Chapter

3 Average

AVERAGE

'Average' is a very simple but effective way of representing an entire group by a single value.

$$\textbf{Average or Mean} = \frac{\text{Sum of given quantities}}{\text{Number of quantities}}$$

To calculate the sum of quantities, they should be in the same unit.

☞ **Shortcut Approach**

If X is the average of $x_1, x_2, x_3 \ldots x_n$ then

(a) The average of $x_1 + a, x_2 + a, x_3 + a, \ldots x_n + a$ is $X + a$.

(b) The average of $x_1 - a, x_2 - a, x_3 - a \ldots x_n - a$ is $X - a$

(c) The average of $ax_1, ax_2, \ldots ax_n$ is aX, provied $a \neq 0$

(d) The average of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots \frac{x_n}{a}$ is $\frac{x}{a}$, provided $a \neq 0$

See Example : Refer ebook Solved Examples/Ch-3

Average of a group consisting two different groups when their averages are known :

Let Group A contains m quantities and their average is a and Group B contains n quantities and their average is b, then **average of group C** containing m + n quantities $= \frac{ma + mb}{m + n}$

WEIGHTED AVERAGE

If we have two or more groups of members whose individual averages are known, then combined average of all the members of all the groups is known as weighted average. Thus if there are k groups having member of number $n_1, n_2, n_3, \ldots\ldots n_k$ with averages $A_1, A_2,$

A_3........A_k respectively then weighted average.

$$A_w = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + ... + n_k A_k}{n_1 + n_2 + n_3 + ... + n_k}$$

☞ **Shortcut Approach**

If, in a group, one or more new quantities are added or excluded, then the new quantity or sum of added or excluded quantities = [Change in no. of quantities × original average] ± [change in average × final no. of quantities]

Take +ve sign if quantities added and

take –ve sign if quantities removed.

See Example : Refer ebook Solved Examples/Ch-3

AVERAGE SPEED IF EQUAL DISTANCES ARE TRAVELLED BY TWO DIFFERENT SPEEDS

If a car travels at a speed S_1 from A to B and at a speed S_2 from B to A. Then

$$\boxed{\text{Average speed} = \frac{2\,S_1 \,.\, S_2}{S_1 + S_2}}$$

The above formula can be found out as follows:
If distance between A and B is d, then

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2d}{\frac{d}{S_1} + \frac{d}{S_2}} = \frac{2}{\frac{1}{S_1} + \frac{1}{S_2}} = \frac{2\,S_1 \,.\, S_2}{S_2 + S_1}$$

AVERAGE SPEED IF EQUAL DISTANCES ARE TRAVELLED BY THREE DIFFERENT SPEEDS

$$\text{Average speed } \frac{3xyz}{xy + yz + zx}$$

Where x, y and 2 are these different speeds.

REMEMBER

- Average of first n natural numbers $= \frac{(n+1)}{2}$
- Average of first n consecutive × 2 even numbers = (n + 1)
- Average of first n consecutive × 2 odd numbers = n
- Average of consecutive numbers $= \frac{\text{First number} + \text{Last number}}{2}$
- Average of 1 to n odd numbers $= \frac{\text{Last odd number} + 1}{2}$
- Average of 1 and n even numbers $= \frac{\text{Last even number} + 2}{2}$
- Average of squares of first n natural numbers $= \frac{(n+1)(2n+1)}{6}$
- Average of the cubes of first n natural numbers $= \frac{n(n+1)^2}{4}$
- Average of n multiples of any number $= \frac{\text{Number} \times (n+1)}{2}$
- If n is odd: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.
- If n is even: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the average of the middle two numbers.
- The average of squares of first n consecutive even number is $\frac{2(n+1)(2n+1)}{3}$.
- The average of squares of consecutive even numbers till n is $\frac{(n+1)(n+2)}{3}$.
- The average of square of consecutive odd numbers till n is $\frac{n(n+2)}{3}$.
- If the average of n consecutive numbers is m, then the difference between the smallest and the largest number is $2(n-1)$.

If a person or a motor car covers three equal distances at the speed of x km/h, y km/h and z km/h, respectively, then for the entire journey average speed of the person or motor car is $\left(\frac{3xzy}{xy + yz + zx}\right)$ km/h.

☞ Shortcut Approach

- If average of n observations is a but the average becomes b when one observation is eliminated, then
  Value of eliminated observation = n (a – b) + b
- If average of n observations is a but the average becomes b when a new observation is added, then
  Value of added observation = n (b – a) + b. We have n observations out of which some observations (a_1, a_2, a_3, ...) are replaced by some other new observations in this way, if the average increases or decreases by b, then
  Value of new observations = a ± nb
  where, $a = a_1 + a_2 + a_3 + ...$

See Example : Refer ebook Solved Examples/Ch-3

NOTE : *In this formula, the signs of '+' and '–' depend upon the increment or decrement in the average.*

ebooks Reference		*Page No.*
Solved Examples	–	S-9-11
Exercises with Hints & Solutions	–	E-24-28
Chapter Test	–	5-6
Past Solved Papers		