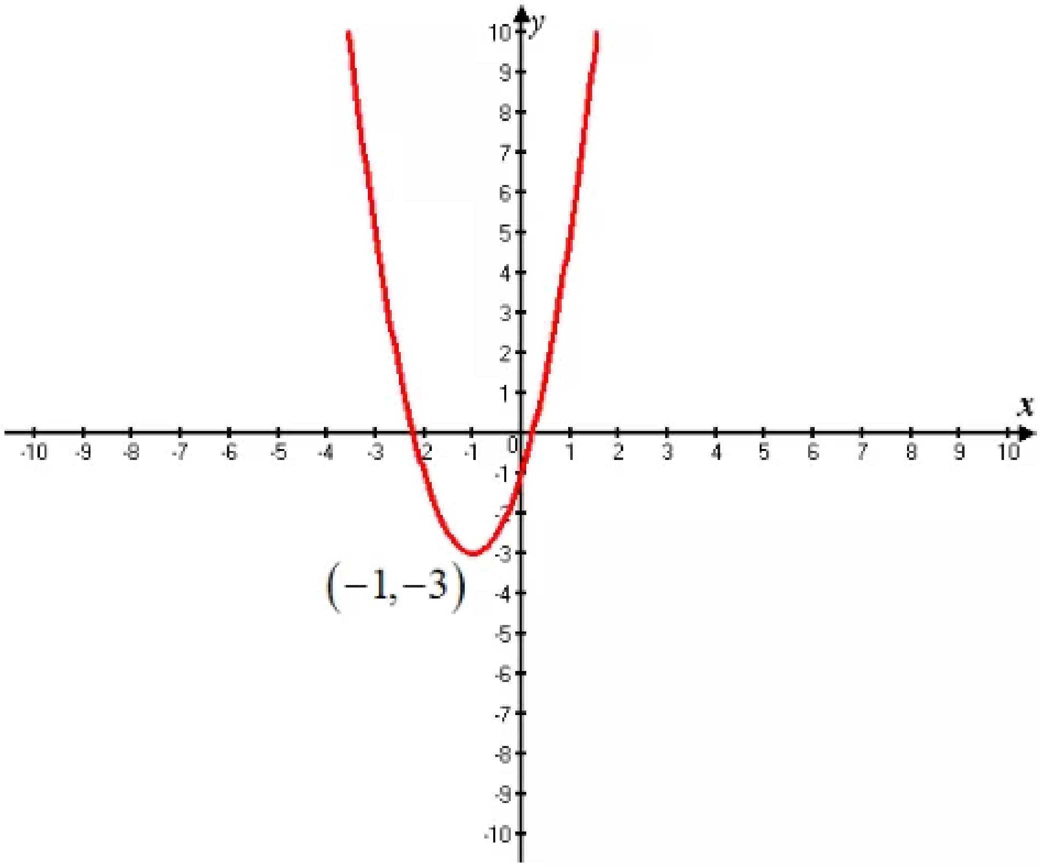
We get $a = 2, h = -1, k = -3$

The vertex is $(h, k) = (-1, -3)$

The axis of symmetry is $x = h \Rightarrow x = -1$

Here $a = 2 > 0$ so the graph opens up

The following diagram contains the graph of the function $y=2(x+1)^2-3$



Answer 8gp.

Suppose that, the path of a football can be modeled by the function
$y=-0.025x(x-50)$
Where x is the horizontal distance (in yards) and y is the corresponding height (in yards).
Here it is need to find the maximum height of the foo ball.

To find the maximum height of the foot ball, we need to calculate the coordinate of the vertex.
On rewriting the given function, we have
$y=-0.025(x-0)(x-50)$
By comparing the equation $y=-0.025(x-0)(x-50)$ with $y=a(x-p)(x-q)$, we get
$a=-0.025, p=0$ and $q=50$
Identify the $x-$ intercepts. Because $p=0$ and $q=50$, the $x-$ intercepts are occur at the points $(0,0)$ and $(50,0)$.

The axis of symmetry is halfway between $(p,0)$ and $(q,0)$ has equation $x = \frac{p+q}{2}$.

$$\begin{aligned} x &= \frac{p+q}{2} \\ &= \frac{0+50}{2} \\ &= \frac{50}{2} \\ &= 25 \end{aligned}$$

And

$y(25) = -0.025(25-0)(25-50)$ By substuting $x = 25$ in y

$$\begin{aligned} &= -0.025(25)(-25) \\ &= 0.025(625) \\ &= 15.625 \end{aligned}$$

So the vertex is $(x, y) = (25, 15.625)$

Since, y coordinate of the vertex represents the maximum height of the football, therefore, the maximum height is 15.625 yards

Answer 9e.

STEP 1 The graph of a quadratic function in vertex form $y = a(x-h)^2 + k$ has its vertex at (h, k) and $x = h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.

On comparing the given equation with the vertex form, we find that $a = -2$, $h = 1$, and $k = -5$. Thus, the vertex is $(h, k) = (1, -5)$ and the axis of symmetry is $x = 1$.

Since $a < 0$, the parabola opens down.

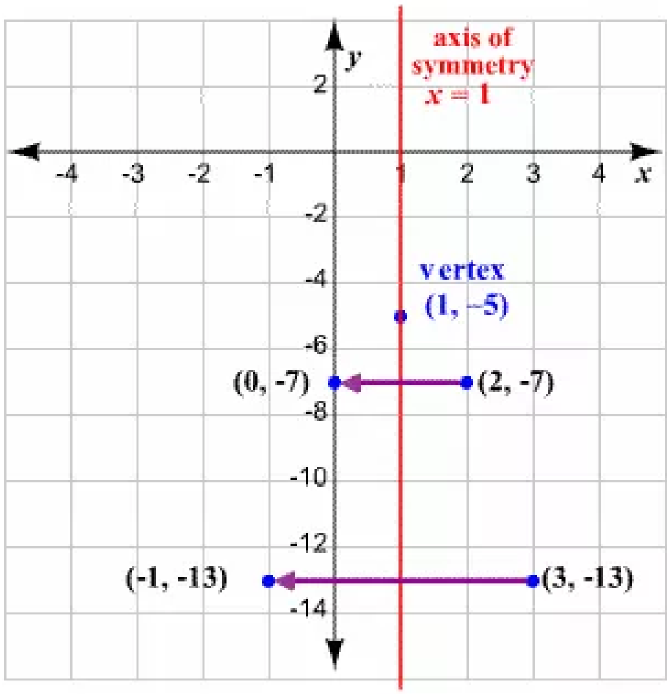
STEP 3 Evaluate the function for two values of x.

$x = 2$: $y = -2(2-1)^2 - 5 = -7$

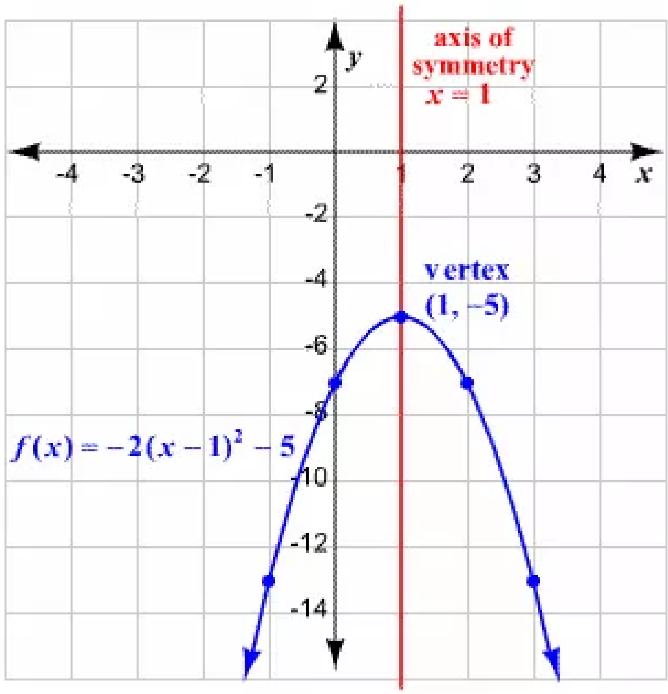
$x = 3$: $y = -2(3-1)^2 - 5 = -13$

Thus, $(2, -7)$ and $(3, -13)$ are two points on the graph.

Now, plot the points (2, –7) and (3, –13) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.



Answer 9gp.

Apply the FOIL method and multiply.

$$y = -[x(x) + x(-7) + (-2)(x) + (-2)(-7)]$$
$$= -(x^2 - 7x - 2x + 14)$$

Combine the like terms.

$$-(x^2 - 7x - 2x + 14) = -[x^2 + (-7 - 2)x + 14]$$
$$= -[x^2 + (-9)x + 14]$$
$$= -(x^2 - 9x + 14)$$

Apply the distributive property.

$$-(x^2 - 9x + 14) = -1(x^2 - 9x + 14)$$
$$= (-1)(x^2) + (-1)(-9x) + (-1)(14)$$
$$= -x^2 + 9x - 14$$

Thus, the given function can be written in standard form as $y = -x^2 + 9x - 14$.

Answer 10e.

Consider the function

$$y = -\frac{1}{4}(x+2)^2 + 1$$

By comparing with the quadratic function in the vertex form,

$$y = a(x-h)^2 + k.$$

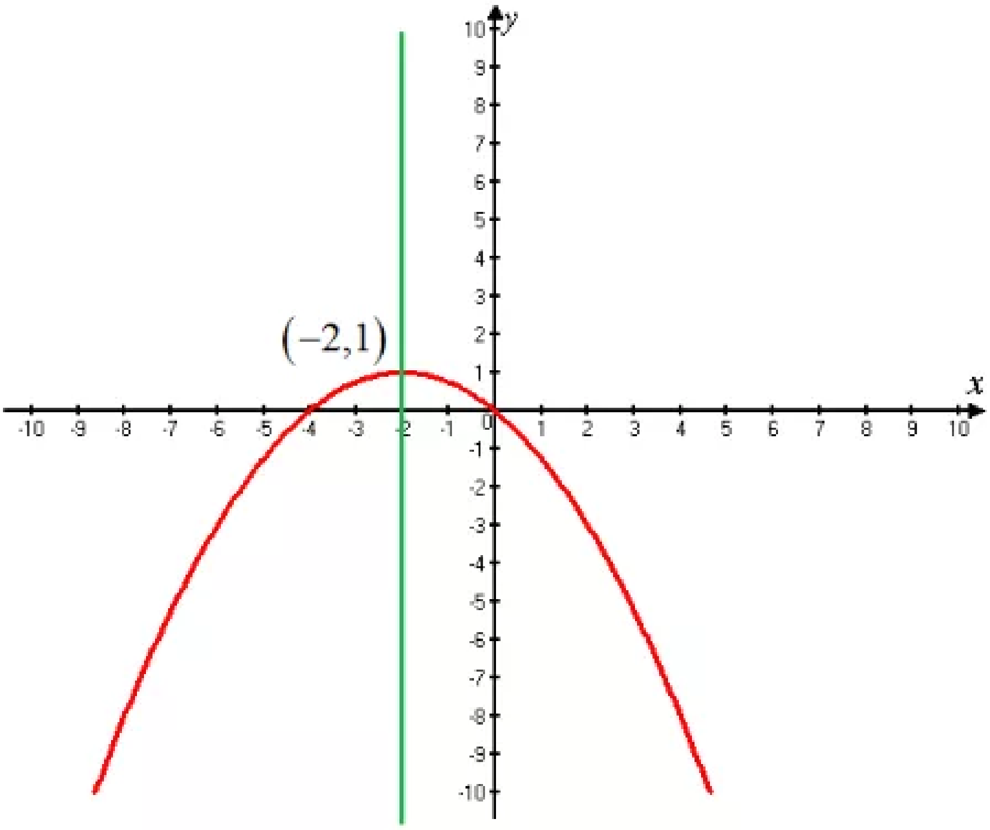
We get $a = -\frac{1}{4}, h = -2, k = 1$

The vertex is $(h,k) = (-2,1)$

The axis of symmetry is $x = h \Rightarrow -2$

Here $a = -\frac{1}{4} < 0$ so the graph opens down

The following diagram contains the graph of the function $y=-\frac{1}{4}(x+2)^2+1$



Answer 10gp.

Write the following quadratic function is standard form.

$$y=-4(x-1)(x+3)$$

In the quadratic function, we need to multiply expressions $x-1$ and $x+3$.

To do this, we use FOIL method that is add the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

$$(x-1)(x+3)=\underset{F}{x^2}+\underset{O}{3x}+\underset{I}{(-x)}+\underset{L}{(-3)}$$

$$=x^2+3x-x-3$$

Now,

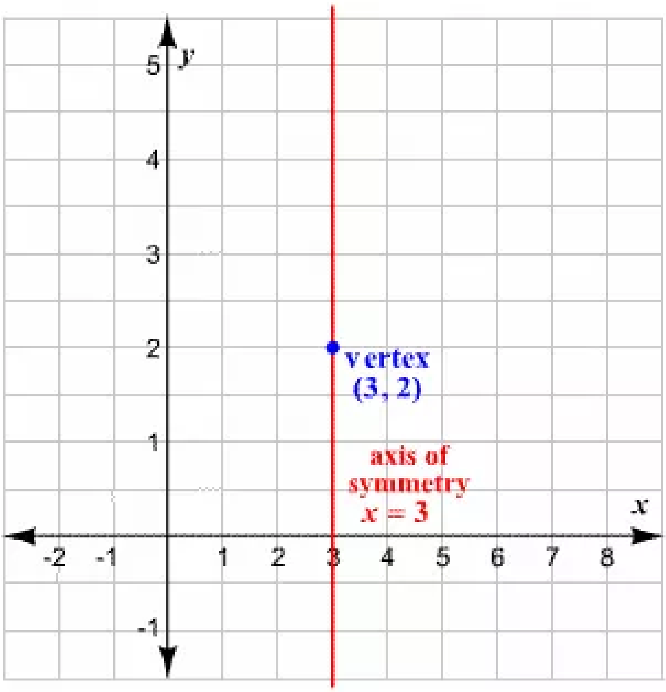
$y=-4(x-1)(x+3)$	Write original function
$=-4(x^2+3x-x-3)$	Multiply using FOIL
$=-4(x^2+2x-3)$	Combine the like terms
$=-4x^2-8x+12$	Distributive property

Answer 11e.

STEP 1 The graph of a quadratic function in vertex form $y = a(x - h)^2 + k$ has its vertex at (h, k) and $x = h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.
On comparing the given equation with the vertex form, we find that $a = \frac{1}{2}$, $h = 3$, and $k = 2$. Thus, the vertex is $(h, k) = (3, 2)$ and the axis of symmetry is $x = 3$.
Since $a > 0$, the parabola opens up.

STEP 2 Plot the vertex (3, 2) on a coordinate plane and draw the axis of symmetry $x = 3$.



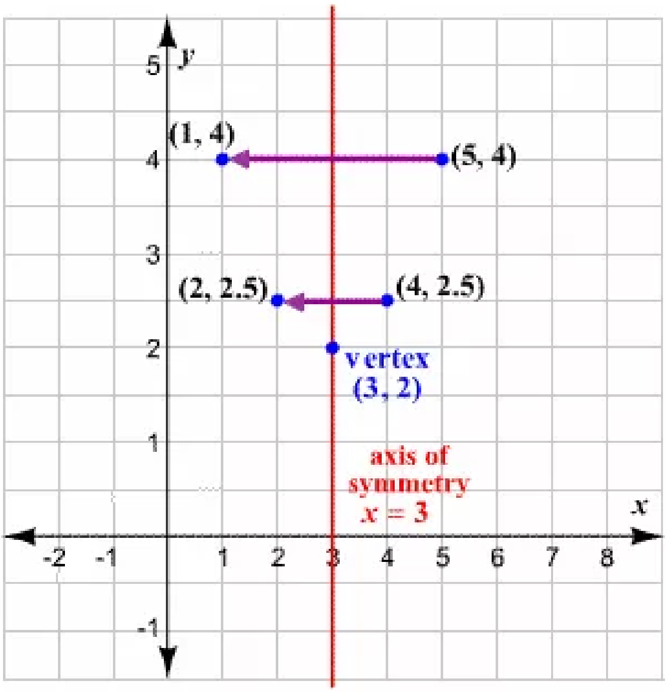
STEP 3 Evaluate the function for two values of x.

$$x = 4: \quad y = \frac{1}{2}(4 - 3)^2 + 2 = 2.5$$

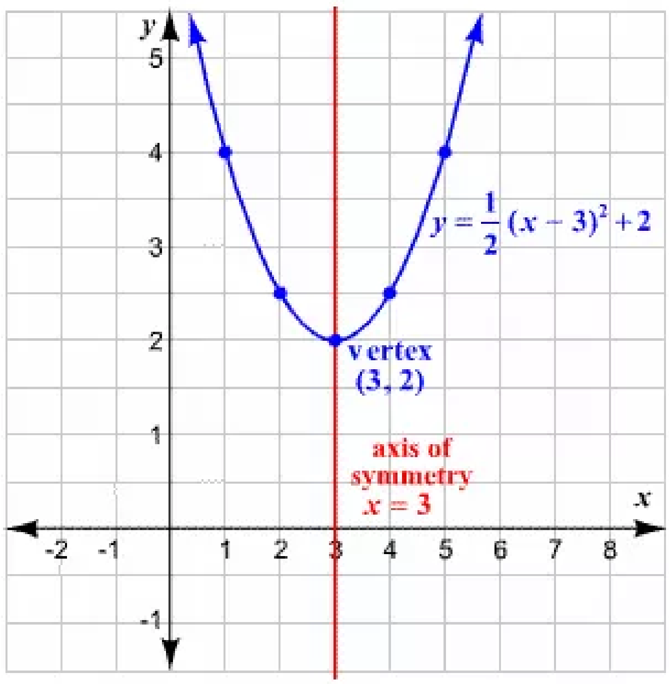
$$x = 5: \quad y = \frac{1}{2}(5 - 3)^2 + 2 = 4$$

Thus, (4, 2.5) and (5, 4) are two points on the graph.

Now, plot the points (4, 2.5) and (5, 4) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.



Answer 11gp.

Apply the FOIL method and multiply.

$$f(x) = 2[x(x) + x(4) + (5)(x) + (5)(4)]$$
$$= 2(x^2 + 4x + 2x + 20)$$

Combine the like terms.

$$2(x^2 + 4x + 2x + 20) = 2[x^2 + (4+2)x + 20]$$
$$= 2(x^2 + 6x + 20)$$

Apply the distributive property.

$$2(x^2 + 6x + 20) = 2(x^2) + 2(6x) + 2(20)$$
$$= 2x^2 + 12x + 40$$

Thus, the given function can be written in standard form as $f(x) = 2x^2 + 12x + 40$.

Answer 12e.

Consider the function

$$y = 3(x+2)^2 - 5$$

By comparing with the quadratic function in the vertex form,

$$y = a(x-h)^2 + k.$$

We get $a = 3, h = -2, k = -5$

The vertex is $(h,k) = (-2,-5)$

Therefore the answer is option (B).

Answer 12gp.

Write the following quadratic function is standard form.

$$y = -7(x-6)(x+1)$$

In the quadratic function, we need to multiply expressions $x-6$ and $x+1$.
To do this, we use FOIL method that is add the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

$$\begin{aligned} & \quad\ \ \text{F} \quad \text{O} \quad \text{I} \quad\ \ \text{L} \\ (x-6)(x+1) &= x^2 + x + (-6x) + (-6) \\ &= x^2 + x - 6x - 6 \end{aligned}$$

Now,

$y = -7(x-6)(x+1)$	Write original function
$= -7(x^2 + x - 6x - 6)$	Multiply using FOIL
$= -7(x^2 - 5x - 6)$	Combine the like terms
$= -7x^2 + 35x + 42$	Distributive property

Answer 13e.

STEP 1 The intercept form of a quadratic function is $y = a(x - p)(x - q)$, where p and q are the x-intercepts and $x = \frac{p+q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.
On comparing the given equation with the intercept form, we find that $a = 1$, $p = -3$, and $q = 3$. Thus, the x-intercepts occur at $(-3, 0)$ and $(3, 0)$.
Since $a > 0$, the parabola opens up.

STEP 2 Then, find the coordinates of the vertex. Substitute for p and q in $x = \frac{p+q}{2}$ and evaluate.

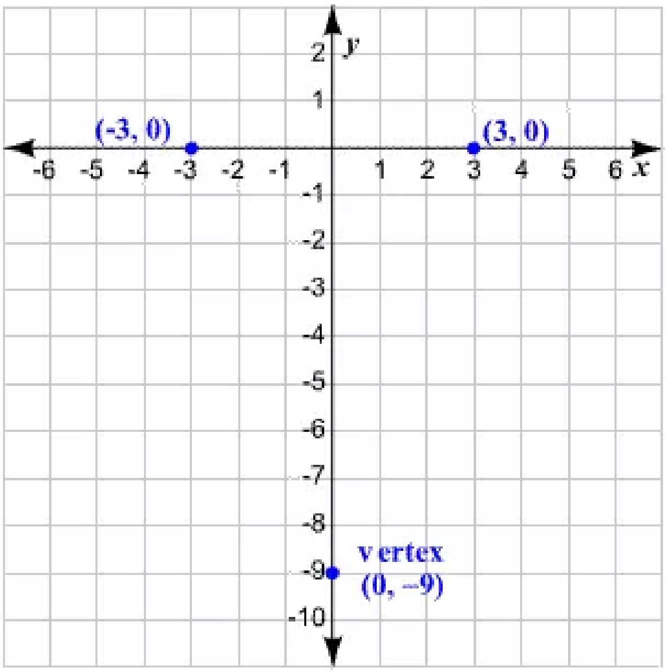
$$x = \frac{-3+3}{2} = 0$$

Substitute 0 for x in the given function and evaluate y.

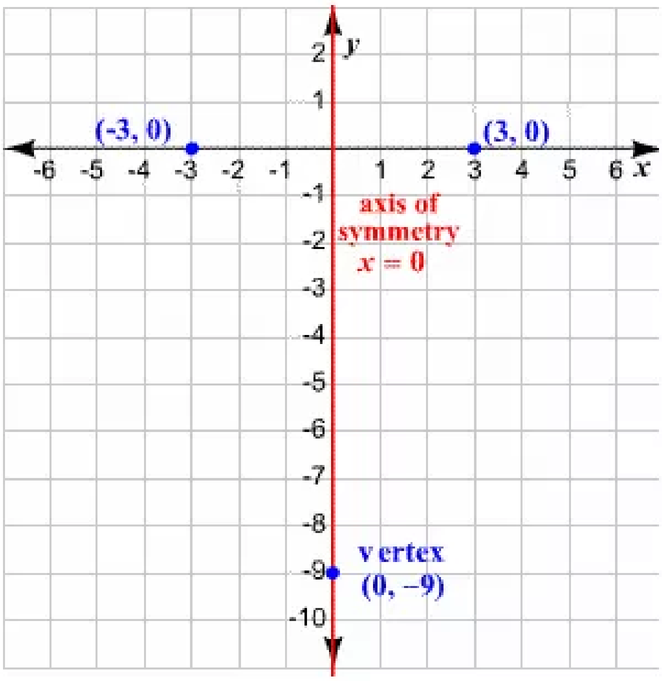
$$\begin{aligned} y &= (0+3)(0-3) \\ &= (3)(-3) \\ &= -9 \end{aligned}$$

Thus, the vertex is $(0, -9)$.

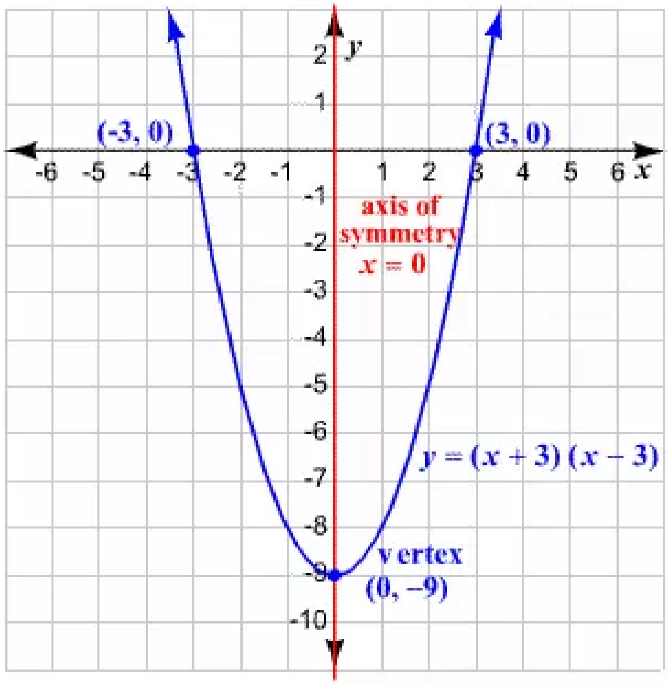
STEP 3 Now, plot the vertex and the points where the x-intercepts occur on a coordinate plane.



Draw the axis of symmetry $x = 0$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 13gp.

Rewrite $(x+5)^2$.

$y=-3(x+5)(x+5)-1$

Apply the FOIL method and multiply.

$$-3(x+5)(x+5)-1 = -3\left[x(x)+x(5)+5(x)+5(5)\right]-1$$
$$= -3\left(x^2+5x+5x+25\right)-1$$

Combine the like terms.

$$-3\left(x^2+5x+5x+25\right)-1 = -3\left[x^2+(5+5)x+25\right]-1$$
$$= -3\left(x^2+10x+25\right)-1$$

Apply the distributive property.

$$-3\left(x^2+10x+25\right)-1 = (-3)\left(x^2\right)+(-3)(10x)+(-3)(25)-1$$
$$= -3x^2-30x-75-1$$

Combine the like terms.

$$-3x^2-30x-75-1 = -3x^2-30x+(-75-1)$$
$$= -3x^2-30x-76$$

Thus, the given function can be written in standard form as

$y=-3x^2-30x-76$.

Answer 14e.

Consider the function

$$y=(x+1)(x-3)$$

By comparing with the quadratic function in the intercept form,

$$y=a(x-p)(x-q).$$

We get $a=1, p=-1, q=3$

The x-intercepts are p and q, that is -1 and 3

Thus the x-intercepts occur at the points $(p,0),(q,0)$,

That is $\boxed{(-1,0) \text{ and } (3,0)}$.

The coordinate of the vertex

$$x=\frac{p+q}{2}$$
$$=\frac{-1+3}{2}$$
$$=1$$

Substitute $x=1$ in $y=(x+1)(x-3)$

$$y=(1+1)(1-3)$$
$$=(2)(-2)$$
$$=-4$$

So, the vertex is $\boxed{(1,-4)}$.

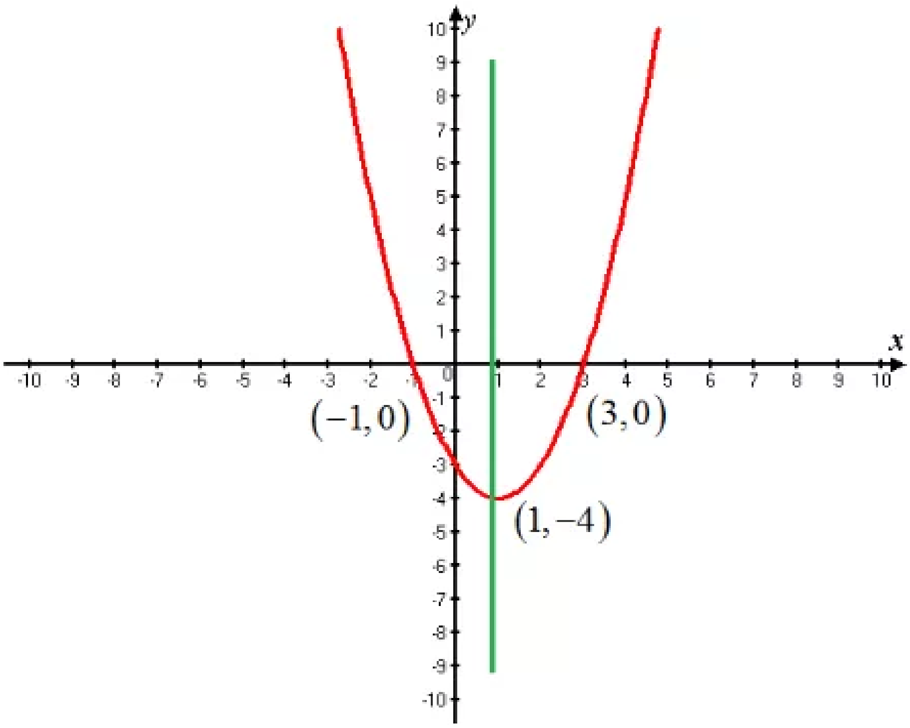
And

The axis of symmetry is $x=\frac{p+q}{2} \Rightarrow \boxed{x=1}$.

Here $a=1>0$, the graph opens up

Now draw a parabola through the vertex and the points where the x-intercepts occur.

The following diagram contains the graph of the function $y=(x+1)(x-3)$



Answer 14gp.

Write the following quadratic function is standard form.

$g(x)=6(x-4)^2-10$

$g(x)=6(x-4)(x-4)-10$

In the quadratic function, we need to multiply expressions $x-4$ and $x-4$.

To do this, we use FOIL method that is add the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

$$(x-4)(x-4)=\overset{F}{x^2}+\overset{O}{(-4x)}+\overset{I}{(-4x)}+\overset{L}{16}$$

$$=x^2-4x-4x+16$$

Now,

$g(x)=6(x-4)(x-4)-10$

$=6(x^2-4x-4x+16)-10$ Multiply using FOIL

$=6(x^2-8x+16)-10$ Combine the like terms

$=6x^2-48x+96-10$ Distributive property

$=6x^2-48x+86$ Combine the like terms

So, the required standard form is $g(x)=\boxed{6x^2-48x+86}$

Answer 15e.

STEP 1 The intercept form of a quadratic function is $y=a(x-p)(x-q)$, where p and q are the x-intercepts and $x=\dfrac{p+q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.

On comparing the given equation with the intercept form, we find that $a=3, p=-2$, and $q=-6$. Thus, the x-intercepts occur at $(-2, 0)$ and $(-6, 0)$. Since $a>0$, the parabola opens up.

STEP 2 Then, find the coordinates of the vertex. Substitute for p and q in $x=\dfrac{p+q}{2}$ and evaluate.

$$x=\frac{-2+(-6)}{2}=-4$$

Substitute -4 for x in the given function and evaluate y.

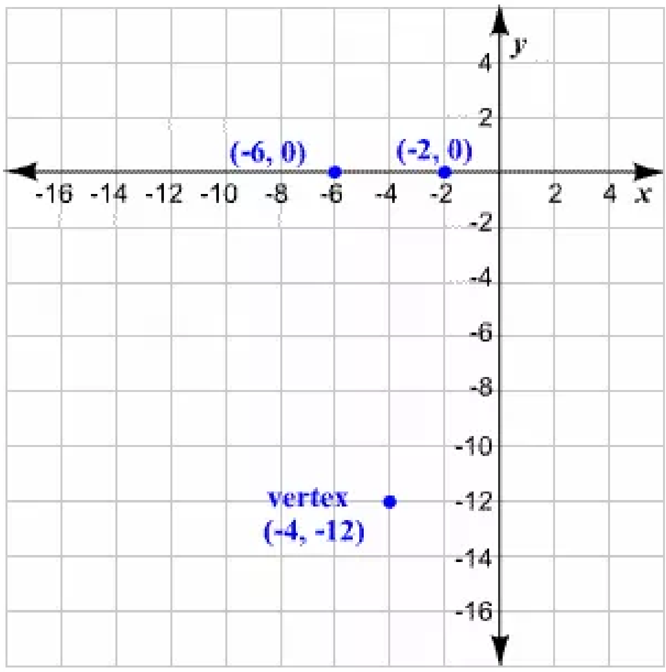
$y=3(-4+2)(-4+6)$

$=3(-2)(2)$

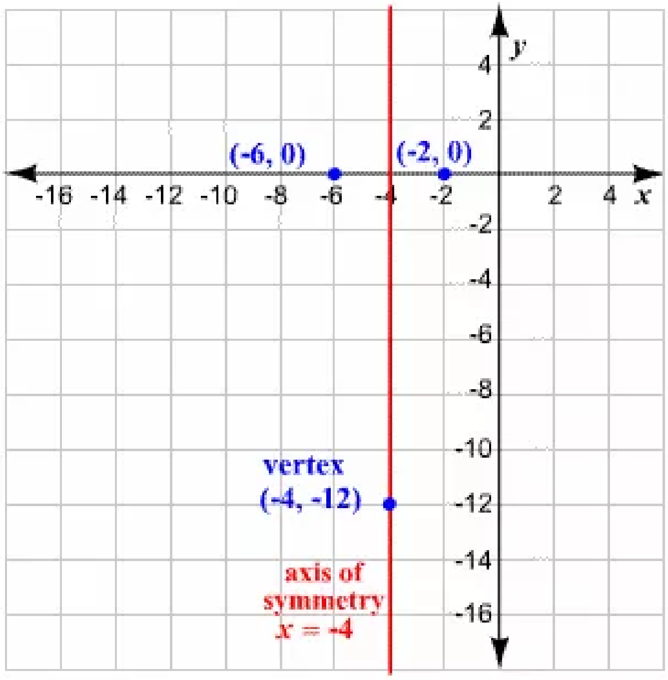
$=-12$

Thus, the vertex is $(-4, -12)$.

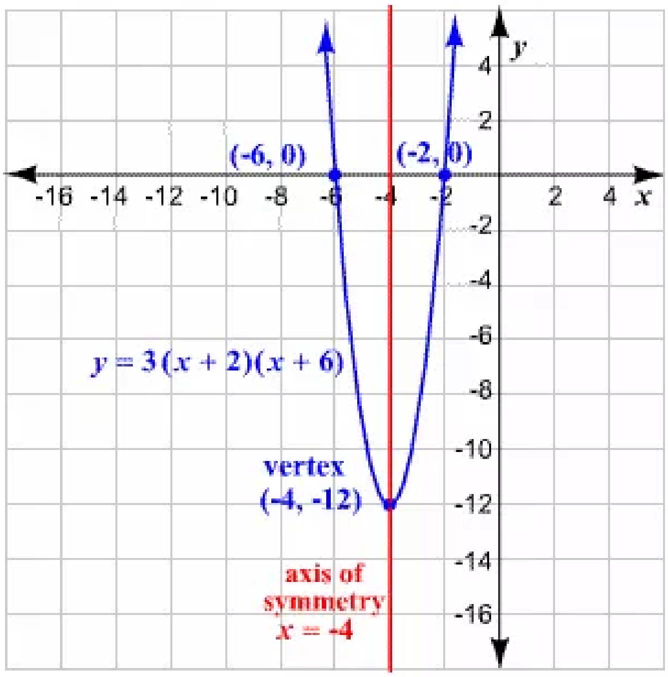
STEP 3 Now, plot the vertex and the points where the x-intercepts occur on a coordinate plane.



Draw the axis of symmetry $x = -4$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 15gp.

Rewrite $(x+2)^2$.

$f(x) = -(x+2)(x+2) + 4$

Apply the FOIL method and multiply.

$$-(x+2)(x+2)+4 = -[x(x)+x(2)+2(x)+2(2)]+4$$
$$= -(x^2+2x+2x+4)+4$$

Combine the like terms.

$$-(x^2+2x+2x+4)+4 = -[x^2+(2+2)x+4]+4$$
$$= -(x^2+4x+4)+4$$

Apply the distributive property and simplify.

$$-(x^2+4x+4)+4 = -x^2-4x-4+4$$
$$= -x^2-4x$$

Thus, the given function can be written in standard form as $f(x) = -x^2 - 4x$.

Answer 16e.

Consider the function

$$f(x)=2(x-5)(x-1)$$

By comparing with the quadratic function in the intercept form,

$$y=a(x-p)(x-q).$$

We get $a=2, p=5, q=1$

The x-intercepts are p and q, that is 5 and 1

Thus the x-intercepts occur at the points $(p,0),(q,0)$,

That is $\boxed{(5,0) \text{ and } (1,0)}$.

The coordinate of the vertex

$$\begin{aligned} x &= \frac{p+q}{2} \\ &= \frac{5+1}{2} \\ &= 3 \end{aligned}$$

Substitute $x=3$ in $f(x)=2(x-5)(x-1)$

$$\begin{aligned} y &= 2(3-5)(3-1) \\ &= 2(-2)(2) \\ &= -8 \end{aligned}$$

So, the vertex is $\boxed{(3,-8)}$.

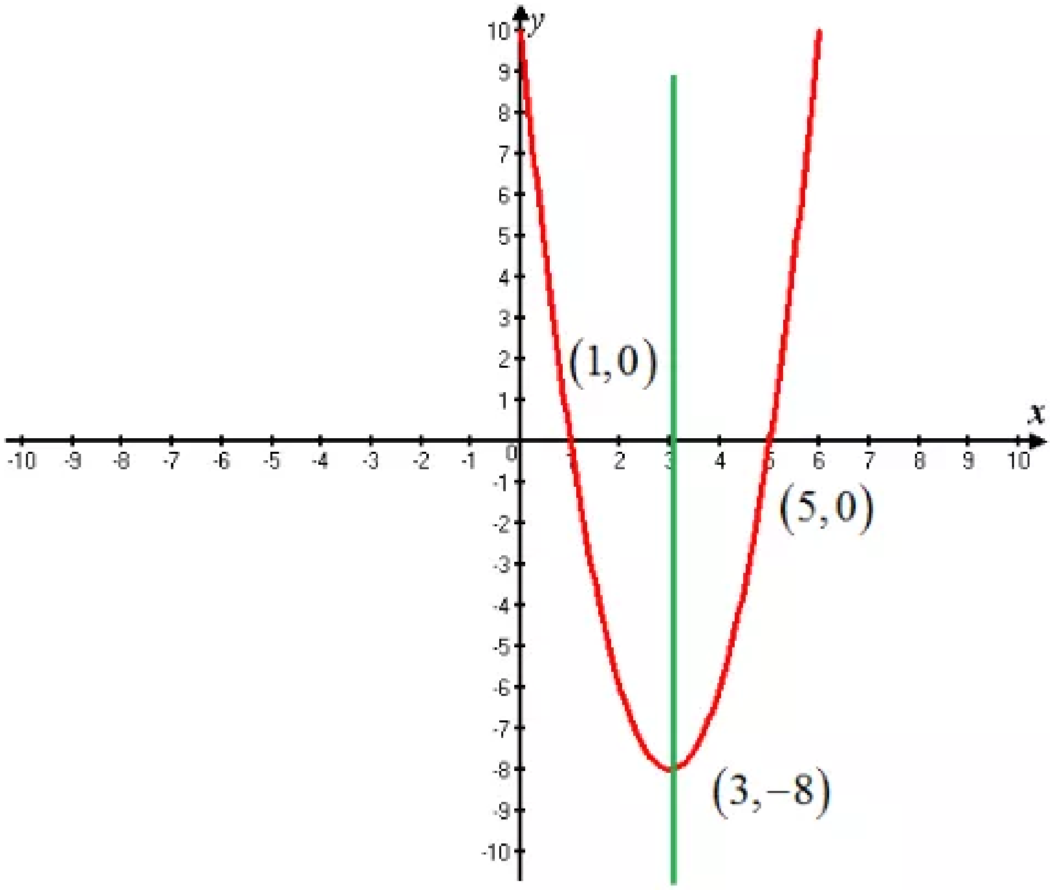
And

The axis of symmetry is $x=\frac{p+q}{2} \Rightarrow \boxed{x=3}$.

Here $a=2>0$, so the graph opens up

Now draw a parabola through the vertex and the points where the x-intercepts occur.

The following diagram contains the graph of the function $f(x)=2(x-5)(x-1)$



Answer 16gp.

Write the following quadratic function is standard form.

$$y=2(x-3)^2+9$$

$$y=2(x-3)(x-3)+9 \qquad \text{Rewrite } (x-3)^2$$

In the quadratic function, we need to multiply expressions $x-3$ and $x-3$.

To do this, we use FOIL method that is add the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

$$\begin{aligned}(x-3)(x-3) &= \overset{F}{x^2}+\overset{O}{(-3x)}+\overset{I}{(-3x)}+\overset{L}{9} \\ &= x^2-3x-3x+9\end{aligned}$$

Now,

$$
\begin{aligned}
y &= 2(x-3)(x-3)+9 \\
&= 2\left(x^2-3x-3x+9\right)+9 && \text{Multiply using FOIL} \\
&= 2\left(x^2-6x+9\right)+9 && \text{Combine the like terms} \\
&= 2x^2-12x+18+9 && \text{Distributive property} \\
&= 2x^2-12x+27 && \text{Combine the like terms}
\end{aligned}
$$

So, the required standard form is $y = \boxed{2x^2-12x+27}$

Answer 17e.

STEP 1 The intercept form of a quadratic function is $y = a(x-p)(x-q)$, where p and q are the x-intercepts and $x = \dfrac{p+q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.
On comparing the given equation with the intercept form, we find that $a = -1, p = 4$, and $q = -6$. Thus, the x-intercepts occur at (4, 0) and (–6, 0). Since $a < 0$, the parabola opens down.

STEP 2 Then, find the coordinates of the vertex. Substitute for p and q in $x = \dfrac{p+q}{2}$ and evaluate.

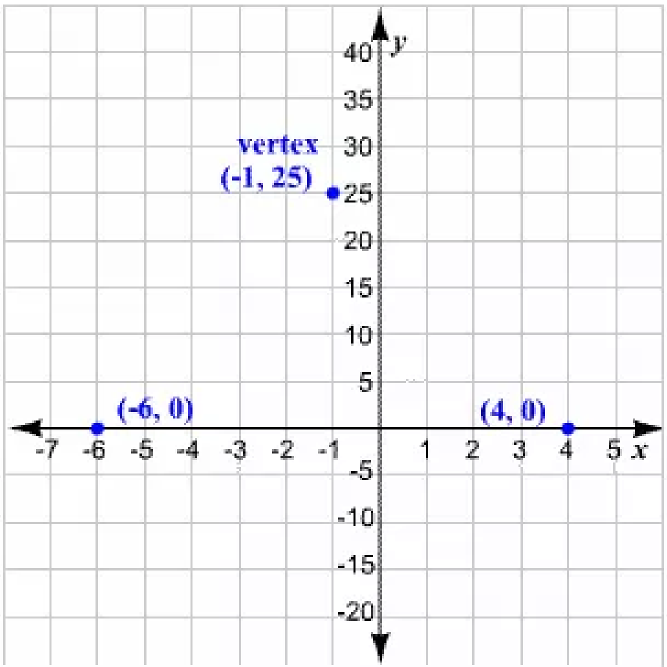
$$x = \frac{4+(-6)}{2} = -1$$

Substitute –4 for x in the given function and evaluate y.

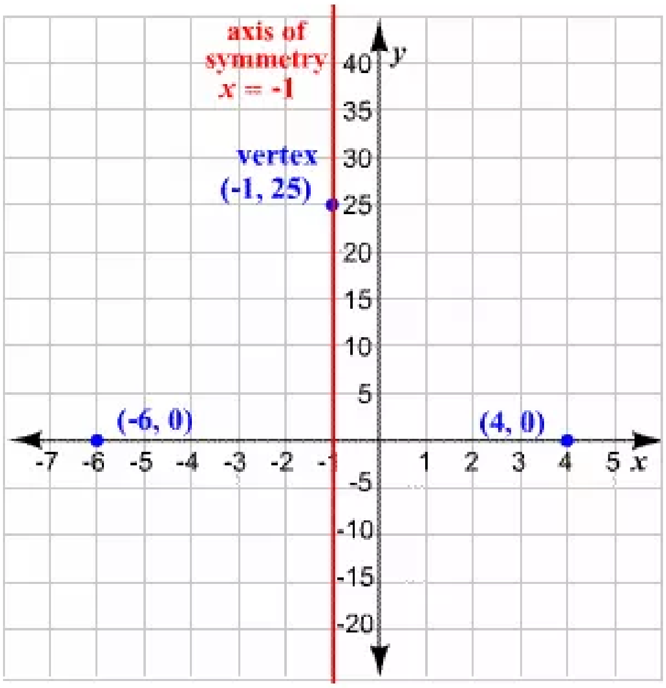
$$
\begin{aligned}
y &= -(-1-4)(-1+6) \\
&= -(-5)(5) \\
&= 25
\end{aligned}
$$

Thus, the vertex is (–1, 25).

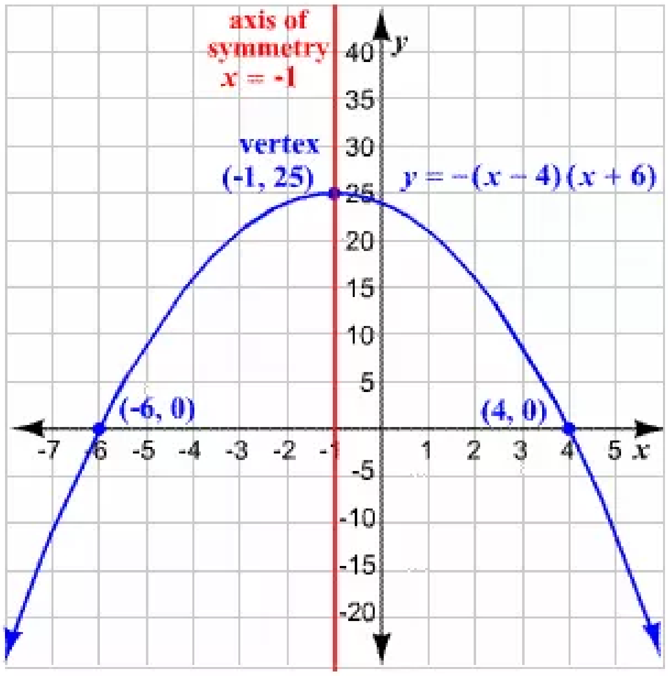
STEP 3 Now, plot the vertex and the points where the x-intercepts occur on a coordinate plane.



Draw the axis of symmetry $x = -1$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 18e.

Consider the function

$$g(x)=-4(x+3)(x+7)$$

By comparing with the quadratic function in the intercept form,

$$y=a(x-p)(x-q).$$

We get $a=-4, p=-3, q=-7$

The x-intercepts are p and q, that is -3 and -7

Thus the x-intercepts occur at the points $(p,0),(q,0)$,

That is $\boxed{(-3,0) \text{ and } (-7,0)}$.

The coordinate of the vertex

$$x=\frac{p+q}{2}$$

$$x=\frac{-3-7}{2}$$

$$=\frac{-10}{2}$$

$$=-5$$

Substitute $x=-5$ in $g(x)=-4(x+3)(x+7)$

$$y=-4(-5+3)(-5+7)$$

$$=-4(-2)(2)$$

$$=16$$

So, the vertex is $\boxed{(-5,16)}$

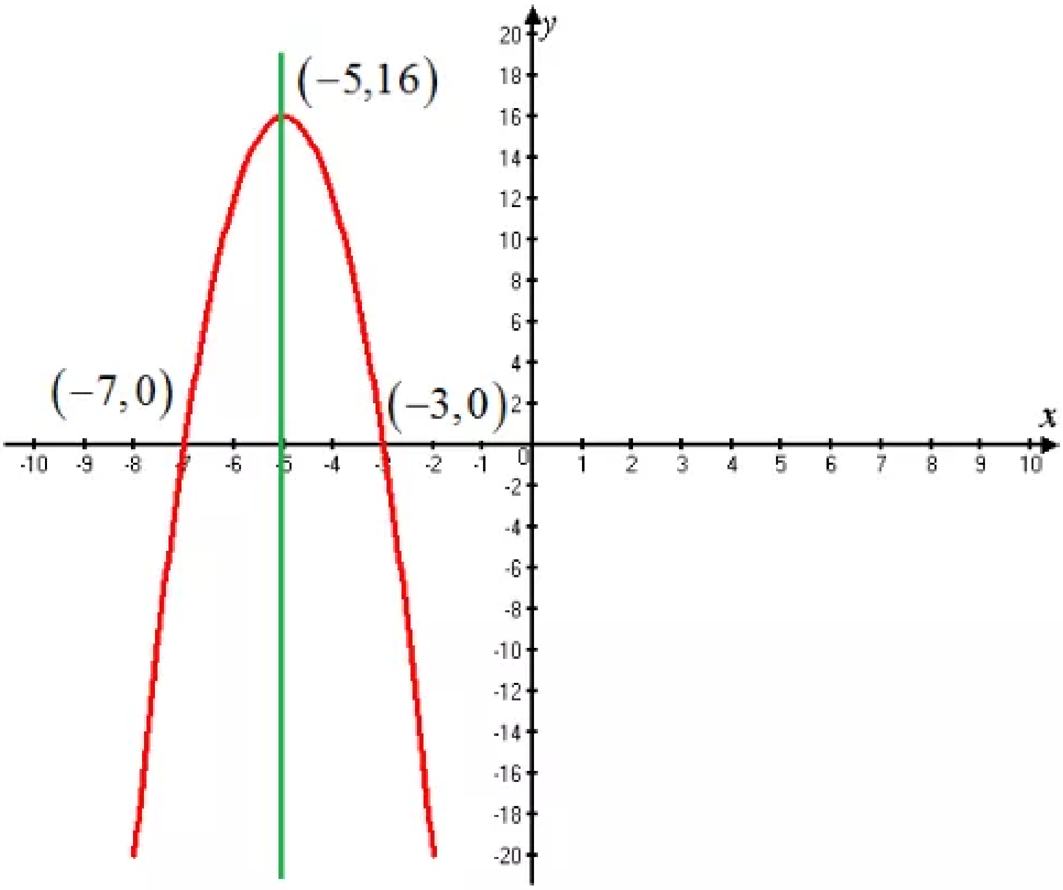
And

The axis of symmetry is $x=\frac{p+q}{2} \Rightarrow \boxed{x=-5}$.

Here $a=-4<0$, so the graph opens down

Now draw a parabola through the vertex and the points where the x-intercepts occur.

The following diagram contains the graph of the function $g(x)=-4(x+3)(x+7)$



Answer 19e.

STEP 1 The intercept form of a quadratic function is $y = a(x - p)(x - q)$, where p and q are the x-intercepts and $x = \frac{p+q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.
On comparing the given equation with the intercept form, we find that $a = 1$, $p = -1$, and $q = -2$. Thus, the x-intercepts occur at $(-1, 0)$ and $(-2, 0)$. Since $a > 0$, the parabola opens up.

STEP 2 Then, find the coordinates of the vertex. Substitute for p and q in

$$x = \frac{p+q}{2}$$ and evaluate.

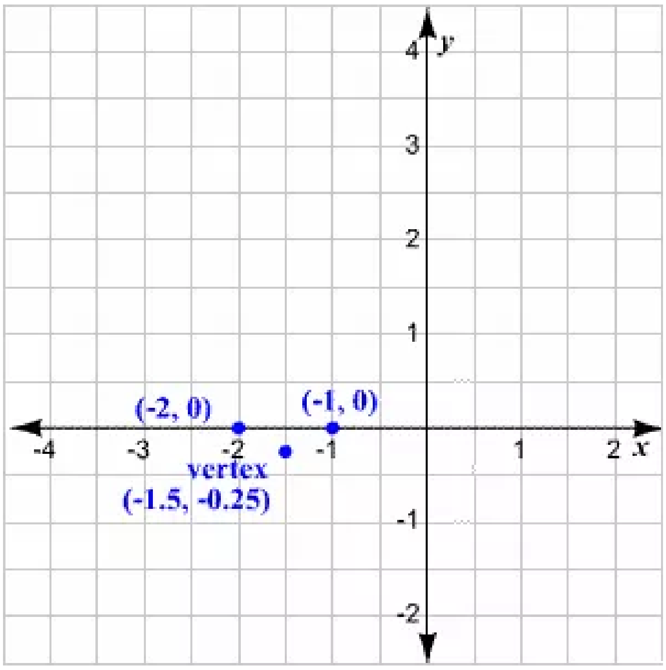
$$x = \frac{-1+(-2)}{2} = -1.5$$

Substitute -1.5 for x in the given function and evaluate y.

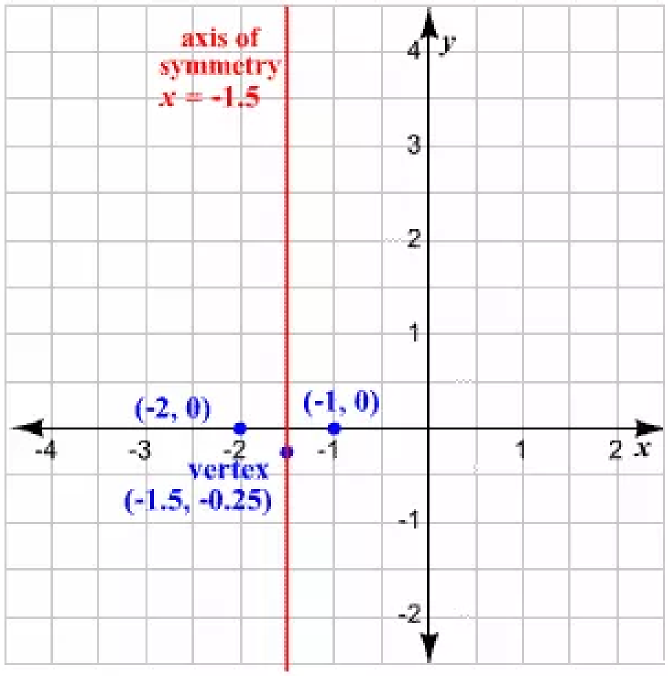
$$y = (-1.5+1)(-1.5+2)$$
$$= (-0.5)(0.5)$$
$$= -0.25$$

Thus, the vertex is $(-1.5, 0.25)$.

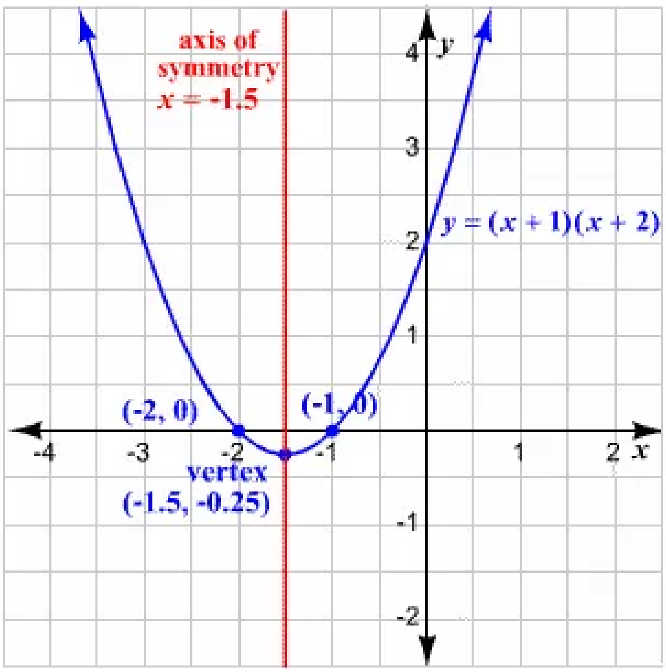
STEP 3 Now, plot the vertex and the points where the x-intercepts occur on a coordinate plane.



Draw the axis of symmetry $x = -1.5$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 20e.

Consider the function

$$f(x) = -2(x-3)(x+4)$$

By comparing with the quadratic function in the intercept form,

$$y = a(x-p)(x-q).$$

We get $a=-2, p=3, q=-4$

The x-intercepts are p and q , that is 3 and -4

Thus the x-intercepts occur at the points $(p,0),(q,0)$,

That is $\boxed{(3,0) \text{ and } (-4,0)}$.

The coordinate of the vertex

$$x = \frac{p+q}{2}$$
$$= \frac{-3+4}{2}$$
$$= \frac{-1}{2}$$

Substitute $x = \frac{-1}{2}$ in $f(x) = -2(x-3)(x+4)$

$$y = -2\left(\frac{-1}{2} - 3\right)\left(\frac{-1}{2} + 4\right)$$
$$= -2\left(\frac{-7}{2}\right)\left(\frac{7}{2}\right)$$
$$= -2\left(\frac{-7}{2}\right)\left(\frac{7}{2}\right)$$
$$= \left(\frac{49}{2}\right)$$

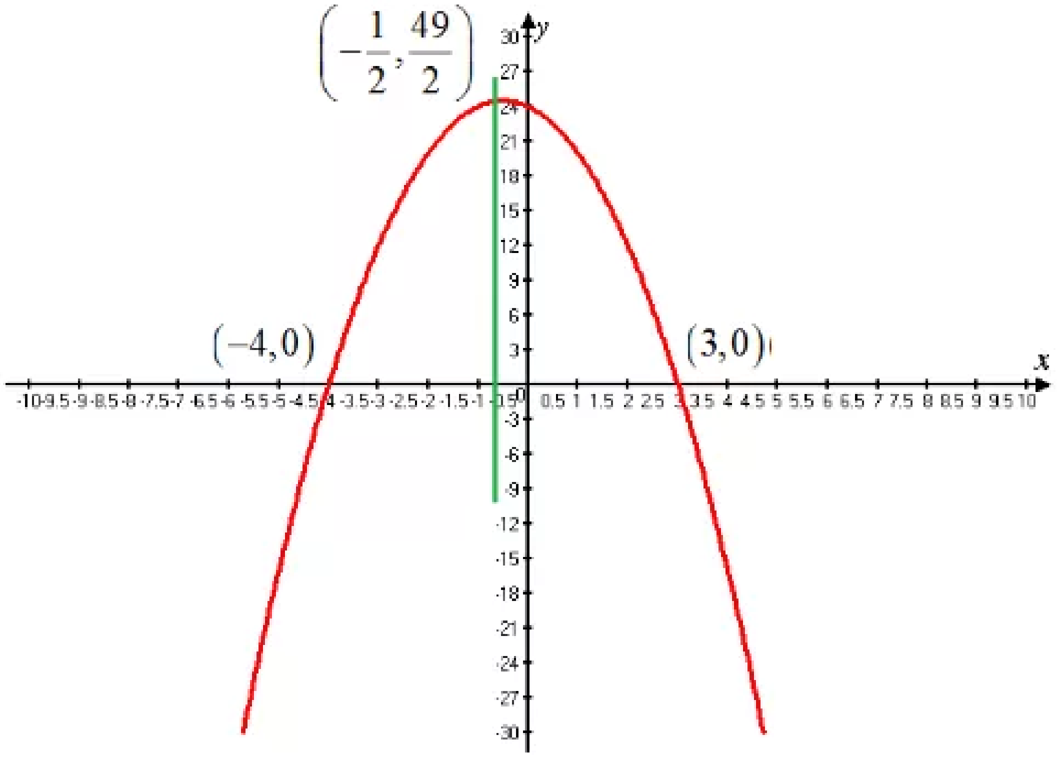
So, the vertex is $\boxed{\left(-\frac{1}{2}, \frac{49}{2}\right)}$.

The axis of symmetry is $x = \frac{p+q}{2} \Rightarrow x = \boxed{-\frac{1}{2}}$.

Here $a = -2 < 0$, so the graph opens down

Now draw a parabola through the vertex and the points where the x-intercepts occur.

The following diagram contains the graph of the function $f(x) = -2(x-3)(x+4)$



Answer 21e.

STEP 1 The intercept form of a quadratic function is $y = a(x - p)(x - q)$, where p and q are the x-intercepts and $x = \frac{p+q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.
On comparing the given equation with the intercept form, we find that $a = 4$, $p = 7$, and $k = -2$. Thus, the x-intercepts occur at (2, 0) and (−2, 0). Since $a > 0$, the parabola opens up.

STEP 2 Then, find the coordinates of the vertex. Substitute for p and q in

$x = \dfrac{p+q}{2}$ and evaluate.

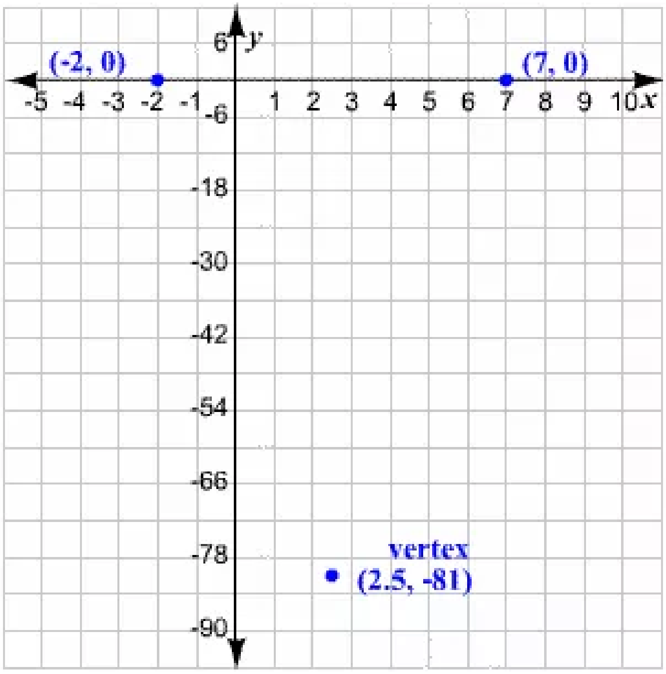
$$x = \frac{7+(-2)}{2} = 2.5$$

Substitute 2.5 for x in the given function and evaluate y.

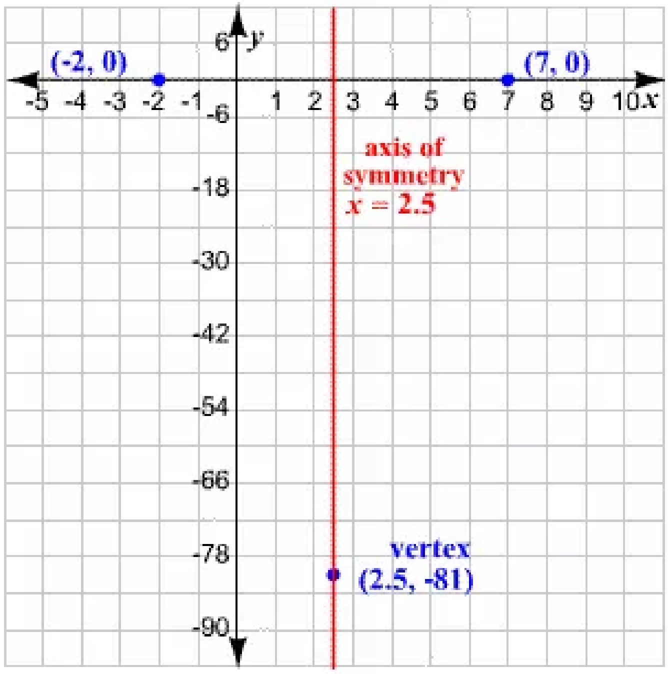
$$\begin{aligned} y &= 4(2.5-7)(2.5+2) \\ &= 4(-4.5)(4.5) \\ &= -81 \end{aligned}$$

Thus, the vertex is (2.5, –81).

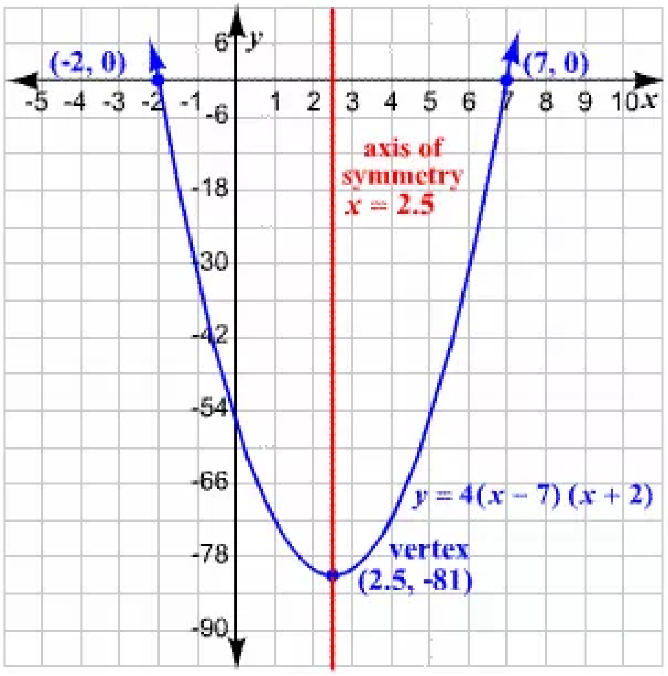
STEP 3 Now, plot the vertex and the points where the x-intercepts occur on a coordinate plane.



Draw the axis of symmetry $x = 2.5$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 22e.

Consider the function

$$y = -(x-6)(x+4)$$

By comparing with the quadratic function in the intercept form,

$$y = a(x-p)(x-q).$$

We get $a = -1, p = 6, q = -4$

The x-intercepts are p and q, that is 6 and -4

The coordinate of the vertex

$$\begin{aligned} x &= \frac{p+q}{2} \\ &= \frac{6-4}{2} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

Substitute $x = 1$ in $y = -(x-6)(x+4)$

$$\begin{aligned} y &= -(1-6)(1+4) \\ &= -(-5)(5) \\ &= 25 \end{aligned}$$

Thus,

The vertex is $(1, 25)$

Therefore the answer is option (A).

Answer 23e.

The x-intercepts of the graph of a quadratic function in the form $y = a(x-p)(x-q)$ are p and q.

On comparing the given function with the intercept form, we get $p = 2$ and $q = -3$. Thus, the x-intercepts of the function are 2 and -3. The error is that the signs have been interchanged.

Answer 24e.

Consider the function

$$y=(x+4)(x+3)$$

Need to write the following quadratic function is standard form.

In the quadratic function, we need to multiply expressions $(x+4)$ and $(x+3)$.

To do this, we use FOIL method that is add the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

$$\begin{array}{l} \quad\;\; F \quad O \quad I \quad L \\ y=x^2+3x+4x+12 \end{array}$$

Combine like terms

$$y=x^2+7x+12$$

Therefore

$$y=\boxed{x^2+7x+12}.$$

Answer 25e.

Apply the FOIL method and multiply.

$$\begin{aligned} y &= x(x)+x(3)+(-5)(x)+(-5)(3) \\ &= x^2+3x-5x-15 \end{aligned}$$

Combine the like terms.

$$\begin{aligned} x^2+3x-5x-15 &= x^2+(3-5)x-15 \\ &= x^2+(-2)x-15 \\ &= x^2-2x-15 \end{aligned}$$

Thus, the given function can be written in standard form as $y=x^2-2x-15$.

Answer 26e.

Consider the quadratic function

$$h(x)=4(x+1)(x-6)$$

We need to write the given quadratic function is standard form.

$h(x)=4(x+1)(x-6)$	Write original function.
$h(x)=4(x^2-6x+x-6)$	Multiply using FOIL
$h(x)=4(x^2-5x-6)$	Combine like terms.
$h(x)=4x^2-20x-24$	Distributive property

Therefore, the standard form of the given quadratic function $h(x)=4(x+1)(x-6)$ is

$$\boxed{h(x)=4x^2-20x-24}.$$

Answer 27e.

Apply the FOIL method and multiply.

$$y = -3[x(x) + x(-4) + (-2)(x) + (-2)(-4)]$$
$$= -3(x^2 - 4x - 2x + 8)$$

Combine the like terms.

$$-3(x^2 - 4x - 2x + 8) = -3[x^2 + (-4 - 2)x + 8]$$
$$= -3[x^2 + (-6)x + 8]$$
$$= -3(x^2 - 6x + 8)$$

Apply the distributive property.

$$-3(x^2 - 6x + 8) = (-3)(x^2) + (-3)(-6x) + (-3)(8)$$
$$= -3x^2 + 18x - 24$$

Thus, the simplified form of the given function is $y = -3x^2 + 18x - 24$.

Answer 28e.

Consider the quadratic function

$$f(x) = (x+5)^2 - 2$$

We need to write the given quadratic function is standard form.

$f(x) = (x+5)^2 - 2$	Write original function.
$f(x) = (x+5)(x+5) - 2$	Rewrite $(x+5)^2$.
$f(x) = (x^2 + 5x + 5x + 25) - 2$	Multiply using FOIL
$f(x) = x^2 + 10x + 25 - 2$	Combine like terms.
$f(x) = x^2 + 10x + 23$	Simplify.

Therefore, the standard form of the given quadratic function $f(x) = (x+5)^2 - 2$ is

$$\boxed{f(x) = x^2 + 10x + 23}.$$

Answer 29e.

Rewrite $(x - 3)^2$.

$$y = (x - 3)(x - 3) + 6$$

Apply the FOIL method and multiply.

$$(x - 3)(x - 3) + 6 = x(x) + x(-3) + (-3)(x) + (-3)(-3) + 6$$
$$= x^2 - 3x - 3x + 9 + 6$$

Combine the like terms.

$$x^2 - 3x - 3x + 9 + 6 = x^2 + (-3 - 3)x + (9 + 6)$$
$$= x^2 - 6x + 15$$

Thus, the given function can be written in standard form as $y = x^2 - 6x + 15$.

Answer 30e.

Consider the quadratic function

$$g(x) = -(x+6)^2 + 10$$

We need to write the given quadratic function is standard form.

$g(x) = -(x+6)^2 + 10$	Write original function.
$g(x) = -(x+6)(x+6) + 10$	Rewrite $(x+6)^2$.
$g(x) = -(x^2 + 6x + 6x + 36) + 10$	Multiply using FOIL
$g(x) = -(x^2 + 12x + 36) + 10$	Combine like terms.
$g(x) = -x^2 - 12x - 36 + 10$	Distributive property
$g(x) = -x^2 - 12x - 26$	Simplify.

Therefore, the standard form of the given quadratic function $g(x) = -(x+6)^2 + 10$ is

$$\boxed{g(x) = -x^2 - 12x - 26}.$$

Answer 31e.

Rewrite $(x + 3)^2$.

$y = 5(x + 3)(x + 3) - 4$

Apply the FOIL method and multiply.

$$5(x + 3)(x + 3) - 4 = 5[x(x) + x(3) + 3(x) + 3(3)] - 4$$
$$= 5(x^2 + 3x + 3x + 9) - 4$$

Combine the like terms.

$$5(x^2 + 3x + 3x + 9) - 4 = 5[x^2 + (3 + 3)x + 9] - 4$$
$$= 5(x^2 + 6x + 9) - 4$$

Apply the distributive property.

$$5(x^2+6x+9)-4 = 5(x^2)+5(6x)+5(9)-4$$
$$= 5x^2+30x+45-4$$

Combine the like terms.

$$5x^2+30x+45-4 = 5x^2+30x+(45-4)$$
$$= 5x^2+30x+41$$

Thus, the given function can be written in standard form as $y=5x^2+30x+41$.

Answer 32e.

Consider the quadratic function

$$f(x)=12(x-1)^2+4$$

We need to write the given quadratic function is standard form.

$f(x)=12(x-1)^2+4$	Write original function.
$f(x)=12(x-1)(x-1)+4$	Rewrite $(x-1)^2$.
$f(x)=12(x^2-x-x+1)+4$	Multiply using FOIL
$f(x)=12(x^2-2x+1)+4$	Combine like terms.
$f(x)=12x^2-24x+12+4$	Distributive property
$f(x)=12x^2-24x+16$	Simplify.

Therefore, the standard form of the given quadratic function $f(x)=12(x-1)^2+4$ is

$$\boxed{f(x)=12x^2-24x+16}.$$

Answer 33e.

For finding the minimum or maximum value, we have to identify the vertex of the graph of the function.

The vertex form of a quadratic function is $y=a(x-h)^2+k$, where (h, k) is the vertex. On comparing the given function with the vertex form, we get $a=3$, $h=3$, and $k=-4$. Thus, the vertex is $(3, -4)$. Since $a > 0$, the graph opens up and so the function has a minimum value.

Therefore, the minimum value of the given function is -4.

Answer 34e.

Consider the quadratic function

$$g(x) = -4(x+6)^2 - 12$$

Then rewrite above function is $g(x) = -4(x-(-6))^2 - 12$

By comparing with the quadratic function in the vertex form, $y = a(x-h)^2 + k$

We get $a = -4, h = -6, k = -12$

Since $a(=-4) < 0$,

We have the function $g(x) = -4(x+6)^2 - 12$ has a maximum value.

In order to find the maximum value, we need to calculate the coordinates of the vertex.
The vertex is

$$(h, k) = (-6, -12)$$

Therefore, the maximum value of the given function $g(x) = -4(x+6)^2 - 12$ is $\boxed{-12}$.

Answer 35e.

For finding the minimum or maximum value, we have to identify the vertex of the graph of the function.

The vertex form of a quadratic function is $y = a(x - h)^2 + k$, where (h, k) is the vertex. On comparing the given function with the vertex form, we get $a = 15$, $h = 25$, and $k = 130$. Thus, the vertex is (25, 130). Since $a > 0$, the graph opens up and so the function has a minimum value.

The minimum or maximum value of a function is the y-coordinate of the vertex. Therefore, the minimum value of the given function is 130.

Answer 36e.

Consider the quadratic function

$$f(x) = 3(x+10)(x-8)$$

Then rewrite above function is $f(x) = 3(x-(-10))(x-8)$

By comparing with the quadratic function in the vertex form, $y = a(x-p)(x-q)$

We get $a = 3, p = -10, q = 8$

Since $a(=3) > 0$,

We have the function $f(x) = 3(x+10)(x-8)$ has a minimum value.

In order to find the minimum value, we need to calculate the coordinates of the vertex.
The vertex is

$$\left(\frac{p+q}{2}, \frac{-a(p-q)^2}{4}\right) = \left(\frac{-10+8}{2}, \frac{-3(-10-8)^2}{4}\right)$$
$$= \left(\frac{-2}{2}, \frac{-3(324)}{4}\right)$$
$$= (-1, -3(81))$$
$$= (-1, -243)$$

Therefore, the minimum value of the given function $f(x) = 3(x+10)(x-8)$ is $\boxed{-243}$.

Answer 37e.

For finding the minimum or maximum value, we have to identify the vertex of the graph of the function.

A quadratic function of the form $y = a(x-p)(x-q)$ is said to be in intercept form. The x-coordinate of the vertex of the graph of a function in this form is $x = \frac{p+q}{2}$.

On comparing the given function with the intercept form, we get $a = -1$, $p = 36$, and $q = -18$. Since $a < 0$, the graph opens down and so the function has a maximum value.

Substitute for p and q in $x = \frac{p+q}{2}$ and simplify.

$$x = \frac{36 + (-18)}{2}$$
$$= \frac{18}{2}$$
$$= 9$$

The x-coordinate of the vertex is 9.

The minimum or maximum value of a function is the y-coordinate of the vertex. For finding the y-coordinate, substitute 9 for x in the given function and simplify.

$$y = -(9-36)(9+18)$$
$$= -(-27)(27)$$
$$= 729$$

Therefore, the maximum value of the given function is 729.

Answer 38e.

Consider the quadratic function

$$y = -12x(x-9)$$

Then rewrite above function is $y = -12(x-0)(x-9)$

By comparing with the quadratic function in the vertex form, $y = a(x-p)(x-q)$

We get $a = -12, p = 0, q = 9$

Since $a(=-12) < 0$,

We have the function $y = -12x(x-9)$ has a maximum value.

In order to find the maximum value, we need to calculate the coordinates of the vertex.

The vertex is

$$\left(\frac{p+q}{2}, \frac{-a(p-q)^2}{4}\right) = \left(\frac{0+9}{2}, \frac{-(-12)(0-9)^2}{4}\right)$$
$$= \left(\frac{9}{2}, \frac{12(81)}{4}\right)$$
$$= \left(\frac{9}{2}, 3(81)\right)$$
$$= \left(\frac{9}{2}, 243\right)$$

Therefore, the maximum value of the given function $y = -12x(x-9)$ is $\boxed{243}$.

Answer 39e.

For finding the minimum or maximum value, we have to identify the vertex of the graph of the function.

A quadratic function of the form $y = a(x-p)(x-q)$ is said to be in intercept form. The x-coordinate of the vertex of the graph of a function in this form is $x = \frac{p+q}{2}$.

On comparing the given function with the intercept form, we get $a = 8, p = 0$, and $q = -15$. Since $a > 0$, the graph opens up and so the function has a minimum value.

Substitute for p and q in $x = \frac{p+q}{2}$ and simplify.

$$x = \frac{0+(-15)}{2}$$
$$= -\frac{15}{2}$$
$$= -7.5$$

The x-coordinate of the vertex is -7.5.

The minimum or maximum value of a function is the y-coordinate of the vertex. For finding the y-coordinate, substitute -7.5 for x in the given function and simplify.

$$y = 8(-7.5)(-7.5+15)$$
$$= 8(-7.5)(7.5)$$
$$= -450$$

Therefore, the minimum value of the given function is -450.

Answer 40e.

Consider the quadratic function

$$y = 2(x-3)(x-6)$$

By comparing with the quadratic function in the vertex form, $y = a(x-p)(x-q)$

We get $a = 2, p = 3, q = 6$

Since $a(=2) > 0$,

We have the function $y = 2(x-3)(x-6)$ has a minimum value.

In order to find the minimum value, we need to calculate the coordinates of the vertex. The vertex is

$$\left(\frac{p+q}{2}, \frac{-a(p-q)^2}{4}\right) = \left(\frac{3+6}{2}, \frac{-2(3-6)^2}{4}\right)$$
$$= \left(\frac{9}{2}, \frac{-2(9)}{4}\right)$$
$$= \left(\frac{9}{2}, -\frac{9}{2}\right)$$

Therefore, the minimum value of the given function $y = 2(x-3)(x-6)$ is $\boxed{-\frac{9}{2}}$.

Answer 41e.

For finding the minimum or maximum value, we have to identify the vertex of the graph of the function.

A quadratic function of the form $y = a(x - p)(x - q)$ is said to be in intercept form. The x-coordinate of the vertex of the graph of a function in this form is $x = \dfrac{p+q}{2}$.

On comparing the given function with the intercept form, we get $a = -5$, $p = -9$, and $q = 4$. Since $a < 0$, the graph opens down and so the function has a maximum value.

Substitute for p and q in $x = \dfrac{p+q}{2}$ and simplify.

$$\begin{aligned} x &= \frac{-9+4}{2} \\ &= -\frac{5}{2} \\ &= -2.5 \end{aligned}$$

The x-coordinate of the vertex is -2.5.

The minimum or maximum value of a function is the y-coordinate of the vertex. For finding the y-coordinate, substitute -2.5 for x in the given function and simplify.

$$\begin{aligned} g(-2.5) &= -5(-2.5+9)(-2.5-4) \\ &= -5(6.5)(-6.5) \\ &= 211.25 \end{aligned}$$

Therefore, the maximum value of the given function is 211.25.

Answer 42e.

Consider the function

$y = a(x-h)^2 + k$ Where $a=1, h=3, k=-2$

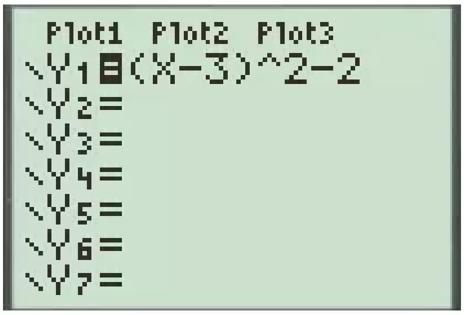
Substitute $a=1, h=3, k=-2$ in the original equation

$y = a(x-h)^2 + k$, get

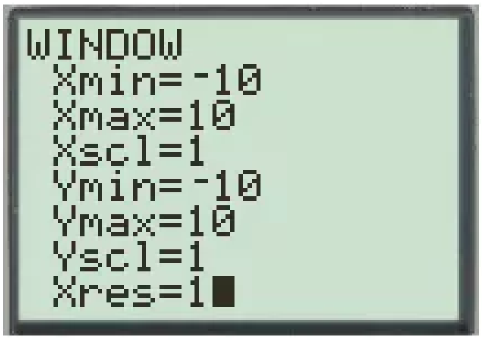
$y = (x-3)^2 - 2$

Use T1-83(graphing) calculator:

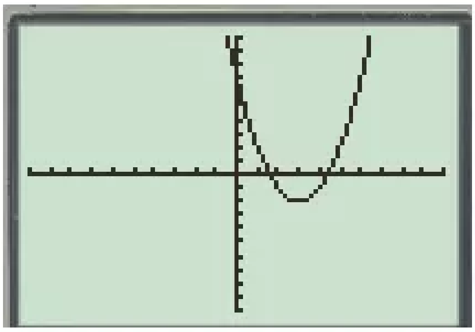
First enter the equation $y = (x-3)^2 - 2$



Enter window settings



Finally enter the graph button



(a) If a changes to -3, then

Substitute $a=-3, h=3, k=-2$ in the original equation

$y=a(x-h)^2+k$, get

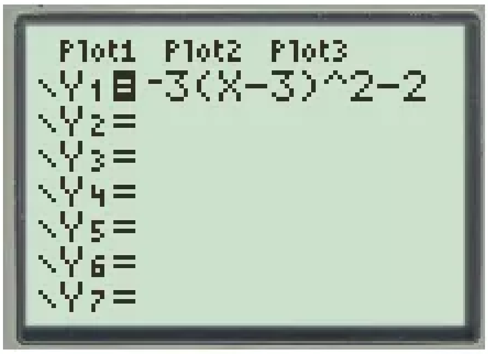
$y=-3(x-3)^2-2$

Thus,

The graph of the function will be narrower and it will face downwards.

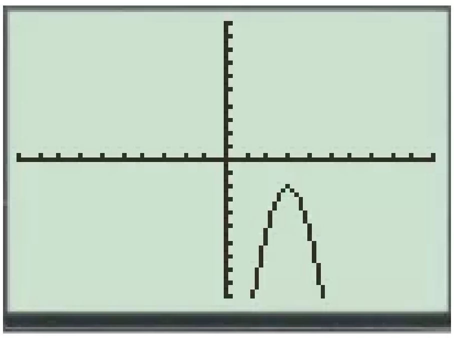
Use T1-83(graphing) calculator:

First enter the equation $y=-3(x-3)^2-2$



Enter window settings

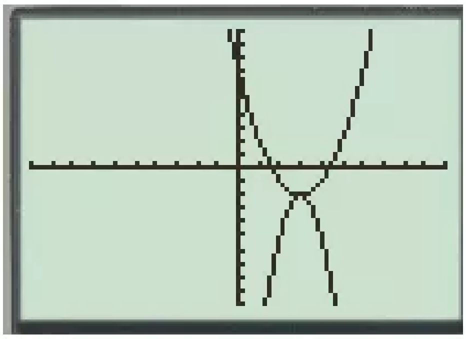
Finally enter the graph button



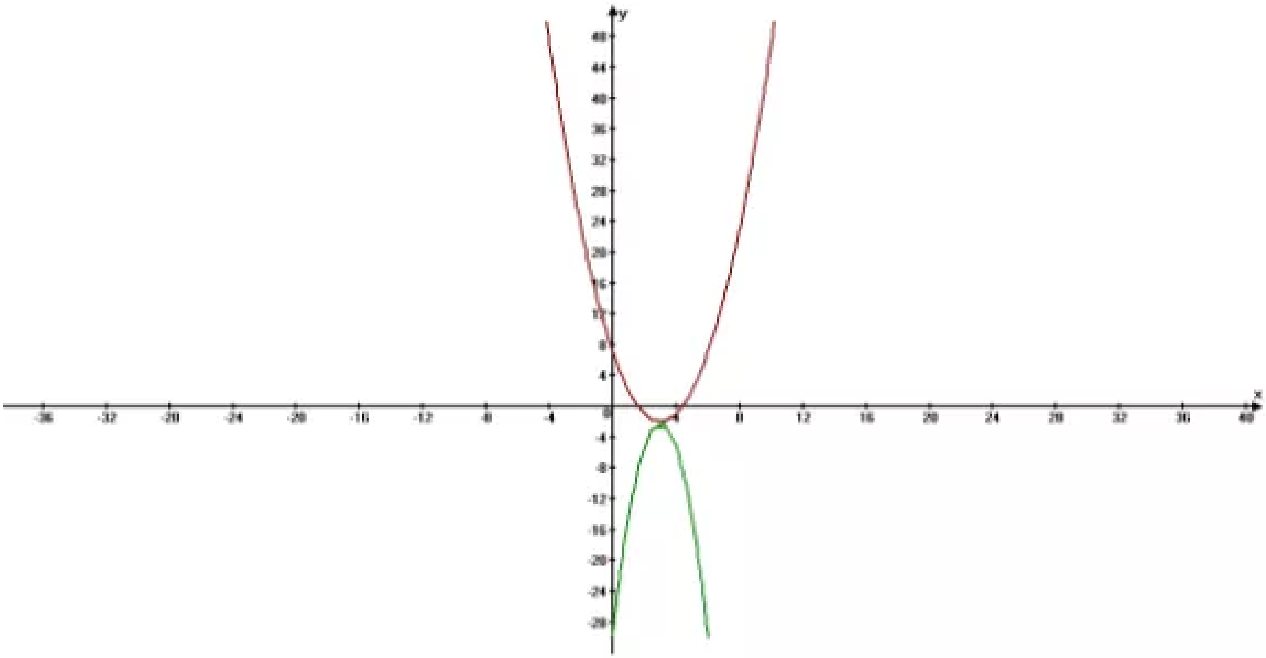
First enter the equations $y=(x-3)^2-2$ and $y=-3(x-3)^2-2$

Enter window settings

Finally enter the graph button



Clear the graph with both equations $y=(x-3)^2-2$ and $y=-3(x-3)^2-2$



In the above diagram the graph of $y=-3(x-3)^2-2$ is represented by green curve and the graph of $y=(x-3)^2-2$ is represented by red curve.

Hence our prediction about the behavior of the graph after applying the given changes is correct.

(b) If h changes to -1, then

Substitute $a=1, h=-1, k=-2$ in the original equation

$y=a(x-h)^2+k$, get

$y=(x+1)^2-2$

Thus,

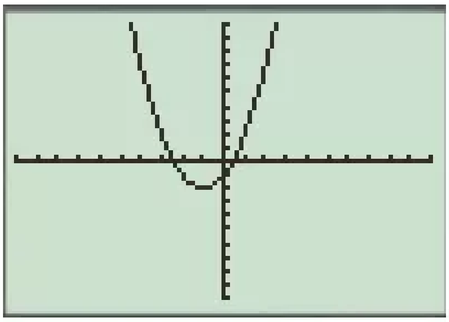
The graph of the function will be translated horizontally through a distance of 4 units along the direction of negative x-axis.

Use T1-83(graphing) calculator:

First enter the equation $y=(x+1)^2-2$

Enter window settings

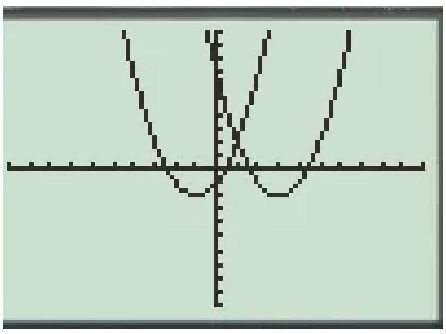
Finally enter the graph button



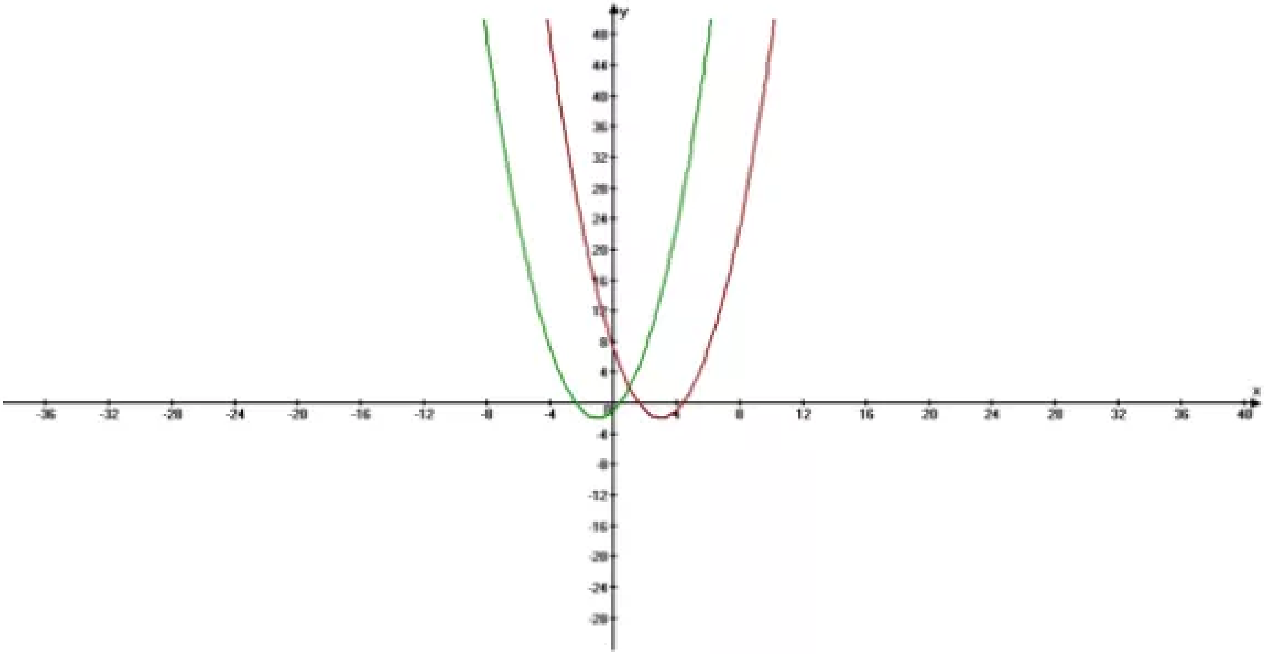
First enter the equations $y=(x+1)^2-2$ and $y=(x-3)^2-2$

Enter window settings

Finally enter the graph button



Clear graph



In the above diagram, the graph of $y=(x+1)^2-2$ is represented by green curve and the graph of $y=(x-3)^2-2$ is represented by red curve.

Hence our prediction about the behavior of the graph after applying the given changes is correct.

(c) If k changes to 2, then

Substitute $a=1, h=3, k=2$ in the original equation

$y=a(x-h)^2+k$, get

$y=(x-3)^2+2$

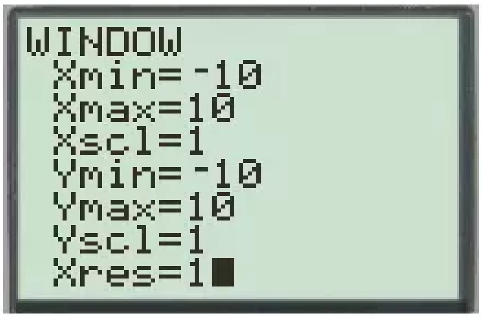
Thus,

The graph of the function will be translated vertically through a distance of 4 units along the direction of positive y-axis.

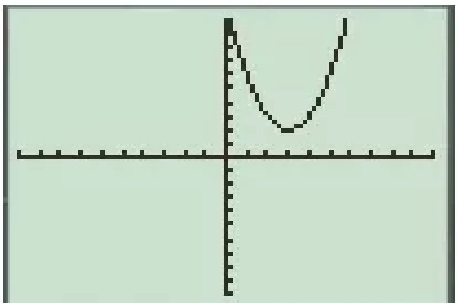
Use T1-83(graphing) calculator:

First enter the equation $y=(x-3)^2+2$

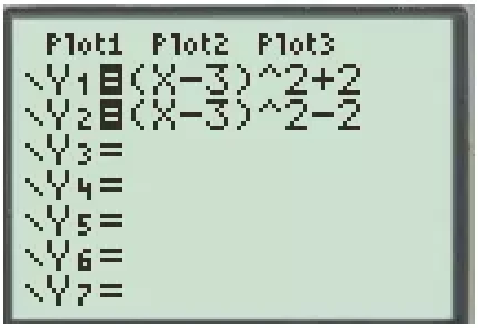
Enter window settings



Finally enter the graph button

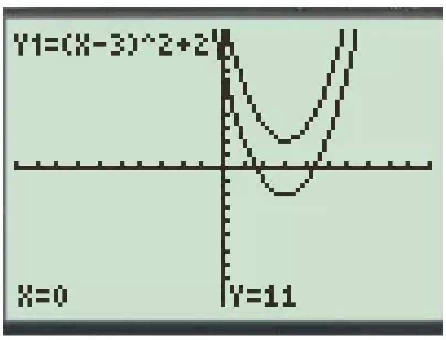


First enter the equations $y=(x-3)^2+2$ and $y=(x-3)^2-2$.

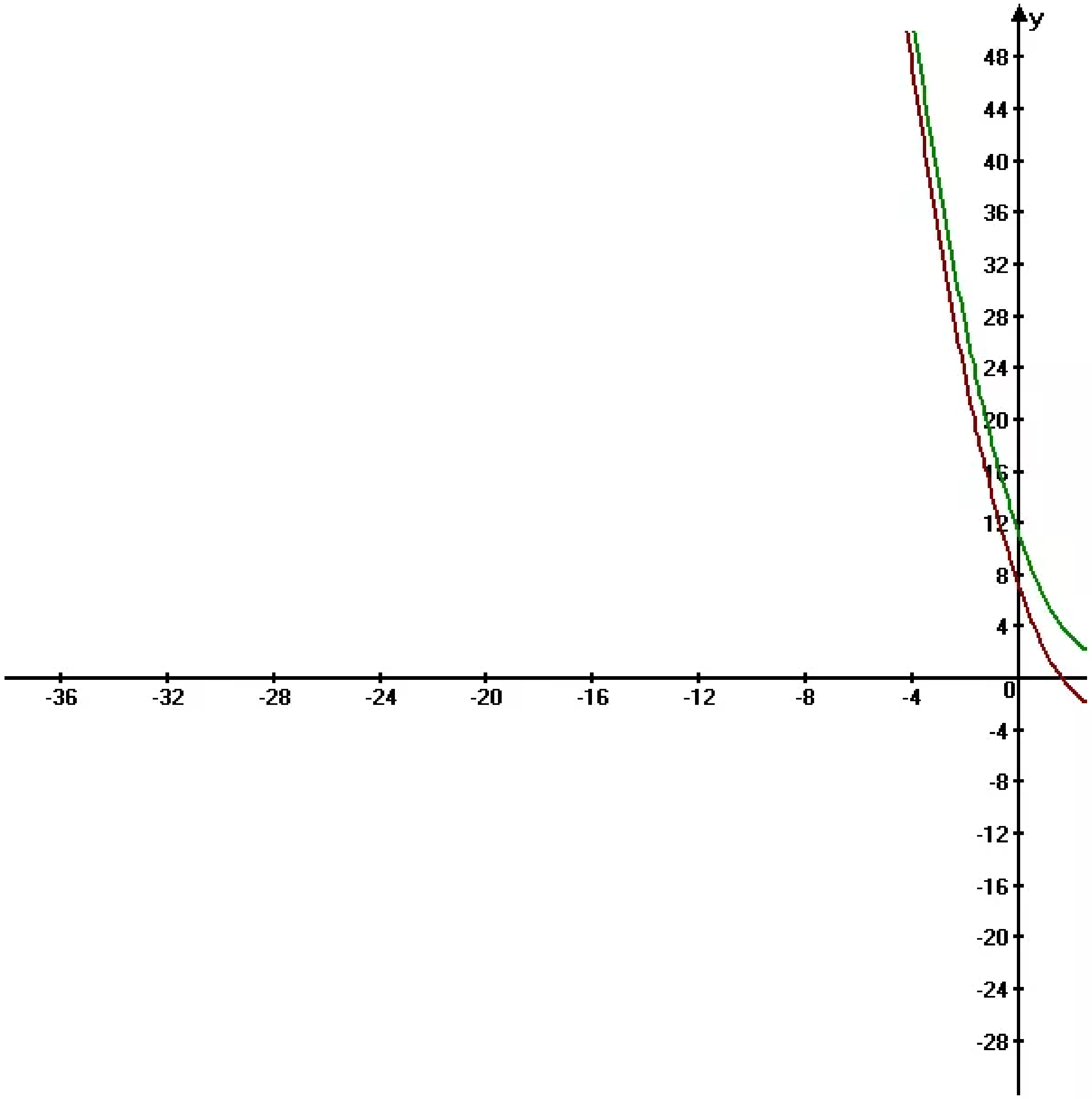


Enter window settings

Finally enter the graph button

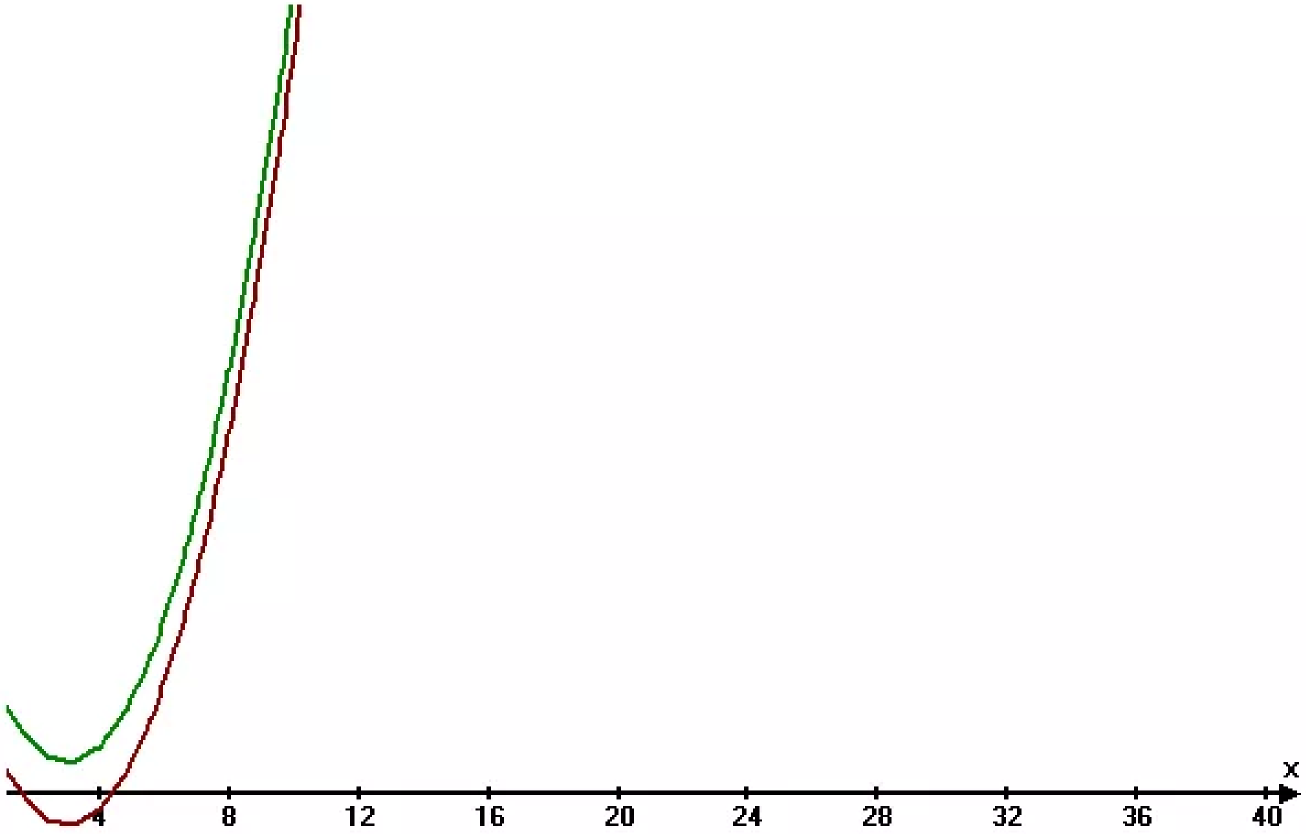


Clear graph



In the above diagram, the graph of $y=(x-3)^2+2$ is represented by green curve and the graph of $y=(x-3)^2+2$ is represented by red curve.

Hence our prediction about the behavior of the graph after applying the given changes is correct.

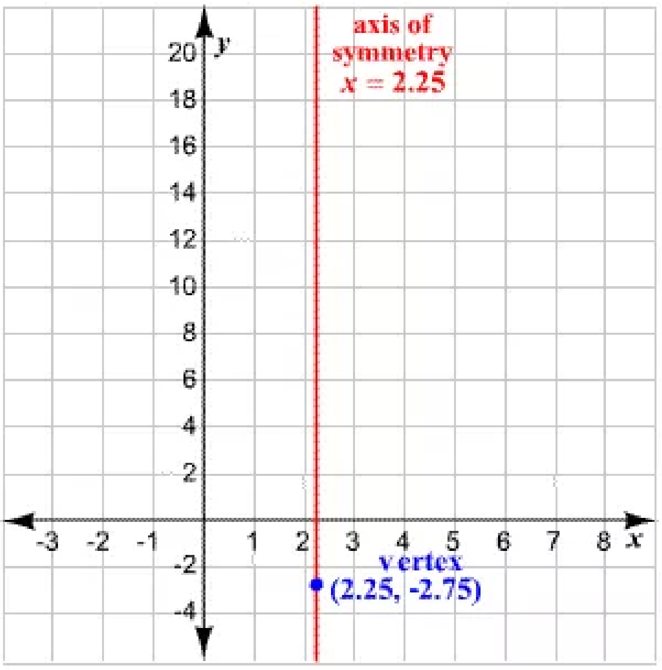


Answer 43e.

STEP 1 The graph of a quadratic function in vertex form $y = a(x - h)^2 + k$ has its vertex at (h, k) and $x = h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.
On comparing the given equation with the vertex form, we find that $a = 5$, $h = 2.25$, and $k = -2.75$. Thus, the vertex is $(h, k) = (2.25, -2.75)$ and the axis of symmetry is $x = 2.25$.
Since $a > 0$, the parabola opens up.

STEP 2 Plot the vertex (2.25, −2.75) on a coordinate plane and draw the axis of symmetry $x = 2.25$.



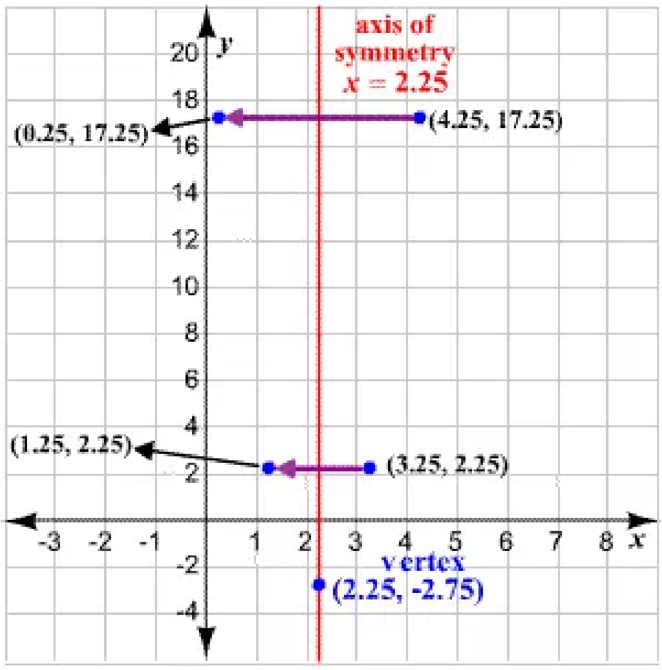
STEP 3 Evaluate the function for two values of x.

$$x = 3.25: \quad y = 5(3.25 - 2.25)^2 - 2.75 = 2.25$$

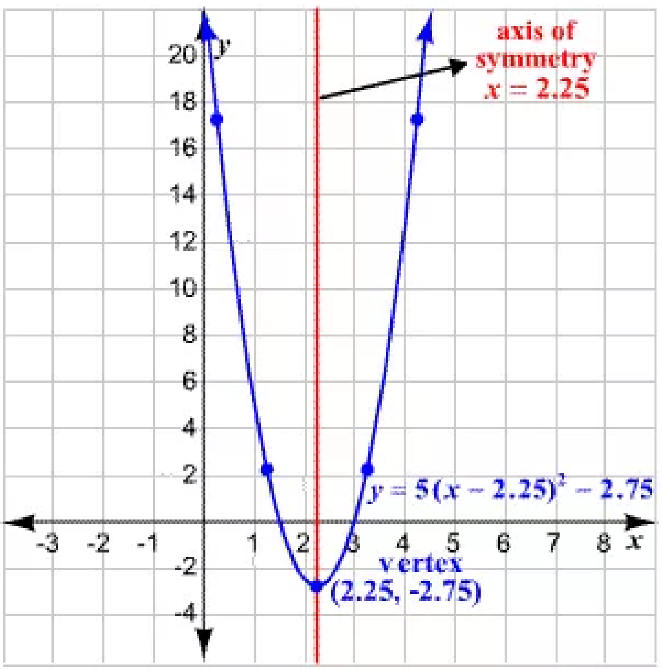
$$x = 4.25: \quad y = 5(4.25 - 2.25)^2 - 2.75 = 17.25$$

Thus, (3.25, 2.25) and (4.25, 17.25) are two points on the graph.

Now, plot the points (3.25, 2.25) and (4.25, 17.25) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.



Answer 44e.

Consider the function,

$$g(x) = -8(x+3.2)^2 + 6.4$$

Compare with the quadratic function in the vertex form,

$$y = a(x-h)^2 + k$$

Then, $a = -8, h = -3.2, k = 6.4$

The vertex,

$$(h,k) = (-3.2, 6.4)$$

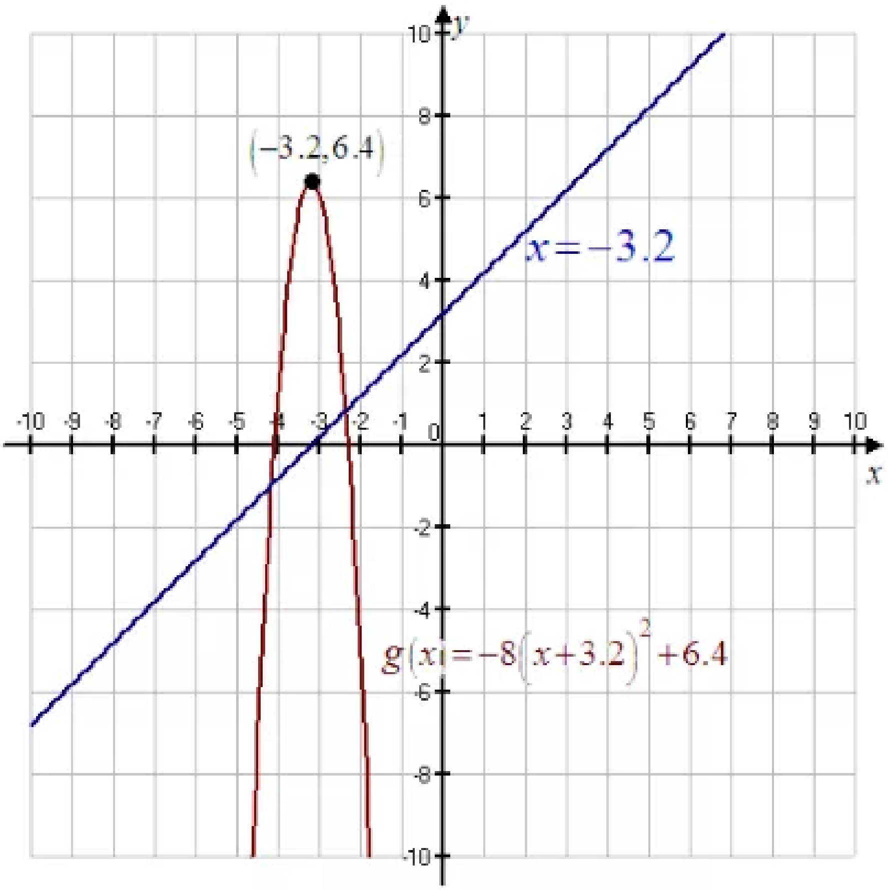
The axis of symmetry is,

$$x = h$$

$$x = -3.2$$

Here, $a < 0$ therefore, the graph opens down.

Sketch the graph of the function $g(x) = -8(x+3.2)^2 + 6.4$.

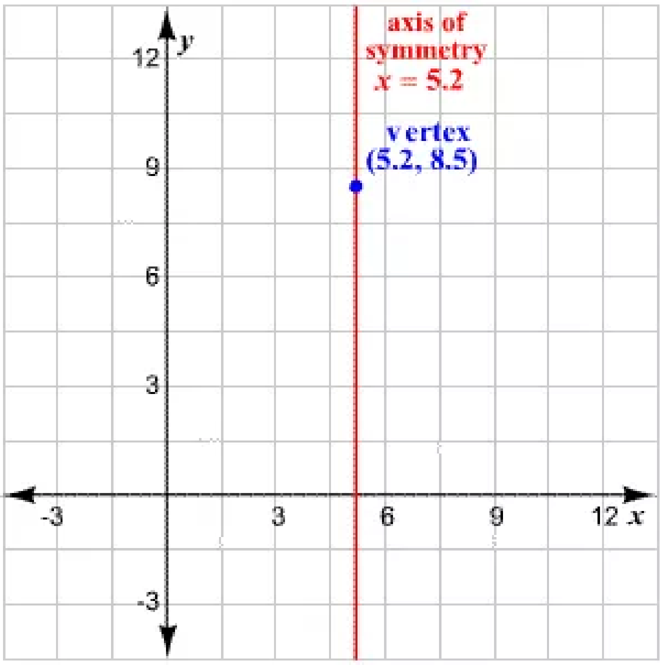


Answer 45e.

STEP 1 The graph of a quadratic function in vertex form $y = a(x-h)^2 + k$ has its vertex at (h, k) and $x = h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.
On comparing the given equation with the vertex form, we find that $a = -0.25$, $h = 5.2$, and $k = 8.5$. Thus, the vertex is $(h, k) = (5.2, 8.5)$ and the axis of symmetry is $x = 5.2$.
Since $a < 0$, the parabola opens down.

STEP 2 Plot the vertex (5.2, 8.5) on a coordinate plane and draw the axis of symmetry $x = 5.2$.



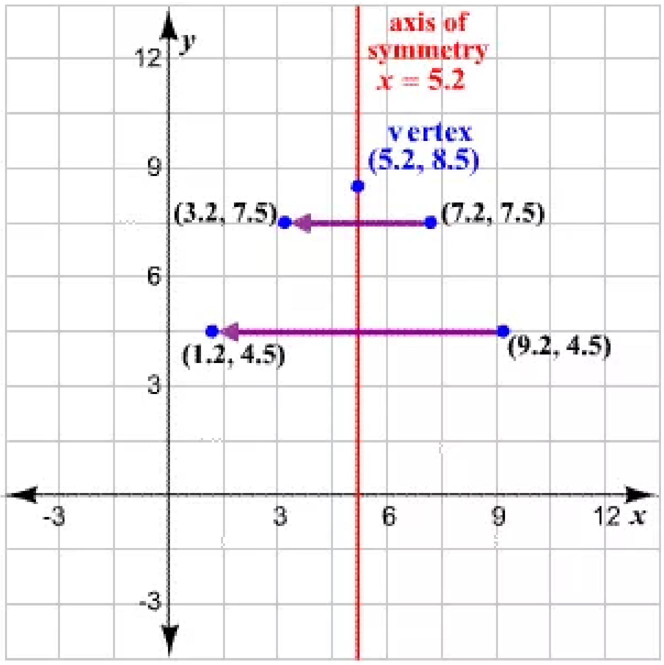
STEP 3 Evaluate the function for two values of x.

$$x = 7.2: \quad y = -0.25(7.2-5.2)^2 + 8.5 = 7.5$$

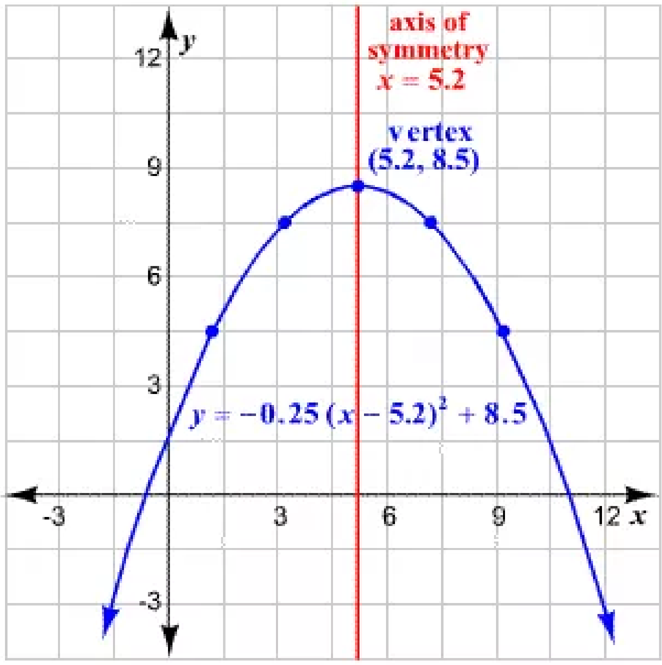
$$x = 9.2: \quad y = -0.25(9.2-5.2)^2 + 8.5 = 4.5$$

Thus, (7.2, 7.5) and (9.2, 4.5) are two points on the graph.

Now, plot the points (7.2, 7.5) and (9.2, 4.5) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.



Answer 46e.

Consider the function

$$y=-\frac{2}{3}\left(x-\frac{1}{2}\right)^2+\frac{4}{5}$$

By comparing with the quadratic function in the vertex form $y=a(x-h)^2+k$

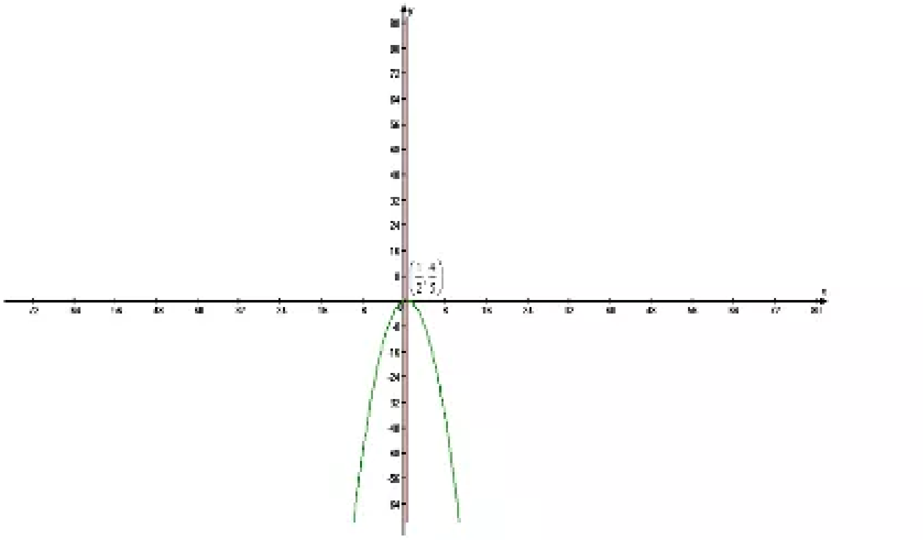
Found $a=-\frac{2}{3}, h=\frac{1}{2}, k=\frac{4}{5}$

The vertex is $(h,k)=\left(\frac{1}{2},\frac{4}{5}\right)$

The axis of symmetry is

$$x=h \Rightarrow x=\frac{1}{2}$$

The following diagram contains the graph of the function $y=-\frac{2}{3}\left(x-\frac{1}{2}\right)^2+\frac{4}{5}$



Answer 47e.

STEP 1 The intercept form of a quadratic function is $y=a(x-p)(x-q)$, where p and q are the x-intercepts and $x = \frac{p+q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.
On comparing the given equation with the intercept form, we find that $a = -\frac{3}{4}, p=-5$, and $k=-8$. Thus, the x-intercepts occur at (–5, 0) and (–8, 0). Since $a < 0$, the parabola opens down.

STEP 2 Then, find the coordinates of the vertex. Substitute for p and q in

$$x = \frac{p+q}{2}$$ and evaluate.

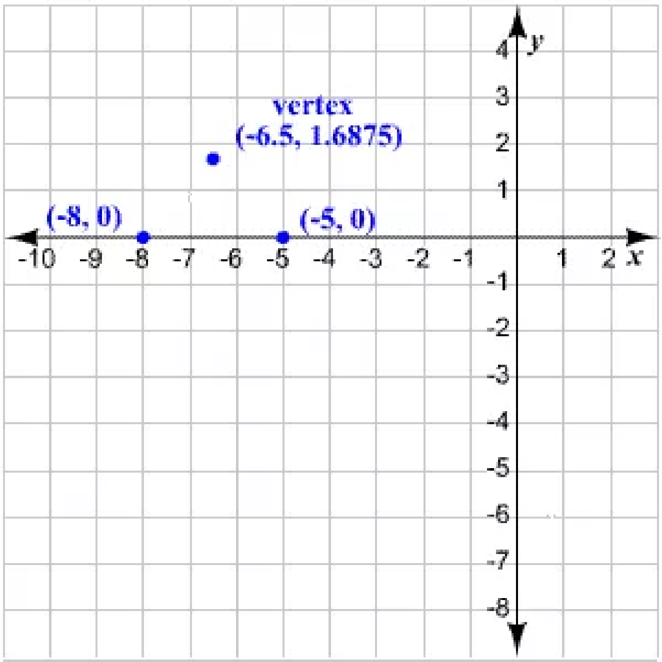
$$x = \frac{-5+(-8)}{2} = -6.5$$

Substitute −6.5 for x in the given function and evaluate y.

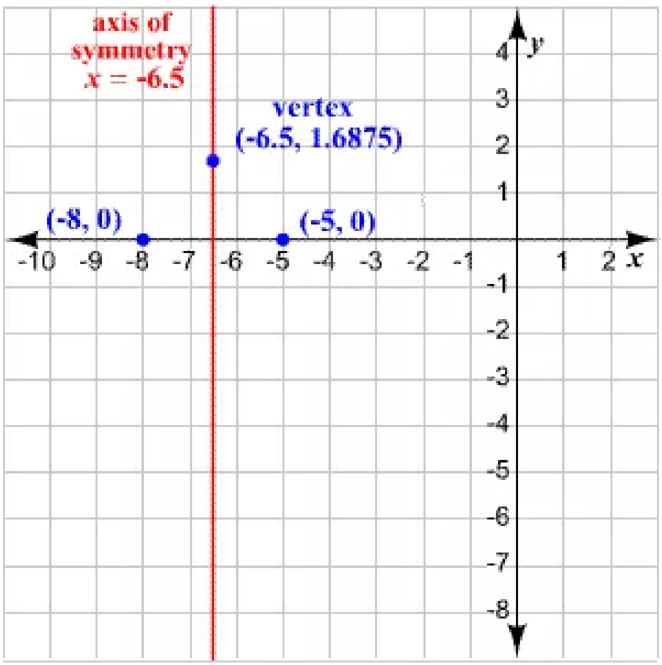
$$y = -\frac{3}{4}(-6.5+5)(-6.5+8)$$
$$= -\frac{3}{4}(-1.5)(1.5)$$
$$= 1.6875$$

Thus, the vertex is (−6.5, 1.6875).

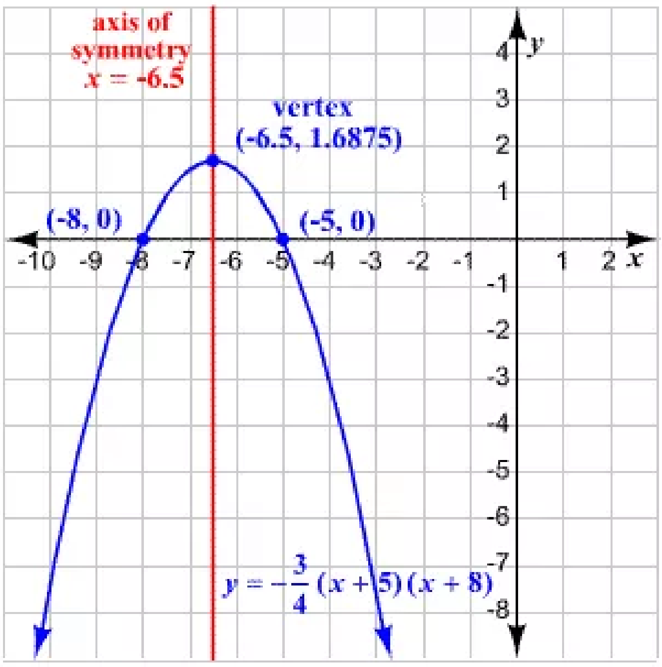
STEP 3 Now, plot the vertex and the points where the x-intercepts occur on a coordinate plane.



Draw the axis of symmetry $x = -6.5$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 48e.

Consider the function

$$g(x)=\frac{5}{2}\left(x-\frac{4}{3}\right)\left(x-\frac{2}{5}\right)$$

By comparing with the quadratic function in the intercept form $y=a(x-p)(x-q)$

Found $a=\frac{5}{2}, p=\frac{4}{3}, q=\frac{2}{5}$

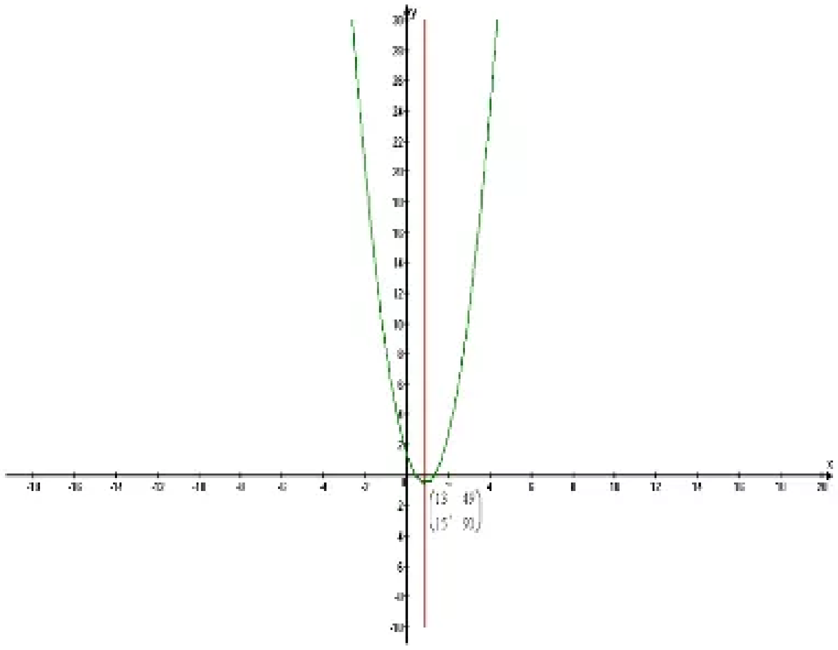
The vertex is $\left(\frac{p+q}{2}, \frac{-a(p-q)^2}{4}\right)=\left(\frac{\frac{4}{3}+\frac{2}{5}}{2}, \frac{-\frac{5}{2}\left(\frac{4}{3}-\frac{2}{5}\right)^2}{4}\right)$

$$=\left(\frac{13}{15}, -\frac{49}{90}\right)$$

The axis of symmetry is $x=\frac{p+q}{2}$

$$\Rightarrow x=\frac{13}{15}$$

The following diagram contains the graph of the function $y=\frac{5}{2}\left(x-\frac{4}{3}\right)\left(x-\frac{2}{5}\right)$



Answer 49e.

The intercept form of a quadratic function is $y = a(x - p)(x - q)$, where p and q are the x-intercepts. The equation for the axis of symmetry is $x = \frac{p+q}{2}$.

The axis of symmetry for the required functions is given as $x = 3$. Thus,

$$\frac{p+q}{2} = 3$$

or

$$p + q = 6.$$

Therefore, for any quadratic function with $x = 3$ as the axis of symmetry, the sum of the x-intercepts will be 6.

Choose any two pair of values for p and q so that their sum is 6.

$p = 2, q = 4$

$p = 5, q = 1$

Since $a \neq 0$, choose any two nonzero values for a. Let them be 1 and 2.

Thus, two different quadratic functions in intercept form whose graphs have axis of symmetry $x = 3$ are $y = (x - 2)(x - 4)$ and $y = 2(x - 5)(x - 1)$.

Answer 50e.

Consider the function

$$y = a(x-h)^2 + k$$

Let us write the above quadratic function in standard form

$$y = a(x-h)^2 + k$$

$$\Rightarrow \quad y = a(x^2 - 2hx + h^2) + k \qquad \text{(By applying distributive property)}$$

$$\Rightarrow \quad y = ax^2 - 2ahx + ah^2 + k$$

$$\Rightarrow \quad y = ax^2 + (-2ah)x + (ah^2 + k)$$

Comparing the given function with $y = ax^2 + bx + c$

Then $a = a, b = -2ah, c = ah^2 + k$

x-coordinate of the vertex is $-\frac{b}{2a} = -\frac{(-2ah)}{2a}$

$$\Rightarrow -\frac{(-2ah)}{2a} = h$$

The vertex of the function is at $x = h$

Consider the function

$$y=a(x-p)(x-q)$$

Let us write the above quadratic function in standard form

$$y=a(x-p)(x-q)$$

$\Rightarrow \quad y=a\left(x^2-qx-px+pq\right)$ (Multiply using FOIL)

$\Rightarrow \quad y=a\left(x^2-(p+q)x+pq\right)$ (By applying distributive property)

$\Rightarrow \quad y=ax^2-a(p+q)x+apq$

Comparing the given function with $y=ax^2+bx+c$

Then $a=a, b=-a(p+q), c=apq$

x-coordinate of the vertex is

$$-\frac{b}{2a}=-\frac{(-a(p+q))}{2a}$$

$$=\frac{p+q}{2}$$

The vertex of the function is at $x=\dfrac{p+q}{2}$

Answer 51e.

In this case, the maximum height is the same as the maximum value of the function. We know that the maximum value of a quadratic function is the y-coordinate of the vertex.

The vertex form of a quadratic function is $y=a(x-h)^2+k$, where (h, k) is the vertex. On comparing the given function with the vertex form, we get $a=-0.03$, $h=14$, and $k=6$. Thus, the vertex is (14, 6) and so the maximum value of the function is 6.

Therefore, the maximum height is 6 ft.

The x-coordinate of the vertex represents the horizontal distance traveled by the animal when it reaches the maximum height.

Since the x-coordinate of the vertex is 14, the jump is 14 ft long.

Answer 52e.

Consider the arch of a bridge forms a parabola with equation

$y=-0.016(x-52.5)^2+45$ where x is the horizontal distance (meters) from the Arch's left end and y is the distance (meters) from the base of the arch

Let λ be the width of the arch

Then $y(\lambda)=y(0)$

$\Rightarrow \quad -0.016(\lambda-52.5)^2+45=-0.016(52.5)^2+45$

$\Rightarrow \quad (\lambda-52.5)^2=(52.5)^2$

$\Rightarrow \quad \lambda-52.5=\pm 52.5$

$\Rightarrow \quad \lambda=52.5\pm 52.5$

$\Rightarrow \quad \lambda=0,105$ (But $\lambda\neq 0$)

Therefore the width of the arch is 105 meters

Answer 53e.

a) In this case, the field's width is the same as the width of the parabola. We know that the width of a parabola is the difference between its x-intercepts.

The x-intercepts of the graph of a quadratic function in the form $y = a(x - p)(x - q)$ are p and q. The given function can be rewritten as $y = -0.000234(x - 0)(x - 160)$. On comparing, we get $a = -0.000234$, $p = 0$, and $q = 160$.

Thus, we can conclude that the field's width is 160 ft.

b) The maximum height of the field's surface is the maximum value of the given function. The maximum value of a quadratic function is the y-coordinate of the vertex.

First, find the x-coordinate of the vertex. The x-coordinate of the vertex of a function in intercept form is given by $x = \frac{p+q}{2}$.

Substitute for p and q, and simplify.

$$\begin{aligned} x &= \frac{0+160}{2} \\ &= \frac{160}{2} \\ &= 80 \end{aligned}$$

The x-coordinate of the vertex is 80.

Now, find the y-coordinate of the vertex. For this, substitute 80 for x in the given function and simplify.

$$\begin{aligned} y &= -0.000234(80)(80-160) \\ &= -0.000234(80)(-80) \\ &\approx 1.5 \end{aligned}$$

The y-coordinate of the vertex is about 1.5. Therefore, the field's surface has a maximum height of about 1.5 ft.

Answer 54e.

Consider two quadratic functions

$$y=-0.5(x-6)^2+18 \text{ and } y=-1.17(x-6)^2+42$$

Take the function $y=-0.5(x-6)^2+18$

By comparing with the quadratic function in the vertex form $y=a(x-h)^2+k$

Then $a=-0.5, h=6, k=18$

The vertex is $(h,k)=(6,18)$

Therefore the maximum value of this function is 18

Take the function $y=-1.17(x-6)^2+42$

By comparing with the quadratic function in the vertex form $y=a(x-h)^2+k$

Then $a=-1.17, h=6, k=42$

The vertex is $(h,k)=(6,42)$

Therefore the maximum value of this function is 42

Clearly the second quadratic function has the larger maximum value of 42

And it depends only on k

Answer 55e.

A kernel of popcorn contains water that expends when the kernel is heated causing it to pop.

The equations below gives the popping volume y (in cubic centimeters per gram)

Of popcorn with moisture content x (as a percent of the popcorn's weight)

Hot-air popping:

$$y=-0.761(x-5.52)(x-22.6)$$

Hot-oil popping:

$$y=-0.652(x-5.35)(x-21.8)$$

(a)

For hot-air popping find the moisture content that maximizes popping volume and the maximum volume.

Given the function

$$y=-0.761(x-5.52)(x-22.6)$$

By comparing with the quadratic function in the intercept form $y=a(x-p)(x-q)$

Then $a=-0.761, p=5.52, q=22.6$

The vertex is $\left(\frac{p+q}{2}, \frac{-a(p-q)^2}{4}\right)=(14.06, 55.5)$

Therefore, the moisture content that maximizes popping volume is 14.06% and the maximum volume is $55.5 \text{ cm}^3/g$

(b)

For hot-oil popping, we need to find the moisture content that maximizes popping volume and the maximum volume.

Given the function $y=-0.652(x-5.35)(x-21.8)$

By comparing with the quadratic function in the intercept form is

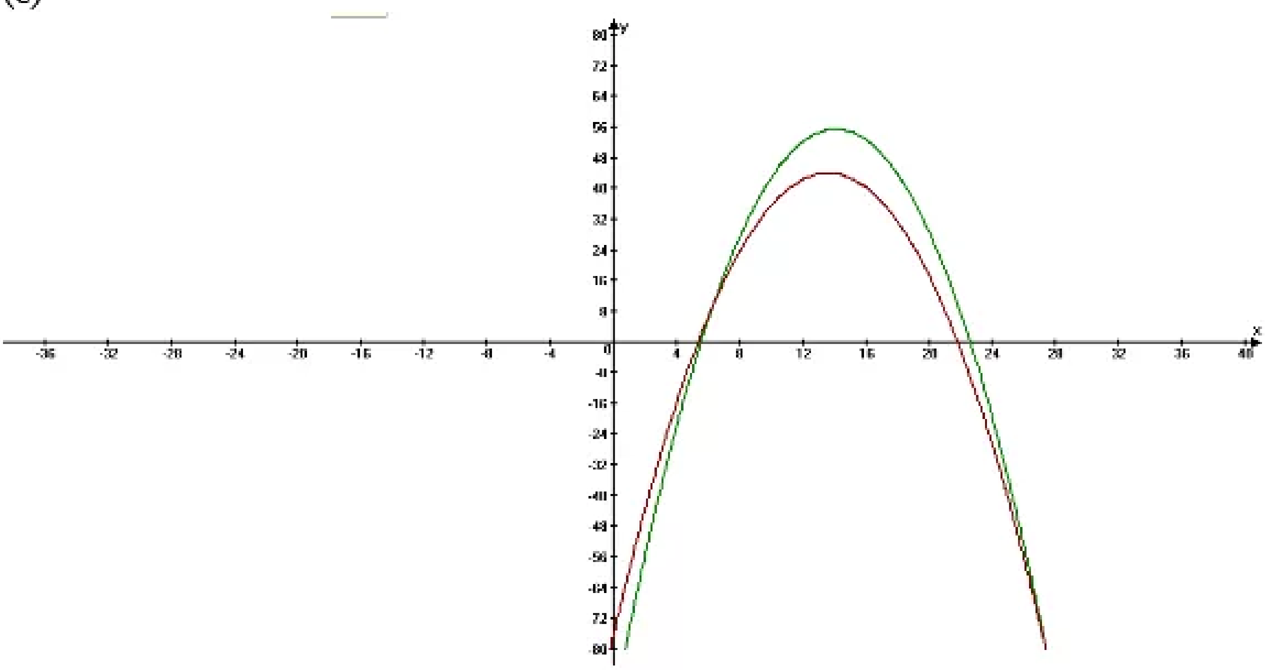
$$y=a(x-p)(x-q)$$

Then $a=-0.652, p=5.35, q=21.8$

The vertex is $\left(\frac{p+q}{2}, \frac{-a(p-q)^2}{4}\right)=(13.575, 44.1)$

Therefore the moisture content that maximizes popping volume is 13.575% and the maximum volume is $44.1\ \text{cm}^3/\text{g}$

(c)



In the above diagram the graph of $y=-0.761(x-5.52)(x-22.6)$ is represented by green curve and the graph of $y=-0.652(x-5.35)(x-21.8)$ is represented by red curve

The domain and range of $y=-0.761(x-5.52)(x-22.6)$ are $[5.52, 22.6]$ and $[0, 55.5]$ respectively

The domain and range of $y=-0.652(x-5.35)(x-21.8)$ are $[5.35, 21.8]$ and $[0, 44.1]$ respectively

The domain of both functions are identified by considering those points for which the graph lies on or above x-axis

The range of both functions are identified by considering the points varying from 0 to their respective maximum values

Answer 56e.

A flying fish reaches a maximum height of 5 feet after flying a horizontal distance of 33 feet

Take the quadratic function

$$y = a(x-h)^2 + k$$

That models the flight path assuming the fish leaves the water at $(0,0)$

Clearly the vertex of this quadratic function is $(h,k) = (5,33)$

Hence the function is $y = a(x-5)^2 + 33$

Put $(x,y) = (0,0)$

$$\Rightarrow \quad 0 = a(-5)^2 + 33$$
$$\Rightarrow \quad 0 = 25a + 33$$
$$\Rightarrow \quad a = -\frac{33}{25}$$

Therefore the quadratic function is

$$y = -\frac{33}{25}(x-5)^2 + 33$$

And a determines how long the flight can be

h determines at which point the height can be maximum and k determines the maximum height.

Answer 57e.

Add 5 to both sides of the given equation.

$$x - 5 + 5 = 0 + 5$$
$$x = 5$$

The solution is 5.

CHECK

Substitute 5 for x in the original equation.

$$x - 5 = 0$$
$$5 - 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

The solution checks.

Answer 58e.

Consider the equation

$$2x+3=0 \quad \text{......(1)}$$

$\Rightarrow \quad (2x+3)+(-3)=0+(-3)$ Adding both sides -3

$\Rightarrow \quad 2x+(3-3)=-3$

$\Rightarrow \quad 2x+0=-3$

$\Rightarrow \quad 2x=-3$ Dividing both sides with 2

$\Rightarrow \quad 2x\left(\frac{1}{2}\right)=-3\left(\frac{1}{2}\right)$

$\Rightarrow \quad x=-\frac{3}{2}$ is the only solution of the given equation

Answer 59e.

Add $5x$ to each side of the given equation.

$$23x - 14 + 5x = -5x - 7 + 5x$$

$$28x - 14 = -7$$

Add 14 to each side.

$$28x - 14 + 14 = -7 + 14$$

$$28x = 7$$

Divide each side by 28.

$$\frac{28x}{28} = \frac{7}{28}$$

$$x = \frac{1}{4}$$

The solution is $\frac{1}{4}$.

CHECK

Substitute $\frac{1}{4}$ for x in the original equation.

$$23x - 14 = -5x - 7$$

$$23\left(\frac{1}{4}\right) - 14 \stackrel{?}{=} -5\left(\frac{1}{4}\right) - 7$$

$$\frac{23}{4} - 14 \stackrel{?}{=} -\frac{5}{4} - 7$$

$$-\frac{33}{4} = -\frac{33}{4} \checkmark$$

Thus, the solution checks.

Answer 60e.

Consider the equation

$$-5(3x+4)=17x+2$$

$$-5(3x+4)=17x+2 \quad \text{By applying distributive property}$$

$$\Rightarrow -15x-20=17x+2$$

$$\Rightarrow (-15x-20)+(-17x+20)=(17x+2)+(-17x+20)$$

$$\Rightarrow (-15x-17x)-(-20+20)=(17x-17x)+(2+20)$$

$$\Rightarrow -32x=22$$

$$\Rightarrow x=-\frac{11}{16} \text{ is the only solution of the given equation}$$

Answer 61e.

Write equivalent equations.

$x - 9 = 16$ or $x - 9 = -16$

Add 9 to each side of both the equations.

$x - 9 + 9 = 16 + 9$ or $x - 9 + 9 = -16 + 9$

Simplify.

$x = 25$ or $x = -7$

The solutions are 25 and −7.

Answer 62e.

Consider the equation

$$|4x+9|=27$$

$$\Rightarrow 4x+9=\pm 27$$

$$\Rightarrow 4x=-9\pm 27$$

$$\Rightarrow 4x=-36,18$$

$$\Rightarrow x=-9,\frac{9}{2} \text{ are the solutions of the given equation}$$

Answer 63e.

Write equivalent equations.

$7 - 2x = 1$ or $7 - 2x = -1$

Subtract 7 from each side of both the equations.

$7 - 2x - 7 = 1 - 7$ or $7 - 2x - 7 = -1 - 7$

Simplify.

$-2x = -6$ or $-2x = -8$

Divide each side of both the equations by -2.

$$\frac{-2x}{-2} = \frac{-6}{-2} \quad \text{or} \quad \frac{-2x}{-2} = \frac{-8}{-2}$$

Simplify.

$$x = 3 \quad \text{or} \quad x = 4$$

The solutions are 3 and 4.

Answer 64e.

Consider the equation,

$$|3-5x| = 7$$

That is

$$|5x-3| = 7 \qquad (\text{If } |x| = a \text{ then } x = \pm a)$$

$$5x-3 = \pm 7$$

$$5x-3+3 = \pm 7+3 \qquad (\text{Add 3 on both sides})$$

$$5x = 3 \pm 7$$

$$5x = -4, 10 \qquad (\text{Simplify})$$

$$x = \boxed{-\frac{4}{5}, 2} \quad \text{are the solutions .}$$

Answer 65e.

First, substitute for A and B in $2A + B$.

$$2A + B = 2\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix}$$

For multiplying a matrix by a scalar, multiply each element in the matrix by the scalar.

$$2\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix} = \begin{bmatrix} 2(-1) & 2(3) \\ 2(2) & 2(-5) \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix}$$

Simplify.

$$\begin{bmatrix} 2(-1) & 2(3) \\ 2(2) & 2(-5) \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ 4 & -10 \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix}$$

For adding two matrices, add elements in corresponding positions.

$$\begin{bmatrix} -2 & 6 \\ 4 & -10 \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix} = \begin{bmatrix} -2+2 & 6+(-6) \\ 4+3 & -10+8 \end{bmatrix}$$

Simplify.

$$\begin{bmatrix} -2+2 & 6+(-6) \\ 4+3 & -10+8 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 7 & -2 \end{bmatrix}$$

Therefore,

$$2A + B = \begin{bmatrix} 0 & 0 \\ 7 & -2 \end{bmatrix}.$$

Answer 66e.

Consider,

$$B=\begin{bmatrix}2 & -6\\ 3 & 8\end{bmatrix} \text{ and } C=\begin{bmatrix}-1 & 4\\ -2 & 3\end{bmatrix}$$

Then $3(B+C)$

$$=3\left(\begin{bmatrix}2 & -6\\ 3 & 8\end{bmatrix}+\begin{bmatrix}-1 & 4\\ -2 & 3\end{bmatrix}\right)$$ (Add corresponding terms)

$$=3\begin{bmatrix}2+(-1) & -6+4\\ 3+(-2) & 8+3\end{bmatrix}$$

$$=3\begin{bmatrix}1 & -2\\ 1 & 11\end{bmatrix}$$ (Multiply every element by 3)

$$=\begin{bmatrix}3(1) & 3(-2)\\ 3(1) & 3(11)\end{bmatrix}$$

$$=\boxed{\begin{bmatrix}3 & -6\\ 3 & 33\end{bmatrix}}$$

Answer 67e.

First, substitute for D and E in $D-4E$.

$$D-4E=\begin{bmatrix}3 & 0 & -1\\ 6 & 1 & 4\end{bmatrix}-4\begin{bmatrix}1 & -2 & 4\\ 3 & -1 & 5\end{bmatrix}$$

For multiplying a matrix by a scalar, multiply each element in the matrix by the scalar.

$$\begin{bmatrix}3 & 0 & -1\\ 6 & 1 & 4\end{bmatrix}-4\begin{bmatrix}1 & -2 & 4\\ 3 & -1 & 5\end{bmatrix}=\begin{bmatrix}3 & 0 & -1\\ 6 & 1 & 4\end{bmatrix}-\begin{bmatrix}4(1) & 4(-2) & 4(4)\\ 4(3) & 4(-1) & 4(5)\end{bmatrix}$$

Simplify.

$$\begin{bmatrix}3 & 0 & -1\\ 6 & 1 & 4\end{bmatrix}-\begin{bmatrix}4(1) & 4(-2) & 4(4)\\ 4(3) & 4(-1) & 4(5)\end{bmatrix}=\begin{bmatrix}3 & 0 & -1\\ 6 & 1 & 4\end{bmatrix}-\begin{bmatrix}4 & -8 & 16\\ 12 & -4 & 20\end{bmatrix}$$

For subtracting two matrices, subtract elements in corresponding positions.

$$\begin{bmatrix}3 & 0 & -1\\ 6 & 1 & 4\end{bmatrix}-\begin{bmatrix}4 & -8 & 16\\ 12 & -4 & 20\end{bmatrix}=\begin{bmatrix}3-4 & 0-(-8) & -1-16\\ 6-12 & 1-(-4) & 4-20\end{bmatrix}$$

Simplify.

$$\begin{bmatrix}3-4 & 0-(-8) & -1-16\\ 6-12 & 1-(-4) & 4-20\end{bmatrix}=\begin{bmatrix}-1 & 8 & -17\\ -6 & 5 & -16\end{bmatrix}$$

Therefore,

$$D-4E=\begin{bmatrix}-1 & 8 & -17\\ -6 & 5 & -16\end{bmatrix}.$$

Answer 68e.

Consider,

$$A = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix}$$

The product of matrices is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix} \quad \text{...... (1)}$$

Now,

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -2+9 & 6+24 \\ 4-15 & -12-40 \end{bmatrix} \quad \text{(Using equation (1))}$$

$$= \boxed{\begin{bmatrix} 7 & 30 \\ -11 & -52 \end{bmatrix}}$$

Answer 69e.

First, substitute for A, B, and C in $A(B-C)$.

$$A(B-C) = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\left(\begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}\right)$$

For subtracting two matrices, subtract elements in corresponding positions.

$$\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\left(\begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}\right) = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 2-(-1) & -6-4 \\ 3-(-2) & 8-3 \end{bmatrix}$$

Simplify.

$$\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 2-(-1) & -6-4 \\ 3-(-2) & 8-3 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 2+1 & -6-4 \\ 3+2 & 8-3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 3 & -10 \\ 5 & 5 \end{bmatrix}$$

Multiply the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Then, add the products and put the result in the first row, first column of the product matrix.

$$\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 3 & -10 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} (-1)(3)+3(5) & \\ & \end{bmatrix}$$

Now, write an expression for the element in the first row, second column. For this, multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Then, add the products and put the result in the first row, second column of the product matrix.

$$\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 3 & -10 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} (-1)(3)+3(5) & (-1)(-10)+3(5) \\ & \end{bmatrix}$$

Similarly, find the elements in the second row of the product matrix.

$$\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 3 & -10 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} (-1)(3)+3(5) & (-1)(-10)+3(5) \\ 2(3)+(-5)(5) & 2(-10)+(-5)(5) \end{bmatrix}$$

Finally simplify the product matrix.

$$\begin{bmatrix} (-1)(3)+3(5) & (-1)(-10)+3(5) \\ 2(3)+(-5)(5) & 2(-10)+(-5)(5) \end{bmatrix} = \begin{bmatrix} 12 & 25 \\ -19 & -45 \end{bmatrix}$$

Thus, we get:

$$A(B-C) = \begin{bmatrix} 12 & 25 \\ -19 & -45 \end{bmatrix}.$$

Answer 70e.

Consider,

$$C = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix} \text{ and } D = \begin{bmatrix} 3 & 0 & -1 \\ 6 & 1 & 4 \end{bmatrix}$$

The product of matrices is defined as

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix} \quad \text{......} \; (1)$$

Then $4(CD)$

$$= 4\begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 & -1 \\ 6 & 1 & 4 \end{bmatrix}$$

$$= 4\begin{bmatrix} -3+24 & 0+4 & 1+16 \\ -6+18 & 0+3 & 2+12 \end{bmatrix} \quad \text{(Using equation (1))}$$

$$= 4\begin{bmatrix} 21 & 4 & 17 \\ 12 & 3 & 14 \end{bmatrix}$$

$$= \begin{pmatrix} 4(21) & 4(4) & 4(17) \\ 4(12) & 4(3) & 4(14) \end{pmatrix}$$

$$= \boxed{\begin{bmatrix} 84 & 16 & 68 \\ 48 & 12 & 56 \end{bmatrix}} \quad \text{(Multiply every element by 4)}$$